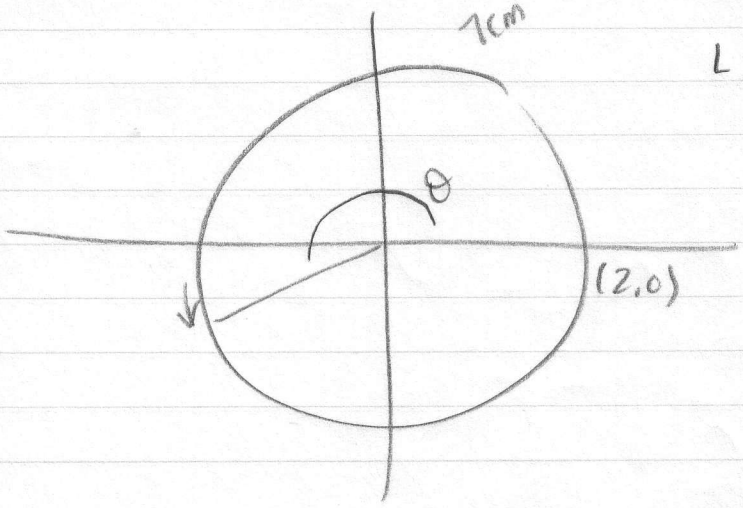
Math Review Session #2 5/15/05

- Trig

Review
- unit circle
- Graphs of sin, cos, tan
- Values of sin, cos, and tan at $0, \pi/6, \pi/4, \pi/3, \pi/2$, etc.
- Basic trig identities
- Differentiation of trig and inverse trig functions.
- Solving trig equations
- Applying trigonometry to optimization and related rates problems.
- Solving Δs and using Law of Sines and Cosines

① Unit Circle Problem

- A bug is crawling around a circle w/ equation $x^2+y^2=4$. It starts at (2,0) and proceeds counterclockwise. It carefully measures the distance that it has travelled and stops crawling after 7 cm.



L (arc length) $= r\theta$

$\theta = \frac{L}{r} = \frac{7}{2}$ rad.

$\frac{7}{2} = \frac{180}{\pi} \approx$

200.535°

(a) Through what angle in degrees has the bug crawled? 200.5°

(b) approximate x and y coordinates of the bug?

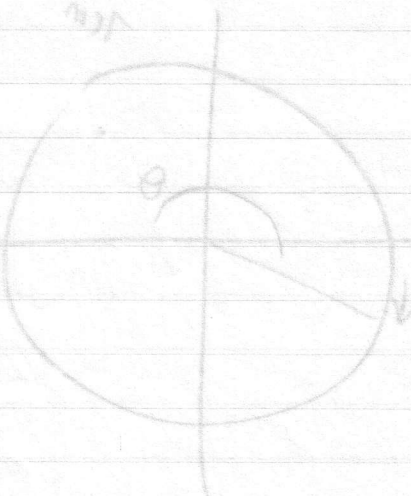
$x = 2\cos\theta \approx -1.873$

$y = 2\sin\theta \approx -0.702$

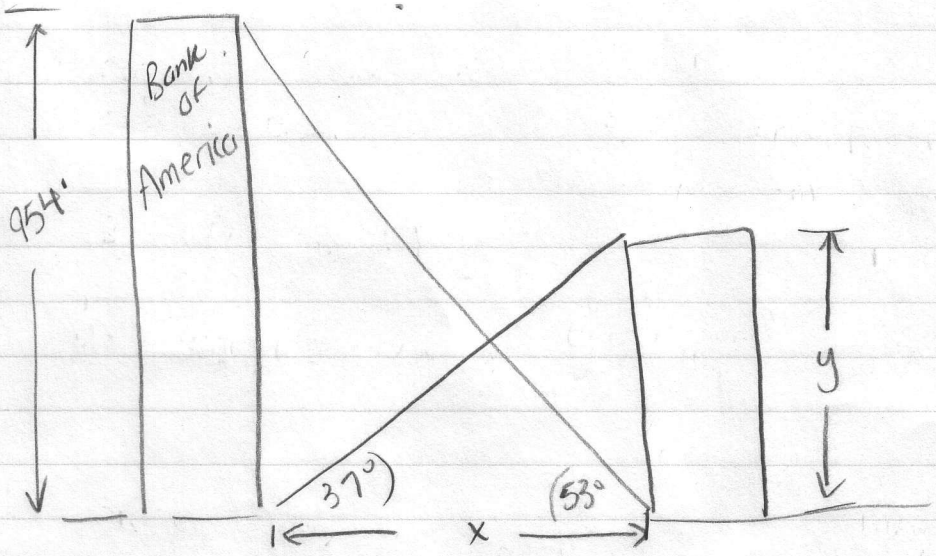
$(2\cos\theta, 2\sin\theta)$

(c) Imagine drawing a line ... tangent

Slope = $\tan\theta \approx 0.375$



② Solving triangles



$\tan 53° = \frac{954}{x}$ $\quad x = \frac{954}{\tan 53°} \approx 718.891$ ft.

$\tan 37° = \frac{y}{x}$ $\quad x \tan 37° = 718.891 \cdot \tan 37°$

≈ 541.723 ft.

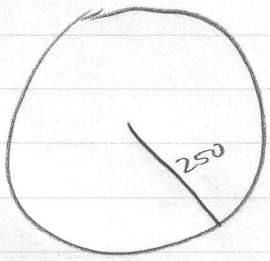
③ Sinusoidal Functions

The London Ferris Wheel - diameter of 500 ft
Completes one revolution in 20 min.

(a) rotating $\underline{18}$ °/min $\quad \frac{360}{20} = 18$

(b) Let t, the time in minutes, be zero when you are at the 6:00 position. Write θ, measure from the 3:00 position, as a function of t.

$\theta = 18t - 90°$



(c) h(t) height above the ground

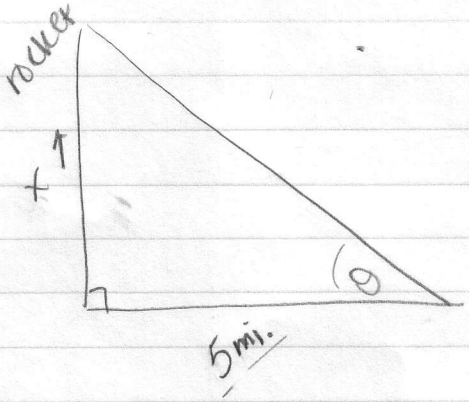
$250 \sin(18t - 90) + 250$

↑ radius of the ferris wheel

(d) Graph h(t). What is the period, the balance value, and the amplitude?

amplitude = 250
b.v. = 250
period = 20 min

④ Trig and Related Rates Problems



Use related rates to find the velocity of the rocket at this instant.

$\tan\theta = \frac{x}{5}$ (tan) is very useful w/ such problems.

$5\tan\theta = x$

$5\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$\frac{d\theta}{dt} = 3°/\text{sec.}$

$\sec\theta = ?$ $= \frac{\text{hyp.}}{\text{adj.}}$ $\sec 60° = \frac{1}{\cos 60°} = \frac{1}{1/2} = 2$

$\frac{dx}{dt} =$

$= 5 \cdot 4 \cdot 3° = \cancel{60\ \text{mi./sec.}}$

$5 \cdot 4 \cdot \frac{\pi}{60} = \pi/3$ mi./sec.
(about 1 mi/sec)

⑤ Solving Trig Equations

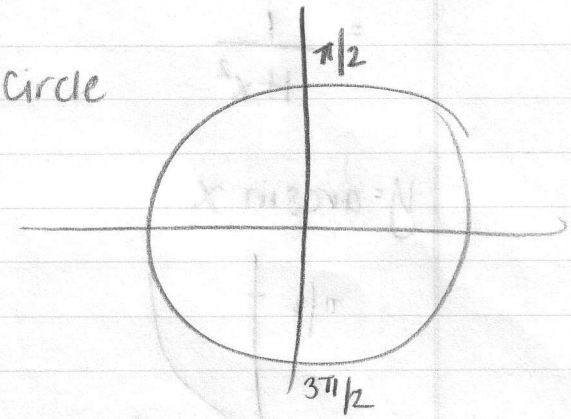
for $0 \le \theta \le 2\pi$

(a) $2\sin t \cos t - \cos t = 0$

$\cos t (2\sin t - 1) = 0$

$\cos t = 0$ or $\sin t = \frac{1}{2}$

-Draw a unit circle



$\cos = 0$ at $\frac{\pi}{2}, 3\pi/2$

$\sin t = \frac{1}{2}$ at $\pi/6$ and $\frac{5\pi}{6}$

Double-Angle Formulas Essential!

(b) $\sin 2t - \cos t = 0$

$2\sin t \cos t - \cos t = 0$

(c) $2\cos^2 t = \sin t + 1$

$2(1 - \sin^2 t) = \sin t + 1$

$2\sin^2 t + \sin t - 1 = 0$

$(2\sin t - 1)(\sin t + 1) = 0$

$\sin t = \frac{1}{2}$ or $\sin t = -1$

$\pi/6, 5\pi/6, 3\pi/2$

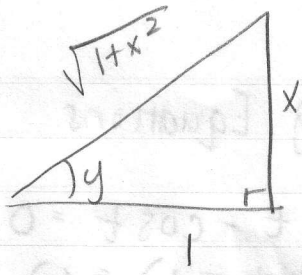
⑥ Inverse Trig Functions

derivative of $y = \arctan x$

$y = \arctan x$ $\quad y' = \frac{1}{1+x^2}$ $\Rightarrow$ now, derive that.

$\tan y = x$ $\quad \sec^2 y \frac{dy}{dx} = 1$ $\quad \frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$.

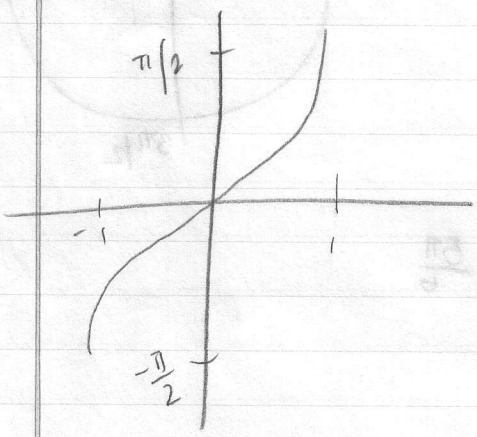
$\tan y = \frac{x}{1}$



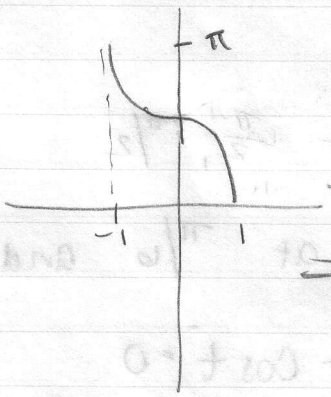
$= \cos^2 y$

$= \frac{1}{1+x^2}$

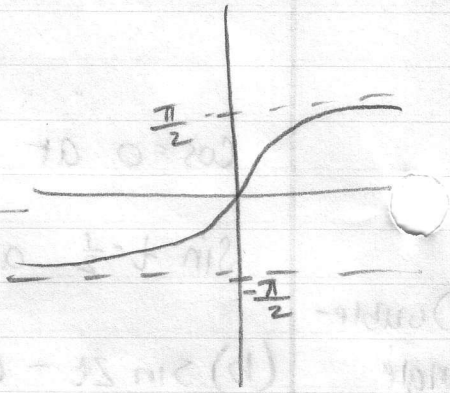
$y = \arcsin x$



$y = \arccos x$



$y = \arctan x$



This should help!