Chapter 17: Implicit Differentiation

Logarithmic Differentiation

General Strategy:

1. Take the natural log of both sides of the equation (remember that ln is not defined for negative arguments)

2. Simplify the expression, move down exponents as per rules of logarithms

3. Take the derivative of both sides remembering to use implicit differentiation

4. Solve for the derivative you are looking for

5. Rewrite the expression explicitly

Logarithmic differentiation has two main uses: differentiating an expression with a variable in the base and the exponent, or for differentiating complicated products or quotients since logs can easily simplify them into sums and differences.

Example 1

Find the derivative of $y = (\tan x)^{3x^2+4x}$ with respect to x. If we only look at some interval say $[0, \frac{\pi}{2}]$ then $\tan x > 0$ so we will proceed with logarithmic differentiation following the steps above:

1. $\ln y = \ln(\tan x)^{4x^2+4x}$

2. $\ln y = (3x^2 + 4x)\ln(\tan x)$

3. $\frac{1}{y}y' = (6x + 4)(\tan x) + (3x^2 + 4x)(\sec^2 x)$

4. $y' = y((6x + 4)\tan x + (3x^2 + 4x)\sec^2 x)$

5. $y' = (\tan x)^{3x^2+4x}((6x + 4)\tan x + (3x^2 + 4x)\sec^2 x)$

Example 2

Find the derivative of $y = \frac{f(x)\cdot g(x)}{h(x)\cdot(k(x)}$ For any functions $f, g, h, k(x)$ assuming that $y > 0$. This assumptions lets us use logarithmic differentiation as above:

1. $\ln y = \ln(\frac{f(x)\cdot g(x)}{h(x)\cdot(k(x)})$

2. $\ln y = \ln f(x) + \ln g(x) - \ln h(x) - \ln k(x)$

3. $\frac{1}{y}y' = \frac{1}{f(x)}f' + \frac{1}{g(x)}g' - \frac{1}{h(x)}h' - \frac{1}{k(x)}k'$

4. $y' = y(\frac{1}{f(x)}f' + \frac{1}{g(x)}g' - \frac{1}{h(x)}h' - \frac{1}{k(x)}k')$

5. $y' = (\frac{f(x) \cdot g(x)}{h(x) \cdot k(x)})(\frac{1}{f(x)}f' + \frac{1}{g(x)}g' - \frac{1}{h(x)}h' - \frac{1}{k(x)}k')$

Implicit Differentiation

General Strategy:

1. Take the derivative of both sides of the expression remembering what is a function of which variable and using the chain rule as necessary

2. Algebraically solve for the desired derivative.

Example 3

Find $\frac{dy}{dx}$ from the expression of the shifted circle $(x-a)^2+(y-b)^2 = c^2$ for some constants a, b and c.

1. $x^2 - 2ax + a^2 + y^2 - 2by + b^2 = c^2$ Expanding the squared quantities

2. $2x - 2a + 2y\frac{dy}{dx} - 2b\frac{dy}{dx} = 0$ Taking the derivative with respect to x

3. $\frac{dy}{dx}(2y - 2b) = 2a - 2x$ Collecting $\frac{dy}{dx}$ terms

4. $\frac{dy}{dx} = \frac{a-x}{y-b}$ Solving for the derivative