Name:________________________________

Math Xb Final Exam Part One
Friday, May 21, 2004

Please circle your section:

Derek Bruff	Ken Chung	Thomas Judson
Will Adams (CA)	Jennie Schiffman (CA)	Ashley Hinerman-Mulroy (CA)
MWF 10–11	MWF 10–11	MWF 11–12

Kazem Mahdavi	Derek Bruff
Maria van Wagenberg (CA)	Connie Zong (CA)
MWF 11–12	MWF 12–1

Part One Scores

Problem Number	Possible Points	Score
1	16	
2	16	
Total	32	

Final Exam Scores

	Possible Points	Score
Part One	32	
Part Two	68	
Total	100	

Directions—Please read carefully! You *are not allowed* to use a calculator or any other aids on this part of the exam. Be sure to write neatly—illegible answers will receive little or no credit. When you are finished with this part of the exam, you may turn it in to the proctor and receive the second part of the exam. Once you have turned in this first part of the exam, you may not look at it again, so be sure you have finished it completely before turning it in. The second part of the exam consists of eight problems, and you may use a calculator on the second part of the exam. **Good luck!**

1. Find the derivative of each of the following functions. You need not simplify your answers.

   (a) $f(x) = 2x^2 - x^{3/2} + x - \pi + \dfrac{1}{x}$

   (b) $f(x) = 2x\sqrt{x}\tan x$

(c) $f(x) = \dfrac{\sin x + \cos x}{\ln x}$

(d) $f(x) = e^{\sec^2 x}$

2. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{1}{1+x^2}\,dx$

(b) $\displaystyle\int \tan x\,dx$

(c) $\int_1^2 xe^{x^2}, dx$

(d) $\int x\sqrt{2x-1}\, dx$

Name:________________________________

Math Xb Final Exam Part Two
Friday, May 21, 2004

Please circle your section:

Derek Bruff Will Adams (CA) MWF 10–11	Ken Chung Jennie Schiffman (CA) MWF 10–11	Thomas Judson Ashley Hinerman-Mulroy (CA) MWF 11–12
Kazem Mahdavi Maria van Wagenberg (CA) MWF 11–12	Derek Bruff Connie Zong (CA) MWF 12–1	

Part Two Scores

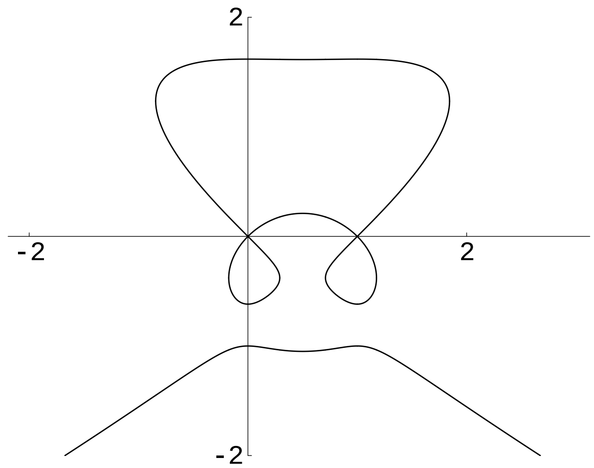
Problem Number	Possible Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	12	
7	8	
8	8	
Total	68	

Directions—Please read carefully! You *are allowed* to use a calculator on this part of the exam, but no other aids are allowed. Read each problem carefully and make sure to answer the specific questions asked. Some questions ask you to justify or explain your answers. You must do so on to receive full credit on these questions. Be sure to write neatly—illegible answers will receive little or no credit. You may not return to the first part of the exam. **Good luck!**

1. The curve with equation

$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

has been likened to a bouncing wagon, as you can see in the graph below. Find the exact x-coordinates of the points at which the curve has horizontal tangent lines. Justify your answer using calculus. (Estimating the x-coordinates from the graph is not sufficient.)



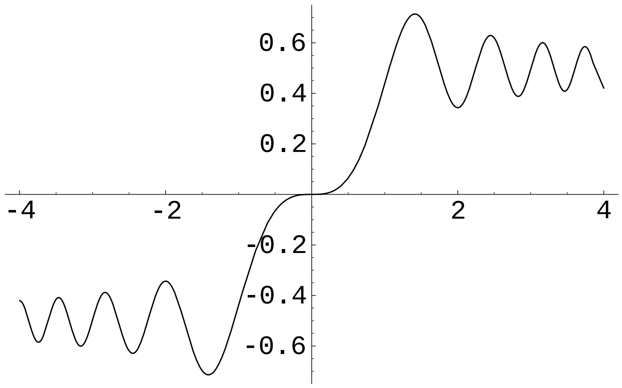
2. A student plans to obtain her Ph.D. and join the faculty at Harvard ten years from now. She would like to save enough money to buy an apartment in Cambridge at that time. She plans to put a fixed amount of money in her savings account *at the beginning of each month* for ten years. Assuming an annual interest rate of 6 percent *compounded monthly*, how much should she deposit each month in order to have 1 million dollars available for her apartment immediately *after her last deposit*?

3. A rocket is launched vertically and is tracked by a radar station located on the ground 5 miles from the launch pad. Suppose that the elevation angle θ of the line of sight to the rocket is increasing at 3° per second ($\pi/60$ radians per second) when $\theta = 60^\circ$ ($\pi/3$ radians). Use related rates to find the velocity of the rocket at this instant. Include appropriate units in your answer.

4. The Fresnel function

$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right)\, dt$$

is named after the French physicist Augustin Fresnel (1788–1827), who is famous for his work in optics. This function first appeared in Fresnel's theory of the diffraction of light.



(a) Find $S'(x)$. Justify your answer.

(b) Use L'Hôpital's Rule to evaluate $\lim_{x \to 0} \frac{S(x)}{x^3}$.

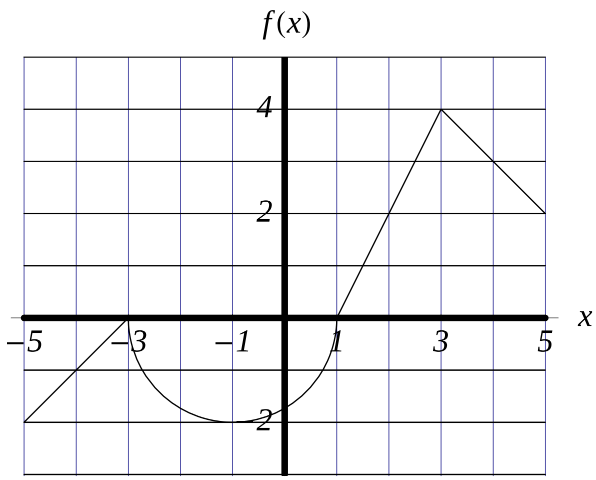
5. The rate at which the world's oil is being consumed is increasing. Suppose that the rate (measured in billions of barrels per year) is given by the function $r(t)$, where t is measured in years and $t = 0$ represents January 1, 1990.

   (a) Write a definite integral that represents the total quantity of oil used between *the start of* 1990 and *the start of* 1995.

   (b) Suppose that $r(t) = 32e^{0.05t}$. Find the approximate value for the definite integral from part (a) using a right-hand sum with $n = 5$ subintervals.

   (c) Interpret each of the five terms in the sum from part (b) in terms of oil consumption.

6. Let $F(x) = \int_{-3}^{x} f(t)\, dt$, where f is the function whose graph is given below. Note that the graph of f is made up of straight lines and a semicircle. Also note that -3 is the lower limit of integration in the definition of F.



(a) Identify the x-values of all critical points of F in the interval $(-5, 5)$.

(b) On what interval(s) in $(-5, 5)$ is F decreasing? Justify your answer.

(c) At what x-values in the interval $(-5, 5)$, if any, does f have a local maximum? Justify your answer.

(d) At what x-values in the interval $(-5, 5)$, if any, does f have a local minimum? Justify your answer.

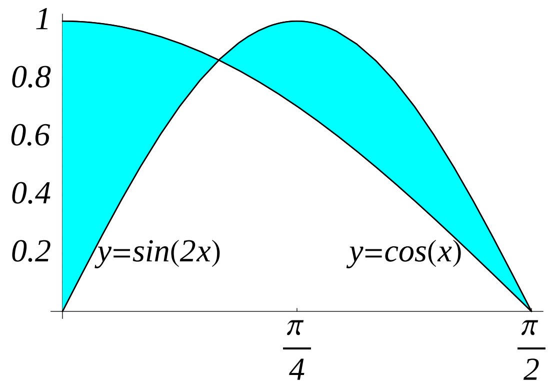
(e) Find each of the following values. If a value is not defined, explain why not.

i. $F(1)$

ii. $F'(1)$

iii. $F''(1)$

7. Find the area of the region enclosed by the curves $y = \cos x$ and $y = \sin 2x$ between $x = 0$ and $x = \frac{\pi}{2}$. That is, find the area of the shaded region graphed below.



8. Psychologists interested in learning theory study learning curves. A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of training time t. The derivative dP/dt represents the rate at which performance improves.

   If M is the maximum level of performance of which the learner is capable, then

$$\frac{dP}{dt} = k(M - P)$$

   is one reasonable model for learning.

   (a) Interpret the differential equation given above for dP/dt in the context of learning curves. That is, explain in words what the differential equation tells us about the rate at which performance improves.

   (b) Explain why the differential equation given above might be seen as a reasonable model of learning.