(when $x = 50$, $z^2 = 900 + 2500 = 3400$ so $\cos^2\theta = \frac{900}{3400}$)

9. $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x = 0 \Leftrightarrow \sin(x) = 0 \Leftrightarrow x = k\pi$.
$f''(x) = \sec x \tan x \tan x + \sec x \sec^2 x = \sec x(\tan^2 x + \sec^2 x)$.
$f''(2\pi k) = 1(0+1) > 0$ so f has a local minimum at $2\pi k$, while
$f''(\pi + 2\pi k) = -1(0+1) < 0$ so f has a local maximum at $\pi + 2\pi k$.

10. $y = \tan x \Rightarrow y' = \sec^2 x \Rightarrow y'' = 2\sec x(\sec x \tan x) = \frac{2\sin x}{\cos^3 x}$. Around $x = \pi k$, $\sin x$ changes sign while $\cos x$ doesn't change sign so y'' changes sign at $x = \pi k$, thus $\tan x$ has inflection points at $x = \pi k$. (Note $\tan x$ has vertical asymptotes at $x = \frac{\pi}{2} + \pi k$.)

11.(a) $f(x) = 3\cos x + 2\sin x$ has period 2π.

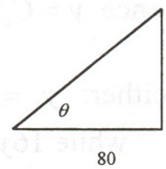
(b) $f'(x) = -3\sin x + 2\cos x = 0 \Rightarrow \tan x = \frac{2}{3} = \frac{-2}{-3}$ hence critical points are when $\sin x = \frac{2}{\sqrt{13}}$ and $\cos x = \frac{3}{\sqrt{13}}$, or $\sin x = \frac{-2}{\sqrt{13}}$ and $\cos x = \frac{-3}{\sqrt{13}}$. Using these values we get $f(crit\ pt) = 3(\frac{3}{\sqrt{13}}) + 2(\frac{2}{\sqrt{13}}) = \sqrt{13}$ or $3(\frac{-3}{\sqrt{13}}) + 2(\frac{-2}{\sqrt{13}}) = -\sqrt{13}$. Max $= \sqrt{13}$, min $= -\sqrt{13}$.

12.(a) $x(\theta) = \frac{v_0^2}{g}\sin(2\theta) \Rightarrow x'(\theta) = \frac{v_0^2}{g}\cos(2\theta)2 = 0 \Leftrightarrow \cos(2\theta) = 0 \Leftrightarrow \theta = \frac{\pi}{4}$

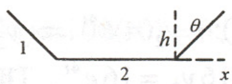
$x_{\max} = \frac{v_0^2}{g}\sin(2(\frac{\pi}{4})) = \frac{v_0^2}{g}$

(b) $\frac{100mi}{hr} = \frac{100mi}{hr}\frac{5280\,ft}{mi}\frac{100m}{328\,ft}\frac{1hr}{3600\sec} \cong \frac{44.7m}{\sec} \Rightarrow x_{\max} \cong \frac{(44.7\,{}^{m}/_{\sec})^2}{9.8\,{}^{m}/_{\sec^2}} \cong 204m$.
($\cong 669\,ft$ which is not very realistic as wind resistance is not taken into account.)

13. $y = 80\tan\theta \Rightarrow \frac{dy}{dt} = 80\sec^2\theta\frac{d\theta}{dt}$. At the instant the runner is 100ft away $\cos\theta = \frac{80}{100} = \frac{4}{5}$. Hence $9 = 80\frac{25}{16}\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = 0.072\frac{rad}{\sec} \cong 0.01146\frac{rev}{\sec}$



14. $V = 8(2+x)h = 8(2+\sin\theta)\cos\theta$
$V' = 8[\cos\theta(\cos\theta) + (2+\sin\theta)(-\sin\theta)] = 8[-2\sin^2\theta - 2\sin\theta + 1]$
$V' = 0 \Leftrightarrow \sin\theta = \frac{2\pm\sqrt{12}}{-4} = \frac{1\pm\sqrt{3}}{-2} \Rightarrow \theta \cong 0.3747\,rad \cong 21.47^\circ$ (negative sin is unrealistic)



15. $\frac{y}{0.5} = \tan\theta \Rightarrow y = 0.5\tan\theta \Rightarrow \frac{dy}{dt} = 0.5\sec^2\theta\frac{d\theta}{dt}$
At the instant the window is one kilometer from the light
$\cos\theta = \frac{1}{2} \Rightarrow \sec^2\theta = 4 \Rightarrow \frac{dy}{dt} = 0.5(4)[6(2\pi)] = 24\pi \cong 75.4\frac{km}{min}$