

Appendix F

L'Hopital's Rule

1. Use L'Hopital's rule since it is of the form $\frac{\infty}{\infty}$: $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{1}{1} = 0$.
2. $\lim_{x \rightarrow 0^+} \frac{\ln x}{x} = -\infty$ since it is of the form $\frac{-\infty}{0^+}$.
3. Use L'Hopital's rule since it is of the form $\frac{\infty}{\infty}$: $\lim_{x \rightarrow \infty} \frac{x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$.
4. $\lim_{x \rightarrow 0} \frac{x}{e^{-x}} = \frac{0}{1} = 0$.
5. Use L'Hopital's rule three times: $\lim_{x \rightarrow \infty} \frac{100x^3}{e^x} = \lim_{x \rightarrow \infty} \frac{300x^2}{e^x} = \lim_{x \rightarrow \infty} \frac{600x}{e^x} = \lim_{x \rightarrow \infty} \frac{600}{e^x} = 0$.
6. Use L'Hopital's rule twice: $\lim_{x \rightarrow \infty} \frac{\ln(5+e^x)}{3x} = \lim_{x \rightarrow \infty} \frac{e^x}{3} = \lim_{x \rightarrow \infty} \frac{e^x}{15+3e^x} = \lim_{x \rightarrow \infty} \frac{e^x}{3e^x} = \frac{1}{3}$.
7. (a) $\lim_{t \rightarrow 0} \frac{t^2+3}{2t^3+100t+1} = \frac{0+3}{0+0+1} = 3$.
(b) Use L'Hopital's rule twice: $\lim_{t \rightarrow \infty} \frac{t^2+3}{2t^3+100t+1} = \lim_{t \rightarrow \infty} \frac{2t}{6t^2+100} = \lim_{t \rightarrow \infty} \frac{2}{12t} = 0$.
8. Use L'Hopital's rule: $\lim_{x \rightarrow 0} \frac{2t}{t^3} = \lim_{x \rightarrow 0} \frac{2}{3t^2} = \infty$.
9. Rewrite and then use L'Hopital's rule: $\lim_{x \rightarrow \infty} 2x \cdot e^{-x} = \lim_{x \rightarrow \infty} \frac{2x}{e^x} = \lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$.
10. $\lim_{x \rightarrow -\infty} \frac{e^x+2}{x} = 0$ since it is of the form $\frac{0+2}{-\infty}$.
11. $\lim_{x \rightarrow \infty} \frac{r^x}{x} = 0$ since it is of the form $\frac{0}{\infty}$ (note: $\lim_{x \rightarrow \infty} r^x = 0$ because $0 < r < 1$).
12. $\lim_{x \rightarrow 0.5} \frac{\ln(1-2x)}{2x-1}$ is undefined. Note: $\lim_{x \rightarrow 0.5^+} \frac{\ln(1-2x)}{2x-1} = -\infty$ since it is of type $\frac{-\infty}{0^+}$.
13. Use L'Hopital's rule and simplify : $\lim_{x \rightarrow \infty} \frac{e^{2x}+7}{5e^{3x}-10} = \lim_{x \rightarrow \infty} \frac{2e^{2x}}{15e^{3x}} = \lim_{x \rightarrow \infty} \frac{2}{15e^x} = 0$
14. $\lim_{t \rightarrow 3} \frac{\ln(t-3)}{t^2-t-6}$ is undefined. $\lim_{t \rightarrow 3^+} \frac{\ln(t-3)}{t^2-t-6} = -\infty$ since it is of the form $\frac{-\infty}{0^+}$