15.(a) Use L'Hopital's rule and simplify: $\lim_{x\to\infty}\frac{\ln x}{x^n}=\lim_{x\to\infty}\frac{\frac{1}{x}}{n\cdot x^{n-1}}=\lim_{x\to\infty}\frac{1}{n\cdot x^n}=0$.

(b) Use L'Hopital's rule n times:

$$\lim_{x\to\infty}\frac{e^{nx}}{x^n}=\lim_{x\to\infty}\frac{n\cdot e^{nx}}{n\cdot x^{n-1}}=\lim_{x\to\infty}\frac{n\cdot n\cdot e^{nx}}{n(n-1)\cdot x^{n-2}}=\lim_{x\to\infty}\frac{n\cdot n\cdot n\cdot e^{nx}}{n(n-1)(n-2)\cdot x^{n-3}}=\ldots..=\lim_{x\to\infty}\frac{n^n\cdot e^{nx}}{n!}=\infty.$$

16. Since $\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k=e\neq 0$ the series must diverge.

17.(a) Use L'Hopital's rule: $\lim_{x\to\infty}\frac{x}{\ln x}=\lim_{x\to\infty}\frac{1}{\frac{1}{x}}=\lim_{x\to\infty}x=\infty$.

(b) Since $\lim_{k\to\infty}\frac{k}{\ln k}=\infty\neq 0$ the series must diverge.

18. Use L'Hopital's rule: $\lim_{x\to\infty}\frac{x^3+10000x}{3^x}=\lim_{x\to\infty}\frac{3x^2+10000}{\ln 3\cdot 3^x}=\lim_{x\to\infty}\frac{6x}{(\ln 3)^2 3^x}=\lim_{x\to\infty}\frac{6}{(\ln 3)^3 3^x}=0$.

19. $\lim_{x\to 0^+}e^x\ln x=-\infty$ since it is of type $1\cdot(-\infty)$.

20. Rewrite $\left(1+\frac{5}{x}\right)^{3x}=\left(\left(1+\frac{5}{x}\right)^{\frac{x}{5}}\right)^{15}$ and substitute $k=\frac{x}{5}$,

then $\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{3x}=\lim_{x\to\infty}\left(\left(1+\frac{5}{x}\right)^{\frac{x}{5}}\right)^{15}=\lim_{k\to\infty}\left(\left(1+\frac{1}{k}\right)^{k}\right)^{15}=e^{15}$.

21. Rewrite and then use L'Hopital's rule: $\lim_{x\to\infty}e^{-x}\ln x=\lim_{x\to\infty}\frac{\ln x}{e^x}=\lim_{x\to\infty}\frac{\frac{1}{x}}{e^x}=\lim_{x\to\infty}\frac{1}{x\cdot e^x}=0$

22. $f(x)=x^2\ln x\Rightarrow f'(x)=2x\ln x+x^2\frac{1}{x}=x(2\ln x+1)$. Critical points occur when $x=0$ or $2\ln x+1=0\Rightarrow x=0$ or $x=e^{-1/2}\approx 0.60653$. A local minimum occurs when $x=e^{-1/2}$. $x=0$ is not in the domain of f.

Using L'Hopital's rule $\lim_{x\to 0^+}x^2\ln x=\lim_{x\to 0^+}\frac{\ln x}{x^{-2}}=\lim_{x\to 0^+}\frac{1/x}{-2x^{-3}}=\lim_{x\to 0^+}\frac{x^2}{-2}=0$. The graph shows the height approaching zero as x approaches zero from the right.