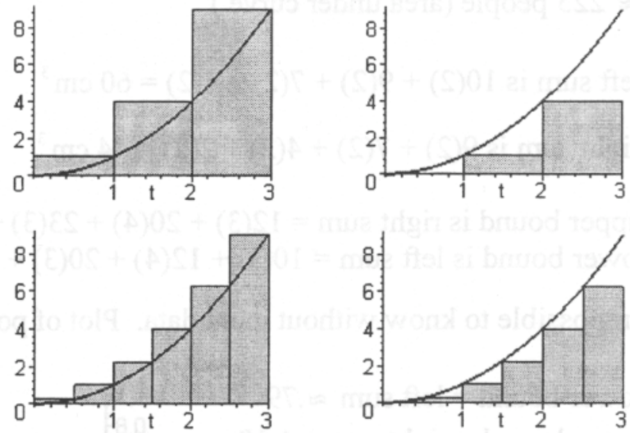
22.1 Finding Net Change in Amount: Physical and Graphical Interplay

2. (a) left sum is $10(2) + 9(2) + 7(2) + 4(2) = 60 \text{ cm}^3$

   (b) right sum is $9(2) + 7(2) + 4(2) + 2(2) = 44 \text{ cm}^3$

8. (a) $UB = 1(1) + 4(1) + 9(1) = 14ft$

   (b) $LB = 0(1) + 1(1) + 4(1) = 5ft$



   (c) $\frac{1}{4}(\frac{1}{2}) + 1(\frac{1}{2}) + \frac{9}{4}(\frac{1}{2}) + 4(\frac{1}{2}) + \frac{25}{4}(\frac{1}{2}) + 9(\frac{1}{2})$
   $= \frac{91}{8} = UB$
   $0(\frac{1}{2}) + \frac{1}{4}(\frac{1}{2}) + 1(\frac{1}{2}) + \frac{9}{4}(\frac{1}{2}) + 4(\frac{1}{2}) + \frac{25}{4}(\frac{1}{2})$

   $= \frac{55}{8} = LB$

   (d) The difference between R_n and L_n is $(\textit{right height} - \textit{left height})(3/n)$
   difference for n = 50 is $9(3/50) = 0.54$ difference for n = 100 is $9(3/100) = 0.27$

   (e) To make $9(3/n) \le 0.01$ we need $n \ge \frac{27}{0.01} = 2700$

9. Upper bound. Draw a picture with a decreasing graph. A left sum approximation will have rectangles whose top edge will be a velocity level ≥ that of the trekker. Hence left sum approximate distance will be greater than actual distance. OR we can think of the distance traveled as being equivalent to the area under the curve.

22.2

7. In what follows, we assume the line starts to form at 10 AM.

   (a) $\int_{10}^{11} r(t)dt - \int_{10}^{11} 30dt$ (b) $\int_{10}^{14} r(t)dt - \int_{10}^{14} 30dt$

   (c) $\int_{8}^{16} r(t)dt$ (d) $\int_{8}^{16} r(t)dt$ (closed doors but kept serving)

   (e) $\int_{10}^{15} r(t)dt - \int_{10}^{15} 30dt$ (f) $\left(\int_{10}^{12} r(t)dt - \int_{10}^{12} 30dt\right)\frac{1}{30}$

   (g) $\left(\int_{10}^{16} r(t)dt - \int_{10}^{16} 30dt\right)\frac{1}{30}$

8. (a) $R_4 < R_{20} < R_{100} < \int_0^2 \frac{1}{x+1}dx < L_{100} < L_{20} < L_4$.

   (b) $|R_4 - L_4| = \left|\frac{1}{3}\frac{2}{4} - \frac{1}{1}\frac{2}{4}\right| = \frac{1}{3} = \frac{4}{3\cdot 4}$.

   (c) $|R_{100} - L_{100}| = \left|\frac{1}{3}\frac{2}{100} - \frac{1}{1}\frac{2}{100}\right| = \frac{4}{3\cdot 100}$

   (d) $|R_n - L_n| = \left|\frac{1}{3}\frac{2}{n} - \frac{1}{1}\frac{2}{n}\right| = \frac{4}{3\cdot n} < 0.05 \Leftrightarrow \frac{4}{0.15} < n \Rightarrow 27 \le n$

   (e) $R_4 = \sum_{k=1}^{4}(\frac{1}{\frac{K}{2}+1})\frac{1}{2} = (\frac{1}{\frac{1}{2}+1})\frac{1}{2} + (\frac{1}{\frac{2}{2}+1})\frac{1}{2} + (\frac{1}{\frac{3}{2}+1})\frac{1}{2} + (\frac{1}{\frac{4}{2}+1})\frac{1}{2} = \frac{19}{20}$