Math Xb Spring 2005
Final Exam Review Guide

1 Topics

The final exam will cover Chapter 17, Chapters 19 through 25 (omitting §20.7), §26.1, §30.1, §30.2, §30.3, and Appendix F in our textbook, *Calculus: An Integrated Approach to Functions and Their Rates of Change*. In particular, you will be responsible for the following topics.

CHAPTER 17: IMPLICIT DIFFERENTIATION AND ITS APPLICATIONS

- To use logarithmic differentiation to find the derivatives of functions of the form $f(x)^{g(x)}$

- To use implicit differentiation to find $\frac{dy}{dx}$ given an equation involving x and y

- To find the slope of a tangent line to a given curve (described by an equation involving x and y) at a given point

- To find the points on a curve (described by an equation involving x and y) at which the tangent line has a given slope

- To understand how to use a relationship between two or more variables that depend on time to find a relationship between the rates of change of those variables (that is, be able to do "related rates" problems)

CHAPTERS 19 & 20: TRIGONOMETRY

- To understand sine and cosine as functions of arc length on the unit circle

- To approximate sine, cosine, and tangent values given a calibrated unit circle

- To be familiar with the graphs of the sine, cosine, and tangent functions

- To understand what it means for a function to be periodic and be able to identify when functions are periodic

- To identify the balance value, amplitude, and period of a sinusoidal function given its formula or graph

- To use trig functions to model other functions

- To understand the interpretation of $\tan x$ as the slope of a certain line.

- To understand the relationship between angles and arc length.

- To use circle symmetry when finding trig function values.

- To understand the relationship between sine, cosine, and tangent and right triangles.

- To know the sine, cosine, and tangent values of $\frac{\pi}{6}$, $\frac{\pi}{4}$, and $\frac{\pi}{3}$.

- To "solve" triangles, that is, to determine all angles and sides of a triangle from some given information.

- To understand the inverse trig functions $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ and their domains and ranges.

- To simplify expressions involving inverse trig functions by using triangles

- To solve equations involving trig functions on both restricted and unrestricted domains

- To be able to find the area of a triangle given the lengths of two sides and the measure of the angle between them.

- To be able to apply the Law of Cosines and the Law of Sines.

- To know the identities listed on the Trig Identities Handout from March 7th (available on the website – look at the schedule for March 7th).

CHAPTER 21: DIFFERENTIATION OF TRIG FUNCTIONS

- Know the derivatives of the six trigonometric functions, and be able to find derivatives of more complex functions that involve trigonometric functions.

- Be able to solve word problems involving trigonometry. These include related rates, optimization, curve-sketching, and other problems. You need to be able to look at a problem and determine what kind of problem it is so that you can decide what strategy to use.

- Know how to find the derivatives of the inverse trig functions using implicit differentiation.

- Know the derivatives of $\sin^{-1}$, $\tan^{-1}$, and $\cos^{-1}$ and be able to use them to find derivatives of more complex functions involving inverse trig functions.

APPENDIX F: L'HÔPITAL'S RULE

- Recognize the indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^{∞}, ∞^{0}, and 0^{0}.

- Evaluate the indeterminate forms $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^{∞}, ∞^{0}, and 0^{0} using L'Hôpital's Rule.

CHAPTER 22: NET CHANGE, AREA, AND THE DEFINITE INTEGRAL

- Understand the relationship between the definite integral and the questions

  1. Given a rate function, how do we calculate the net change in amount?
  2. How do we calculate the signed area between the graph of a function and the horizontal axis?

  and be able to use these relationships to solve problems

- Be able to approximate a definite integral, net change, or signed area using left- and right-hand sums.

- Be able to evaluate a definite integral, net change, or signed area using geometric area formulas.

- Know the integral properties on p. 738 and be able to use them simplify or alter integrals and to write inequalities involving integrals.

CHAPTER 23: THE AREA FUNCTION

- Be able to find the area function $_cA_f(x)$ for a function $f(t)$.

- Interpret area function as a net change function.

- Know that two area functions $_aA_f$ and $_bA_f$ differ by a constant and why.

- Determine where an area function A_f is increasing, decreasing, concave up, and concave down by examining f.

- Know the statement of, understand, and be able to use the Fundamental Theorem of Calculus, Part 1.

CHAPTERS 24: THE FUNDAMENTAL THEOREM OF CALCULUS

- To know the Fundamental Theorem of Calculus, version 2.

- To apply the Fundamental Theorem of Calculus to evaluate definite integrals.

- To use properties of definite integrals to assist in evaluating definite integrals.

- To compute the average value of a function on a given interval.

- To interpret the average value of a function in a particular context (e.g., velocity or speed).

CHAPTER 25 & §26.1: FINDING ANTIDERIVATIVES AND APPROXIMATING DEFINITE INTEGRALS

- To evaluate simple definite and indefinite integrals. (See the list on page 784.)

- To evaluate more complex definite and indefinite integrals using algebra and/or the substitution rule.

- Be able to approximate a definite integral using Left-Hand Sums, Right-Hand Sums, Trapezoidal Sums, and Midpoint Sums.

- Be able to sketch illustrations of L_n, R_n, T_n, and M_n.

- Know for an increasing function f, $L_n \leq \int_a^b f\, dx \leq R_n$, and vice-versa for a decreasing function, and be able to use this to determine an upper bound on the error from using R_n or L_n.

- Know that for a function f which is concave up, $M_n \leq \int_a^b f\, dx \leq T_n$ and vice-versa for a concave-down function, and be able to use this to determine an upper bound on the error from using M_n or T_n.

§15.2, §31.1, §31.3, AND IN-CLASS HANDOUT: DIFFERENTIAL EQUATIONS

- To use the differential equation $\frac{dy}{dt} = ky$ and its solution $y(t) = Ce^{kt}$ to model and analyze exponential growth and decay problems.

- To solve the differential equation $\frac{dy}{dt} = ky$ given an initial condition.

- To determine if a function $y = f(x)$ is a solution to a given differential equation.

- To interpret a given differential equation in the context of a particular application.

- To write a differential equation which models a given situation.

- To draw a slope field for a given differential equation, or match a differential equation with a slope field

- Be able to identify equilibrium solutions for differential equations

- Be able to determine when equilibria are stable or unstable

- Be able to analyze a differential equation $\frac{dy}{dt}$ qualitatively, sketching representative solution curves and determining what happens for different initial conditions as $t \to \infty$

2   Suggested Exercises

CHAPTER 17

- §17.1 #1–4

- §17.2 #1–4, 7

- §17.3 #1–6, 9–12

- §17.4 #1–14 (don't forget that there are also related rates problems in §21.3)

CHAPTERS 19 & 20

- §19.1 #1, 3, 4, 5

- §19.2 #1–4, 6, 7, 9–15, 17 − 19

- §19.3 #2–4, 7, 8, 10, 12

- §19.4 #2–8, 10, 11
- §20.1 #1–6, 9
- §20.2 #2–9
- §20.3 #1, 4–14
- §20.4 #1–12, 21, 22
- §20.5 #1, 4–9
- §20.6 #3–9

Chapter 21

- §21.2 #1, 2, 6–15
- §21.3 #1, 3, 8, 12–15, 17, 18, 23
- §21.4 #1–9

Appendix F

- App. F #1–15, 18–21

Chapter 22

- §22.1 #1–3, 7–9
- §22.2 #1, 4–8
- §22.3 #1–7
- §22.4 #1–5, 7–9

Chapters 23 & 24

- §23.1 #1–4
- §23.2 #1–3
- §23.3 #1–4
- §24.1 #2–8, 11–13
- §24.2 #1–6, 8–11

Chapter 25 & §26.1

- §25.1 #2–14 17, 18
- §25.2 #1–27
- §25.3 #1–11
- §26.1 #3, 4, 6, 7, 10,

§15.2, §31.1, §31.3, and In-Class Handout on Slope-Fields

- §15.2 #3 – 12, 16, 17
- §31.1 #1 – 7
- §31.3 #1–7, 19–22
- Handout on Slope Fields: #1, 2, 4, 5, 6, 8, 9–14