21.1

Problem 4

- a) $\frac{d}{dx}\cos x|_{x=0} = \lim_{h\to 0}\frac{\cos(0+h)-\cos(0)}{h} = \lim_{h\to 0}\frac{\cos(h)-1}{h}$
- b) $\approx \frac{\cos(0.01)-1}{0.01} \approx -0.005$ or $\approx \frac{\cos(0.001)-1}{0.001} \approx -0.00005 \to 0$

21.2

Problem 1

$\frac{d}{dx}\tan x = \frac{d}{dx}(\frac{\sin x}{\cos x}) = \frac{\cos x\cos x - \sin x(-\sin x)}{(\cos x)^2} = \frac{1}{\cos^2 x} = \sec^2 x$

Problem 2

$\frac{d}{dx}\sec x = \frac{d}{dx}(\frac{1}{\cos x}) = \frac{\cos x\dot{0} - 1\dot{(}-\sin x)}{(\cos x)^2} = \frac{sinx}{\cos^2 x} = \frac{1}{\cos x}\frac{\sin x}{\cos x}x$

21.3 - first assignment

Problem 7

The tangent line approximation to $f(x) = \sin(x)$ is $y = x$. $f'(x) = \cos x$, $f''(x) = -\sin x$. f'' is positive to the left of zero and negative to the right of zero, hence f is concave up to the left of zero and concave down to the right of zero.

	Actual	Approximation	Approximation relation to actual
a) sin 0.2	=0.198	≈ 0.2	too large
b) sin 0.1	=0.0998	≈ 0.1	too large
c) sin 0.01	=0.00999	≈ 0.01	too large
d) sin (-0.01)	=-0.0998	≈ -0.1	too small

Problem 8

$\tan\theta = \frac{x}{30} \Rightarrow$
$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt} \Rightarrow$
$\frac{d\theta}{dt} = \cos^2\theta\frac{1}{30}\frac{dx}{dt} \Rightarrow$
$\frac{d\theta}{dt}|_{x=50} = \frac{900}{3400}\frac{1}{30ft}(-46\frac{ft}{sec}) \approx -.4059\frac{rad}{sec}$

Problem 15

$\frac{y}{0.5} = \tan\theta \Rightarrow$
$y = 0.5\tan\theta \Rightarrow$
$\frac{dy}{dt} = 0.5\sec^2\theta\frac{d\theta}{dt}$
At the instant the window is one kilometer from the light
$\cos\theta = \frac{1}{2} \Rightarrow \sec^2\theta = 4 \Rightarrow$
$\frac{dy}{dt} = 0.5(4)[6(2\pi)] = 24\pi \approx 75.4\frac{km}{min}$

Problem 18

At $t = 2$ the wheel will have completed 14 revolutions so the point will be back on the positive x axis. Movment is vertical so the horizontal component to the velocity will be zero.

Problem 23

$z^2 = x^2 + y^2 \Rightarrow$
$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$.
We know that $\frac{dx}{dt} = -5\frac{ft}{sec}$ and $\frac{dy}{dt} = 10\frac{ft}{sec}$ so
a) When $x = 200$ and $y = 100, z = 100\sqrt{5}$ hence $100\sqrt{5}\frac{dz}{dt} = 200(-5) + 100(10) = 0$, thus the distance between them is neither increasin or decreasing.
b) $\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2} \Rightarrow$
$\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}\cos^2\theta$.
At that instant$\cos\theta = \frac{200}{100\sqrt{5}}$, thus
$\frac{d\theta}{dt} = \frac{200(10) - 100(-5)}{200^2}(\frac{2}{\sqrt{5}})^2 = \frac{2500\times 4}{40000\times 5} = \frac{5}{100}\frac{rad}{sec}$

21.3 - Second Assignment

Problem 1

- a) $f'(x) = 1 + 2\cos x = 0 \iff \cos x = -\frac{1}{2} \iff x = \frac{2}{3}\pi + k(2\pi)$ or $x = \frac{4}{3}\pi + k(2\pi)$
- b) decreasing on $[\frac{2}{3}\pi + 2k\pi, \frac{4}{3}\pi + 2k\pi]$
  increasing on $[-\frac{2}{3}\pi + 2k\pi, \frac{2}{3}\pi + 2k\pi]$
- c) local maxima at $\frac{2}{3}\pi + 2k\pi$ and local minima at $\frac{4}{3}\pi + 2k\pi$.
- d) no global max or min as keeps on climbing
- e) $f''(x) = -2\sin x = 0$ for $x = k\pi$. Finding the zeros shows us that f is concave up on $[(2n-1)\pi, 2n\pi]$ and concave down on $[2n\pi, (2n+1)\pi]$

Problem 3

$f'(x) = -\sin x + \sqrt{3}\cos x = 0 \iff \tan x = \sqrt{3}$
$\Rightarrow x = \frac{x}{3}, \frac{4\pi}{3}$

Problem 5

$f'(x) = -2\sin 2x + 2\sin x = -4\sin x\cos x + 2\sin x =$
$2\sin x(-2\cos x + 1) = 0 \iff \sin x = 0$ or $\cos x = \frac{1}{2}$
$\Rightarrow x = 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

Problem 6

$f' = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x) = 0 \iff$
$\tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

21.4

Problem 21.4

Problem 1

$y' = 3\sec^2 x - 4\frac{1}{1}1 + x^2$

Problem 2

$f'(x) = 3\frac{1}{1+(2\sqrt{x})^2}2\frac{1}{2}x^{-\frac{1}{2}} = \frac{3}{(1+4x)\sqrt{x}}$

Problem 3

$y' = \cos x \sin^{-1} x + \sin x \frac{1}{\sqrt{1-x^2}}$

Problem 4

$y = (\tan^{-1} x))^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}(\tan^{-1} x)^{-\frac{1}{2}} \frac{1}{1+x^2} = \frac{1}{2(1+x^2)\sqrt{\tan^{-1} x}}$

Problem 5

$y' = (1)\tan^{-1} x + x\frac{1}{1+x^2} = \tan^{-1} x + \frac{x}{1+x^2}$

Question 6

$y = \frac{1}{e}\tan^{-1} e^x \Rightarrow y' = \frac{1}{e}\frac{1}{1+(e^x)^2}e^x = \frac{e^{x-1}}{1+e^{2x}}$

21.3 - Third Assignment

Problem 2

- a) $\cos x$ has period $2\pi, \sin 2x$ has period π, so it also repeats in 2π, so f has period 2π.
- b) $f'(x) = \sin x + \cos 2x = \sin x + 1 - 2\sin^2 x = (1 + 2\sin x)(1 - \sin x) = 0 \iff x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
  $\frac{\pi}{2}$ is a stationary point, $\frac{7\pi}{6}$ is a global maximum point and $\frac{11\pi}{6}$ is a global minimum point.

Problem 10

$y = \tan x \Rightarrow y' = \sec^2 x \Rightarrow y^n = 2\sec x(\sec x \tan x) = \frac{2\sin x}{\cos^2 x}$. Around $x = \pi k, \sin x$ changes sign while $\cos x$ doesn't change sign so y^n changes sign at $x = \pi k$, thus $\tan x$ has inflection points at $x = \pi k$. (Note $\tan x$ has vertical asymptotes at $x = \frac{\pi}{2} + \pi k$.)

Problem 12

- a) $x(\theta) = \frac{v_0^2}{g}\sin(2\theta) \Rightarrow x'(\theta) = \frac{v_0^2}{g}\sin(2\theta)2 = 0 \Longleftrightarrow \theta = \frac{\pi}{4}$
  $x_{max} = \frac{v_0^2}{g}\sin(2(\frac{\pi}{4})) = \frac{v_0^2}{g}$
- b) $\frac{100mi}{hr} = \frac{100mi}{hr}\frac{5280ft}{mi}\frac{100m}{328ft}\frac{1hr}{3600sec} \approx \frac{44.7m}{sec} \Rightarrow x_{max} \approx \frac{(44.7\frac{m}{sec})^2}{9.8\frac{m}{sec^2}} \approx 204m.$
  This is not very realistic as wind resistance is not taken into account.

Problem 14

$V = 8(2 + x)h = 8(2 + \sin\theta)\cos\theta$
$V'8[\cos\theta(\cos\theta) + (2 + \sin\theta)(-\sin\theta)] = 8[-2\sin^2\theta - 2\sin\theta + 1]$
$V' = 0 \Longleftrightarrow \sin\theta = \frac{2\mp\sqrt{12}}{-4} = \frac{1\mp\sqrt{3}}{-2} \Rightarrow \theta \approx 0.3747$. The negative solution is unrealistic.