Solutions to #2 and #3 on 2004 Final Exam

Solution to #2

Let M = amount put into savings account at the beginning of each month. The monthly interest rate r is $r = 6/12\% = .5\%$. The total number of deposits is 120.

Just after the 120th deposit, the student has in the account

$$S = M + M(1.005) + M(1.005)^2 + \cdots + M(1.005)^{119}$$

(If you have a hard time coming up with this on your own, try thinking through how much the student has in the account after the third month or the sixth month, writing it carefully as a sum and looking for the pattern.)

We find that

$$S = \frac{M - M(1.005)^1 20}{1 - 1.005} = M(163.879)$$

Thus, if we want S to be equal to \$1,000,000, we solve and find that $M = 6102.06$.

Solution to #3

Let h = the height of the rocket. We know that $\frac{d\theta}{dt} = \frac{\pi}{60}$ radians/sec at the moment when $\theta = \pi/3$. We want to find $\frac{dh}{dt}$ at that same instant.

An equation relating θ and h is

$$\tan \theta = \frac{h}{5}$$

We differentiate both sides of this to get

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{5} \frac{dh}{dt}$$

We plug in the information that we have about the instant when $\theta = \pi/3$ (please remember that when we are differentiating, we *must* use angle measurements in radians) to get

$$(2)^2 \left(\frac{\pi}{60} \right) = \frac{1}{5} \frac{dh}{dt}$$

$$1.05 \text{ miles/sec} \approx \frac{dh}{dt}$$