Name:_____

Math Xb Midterm II Part 1
Spring 2006

Please circle your section:

Thomas Barnet-Lamb	Jon Bloom	Angela Vierling-Claassen
MWF 10–11	**MWF 10–11**	MWF 10–11

Bret Benesh	Jon Bloom
MWF 11-12	**MWF 11-12**

Part One Scores

Problem Number	Possible Points	Score
1	9	
2	6	
3	12	
4	8	
5	7	
Total	42	

Midterm II Scores

	Possible Points	Score
Part One	42	
Part Two	58	
Total	100	

Directions—Please read carefully!

You *are not allowed* to use a calculator or any other aids on this part of the exam. When you are finished with this part of the exam, you may turn it in to the proctor. Once you have turned in this first part of the exam, you may not look at it again, so be sure you have finished it completely before turning it in. **You may not use a calculator until you have turned in this first part of the exam.**

To receive full credit on a problem, you will need to justify your answers carefully—**unsubstantiated answers will receive little or no credit** (except if the directions for that question specifically say no justification is necessary). Be sure to **write neatly—illegible answers will receive little or no credit**. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers.

Good luck!

1. (9 points, 3 points each) Find each of the following limits.

(a) $\lim_{x \to 0} \dfrac{e^{3x} + x}{e^x + 3}$

(b) $\lim_{x \to 0} \dfrac{e^{3x} + x - 1}{e^x - 1}$

(c) $\lim_{x \to \infty} \dfrac{x^2}{1.001^x}$

2. (6 points, 3 points each) Compute the following derivatives.

(a) $f'(x)$, where $f(x) = \arcsin(x)(x^2 - 1)$

(b) $h'(x)$, where $h(x) = \arctan(\arccos(x))$

3. (12 points, 3 points each) Find the following definite and indefinite integrals.

(a) $\int [(x^6+1)^9 + (x^6+1)^8 + (x^6+1)^7](6x^5)dx$

(b) $\int \frac{\arcsin x}{\sqrt{1-x^2}} dx$

(c) $\int \left(\frac{10}{x} + 1 + e^x\right) dx$

(d) $\int_0^5 \left(5\sin x + \sqrt{x}\right) dx$

4. (8 points) Here is a list of expressions involving integrals. Some pairs of expressions are equal. Pair up any equal expressions below. You don't have to show your work.

(a) $2\int_0^3 \arctan(x)dx + \int_3^6 2\arctan(x)dx$

(b) $\int_0^{1/2} (\arctan(x) - \arcsin(x))\,dx + \int_{1/2}^1 \arctan(x)dx$

(c) $\int_0^{10} \arctan(x)dx$

(d) $\int_0^3 \arctan(x)dx$

(e) $2\int_0^5 \arctan(x)dx + 2\int_5^6 \arctan(x)dx$

(f) $\int_0^{1/2} (\arctan(x) + \arcsin(x))\,dx + \int_{1/2}^1 \arctan(x)dx$

(g) $\frac{1}{2}\int_0^{10} 2\arctan(x)dx$

(h) $\int_0^2 \arctan(x)dx - \int_3^2 \arctan(x)dx$

(i) $\int_0^1 \arctan(x)dx + \int_{1/2}^0 \arcsin(x)dx$

_____ is equal to _____

_____ is equal to _____

_____ is equal to _____

_____ is equal to _____

5. (7 points) Put the following in *ascending* order in the spaces provided below. You do not need to justify your solution. [*Hint*: Think about which expressions are positive, which are negative, and which are zero. A picture may be helpful.]

(a) $\displaystyle\int_4^9 \ln t \; dt$

(b) $\ln 4 + \ln 5 + \ln 6 + \ln 7 + \ln 8$

(c) $\ln 5 + \ln 6 + \ln 7 + \ln 8 + \ln 9$

(d) Zero

(e) $\displaystyle\sum_{k=0}^{9} \frac{\ln(4 + \frac{k}{2})}{2}$

(f) $\ln(4/9)$

(g) $\displaystyle\lim_{h \to 0} \frac{\ln(4+h) - \ln 4}{h}$

_____ $\leq$ _____ $\leq$ _____ $\leq$ _____ $\leq$ _____ $\leq$ _____ $\leq$ _____

Name:_____

Math Xb Midterm II Part 2
Spring 2006

Please circle your section:

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Bret Benesh	Jon Bloom
MWF 11-12	**MWF 11-12**

Part Two Scores

Problem Number	Possible Points	Score
1	12	
2	10	
3	12	
4	12	
5	12	
Total	58	

Directions—Please Read Carefully! You *are* allowed to use a calculator on this part of the exam, but no other aids are allowed. **You cannot start to use your calculator until you have turned in the first part of the exam.** Be sure to write neatly—illegible answers will receive little or no credit.

To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit (except if the directions for that question specifically say no justification is necessary). **Write neatly**—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. **Good Luck!!!**

1. (12 points) A ball is thrown up in the air from an initial height of 0 ft and with an initial velocity of 64 ft/sec. It travels straight up and straight down to its original position. The acceleration due to gravity is -32 ft/sec^2. Therefore the velocity at time t is given by $v(t) = -32t + 64$.

   (a) What is the average velocity of the ball between $t = 0$ and $t = 3$?

   (b) What is the average speed of the ball between $t = 0$ and $t = 3$?

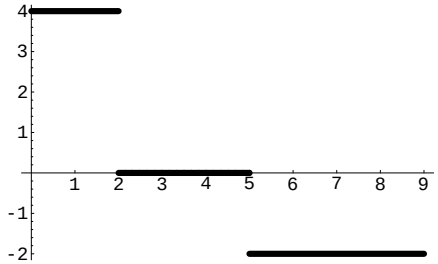
   (c) Find an equation for the height $h(t)$ of the ball (remember the ball is thrown from an initial height of 0).

   (d) What is the average height of the ball while it is in the air?

2. (10 points) Estimate the area under the graph of $y = x^3 + 2$ between $x = 1$ and $x = 9$ using four intervals and a left-hand sum.

3. (12 points) At noon, Gob rides from his home to the banana stand on his Segway. Since it is downhill, he rides at 4 miles per hour, and makes the trip in two hours. He then spends three hours in front of the banana stand performing magic. Finally, he heads home. His trip home is uphill, so he can only travel at 2 miles per hour, and it takes him 4 hours. Here is a graph of his velocity $v(t)$, where $t = 0$ corresponds to noon:

Graph of Gob's velocity, $v(t)$:

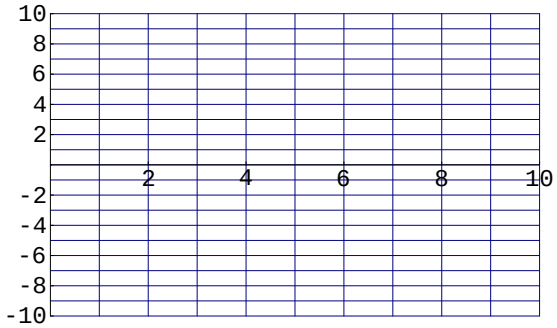


(a) Express the net change in Gob's position from $t = 0$ to $t = 4$ using integral notation (you don't have to find the net change in position, just write down an integral that represents the net change).

(b) Find the net change in Gob's position from $t = 0$ to $t = 6$ (your answer should be a number, don't forget the units).

(c) Write a piece-wise defined formula describing Gob's position, $p(x)$, as a function of time x (your formula should *not* contain an integral sign $\int$).

(d) Graph Gob's position function, $p(t)$, on the axes below. Make your graph as precise as possible.



4. (12 points) Below is a list of values for the function $f(x)$. Note that $f(x)$ has a continuous derivative $f'(x)$:

$f(1) = 3$
$f(-1) = -5$
$f(0) = 2$
$f(2) = 7.5$
$f(8) = 0$
$f(-4) = 1$

(a) Find $\int_2^8 f'(x)\ dx$.

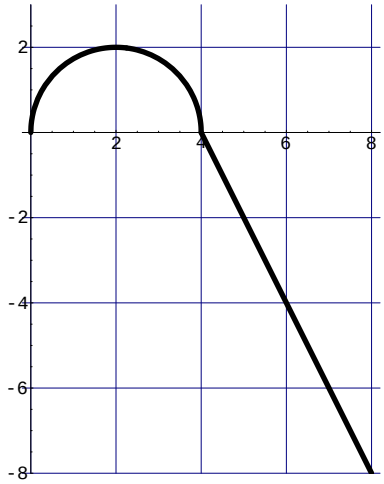
(b) Find $\int_{-1}^{-4} f'(x)\ dx$.

(c) Assume that $\int_2^{10} f'(x)\ dx = 7.7$. Find $\int_8^{10} f'(x)\ dx$.

(d) An antiderivative of $g(x) = \tan x$ is $G(x) = \ln|\sec x|$. Please circle the answer below that best describes $\int_{\frac{\pi}{6}}^{\frac{3\pi}{4}} \tan x\ dx$. *Hint:* Draw a picture before you answer this question.

i. $\ln|\sec\frac{3\pi}{4}| - \ln|\sec\frac{\pi}{6}|$
ii. $-(\ln|\sec\frac{3\pi}{4}| - \ln|\sec\frac{\pi}{6}|)$
iii. It cannot be determined from methods we have learned in class.

5. (12 points) The rate that water is entering a tank is given by the function $f(t)$ drawn below. The function $f(t)$ is measured in gallons/minute and t is measured in minutes past noon.



(a) Find $\int_2^6 f(t)dt$. What does this quantity represent in terms of the physical situation?

(b) In terms of the physical situation of the water in the tank, what does the area function ${}_0A_f(x) = \int_0^x f(t)dt$ represent for $x > 0$?

(c) For what values of x is ${}_0A_f(x)$ decreasing?

(d) Find $\frac{d}{dx}\int_2^x f(t)dt$ at $x = 6$.