Name:_____

**Math Xb Midterm I Part II**
**Spring 2006**

*Please circle your section:*

Thomas Barnet-Lamb     Jon Bloom     Angela Vierling-Claassen
MWF 10–11     **MWF 10–11**     MWF 10–11

Bret Benesh     Jon Bloom
MWF 11-12     **MWF 11-12**

| Problem Number | Possible Points | Score |
| --- | --- | --- |
| 1 | 10 | |
| 2 | 12 | |
| 3 | 12 | |
| 4 | 8 | |
| 5 | 12 | |
| 6 | 10 | |
| 7 | 10 | |
| Total | 74 | |

**Directions—Please Read Carefully!**   You *are* allowed to use a calculator on this part of the exam, but no other aids are allowed. Be sure to write neatly—illegible answers will receive little or no credit.

**To receive full credit on a problem, you will need to justify your answers carefully— unsubstantiated answers will receive little or no credit** (except if the directions for that question specifically say no justification is necessary). **Write neatly**—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. **Good Luck!!!**

1. (10 points, 5 points each)

   (a) In the triangle below, some of the side lengths and angles (in *degrees*) have been marked. What is the length of the side labelled $a$?
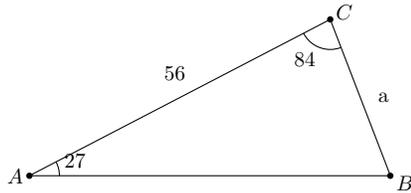
   

   (b) In the triangle below, some of the side lengths and angles (in *degrees*) have been marked. What is the length of the side labelled $a$?
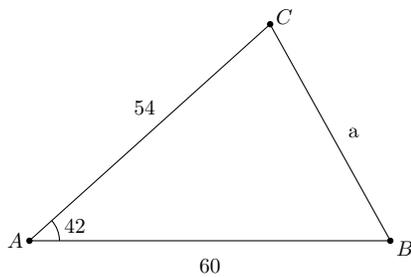
2. (12 points, 3 points each) The cobra population in a jungle can be modeled with a sinusoidal function. The population is at its maximum of 2500 at time t = 0 months and first reaches its minimum of 1500 at time t = 3 months.

(a) Sketch a graph of the snake population over 12 months ($0 \leq t \leq 12$). Label the maximum and minimum points on the graph and sketch and label the balance line.

(b) Write a function P(t) for the graph you sketched in part (a).

(c) Find the first time t when the cobra population is 2250.

(d) Find ALL other times t when the cobra population is 2250 during the first year.
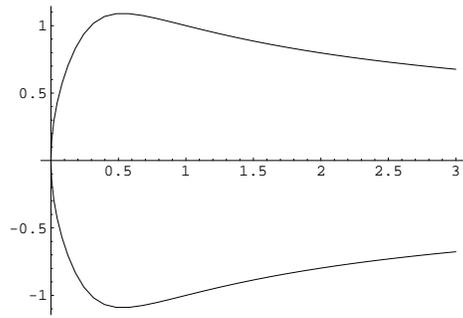
3. (12 points, 4 points each)

(a) I have a set of 5 Russian dolls, which fit inside one another. If I put the dolls in order of size, each is exactly $\frac{4}{5}$ the size of the previous doll. The largest doll is half a yard high. If I lay the dolls head-to-toe, how far do they stretch in total?

(b) Now suppose I have a nesting set of 50 Russian dolls, each doll $\frac{4}{5}$ the size of the previous doll. The largest doll is half a yard high. If I lay the dolls head-to-toe, how far do they stretch in total? Give six decimal places in your answer.

(c) Now suppose that I have a nesting set of infinitely many dolls, each doll $\frac{4}{5}$ the size of the previous doll with the largest doll half a yard high. If the dolls are all lying head-to-toe, how far do they stretch in total (note that the answer here might be infinitely far or the answer might be a finite number – remember to justify whatever answer you give.)

4. (8 points) Find the equation of the tangent line to $(x^2 y^3 + y)^2 = 4x$ at the point $(1,1)$

5. (12 points, 4 points each)

(a) Find
$$\cos(\alpha + \beta) - \cos(\alpha - \beta)$$
in terms of $\sin \alpha$, $\cos \alpha$, $\sin \beta$, and $\cos \beta$ (your answer may involve some or all of these four quantities).

(b) Find an expression for
$$(\sin x + \cos x)^2$$
in terms of $\sin 2x$.

(c) Use trig identities and the trig values you know to find the *exact* value of $\sin 75°$. (Note that you must show how you have used trig identities to receive credit and that a decimal answer will not get you any credit.)

6. (10 points) Gladys Kravitz is convinced that her neighbor is a witch and this time she's going to get proof. She takes a 10-foot ladder over to her neighbor's house where she plans to look through a window and take the photo which will prove that she's not crazy and that her neighbor is really a witch.

Gladys is perched precariously at the top of a 10-foot ladder leaning against the back wall of her neighbor's house. Her husband, Abner, sees what she is up to and comes over to try to stop her. The ladder starts to slide down the wall. Abner is standing on the ground 6 feet away from the wall (behind Gladys). When the base of the ladder hits Abner, the top of the ladder is sliding down the wall at a rate of 4 feet per second. How fast is the base of the ladder moving when it hits Abner?



6 feet
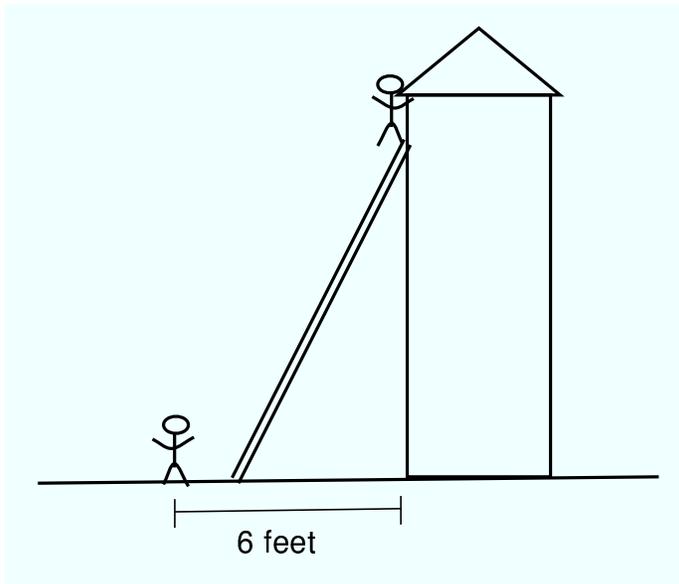
7. (10 points) A rain gutter is to be constructed from a metal sheet of width 30 centimeters and length 5 meters by bending up one-third of the sheet on each side through an angle $\theta$ as shown below. How should $\theta$ be chosen so that the gutter will carry the maximum amount of water? *You must carefully justify your work to get full credit. In particular, you need to show that you have found a maximum.*



5 meters (drawing not to scale)

10 cm    10 cm    10 cm

Fold

$\theta$          $\theta$