CHAPTER 19 - TRIGONOMETRY: INTRODUCING PERIODIC FUNCTIONS

Section 1 - The Sine and Cosine Functions: Definitions and Basic Properties

We start with the unit circle. It's a circle, centered at the origin with radius 1. The x-coordinate of a point on the circle is cos(x) and the y-coordinate is sin(x) where x is the distance traveled along the circle in the counterclockwise direction.

Sine and cosine are unique because they are *periodic* functions. A periodic function is a function in which there is some positive constant k that for any x, $f(x + k) = f(x)$. This means that the function *repeats* itself in periods. The period of a function is the space over which you can basically cut and paste the graph horizontally.

The domains of both sine and cosine are all real numbers and the ranges are both from -1 to 1 inclusive.

Cosine is an even function: $\cos(x) = \cos(-x)$
Sine is an odd function: $\sin(x) = -\sin(-x)$

A few identities (to be expanded upon later) :

$\sin^2(x) + \cos^2(x) = 1$

$\sin(x + 2\pi n) = \sin(x)$
$\cos(x + 2\pi n) = \cos(x)$

$\sin(x) = \cos(x - \pi / 2)$

$\sin(x \pm \pi) = -\sin(x)$
$\cos(x \pm \pi) = -\cos(x)$

Section 2 – Modifying the Graphs of Sine and Cosine

Some things to know about a sine or cosine graph:
balance line – the centrally located horizontal line about which the graph oscillates.
balance value – y-value of the balance line
amplitude – positive number that indicates the max vertical distance between graph and balance line.
period – area over which graph begins to repeat itself.
frequency – reciprocal of the period.
sinusoidal function – a function that is either sine or cosine or something created by shifting, flipping, stretching or shrinking sine or cosine.

Notice this stuff in the function :
$y = A \sin (B (x - C)) + K$
K is the balance value (vertical shift)
A is the amplitude (must be positive)

$(2\pi) / B$ is the period (must be positive)
C is the number of horizontal units shifted (to the right if negative, to the left if positive)

Section 3 - The Function f(x) = tan(x)

tan(x) = sin(x) / cos(x)

graph of tangent has vertical asymptotes. It approaches negative infinity as x approaches $\pi / 2$ from the positive direction. It approaches positive infinity as x approaches $\pi / 2$ from the negative direction.

tangent has a period of π.

Section 4 – Angles and Arc Lengths

some things to know about angles :
initial side – a ray from which you begin measuring
terminal side – it is rotated about its endpoint and the final position to be measured to is the terminal side.
vertex – common point of the initial and terminal sides.
standard position – an angle is in this position when the initial side is along the positive axis, vertex at the origin.
coterminal – two angles are coterminal if their sides coincide.

two ways to measure :
degrees : 360 around a circle
radians : 2π around a circle

from degrees to radians : degree amount * π / 180
from radians to degrees : radian amount * 180 / π

arc length : r _

CHAPTER 20 – TRIGONOMETRY – CIRCLES AND TRIANGLES

Section 1 – Right-Triangle Trigonometry: The Definitions

Similar Triangles have angles of equal measure and thus sides with proportional lengths.

sin = opposite / hypotenuse
cos = adjacent / hypotenuse
tan = oppositive / adjacent
csc = hypotenuse / opposite
sec = hypotenuse / adjacent
cot = adjacent / opposite

two angles are *complementary* if their sum is 90 degrees or $\pi / 2$.

solving a triangle means solving for all three sides and all three angles.

Angle of elevation is the angle from the horizontal up to an object.
Angle of depression is the angle from the horizontal down to an object.

Section 2 – Triangles We Know and Love, and the Information They Give Us

45-45-90:
two legs are of equal length.
each of their lengths equals the hypotenuse divided by the root of 2.

30-60-90:
the smallest leg has length x
the hypotenuse has length 2x
the longer leg has length x * root(3)

Section 3 – Inverse Trigonometric Functions

Do not confuse with reciprocal: $\sin^{-1}$ DOES NOT MEAN 1 / sin.

$\sin^{-1}$ is the angle between $-\pi / 2$ and $\pi / 2$ whose sine is x…domain: [-1,1] = arcsin
$\cos^{-1}$ is the angle between 0 and π whose cosine is x…domain: [-1,1] = arccos
$\tan^{-1}$ is the angle between $-\pi / 2$ and $\pi / 2$ whose tangent is x…domain: $(-\infty,\infty)$ = arctan

Section 4 – Solving Trigonometric Equations

Pretty straightforward. Isolate the trig function on one side of equation. Then take the inverse trig function of both sides to cancel away the trig function and solve for your variable of choice. See examples in the book.

Section 5 – Applying Trigonometry to a General Triangle : The Law of Cosines and The Law of Sines

An *oblique* triangle is a non-right triangle.

Law of Cosines:
$c^2 = a^2 + b^2 - 2 a b \cos C$
use it when you have SSS or SAS

Law of Sines:
$\sin A / a = \sin B / b = \sin C / c$
use it when you have AAS, ASA or SSA

Area:
$.5\, a\, b \sin C$

Section 6 – Trigonometric Identities

See handout of trig identities…
http://www.courses.fas.harvard.edu/~mathxb/worksheets/trig_ids_list.pdf