Name:_____

Math Xb Midterm Examination I
Tuesday, April 19, 2005

Please circle your section:

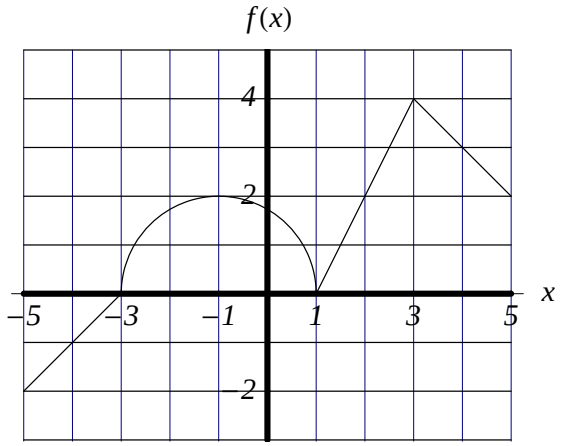
Amanda Alvine Ashley Hinerman-Mulroy (CA) MWF 10–11	Angela Vierling-Claassen Shayak Sarkar (CA) MWF 10–11
Ben Weinkove Connie Zong (CA) MWF 10–11	Thomas Judson Wendy Widman (CA) MWF 11–12

Problem Number	Possible Points	Score
1	12	
2	12	
3	10	
4	11	
5	15	
6	12	
7	11	
8	8	
9	9	
Total	100	

Directions—Please Read Carefully! You have two hours to take this midterm. To receive full credit on a problem, you will need to justify your answers carefully—**unsubstantiated answers will receive little or no credit** (except if the directions for that question specifically say no justification is necessary). Please be sure to write neatly—**illegible answers will receive little or no credit**. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers.

Make sure to use correct mathematical notation. Any answer in decimal form must be accurate to two decimal places, unless otherwise specified. You may use a calculator on this exam, but no other aids are allowed. **Good Luck!**

1. (12 points) The graph of $f(x)$, given below, is made up of straight lines and a semicircle.



(a) Estimate $\int_{-5}^{5} f(x)\,dx$ using a left-hand sum with $n = 5$ subintervals.

(b) Find the exact value of $\int_{-5}^{5} f(x)\,dx$.

(c) If $F(x) = \int_{-1}^{x} f(t)\,dt$, find $F(3)$.

(d) If $F(x) = \int_{-1}^{x} f(t)\,dt$, find $F(-3)$.

2. (12 points)

(a) Suppose f is a continuous function such that $f(1) = 2$ and $f'(1) = 3$. Evaluate

$$\lim_{x \to 1} \frac{[f(x)]^2 - 4}{x^2 - 1}.$$

Be sure to carefully justify your solution.

(b) Show that $\lim_{x \to 0+} (1 + ax)^{1/x} = e^a$, where a is a positive constant. Carefully justify your work.

3. (10 points) The following table gives the emissions, E, of nitrogen oxides in millions of metric tons per year in the U.S.[1] Let t be the number of years since 1940 and $E = f(t)$.

Year	1940	1950	1960	1970	1980	1990
E	6.9	9.4	13.0	18.5	20.9	19.6

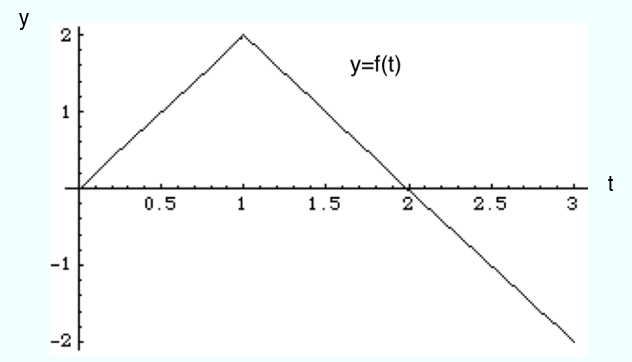
(a) What are the units and the meaning of $\int_0^{50} f(t)\,dt$?

(b) Estimate $\int_0^{50} f(t)\,dt$ using a right-hand sum.

[1] Statistical Abstracts of the U.S., 1992.

4. (11 points) A plot of ground in the shape of a circular sector (a wedge of pie) is to have a border of roses along the straight lines and tulips along the circular arc. Roses cost \$20 per meter and tulips cost \$15 per meter. If the area of the plot is to be 100 square meters, is it possible to minimize the cost of the flowers? If so, what is the minimum cost? If not, why not? [*Hint: The area of a sector is given by* $A = r^2\theta/2$, *and the length of the circular arc is given by* $L = r\theta$, *where* θ *is the angle of the sector in radians.*]

5. (15 points) Consider the function $f(t)$, defined for $0 \leq t \leq 3$ with graph $y = f(t)$ as shown:



Recall that the area function ${}_aA_f(x)$ is given by ${}_aA_f(x) = \int_a^x f(t)dt$.

(a) Describe in words the function ${}_0A_f(x)$ (i.e. what is the input? what is the output?)

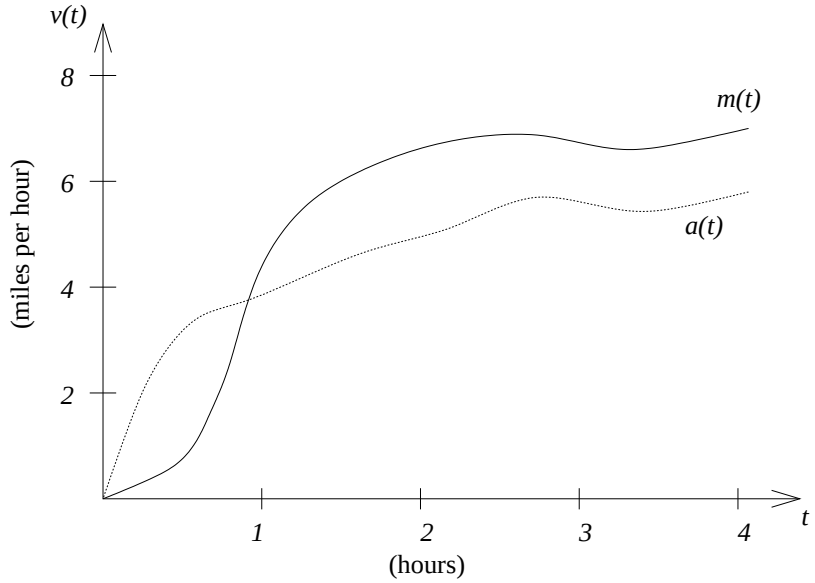
(b) When is ${}_0A_f$ increasing?

(c) When is ${}_0A_f$ concave down?

(d) Find the absolute minimum and absolute maximum of ${}_0A_f$ (you should find both the maximum and minimum values, and the values of t where they occur).

(e) Find an explicit formula for ${}_0A_f(x)$ for x in $[0, 1]$.

6. (12 points) Two friends, Amani and Margaret, ran the 26.2 mile Boston Marathon this year. Amani's velocity is given by $a(t)$ and Margaret's by $m(t)$ with t measured in hours and the velocity in miles/hour. The start time, $t = 0$, is noon, and the two runners begin at the same spot. The graph below shows the first four hours of the race.



(a) Write an integral expressing the number of miles that Amani covered in the first two hours.

(b) Write an expression for the number of miles along the course that Margaret still has left to run at 1:30 pm. Your expression should involve an integral.

(c) Who is ahead after 1 hour of running? Write an integral that expresses the distance between the two friends after 1 hour.

(d) At $t = 0$ the runners begin at the same spot. Let T be the next time that the runners are at the same spot. Write an equation to describe T. (Note that we are not asking you to find or guess T by looking at the graph, just to write an equation that describes T.)

7. (11 points) Venkman and Ray, two members of the Ghost Busters, have caught a ghost in their laser beams. The ghost is floating in the air, 25 feet directly above a ghost trap on the ground. Venkman is to the left of the trap, and walking directly towards it at a speed of 1 ft/sec. Ray is to the right of the trap and walking directly towards it at a speed of 2 ft/sec. As they walk, they keep their beams fixed on the ghost, which remains fixed.

   At the point when Venkman is 8 feet from the trap, and Ray is 5 feet from the trap, at what rate is the angle between their beams decreasing? You can ignore the heights of Venkman and Ray (in other words, pretend the beams start from the ground). [*Hint: The angle between the beams really consists of two angles – one that Venkman's beam makes with the vertical line from the ghost to the trap, and one that Ray's beam makes with the same vertical line. You'll probably want to consider these two angles separately.*]

8. (8 points) Use the properties of the definite integral and the sine function to show that

$$\frac{\pi}{6} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq \frac{\pi}{3}.$$

without evaluating the integral $\displaystyle\int_{\pi/6}^{\pi/2} \sin x \, dx$.

9. (9 points) If $y = \cot^{-1} x$, show that

$$\frac{dy}{dx} = \frac{-1}{1+x^2}.$$

Use the definition of $\arctan x$ as well as the derivative of the tangent function. Be sure to carefully justify your answer.