20.6 Trigonometric Identities

4. (*i*) b: $1-\cos^2 x = \sin^2 x$

(*ii*) a: $1-\sin^2 x = \cos^2 x$

(*iii*) b: $\frac{1}{2}-\frac{1}{2}\cos 2x = \frac{1}{2}(1-\cos 2x) = \sin^2 x$

(*iv*) c: $-1+\sec^2 x = \tan^2 x$

(*v*) b: $1-\sin^2(x+\frac{\pi}{2}) = \cos^2(x+\frac{\pi}{2})$
$= (\cos(x+\frac{\pi}{2}))^2 = (\cos x\cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2})^2$
$= (-\sin x)^2 = \sin^2 x$

(*vi*) f: $\sin(x-\frac{\pi}{2}) = \sin x\cos\frac{\pi}{2} - \cos x \sin\frac{\pi}{2}$
$= -\cos x$

(*vii*) a: $\frac{1}{2}+\frac{1}{2}\cos 2x = \frac{1}{2}(1+\cos 2x) = \cos^2 x$

(*viii*) d: $\cos(x-\frac{\pi}{2}) = \cos x\cos\frac{\pi}{2} + \sin x\sin\frac{\pi}{2}$
$= \sin x$

5. $\tan(A+B) = \dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{(\sin A\cos B + \cos A\sin B)\frac{1}{\cos A\cos B}}{(\cos A\cos B - \sin A\sin B)\frac{1}{\cos A\cos B}} = \dfrac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$

$= \dfrac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1-\frac{\sin A\sin B}{\cos A\cos B}} = \dfrac{\tan A + \tan B}{1-\tan A\tan B}$. The second proof is similar with all second terms of opposite sign.

6. $\tan 2x = \tan(x+x) = \dfrac{\tan x + \tan x}{1-\tan x\tan x} = \dfrac{2\tan x}{1-\tan^2 x}$

9. $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{\frac{1}{2}(1-\cos 2x)}{\frac{1}{2}(1+\cos 2x)} = \dfrac{1-\cos 2x}{1+\cos 2x}$