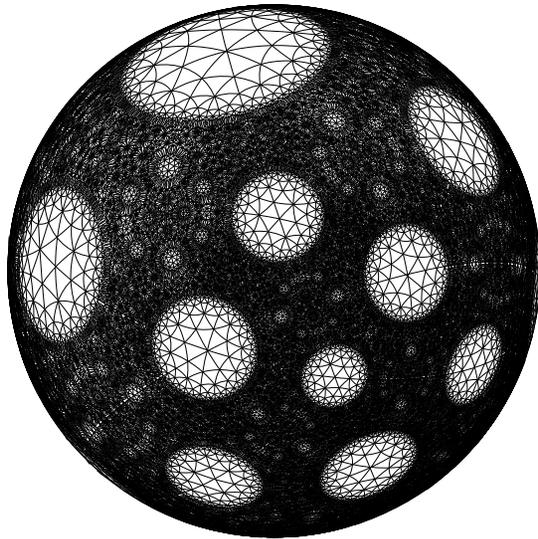


Rigidity for planes in hyperbolic 3-manifolds



Curtis T McMullen
Harvard University

Totally geodesic planes

Euclidean Planes \mathbb{R}^2 \square (space of lattices L in \mathbb{R}^3)

Littlewood

Hyperbolic Planes \mathbb{H}^2 \square (space of lattices L in \mathbb{R}^3)

Oppenheim

Hyperbolic Planes \mathbb{H}^2 \square (moduli space \mathcal{M}_g)

Teichmüller

Hyperbolic Planes \mathbb{H}^2 \square (3-manifold \mathbb{H}^3/Γ)

Ratner-Shah / Thurston

Rigidity of planes

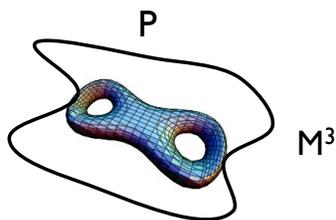
Hyperbolic plane \mathbb{H}^2 \square $M = \mathbb{H}^3/\Gamma$

image = immersed plane $P \subset M$

Shah, Ratner

M is compact $\Rightarrow P$ is dense (typical) or

P is a closed surface (miracle)



Infinite volume 3-manifolds

Does rigidity persist??

No. For $M = S \times \mathbb{R}$, cylinders cause problems

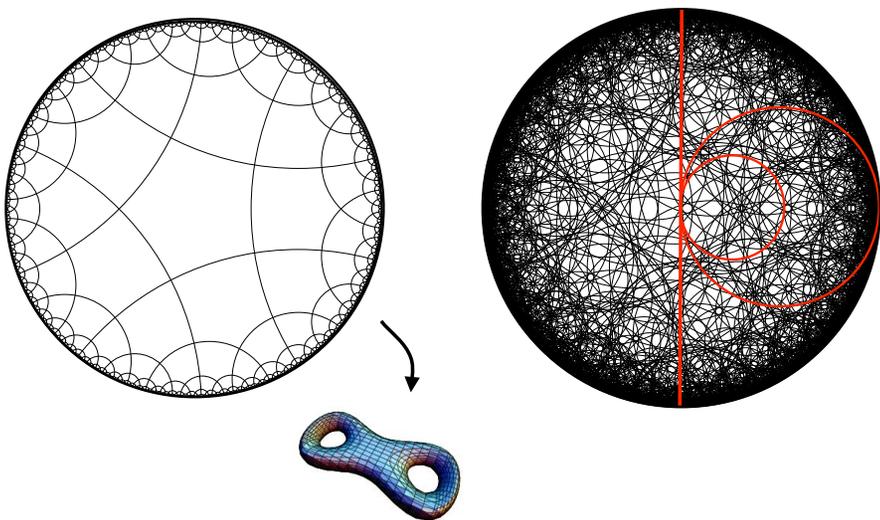
M, Mohammadi, Oh

Yes. Rigidity persists for acylindrical 3-manifolds.

Topology of 3-manifolds enters

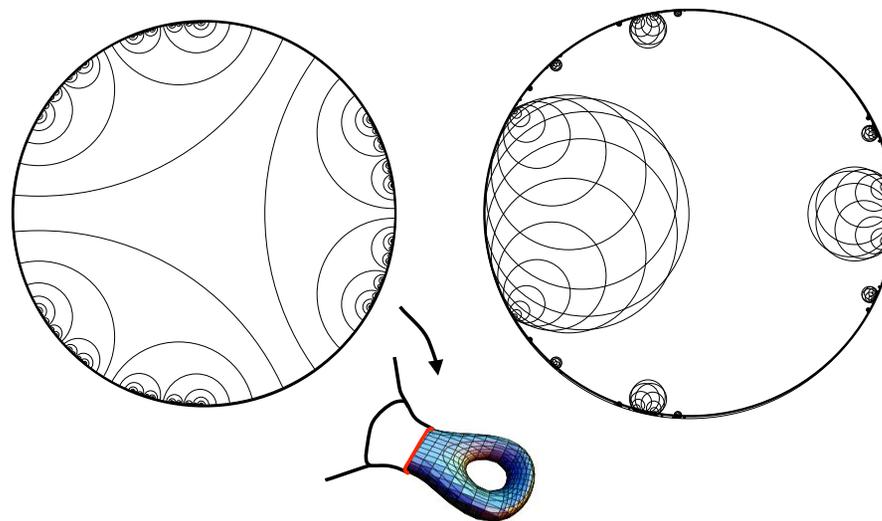
Finite volume 2-manifolds

Horocycles are closed or dense *Hedlund*



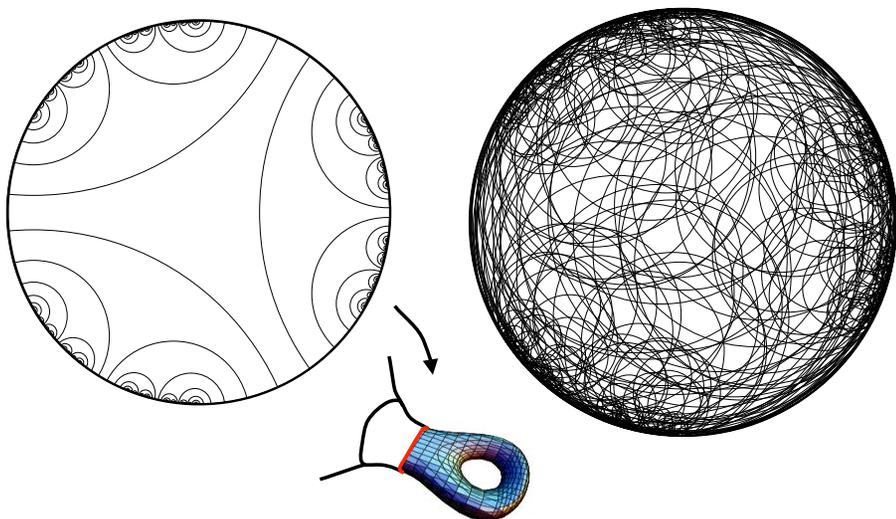
Infinite volume 2-manifolds

Horocycles are closed or dense *Dalbo*



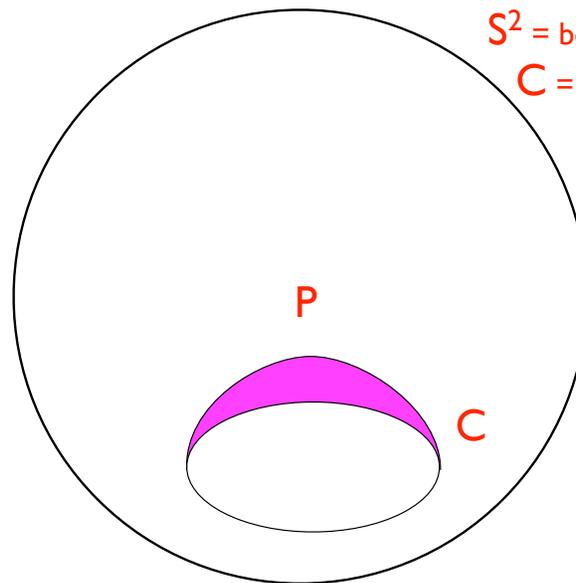
Infinite volume 2-manifolds

Horocycles are closed or dense *Dalbo*

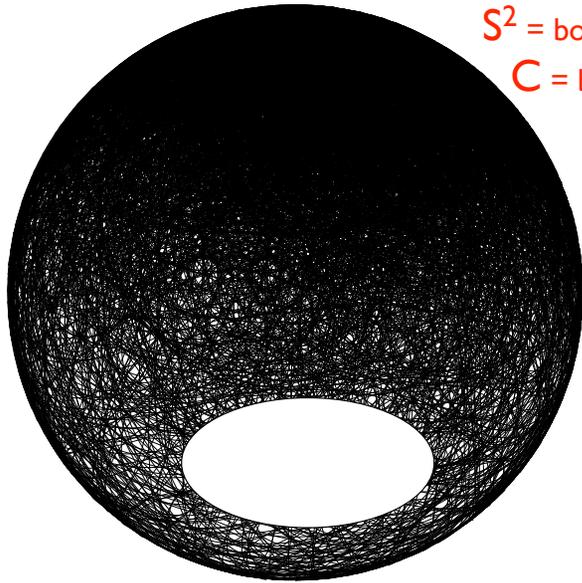


Finite volume 3-manifolds

S^2 = boundary of H^3
 C = boundary of plane



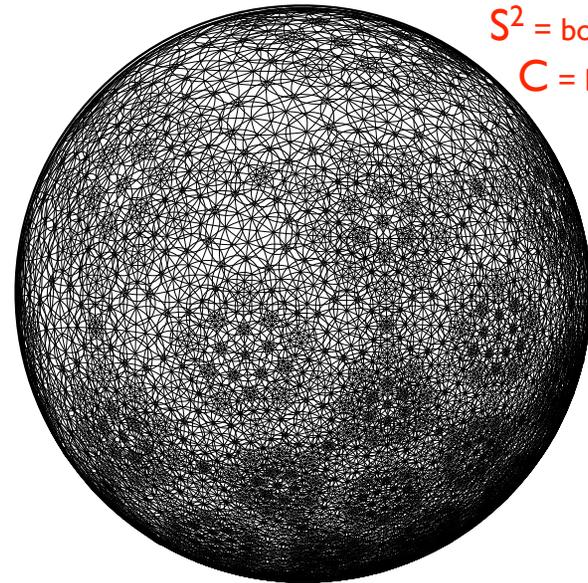
Finite volume 3-manifolds



$S^2 = \text{boundary of } \mathbb{H}^3$
 $C = \text{boundary of plane}$

$$M = \Gamma \setminus \mathbb{H}^3$$
$$\Gamma \cdot C$$

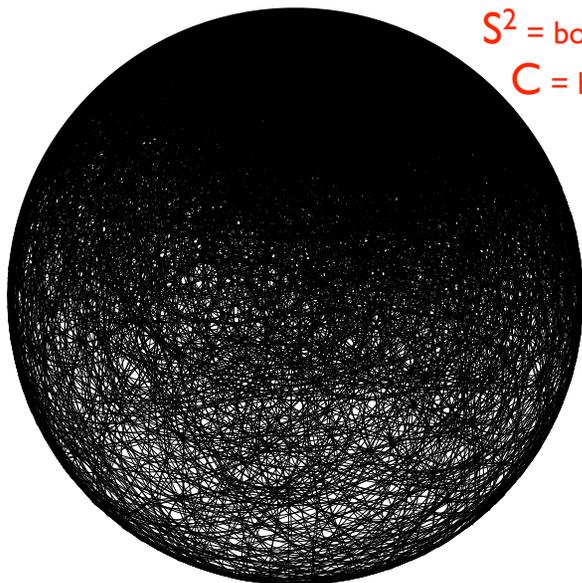
Another immersed plane



$S^2 = \text{boundary of } \mathbb{H}^3$
 $C = \text{boundary of plane}$

$$M = \Gamma \setminus \mathbb{H}^3$$
$$\Gamma \cdot C$$

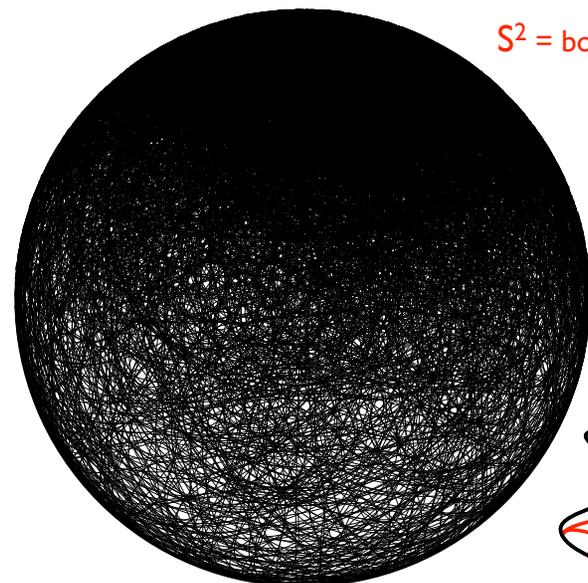
Immersed plane



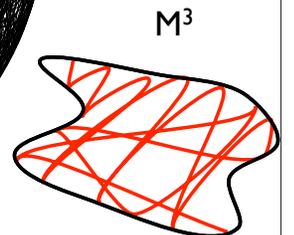
$S^2 = \text{boundary of } \mathbb{H}^3$
 $C = \text{boundary of plane}$

$$M = \Gamma \setminus \mathbb{H}^3$$
$$\Gamma \cdot C$$

Dense plane in M

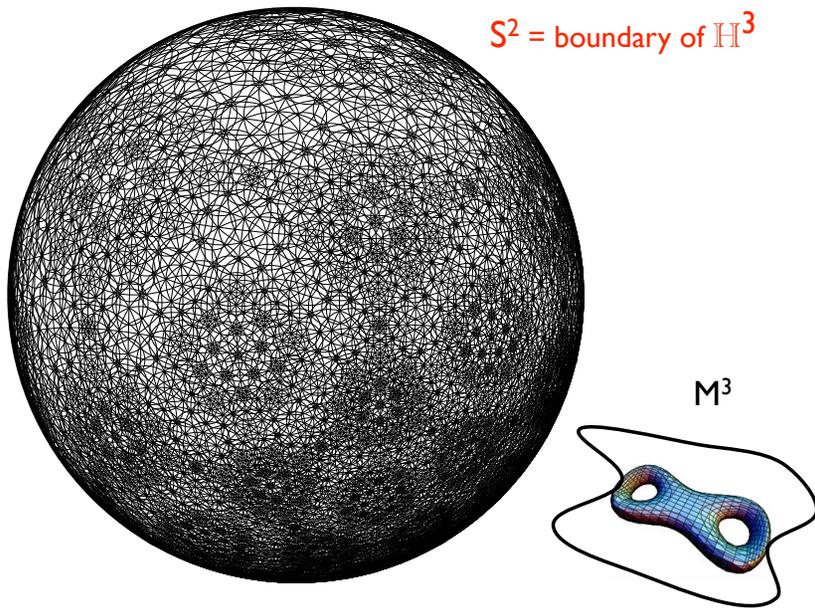


$S^2 = \text{boundary of } \mathbb{H}^3$

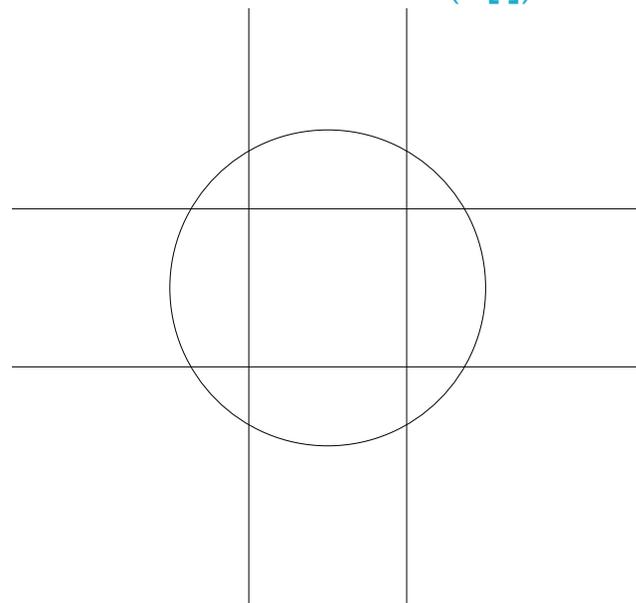


Closed, totally geodesic surface in M

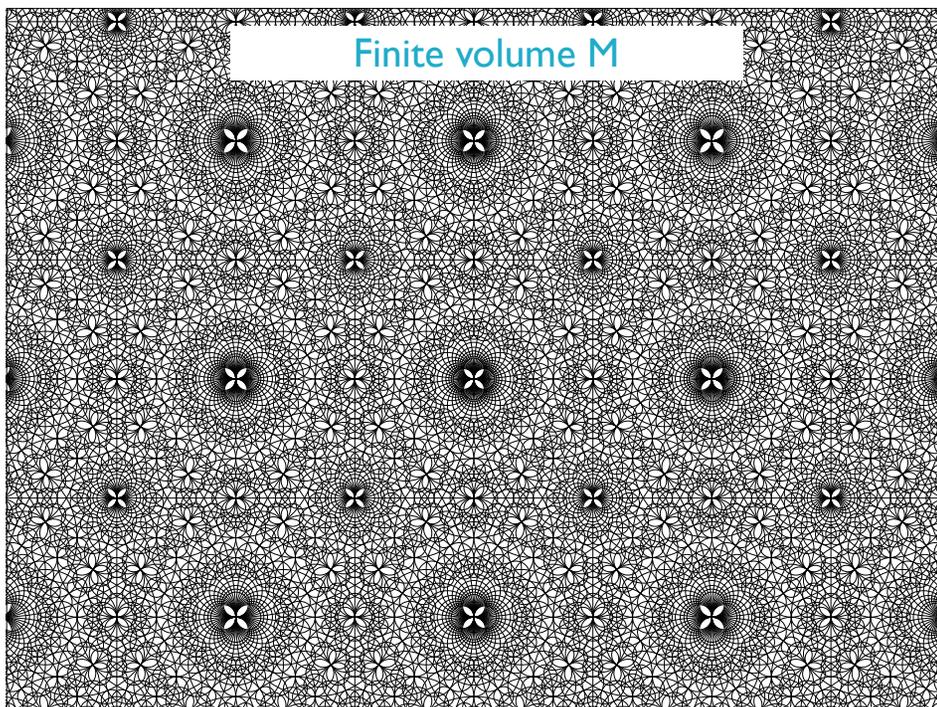
$S^2 = \text{boundary of } \mathbb{H}^3$



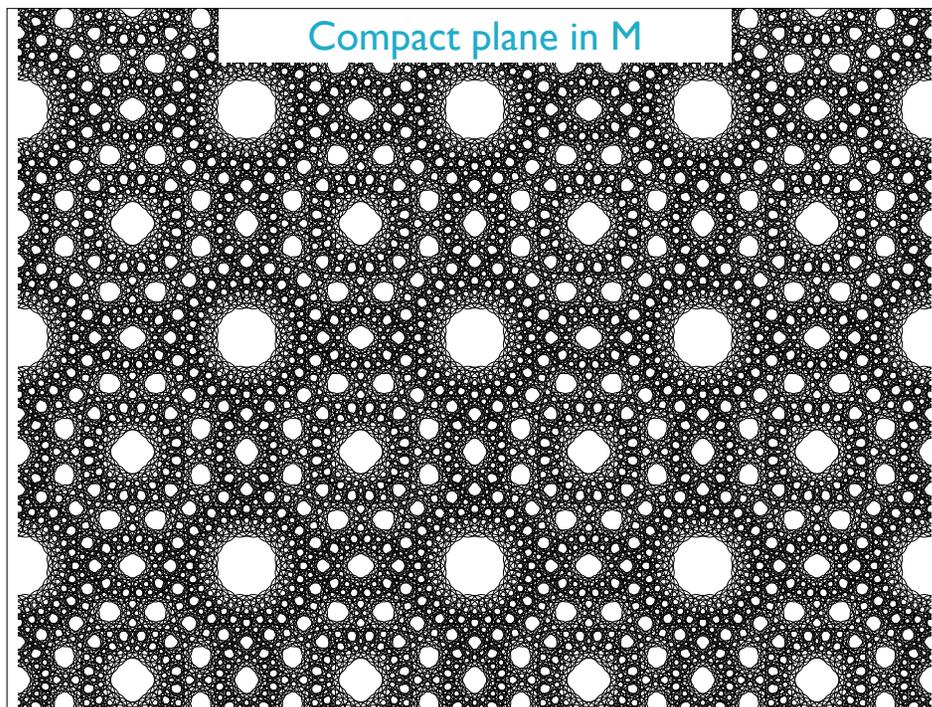
Arithmetic M : $SL_2(\mathbb{Z}[i])$

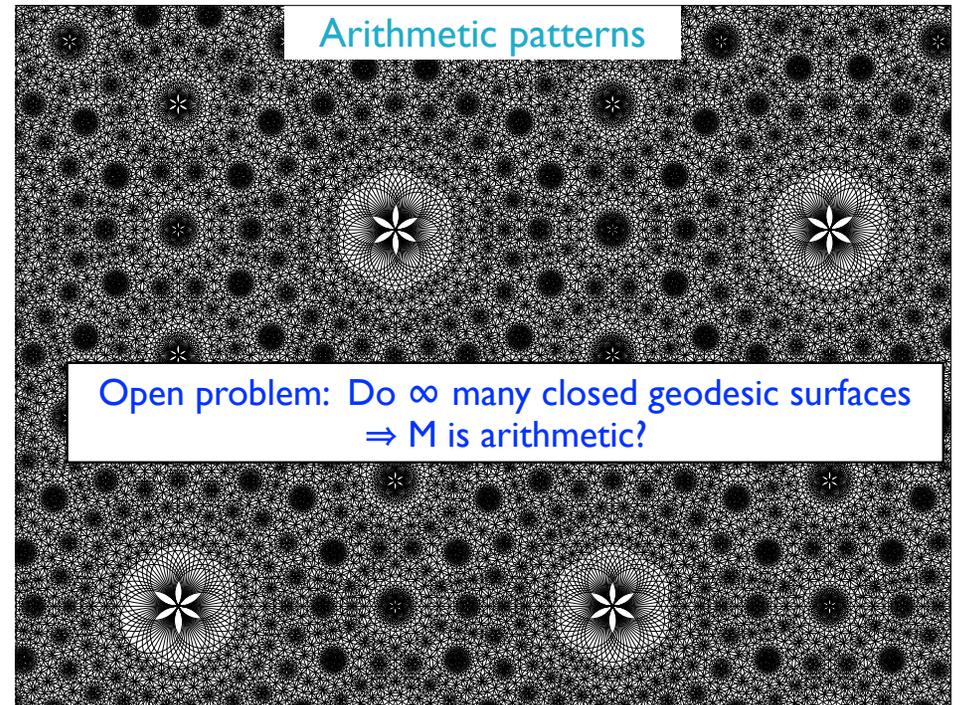
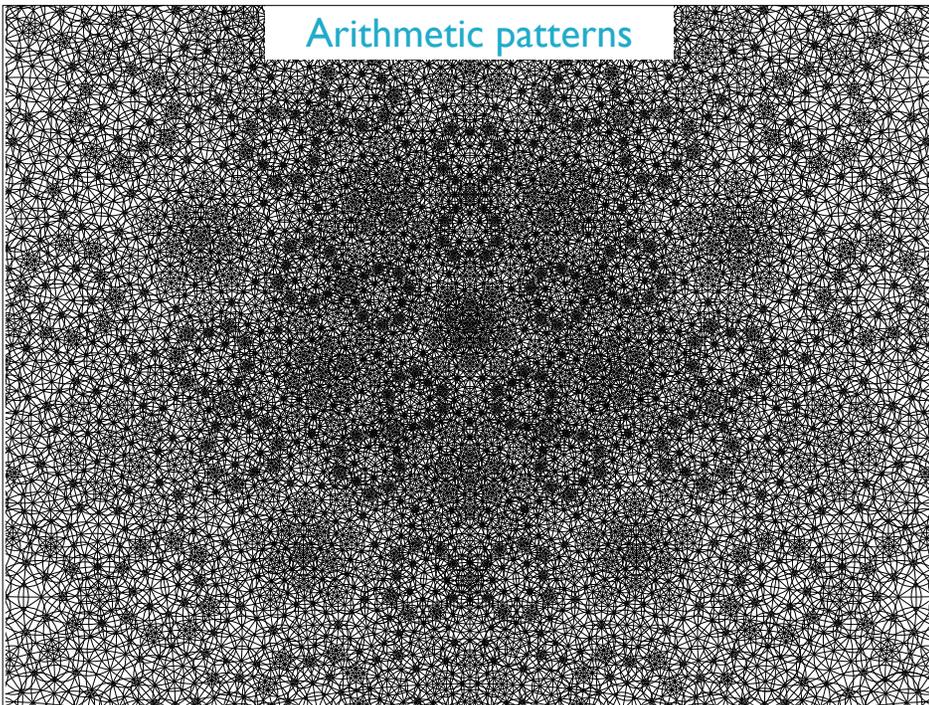
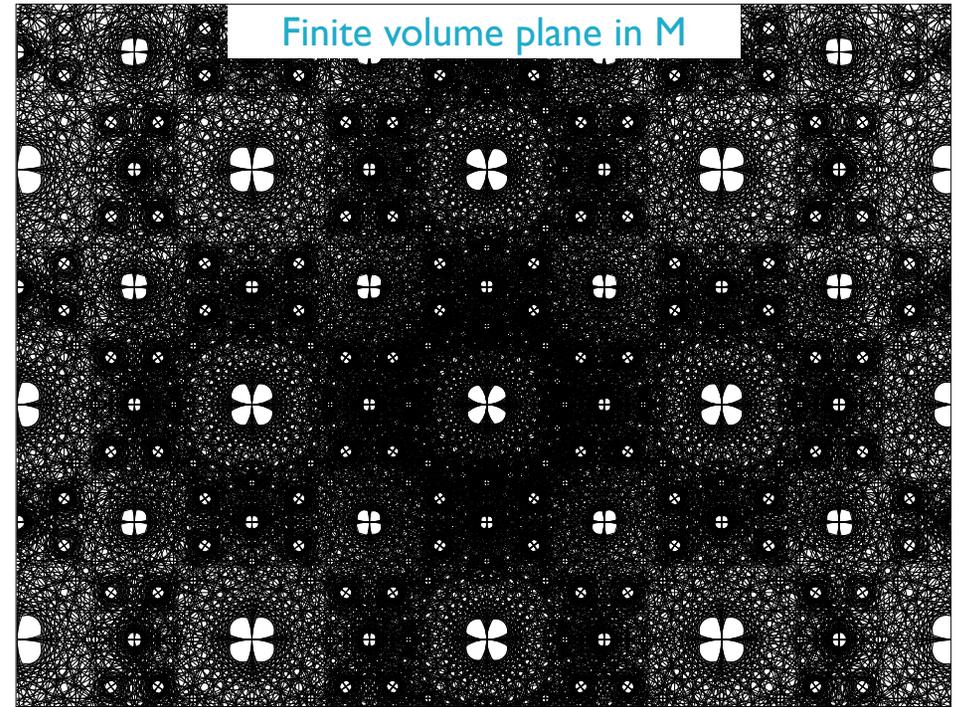
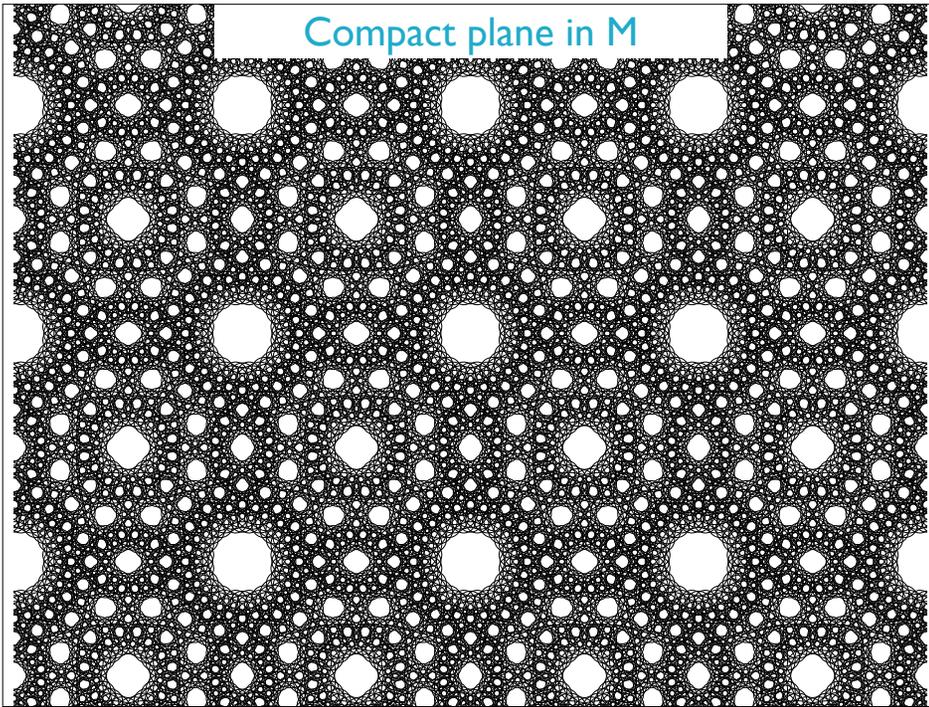


Finite volume M

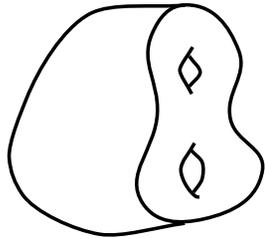


Compact plane in M



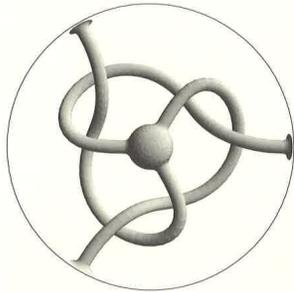


3-manifolds with geodesic boundary

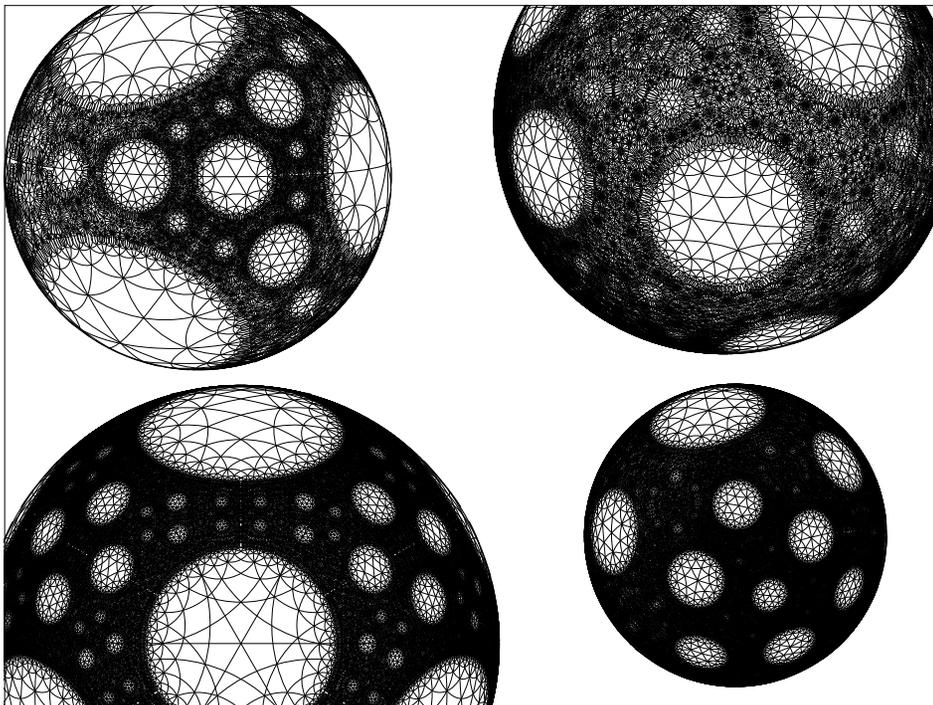
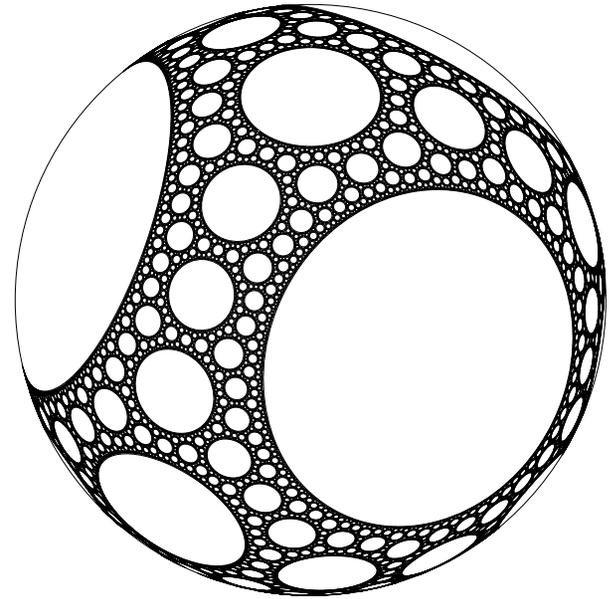


$$M = S^3 - Y$$

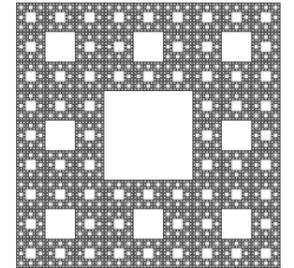
Photograph courtesy of Heilmann Ferguson



3-manifolds with geodesic boundary

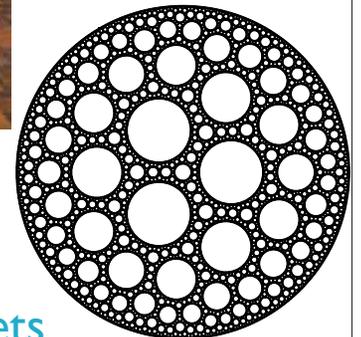


Sierpinski carpets



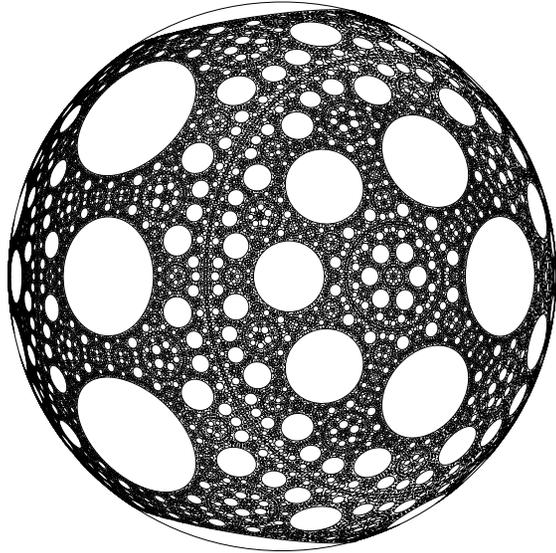
$$\Lambda = S^2 \setminus \bigcup D_i$$

$$\text{diam}(D_i) \rightarrow 0$$



1 dim Cantor sets

M is acylindrical \Leftrightarrow
 Limit set Λ is a Sierpinski carpet



Acylindrical with pleated boundary

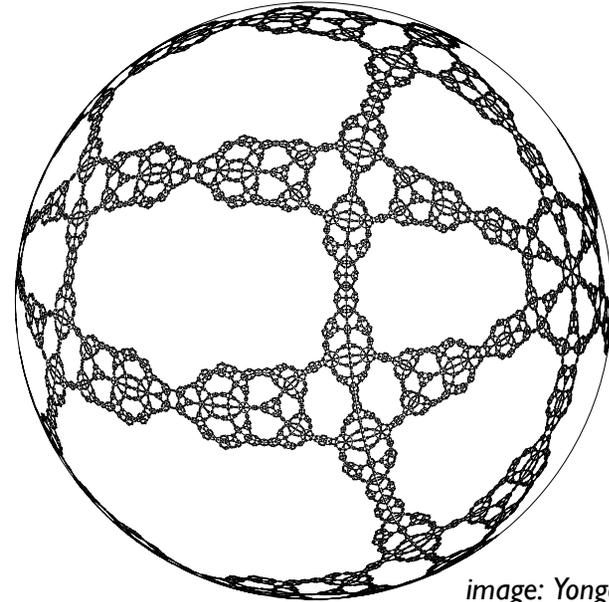
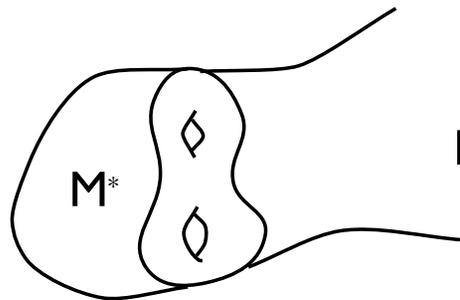


image: Yongquan Zhang

Rigidity Theorem (MMO)



plane =
 $P^* = P \cap M^*$

If M is convex cocompact and acylindrical,
 then any plane P^* in M^* is closed or dense.

Rigidity Theorem (MMO)



plane =
 $P^* = P \cap M$

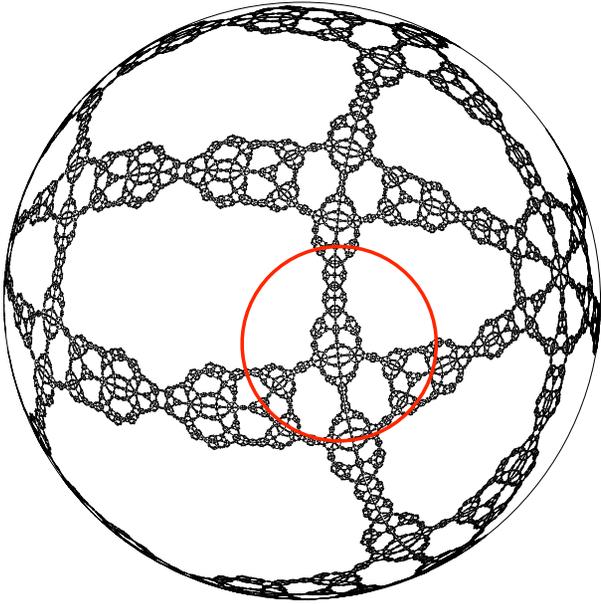
P^* closed \Rightarrow

P^* is a properly immersed surface in M^*

$\pi_1(P^*)$ is nonelementary

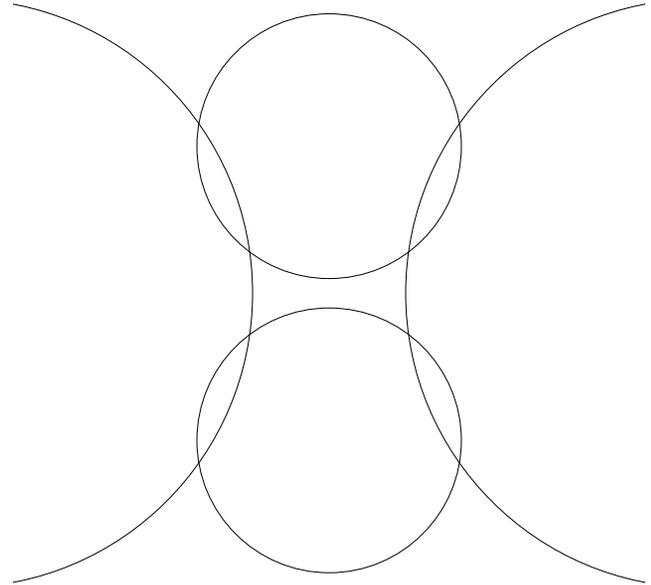
There are at most countably many such planes
 If infinitely many, then $P_i^* \rightarrow M^*$ (equidistribution)

Proof: Slices of Carpets

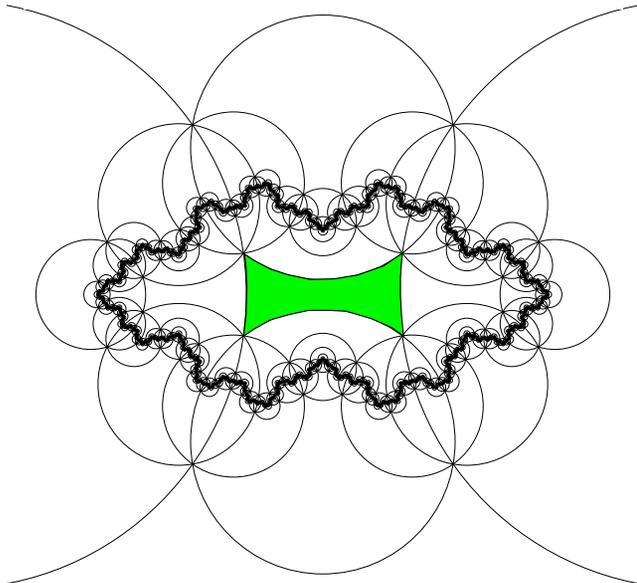


Key:
Thick
carpets have
thick slices

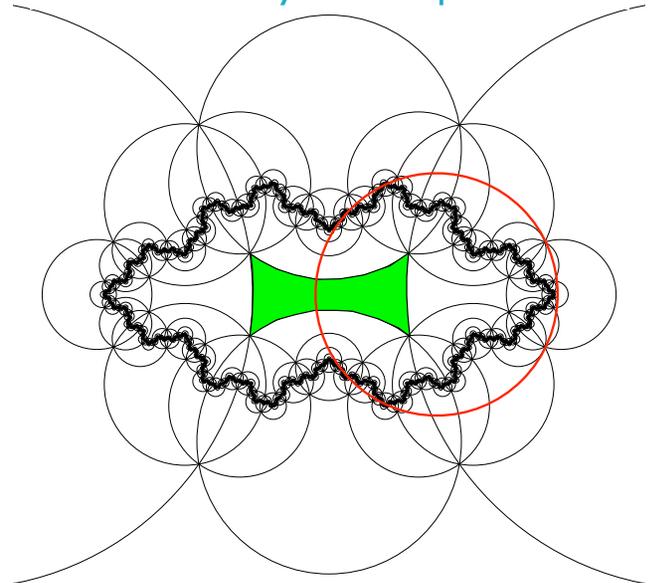
Coda: Surface groups



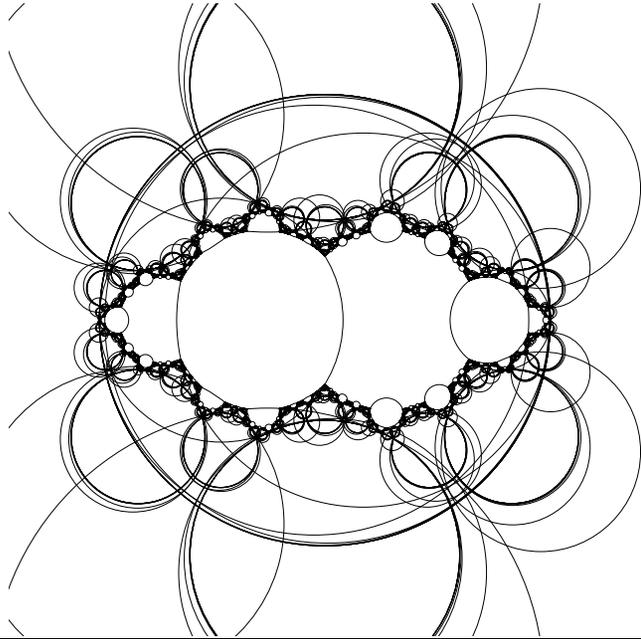
Limit set and fundamental domain



Delicately chosen plane



P^* proper but P not closed!



Q. Even in the case of compact M

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what are the possibilities for
the fundamental group of P ?