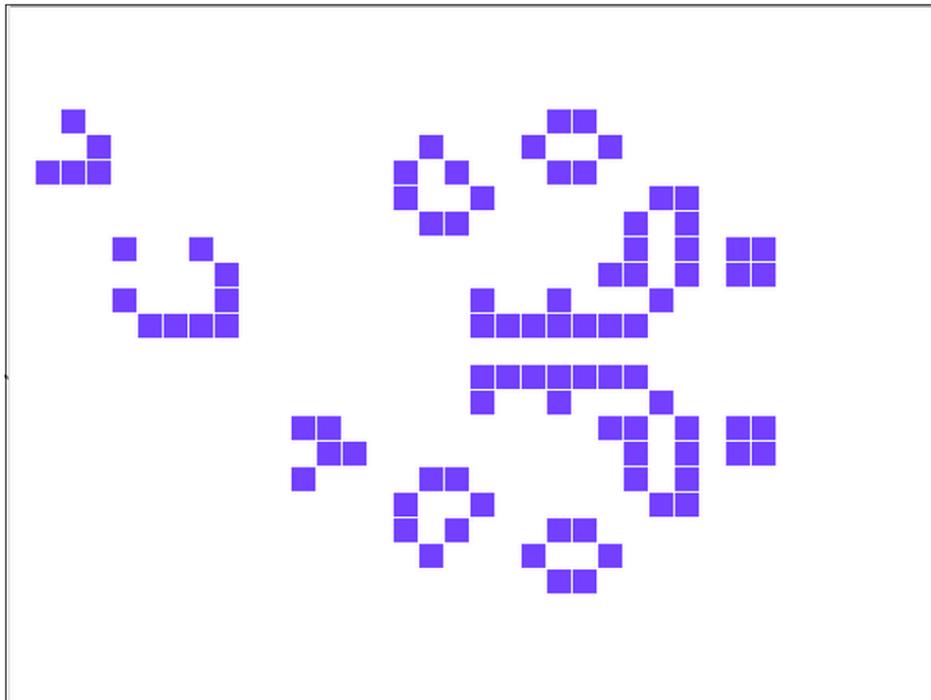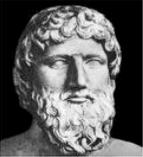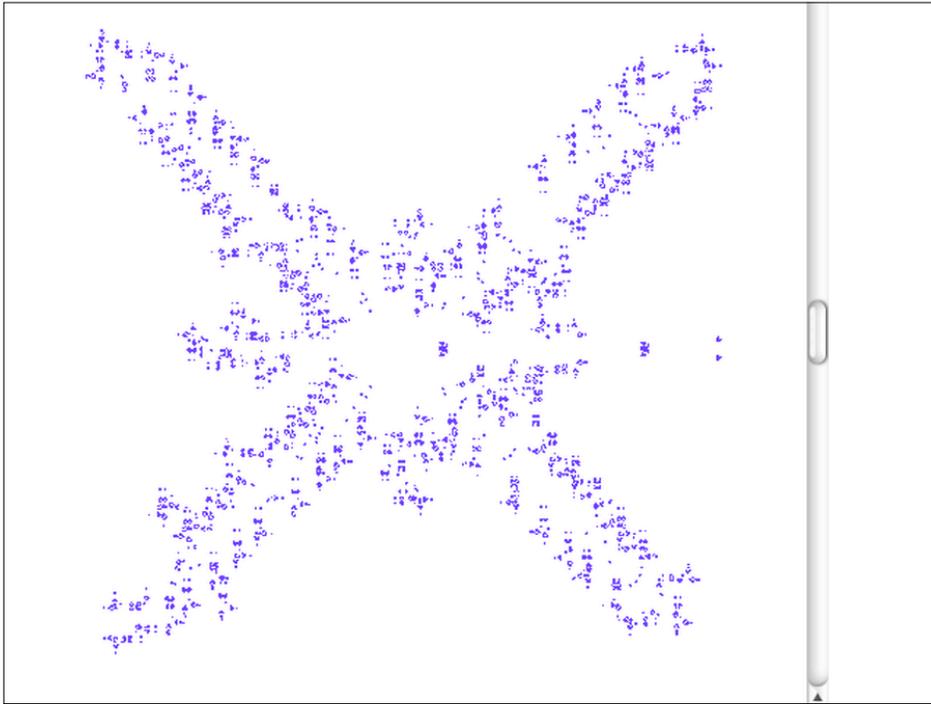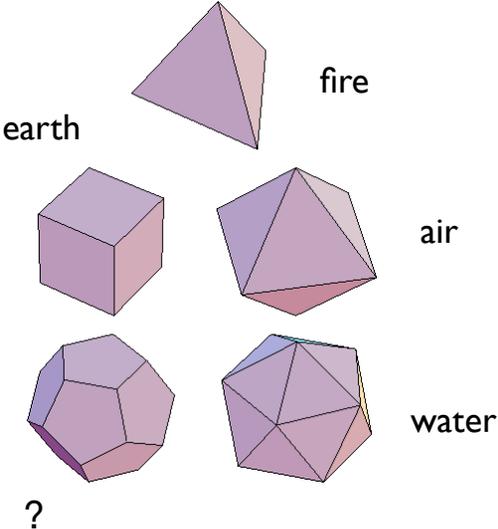# A handful of metaphors

Curtis T McMullen
Harvard University
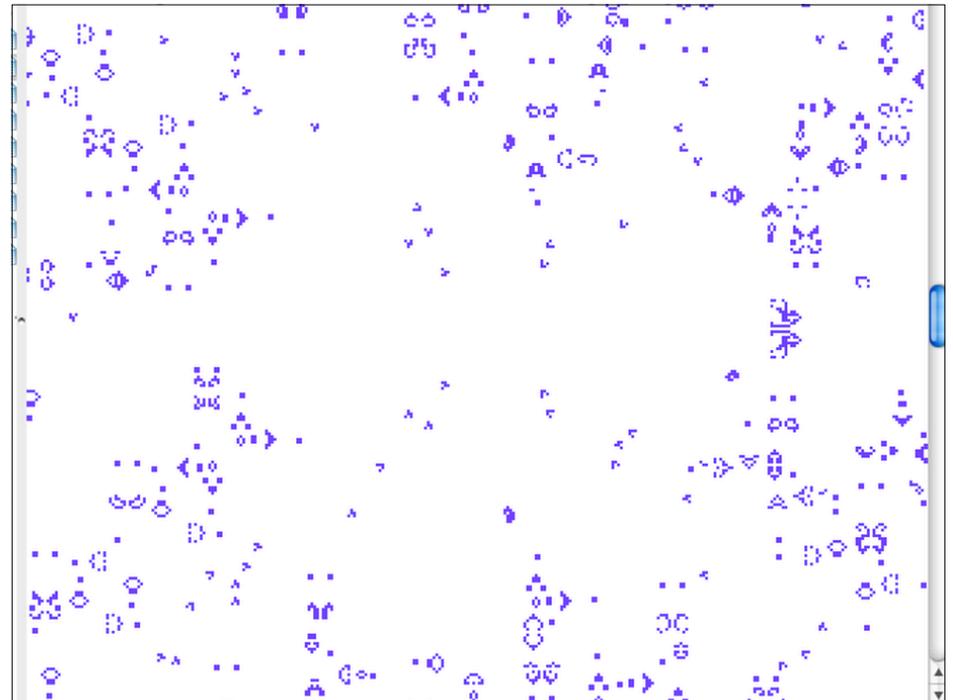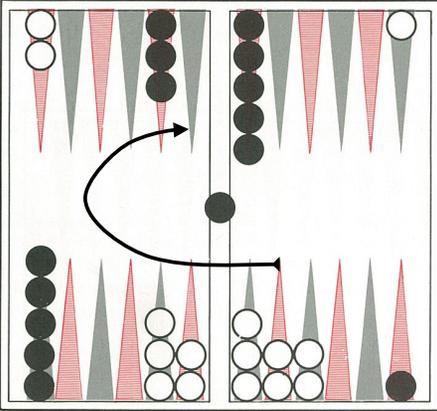
Plato
360 BC

fire

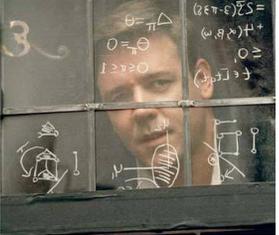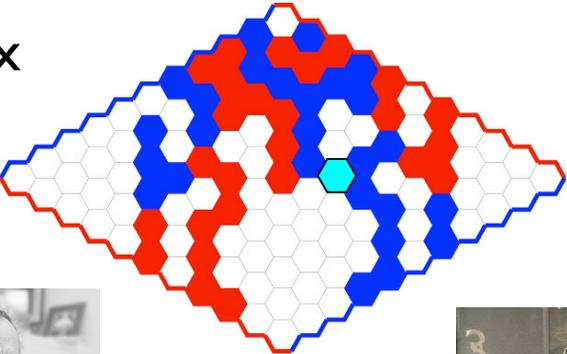earth

air

water

?

A random element enters...

And so I say,
The atoms must a little swerve at times-
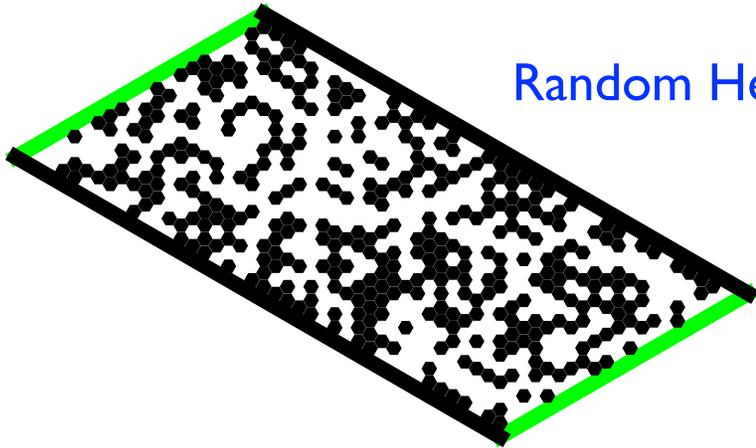
Lucretius, 50 BC
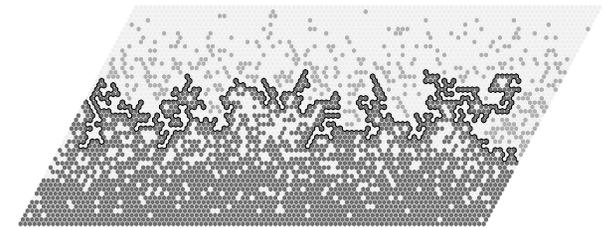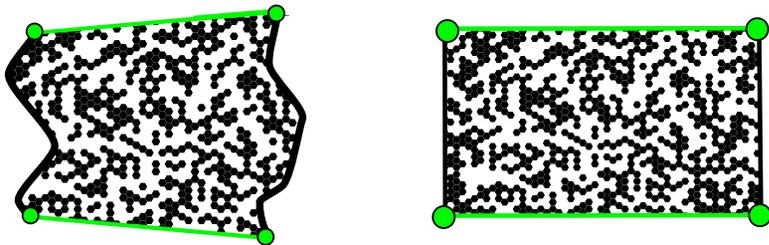
Hex

First player wins

Random Hex

Black or white with equal probability

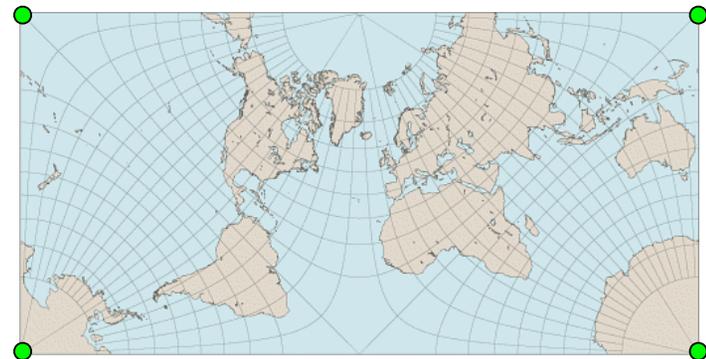What are the chances black wins?

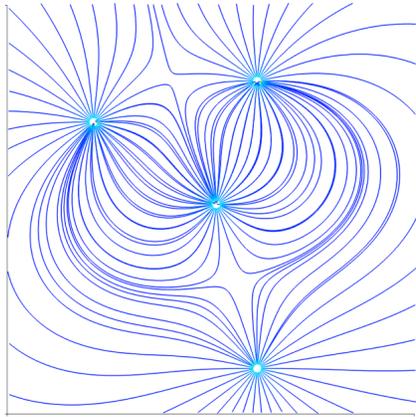# Critical percolation

Conformal Invariance?

The chance that black wins
-- in the continuum limit --
is a conformal invariant.

Riemann mapping theorem

Physics and conformal mappings
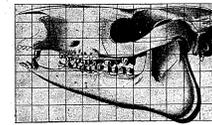

Biological Conformal Mapping

Fig. 538. *Scorpaena* sp. Fig. 524. *Antigonia capros.*

Fig. 538. Skull of *Hyrachyus agrarius*. After Osborn.

differences are very strikingly brought out by the series of conformal transformations which I have represented in Fig. 540. In this case it will perhaps be noticed that the correspondence is not always quite accurate in small details. It could easily be made
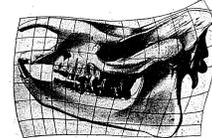
Fig. 539. Skull of *Aceratherium tridactylum*. After Osborn.

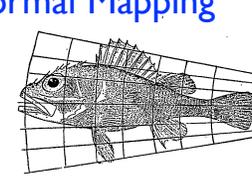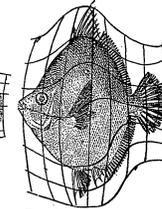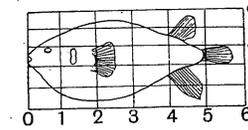Fig. 550. Skull of chimpanzee. Fig. 551. Skull of baboon.

and constitute a simple "projection" of our human skull. The network represented in Fig. 549 constitutes such a projection of

Fig. 548. Human skull.

Fig. 526. *Orthagoriscus.*

Thompson, 1917

Round Boards

Probability of crossing =

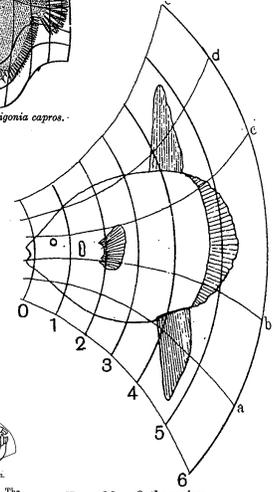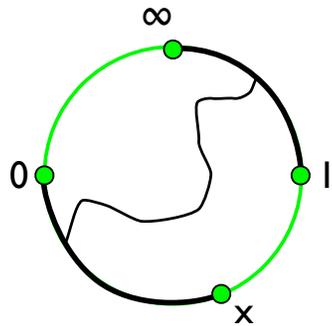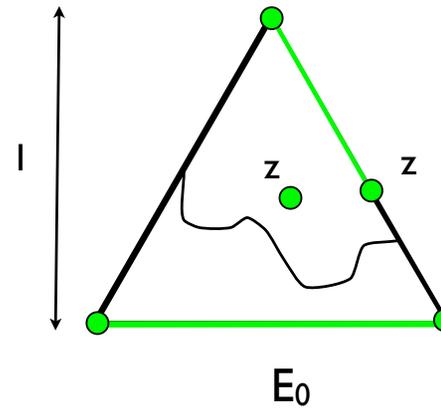$$\frac{3\Gamma(2/3)}{\Gamma(1/3)^2}x^{1/3}\,_2F_1\left(\frac{1}{3},\frac{2}{3},\frac{4}{3};x\right)$$

conformal field theory

Triangular boards

Probability of crossing
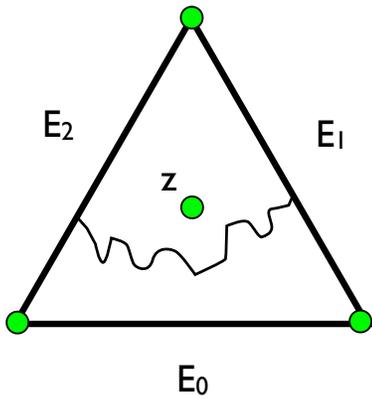
$f_0(z)$ = height of z

E_0

= probability z is blocked from E_0

Smirnov's Idea: Same for z *inside* triangle

Triangular Cauchy-Riemann equations

$$\frac{df_i}{dz} = \omega \frac{df_{i+1}}{dz}$$

$$\Rightarrow f_i(z) = \text{Im}(\omega^i F(z)).$$

boundary conditions $\Rightarrow$ $F(z) = z$

$\Rightarrow$ Cardy's formula and conformal invariance

Other lattices??