Illustrating Infinity

Curtis T McMullen
Harvard University

1961

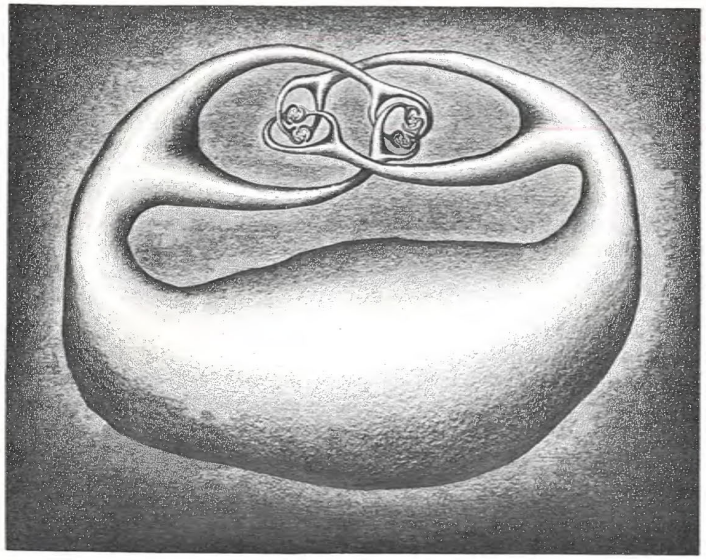
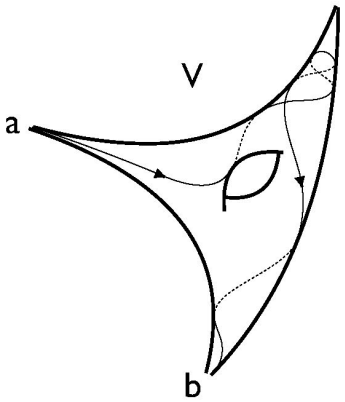
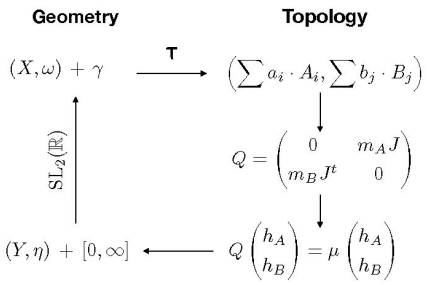
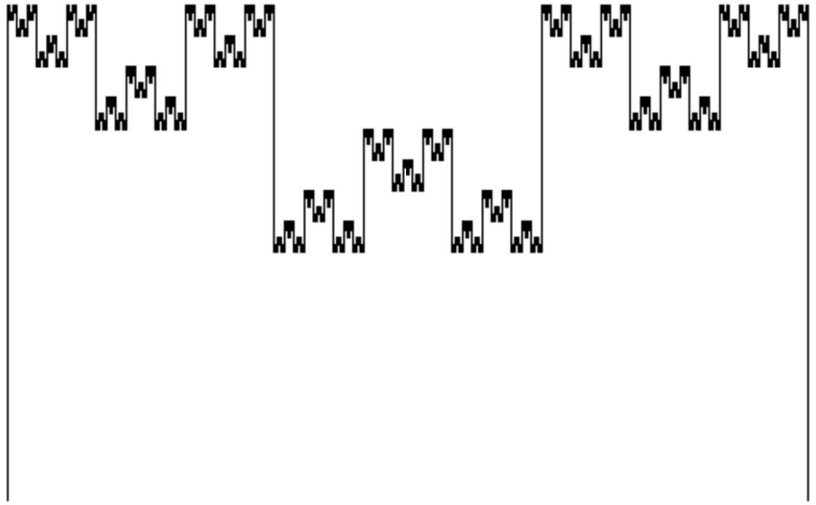


FIG. 4–11. The Alexander horned sphere.

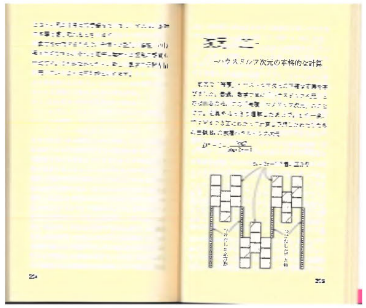


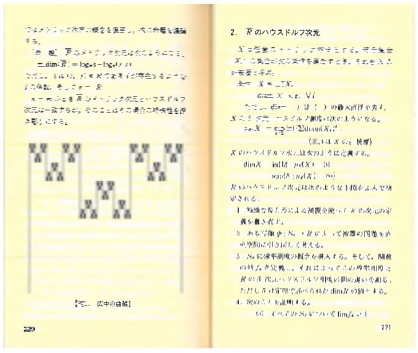


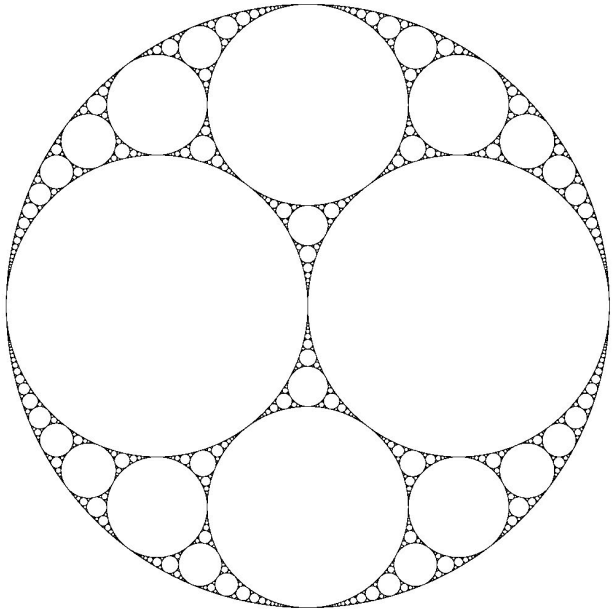
1984



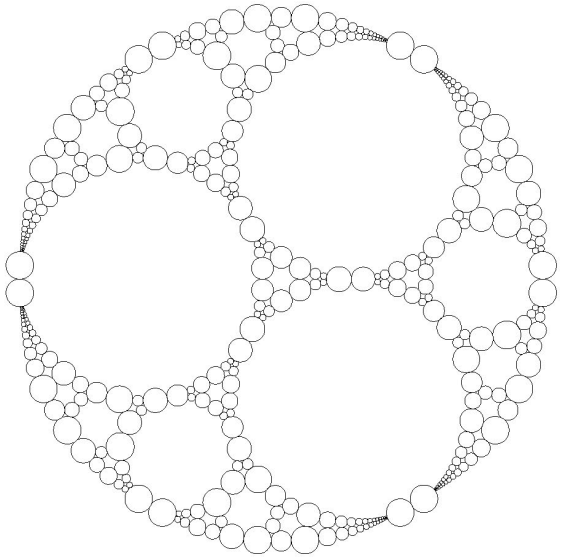
Dimension = 1.34968… = $\log_2(1+2^{\log_3 2})$





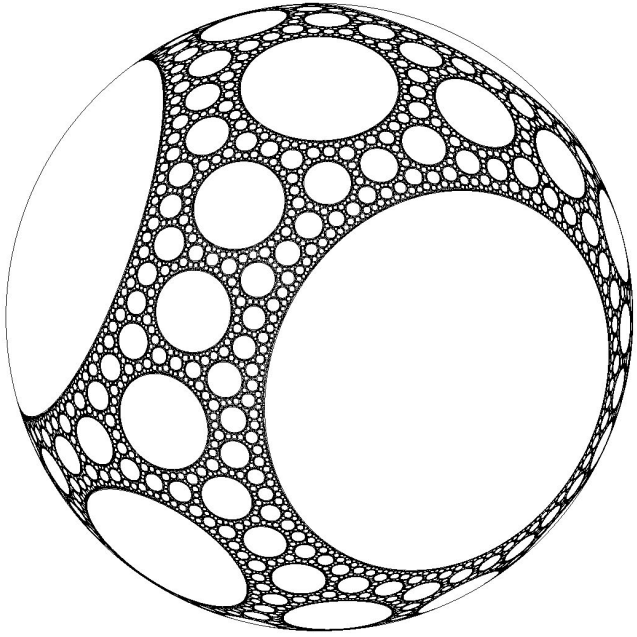


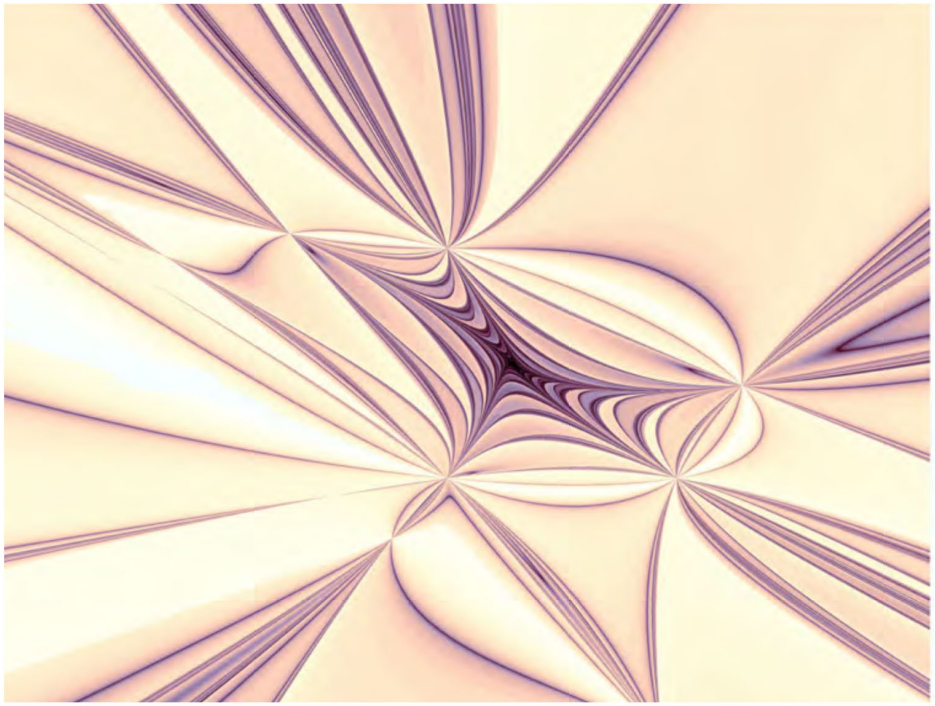
Dimension =
1.305688…

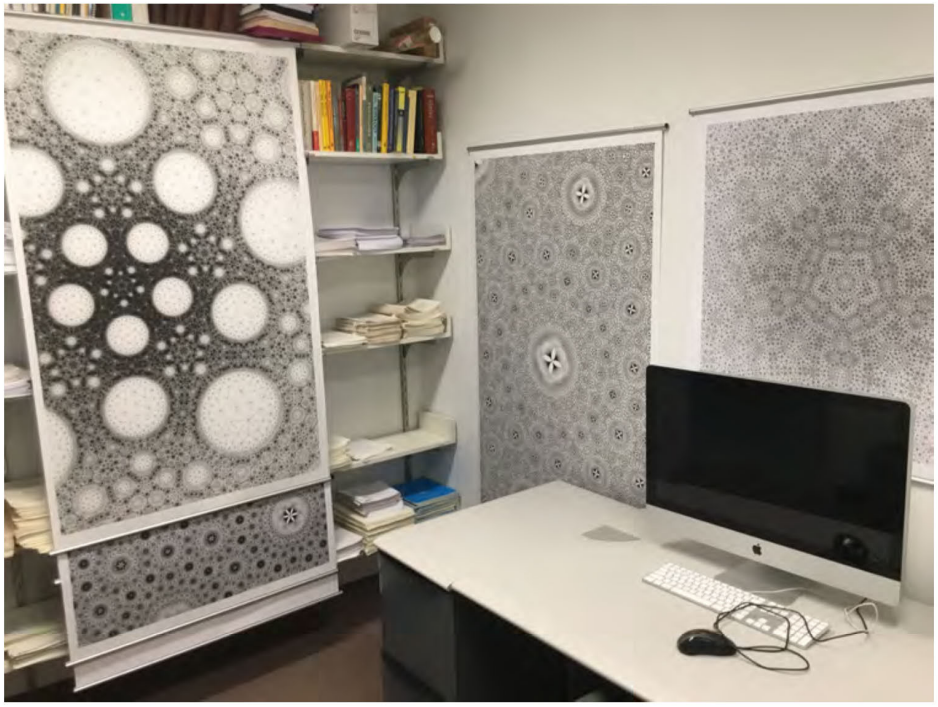


Markov
partition

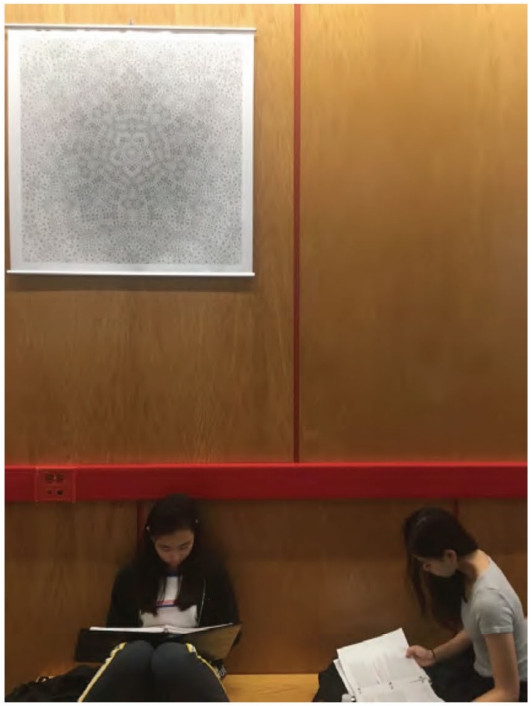
1995

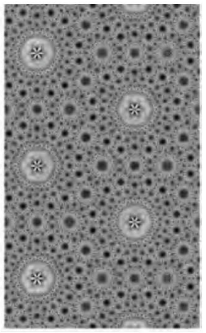


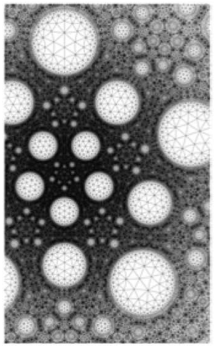


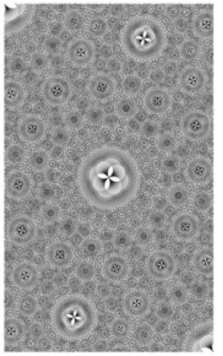


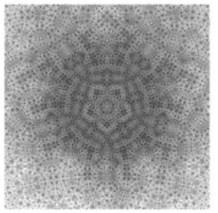

2019

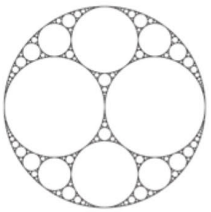


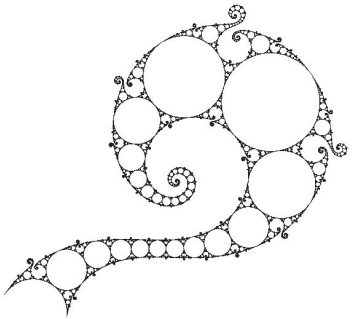


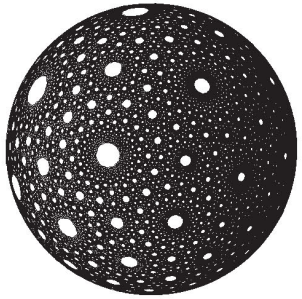


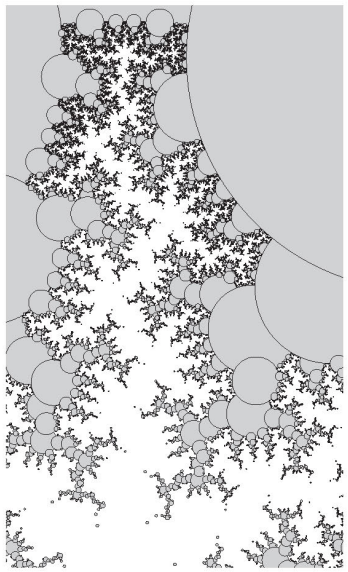






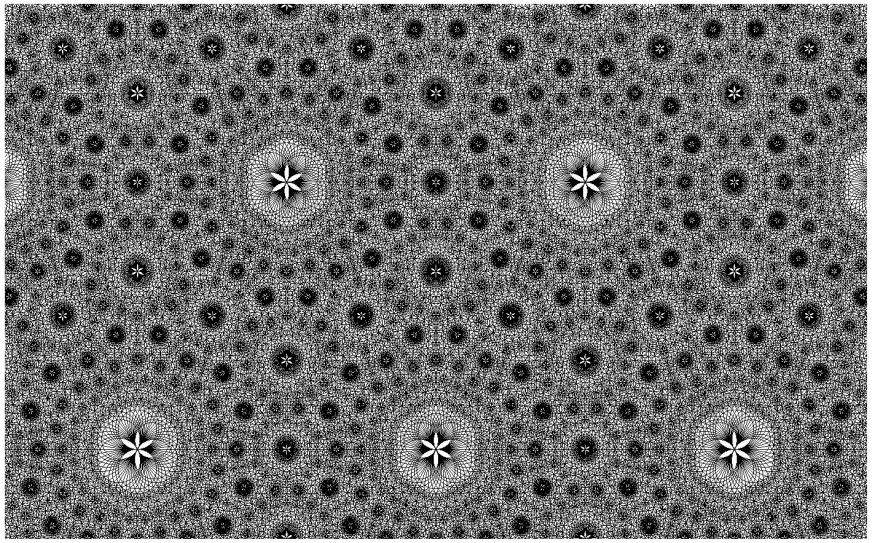


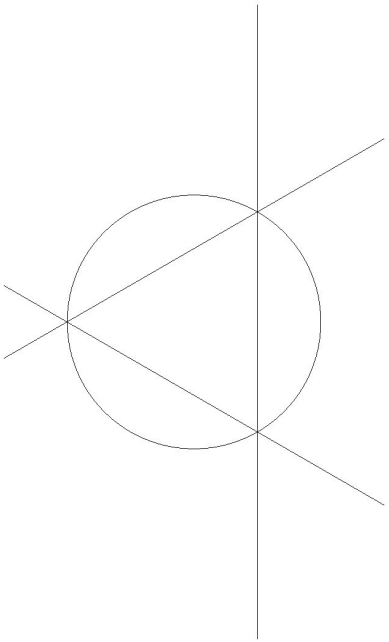


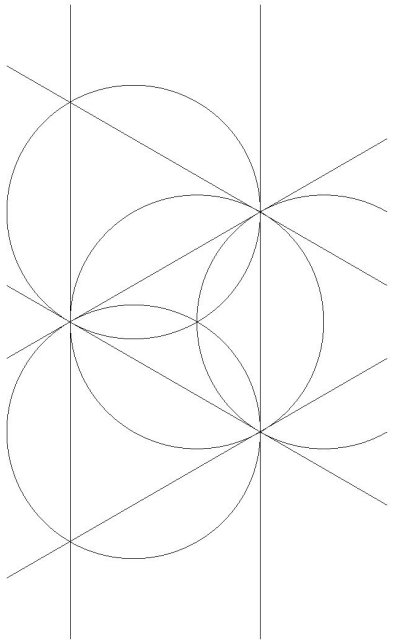


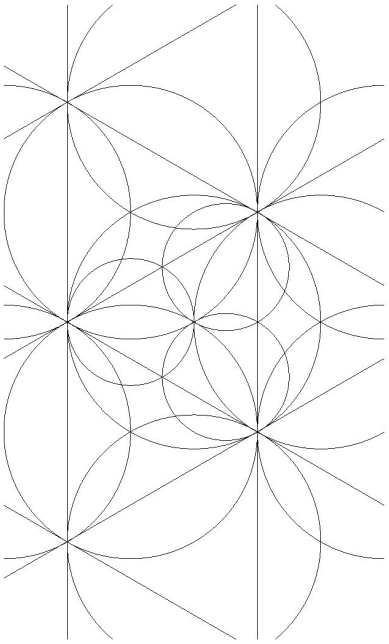
How does it work?

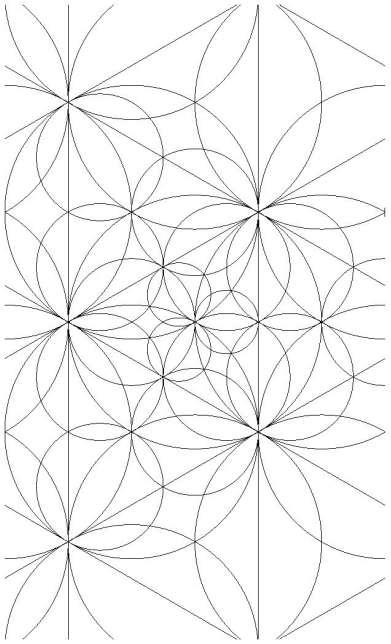
```
#
#  Noncompact arithmetic tetrahedron - SL_2(Z[w])
#
lim -e 0.04 -d 50 -w -3 -5 3 5 <<eof | ps2pdf - > hex.pdf
c 0 0 2
r 0 0 2
c 1 0 -0.5
r 1 0 -0.5
c -0.5 0.86602540378443864676 -0.16666666666666
r -0.5 0.86602540378443864676 -0.16666666666666
c -0.5 -0.86602540378443864676 -0.8333333333333
r -0.5 -0.86602540378443864676 -0.8333333333333
eof

open hex.pdf &
```

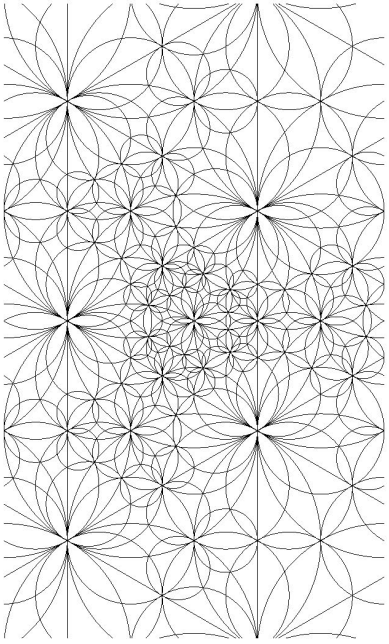


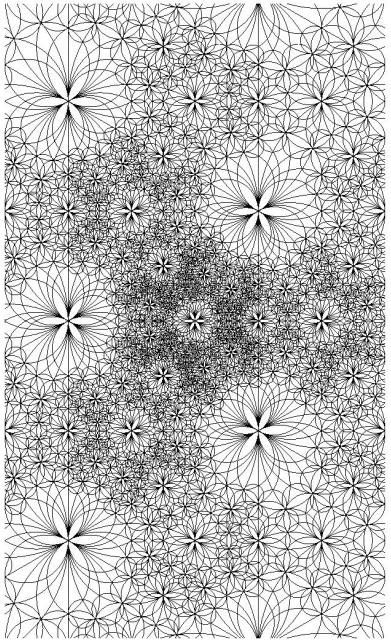


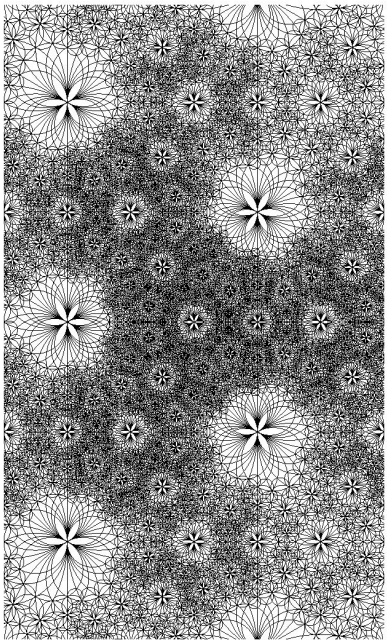


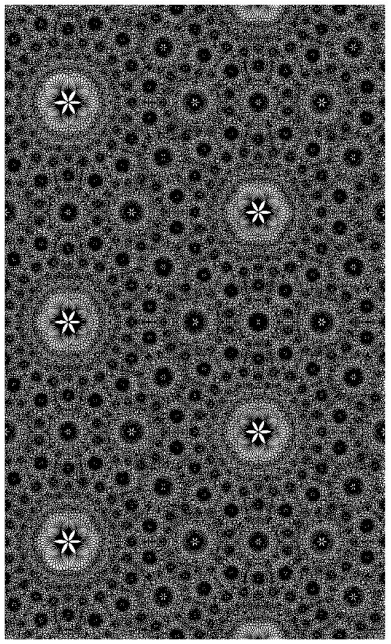




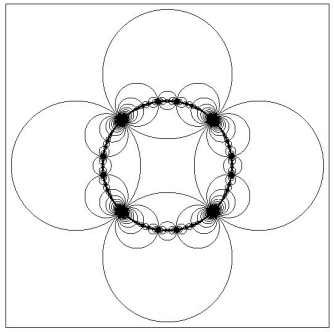


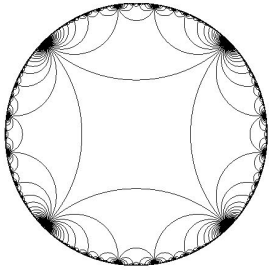


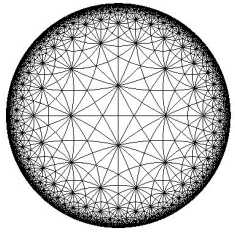


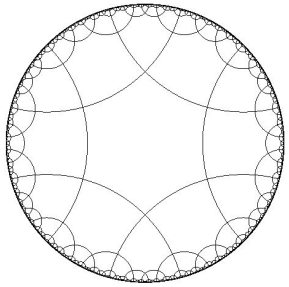


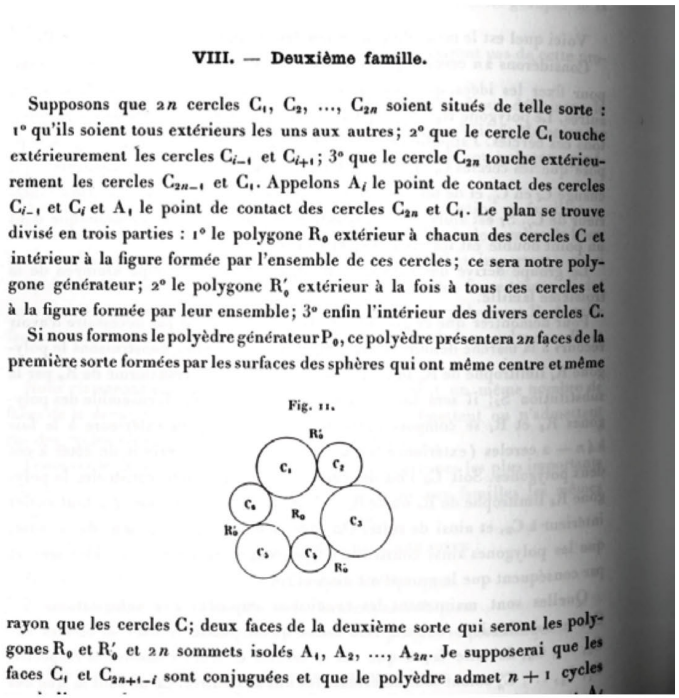
What can it do?











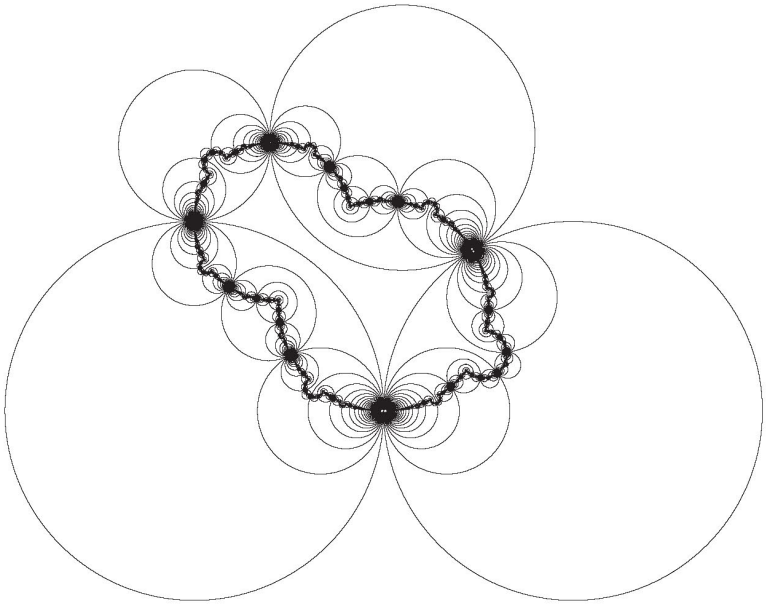
VIII. — Deuxième famille.

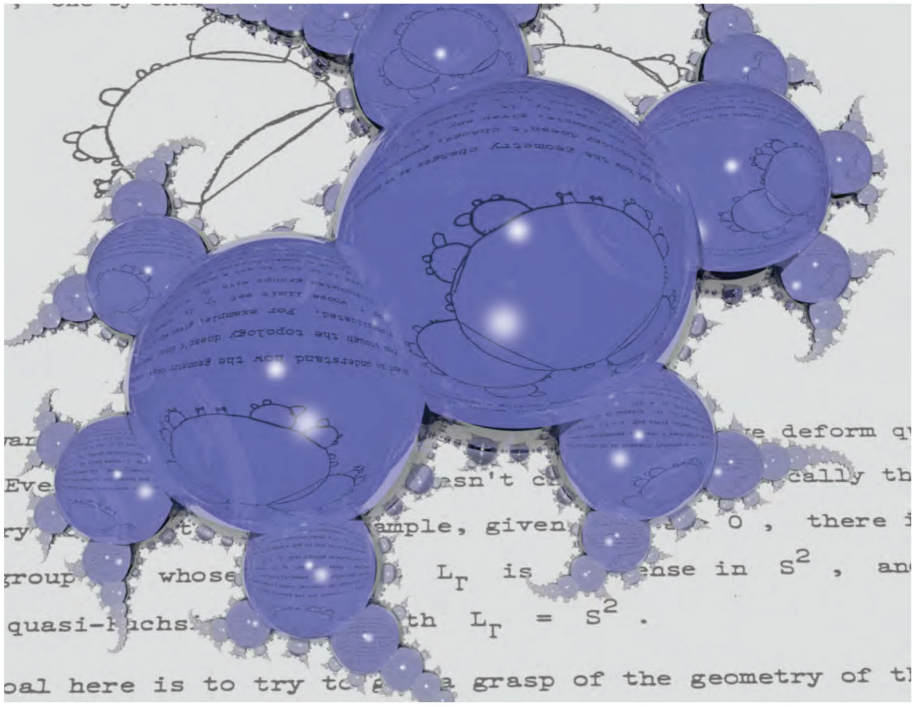
Supposons que $2n$ cercles C_1, C_2, ..., C_{2n} soient situés de telle sorte : 1° qu'ils soient tous extérieurs les uns aux autres; 2° que le cercle C_i touche extérieurement les cercles C_{i-1} et C_{i+1}; 3° que le cercle C_{2n} touche extérieurement les cercles C_{2n-1} et C_1. Appelons A_i le point de contact des cercles C_{i-1} et C_i et A_1 le point de contact des cercles C_{2n} et C_1. Le plan se trouve divisé en trois parties : 1° le polygone R_0 extérieur à chacun des cercles C et intérieur à la figure formée par l'ensemble de ces cercles; ce sera notre polygone générateur; 2° le polygone R'_0 extérieur à la fois à tous ces cercles et à la figure formée par leur ensemble; 3° enfin l'intérieur des divers cercles C.

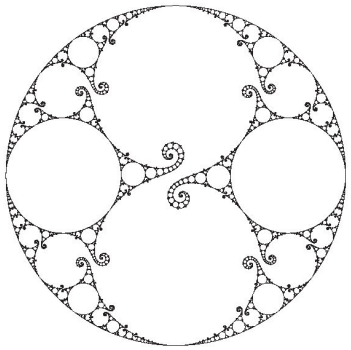
Si nous formons le polyèdre générateur P_0, ce polyèdre présentera $2n$ faces de la première sorte formées par les surfaces des sphères qui ont même centre et même

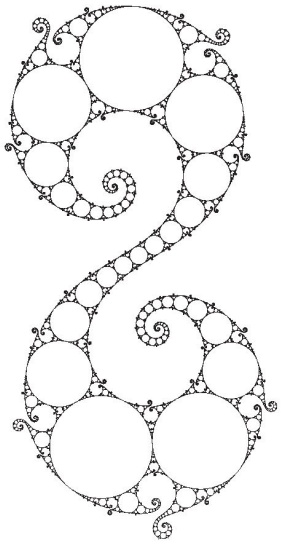
Fig. 11.

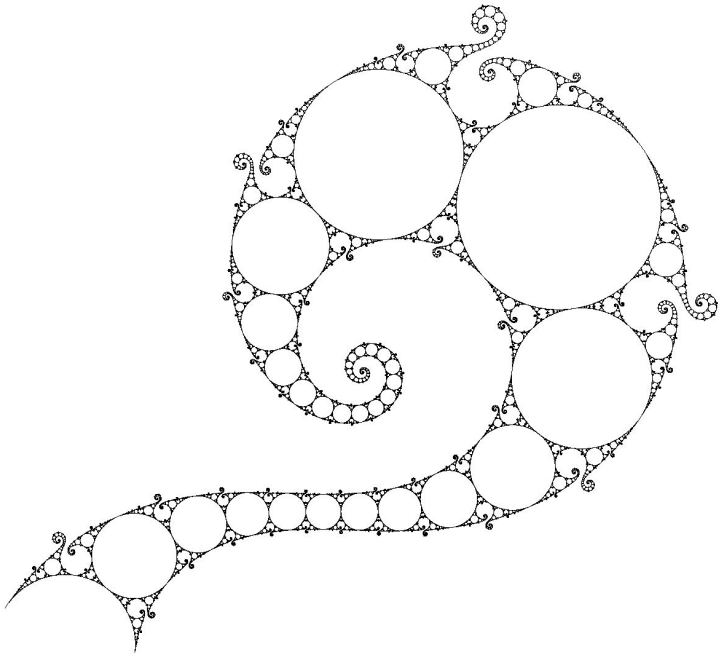
rayon que les cercles C; deux faces de la deuxième sorte qui seront les polygones R_0 et R'_0 et $2n$ sommets isolés A_1, A_2, ..., A_{2n}. Je supposerai que les faces C_i et C_{2n+1-i} sont conjuguées et que le polyèdre admet $n+1$ cycles

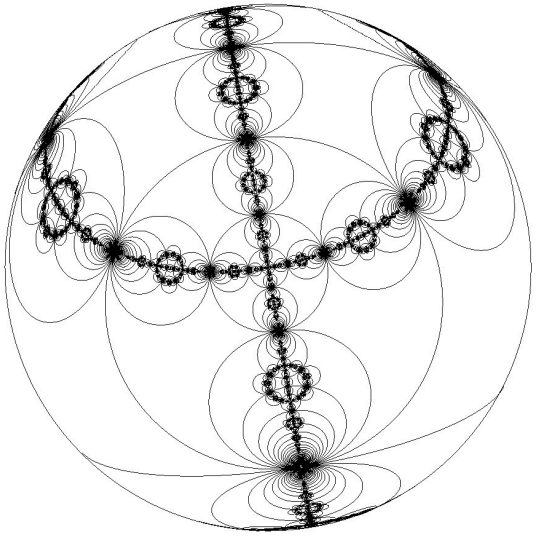


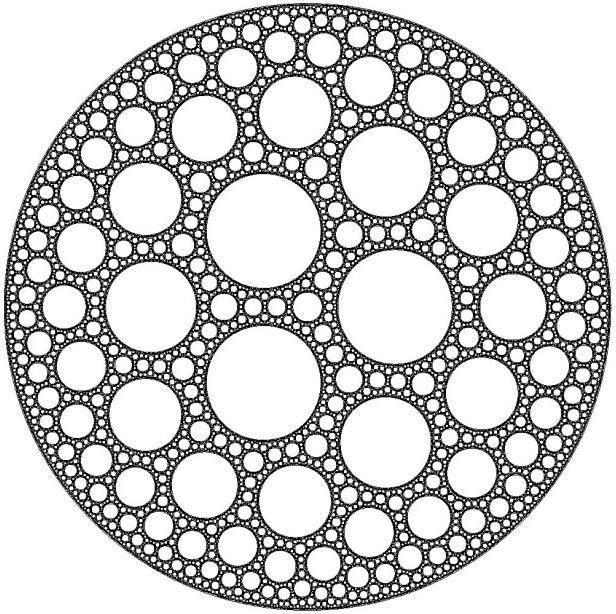


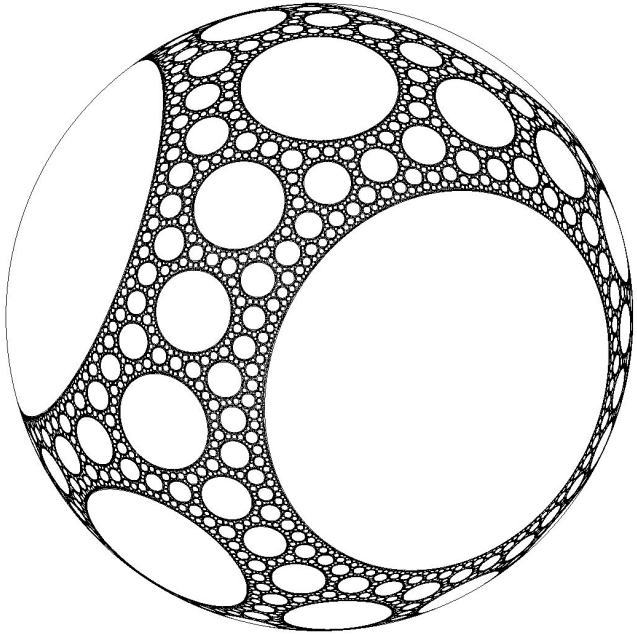


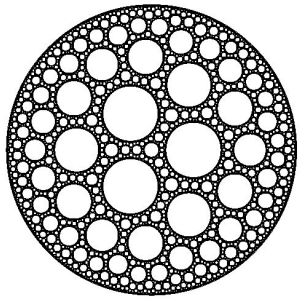


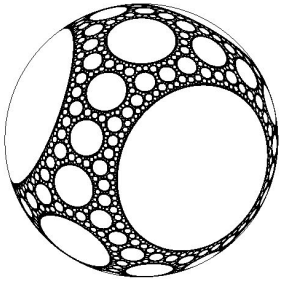


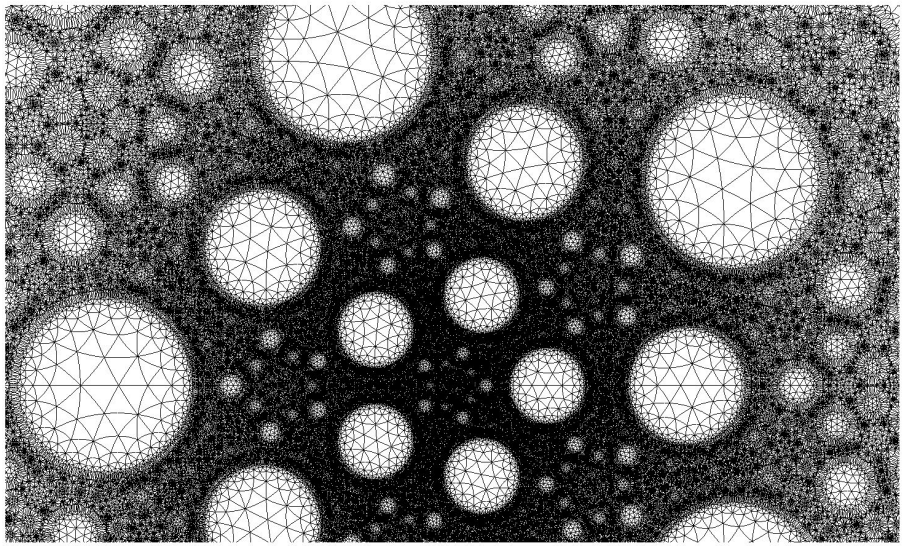


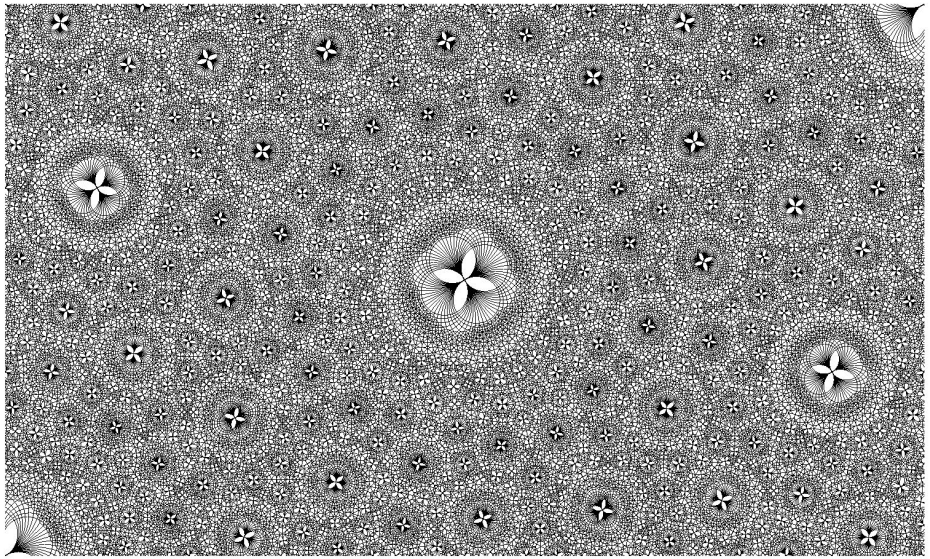


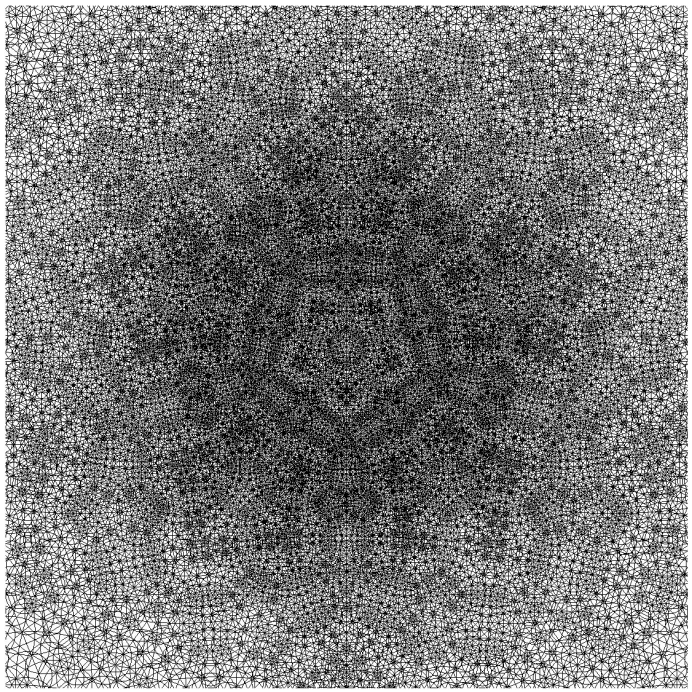


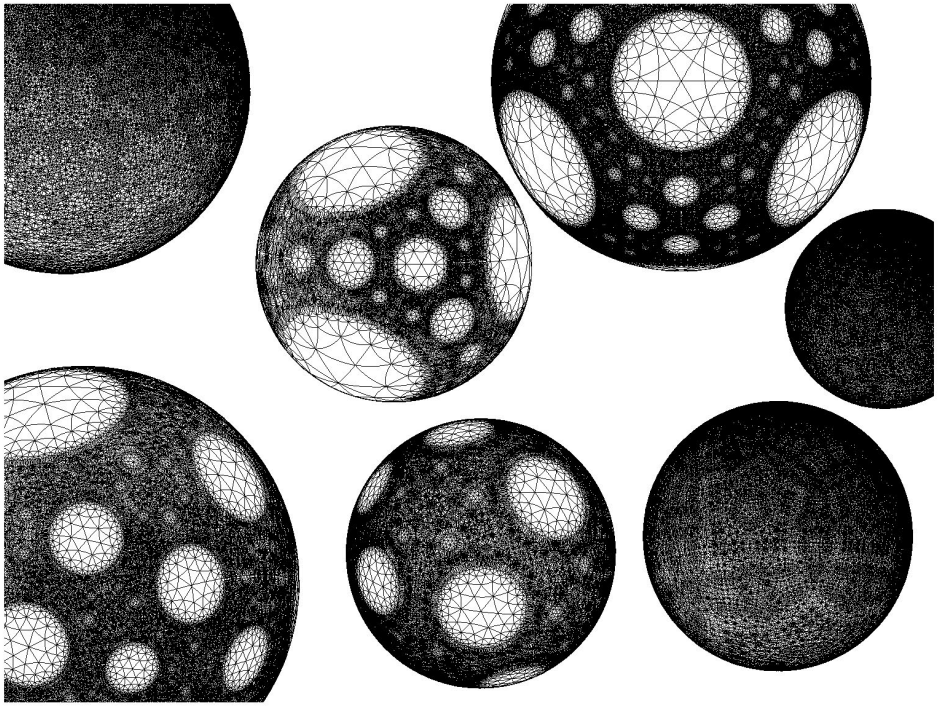


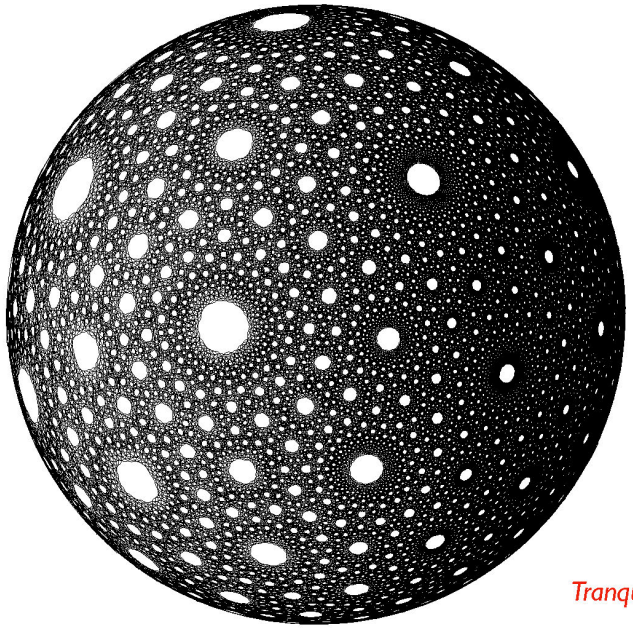












Tranquility base