Homework 10
Real Analysis
Math 212a – Harvard University – Fall 1998
Due Monday, 30 November 1998

Royden: Chapter 8: 49, 52.

   Chapter 9: 8 (assume X, Y are Hausdorff), 23, 41, 47, 48, 50(b).

1. The *Hilbert cube* $X \subset \ell^2(\mathbb{N})$ consists of all sequences such that $|a_n| \leq 1/n$. Prove that any $f \in C(X)$ can be approximated by a function $g(a_1, \ldots, a_m)$ that depends on only finitely many coordinates.

2. Show the same result fails if we replace X by the closed unit ball in $\ell^2(\mathbb{N})$.

3. Let X be an infinite-dimensional Banach space. Show that X does not have a countable basis as a real vector space.