Homework 4
Real Analysis
Math 212a – Harvard University – Fall 1998
Due Friday, 16 October 1998

Royden:
Chapter 3, Problem 31 (Lusin's theorem).
Chapter 4, Problems 9, 14(b), 16, 19.

1. (a) Let R be an integral domain such that $x^2 = x$ for all $x \in R$. Prove that R is isomorphic to $\mathbb{Z}/2$.

   (b) Prove that any prime ideal in $(\mathbb{Z}/2)^X$ is maximal.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be (Lebesgue) integrable. Prove that

$$h(t) = \int |f(x) - f(x+t)| \, dx$$

is a continuous function of t. In particular, $h(t) \to 0$ as $t \to 0$.