Homework 7
Real Analysis
Math 212a – Harvard University – Fall 1998
Due Friday, 6 November 1998

Royden:
Chapter 6: 2, 8, 12, 16, 17, 23.

1. Given $f \in L^p[a,b]$, $1 \le p < \infty$, let $M(t) = m\{x : |f(x)| > t\}$. Show that
$$\int_a^b |f|^p = \int_0^\infty pt^{p-1} M(t)\, dt.$$

2. For $1 \le p, q \le \infty$, consider the ℓ^p and ℓ^q norms on $\mathbb{R}^n$, and calculate
$$\sup_{x \neq 0} \frac{\|x\|_p}{\|x\|_q}.$$

3. Construct a continuous path $p : [0,1] \to L^2[0,1]$ that makes a right-angle turn at every moment. That is, construct p so that $\langle p(t) - p(a), p(t) - p(b) \rangle = 0$ for any $a < t < b$. Here $\langle f, g \rangle = \int_0^1 f(x)g(x)\, dx$.