

Homework 9

Real Analysis

Math 212a – Harvard University – Fall 1998

Due Friday, 20 November 1998

Royden: Chapter 7: 48.

Chapter 8: 16, 23, 42, 45, 53.

Chapter 9: 2 (assume K_1 is Hausdorff), 16.

1. Let $X = \{0, 1\}^{\mathbb{R}}$ with the product topology, and let $p \in X$ be the point with all coordinates zero. Is there a continuous function $f : X \rightarrow \mathbb{R}$ such that $f^{-1}(0) = \{p\}$?
2. Let \mathcal{F} be the set of all continuous convex functions $f(x)$ on $[-1, 1]$ with $|f(x)| \leq 1$. Show that $\mathcal{F}|_{[-r, r]}$ is compact in $C[-r, r]$ for any $r < 1$, but \mathcal{F} is not compact in $C[-1, 1]$.