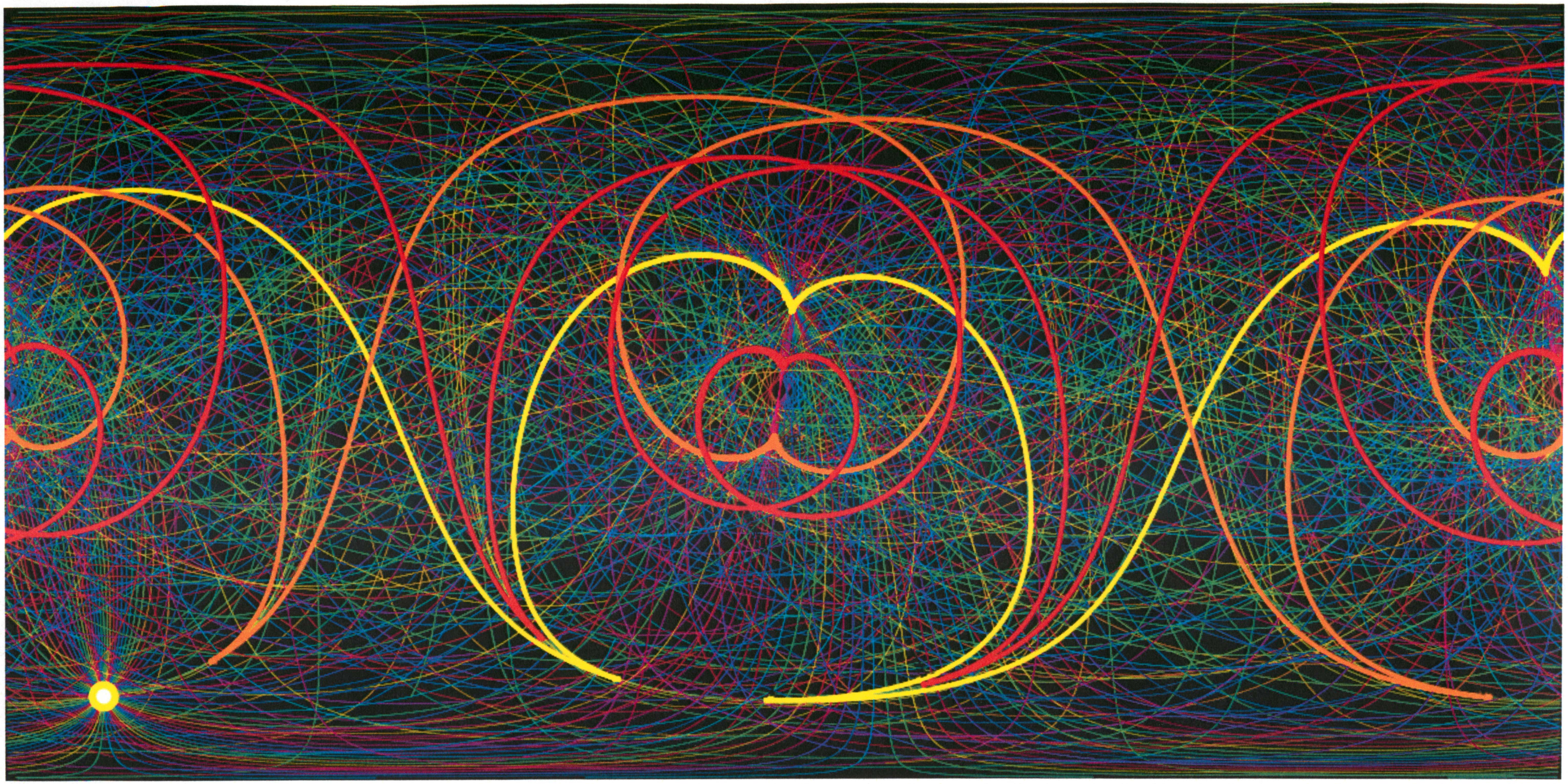
HCRP, 2009

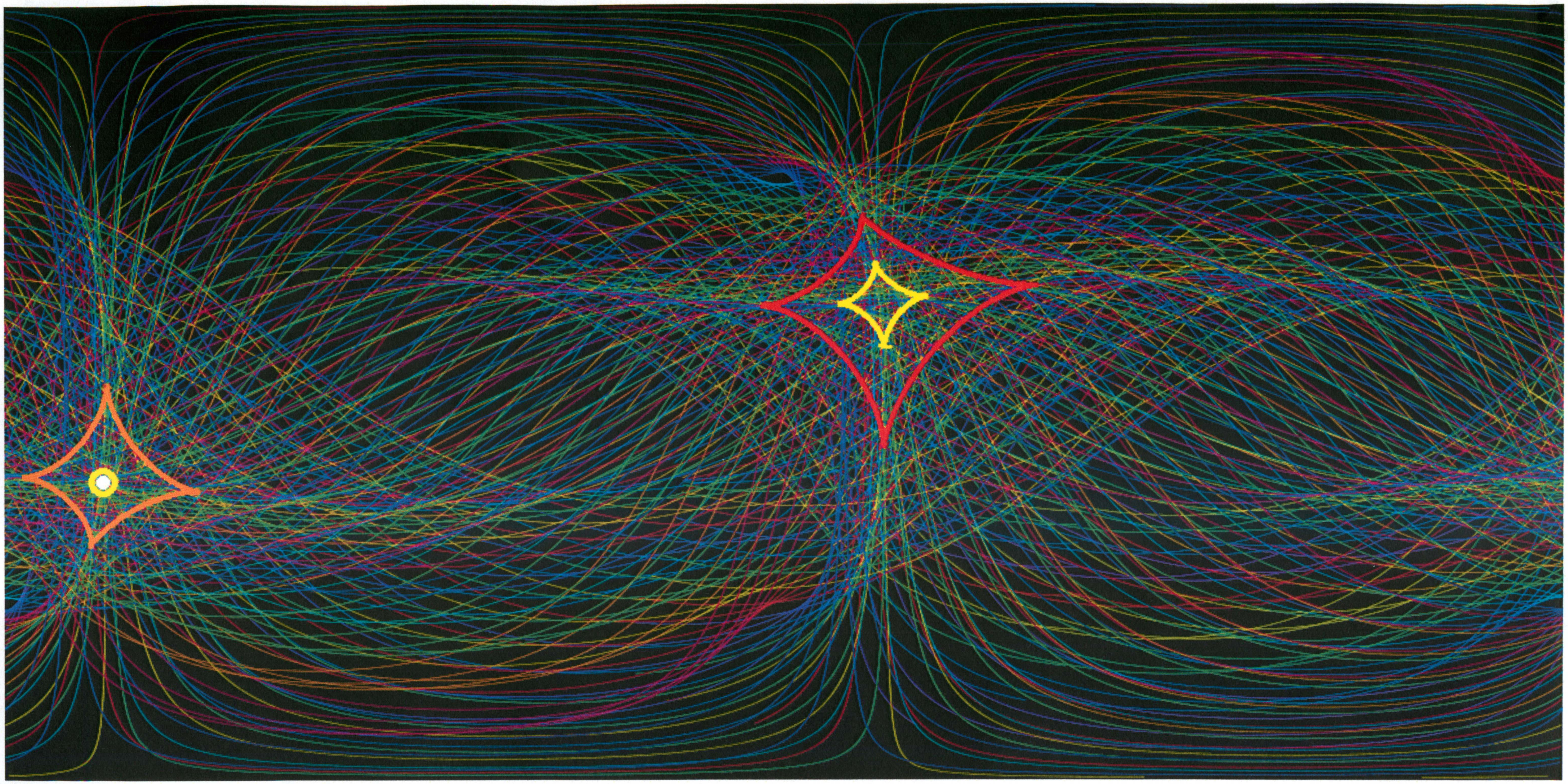
Michael Teodorescu

Jacobi last geometric statement



The exponential map on a spheroid $x^2/4+y^2+z^2=1$. The primary caustic (the first root of the Jacobi field f starting at the initial point) is drawn in yellow, the secondary caustic (the second root of f(g(t)) orange, the ternary red. One clearly sees the 4 cusps as the still unproven Jacobi's last geometric statement claims. 100 geodesics of the 6000 computed geodesics have been drawn. The picture was computed by solving the geodesic equations g''k = -G(i,j,k) g'i g'j (where G is the connection using Einstein notation) in conjunction with the Gauss-Jacobi equation f'= - K(g(t)) f (where K is the curvature of the surface) numerically with Mathematica. Since special needs are required (identifications of the map, assuring that we stay on the energy surface, checking whether the Jacobi field f reaches zero), the differential equations were "hand" integrated using Runge-Kutta and not using built-in DSolve routines. 6000 geodesics g(t) were computed on the ellipsoid and drawn in the spherical coordinate plane with (theta,phi) coordinate

Jacobi last geometric statement

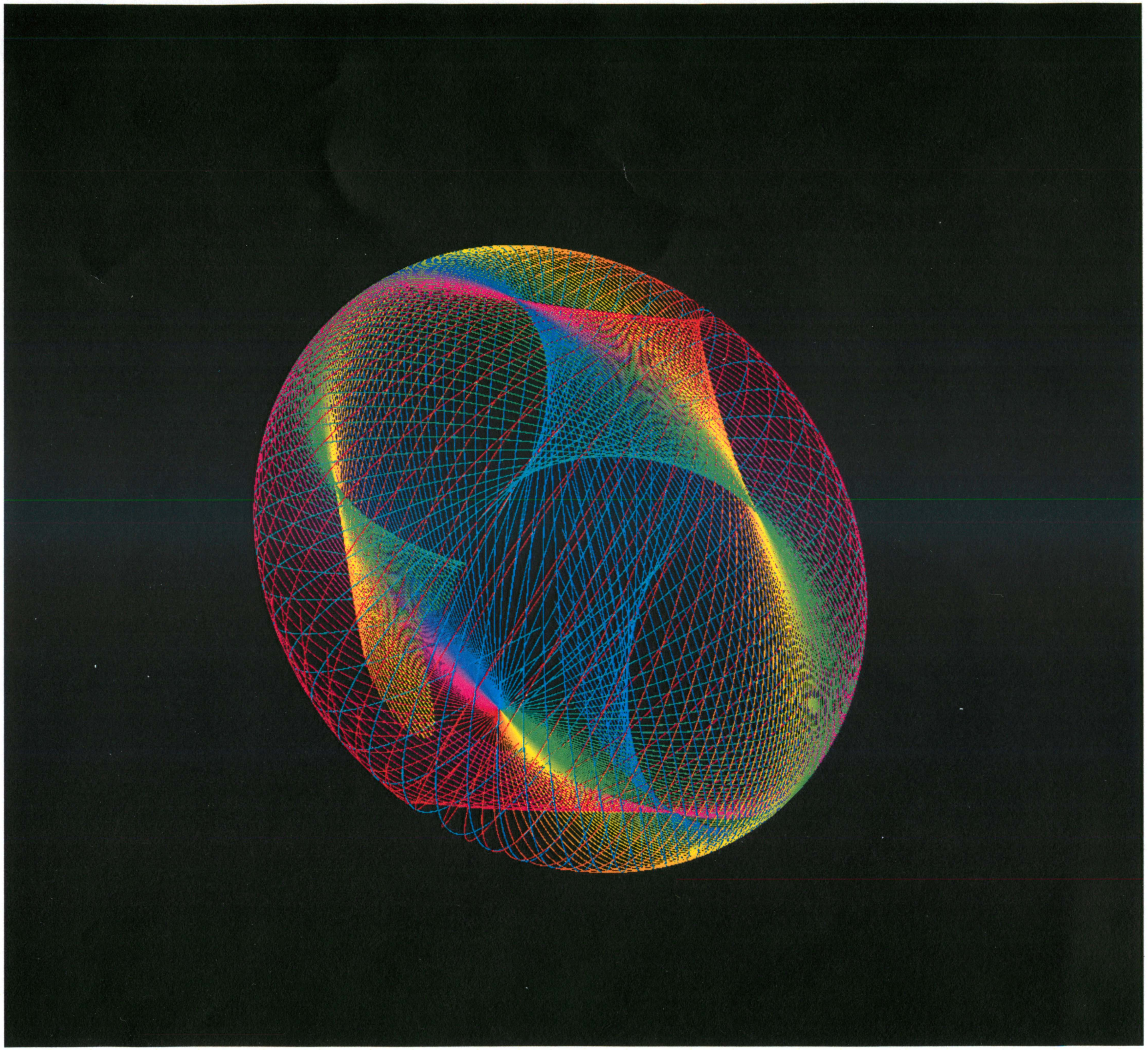


The non-rotationally symmetric ellipsoid

$$x^2/1.12 + y^2/1.062 + z^2 = 1$$

has caustics close to the antipode in the sphere case. We see again the primary, secondary and ternary caustic. 4000 geodesics

Jacobi last geometric statement



Geodesics on a non-rotationally symmetric ellipsoid

May 8, 2009