TOWARDS A TOPOLOGICAL PROOF OF THE FOUR COLOR THEOREM XI

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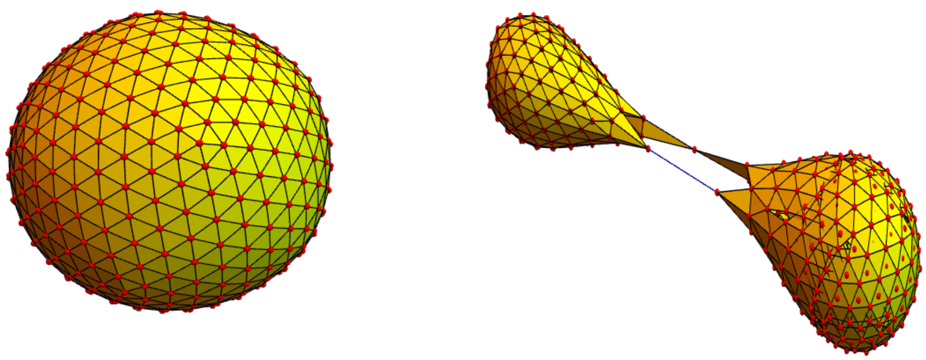
Abstract. We look in more detail at the dual cut graph for $G \in \mathcal{S}_2$. Two vertices are connected in this graph if they are two vertices $\{a, b\}$ such that they are the dual of an edge.



The **dual cut graph** G' of a two-dimensional graph $G = (V, E)$ possibly with boundary (like a sphere, disc or annulus) is the graph whose vertex set is V and whose edges consist of pairs (x, y) such that $\{x, y\}$ is the intersection of the spheres $S(e_1) \cap S(e_2)$, where $e = (e_1, e_2) \in E$.

Question: For which graphs G is G' connected? If it is not connected, can we always make it connected with one cut?
All we need is that we can connect any two odd degree vertices after making some subconnections Proof: given an odd degree vertex x, the

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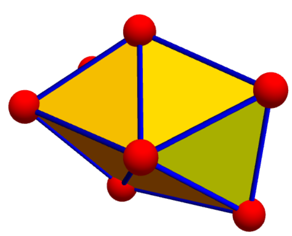
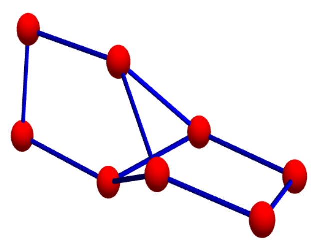
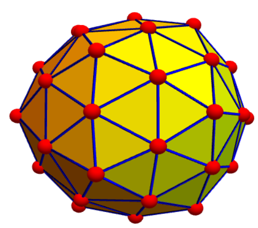
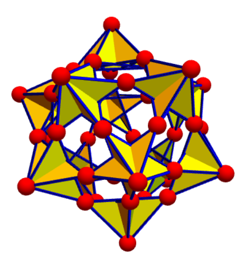
sphere $S(x)$ in G' is again a closed circle. After two subdivisions, we can connect it to the center. Now we can connect to every vertex in distance 2.
An important observation is that if we make a refinement $G \to H$, then this produces a larger graph $H' > G'$ which produces more connections between the old G'. Especially, if G' is connected, then H' is connected.

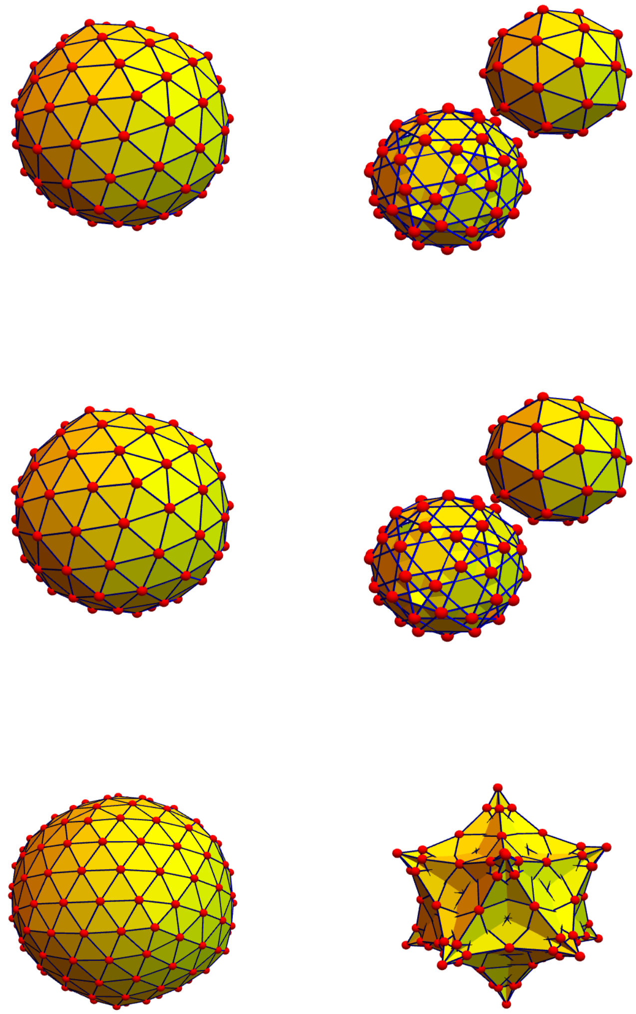
The dual graph does not have to be connected as the octahedron already shows. But also non-Eulerian graphs can lead to non-connected graphs. This is no problem however as we can make a refinement to get connectedness.











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