TOWARDS A TOPOLOGICAL PROOF OF THE FOUR COLOR THEOREM XV

OLIVER KNILL

Abstract. We were stuck as the previous setup could lead to situations where we can not continue. This was due to the realization that we can not always cut connections from V to T as such connections can be part of an already cleaned out sphere. And we had concrete examples where we could not continue. The solution is very easy and also will simplify the programming: first connect T and V, then leave it connected. It also makes the topological setup easier as the complement of the cleaned out region is from then on always contractible once the cleaned inner sphere V and the outer sphere T are connected.

Recall notation: a disc is a graph in $\mathcal{B}_2$. Its boundary is a circular graph $\in \mathcal{S}_1$. The interior is the set of vertices x which have a circular unit sphere $S(x)$.

The edge dual graph of a disc D is denoted by D'. If D is part of a sphere and the complement D^c of the disc is Eulerian, there is a refinement such that the odd degree vertex are only at the boundary. We now need only to work on such discs as we use a new idea: recall that the original graph G which is a two sphere bounding a three dimensional ball B. We start with cleaning out the most inner central sphere to have one vertex for which all edges have even degree. Our cutting game can not cut any edges on the boundary T.

After cleaning the most inner sphere, first connect V with T, The first time we make contact with T, we do not have to worry about not cutting $V - T$ connections. From now on, always keep V and T connected. The part which needs to be completed remains then contractible for the rest of the time, the algorithm runs.

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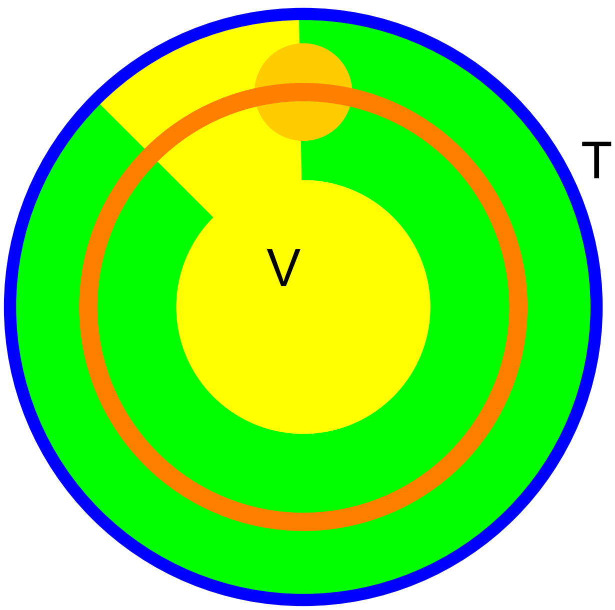
Like this, we do not have to worry about not being able to cut connections from V to T.

This simplifies the story. It had bothered us that there were cases of discs D with a triangular “island” T and cleaned out boundary V for which we can not refine the interior if we are not allowed to cut connections from T to V. The reason why this could happen is that the connections from T to V partition the disc into several disc. While in the case of one disc, Gauss-Bonnet assures that we can clean out, this does not hold any more in the case, when several discs are present: we can have odd degree vertices in two different discs and be in a situation where be are unable to cut through the vertices of T to render the remaining vertices to have even degree.



Department of Mathematics, Harvard University, Cambridge, MA, 02138