# TOWARDS A TOPOLOGICAL PROOF OF THE FOUR COLOR THEOREM III

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ABSTRACT. If the spheres reach the boundary surface $S$, we have constraints. The surface $S(x)$ can not intersect $S$ in a wheel graph. This is no problem as we can pad the outer surface.

Notation from [1, 2] and previous notes.
It seems that we can refine well as long as we don't reach the outer surface $S$. What happens if $x$ is an interior point whose unit sphere $S(x)$ intersects with the outer sphere $S$ in a disc. In that case, we have make cuts first so that the sphere $S(x)$ only intersects in one point. In general, the spheres of $S(x)$ for interior points should not intersect $S$ in an open set.

In the case of the icosahedron for example, after a refinement, all inner vertices $x$ have the property that $S(x)$ can intersect the boundary sphere $S$ in a set which does not contain a wheel.

This looks not a problem as we can first make a cobordism of $S$ with a completed dual graph. Now we have less likely a unit sphere $S(x)$ nearby intersecting with a wheel graph. Since we need to cover all the triangles of $S$, we need to be able to clean out 2 sphere without affecting the boundary. It is clear that we need to be able to clean out a sphere without affecting a connected subgraph which does not contain a wheel graph. But that is exactly needed to change the parity of a point.

The possible intersections of two spheres, where one is contained in the other is large. We can however ask that the intersection is either a vertex, an edge or triangle. Now we have to show that we can make the graph Eulerian, if we assume that a disk $U$ and triangle bounded away is untouchable in the sense that we can not use edges from those

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parts.

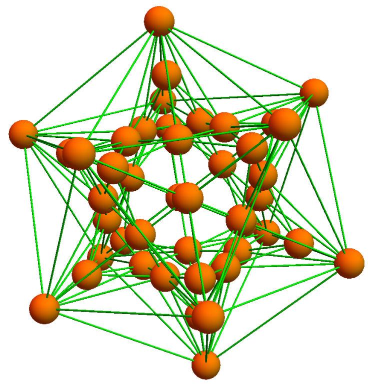

FIGURE 1. Padding the outer surface $S$ with an inner sphere. Since we are not allowed to cut edges of the outer surface. This construction is a cobordism between $G$ and the completed dual $\overline{G}$.

Therefore, we know now. Remarks.

1) We need to be able to refine a sphere under the condition that a disc and an additional triangle is untouchable.

2) We need to show that spheres have maximally triangular intersections with the boundary $S$ is sustainable in the sense that it stays invariant under refinements.

3) The condition that the spheres $S(x)$ near the boundary have only small intersections with the boundary forces the inner neigborhood of $S$ to be larger.

## REFERENCES

[1] O. Knill. Coloring graphs using topology. http://arxiv.org/abs/1410.3173, 2014.
[2] O. Knill. Graphs with eulerian unit spheres. http://arxiv.org/abs/1501.03116, 2015.

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