Thm:

S

$$\boxed{\begin{array}{l} S \in \mathcal{S}_2 \\ \text{can be embedded in} \\ T \in \mathcal{S}_3 \\ \text{Eulerian} \end{array}}$$

= 4 colorable

need only find $B \in \mathcal{B}_3$, $S = \delta B$

Strategy:

now make homology transformations of ball B until it is Eulerian.

How?

int(B)

$U \in \mathcal{B}_3$

int(U) Euclidian

make U larger and larger: to put cut part of S(x) x into U

Notation

$$S = V \cup W$$

$$V \cap W \in \mathcal{S}_2$$

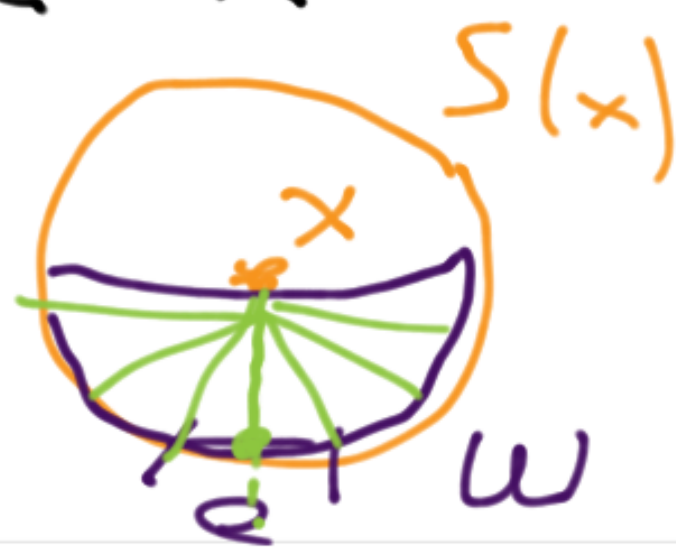
$$V, W \in \mathcal{B}_2$$

V <u>Eulerian part</u>

All vertices in $\mathrm{int}(V)$ have even degree.

Lemma $\boxed{1}$

can cut $S(x) \cap w$, to make x Eulerian



Proof:

Corollary 1

There is a deformation of $S(x)$ rendering it Eulerian.

The edge refinements only modify edges in $\operatorname{int}(w)$.

Lemma 2

By making $S(x)$ Eulerian, we add x into the Eulerian part U.

Distance $d(U, \delta B)$ does not increase.

Corollary 2

After finitely many steps we have moved all vertices of ∂U into U. This eventually decreases $d(u, 5)$

Proof: We never add points to δU.

Consequence:

after finitely many steps $U_k = B$ and all vertices x in $\mathrm{int}(B)$ are Eulerian ($S(x)$ is Eulerian).

Then B is Eulerian.

QED.