MATHEMATICA ROUTINES FOR PRODUCTS AND BARYCENTRIC REFINEMENT

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Abstract. Mathematica routines for investigating the universality.

1. Barycentric refinement

```
TopDim=2;
Cl[s_,k_]:=Module[{n,t,m,u,q,V=VertexList[s],W=EdgeList[s],l},
 n=Length[V]; m=Length[W]; u=Subsets[V,{k,k}]; q=Length[u]; l={};
 W=Table[{W[[j,1]],W[[j,2]]},{j,m}]; If[k==1,l=Table[{V[[j]]},{j,n}],
  If[k==2,l=W,Do[t=Subgraph[s,u[[j]]]; If[Length[EdgeList[t]]==
  Binomial[k,2],l=Append[l,VertexList[t]]], {j,q}]]; l];

Ring[s_,a_]:=Module[{v,n,m,u,X},v=VertexList[s]; n=Length[v];
 u=Table[Cl[s,k],{k,TopDim+1}] /. Table[k->a[[k]],{k,n}];m=Length[u];
 X=Sum[Sum[Product[u[[k,l,m]],
    {m,Length[u[[k,l]]]}],{l,Length[u[[k]]]},{k,m}]];

GR[f_]:=Module[{s={}},Do[Do[If[Denominator[f[[k]]/f[[l]]]==1 && k!=l,
 s=Append[s,k->l]],{k,Length[f]}],{l,Length[f]}];
 UndirectedGraph[Graph[s]]];

GraphProduct[s1_,s2_]:=Module[{f,g,i,fc,tc},
 fc=FromCharacterCode; tc=ToCharacterCode;
 i[l_,n_]:=Table[fc[Join[tc[l],IntegerDigits[k]+48]],{k,n}];
 f=Ring[s1,i["a"],Length[VertexList[s1]]]];
 g=Ring[s2,i["b"],Length[VertexList[s2]]]]; GR[Expand[f*g]]];

NewGraph[s_]:=GraphProduct[s,CompleteGraph[1]];
Bary[s_,n_]:=Last[NestList[NewGraph,s,n]];

TopDim=2; GraphProduct[CompleteGraph[4],StarGraph[4]]
TopDim=3; Bary[CompleteGraph[4],2]
```

Now we compute the eigenvalues

```
Laplace[s_,n_]:=Normal[KirchhoffMatrix[Bary[s,n]]];
Spectrum[s_,n_]:= Sort[Eigenvalues[1.0*Laplace[s,n]]];
F[s_,n_]:=ListPlot[Spectrum[s,n],Filling -> Bottom];
TopDim=2; F[CompleteGraph[3],3]
```

You can copy paste the code online on *http://www.math.harvard.edu/knill/graphgeometry/faqu.html*. See the preprint [1] for the mathematics.

References

[1] O. Knill. The graph spectrum of barycentric refinements. http://arxiv.org/abs/1508.02027, 2015.

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