

Real and Complex Dynamical Systems

edited by

Bodil Branner

and

Poul Hjorth

Mathematical Institute,
The Technical University of Denmark,
Lyngby, Denmark



Springer-Science+Business Media, B.V.

NATO ASI Series

Advanced Science Institutes Series

A Series presenting the results of activities sponsored by the NATO Science Committee, which aims at the dissemination of advanced scientific and technological knowledge, with a view to strengthening links between scientific communities.

The Series is published by an international board of publishers in conjunction with the NATO Scientific Affairs Division

A Life Sciences	Plenum Publishing Corporation
B Physics	London and New York
C Mathematical and Physical Sciences	Kluwer Academic Publishers
D Behavioural and Social Sciences	Dordrecht, Boston and London
E Applied Sciences	
F Computer and Systems Sciences	Springer-Verlag
G Ecological Sciences	Berlin, Heidelberg, New York, London,
H Cell Biology	Paris and Tokyo
I Global Environmental Change	

PARTNERSHIP SUB-SERIES

1. Disarmament Technologies	Kluwer Academic Publishers
2. Environment	Springer-Verlag / Kluwer Academic Publishers
3. High Technology	Kluwer Academic Publishers
4. Science and Technology Policy	Kluwer Academic Publishers
5. Computer Networking	Kluwer Academic Publishers

The Partnership Sub-Series incorporates activities undertaken in collaboration with NATO's Cooperation Partners, the countries of the CIS and Central and Eastern Europe, in Priority Areas of concern to those countries.

NATO-PCO-DATA BASE

The electronic index to the NATO ASI Series provides full bibliographical references (with keywords and/or abstracts) to more than 50000 contributions from international scientists published in all sections of the NATO ASI Series.

Access to the NATO-PCO-DATA BASE is possible in two ways:

- via online FILE 128 (NATO-PCO-DATA BASE) hosted by ESRIN, Via Galileo Galilei, I-00044 Frascati, Italy.
- via CD-ROM "NATO-PCO-DATA BASE" with user-friendly retrieval software in English, French and German (© WTV GmbH and DATAWARE Technologies Inc. 1989).

The CD-ROM can be ordered through any member of the Board of Publishers or through NATO-PCO, Overijse, Belgium.



Proceedings of the NATO Advanced Study Institute on
Real and Complex Dynamical Systems
Hillerød, Denmark
June 20–July 2, 1993

A C.I.P. Catalogue record for this book is available from the Library of Congress

ISBN 978-90-481-4565-2 ISBN 978-94-015-8439-5 (eBook)
DOI 10.1007/978-94-015-8439-5

All Rights Reserved

© 1995 Springer Science+Business Media Dordrecht
Originally published by Kluwer Academic Publishers in 1995
Softcover reprint of the hardcover 1st edition 1995

No part of the material protected by this copyright notice may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage and retrieval system, without written permission from the copyright owner.

TABLE OF CONTENTS

Preface	vii
List of participants	ix
Viviane BALADI: <i>Dynamical Zeta Functions</i>	1
Chris BUDD: <i>The Global Dynamics of Impact Oscillators</i>	27
Chris BUDD: <i>Grazing in Impact Oscillators</i>	47
Adrien DOUADY: <i>Topological Entropy of Unimodal Maps</i>	65
John H. HUBBARD and Ralph W. OBERSTE-VORTH: <i>Hénon Mappings in the Complex Domain</i>	89
Bruce KITCHENS: <i>Symbolic Dynamics, Group Automorphisms and Markov Partitions</i>	133
Silvina P. DAWSON, Roza GALEEVA, John MILNOR and Charles TRESSER: <i>A Monotonicity Conjecture for Real Cubic Maps</i>	165
Colin SPARROW: <i>Dynamics of Ordinary Differential Equations</i>	185
Sebastian van STRIEN: <i>Real Bounds in Complex Dynamics</i>	211
Marcelo VIANA: <i>Homoclinic Bifurcations and Strange Attractors</i>	231
Jean-Christophe YOCCOZ: <i>Introduction to Hyperbolic Dynamics</i>	265
Lai-Sang YOUNG: <i>Ergodic Theory of Differentiable Dynamical Systems</i>	293
Index	337

PREFACE

This volume contains edited versions of 11 contributions given by main speakers at the NATO Advanced Study Institute on Real and Complex Dynamical Systems in Hillerød, Denmark, June 20th - July 2nd, 1993.

The vision of the institute was to illustrate the interplay between two important fields of Mathematics: Real Dynamical Systems and Complex Dynamical Systems. The interaction between these two fields has been growing over the years. Problems in Real Dynamical Systems have recently been solved using complex tools in the real or by extension to the complex. In return, problems in Complex Dynamical Systems have been settled using results from Real Dynamical Systems. The programme of the institute was to examine the state of the art of central parts of both Real and Complex Dynamical Systems, to reinforce contact between the two aspects of the theory and to make recent progress in each accessible to a larger group of mathematicians.

We wish to express our sincere thanks to all lecturers and participants for having helped to make this ASI a success. We acknowledge the extensive amount of time the invited main speakers had to spend to prepare the expository lectures, and to prepare their manuscripts for publication. Special thanks go to Adrien Douady, Sebastian van Strien and Lai-Sang Young for their efforts as members of the Scientific Organizing Committee; moreover, to the conference secretaries Lone Aagesen and Tove Densted for their competent management of all matters great & small during the conference; to the very professional staff at the Apotekerforeningens Kursusejendom (where the Institute was held); and to two local participants, Pia Willumsen and Dan Sørensen for assistance above and beyond the call of mere participation. We would also like to thank Jacob Dylander and acknowledge his patient and careful work of formatting and assembling the manuscripts of this book.

The ASI was funded principally by NATO, with additional support from MIDIT (Modelling, Non-linear Dynamics and Irreversible Thermodynamics) at The Technical University of Denmark, the Danish Natural Science Research Council, the Carlsberg Foundation, the Thomas B. Thrige Foundation, the Otto B. Mønsted Foundation, the Mathematical Institute at The Technical University of Denmark and the Danish Mathematical Society. We would like to thank all these organizations for their support. For its efforts on behalf of this ASI we are most grateful to the Scientific Affairs Division of NATO, particularly to Dr. Luis V. da Cunha, the Director of the ASI programme.

Bodil Branner, *Scientific Director*
Poul Hjorth

LIST OF PARTICIPANTS

Simonette ABENDA
S.I.S.S.A.
Via Beiruth, 4
I-34014 Trieste
ITALY
e-mail abenda@tsmi19.sissa.it

Raphael ALBRECHT
Inst. für Theor. Physik, Lehrstuhl E
Physikalische Inst. der RWTH Aachen
Sommerfeldstrasse
D-52056 Aachen
GERMANY
e-mail raphael@thphys.physik.rwth-aachen.de

Ana Isabel ALONSO DE MENA
E.T.S. Ingenieros Industriales
Dept. Matematica Aplicada F a la In-
genieria
Paseo del Cauce /n
E-47011 Valladolid
SPAIN
e-mail etmat3@cpd.uva.es

Jose FERREIRA ALVES
Faculdade de Ciencias
Grupo de Matematica Pura
Praca Gomes Teixeira
P-4000 Porto
PORTUGAL
e-mail jfalves@fcl.fc.up.pt

Pau ATELA
Department of Mathematics
Smith College
Northampton, MA 01063
UNITED STATES
e-mail patela@sophia.smith.edu

Viviane BALADI
Laboratoire de Mathématiques
École Normale Supérieure de Lyon
46, Allée d'Italie
F-69364 Lyon Cedex 17
FRANCE

and
Math. Department – ETH Zürich
ETH Zentrum
Rämistrasse 101
CH-8092 Zürich
SWITZERLAND
e-mail baladi@math.ethz.ch

Krzysztof BARAŃSKI
Institute of Mathematics
Warsaw University
ul. Banacha 2
PL-02-097 Warsaw
POLAND
e-mail baranski@mimuw.edu.pl

Julia A. BARNES
Math. Dept. CB # 3250
University of North Carolina at Chapel
Hill
Chapel Hill, NC 27599
UNITED STATES
e-mail barnes@math.unc.edu

Bodil BRANNER
Mathematical Institute
The Technical University of Denmark
Building 303
DK-2800 Lyngby
DENMARK
e-mail branner@mat.dtu.dk

Jean-Yves BRIEND
 École Normale Supérieure de Lyon
 46, Allée d'Italie
 F-69364 Lyon Cedex 07
 FRANCE
 e-mail jjbriend@ens.ens-lyon.fr

Henk BRUIN
 Delft University of Technology
 TWI General Mathematics
 Mekelweg 4
 NL-2600 GA Delft
 NETHERLANDS
 e-mail bruin@dutiaw3.tudelft.nl

Morten BRØNS
 Mathematical Institute
 The Technical University of Denmark
 Building 303
 DK-2800 Lyngby
 DENMARK
 e-mail brons@mat.dtu.dk

Chris BUDD
 School of Mathematics
 University of Bristol
 University Walk
 GB-Bristol, BS8 1TW
 UNITED KINGDOM
 e-mail chris.budd@bristol.ac.uk

Xavier BUFF
 École Normale Supérieure
 45 rue d'Ulm
 F-75005 Paris Cedex
 FRANCE
 e-mail buff@ens.ens.fr

Manuel CHAVES
 Grupo de Matematica Pura
 Faculdade de Ciencias
 Praca Gomes Teixeira
 P-4000 Porto
 PORTUGAL
 e-mail machaves@fc.up.pt

Amy CHIU
 Department of Mathematics
 Boston University
 111, Cummington St.
 Boston, MA 02215
 UNITED STATES
 e-mail olympiad@math.bu.edu

Adrien DOUADY
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay
 FRANCE
 e-mail adrien.douady@ens.fr

Jakob DYLANDER
 Mathematical Institute
 University of Copenhagen
 Universitetsparken 5
 DK-2100 Copenhagen Ø
 DENMARK
 e-mail dylander@math.ku.dk

Adam EPSTEIN
 Department of Mathematics
 Cal. Tech.
 Pasadena, CA
 UNITED STATES
 e-mail elago@cunyvms.bitnet

Juan Francisco ESTRADA GARCIA
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay Cedex
 FRANCE

Nuria FAGELLA
 Department of Mathematics
 Boston University
 111, Cummington St.
 Boston, MA 02215
 UNITED STATES
 e-mail nuria@math.bu.edu

FANG Jinqing
 China Institute of Atomic Energy
 P.O. Box 275-27
 Beijing 102413
 P. R. of China

Jérôme FEHRENBACH
 École Normale Supérieure
 45 rue d'Ulm
 F-75005 Paris Cedex
 FRANCE
 e-mail fehrenba@clipper.ens.fr

Bjørn FELSAGER
 Kildegård Gymnasium
 Kildegårdsvej 87
 DK-2900 Hellerup
 DENMARK
 e-mail felsager@mat.dtu.dk

Marguerite FLEXOR
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay Cedex
 FRANCE
 e-mail flexor@matups.matups.fr

Roza GALEEVA
 Department of Mathematics
 UWM P.O. Box 413
 Milwaukee, WI 53201
 UNITED STATES
 e-mail roza@archimedes.math.uwm.edu

Danièle GERARD
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay Cedex
 FRANCE
 e-mail dgerard@matups.matups.fr

Coby GEÿSEL
 University of Amsterdam
 Faculty of Math.F and Computer Science
 Plantage Muidergracht 24
 NL-1018 TV Amsterdam
 NETHERLANDS
 e-mail coby@fwi.uva.nl

Aleksei Antonovich GLUTSUK
 Department of Mathematics
 Moscow State University
 Leninskie Gori
 117234 Moscow
 RUSSIA

Kari HAG
 Department of Mathematics
 The Norwegian Institute of Technology
 N-7034 Trondheim
 NORWAY
 e-mail kari@imf.unit.no

Peter HAISSINSKY
 École Normale Supérieure de Cachan
 61, Avenue du Président Wilson
 F-94230 Cachan Cedex
 FRANCE

Ernst HANSEN
 Mathematical Institute
 University of Copenhagen
 Universitetsparken 5
 DK-2100 Copenhagen Ø
 DENMARK
 e-mail erhansen@math.ku.dk

Kai HANSEN
 Niels Bohr Institute
 University of Copenhagen
 Blegdamsvej 15-17
 DK-2100 Copenhagen Ø
 DENMARK
 e-mail khansen@nbivax.nbi

Jane HAWKINS
 Math. Dept. CB#3250
 University of North Carolina at Chapel
 Hill
 Chapel Hill, NC 27514
 UNITED STATES
 e-mail jmhawk@unc.bitnet

Sandra HAYES
 Mathematisches Institut der
 Technischen Universität München
 Arcisstrasse 21
 D-8000 München
 GERMANY
 e-mail hayes@mathematik.tu-muenchen.de

Florent HIVERT
 École Normale Supérieure
 45 rue d'Ulm
 F-75005 Paris Cedex
 FRANCE
 e-mail hivert@ens.ens.fr

Poul HJORTH
 Mathematical Institute
 The Technical University of Denmark
 Building 303
 DK-2800 Lyngby
 DENMARK
 e-mail hjorth@mat.dtu.dk

Jun HU
 Math. Dept. of CUNY
 Graduate Center
 33, W 42nd Street
 New York, NY 10036
 UNITED STATES
 e-mail huj@cunyvmssl.gc.cuny.edu

John H. Hubbard
 Department of Mathematics
 White Hall
 Cornell University
 Ithaca, NY 14853
 UNITED STATES
 e-mail hubbard@math.cornell.edu

Pascal HUBERT
 École Normale Supérieure
 46 Allée d'Italie
 F-69364 Lyon Cedex
 FRANCE
 e-mail hubert@ens.ens-lyon.fr

Axel HUNDEMÉR
 Mathematisches Institut
 der Technischen Universität München
 Arcisstrasse 21
 D-8000 München 2
 GERMANY

Arne JAKOBSEN
 Department of Mathematics
 The Norwegian Institute of Technology
 N-7034 Trondheim
 NORWAY
 e-mail arneja@imf.unit.no

Emmanuelle JEANDENANS
 Université de Bourgogne
 Département de Mathématiques
 Lab. de Topologie - URA 755, B.P. 138
 F-21004 Dijon Cedex
 FRANCE
 e-mail topolog@satie.u-bourgogne.fr

Habib JELLOULI
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay Cedex
 FRANCE
 e-mail jellouli@matups.matups.fr

Yunping JIANG
 Mathematics Department
 Queens College of CUNY
 65-30 Kissena Bld.
 NY 11367
 UNITED STATES
 e-mail jiang@math.sunysb.edu

Peter JONES
 Department of Mathematics
 Yale University
 New Haven
 CT 06520
 UNITED STATES

Ross JONES
 University of Cambridge
 Department of Pure Maths and Math-
 ematical Statistics
 16, Mill Lane
 GB-Cambridge CB2 1SB
 UNITED KINGDOM
 e-mail rpj12@cus.cam.ac.uk

Mattias JONSSON
 Matematiska Inst. 115
 Kungl. Tekniska Högskolan
 S-10044 Stockholm
 SWEDEN
 e-mail mjo@math.kth.se

Jeremy KAHN
 Department of Mathematics
 University of California
 Berkeley, CA 94720
 UNITED STATES
 e-mail kahn@math.berkeley.edu

Bruce KITCHENS
 Mathematical Sciences Department
 IBM T.J. Watson Research Center
 Yorktown Heights
 New York 10598
 UNITED STATES
 e-mail kitch@watson.ibm.com

Oliver KNILL
 Mathematik Departement
 ETH Zentrum
 CH-8092 Zürich
 SWITZERLAND
 e-mail knill@math.ethz.ch

Carsten KNUDSEN
 Physics Department
 The Technical University of Denmark
 Building 309
 DK-2800 Lyngby
 DENMARK
 e-mail carsten@chaos.fl.dtu.dk

Bernd KRAUSKOPF
 Department of Mathematics
 University of Groningen
 P.O. Box 800
 NL-9700 AV Groningen
 NETHERLANDS
 e-mail bernd@math.rug.nl

Hartje KRIETE
 Lehrstuhl II für Mathematik
 RWTH Aachen
 D-52056 Aachen
 GERMANY
 e-mail kriete@math2.rwth-aachen.de

Harbir LAMBA
 School of Mathematics
 University of Bristol
 University Walk
 GB-Bristol B58 1TW
 UNITED KINGDOM
 e-mail h.lamba@bristol.ac.uk

Jens Christian LARSEN
 Mathematical Institute
 The Technical University of Denmark
 Building 303
 DK-2800 Lyngby
 DENMARK
 e-mail maijcl@vm.uni-c.dk

Laurent LE FLOCH
 Laboratoire de géométrie analytique
 UFR de Mathématiques
 Université de Rennes I
 F-35042 Rennes Cedex
 FRANCE

LI Weigu
 Department of Mathematics
 Moscow State University
 Leninskie Gori
 117234 Moscow
 RUSSIA

Frank LORAY
 Laboratoire de géométrie analytique
 UFR de Mathématiques
 Université de Rennes I
 F-35042 Rennes Cedex
 FRANCE

LUO Jiaqi
 Department of Mathematics
 Cornell University
 Ithaca, NY 14853
 UNITED STATES
 e-mail jiaqi@math.cornell.edu

Stefano LUZZATTO
 Int. School for Advanced Study
 Via Beirut 4
 I-34014 Trieste
 ITALY
 e-mail luzzatto@tsmi19.sissa.it

Lawrence MA
 Inst. des Hautes Etudes Scientifiques
 35, route de Chartres
 F-91440 Bures-sur-Yvette
 FRANCE
 e-mail ma@frihes61.bitnet

Stefano MARMI
 Dipartimento di Matematica U. Dini
 Università di Firenze
 Viale Morgani 67/A
 I-50134 Firenze
 ITALY
 e-mail marmi@firenze.infn.it

John MILNOR
 Institute for Mathematical Sciences
 SUNY
 Stony Brook, NY 11794-3651
 UNITED STATES
 e-mail jack@math.sunysb.edu

Fernando Jorge Soares MOREIRA
 Faculdade de Ciencias
 Grupo de Matematica Pura
 Praca Gomes Teixeira
 P-4000 Porto
 PORTUGAL
 e-mail fsmoreir@fcl.fc.up.pt

Pierre MOUSSA
 Service de Physique Théorique
 Orme des Merisiers
 C.E.-SACLAY
 F-91191 Gif-sur-Yvette Cedex
 FRANCE
 e-mail moussa@amoco.saclay.cea.fr

Shizuo NAKANE
 Tokyo Institute of Polytechnics
 1583 Iiyama, Atsugi-shi
 Kanagawa /243-02
 JAPAN
 e-mail snakane@tansei.cc.u-tokyo.ac.jp

Vincent NAUDOT
 Université de Bourgogne
 Département de Mathématiques
 Laboratoire de Topologie - URA 755
 B.P. 138
 F-21004 Dijon Cedex
 FRANCE

Alec NORTON
 Mathematics Department
 UT Austin
 Austin, TX 78712
 UNITED STATES
 e-mail alec@math.utexas.edu

Ralph W. OBERSTE-VORTH
 University of South Florida
 Department of Mathematics
 Tampa, FL 33620-5700
 UNITED STATES
 e-mail ralph@math.usf.edu

Christopher Shaun PENROSE
 School of Mathematical Sciences
 Queen Mary & Westfield College
 University of London
 Mile End Road
 GB-London E1 4NS
 UNITED KINGDOM
 e-mail csp@maths.qmw.ac.uk

Ricardo PEREZ-MARCO
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay
 FRANCE

Marie-Christine PEROUEME
 École Normale Supérieure de Lyon
 46 Allée d'Italie
 F-69364 Lyon Cedex
 FRANCE
 e-mail mcperoue@umpa.ens-lyon.fr

Carsten LUNDE PETERSEN
 Mathematical Institute
 Århus University
 Ny Munkegade
 DK-8000 Århus C
 DENMARK
 e-mail lunde@frihes61.bitnet

Kevin M. PILGRIM
 Department of Mathematics
 University of California
 Berkeley, CA 94720
 UNITED STATES
 e-mail pilgrim@math.berkeley.edu

Alberto A. PINTO
 Faculdade de Ciências
 Grupo de Matematica Aplicada
 Universidade do Porto
 Rua das Taipas
 P-Porto
 PORTUGAL

Julio E. POISOT MACIAS
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay Cedex
 FRANCE

James T. ROGERS, Jr.
 Department of Mathematics
 Tulane University
 New Orleans, LA 70118
 UNITED STATES
 e-mail mtidamf@vm.tcs.tulane.edu

Hans Henrik RUGH
 École Normale Supérieure de Lyon
 46, Allée d'Italie
 F-69364 Lyon
 FRANCE
 e-mail hhrugh@umpa.ens-lyon.fr

Gustav RYD
 Matematiska Inst. 115
 Kungl. Tekniska Högskolan
 S-10044 Stockholm
 SWEDEN
 e-mail gustav@math.kth.se

Eliane SALEM
 Laboratoire Topologie et Géométrie
 URA CNRS 1408
 Université Paul Sabatier
 118 route de Narbonne
 F-31062 Toulouse Cedex
 FRANCE
 e-mail salem@cict.fr

Duncan SANDS
 University of Cambridge
 Department of Mathematics and Math-
 ematical Statistics
 16, Mill Lane
 GB-Cambridge CB2 1SB
 UNITED KINGDOM
 e-mail sands@dmi.ens.fr

Bahtiyar Özgür SARIOĞLU
Bilkent University
Department of Mathematics
Bilkent 06533 Ankara
TURKEY
e-mail sozgur@trbilun.bitnet

R. Phil SCHAFER
The University of Texas at Austin
Department of Mathematics
Austin, Texas 78712-1082
UNITED STATES
e-mail philip@math.utexas.edu

Dierk SCHLEICHER
Department of Mathematics
Cornell University
Ithaca, NY 14853
UNITED STATES
e-mail dierk@math.cornell.edu

Pierrette SENTENAC
Université de Paris-Sud
Département de Mathématiques, Bât.
425
F-91405 Orsay Cedex
FRANCE
e-mail sentenac@matups.matups.fr

Mitsuhiro SHISHIKURA
Department of Mathematics
Tokyo Institute of Technology
Ohokayama, Meguro
Tokyo 152
JAPAN
e-mail mitsu@math.titech.ac.jp

Jan SKRZYPCZAK
Institute of Mathematics
Warsaw University
ul. Banacha 2
PL-02-097 Warsaw
POLAND
e-mail janskrz@mimuw.edu.pl

Colin T. SPARROW
Department of Mathematics and Math-
ematical Statistics
16, Mill Lane
GB-Cambridge CB2 1SB
UNITED KINGDOM
e-mail [c.t.sparrow@
statslab.cam.ac.uk](mailto:c.t.sparrow@statslab.cam.ac.uk)

Sebastian VAN STRIEN
University of Amsterdam
Faculty of Math. and Computer Sci-
ence
Plantage Muidergracht 24
NL-1018 TV Amsterdam
NETHERLANDS
e-mail strien@fwi.uva.nl

Dan SØRENSEN
Mathematical Institute
The Technical University of Denmark
Building 303
DK-2800 Lyngby
DENMARK
e-mail dan@mat.dtu.dk

TAN Lei
École Normale Supérieure de Lyon
46, Allée d'Italie
F-69364 Lyon Cedex 07
FRANCE
e-mail tanlei@frensl61.bitnet

Hans THUNBERG
Matematiska Inst. 115
Kungl. Tekniska Högskolan
S-10044 Stockholm
SWEDEN
e-mail hasset@math.kth.se

Shigehiro USHIKI
Graduate School of Human and Envi-
ronmental Studies
Kyoto University
606-01 Kyoto
JAPAN
e-mail [ushiki@
platon.kula.kyoto-u.ac.jp](mailto:ushiki@
platon.kula.kyoto-u.ac.jp)

Marcelo VIANA
 Departamento de Matematica Pura
 Faculdade de Ciencias do Porto
 P-4000 Porto
 PORTUGAL
 and
 Instituto de Matemática Pura e Apli-
 cada
 Est. D. Castorina 110
 22460 Rio de Janeiro
 Brasil
 e-mail viana@impa.br

Fabienne VUILLEMIN
 Université de Bourgogne
 Département de Mathématiques
 Laboratoire de Topologie - URA 755
 B.P. 138
 F-21004 Dijon Cedex
 FRANCE
 e-mail topolog@frcub11.bitnet

Marysia T. WEISS
 Hofstra University
 Hempstead, New York 11550
 UNITED STATES
 e-mail matmtw@hofstra.bitnet

Anne Marie WILKINSON
 Department of Mathematics
 University of California
 Berkeley, CA 94720
 UNITED STATES
 e-mail wilkinso@math.berkeley.edu

Pia WILLUMSEN
 Mathematical Institute
 The Technical University of Denmark
 Building 303
 DK-2800 Lyngby
 DENMARK
 e-mail pia@mat.dtu.dk

WU He
 Math Group
 Int. Centre for Theoretical Physics
 P.O. Box 586
 I-34100 Trieste
 ITALY
 e-mail wuhe@ictp.trieste.it

Jean-Christophe YOCCOZ
 Université de Paris-Sud
 Département de Mathématiques
 Bâtiment 425
 F-91405 Orsay
 FRANCE

Lai-Sang YOUNG
 Department of Mathematics
 University of California
 Los Angeles, CA 90024
 UNITED STATES
 e-mail lsy@math.ucla.edu

INDEX

A	
a priori bounds	222
absolute continuity	
of a foliation	327
of the W^s -foliation	331
absolutely continuous	
conditional measures on	
unstable manifolds	315, 328, 331
invariant measure	255, 321
accessible boundary	121
accounting scheme	190, 205
acim	321, 323
adapted norm	268
admissible solutions	198
amalgamation	139
analytic map	
hyperbolic	16
anti-monotonicity	193
approximate	
contractive directions	261
critical set	261
critical values	261
area-conservative case	242, 252
area-dissipativeness	252, 253, 260
attraction	
basin of	253, 263
attractor	324
Axiom A	325, 327, 328
periodic	242
strange	253, 254
of Hénon type	254, 262, 263
prevalence of	262
automorphism	
toral	146
automorphism of subshift	
of finite type	145
autonomous system	186
averaging	
method of	188
Axiom A	
attractor	325, 327, 328
diffeomorphism	9
B	
baker transformation	294, 314
basic set	286
basin of attraction	253, 263
Bernoulli shift	294, 297, 328, 332
Bernoulli transformation	295
bifurcation	
border-collision	52
discontinuous grazing	35
gluing	198, 203, 207
grazing	48
homoclinic	194, 232, 233
associated to a horseshoe	247
Shil'nikov	208
Hopf	188
period-doubling	243
cascade of	244
period-multiplying	206
saddle-node	243
billiard balls	27
billiards	301
bimodal map	166
Birkhoff Ergodic Theorem	294
block map	134
"bones"	173
border-collision bifurcation	52
bound period	256
bound time	
total	257
bounded non-linearity	223
bounded type	212
Branner	213
C	
canonical system of conditional	
probability measures	298

- Cantor set
 thickness of a 250
 set of curves 237
 Carathéodory loop 76
 cascade
 period-doubling . . 192, 193, 197
 of period-doubling bifurcations 244
 Čech cohomology 121
 center manifold 244
 chain recurrence 276, 284
 chain-transitivity 286
 chaotic behaviour 188
 chaotic dynamics 255
 chattering 30
 coefficient of restitution 28
 cohomology equivalence relation . 7
 complex bounds 212
 conditional entropy 296
 conditional probability measures
 canonical system of 298
 cone
 invariant 301, 333
 over the solenoid 118
 cone field 98, 269
 invariant 236
 conjecture
 Williams 141
 conjugacy
 elementary 137
 topological 152, 238
 conservative Hénon family 242
 conservative Standard family . . . 252
 continuation 190
 continuation of a hyperbolic set . 239
 continuity
 of entropy map 85
 Continuum Theorists 130
 contractive direction
 approximate 261
 critical point 254, 259, 260, 262
 critical saddle-node cycle 244–246, 262
 critical set
 approximate 261
 critical subdivision 69
 critical value 259
 approximate 261
 critically finite map 72
 cross-ratio 215, 216
 cross-ratio distortion 217
 crossed mapping 94, 96
 degree of 97
 cubic maps 165
D
 decomposition
 spectral 284
 degree of φ 143
 dense polynomials 125
 derivative
 exponential growth of . 254, 255
 diamond 142
 diffeomorphism
 Axiom A 9
 diffeomorphisms
 parametrized families of . . 232
 differentiability
 intrinsic 251
 dimension
 Hausdorff 248
 dimension formula 319
 dimension of a measure 317
 discontinuity set 49
 discontinuous dynamical system . 27
 discontinuous grazing bifurcation . 35
 dissipativeness
 sectional 242, 252
 doubling map 66, 76
 doubly transitive point 136, 152
 dynamical argument 66, 78
 dynamical stability 239, 246
 dynamical zeta function 1
 weighted 1
 dynamics
 chaotic 255
 symbolic 238
E
 efficient cardinal 65, 67
 elementary conjugacy 137
 elliptic island 252
 entropy 65, 314
 conditional 296
 metric 295
 topological 3, 65, 156, 165

equilibrium states 4
 equivalence
 shift 141
 ergodic decomposition
 of invariant measures 321
 Ergodic Theorem 294
 Subadditive 303
 ergodic transformation 294
 essential spectral radius 4
 expanding map 12, 14, 321
 non-uniformly 323
 expansive homeomorphism 148
 expansive transformation 152
 expansivity 274
 exponential growth
 of the derivative 254, 255
 exponential mixing 11
 external argument 66, 75

F

factor 141, 152
 factor map 141
 finite-to-one 143
 infinite-to-one 143
 Falkner-Skan equation . . 185, 198, 208
 Feigenbaum point 86
 Fibonacci 213
 Fibonacci map 223, 225
 fingers 48
 finite type
 subshift of 134
 automorphism of 145
 finitely presented systems 162
 finite-to-one factor map 143
 folding period 261
 foliation
 absolute continuity of 327
 invariant 239
 regularity of 239
 stable and unstable
 regularity of 247
 strong-stable 245
 Fredholm determinant 3, 13, 14
 generalized 13
 frequency
 resonant 34
 full n -shift 134

G

Gauss map 15
 generalized solenoid 151
 generic 330
 generic hyperbolicity 177
 generic unfolding 232, 241
 $GL(n, \mathbb{Z})$ 147
 global stable manifold . . 282, 312, 326
 global unstable manifold . . 312, 326
 gluing bifurcation 198, 203, 207
 graph transform 310
 graze 47
 grazing bifurcation 48
 discontinuous 35
 grazing impact 47
 Grobman-Hartman Theorem 271
 Grothendieck 14

H

Hausdorff dimension 248
 Hausdorff dimension of a measure 317
 Hausdorff dimension of a set 317
 Hausdorff d -measure 248
 Hawaiian earring 121
 Hénon family
 conservative 242
 Hénon map 90, 185, 193, 332
 Herman 221
 Hertzian law 28
 heteroclinic loop 194, 197
 holonomy map 239, 251
 homeomorphism
 expansive 148
 topologically mixing 291
 transitive 291
 homeomorphisms
 infinite-dimensional space of 114
 homoclinic bifurcation . . 194, 232, 233
 Shil'nikov 208
 homoclinic bifurcation
 associated to a horseshoe 247
 homoclinic orbit 188, 194
 Shil'nikov 194
 homoclinic point 232
 transverse 232-234
 homoclinic tangency . . . 232, 234, 235
 persistence of 251

- Hopf bifurcation 188
 horizontal-like 99
 horseshoe 188, 236, 314
 homoclinic bifurcation
 associated to a 247
Hu 75
Hubbard 213
 hyperbolic analytic map 16
 hyperbolic automorphism 266
 hyperbolic continuation 270, 274
 hyperbolic distance 216
 hyperbolic map 266
 hyperbolic polynomial 102
 hyperbolic set 236, 247
 continuation of 239
 thick 251
 hyperbolicity 266
 generic 177
- I**
- impact 27
 grazing 47
 low velocity 51
 impact map P 29
 impact oscillator 27
 impact side 49
 inductive trick 222
 infinite stretching 31
 infinite-dimensional space
 of homeomorphisms 114
 infinitely renormalizable 211
 infinite-to-one factor map 143
 initial conditions
 sensitive dependence on 253, 255
 intermittent behaviour 48
 interval map
 piecewise monotone 18
 intrinsic differentiability 251
 invariant cone 301, 333
 invariant cone field 236
 invariant foliation 239
 regularity of 239
 invariant measure 294
 absolutely continuous 255, 321
 ergodic decomposition of 321
 for attractors 324
 space of 320
- invariant set 268
 isentrope 168
 itinerary 238
- J**
- Jakobson's Theorem** 324
Jiang 75
Jordan form away from zero 141
Julia set 17, 95
- K**
- Keller** 213, 223
 kneading 172
 angle 66, 79
 invariant 66
 matrix 23
 theory 192, 196, 208
Kobayashi metric 98
Koebe Lemma 215
Koebe Principle 218
Krein-Milman Theorem 320
- L**
- Lakes of Wada** 92, 125
Lebesgue measure 214
 positive 214
Levin 213
 limit capacity 248
 limit point 289
 limit set 289
 line of tangencies 247, 248
 local connectivity 213
 local ergodicity 330
 local product structure 152, 282–284
 local stable manifold 281, 310, 326
 local stable set 152
 local unstable manifold 310, 326
 local unstable set 152
 locally maximal 283
Lorenz equations 185, 187, 208
Lorenz geometric model 196
 low velocity impact 51
Lyapunov chart 306
Lyapunov exponent 299, 314, 333
Lyubich 212
- M**
- magic word** 143
manifold

center 244
 stable 107, 277
 global 282
 local 281
 strong-stable 244
 unstable 107
 maps with singularities 313
 Markov
 condition 159
 extension 21
 matrix 72
 partition 72, 157, 162
 Marstrand's theorem 250
 measure
 dimension of 317
 Hausdorff dimension of 317
 measure preserving transformation 294
 meromorphic extension 5, 8, 9, 14, 19,
 22
 meromorphic function 11, 15
 method of averaging 188
 metric entropy 295
 Milnor 66
 Misiurewicz point 75
 mixing
 exponential 11
 topological 135
 transformation 295
 modulus of an annulus 99, 216
 monotone
 piecewise 167
 monotonicity 165
 of entropy map 85
 mpt 294

N
 negative Schwarzian derivative . . 22
 n -itinerary 67
 non wandering
 point 290
 set 290
 non-uniformly expanding map . . 323
 non-impact side 49
 norm
 adapted 268
 normally contracting saddle-node 244
 Nowicki 213, 223

n -precritical point 69
 n -shift
 full 134
 nuclear operator 14

O
 obstacle
 rigid 28
 one-dimensional 48
 operator
 nuclear 14
 Perron-Frobenius 323
 transfer 2, 323
 orbit following 190
 order type 173
 oscillator
 impact 27
 Oseledec's Theorem 299
 over-Markov packing 72

P
 packing
 over-Markov 72
 parabolic point 75
 parametrized families
 of diffeomorphisms 232
 partition
 Markov 72
 period-doubling
 bifurcation 243
 cascade 192, 193, 197, 244
 periodic attractor 242
 periodic behaviour 255
 periodic orbit 186
 primitive 1
 periodic point 289
 period-multiplying bifurcation . . 206
 Perron value 136
 Perron-Frobenius
 operator 323
 theorems 3
 persistence of homoclinic tangencies 251
 Pesin's formula 314-316
 Petersen 221
 phase ϕ_i 30
 phase space 186
 φ
 degree of 143

- piecewise monotone interval map . 18
 piecewise-monotone map 167
 plateau 86
 Poincaré metric 95, 215
 polynomial
 quadratic 74
 polynomial tuning 81
 pre-loaded 36
 pressure
 topological 4
 prevalence of strange attractors . 262
 prime-orbit theorem 10
 primitive periodic orbit 1
 product structure
 local 282–284
 projective limit 90
 pseudo-orbit 275
 p -telescope 95
 Pustyl'nikov map 27
- Q**
- Q -orbits 198, 200
 quadratic family . . . 240, 243, 245
 quadratic polynomial 74
 quadratic tangency . . . 232, 234, 241
 quadratic-like
 family 240, 252, 254, 258
 map 252
 quasi-conformal
 homeomorphism 221
 map 115
- R**
- ramified covering lamination . . 117
 rational 3
 rational function 9
 real bounds 215
 rectangle 161
 recurrence 289
 chain 276, 284
 regularity of
 invariant foliation 239
 the stable and unstable
 foliations 247
 renormalization 18, 67, 240, 242, 244,
 251
 renormalization theory 211
 repeller 17
- resonances 11
 resonant frequency 34
 restitution
 coefficient of 28
 return 256
 reversible system 201, 206
 Riemann surface laminations . . . 93
 Riemann zeta function 10
 rigid obstacle 28
 rigidity result 213
 Ruelle's inequality 314, 315
- S**
- saddle-node 244
 normally contracting . . . 244
 bifurcation 243
 cycle critical . . . 244–246, 262
 sawtooth
 stunted 170
 scaling laws 225
 Schwarzian derivative 217
 negative 22
 Schwarz's Lemma . 98, 215, 217, 218
 sectional dissipativeness . . . 242, 252
 sensitive dependence
 on initial conditions . . . 253, 255
 Shadowing Lemma 275
 Shannon-Breiman-McMillan
 Theorem 297
 shift
 Bernoulli . . . 294, 297, 328, 332
 suspension of 7
 shift equivalence problem . . . 141
 shift map 238
 Shil'nikov homoclinic bifurcation 208
 Shil'nikov homoclinic orbit . . . 194
 σ_A 3
 Sinai-Ruelle-Bowen measure 315, 318,
 324, 328, 331–333
 Sinai-Ruelle-Bowen measure . . . 11
 singular map
 stable and unstable sets for . 245
 singularity
 square-root 52
 size of U' in U 99
 smallest interval trick 220
 solenoid 120, 150

- 2-adic 150
 generalized 151
 Space Ball 28
 space of invariant measures 320
 spectral decomposition 284
 spectral radius
 essential 4
 splitting algorithm 259, 261
 square-root singularity 52
 SRB measure 315, 318, 324, 328,
 331–333
 s-rectangle 237
 stability
 dynamical 239, 246
 stable and unstable sets
 for singular maps 245
 stable and unstable foliation
 regularity of 247
 stable and unstable manifolds
 for nonuniformly
 hyperbolic systems 311
 stable disk 100, 112
 stable manifold 107, 312
 global 282, 312, 326
 local 281, 310, 326
 Stable Manifold Theorem 277
 for a Fixed Point 309
 stable set 152
 local 152
 Standard family
 conservative 252
 standard map 302
 state splitting 138
 Steenrod homology 121
 sticking region 30
 strange attractor 253, 254
 of Hénon type 254, 262, 263
 prevalence of 262
 stretching
 infinite 31
 strong-stable foliation 245
 strong-stable manifold 244
 stunted sawtooth 170
 sub resonant 35
 Subadditive Ergodic Theorem 303
 subadditive sequence 67
 subdivision 69
 critical 69
 subshift of finite type 3, 134
 automorphism of 145
 Sullivan 212
 Sullivan's renormalization result 220
 super resonant 35
 suspensions of shifts 7
 Świątek 213
 symbolic dynamics 238
- T**
 tangency
 homoclinic 232, 234, 235
 persistence of 251
 line of 247, 248
 quadratic 232, 234, 241
 telescopes 94
 tent map 66, 73
 tessellation 71
 thick hyperbolic set 251
 thickness of a Cantor set 250
 Thurston 66
 topological conjugacy 152, 238
 topological entropy 3, 65, 156, 165
 topological invariant 191, 208
 topological pressure 4
 topological transitivity 295, 326
 topologically critical 69
 topologically
 mixing homeomorphism 291
 topologically
 mixing transformation 135
 topologically
 transitive transformation 135
 toral automorphism 146
 total bound time 257
 trajectory 29
 transfer operator 2, 323
 transition rule 134, 150, 159
 transitive
 doubly 136, 152
 transitive homeomorphism 291
 transitive transformation 152
 transitivity 238
 topological 135, 295, 326
 transverse homoclinic point 232–234

tuning 67, 80, 86
 by blowing up 80
 by modification 81
 polynomial 81
 2-adic solenoid 150

U

unfolding
 generic 232, 241
 uniformly hyperbolic 325
 uniformly hyperbolic attractor 326
 unimodal map 73
 unstable disk 100
 unstable manifold 107, 312
 absolutely continuous
 conditional measures
 on 315, 328, 331
 global 312, 326
 local 310, 326
 unstable set 152
 local 152
 u -rectangle 236

V

variational principle 4, 65

W

wandering domains 108
 weighted dynamical zeta function 1
 wild attractors 212
 existence of 214
 Williams conjecture 141
 word
 magic 143
 W^s -foliation
 absolute continuity of 331

Y

Yoccoz 75, 213
 Yoccoz puzzle 213

Z

zeta function 136, 149
 dynamical 1
 weighted 1
 Riemann 10