

# WIND TOMOGRAPHY IN BINARY SYSTEMS

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## ABSTRACT

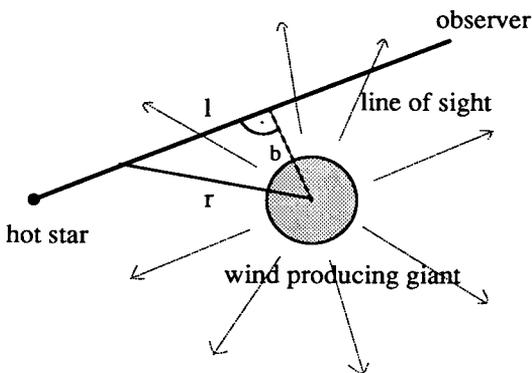
We solve Abel's integral equation by an explicit diagonalization of Abel's operator. This new method is particularly suitable for determining the velocity laws of stellar winds.

### 1. WIND TOMOGRAPHY AND ABEL'S INTEGRAL

Binary systems in which a compact, point-like radiation source shines through the wind of a companion star offer excellent laboratories for experimentally measuring the run of the velocity profile of stellar winds. In reality, such a situation occurs e.g. in symbiotic binaries (Vogel 1991). In the Figure we depict the geometry of the problem. As a function of the orbital phase, the line of sight from the compact star to the observer passes the extended atmosphere of the companion at different impact parameters,  $b$ . The amount of attenuated light is related to the column density  $n(b)$  of scatterers or absorbers in the wind along a line of sight depending on the impact parameter  $b$ . The observation of different column densities as a function of phase thus provides a "tomographic" view of the stellar wind. Assuming a constant mass loss rate  $\dot{M}$ , the column density can be expressed via the continuity equation as :

$$n(b) = A(g)(b) = a \int_b^{\infty} \frac{dr}{rv(r)\sqrt{r^2 - b^2}} = \int_b^{\infty} \frac{1}{\sqrt{r^2 - b^2}} g(r) dr \quad ,$$

where  $a = \frac{2\dot{M}}{4\pi\mu m_H}$  depends on the constant mass loss rate  $\dot{M}$  and the mean molecular weight  $\mu m_H$  in the wind,  $g(r) = a/(rv(r))$ , and  $A$  denotes the Abel integral operator.



The problem is to determine the unknown velocity  $v(r)$  of the stellar wind from the observed column density as a function of the impact parameter, or equivalently, as a function of the orbital phase. This is an inverse problem which are notoriously ill-posed in the sense that small perturbations in the observed data function may result in big errors in the solution. The standard solution to such ill-posed inversion problems are smoothing techniques (for an excellent text book on the subject see Craig and Brown 1986).

## 2. A NEW APPROACH TO ABEL'S EQUATION - DIAGONALIZATION

We have found an explicit diagonalization of Abel's integral operator by the following eigenfunctions (see Knill, Dgani and Vogel 1993 for the detailed derivations):

$$A(\psi_i) = \lambda_i \cdot \psi_i, \quad \text{where} \quad \psi_i(r) = r^{-i}, \quad i \geq 1$$

are the eigenfunctions to the eigenvalues

$$\lambda_i = \int_0^{\pi/2} \cos^{i-1}(\phi) d\phi = \begin{cases} \frac{(i-2)!!}{(i-1)!!} & , \quad i \text{ even} \\ \frac{\pi}{2} \cdot \frac{(i-2)!!}{(i-1)!!} & , \quad i \text{ odd} \end{cases}$$

where  $i!!$  denotes the double factorial of  $n$  which are recursively defined by  $0!! = 1!! = 1, i!! = i \cdot (i-2)!!$ . We note that these eigenvalues constitute a monotonically decreasing series.

## 3. THE INVERSION METHOD

Let's assume for the moment that  $g = \sum_{i=1}^{\infty} g_i r^{-i}$  is analytic in  $[0, \infty)$ . The operator  $A$  then maps  $g$  into the function  $Ag = \sum_{i=1}^{\infty} g_i \lambda_i b^{-i}$ , and the inversion of  $A$  is explicitly given by

$$A^{-1} : \quad \sum_{i=1}^{\infty} n_i b^{-i} \quad \mapsto \quad \sum_{i=1}^{\infty} \frac{n_i}{\lambda_i} r^{-i}$$

This diagonalization again shows that Abel's operator is ill-posed, since it has an unbounded inverse, that means, the data function  $n(b) = \sum_{i=1}^{\infty} n_i b^{-i}$  with a perturbation  $\delta n$ , results in an error for the source function  $g$  of

$$\delta g(r) = \sum_{i=1}^{\infty} \frac{\delta n_i}{\lambda_i} r^{-i},$$

where  $\lambda_i \rightarrow 0$  as  $i \rightarrow \infty$ . Thus,  $\delta g$  can be arbitrarily large for very small errors in the data  $\delta n$ . The simplest solution is to truncate the sum. This leads to an accurate and well-posed inversion on polynomials, if the source function  $g(r)$  is well approximated by a polynomial of the eigenfunctions.

Stellar winds are expected to approach a terminal constant velocity for large distances. Thus, the source function  $g(r) = a/(rv(r))$  asymptotically approaches the first eigenfunction  $1/r$ . Moreover, it can also be shown that all reasonable stellar winds can be well represented by a low order polynomial in  $1/r$ , namely by the first few eigenfunctions. Thus, our method is well suited for the reconstruction of the velocity profile in stellar winds using the following procedure:

- From the accurate data function  $n(b)$ , we only know an experimentally determined data set

$$\{(b_i, n_i(b_i)) \mid i = 1, \dots, N, 0 < b_i < \infty\} \quad .$$

- These noisy data is fitted with a polynomial of order  $K$

$$n^{(K)}(b) = \sum_{i=1}^K n_i^{(K)} b^{-i} \quad .$$

- The fitting polynomial  $n^{(K)}$  is now transformed by  $A^{-1}$  to

$$g^{(K)}(r) = \sum_{i=1}^K \frac{n_i^{(K)}}{\lambda_i} r^{-i} \quad ,$$

with the  $\lambda_i$ 's given above, and from this the velocity law is easily derived.

We have tested our method for different velocity laws representing radiatively driven winds from hot stars, winds from K supergiants, and winds expected from cool, dust driven giants. We found satisfactory results in that the error in the reconstructed function scales linearly with the error in the data.

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