

What is $6 \div 2(2 + 1)$?

*Battles about
mathematical Syntax*

Oliver Knill, Stem club, April 28, 2018

*Plan
for today
7 short
units*

- A. Battles
- B. Experiments
- C. History
- D. More riddles
- E. Social Media
- F. Literature
- G. Conclusions

A) Battles

What is $2x/3y-1$
for $x=9$ and $y=2$?

What did you get?

2

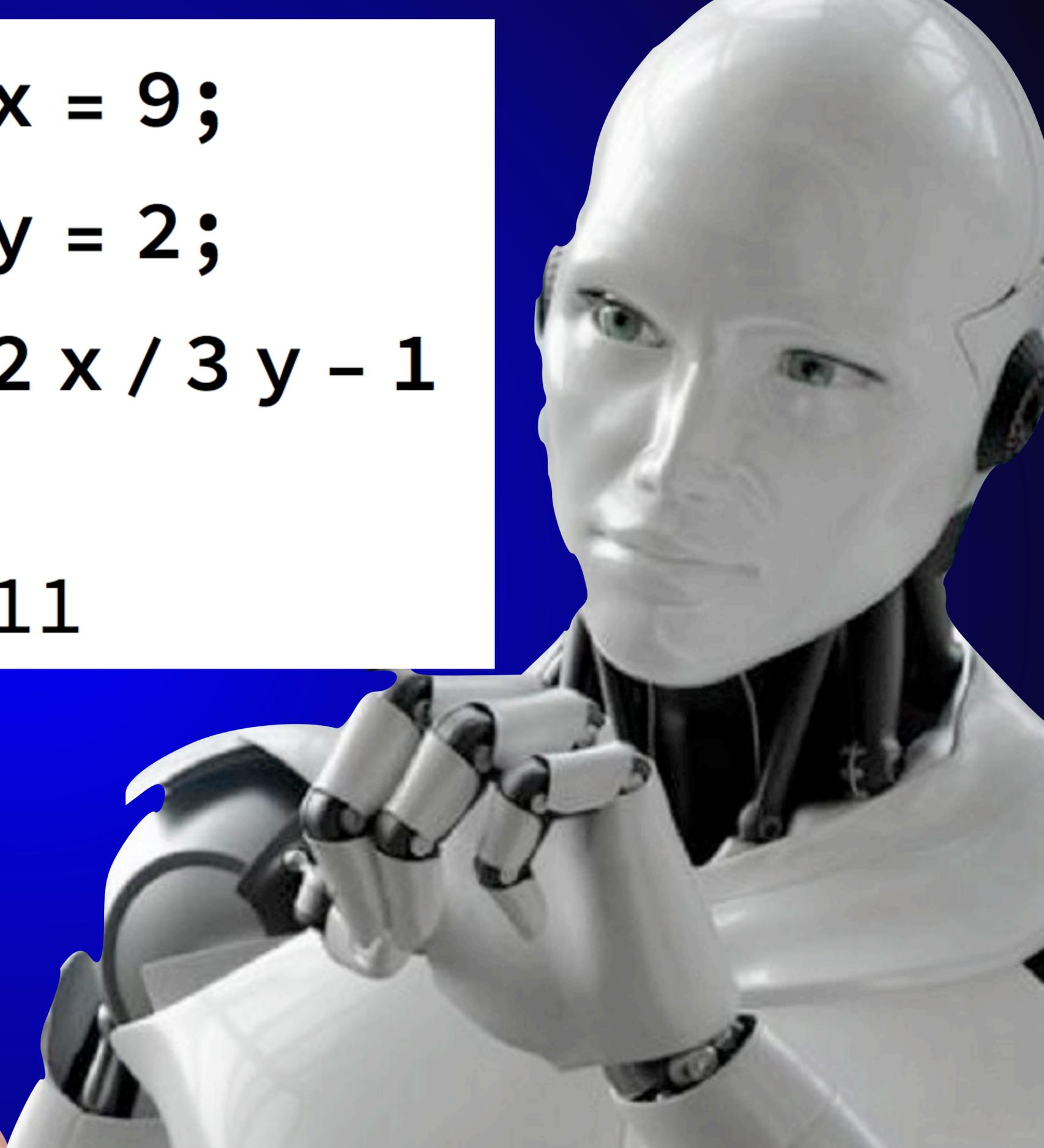
$x = 9;$

$y = 2;$

$2x / 3y - 1$

11

100%
HUMAN



Where did it come
from?

Email from 2014:

"The problem $2x/3y-1$ with $x=9$ and $y=2$ was actually posed for a 5th grade homework question and it is interesting how much debate has occurred between our parents. The students who had help from parents had the answer of 2. The teacher explained that the correct answer was 11."

Please Excuse My

$$2x/3y-1$$

$$x=9$$

$$y=2$$

Website entry

Dear Aunt Sally

Fall 2017

⑨

$$X^3/3 + y = \frac{X^3}{3} + y$$

PEMDAS

Annotations for PEMDAS:

- Parameters (arrow to P)
- Exp (arrow to E)
- Div (arrow to D)
- Add (arrow to A)
- Sub (arrow to S)
- mult (arrow to M)

Please Excuse
My Dear Aunt
Sally

January 2018

You were called an idiot so I thought it fair you should be informed to defend yourself.... Old x 

 **Timothy**
to knill 

@gmail.com>

Jan 18



Reference this link for the debate: <https://www.youtube.com/watch?v=URcUvFIUhQ>

In the silly debate about a YouTube video on the equation $6 \div 2(1+2)$, poster Lennard Church disputed your article <http://www.math.harvard.edu/~knill/pedagogy/ambiguity/>

Timothy, whoever wrote that article is an idiot,

the problem?"

Notation

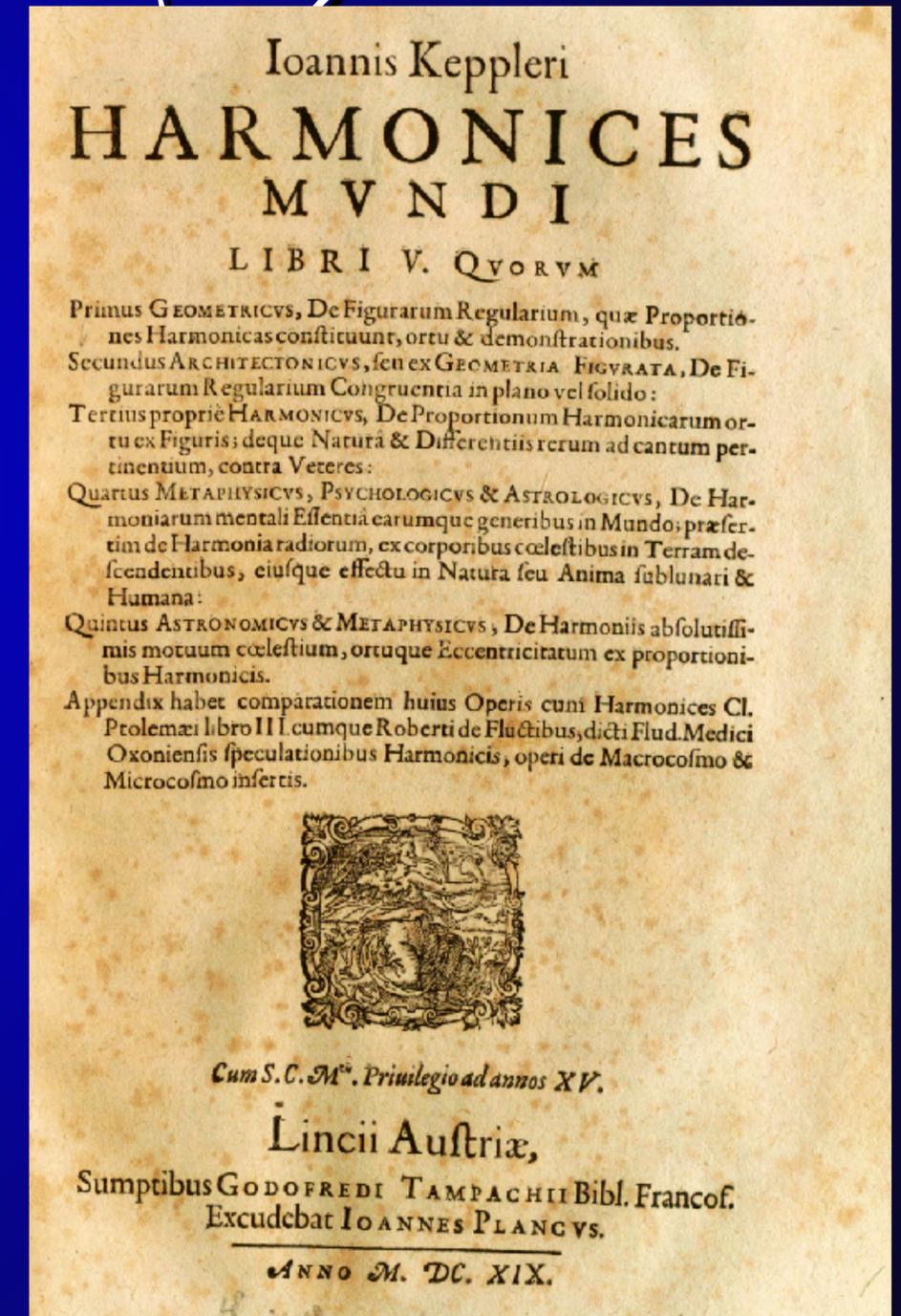
Music and Math



Kepler: 1571-1630

Kepler:
musical
notation

Music
Notation



Music

Prelude

Op. 28, No. 7

Frederic Chopin

Piano

Andantino

p dolce

con pedale

8

mp

mp

rit. e dim. - - - pp

Math

$$(1'') \quad \left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial z}\right)^2 - \frac{2m}{K^2} \left(E + \frac{e^2}{r}\right) \psi^2 = 0 .$$
$$r = \sqrt{x^2 + y^2 + z^2} .$$

Und unser Variationsproblem lautet

$$(3) \quad \left\{ \begin{array}{l} \delta J = \delta \iiint dx dy dz \left[\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial z}\right)^2 - \right. \\ \left. - \frac{2m}{K^2} \left(E + \frac{e^2}{r}\right) \psi^2 \right] = 0 , \end{array} \right.$$

das Integral erstreckt über den ganzen Raum. Man findet daraus in gewohnter Weise

$$(4) \quad \left\{ \begin{array}{l} \frac{1}{2} \delta J = \int df \delta \psi \frac{\partial \psi}{\partial n} - \iiint dx dy dz \delta \psi \left[\Delta \psi + \right. \\ \left. + \frac{2m}{K^2} \left(E + \frac{e^2}{r}\right) \psi \right] = 0 . \end{array} \right.$$

Es muß also erstens

$$(5) \quad \Delta \psi + \frac{2m}{K^2} \left(E + \frac{e^2}{r}\right) \psi = 0$$

1) Es entgeht mir nicht, daß diese Formulierung nicht ganz eindeutig ist.

B) Experiments

What is $6 \div 2(2 + 1)$?

$$(6/2)(2+1) = 3 * 3 = 9 \quad (\text{PEDMAS})$$

$$6/(2*3) = 1 \quad (\text{PEMDAS})$$

What is $4x$ for $x=3$?

43 (decimal writing) or 12 (3×4) ?

What is $4\frac{1}{2}$?

2 (4 times $1/2$) or 4.5?

Is $2 < 2$ true?

*especially in colloquial language,
we mean with "less" often also
"less or equal".*

Is $0 \in \mathbb{N}$?

This is not uniform. Some include 0 in the natural numbers, others not.

ISO 80000-2, Notion of "Whole numbers"

What is $\log(10)$?

Most mathematicians and "grown up" computer algebra systems consider $\log = \ln$, some legacy frameworks in calculus or excel understand with \log the base 10 log.

What is -2^4 ?

The common understanding is 16.

*There can be misunderstanding
in that one understand $(-2)^4=16$.*

Compute $5 * 3!$

*Is it $15!$ or $5 * 6 = 30$. The common understanding is that the factorial is evaluated before everything else.*

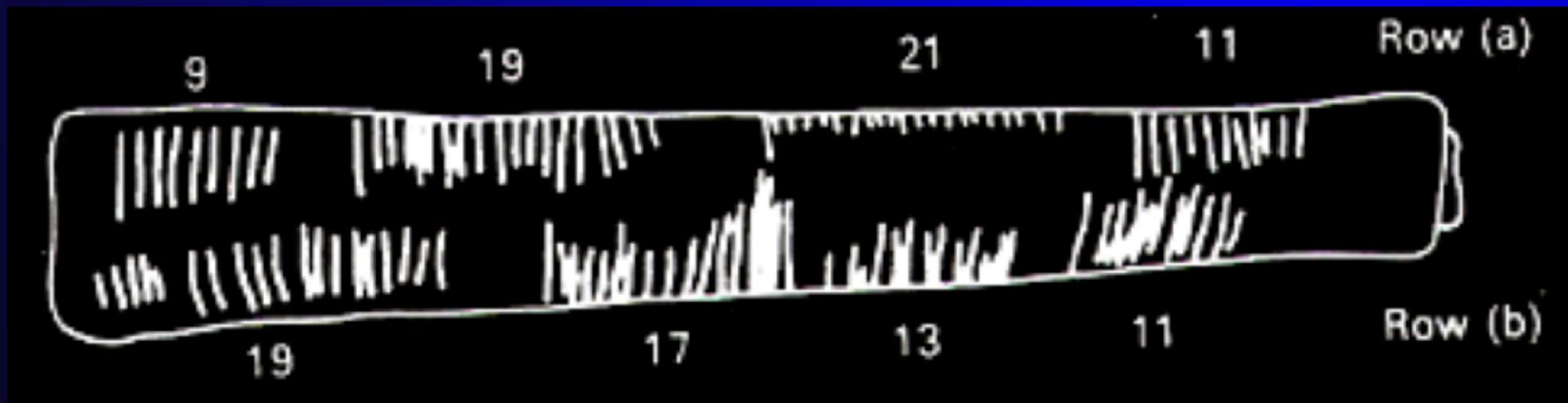
*C) Some History of
Notation*

Since when do Humans
write Math?

Ishango Bone, -20'000 Years



60



primes

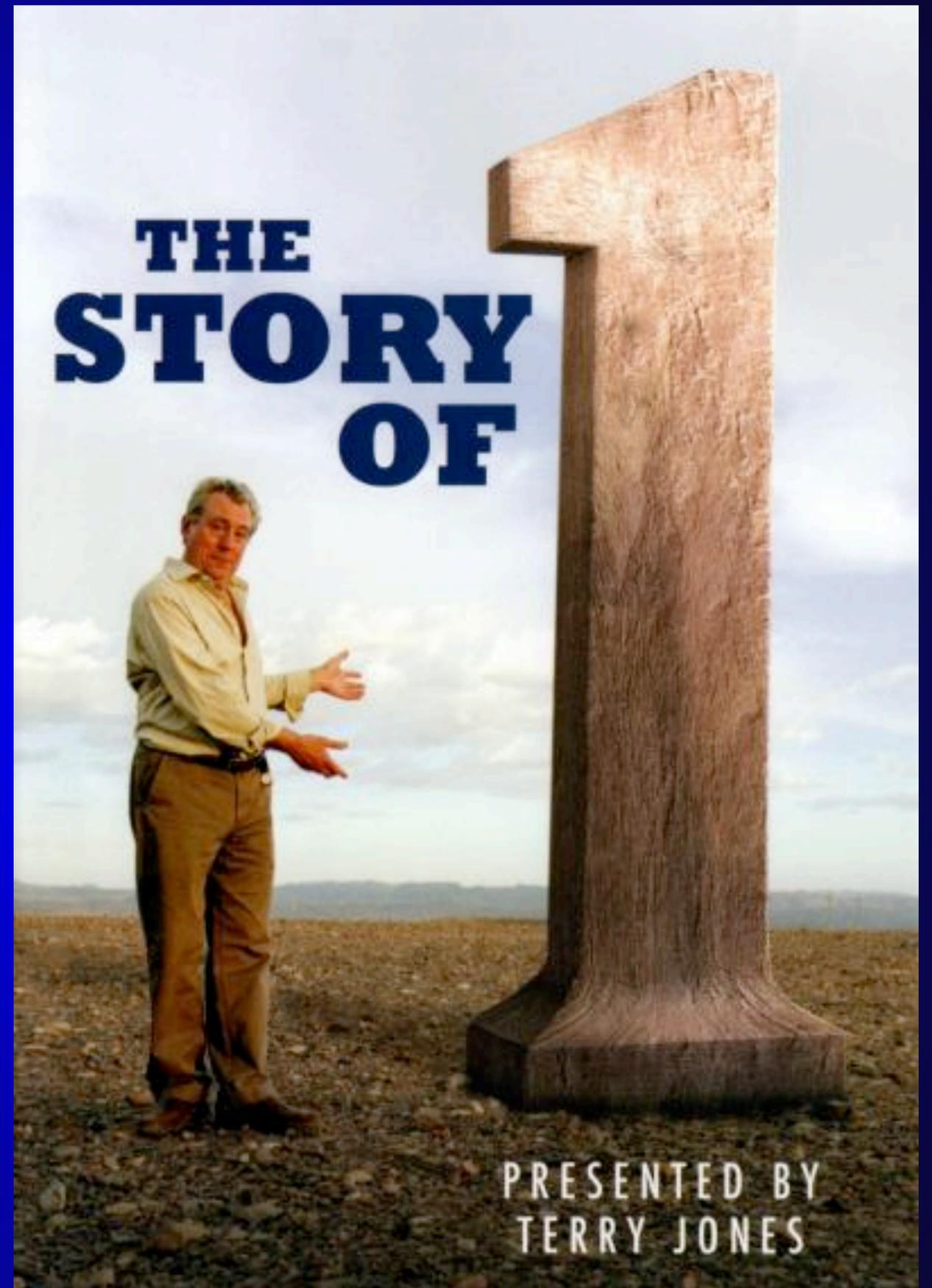
60

cradle of mathematics

Question 1:

Since when do we
know the number 1?

Story of 1





Lebombo

40000 BC



Plympton

1800 BC

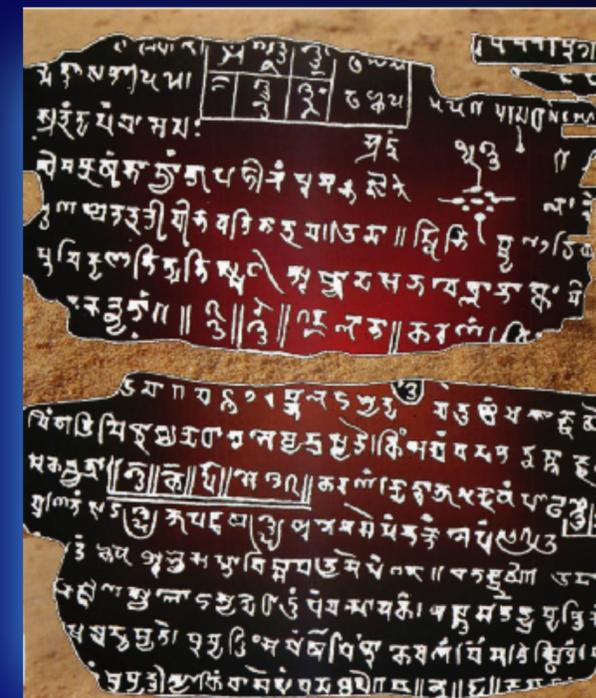


YBC 7289



Maya

150 AD



Bakshali

400 AD



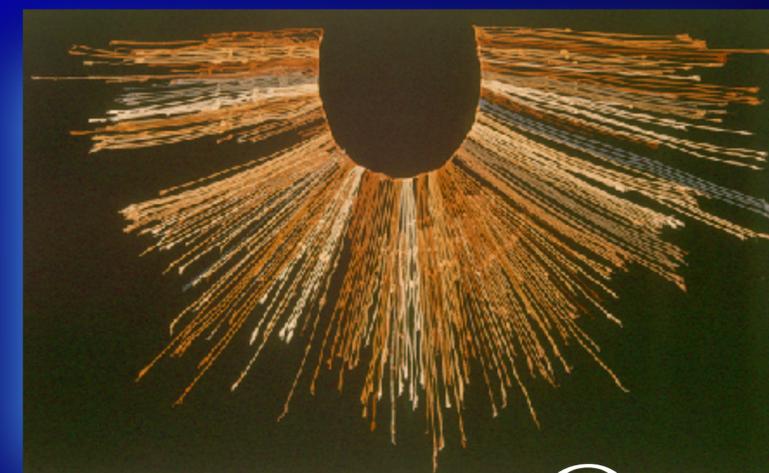
Ishango

20000 BC



Rhind

1600 BC



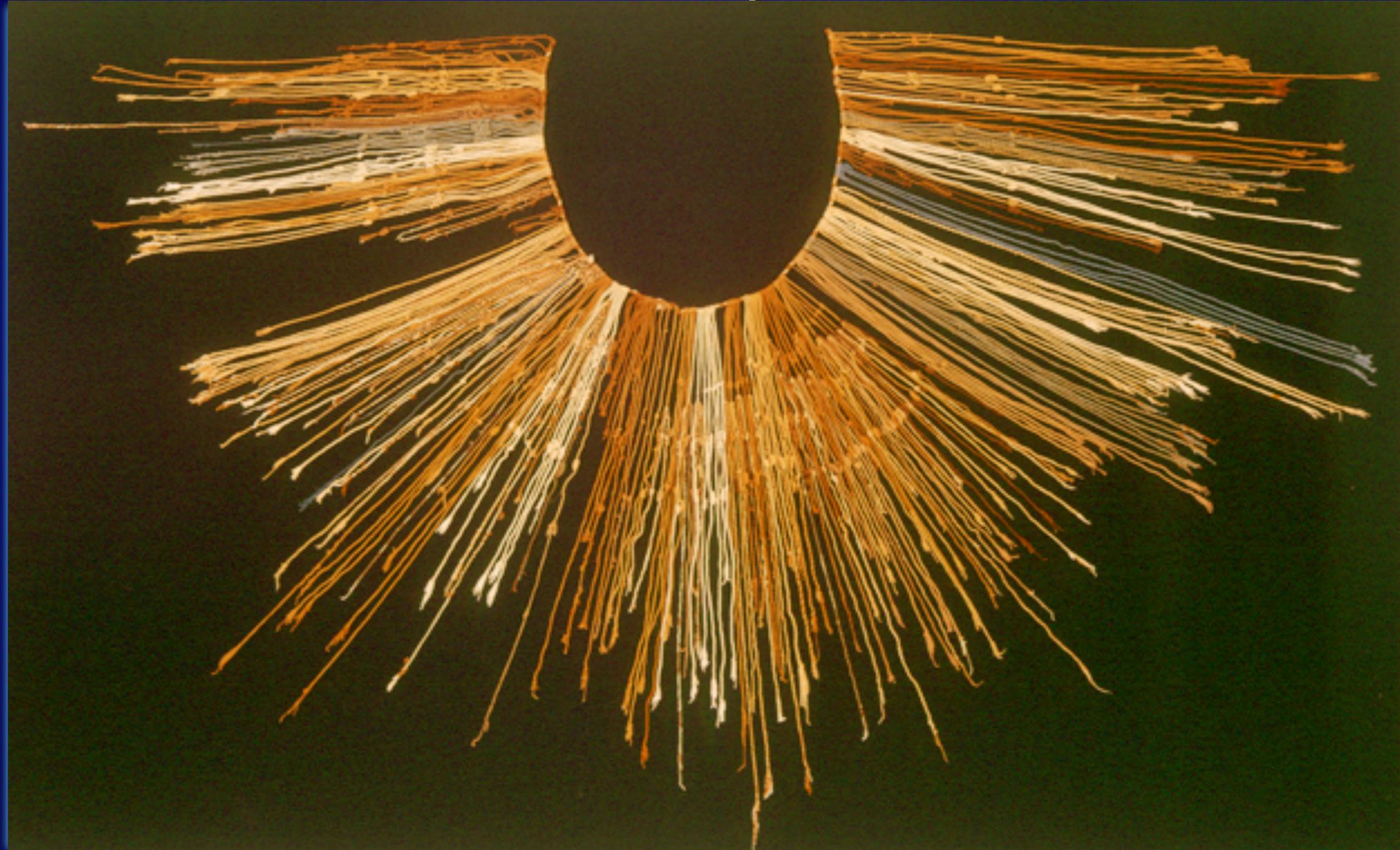
Quipu

1000AD

Question 2:

Since when does one
write 0?

Quipu

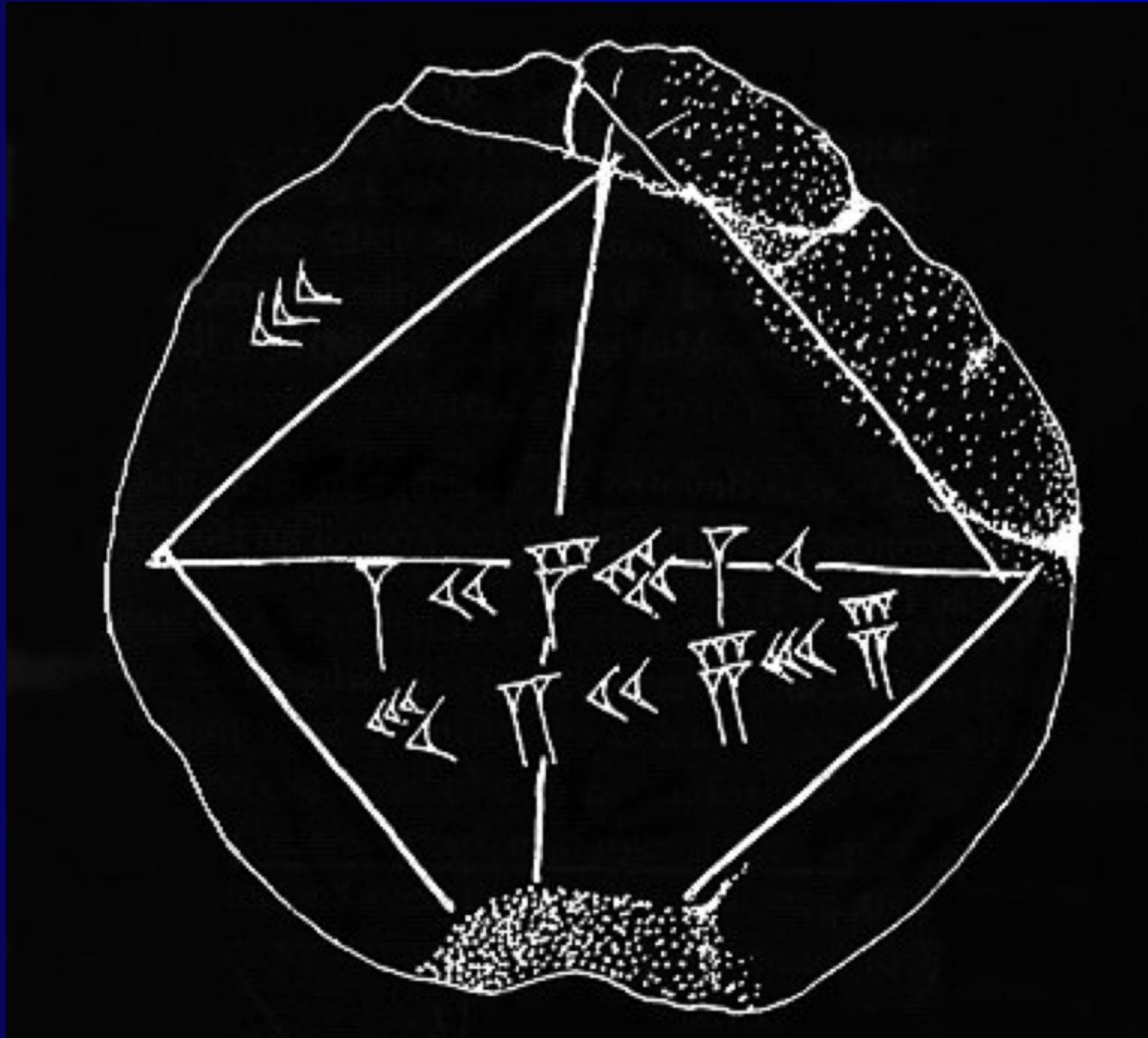


INCAS, ANDEAN CULTURES



YBC 7289

0.30



1.24 51 10

$$1 + 24/60 + 51/3600 + 10/60^3$$

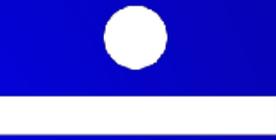
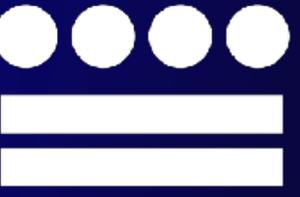
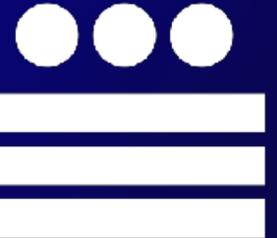
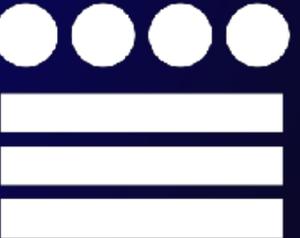
$$0 + 42/60 + 25/3600 + 35/60^3$$

0.42 25 35

Maya Numerals

36 BC

Stela 2 Chiapa de
Corzo, Chiapas

0	1	2	3	4
				
5	6	7	8	9
				
10	11	12	13	14
				
15	16	17	18	19
				



Chiapa de Corzo

Zero

130 AD Ptolemy $\overline{0}$

600 Brahmagupta (rules)

Sifr (Arabic) \longrightarrow Sunya (Sanskrit)

Fibonacci
1170–1250

Zephyrum

\downarrow
Zefiro
Zero

Question 3:

Since when do we
write +?

Answer:

Nicholas Oresme

1320–1382



$$1 + 1/2 + 1/3 + \dots = \infty$$

harmonic series

Question 4:

When was the first
arithmetic printed?

e quante miglia hauera fatto ciaschaduno di loro.
 fa secondo la riegula cosi.

$$\begin{array}{r} 7 \\ 9 \\ \hline 16 \end{array} \text{partitore}$$

$$\begin{array}{r} i \\ 35 \\ 63 \\ \hline 16 \end{array} \text{30mi.}$$

Unde in 30mi. 3.e $\frac{i5}{16}$ se scontrerano.

Se tu vuol sapere quãta miglia hauera fatto ciaschaduno: fa per la riegula del. 3. dicendo
 E primo per quellui da Roma.

$$\begin{array}{r} i i 2 \\ 7 \\ \hline 1 \end{array} \times \frac{250}{1} = \frac{63}{16}$$

$$\begin{array}{r} 250 \\ 63 \\ \hline 750 \\ 500 \\ \hline 15750 \end{array}$$

$$\begin{array}{r} 16) 250 \\ 95 \\ \hline 1650 \\ 1600 \\ \hline 50 \\ 32 \\ \hline 18 \\ 16 \\ \hline 2 \end{array}$$

Quellui che vien da Roma hauera fatto miglia
 i 40.e $\frac{5}{8}$ Doi mettila riegula per
 el corriero da Venexia.

$$\begin{array}{r} i 44 \\ 9 \\ \hline 1 \end{array} \times \frac{250}{1} = \frac{63}{16}$$

$$\begin{array}{r} 44 \\ 35 \\ \hline 794 \\ 875 \\ \hline 1649 \\ 1600 \\ \hline 49 \\ 32 \\ \hline 17 \end{array}$$

1478 Treviso



First printed arithmetic
 1478 at Treviso in Northeastern Italy

FIG. 38.—From the earliest printed arithmetic, 1478

A historical Problem

A courier travels from Rome to Venice in 7 days. An other courier starts at the same time and travels from Venice to Rome in 9 days. The distance from Venice to Rome is 250 miles. In how many days will the couriers meet?

A HISTORY OF ATHEMATICAL NOTATIONS

By

FLORIAN CAJORI, PH.D.

*Professor of the History of Mathematics
University of California*

VOLUME I

NOTATIONS IN ELEMENTARY
MATHEMATICS

THE OPEN COURT COMPANY,
PUBLISHERS,
86, STRAND, LONDON, W.C.2.

Solution

velocity=distance/time

The first courier has speed $250/9$

The second courier has speed $250/7$

Their relative speed is the sum. The time is

$$x = 250 / (250/7 + 250/9) = \\ 1 / (1/7 + 1/9) = 63/16$$

È primo per quelli da Roma.

i i 2

$$\begin{array}{r} \text{ } \\ \text{ } \\ \text{ } \\ \hline 2 \\ \text{ } \\ \hline 1 \end{array}$$

×

$$\begin{array}{r} 250 \\ \hline 1 \end{array}$$

$$\begin{array}{r} \text{---} \\ \text{---} \end{array}$$

$$\begin{array}{r} 63 \\ \hline 16 \end{array}$$

16) *

$$\begin{array}{r} * * 2 \\ * * 9 * 0 \text{ | } i 40 \\ * * 2 * 2 \\ * * * \\ * \end{array}$$

$$\begin{array}{r} 250 \\ \hline 63 \\ \hline 250 \end{array}$$

500

$$\begin{array}{r} \hline 15750 \end{array}$$

Quellui che vien da Roma bauera fatto miglia

1489

4 + 5 Wilt du das wys
 4 — 17 sen oder defigley
 3 + 30 chen / So sumier
 4 — 19 die zentner vnd
 3 + 44 lb vnd was auß
 3 + 22 — ist / das ist mi
Zentner 3 — 11 lb nus dz setz beson
 3 + 50 der vnd werden
 4 — 16 4539 lb (So
 3 + 44 du die zentner
 3 + 29 zu lb gemacht
 3 — 12 hast vnd das /
 3 + 9 + das ist meer

darzu addierest vnd 75 minus. Nun
 solc du für Holz abschlahen allweeg für
 ain legel 24 lb. Vnd das ist 13 mal 24.
 vnd macht 312 lb darzu addier das —
 das ist 75 lb vnd werden 387. Dye sub
 trahier von 4539. Vnd bleyben 4152
 lb. Nun sprich 100 lb das ist ein zentner
 pro 4 fl $\frac{1}{8}$ wie künnen 4152 lb vnd kumē
 171 fl 5 β 4 heller $\frac{2}{3}$ Vñ ist rechte gemacht

Pfeffer



Mercantile Arithmetic, Johannes Widmann Leipzig

D) More riddles

What is $6 / 2(2 + 1)$?

What is

$$\frac{6}{2(2+1)}$$

?

Teacher: What is $11q - q$?

Student: $10q$

Teacher: you are welcome

Teacher: What is $2k+k$?

Student 3000 .

What is $2/2/2$?

What is $1/2x$?

TI-82: $1/(2x)$

TI-83: $(1/2)x$



TI-82



TI-83

What is 2^{2^2} ?

What is 0^0 ?

What is $0/0$?

What is x/x for $x \rightarrow 0$?

What is x^2/x for $x \rightarrow 0$?

What is

$$1 + 2 + 3 + 4 + \dots$$

Answer: $-1/12$

E) Social Media

HP Forum Archive 21

[[Return to Index](#) | [Top of Index](#)]

PEMDAS

Message #1 Posted by [Les Koller](#) on 24 July 2013, 8:09 p.m.

This has probably been discussed here before, but I'd like some input. Without using ANY technology, just your brain and education, what is the value of

$$6 \div 2(1+2)$$

What is your justification? Does your HP Calculator give the same answer? How about your TI?

Thanks for humoring me here...I've never seen this one before.

Re: PEMDAS

*Message #2 Posted by [Kimberly Thompson](#) on 24 July 2013, 8:23 p.m.,
in response to message #1 by [Les Koller](#)*

Les

Social media aside, the answer is in the title PEMDAS. Using the *convention* from my Engineering studies, I get ONE.

Other *conventions* will possibly yield other results.

I learned my MATH from mathematicians, while attaining an Engineer Degree. I'm very happy with their tutoring.

SlideRule

Edited: 25 July 2013, 7:45 a.m.

Re: PEMDAS

Message #8 Posted by [William L. Drylie](#) on 28 July 2013, 3:41 a.m., in response to message #3 by Les Koller

Hi Les;

Physicists and Mathematicians do not do it differently. The answer is = 1. Any competent Algebra text with a review of the basic axioms of Arithmetic will tell you to treat the numerator and denominator as if they were enclosed in parentheses. 6 is one expression and 2(1+2) is another expression. The 2(1+2) gets done first as the 1+2 is enclosed and the parentheses are removed by the distributive property. What is left then is 6/6=1. I showed this to a Physics Professor at Duke University who I am friends with and he said "surely you're joking"? I said "no" and he said "if you are going to make me tell you the answer to that, you will buy me a beer after work, and I may consider not having your degree revoked"!! (laughing) he said "1...you idiot" How shameful!! I actually treated him to a Bratwurst and an 8 dollar German beer at the Bavarian Brat House in Chapel Hill NC. after work and told him the story. The TI's, even with the N-Spire CX CAS, expect you to have a certain level of expertise with entering Algebraic expressions. If you enter (6)/(2*(1+2)) you will get the correct answer of 1. You can see it is also correct as two expressions and it satisfies TI's syntax for Algebraic expressions. My old 48SX shows the answer as 1, written as you stated the original. The hp does the work.

Sincerely, Bill Drylie

Re: PEMDAS

Message #9 Posted by [jep2276](#) on 28 July 2013, 11:23 a.m., in response to message #8 by William L. Drylie

Bill,

Why do people think that 2(x+2) is actually 2*(x+2) and not understanding that 2 is the coefficient of (x+2) and therefore a factor that cannot be separated?



@Jonathan Chaim

Geniuses Only

look close on details

**ReShare to see
you're friends**

$$\begin{array}{c}
 \text{Hexagon with square inside} + \text{Hexagon with square inside} + \text{Hexagon with square inside} = 45
 \end{array}$$

$$\begin{array}{c}
 \text{Bananas} + \text{Bananas} + \text{Hexagon with square inside} = 23
 \end{array}$$

$$\begin{array}{c}
 \text{Bananas} + \text{Clock} + \text{Clock} = 10
 \end{array}$$

$$\begin{array}{c}
 \text{Clock} + \text{Bananas} + \text{Bananas} \times \text{Hexagon with square inside} = ??
 \end{array}$$

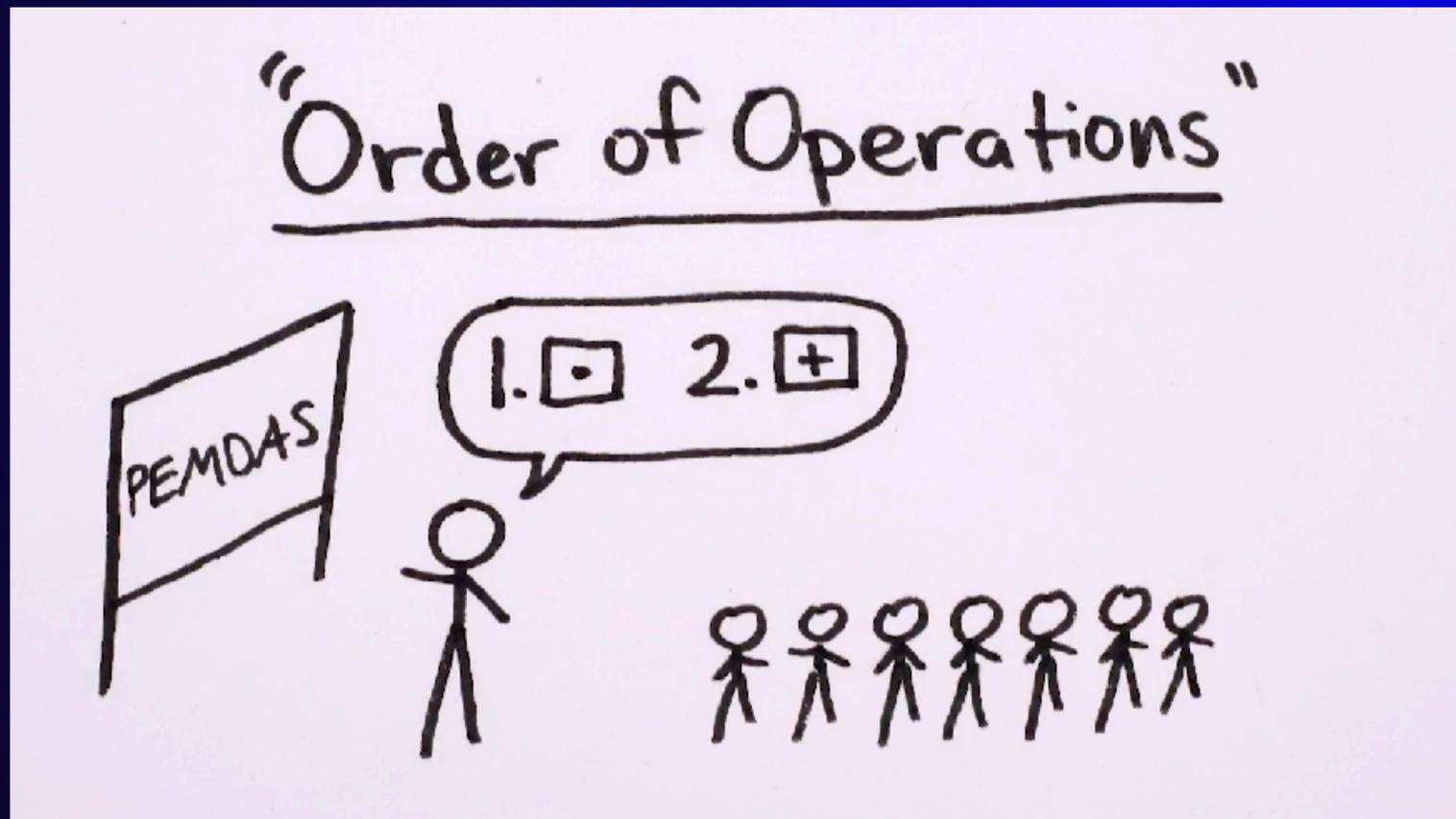
$$6/2(1+2)$$

$$1/2x$$

Youtube

6/2(1+2) or 6÷2(1+2) equals ? Final Thoughts

Youtube: "morally wrong"



One of the Basic Math Rules You Learned In School Is Wrong

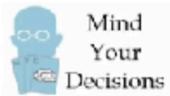
Eric Limer
5/18/13 7:30pm • Filed to: MATH

248.2K 222 3

~~PEMDAS~~

Can You Solve This?

$$6 \div 2(1+2) =$$



What is $6 \div 2(1+2) = ?$ The Correct

7.7M views



25K

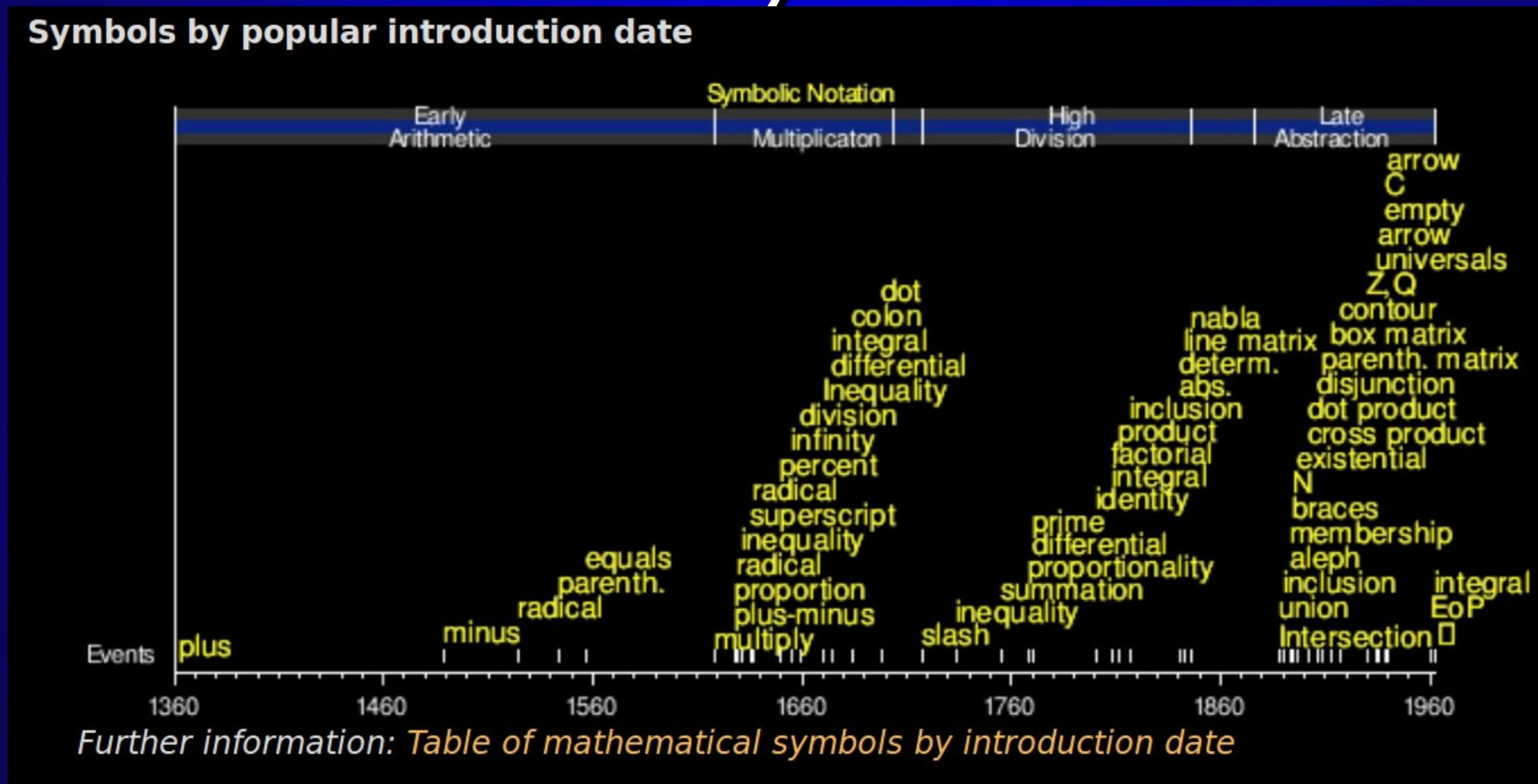


21K

Youtube

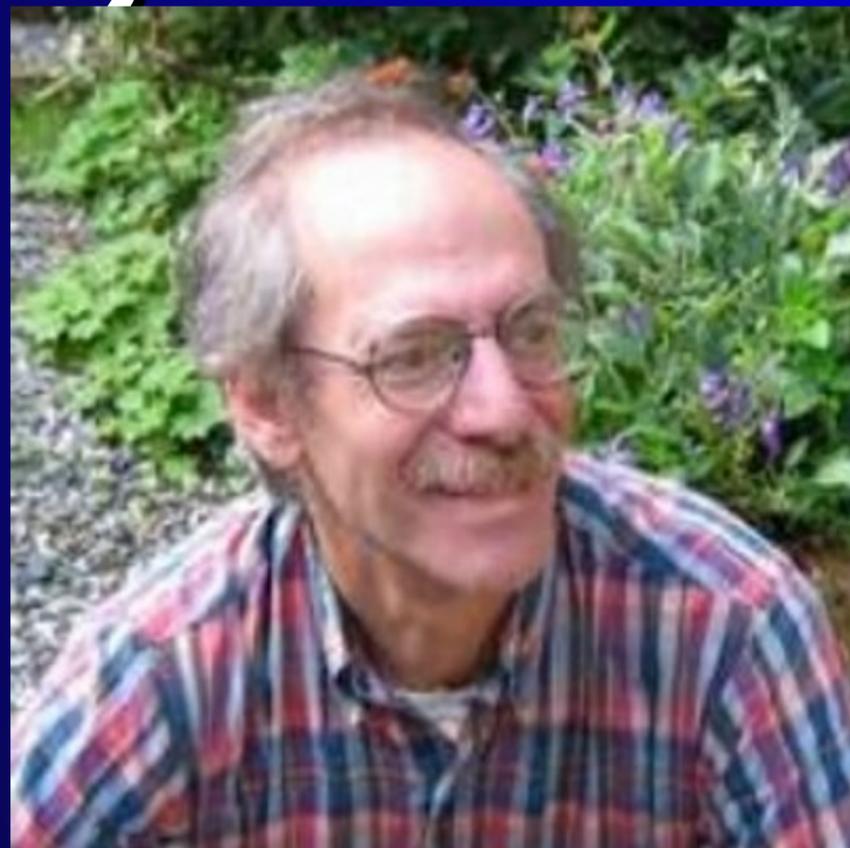
E) Literature

Wikipedia



history of mathematical notation

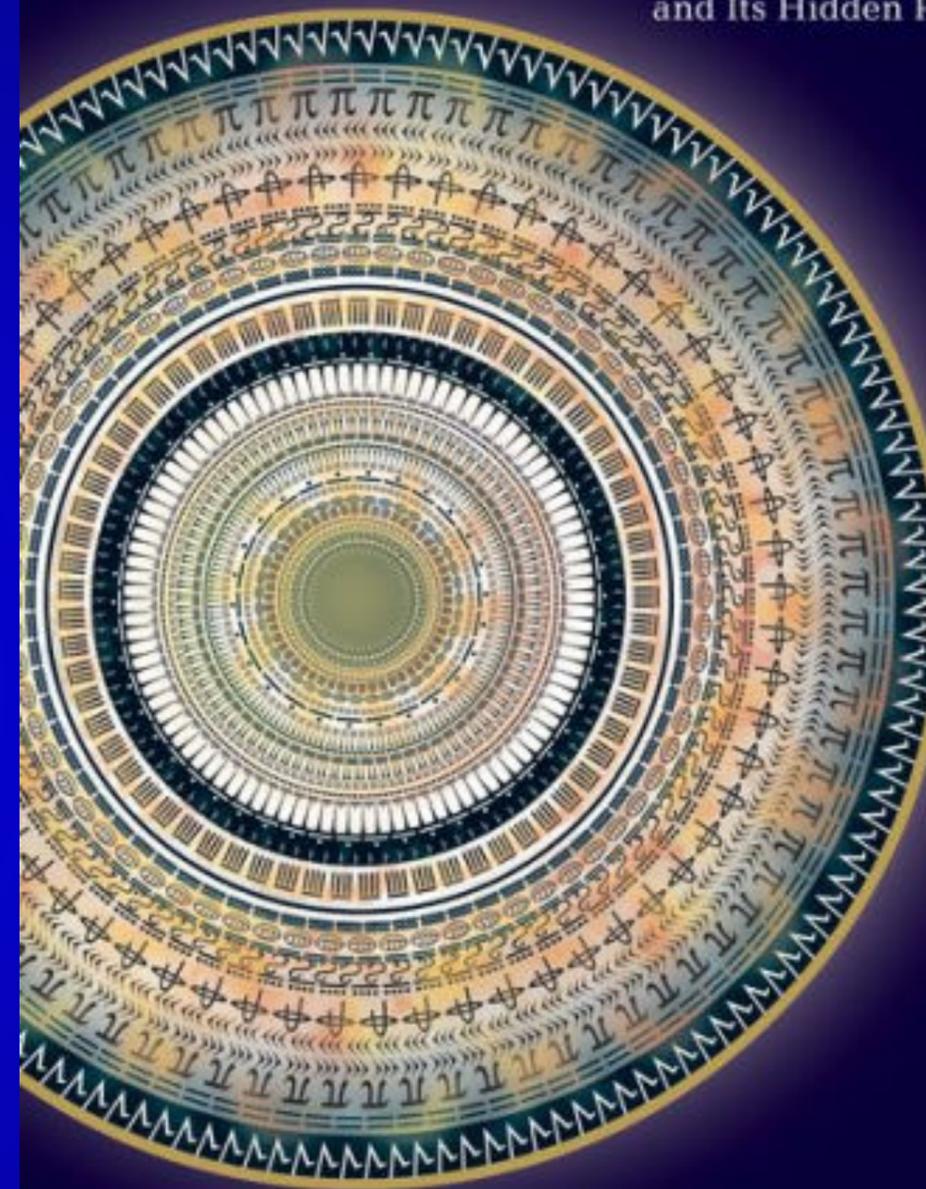
Joseph Mazur



Enlightening Symbols

A Short History of Mathematical Notation

and Its Hidden Powers



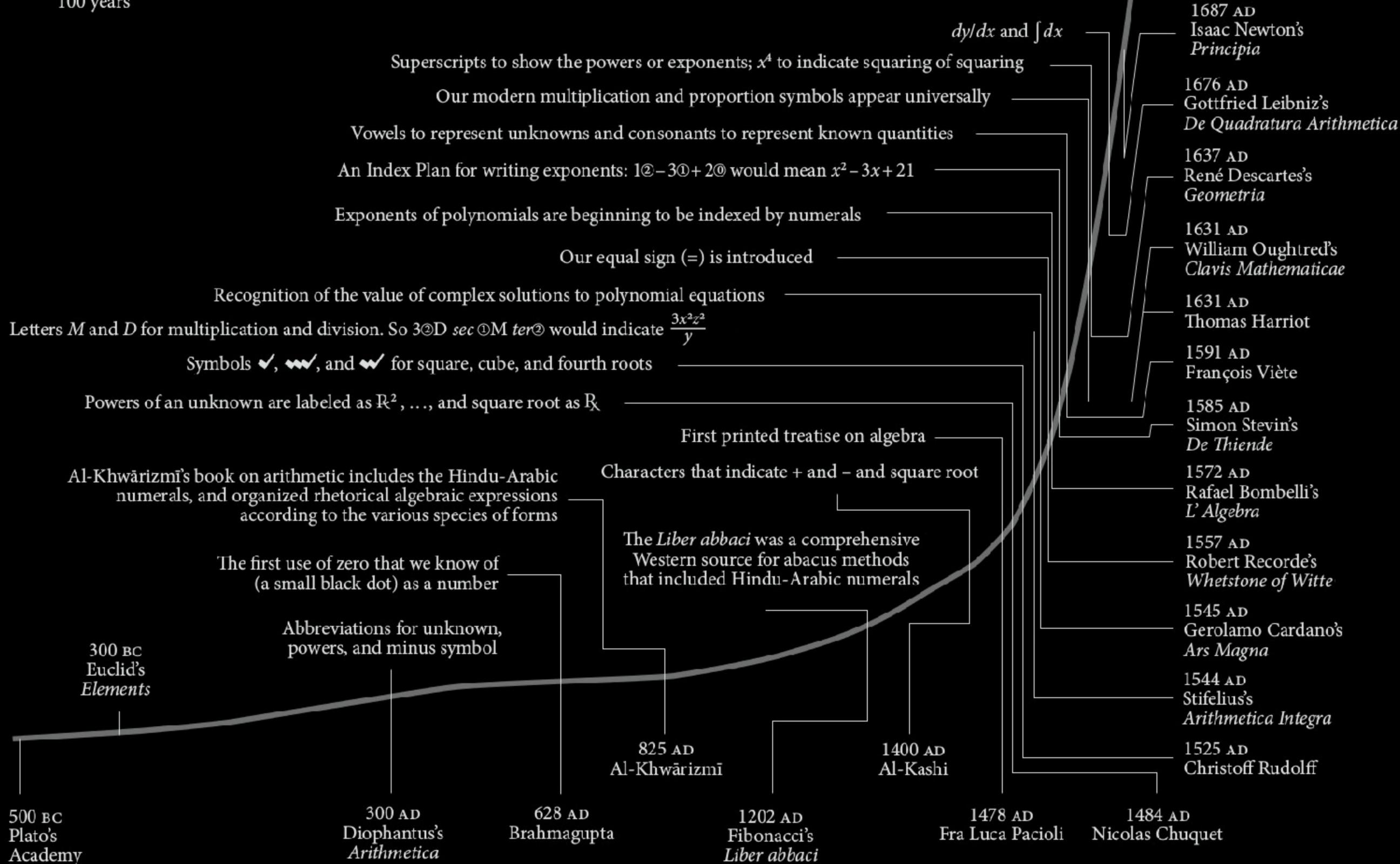
Joseph Mazur

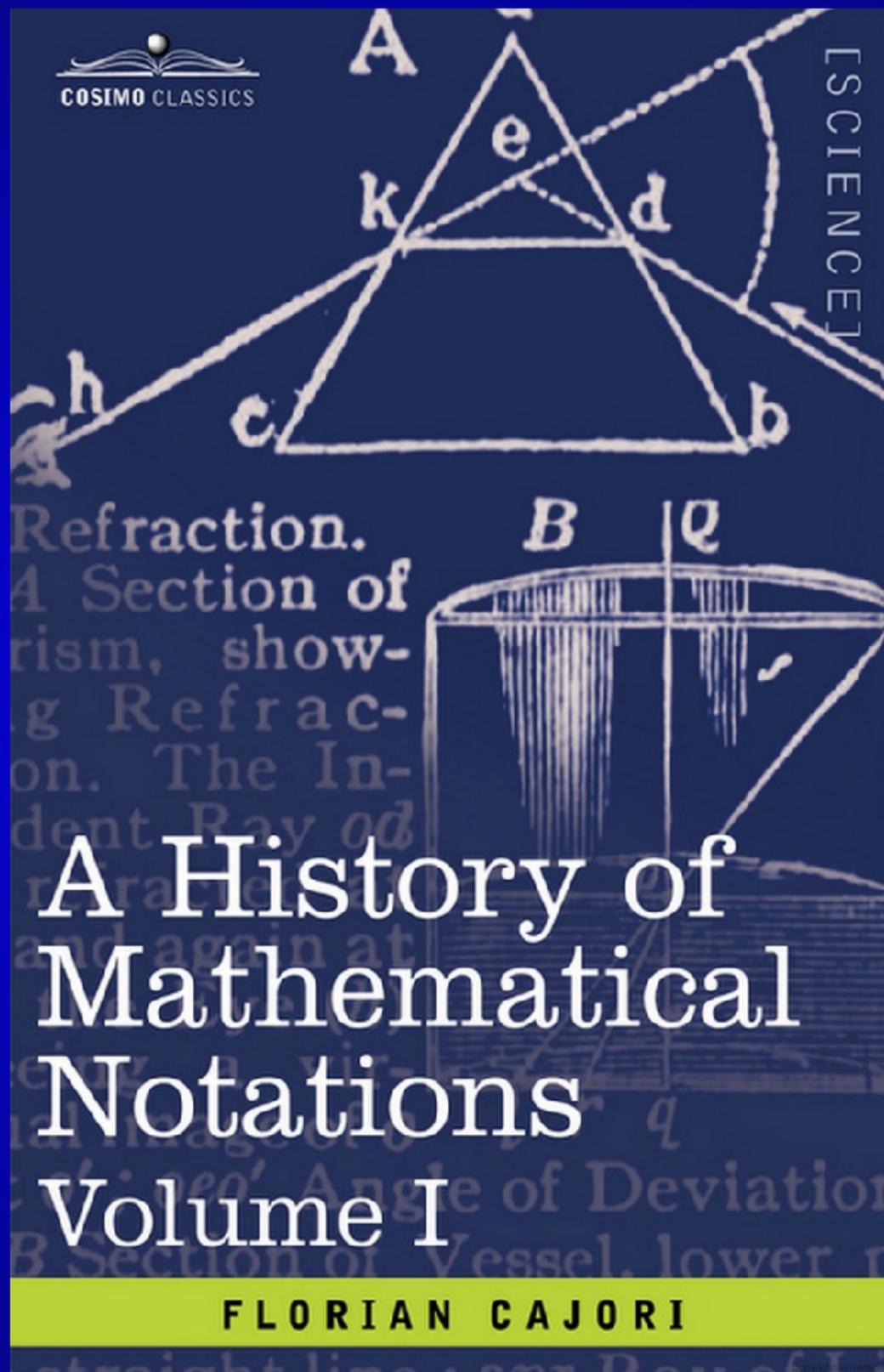
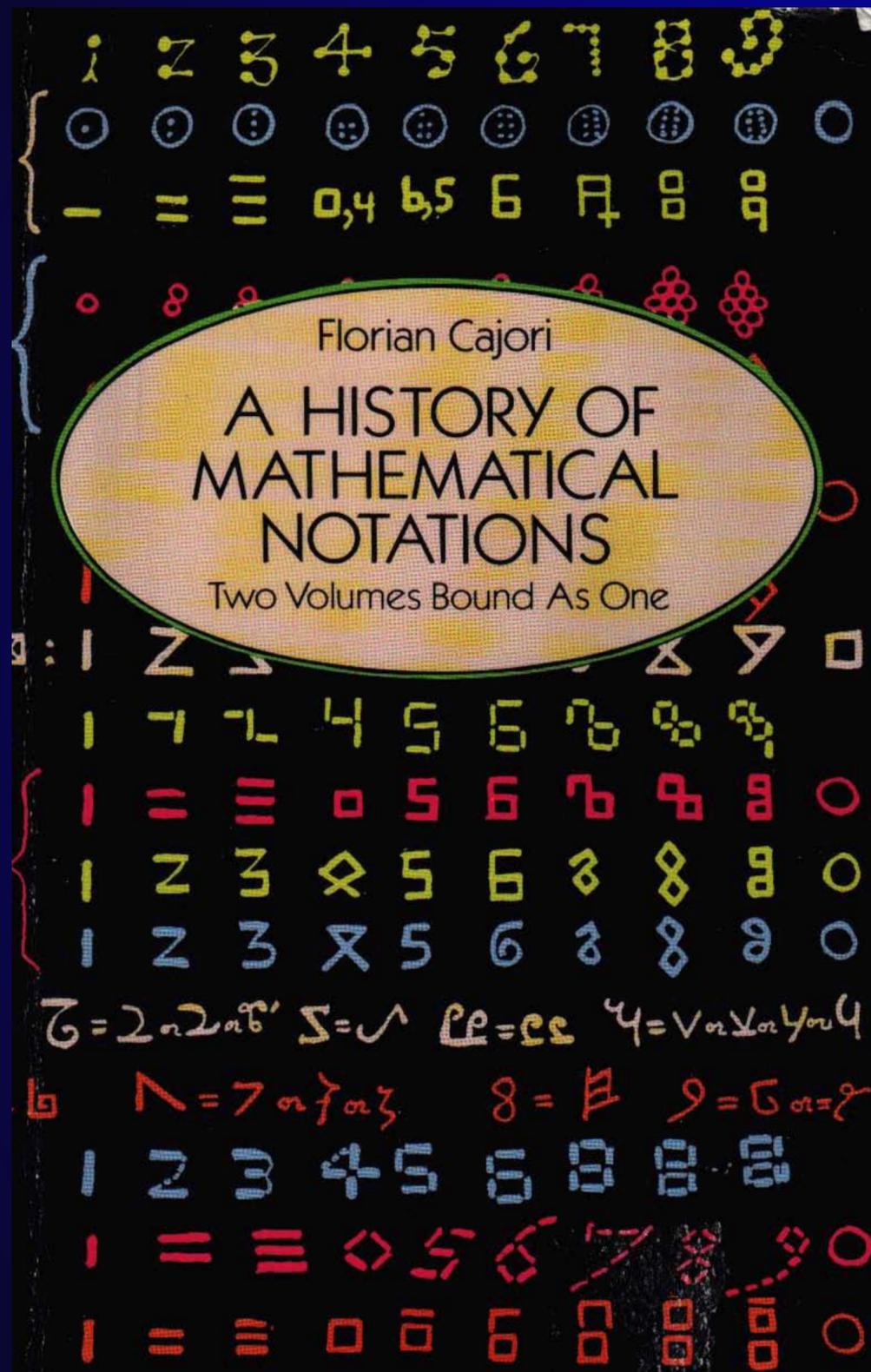
Enlightening symbols

The Significant Initiators

From "enlightening Symbols"

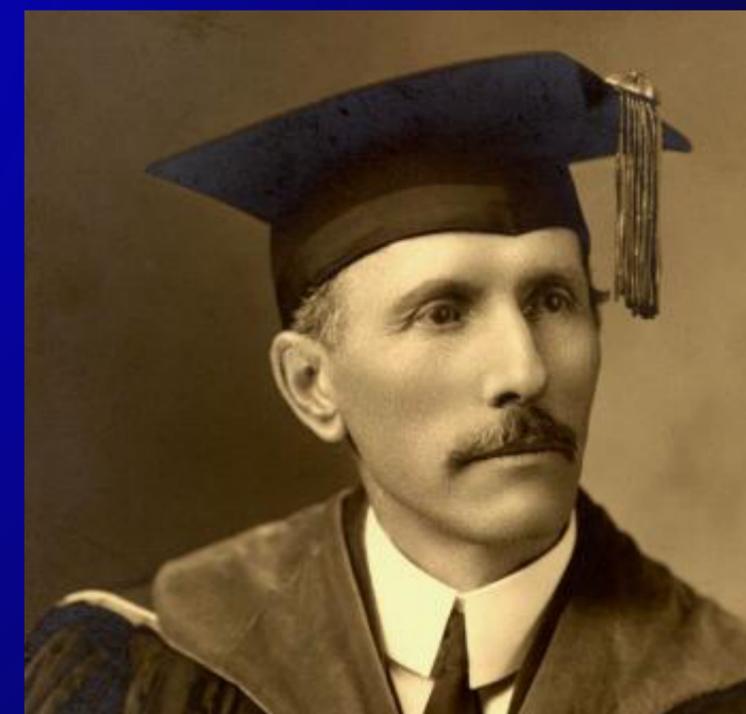
100 years





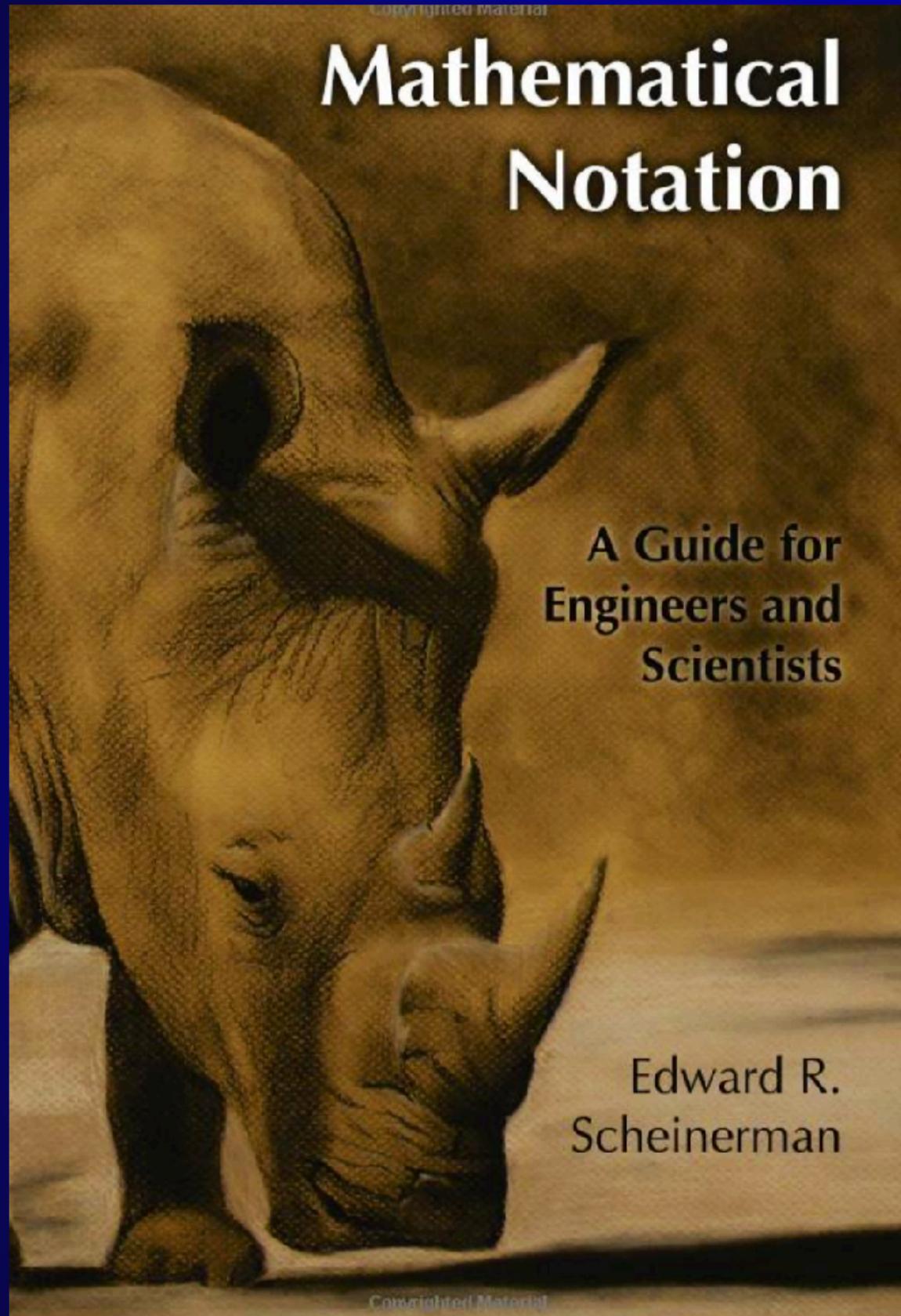
Florian Cajori

1859-1930



242. *Order of operations in terms containing both \div and \times .*—If an arithmetical or algebraical term contains \div and \times , there is at present no agreement as to which sign shall be used first. “It is best to avoid such expressions.”³ For instance, if in $24 \div 4 \times 2$ the signs are used as they occur in the order from left to right, the answer is 12; if the sign \times is used first, the answer is 3.

Some authors follow the rule that the multiplications and divisions shall be taken in the order in which they occur.⁴ Other textbook writers direct that multiplications in any order be performed first, then divisions as they occur from left to right.⁵ The term $a \div b \times b$ is interpreted by Fisher and Schwatt⁶ as $(a \div b) \times b$. An English committee⁷ recommends the use of brackets to avoid ambiguity in such cases.



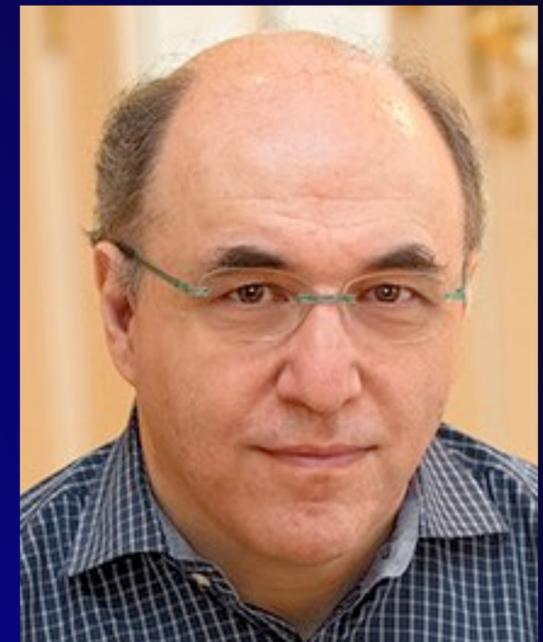
Edward Scheinerman



F) Conclusions

*Be clear. Especially
if no consensus exists*

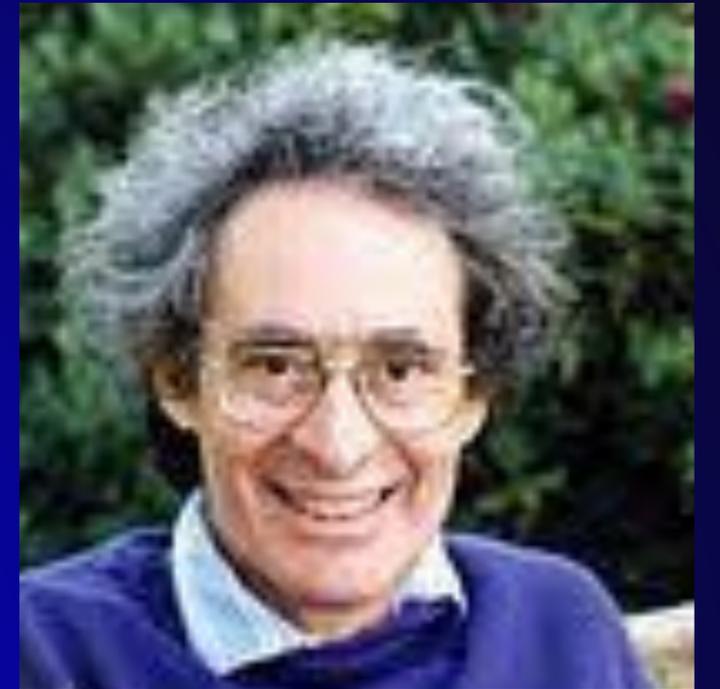
A quote



"Mathematical notation isn't really a problem about mathematics. It's really more a problem in linguistics. It's not about what mathematical notation could conceivably be like; it's about what mathematical notation as it's actually used is actually like—as it's emerged from history and presumably from the constraints of human cognition and so on. And, in fact, I think mathematical notation is a pretty interesting example for the field of linguistics."

S. Wolfram, *Mathematical Notation: Past and Future* (2000)

A quote



"A seemingly modest change of notation may suggest a radical shift in viewpoint. Any new notation may ask new questions"

Barry Mazur, cited in "Enlightening Symbols"

The end