# CALCULUS AND ECONOMICS

Oliver Knill

April 26, 2013

# TOTAL AND MARGINAL COST

$C(x)$ Total Cost

$C'(x)$ Marginal Cost

$x$ = quantity of goods sold

**Example**: Where is the total cost $C(x)$ maximal, if

$$C(x) = x^3 - 4x$$

# THE STRAWBERRY THEOREM

see Heckner/Kretschmer:
Don't worry about Micro

Marginal Cost $f(x)$:
of one Strawberry

Total Cost $F(x)$
of $x$ strawberries

Average Cost $g(x) = F(x)/x$

Expected cost of one strawberry if x strawberrys are present.

# SUMMARY SO FAR

TOTAL COST: $F(x)$

MARGINAL COST: $f(x)$

AVERAGE COST: $g(x)=F(x)/x$

Formulas to remember:

$$F'(x) = f(x)$$
$$F(x)/x = g(x)$$

x

x=1 x=2 x=3 x=4 x=5 x=6

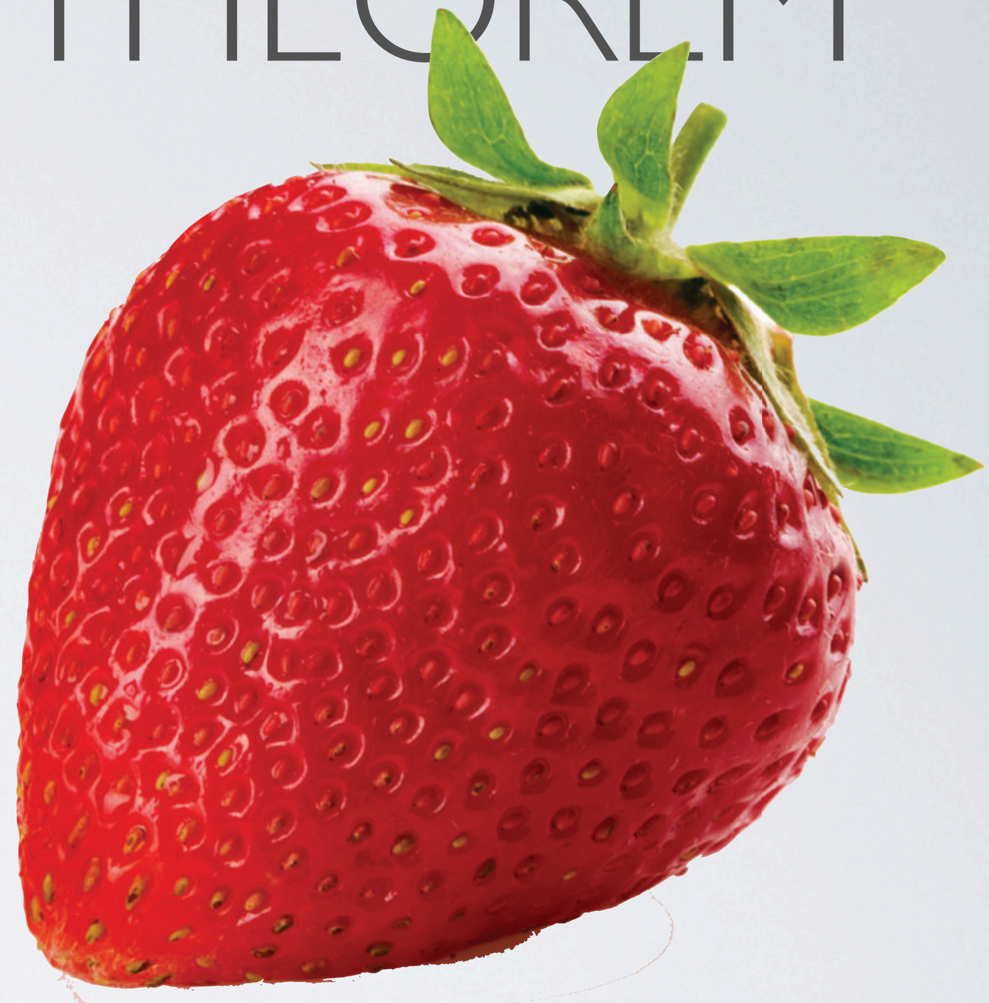
total cost F(3):
cost of the strawberry 3

marginal cost f(3):
cost of the strawberry 3

# THE STRAWBERRY THEOREM

Critical points of the average cost are break even points.

# THE STRAWBERRY THEOREM



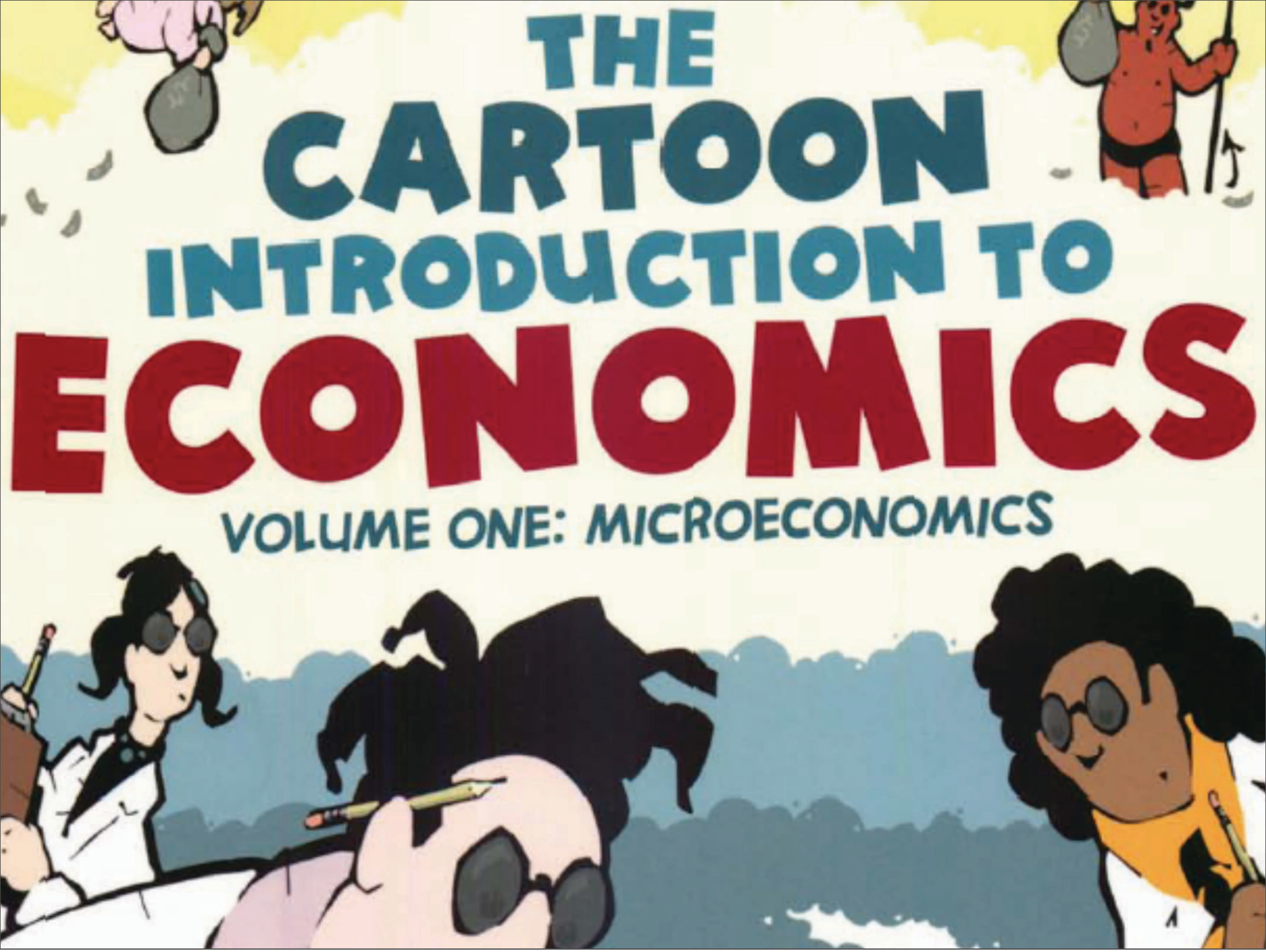
$$f=g \quad \Leftrightarrow \quad g'=0$$

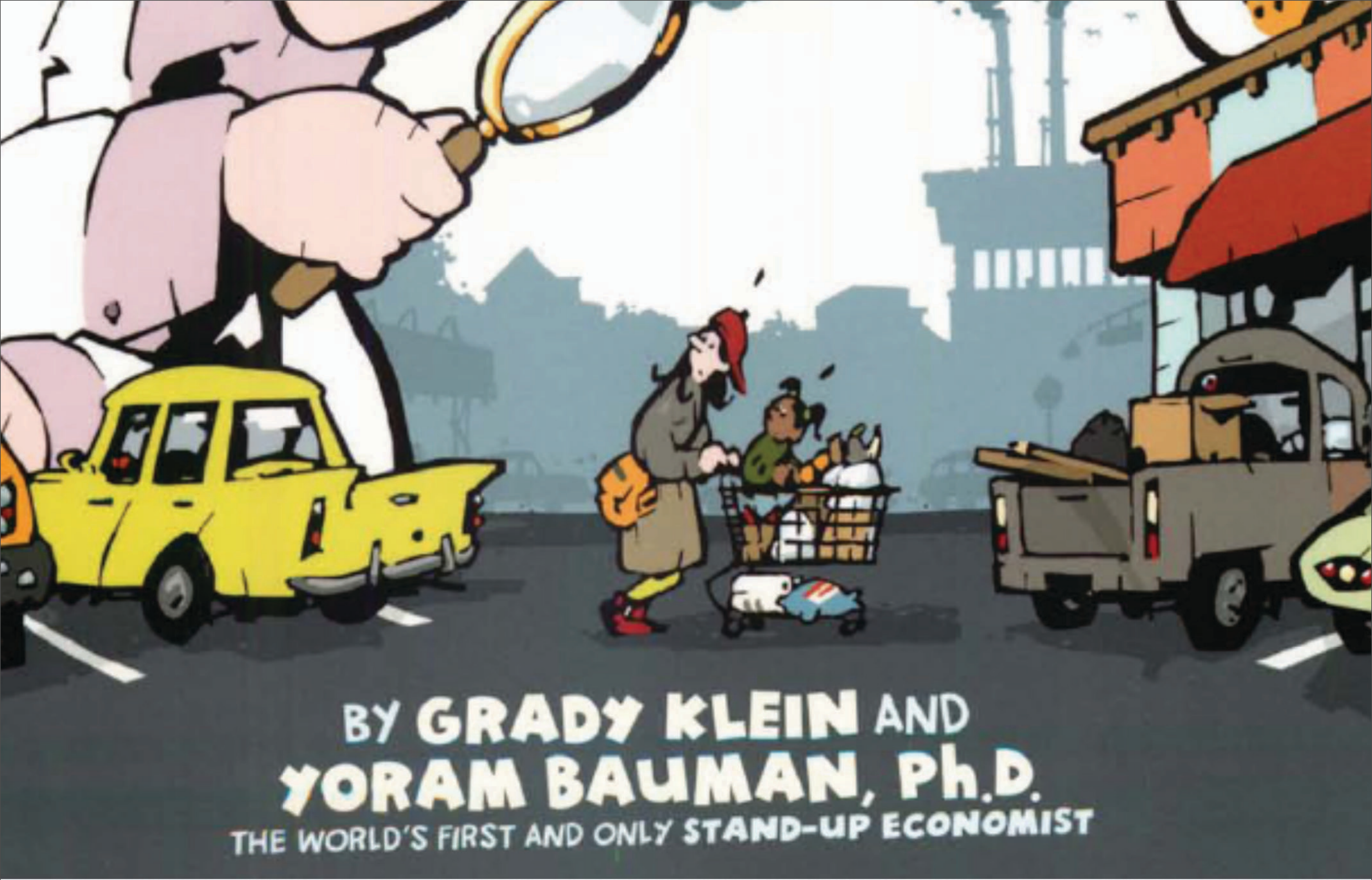
Proof?

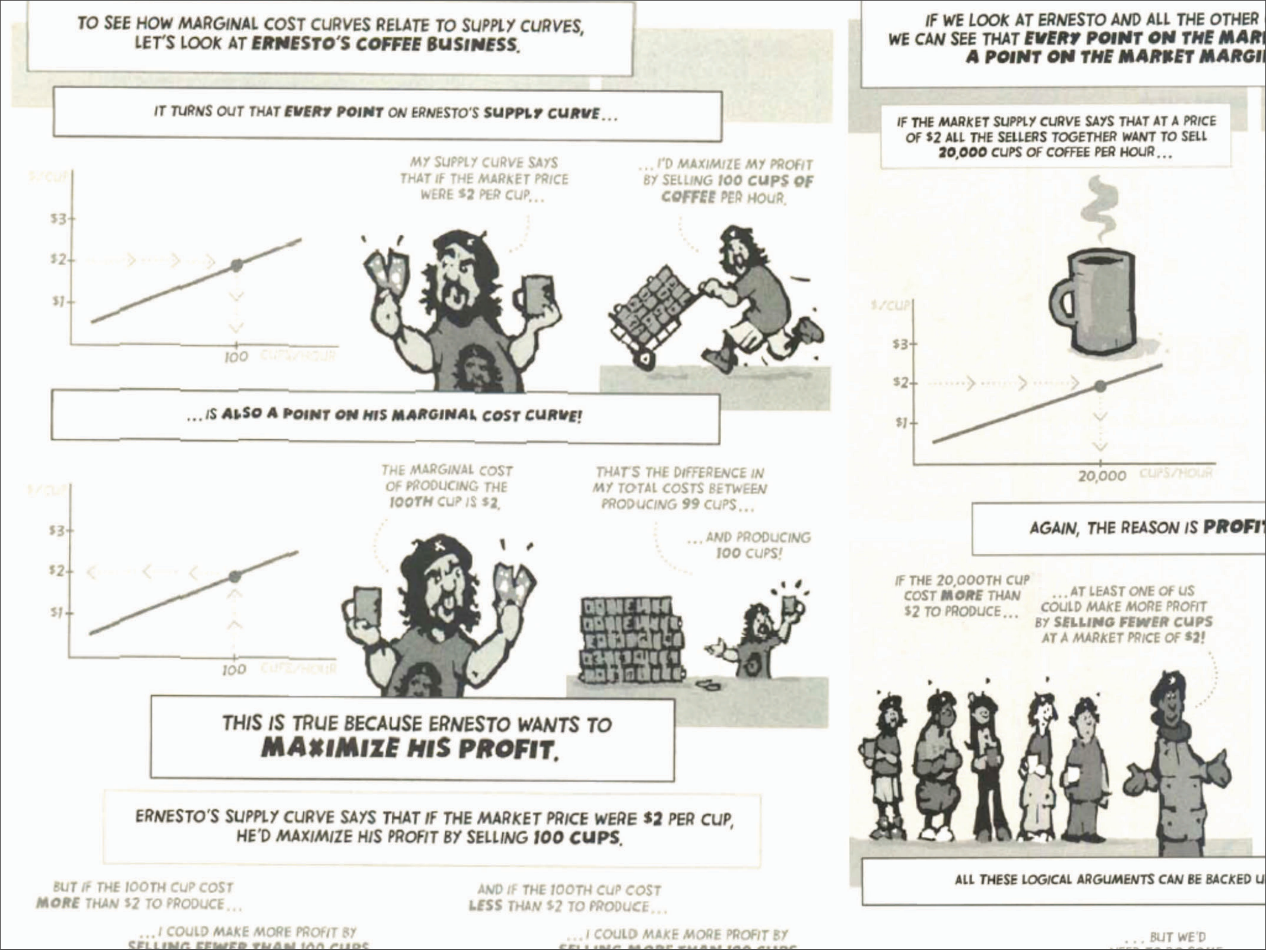
# PROOF:

$$g'=(F/x)'=F'/x-F/x^2 \;\; =0$$

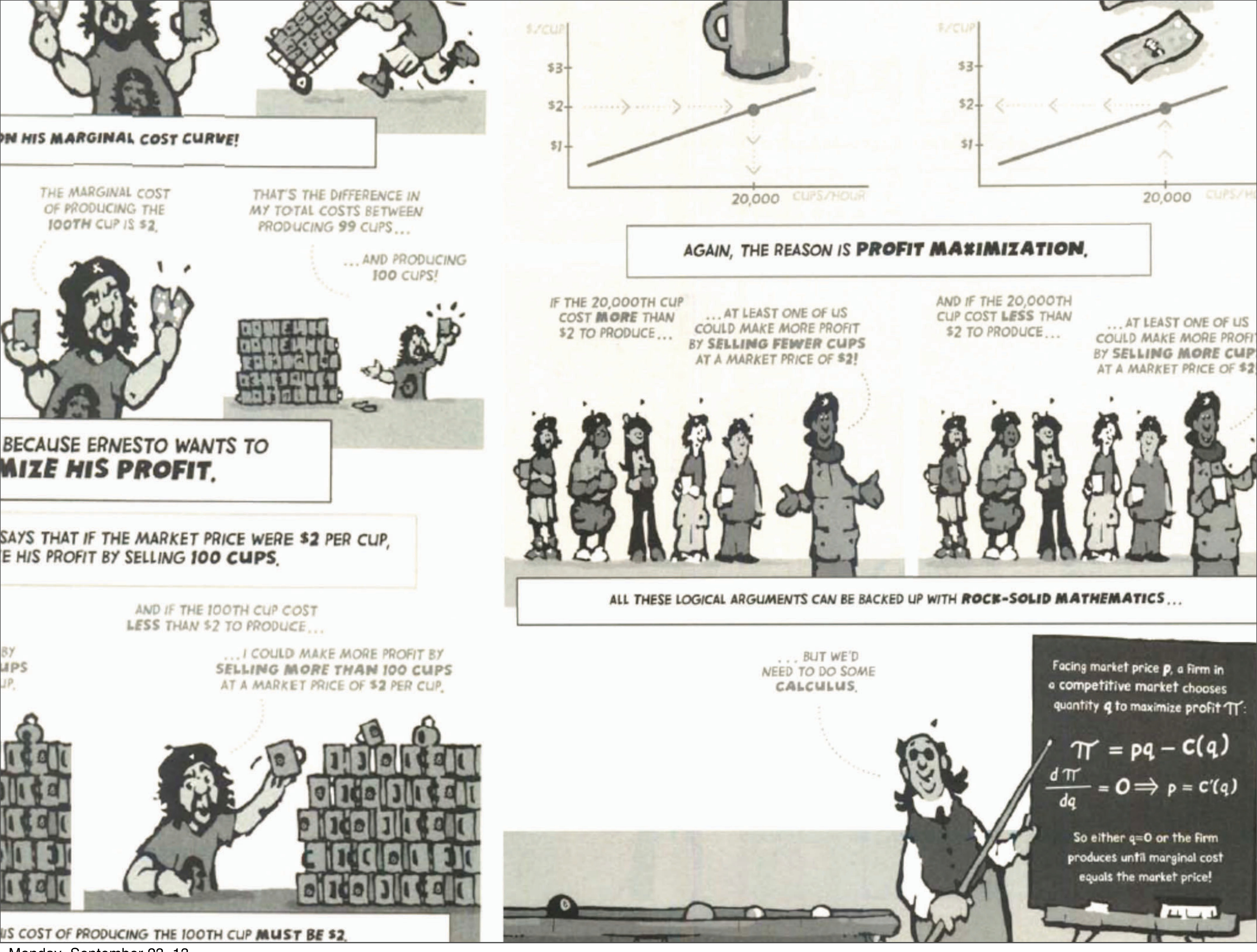
This means f x=F


THE CARTOON INTRODUCTION TO ECONOMICS
VOLUME ONE: MICROECONOMICS


BY GRADY KLEIN AND
YORAM BAUMAN, Ph.D.
THE WORLD'S FIRST AND ONLY STAND-UP ECONOMIST




Monday, September 23, 13



N HIS MARGINAL COST CURVE!
THE MARGINAL COST OF PRODUCING THE 100TH CUP IS $2.
THAT'S THE DIFFERENCE IN MY TOTAL COSTS BETWEEN PRODUCING 99 CUPS...
...AND PRODUCING 100 CUPS!
BECAUSE ERNESTO WANTS TO
MIZE HIS PROFIT.
SAYS THAT IF THE MARKET PRICE WERE $2 PER CUP,
E HIS PROFIT BY SELLING 100 CUPS.
AND IF THE 100TH CUP COST LESS THAN $2 TO PRODUCE...
...I COULD MAKE MORE PROFIT BY SELLING MORE THAN 100 CUPS AT A MARKET PRICE OF $2 PER CUP.
IS COST OF PRODUCING THE 100TH CUP MUST BE $2.
$/CUP
$3
$2
$1
20,000
CUPS/HOUR
$/CUP
$3
$2
$1
20,000
AGAIN, THE REASON IS PROFIT MAXIMIZATION.
IF THE 20,000TH CUP COST MORE THAN $2 TO PRODUCE...
...AT LEAST ONE OF US COULD MAKE MORE PROFIT BY SELLING FEWER CUPS AT A MARKET PRICE OF $2!
AND IF THE 20,000TH CUP COST LESS THAN $2 TO PRODUCE...
...AT LEAST ONE OF US COULD MAKE MORE PROFI BY SELLING MORE CUP AT A MARKET PRICE OF $2
ALL THESE LOGICAL ARGUMENTS CAN BE BACKED UP WITH ROCK-SOLID MATHEMATICS...
...BUT WE'D NEED TO DO SOME CALCULUS.
Facing market price p, a firm in a competitive market chooses quantity q to maximize profit π:
$\pi = pq - C(q)$
$\frac{d\pi}{dq} = 0 \Rightarrow p = C'(q)$
So either q=0 or the firm produces until marginal cost equals the market price!

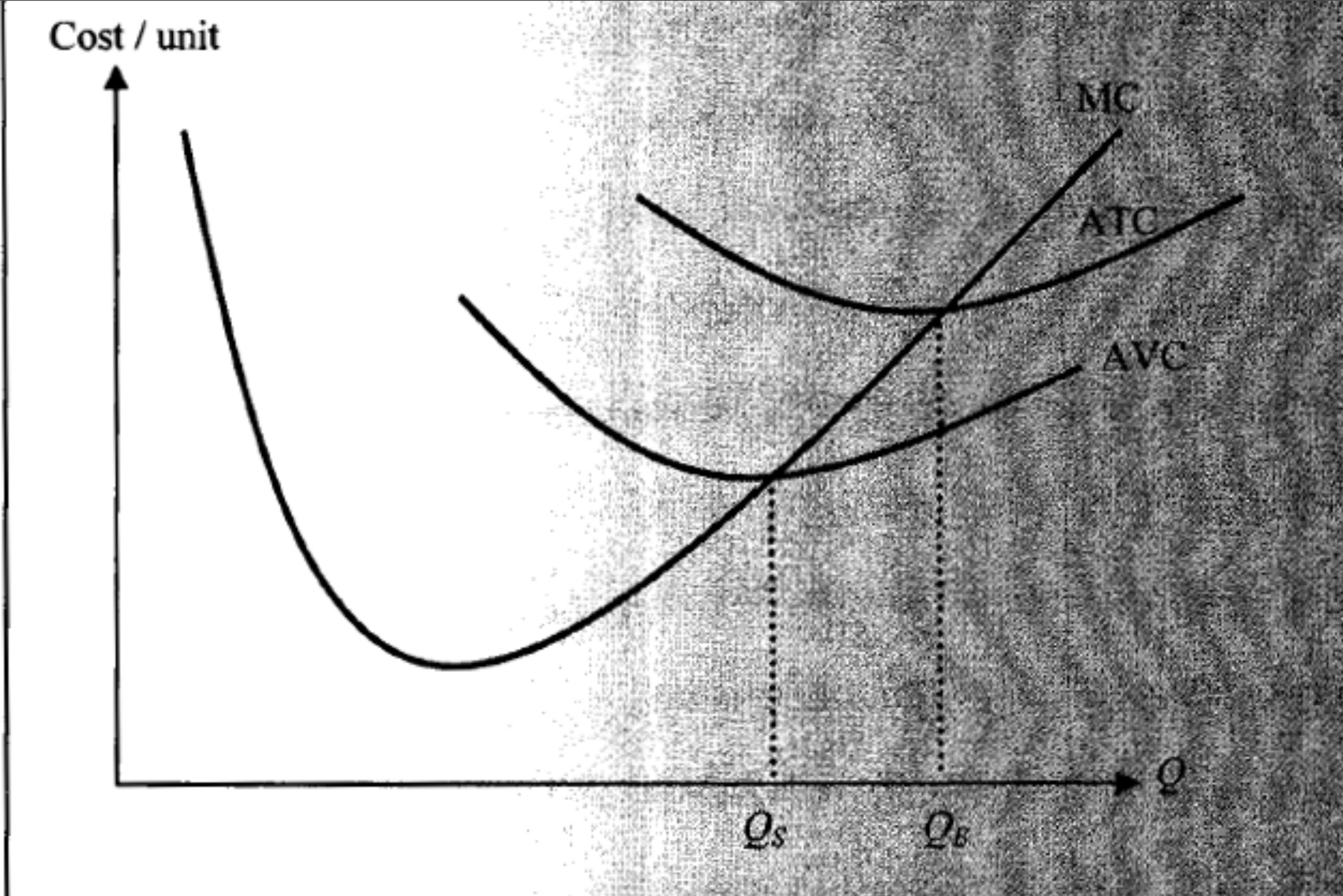



**Fig. 9.4.** The marginal cost MC curve cuts through average variable cost AVC and average total cost ATC curves at their respective minima. These points are the shutdown and break-even points, respectively

You take one strawberry after another and place them on a scale that tells you the average weight of all strawberries. The first strawberry that you

**Mathematical Proof**

Rather than blindly trusting the intuition above, we can also prove our analysis mathematically. Let us perform this proof for the intersection of MC and ATC. Our first step is to compute the derivative of ATC with respect to $Q$ and set this equal to zero to find the curve's critical point, here the minimum:

$$\frac{d\mathrm{ATC}}{dQ} = 0 \tag{9.14}$$

In order to make Equation 9.14 usable, let us substitute TC/$Q$ for ATC. Therefore, we get:

$$\frac{d(\frac{\mathrm{TC}}{Q})}{dQ} = 0 \tag{9.15}$$

To avoid complicated calculus, let us reformulate the numerator as a product:

$$\frac{d(\mathrm{TC} \cdot Q^{-1})}{dQ} = 0 \tag{9.16}$$

Remembering the **product rule**, we differentiate TC $\cdot$ $Q^{-1}$ with respect to tion. When looking at Equation 9.17, we notice that the very first term is MC and so we can write:

$$\mathrm{MC} \cdot Q^{-1} - \mathrm{TC} \cdot Q^{-2} = 0 \tag{9.18}$$

As a final step we multiply both sides by $Q$ and write the second term as a fraction:

$$\mathrm{MC} - \frac{\mathrm{TC}}{Q} = 0 \tag{9.19}$$

Since, by definition, TC/$Q$ is equal to ATC, we finalise our equation to become:

$$\mathrm{MC} - \mathrm{ATC} = 0 \tag{9.20}$$

Now our task of proving that ATC is equal to MC when ATC is at its minimum is easy. Having taken the derivative of ATC in Equation 9.14 to show its minimum, we have worked all the way to Equation 9.20. This last Equation will hold true, i.e. will correspond to a minimum of the ATC curve when we set MC and ATC equal to each other. Hence, when MC is equal to ATC, ATC is at its minimum. The same mathematical steps can be followed to prove the intersection of AVC and MC at the minimum of AVC.