# Calculus and Music

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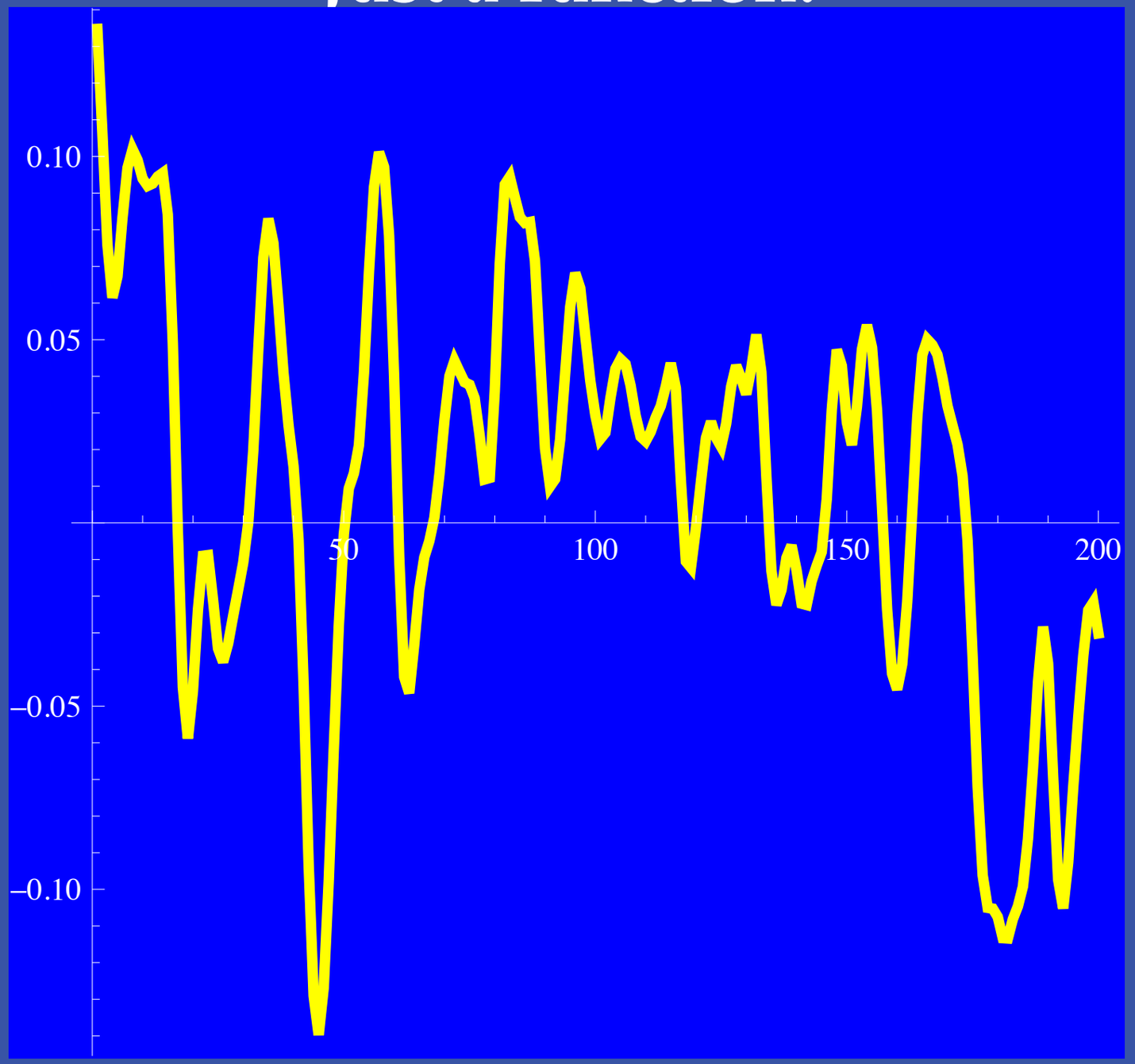
April 22, 2013

# What is Music

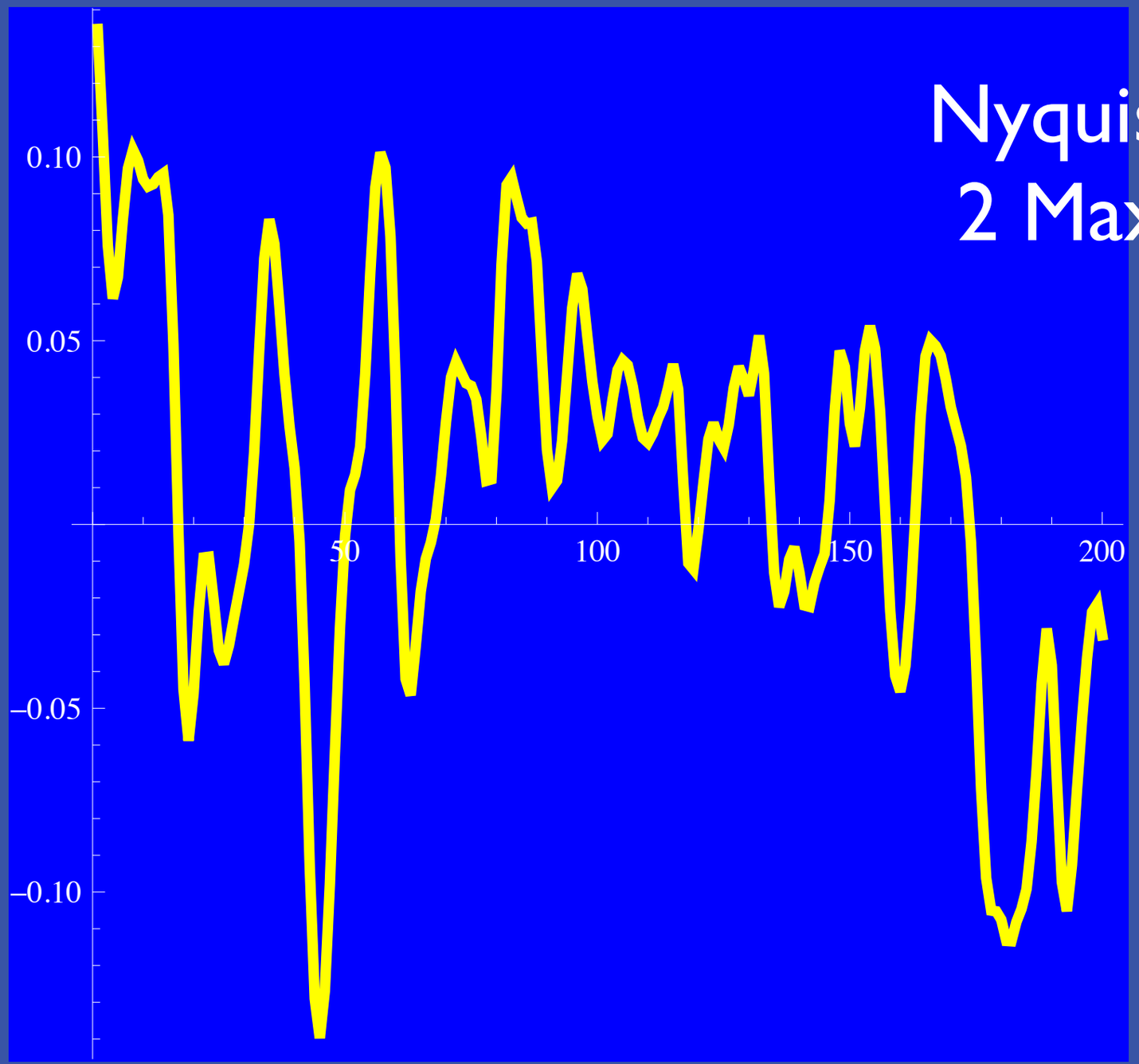
## Just a function!



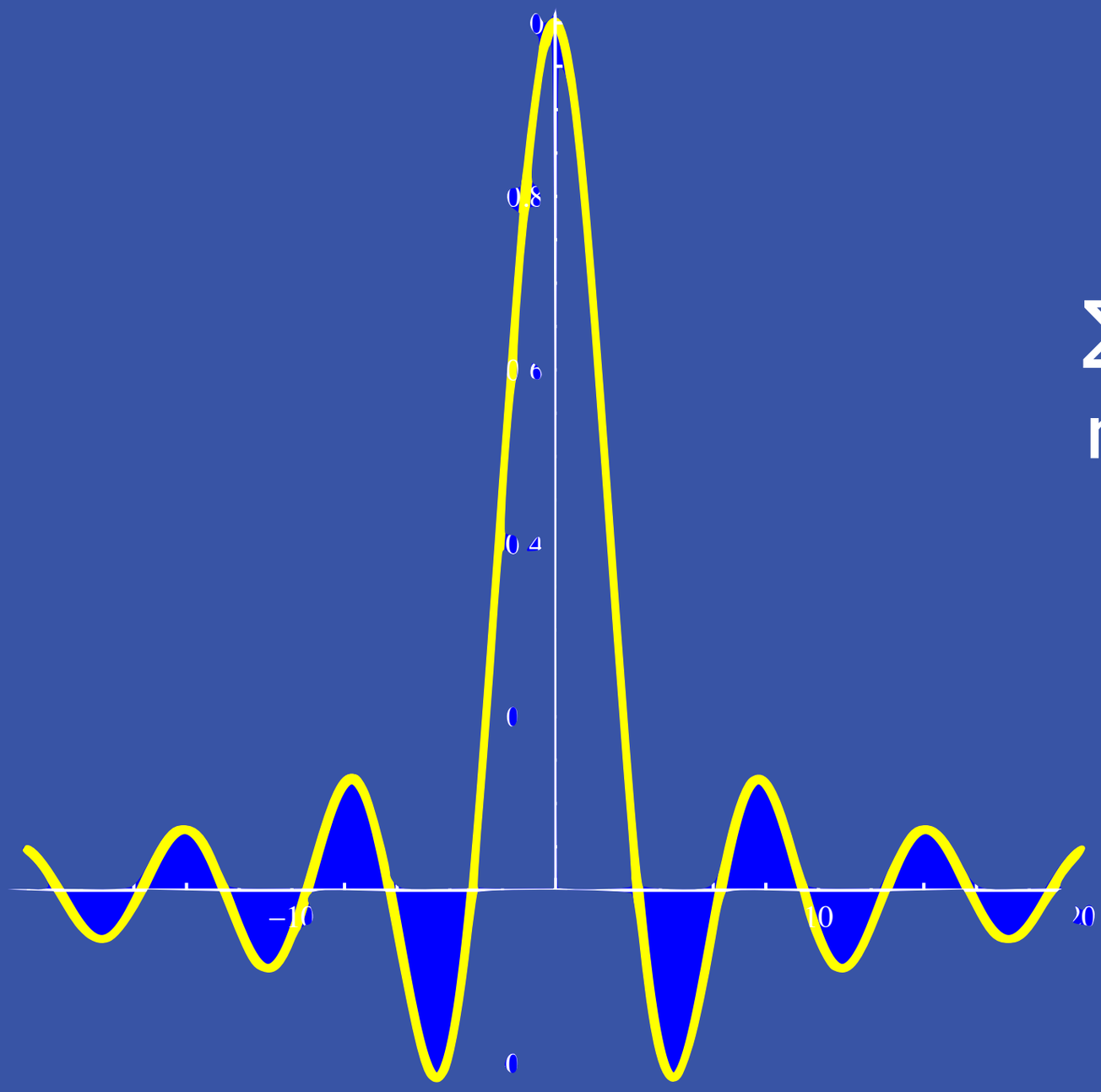
# Sampling

44.1 KHz
192 KHz

Nyquist Shannon
2 MaxFrequncy



# Reconstruction

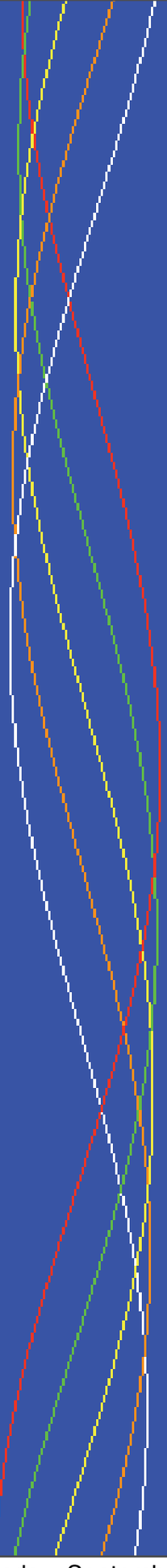



Whittacker-Shannon formula

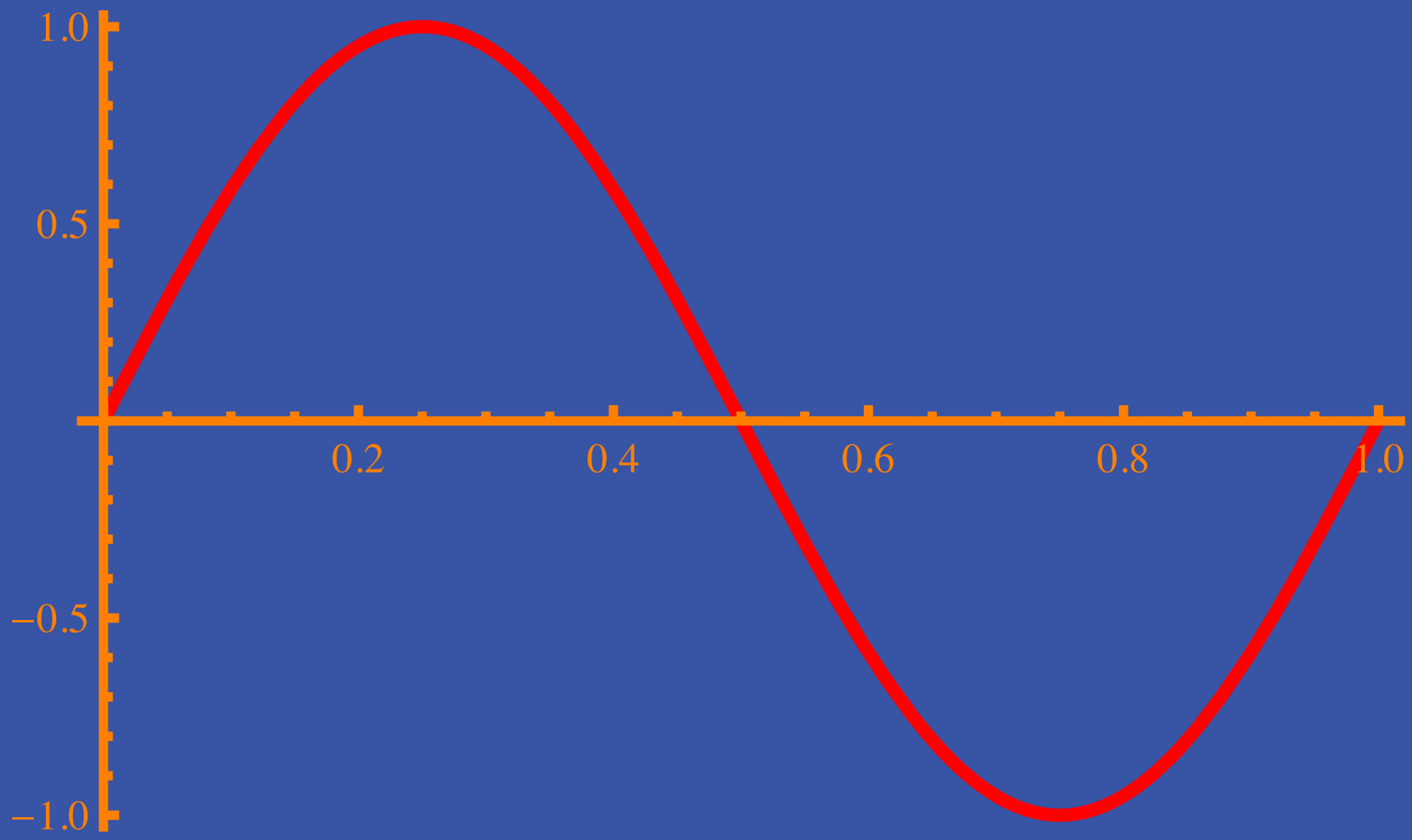
$$\sum_n x(n)\ \mathrm{sinc}(t-n)$$

# A Wave Form

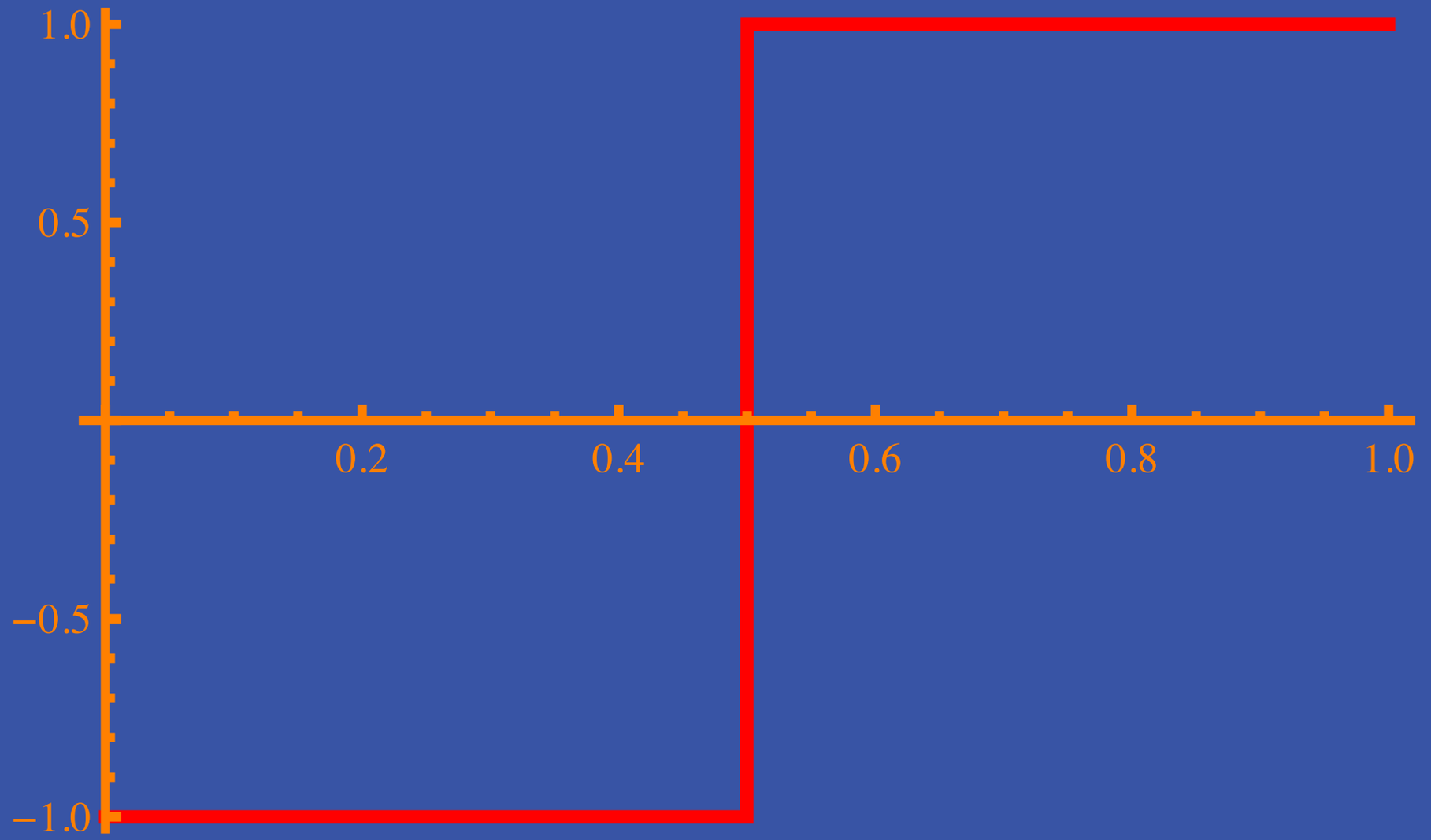
## Bilding block



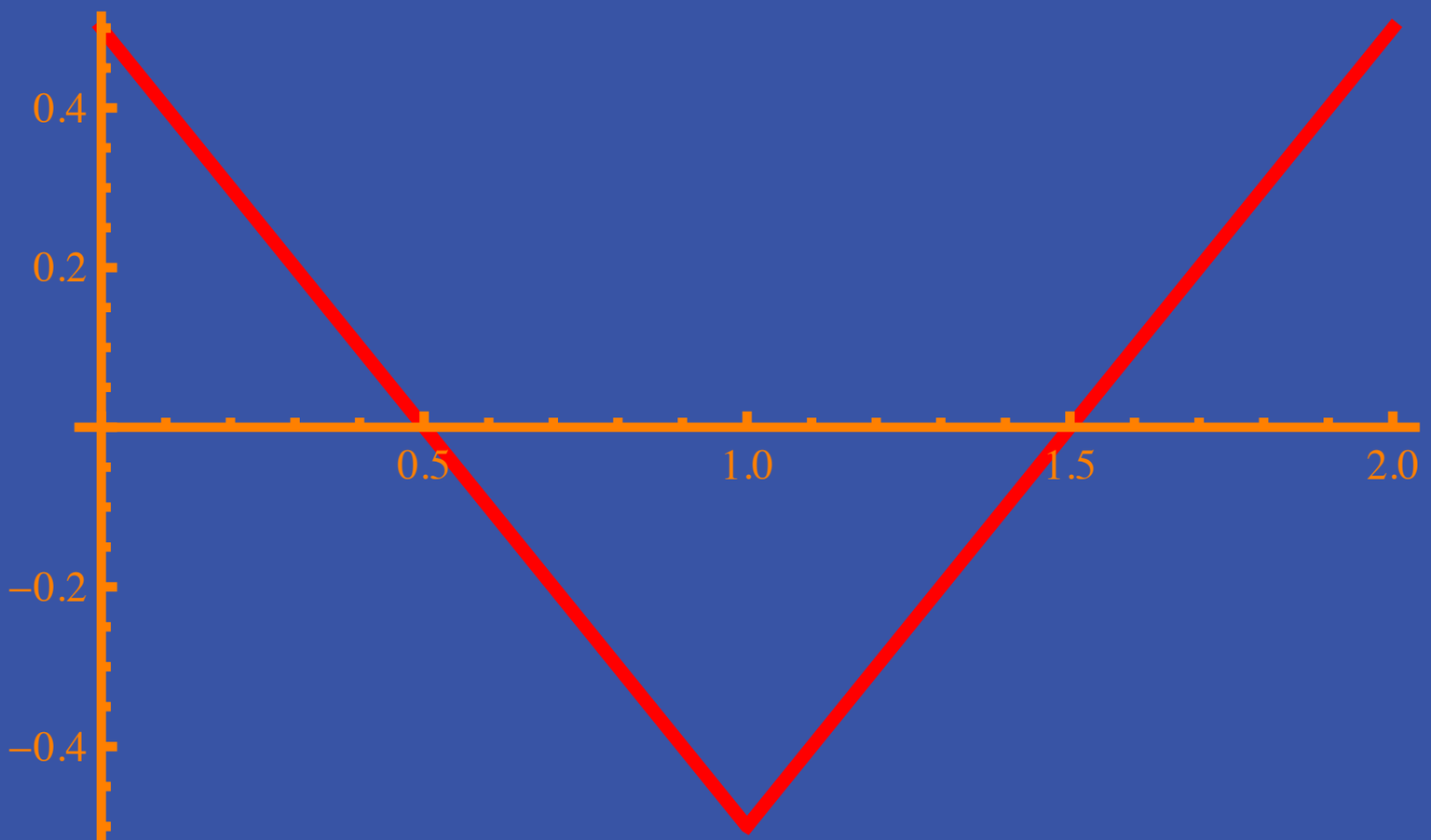
# Sin



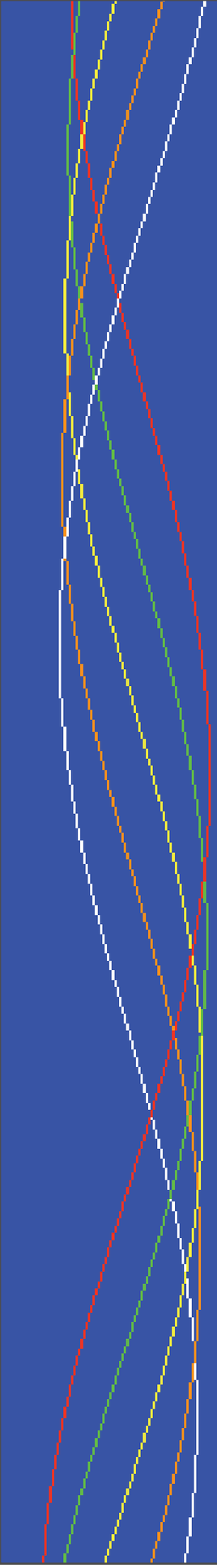
# Sign

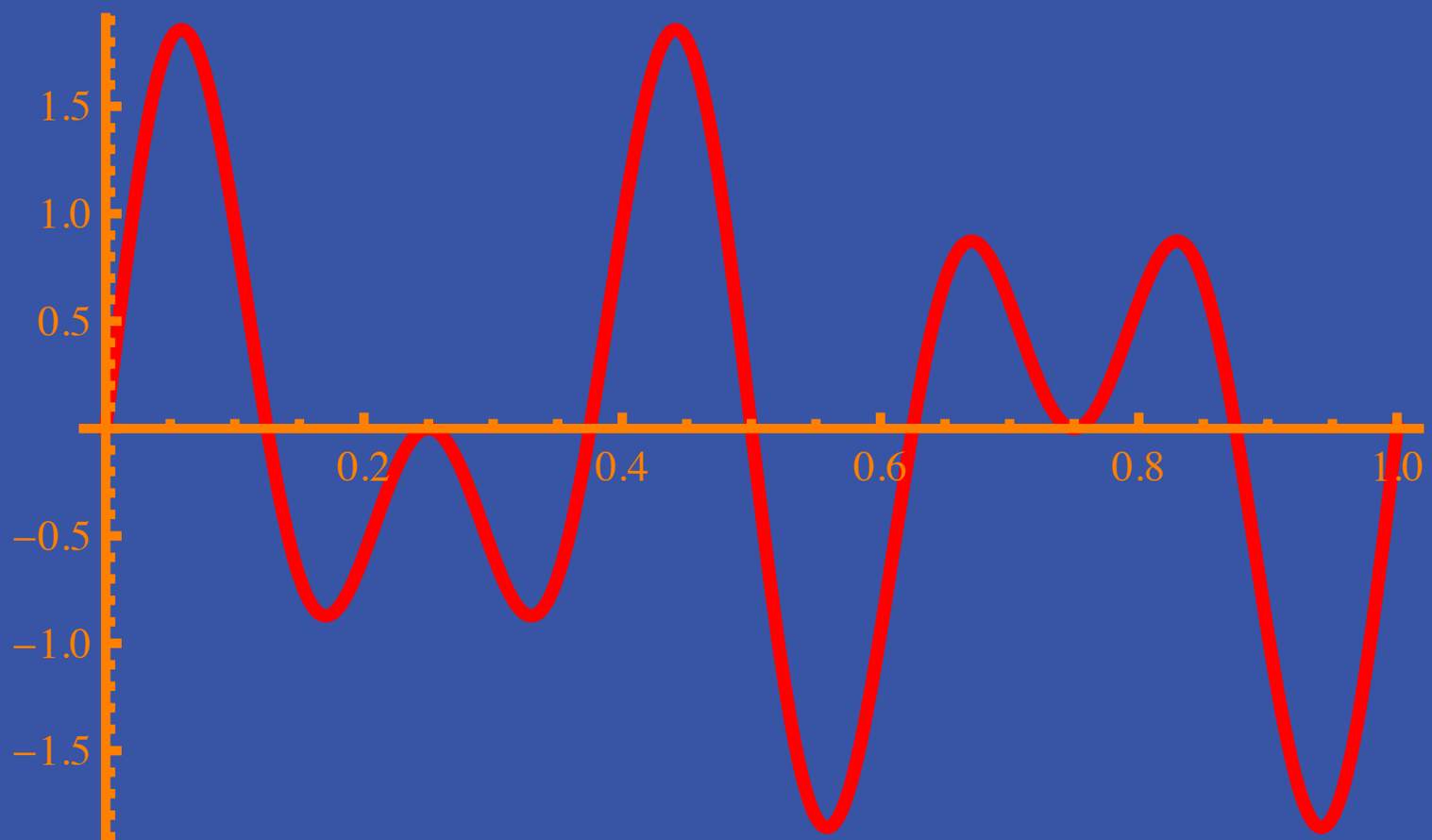


# Abs

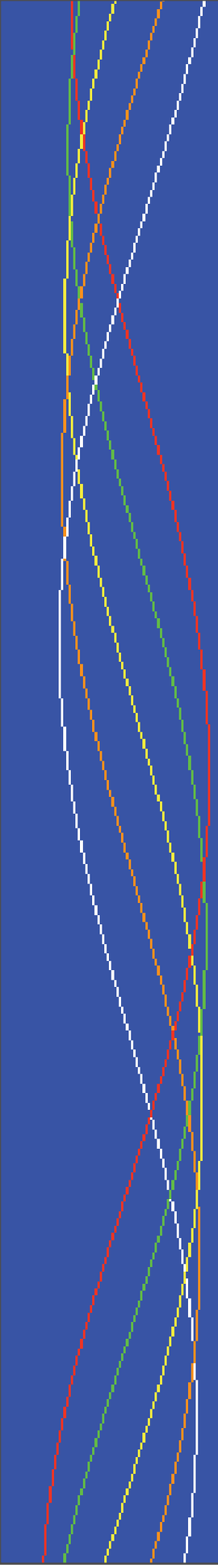


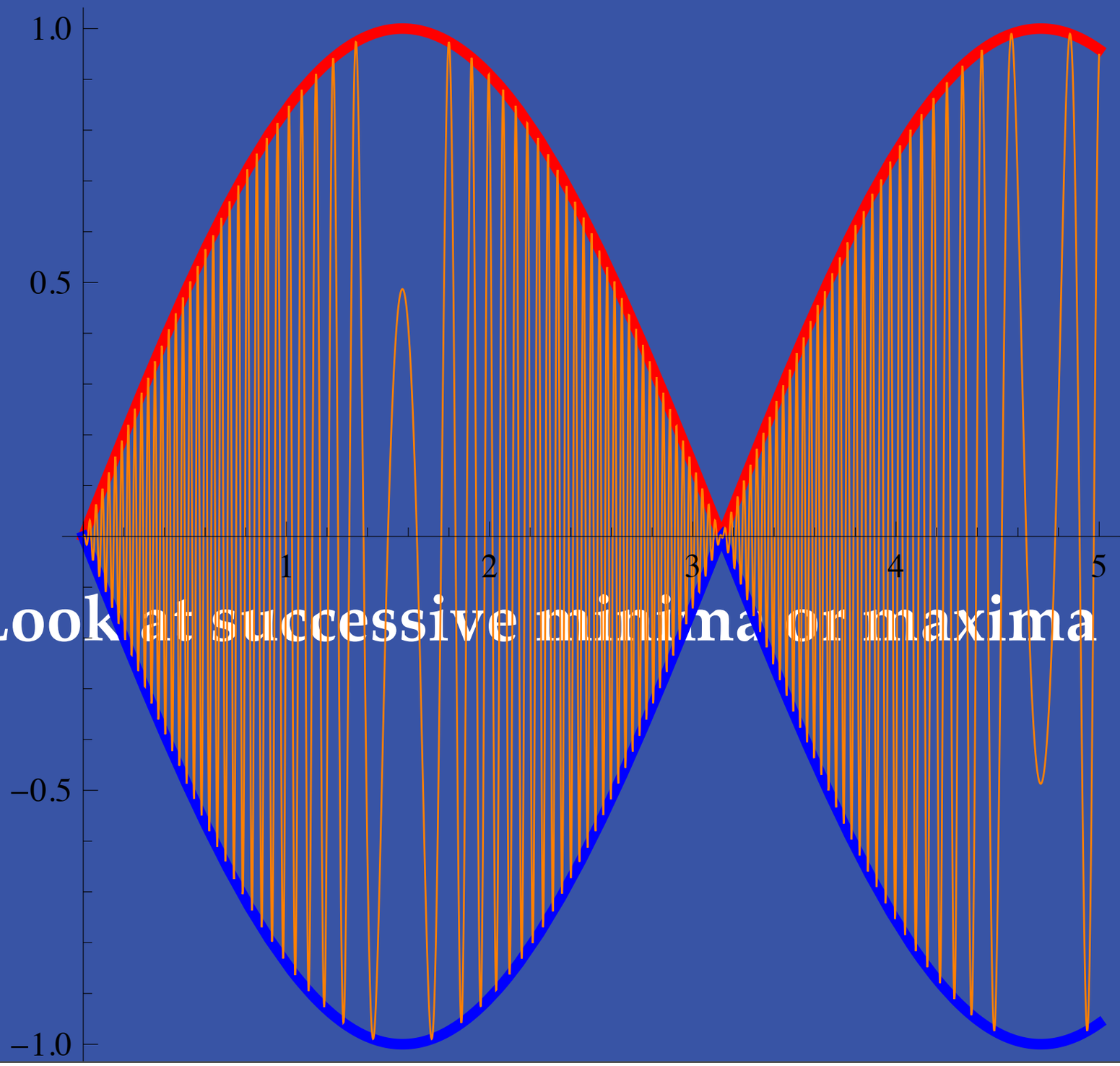
# Sum





# Hull

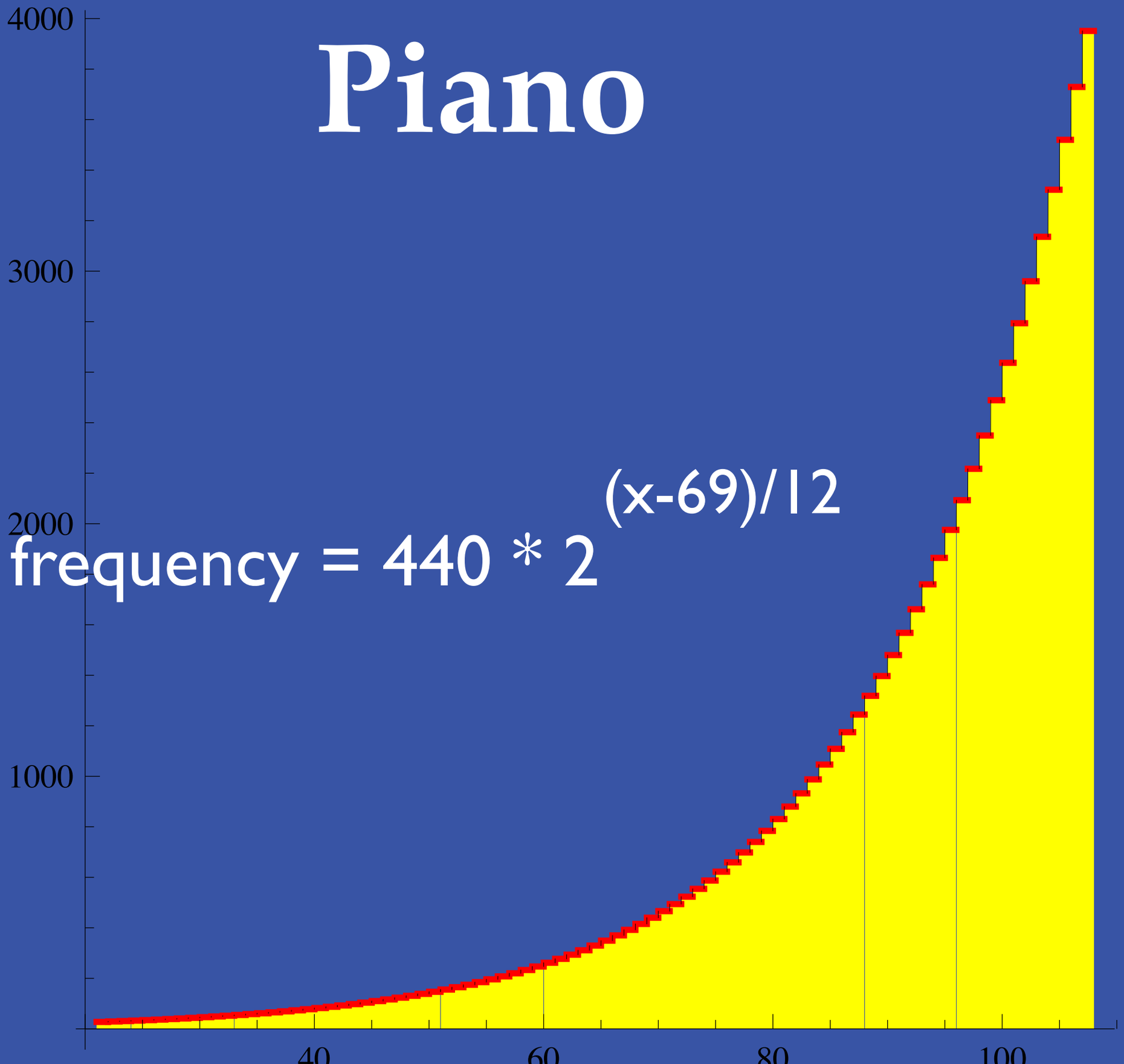





Look at successive minima or maxima

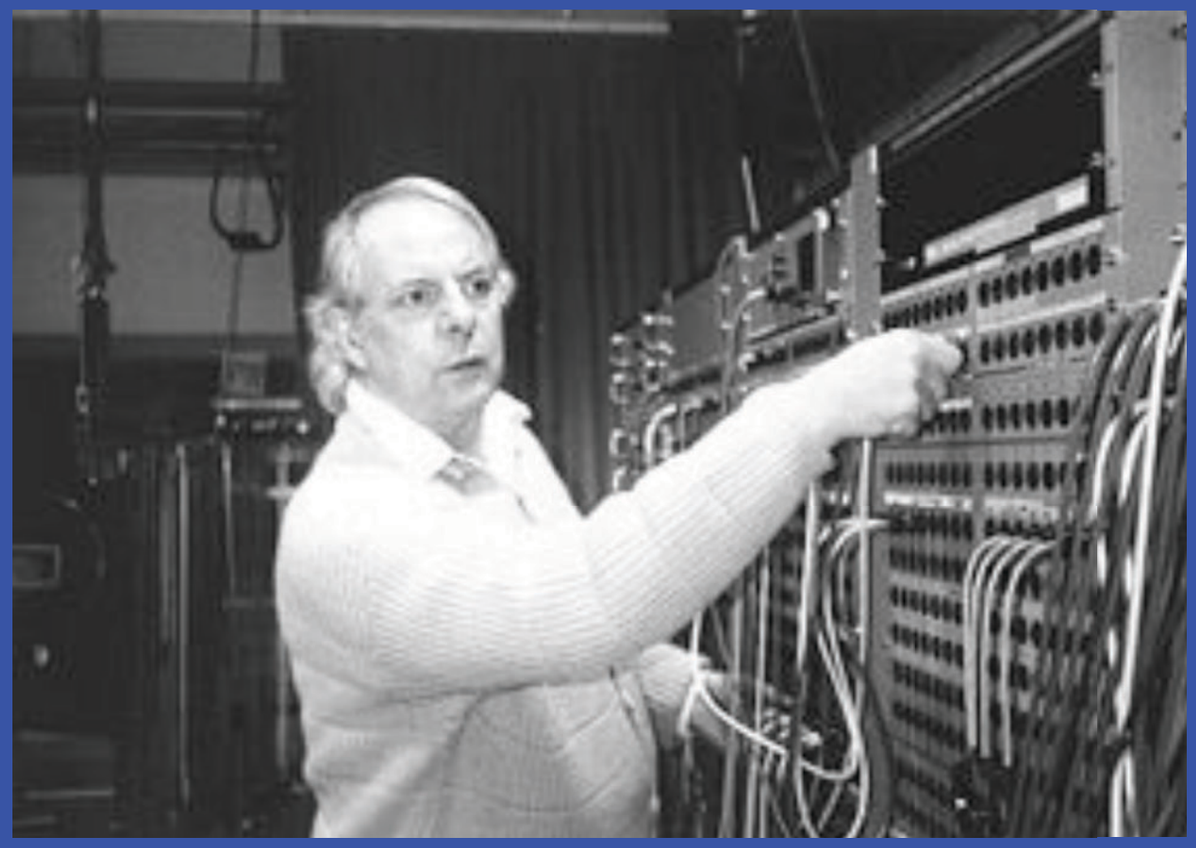
# Piano

$$frequency = 440 * 2^{(x-69)/12}$$



# 5 Tone

## Frequency change is $5^{1/5}$

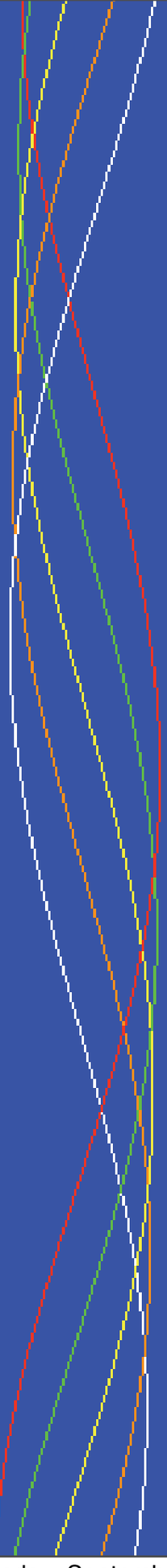


## Stockhausen: 1928-2007

# Stockhausen

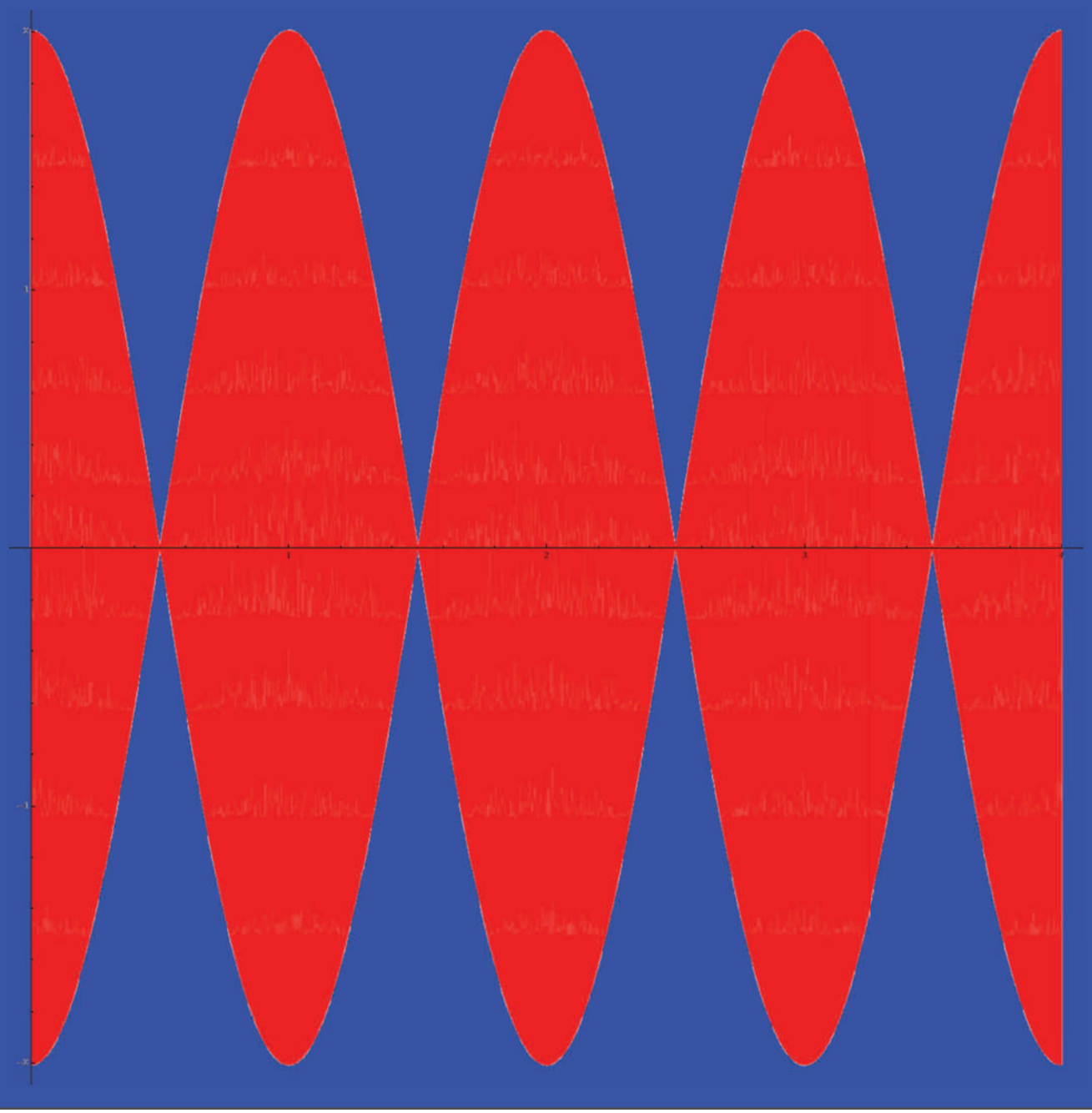
# 16 Tone

## Frequency change is $2^{1/16}$



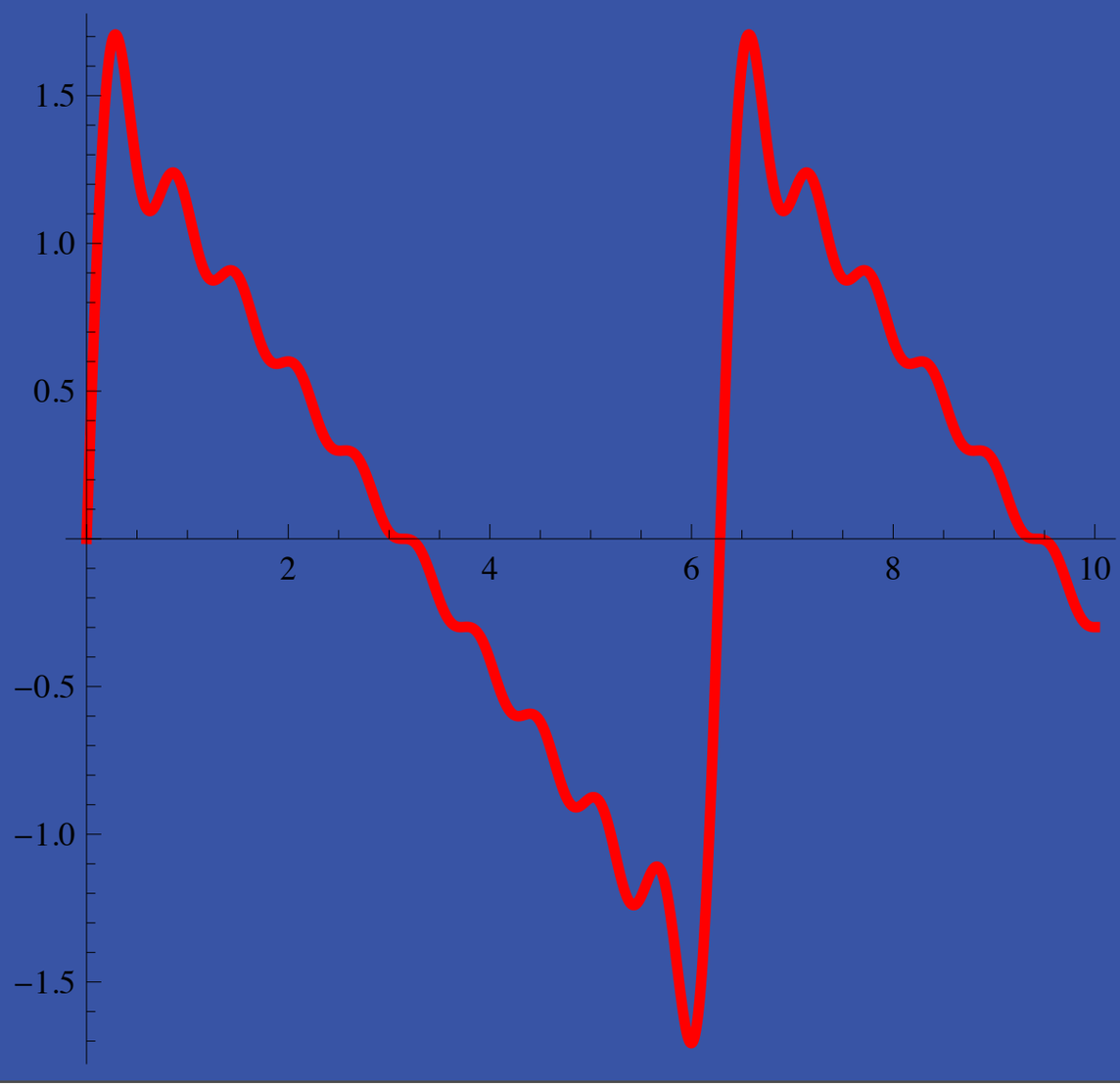
# Interference

$$f(x) = \sin(300\,x) + \sin(301\,x)$$



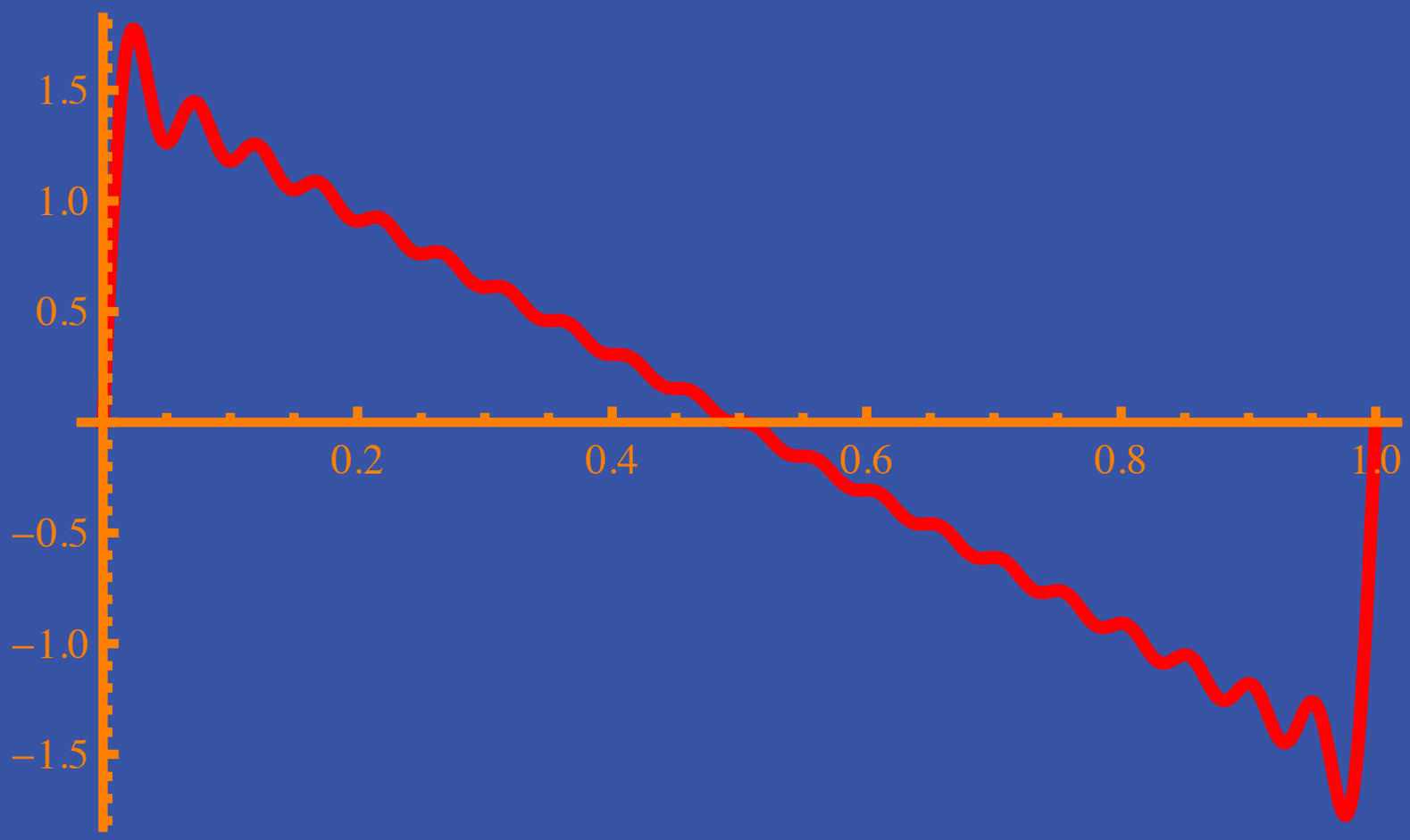
# Approximating functions

$$f(x) = \sin(x)/1 + \sin(2x)/2 + \sin(3x)/3 + \ldots + \sin(10x)/10$$

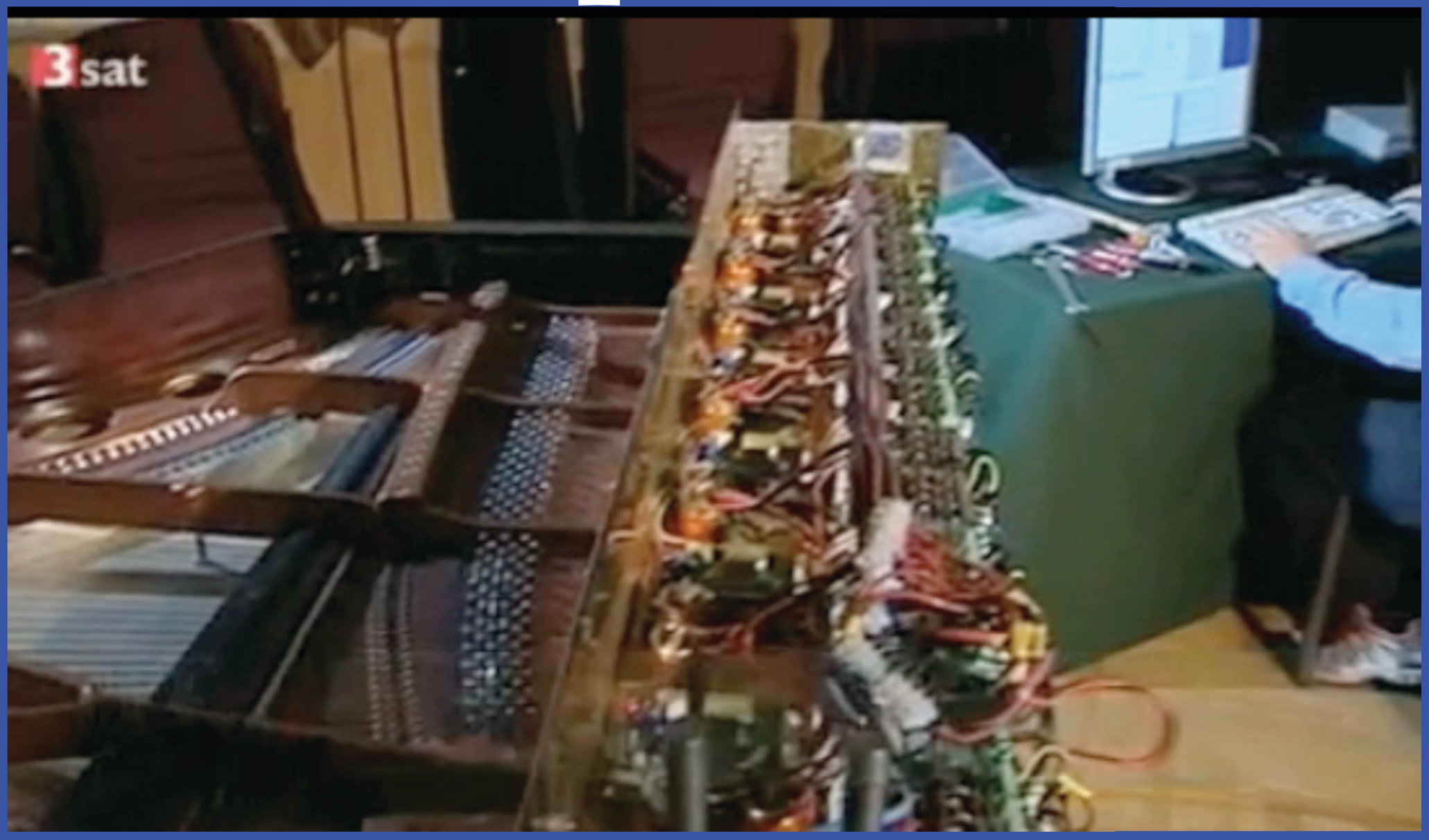


# Approximating functions

$$f(x) = \sin(x)/1 + \sin(2x)/2 + \sin(3x)/3 + \ldots + \sin(20x)/20$$



# Sound Decomposition




Speaking Piano, Source: 3Sat, 2009

# Problem 1: How does this sound?

$$f(x) = \sin(10000\sqrt{x})$$

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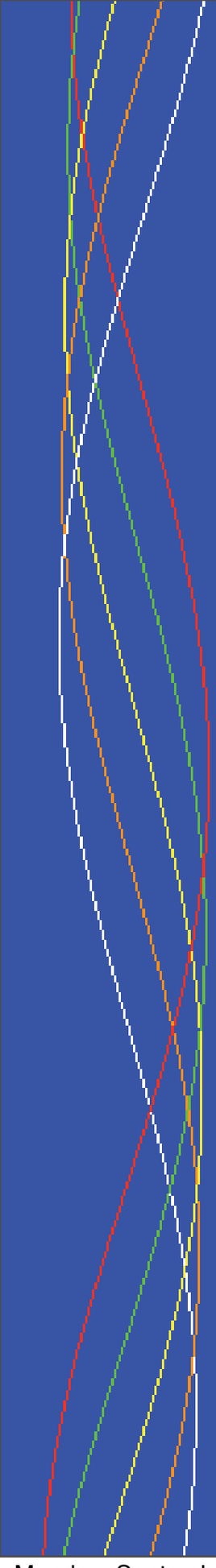
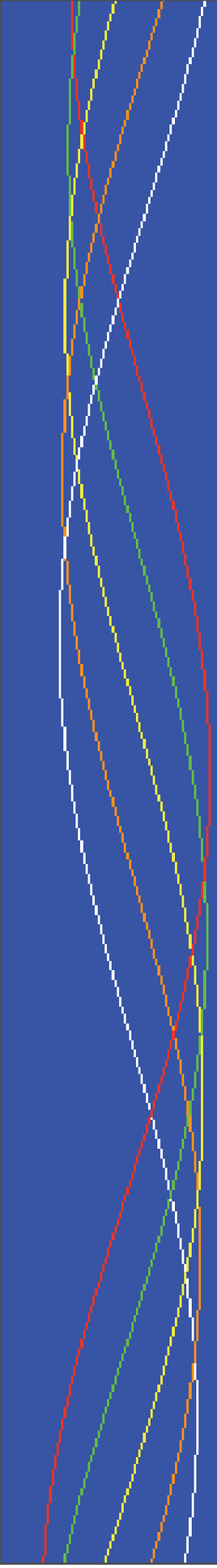


# Problem 2: How does this sound?

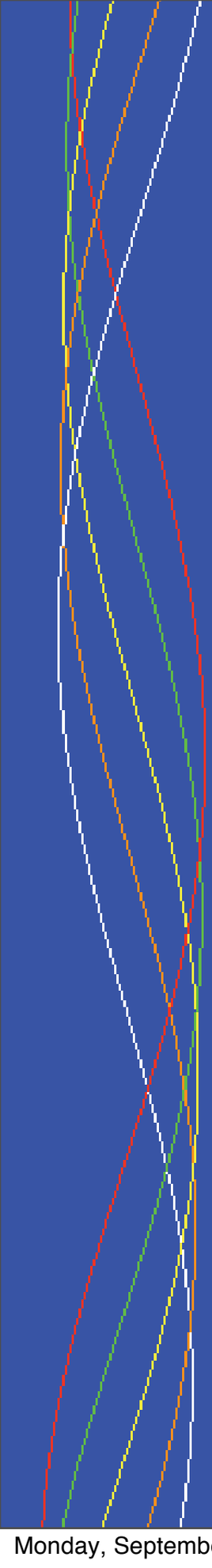
$$f(x) = \sin(1000\ x^*x)$$



$$f(x) = \sin(1000\ x^{*}x)$$

# Problem 3: How does this sound?

$$f(x) = \sin(\sin(1000x)$$

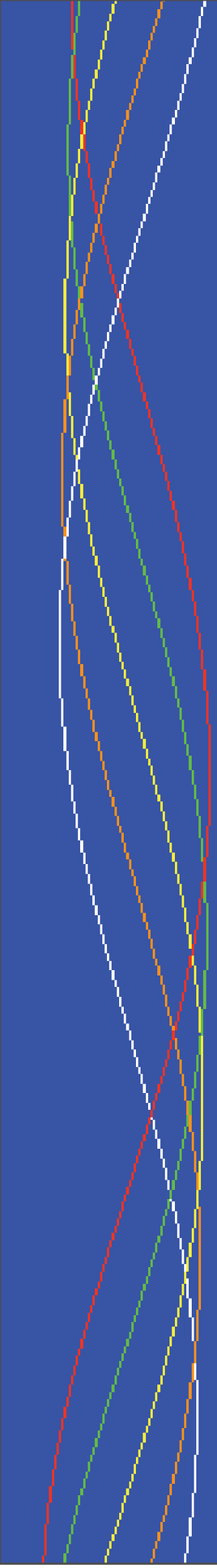


$$f(x) = \sin(\sin(1000x)$$

# Frequency modulation

# Problem 4: How does this sound?

$$f(x) = \sin(x)\,\sin(1000\,x)$$



$$f(x) = \sin(x)\,\sin(1000\,x)$$

# Amplitude modulation

# Sound Functions

# More operations

# Differentiate

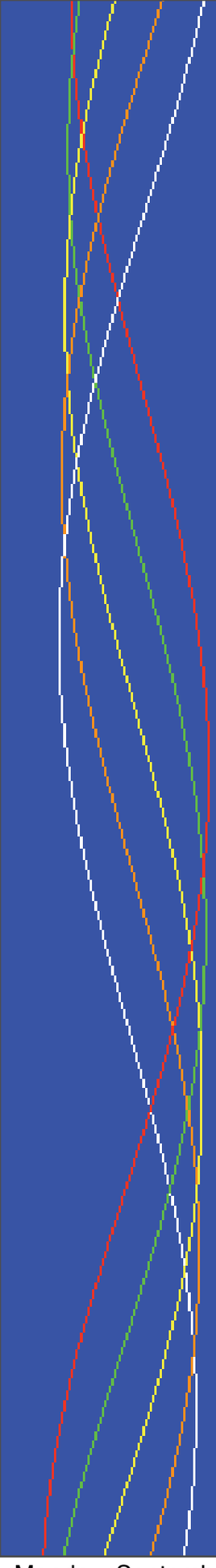
# Sound Functions

## Differentiate

# Sound Functions

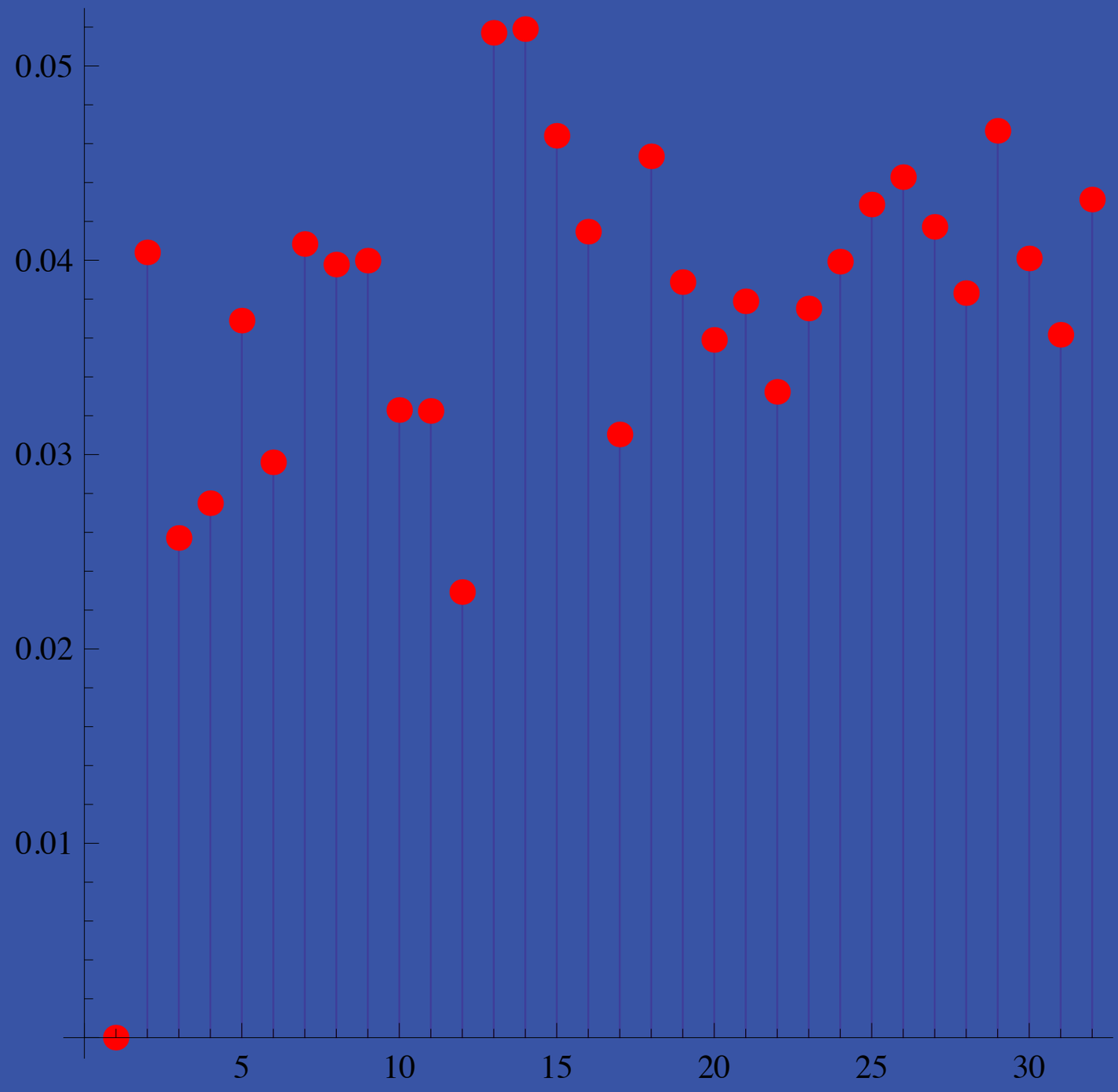
Here is the result

# Stereo difference



# Palindrome

# Palindrome

# Filtering

# Which Scale is Best?