Parameters for communicating Mathematical Ideas

Oliver Knill, Harvard University

Professional Norms in Mathematics, 9/21/2019

Part 1: Preceptors!


2017


2001

Robin
Gottlieb

2001

2019

We are part of the invisible university

Thanks!

What do we do?

Preceptor faculty position

teach	teaching service courses
train	mentor, coach, advisor
tinker	support, develop, create

Minus and Plus

non-tenured

carpe diem

rel. low status

no pressure

relativ low pay

satisfaction

good things

- no publication pressure
- work with lots of great students
- work with great colleagues
- wear different hats
- branch out, i.e. pedagogy, outreach,art
- junior faculty stay young!

We are wearing different hats

Harvard Mathematics Department Undergraduate Events Overview

Department of Mathematics FAS Harvard University One Oxford Street Cambridge MA 02138 USA Tel: (617) 495-2171 Fax: (617) 495-5132

Undergraduate Events page

The Undergraduate Events Committee oversees, organizes, and advertises math-related talks and events put on the department and student math organizations. We aim to engage the undergraduate math community outside the classroom and create a welcoming and inclusive environment in the department, particularly for under-represented grou in math. On this page, we link to some events or event series which take place at the Harvard Math department and near by. To advertise an event on the main page please em ``webmaster at math dot harvard dot edu". To advertise a talk at the seminar page please email "seminars at math dot harvard dot edu". The current committee members are



Flor Orosz Hunziker orosz@math.harvard.edu Stepan Paul spaul@math.harvard.edu Voula Collins vcollins@math.harvard.edu

Math Memos

Monthly Math Memos: 9/2019.

Math Table

The Math Table: is a colloquium for undergraduate speakers with dinner provided (every other Wednesday, 6pm, in SC 507).

Open Neighborhood Seminar

- Facebook: @ HarvardMathUE
- Instagram: @ HarvardMathUE
- Twitter: @ HarvardMathUE

Undergraduate Events

Today September 2019 Print Week Month Agenda

Sun	Mon	Tue	Wed	Thu	Fri	Sat
Sep 1	2	3	4	5	6	
		9am Calculus Advisi	9am Calculus Advisi	10am Calculus Advis	10am Calculus Advis	

The undergraduate Mathematics Colloquium (aka Math Table) sponsored by the Mathematics Department, meets every other Wednesday night at 6 PM in Science Center 507. Talks are catered with dinner starting at 6pm, and a half-hour talk starting at 6:30pm. Each week, students give talks on diverse topics of pure and applied mathematics. Students, irrespective of their mathematical backgrounds, are strongly encouraged to attend and/or to give talks. Unlike in years past, faculty talks aimed at students will take place in the Open Neighborhood Seminar, (here are older events), which meets on the other Wednesday nights at 4:30pm in SC 507. Please check out the

Undergraduate events page

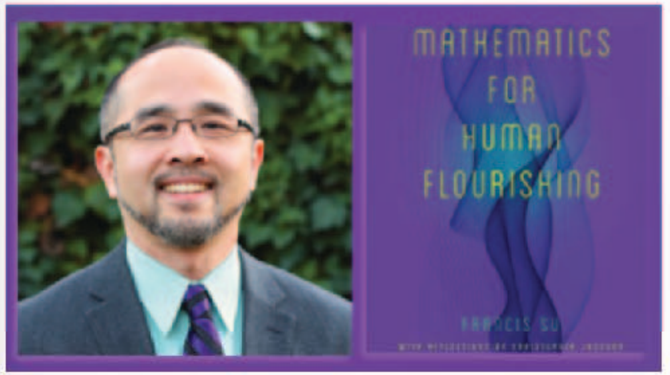
for a calendar. Students interested in giving a talk can contact Stepan Paul Flor Orosz Hunziker or Voula Collins We are happy to suggest ideas for talk topics and work with you on preparing a talk!

Flor Orosz Hunziker
orosz@math

Stepan Paul
spaul@math

Voula Collins
vcollins@math

Next Open-Neighborhood Seminar/Mathtable



Math table of September 25, 2019

Speaker: Francis Su (Harvey Mudd) and former president of the MAA will be giving a ``Why Do Math?" talk on September 25th at 5:30pm **Title.** Mathematics for Human Flourishing
Abstract. How is math tied to what it means to be human? Why does the practice of mathematics often fall short of our ideals and hopes? I'll describe how math helps people flourish, regardless of what they do with their lives or careers, because it meets basic human desires and builds virtues that contribute to a life well-lived. And thus math belongs to everyone. I'll also share what I've learned from a prison inmate who has helped me re-think what it means to do math well.
The second talk is a special Open Neighborhood Seminar on Thursday September 26th at 5:30 in SC507. The Title is: Sperner's lemma: topological combinatorics solving social science problems Abstract: Sperner's lemma is an elementary combinatorial result with a topological flavor, and it has many spectacular applications, including an elementary proof of the Brouwer fixed point theorem, and a solution to the classical cake-cutting problem: how to divide an object fairly among n people. I will trace the history of some generalizations of Sperner's lemma, including some very recent applications to divide rent fairly among roommates.

As for myself

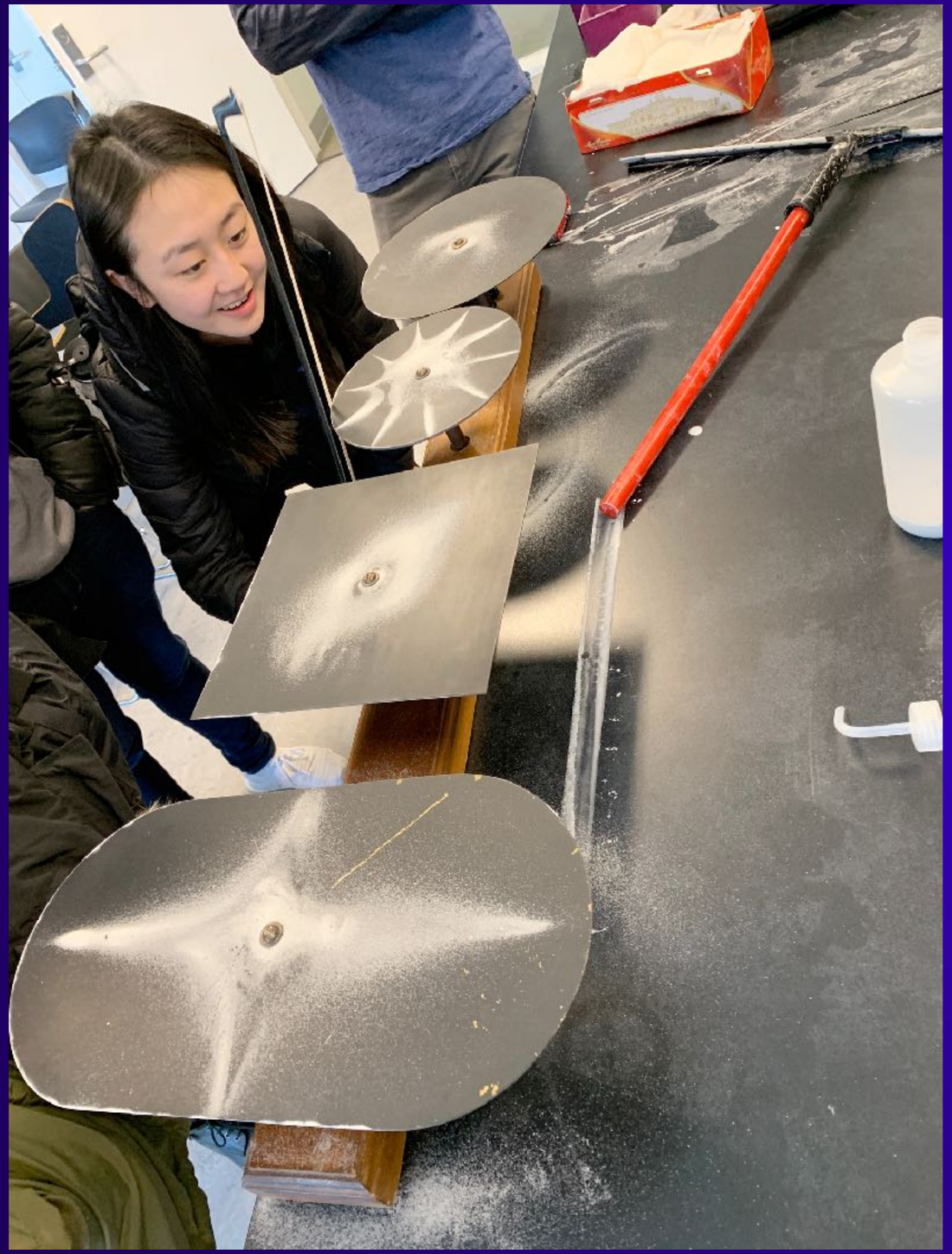
Teaching

College Summer School

Extension School


Unit 7
Curves
① Parametrization
② Matching
③ Velocity, Acceleration
Parametrization
$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} t\cos t \\ t\sin t \end{bmatrix}$
defines a curve in $\mathbb{R}^2$
t is a parameter
Think about t as "time". t is defined on a parameter domain $[a,b]$ or $\mathbb{R}$
Spiral
Archimedes
Example
a)
circle
$\vec{r}(t) = \begin{bmatrix} \cos t \\ \sin t \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$
$\vec{F}(t) = \begin{bmatrix} \cos t^2 \\ \sin t^2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$
also a circle.
b)
Curve in $\mathbb{R}^3$
$\vec{r}'(t)$ acceleration
Newton $m\vec{r}''(t) = \vec{F}$
$\vec{r}'''(t)$ = jerk
$\vec{r}''''(t)$ = snap
$\vec{r}'''''(t)$ = crackle
$\vec{r}''''''(t)$ = pop
$\vec{r}^{(n)}(t)$ = Hazard
Integration
If $\vec{r}'(t)$ is given.
What is C?
$\vec{r}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + C = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$
$C = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$
$\vec{r}'(t) = \begin{bmatrix} t+4 \\ \sin t \end{bmatrix}$, $\vec{r}(0) = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$
where are we at $t=1$?
$\vec{r}(t) = \int_0^t \vec{r}'(s)\,ds + \vec{r}(0)$
More important:
given $\vec{r}''(t)$, $\vec{r}(0), \vec{r}'(0)$
what is $\vec{r}(t)$?
$\vec{r}'(t) = \int_0^t \vec{r}''(s)\,ds + \vec{r}'(0)$
$\vec{r}(t) = \int_0^t \vec{r}'(s)\,ds + \vec{r}(0)$
Free Fall
$\vec{r}''(t) = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$
Zeno
$h = 1.66 \cdot 10^{-35}$
10^{-44} Seconds
Velocity
$\vec{F}(t)$
$\vec{r}'(t)$

Explore




Test time
Encoder

Support

administration IT

40 years of Education Technology Timelines

word wide web → HTML5

flash → WebGL

java → javascript

Computer Games → Twitter

Educational TV → VHS → CD → DVD → YouTube

since 30s: slide projectors → 3Dprint

since 40ies: overhead projectors

since 60s: PRS

power point → keynote

Computer algebra systems

Macsyma → Maxima

Reduce → Mathematica, Maple, Matlab

Cayley → Magma

Pocket calculators

Programmable calculators

Graphing calculators

Calculators with CAS

Ipad Alpha

70s 80s 90s now

tinker with technology

Advertisement for math

Mathematics in Movies

This is a collection of movie clips in which Mathematics appears. The site has been changed in 2010 to HTML5 video so that it is accessible by all devices. There is always also a direct video links. It can be include a clip into a presentation, chose the quicktime version. (Media RSS link). Since the format which was chosen in 2006 were small scale movies (bandwidth was still an issue at that time), the movies are now added as separate pages for various additional movie clips, which is easier to maintain an extend than an ever growing database and allows on each page also to adapt the quality of the movie and to include additional content.

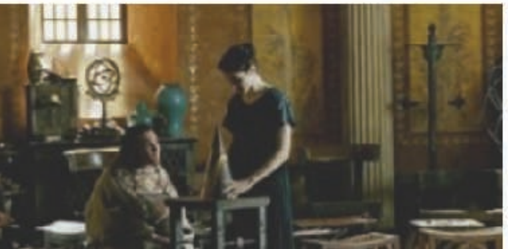
Date: March 2006 - August 2018
by:
Oliver Knill
Department of Mathematics
Harvard University

Math in Movies

- Answer to ultimate question: 42
- Eigenvalues of Moebius strip
- Scrumtrulescent
- The number 73
- 3D printing in Ocean 8
- Time machine blackboards
- Rubik Cube fighting back
- Logic in "Catch 22"
- Relativity in "Interstellar"

- Math in Dr Zhivago movie
- Sum of arithmetic series
- Physics in "Theory of Everything"
- Monkey theorem in "After the Dark"
- Fermat's room
- Chaos in "Chaos theory"
- Number theory in "The man who knew infinity"
- Geometry in "Suicide Squad"

- Counting in "Clan of the Cave Bear"
- Calc and Lin Alg in the ``Thinning"
- Hidden Figures
- Trachtenberg method in "Gifted"
- Percentages in "Guardians of the Galaxy"
- Addition in "The Warmachine"
- Entropy in "Arrival"
- Graph theory in "Goodwill Hunting"



Agora (Appollonian cones) [IMDb link]

Hypatia and her Apollonian Cones.
2009
To the movie. Direct media links: Quicktime MP4, Webm and Ogg Vorbis.

Agora (Relative motion experiment) [IMDb link]

Hypatia makes an experiment with relative motion
2009
To the movie. Direct media links: Quicktime MP4, Webm and Ogg Vorbis.

Oliver Knill

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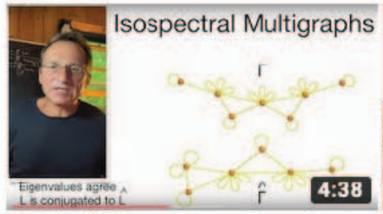
Youtube

Uploads PLAY ALL

SORT BY



Energized Simplicial Complexes

181 views • 2 weeks ago

CC

Harvard Science Center by Drone

167 views • 3 weeks ago

Tough Math!

217 views • 1 month ago

Why Povray is great.

487 views • 1 month ago

Tesseract, the 4D Cube

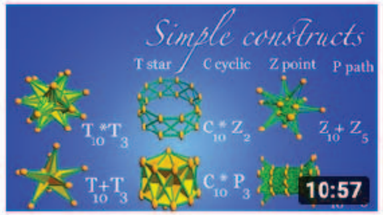
159 views • 1 month ago

Forth of July Lecture on Curves

264 views • 2 months ago

Parametrized Poincare-Hopf

Rheinfall Impressions 2019



On Numbers and Graphs

Dehn-Sommerville and

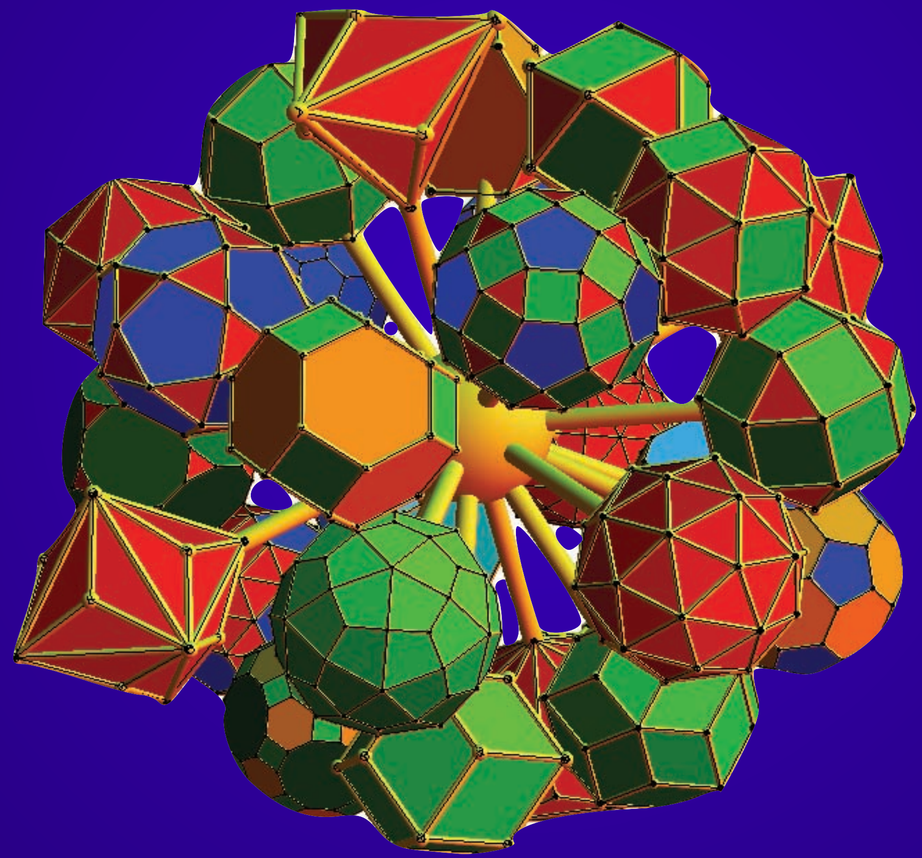
Average Simplex Cardinality

Chicken Love Roast Beef!

technology


nice projects

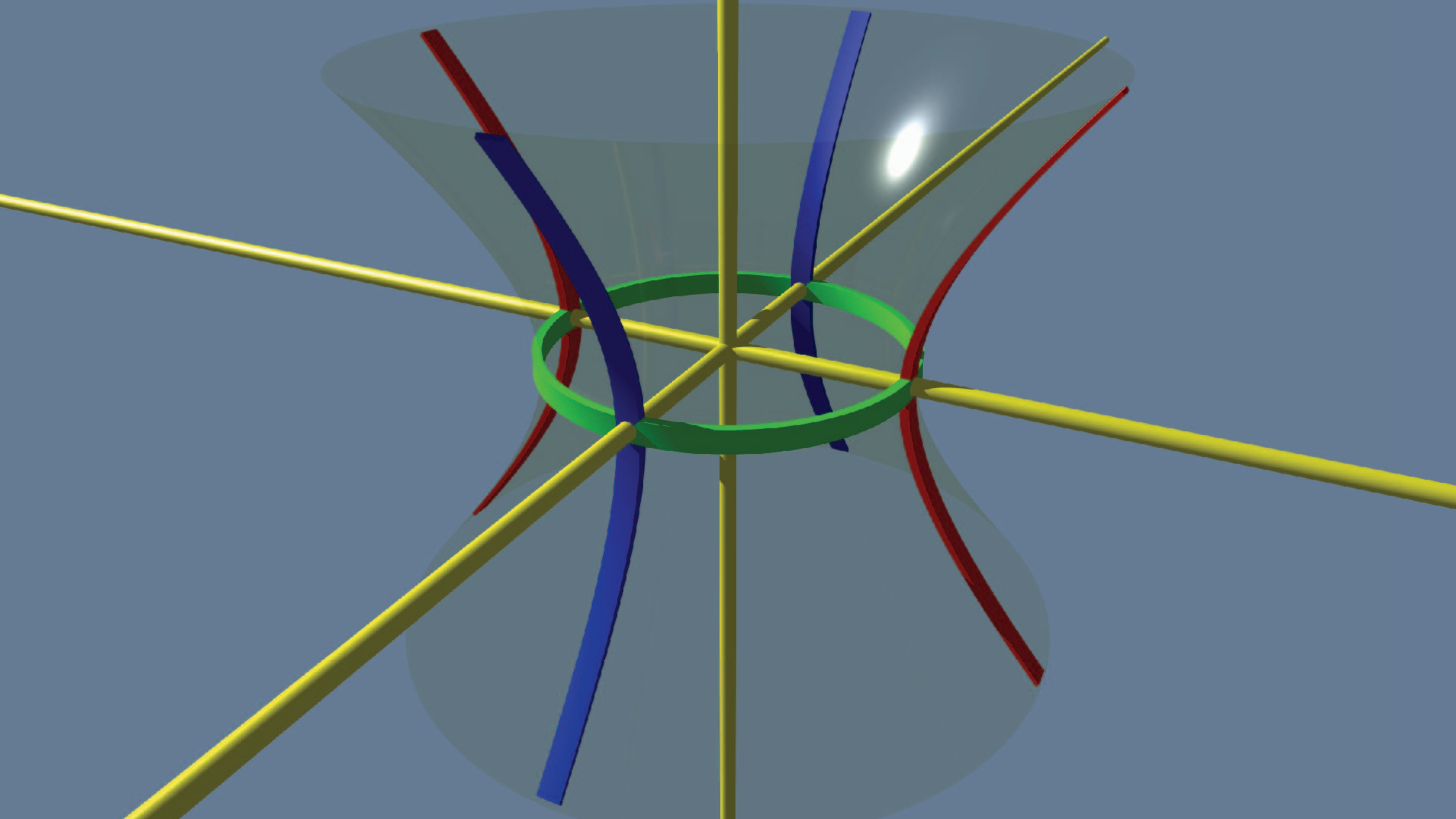
Archimedean



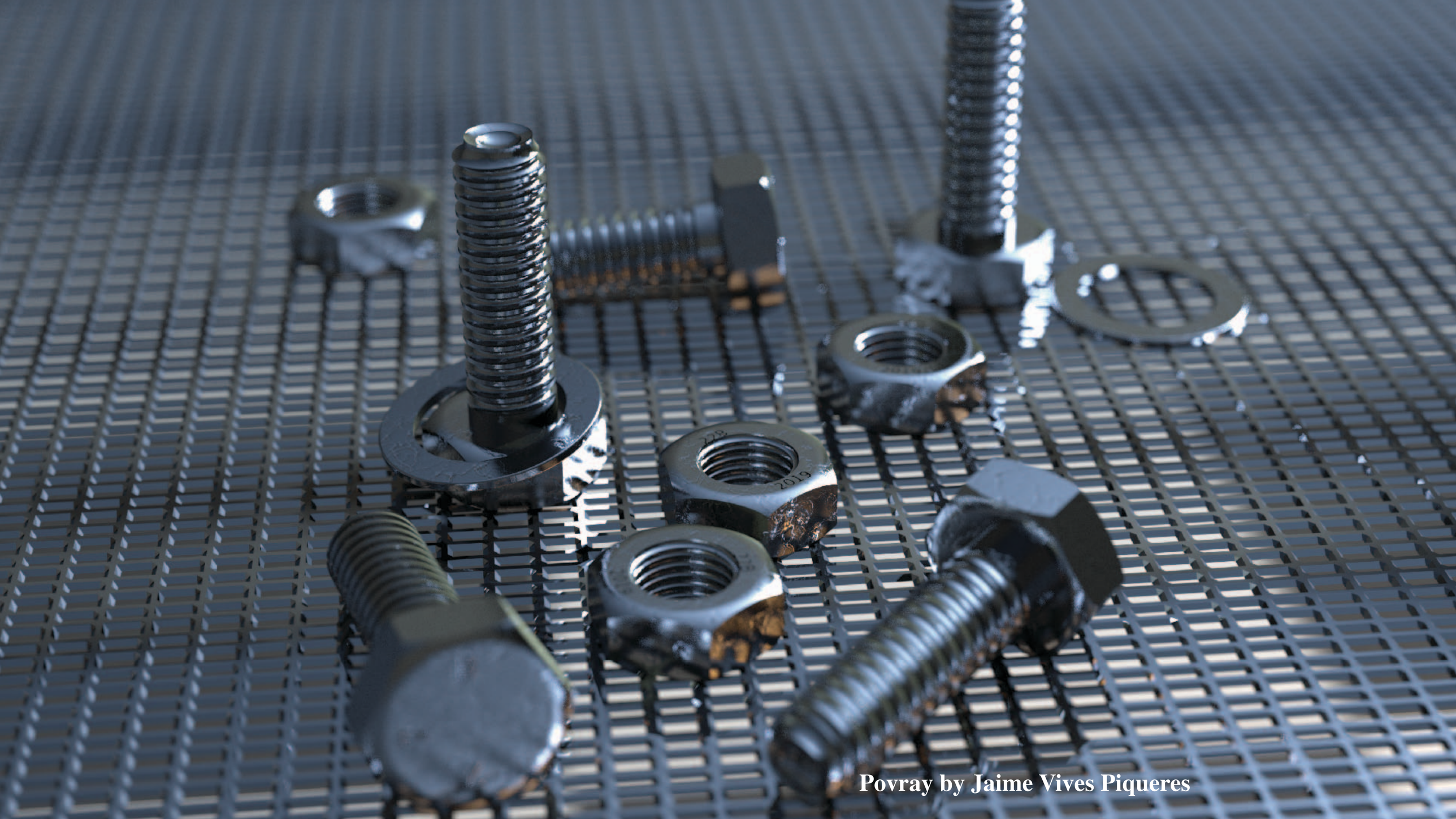
2013

Gregor Luetolf


Graphics



```
#include "colors.inc"
#include "textures.inc"

camera          { location  <2,2,-4> up  <0,1,0> right <4/3,0,0> look_at <0,0,0> }
light_source    { <-40, 30, -10>     color rgb <1,1,1>                           }

#declare rd=seed(0);
#declare z_plane1 = plane { <0,0,1>, 20
      texture{T_Tile1 translate rand(rd)*10}
      texture{T_Spots translate rand(rd)*10 scale 1+rand(rd)*2}
}

#declare z_plane1 = intersection{ plane { <1,0,0>,  -2 texture{phong1} } box   { <-3,-1,-3>,<3,1,3> } }
#declare z_plane2 = intersection{ plane { <0,1,0>,  -1 texture{phong2} } box   { <-3,-1,-3>,<3,1,3> } }
#declare z_plane3 = intersection{ plane { <0,0,1>,  -2 texture{phong3} } box   { <-3,-1,-3>,<3,1,3> } }

#declare axis= union {
  cylinder { <-10,0,0>, <10,0,0>, 0.04 texture{ phong0 } }
  cylinder { <0,-10,0>, <0,10,0>, 0.04 texture{ phong0 } }
  cylinder { <0,0,-10>, <0,0,10>, 0.04 texture{ phong0 } }
}

#declare cc=0.05;

#declare S1 = poly {2, <1,0,0,0,-1,0,0,1,0,-0.3> scale 1.5  hollow
             clipped_by { sphere { <0,0,0> 2.0 } } }

#declare trace_z=difference {intersection {object{ S1} box {<-100,-100,-cc>,<100,100,cc> }} object{S1 scale 0.95}}
#declare trace_y=difference {intersection {object{ S1} box {<-100,-cc,-100>,<100,cc,100> }} object{S1 scale 0.95}}
```


Povray by Jaime Vives Piqueres


Povray by Jaime Vives Piqueres


Povray by Gilles Tran

Plumber Stories

Oliver Knill, April 27, 2019
April 25'th is Plumbers day, Back to Random and Silly.

Commedy

he following stories were told on April 26, 2019 at the ``Faculty Lounge show" organized by the **Harvard College Stand Up Comic Society** (HCSUCS). It even gave a certificat! Harvard faculty performed there to raise oney for the **Harvard Square Homeless Shelter** and the **Y2Y Youth Homeless Shelter**. As a complete novice in this art of performance, I was coached three times by members of society and had even a 1-1 rehearsal in ne of the lecture hall. Especially **Lincoln Sorscher**, **Kellen Dugan** and **Mariana Garza** fed a lot of joke suggestions and insight on what makes comedy. I only adapted jokes which kept the stories true. As commented on ater, there were some exaggerations. The stories were trimmed then for a 7 minute total performance. (During the rehearsal, I still needed 8 minutes but could trim it down a bit.) As one can see, there is obviously not much alent for comedy here. But we worked hard: Here (txt file) were some early idea notes and here (txt file) were some suggestion notes by the HCSUCS team. Below is the final text which I rehearsed for the performance. I ould not be good enough to improvise this. That is an other level. Some of the students did it as ``appetizers" before the main 6 performances.

The broken pinky

A couple of weeks ago, I broke my Pinky! While running near the Charles river, I had fallen over a root, probably a square root. The little finger stood out a 90 degree angle. I did not need a protractor to see that this is fucked up! I went to Harvard University Services, payed my 30 dollars co-payment, was sent to x-ray and saw the doctor who looked at the finger and said: that is fucked up. You have to go the Mt Auburn emergency services! I packed my things and walked over to Mt Auburn Hospital, just happy of not having broken the third finger. Lots of people in the emergency room. Some more damaged than me. A cook told me that he had cut half has arm off and had been bleeding there for 3 hours. I payed my 30 dollars co-payment, saw the doctor who said: this is fucked up. You have to see a surgeon! They numbed up my finger and stuck it into a splint and sent me home. A couple of days later, I met the surgeon, payed the 30 dollars co-payment. The doctor looked at it and said: this is fucked up! I need more time. You have to come back later. I came back, payed my 30 dollars co-payment, and then, like a good plumber, the pieces of the finger were put together again. This was my first plumber story. Did you know that yesterday was national plumber day? Actually, national ``Hug a plumbers day"!


Story telling
Set Marker
Hover Mode
Boost

Outreach

The Adventure of Teaching Algebra

This one-day workshop was organized by the Texas Valley Communities Foundation and the Texas Graduate Center, which is affiliated with the Math for teaching degree program at the Harvard extension school. Thanks to **Mary Alice Reyes** and **Adriana Lopez** from the Texas Graduate Center for arranging and organizing that (and dinner). It was an inspiring workshop with amazing contributions from the class which I still have to digest. There were almost 30 teachers present. (Photos by the center: pic1, pic2, pic3.) Some handouts are to the right. In the wake of the preparations, I also mixed in a bit of algebra in my current passion for geometry on graphs.

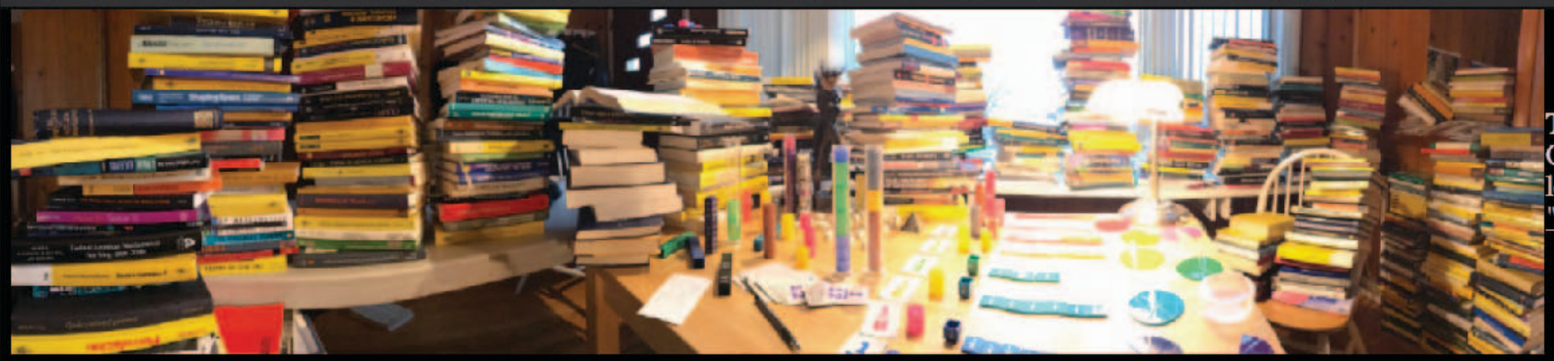
Notes [PDF], Warm up [PDF], Unit 1 [PDF], Unit 2 [PDF], Unit 3 [PDF],Unit 4 [PDF], Postscript [PDF]. Feb 3, 2017: Hidden figures shows the importance of algebra skills:

More about the math. See also 10-2=20 [Jun 2, 2017] and Percentages [Jun 13, 2017].

Leonard Euler who lived from 1707 to 1783 is the grand master of Peda the realm of algebra. Euler also invented graph theory (Koenigsberg Brid Problem), seeds of topology (Euler characteristic etc) and so many other He is probably the most inspiring mathematician ever, not only because theorems and formulas v-e+f=2,exp(i π)+1=0,1+1/4+1/9+1/16+...=$\pi^2/6$ also because of his outreach and his passion for making it accessible. Eul walked the talk, like many of the teachers who throw their energy into th cause of teaching. Euler's contributions to algebra pedagogy was not only writing his textbook in Algebra but producing a gold standard in clarity w hard to surpass. It is one of the most successful textbooks of all times.

I have here PDF files of Euler's Algebra textbook:

English translation [PDF] (with notes of Bernoulli and Lagrange)	German Part I [PDF],	German Part II [PDF]



The picture to the right was taken in my Office. It is part of this panorama. To the left (click for more photos), we see the "Fraction lab".

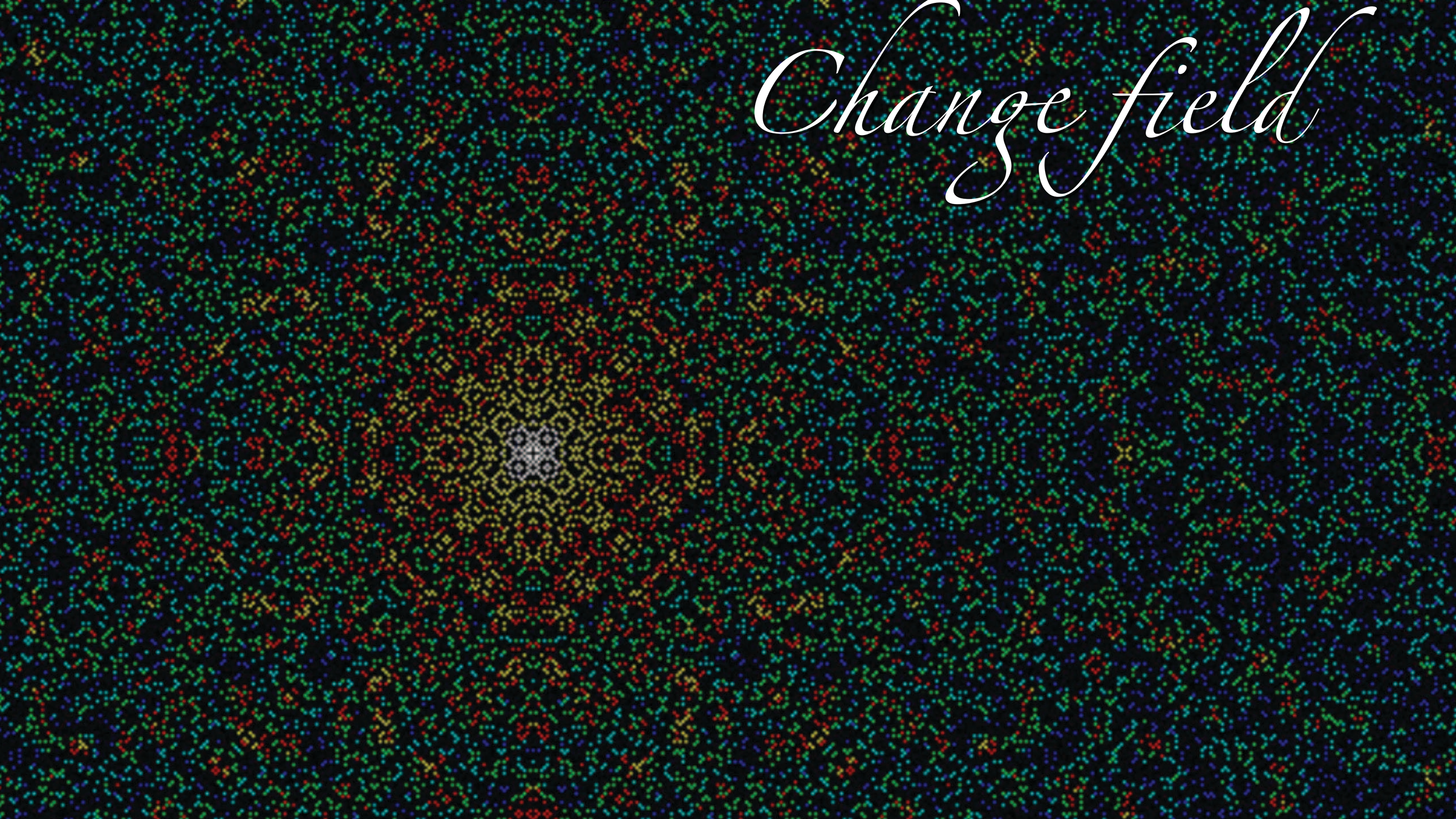
Some photos from the workshop. I hope to link later to more, done by the graduate center.

There is also great food in the Texas valley. Here is a photo from a restaurant (thanks to Claudia from the Texas Valley Communities Foundation) for that lunch:


Different math
stratosphere
main stream
uncharted

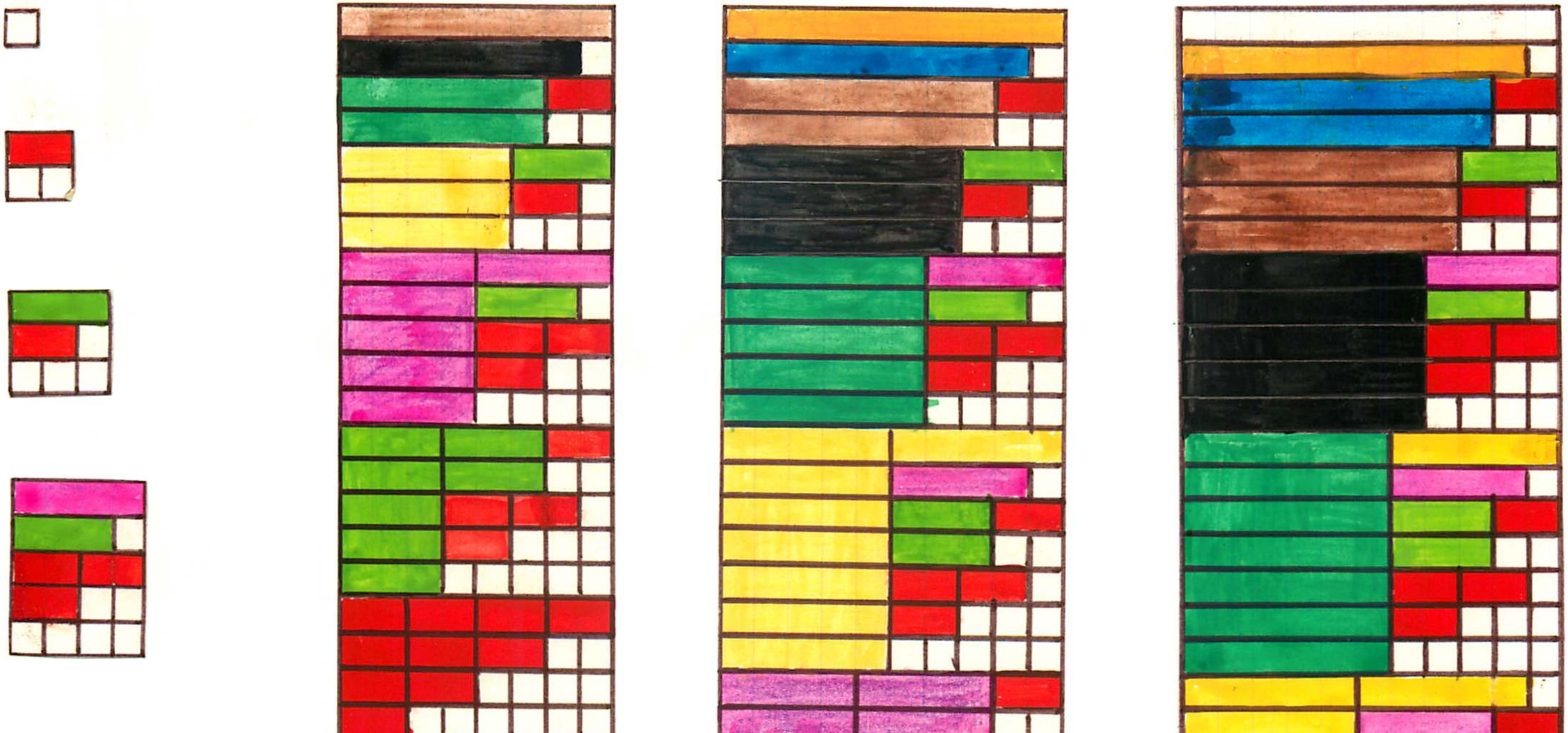
off the chart

uncharted territory


Change field

Accessible Math

2. "p(n)"



DORIS LEUTHARD
Bundesratspräsidentin

«DIE NEUGIERDE STEHT AN ERSTER STELLE»

Kennen Sie die Schweizer Jugend? Dann kennen Sie die Schweiz! Neugierig und mutig, offen und praktisch – so sind wir Schweizer. So lösen wir die grossen Herausforderungen. So packt auch die Jugend seit 50 Jahren unter dem Dach der Stiftung «Schweizer Jugend forscht» kühn zu.
Herzliche Gratulation!

Ohne Forschung bleiben wir stehen. Ohne die Erfindung der archimedischen Schraube 220 vor Christus hätte heute wohl mancher Klärmeister seine Probleme. Damit wir die Knacknüsse der Zukunft bewältigen können, reicht es nicht, die Lösungen von heute als Antworten für morgen zu präsentieren. Unsere jugendlichen Forscherinnen und Forscher zeigen Jahr für Jahr neue Wege, neue Denkansätze und lassen uns staunen. So werden wir Resultate für die Zukunft finden! Die Jugend mit ihrem unverbrauchten Blick auf das Leben; die Stiftung strukturgebend und motivierend.
Ziel jeder Forschungstätigkeit muss es sein, Wohlfahrt und Wohlstand zu fördern und damit die Lebensqualität aller zu erhöhen. Das ist uns bisher gut gelungen. Die sich verknappenden Ressourcen auf unserem Planeten bringen neue Herausforderungen. An der Schwelle hin zu einer digitalen Gesellschaft funktionieren herkömmliche Rezepte nicht mehr. Spannende Fragestellungen für die nächsten 50 Jahre gibt es somit zuhauf.

Liebe Jugendliche, lasst euch den Blick nicht durch Mauern und Zäune verbauen. Orientiert euch an Galileo Galilei: «Die Neugier steht immer an erster Stelle eines Problems, das gelöst werden will.»

OLIVER KNILL
Mathematiker,
Harvard University

«EINE GROSSARTIGE ERFAHRUNG»

Die Teilnahme am Wettbewerb Schweizer Jugend forscht war eine Weichenstellung. Das Projekt zeigte mir, dass ich Freude an der Mathematik habe, auch unabhängig von Erfolg. Das hat sich bis heute nicht geändert und gab mir auch später Kraft. Mein Weg zeigt, dass es nicht unbedingt eine traditionelle Forscherkarriere sein muss, um trotzdem weiter im Gebiet arbeiten zu können, das mich begeistert. Ich betrachte mich heute als Amateur-Forscher, und die Freude am mathematischen Arbeiten ist die gleiche geblieben wie beim Arbeiten an der «Anschaulichen Zahlentheorie», die ich bei Schweizer Jugend forscht eingereicht habe.

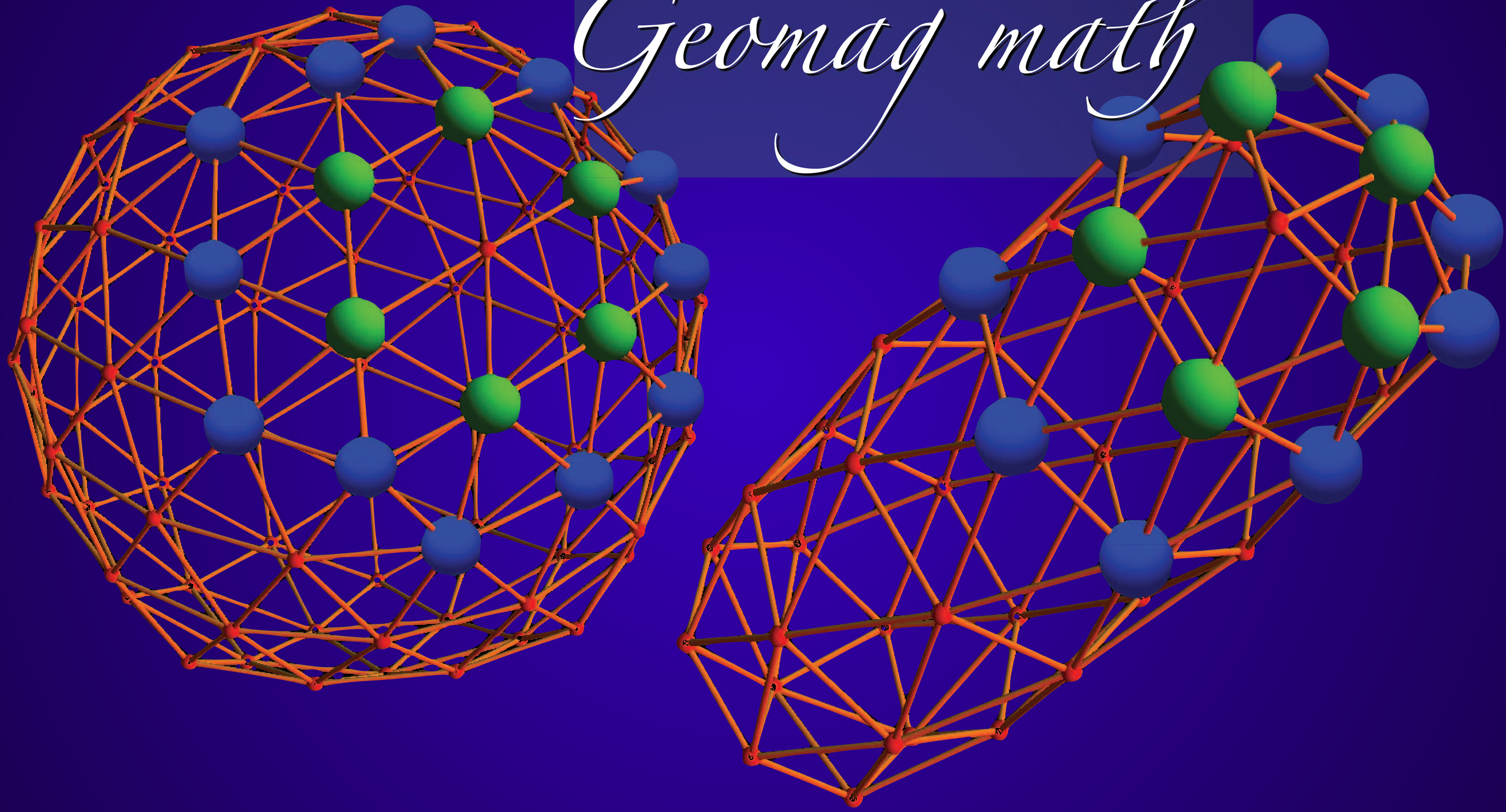
Das Herumexperimentieren ist Erholung und Bedürfnis geblieben. Als junger Forscher hatte ich mich natürlich auch überschätzt. Die Bewertungen durch die Experten und Expertinnen haben mich auf den Boden gebracht. Es mag paradox wirken, doch zu viel Ermuti-

Der Wettbewerb hat mich auch in Kontakt mit anderen «Forschernaturen» aus der Schweiz gebracht. Einige dieser Kontakte halten seit Jahrzehnten. Ich befreundete mich mit einer Gruppe aus der Westschweiz. Die gemeinsame Reise der Teilnehmenden nach Paris hat diese Freundschaft aufgebaut und zementiert. Wir haben uns viel geschrieben und uns Jahre nach dem gemeinsamen Abenteuer noch besucht. Diese Begegnungen gehören bis heute für mich zu den tollsten Ereignissen meines Lebens.

Oliver Knill studierte und promovierte an der ETH Zürich in Mathematik und unterrichtet seit 2000 an der Universität Harvard. 1981 reichte er bei Schweizer Jugend forscht seine Forschungsarbeit mit dem Titel «Anschauliche Additive Zahlentheorie» ein – sie hat 154 Seiten und ist seinem ehemaligen Mathematiklehrer gewidmet.


GEOMETRY
TYCHO & KEPLER
Solve It
TIME TRAVEL
IAN STEWART
Graph Theory and Complex Networks
THE HISTORY OF STATISTICS

Geomag math



Mickey Mouse Theorem

Mickey Mouse Theorem

Let G be a discrete manifold. Assume all embedded wheel graphs have less than 6 vertices. Then G is a discrete sphere.

Classical analogues

Heinz Hopf

Harry Rauch

John Synge

Wilhelm Klingenberg

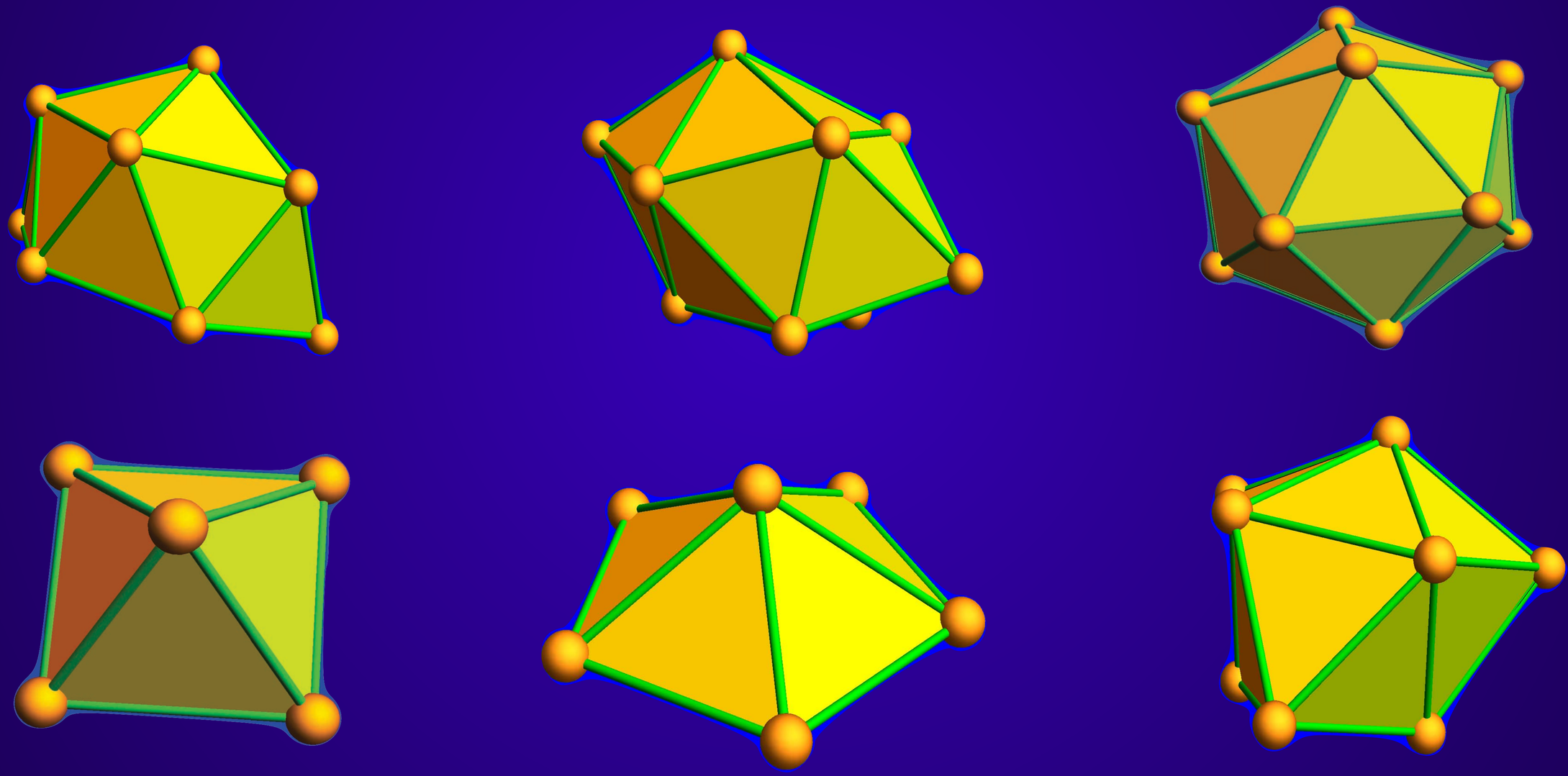
Marcel Berger

A positive curvature manifold satisfying a pinching condition is a sphere.

An orientable positive curvature manifold is simply connected

In 2D:

"6 mice"



About non-tenured positions

Backgrond statistics

http://www.ams.org/Excerpt_PhDs_Employment.pdf

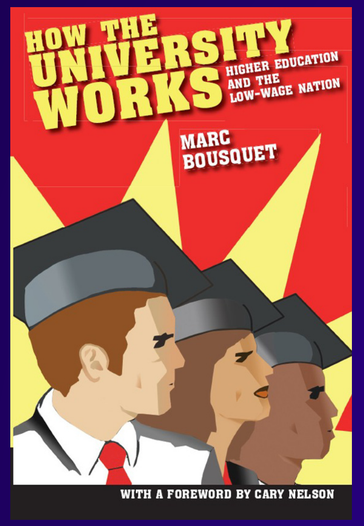
2005-2015 50 % increase of math PhD's

2018: 73 percent of all faculty positions are off the tenure track

https://nces.ed.gov/ipeds/

Full-time, non-tenure-track faculty members at doctoral, master's and baccalaureate institutions: some 38 percent of these full-time instructors are on annual contracts. Some 58 percent are either on multiyear (20 percent) or indefinite or at-will contracts (38 percent).

https://www.insidehighered.com/news/2018/10/12/about-three-quarters-all-faculty-positions-are-tenure-track-according-new-aaup



2008

From Bousquet

Tough Job market

Why in this conference on professional norm?

Implicit Bias

racial

gender

orientation

institution

age

title

also due to confirmation bias

Dr Fox lecture

we are in summer 1970 at a faculty retreat near Lake Tahoe

Similarly as positive bias, we have to be aware of negative bias.

End of Part 1

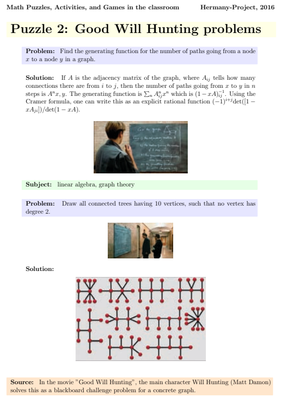
Part 2

Taxonomies

Norms

Taxonomy

Paul Hermany


Math Puzzles, Activities, and Games in the classroom Hermany-Project, 2016

Puzzle 2: Good Will Hunting problems

Problem: Find the generating function for the number of paths going from a node x to a node y in a graph.

Solution: If A is the adjacency matrix of the graph, where A_{ij} tells how many connections there are from i to j, then the number of paths going from x to y in n steps is $A^n x, y$. The generating function is $\sum_n A_{xy}^n x^n$ which is $(1 - xA)_{xy}^{-1}$. Using the Cramer formula, one can write this as an explicit rational function $(-1)^{xy} \det(\{1 - xA_{xy}\})/\det(1 - xA)$.

Subject: linear algebra, graph theory

Problem: Draw all connected trees having 10 vertices, such that no vertex has degree 2.

Solution:

Source: In the movie "Good Will Hunting", the main character Will Hunting (Matt Damon) solves this as a blackboard challenge problem for a concrete graph.

Grade Level

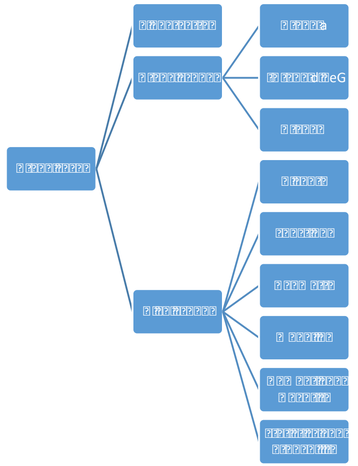


Figure 11: Taxonomy Visualization - 15 Grade Levels from the Comma

1 R Recreational M Mathematics DB Database Fox, Goose, and a Bag of Beans

2 Prompt

3

4 Solution

1. The farmer takes the goose to the other side of the river and leaves it there.
2. The farmer then takes the bag of beans to the other side and feeds one to the goose.
3. The farmer then takes the fox to the other side and leaves it with the bag of beans.
4. Lastly, the farmer takes the goose to the other side.

Math Puzzles, Activities, and Games in the classroom Hermany-Project, 2016

Puzzle 3: Miller puzzle

Problem: Find a solution to the system or equations

$$a_1(10a_2 + a_3) = 100a_4 + 10a_5 + a_6) = 100a_4 + 10a_5 + a_6 = (10a_7 + a_8) * a_9$$

where $(a_1, \ldots, a_9)$ is a permutation of the 9 numbers $1, \ldots, 9$. In short $a_1 * a_2a_3 = a_4a_5a_6 = a_7a_8 * a_9$.

Solution: Brute force search shows that there are exactly 4 solutions:
$\vec{a} = (2, 7, 8, 1, 5, 6, 3, 9, 4)$
$\vec{a} = (3, 5, 8, 1, 7, 4, 2, 9, 6)$
$\vec{a} = (4, 3, 9, 1, 5, 6, 7, 8, 2)$
$\vec{a} = (6, 2, 9, 1, 7, 4, 5, 8, 3)$

Subject: arithmetic, addition and multiplication

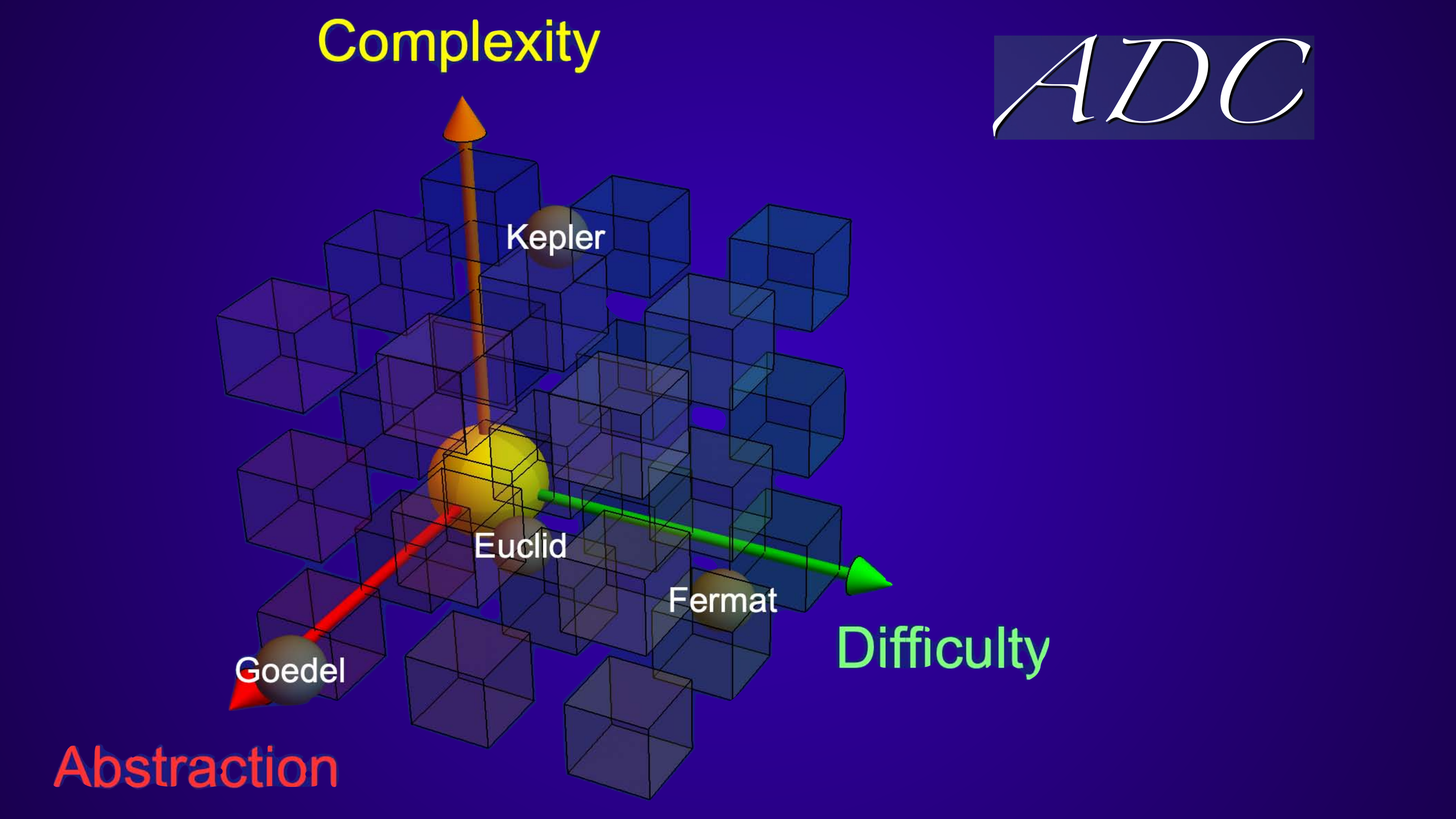
Source: This is one of the classical Canterbury puzzles.
http://www.transum.org/software/SW/Starter_of_the_day/students/hot/MillersPuzzle.asp

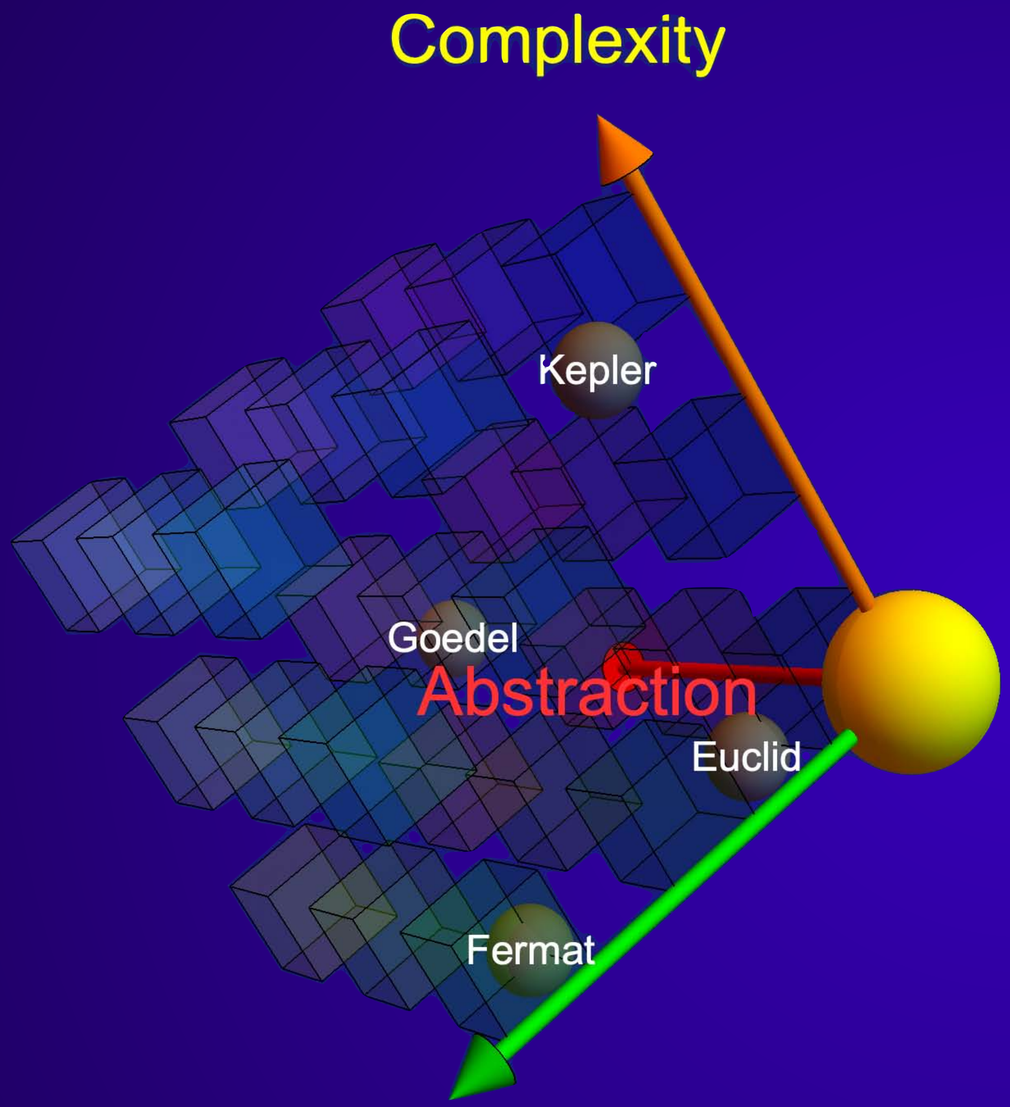
R Recreational M Mathematics DB Database Title Rubik's Cube

Keywords (Folksonomy)

14 Prompt

Solve the cube.


Complexity
ADC
Kepler
Euclid
Fermat
Goedel
Difficulty
Abstraction

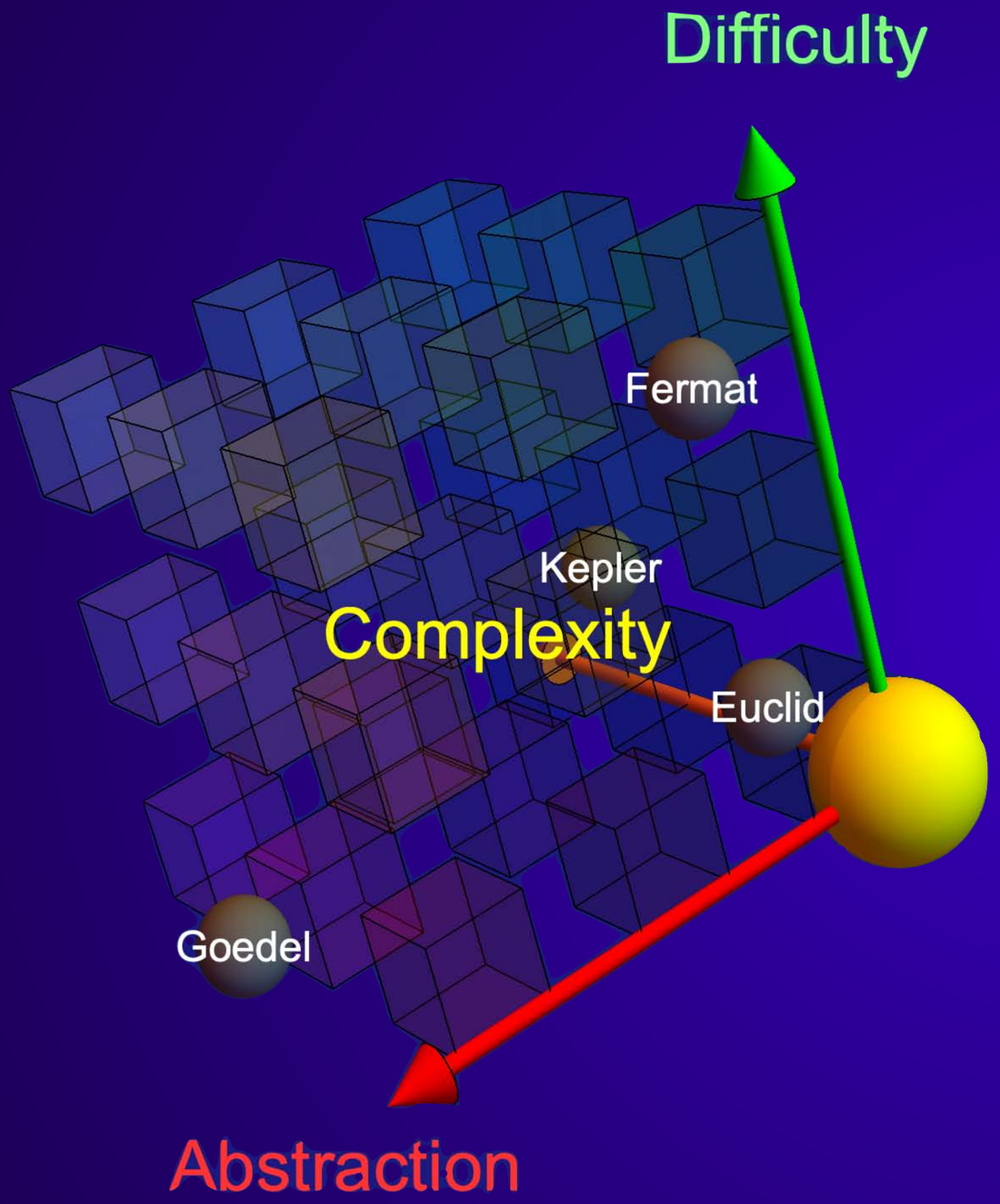


Complexity

"how tedious? "

solve

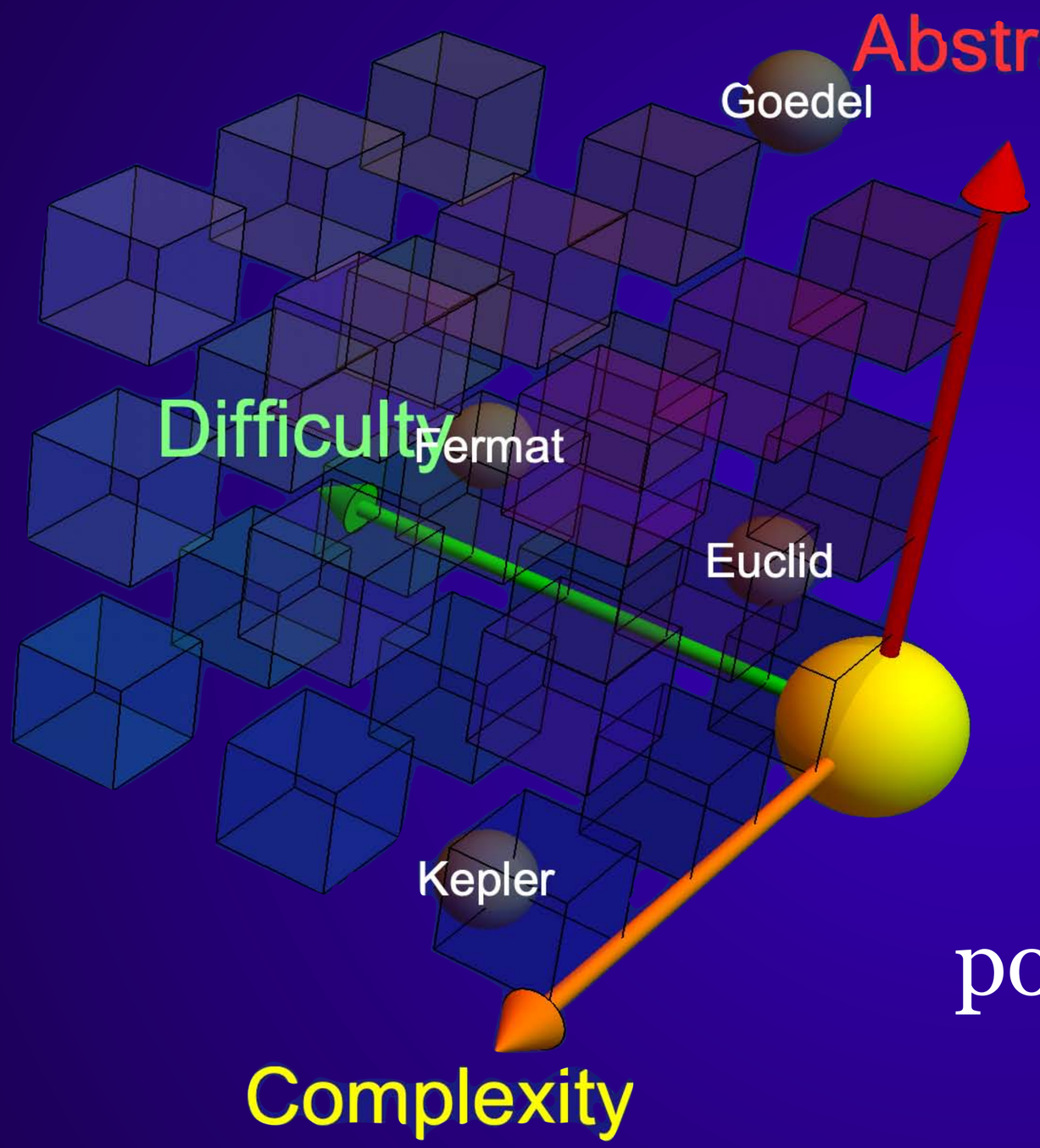
$$x^3 + 3x + 7x^2 + 3x^3 + x^4 = 0$$



Difficulty

"needs idea"

the roots of a palindromic polynomial are invariant under the map T(x)=1/x.



Abstraction

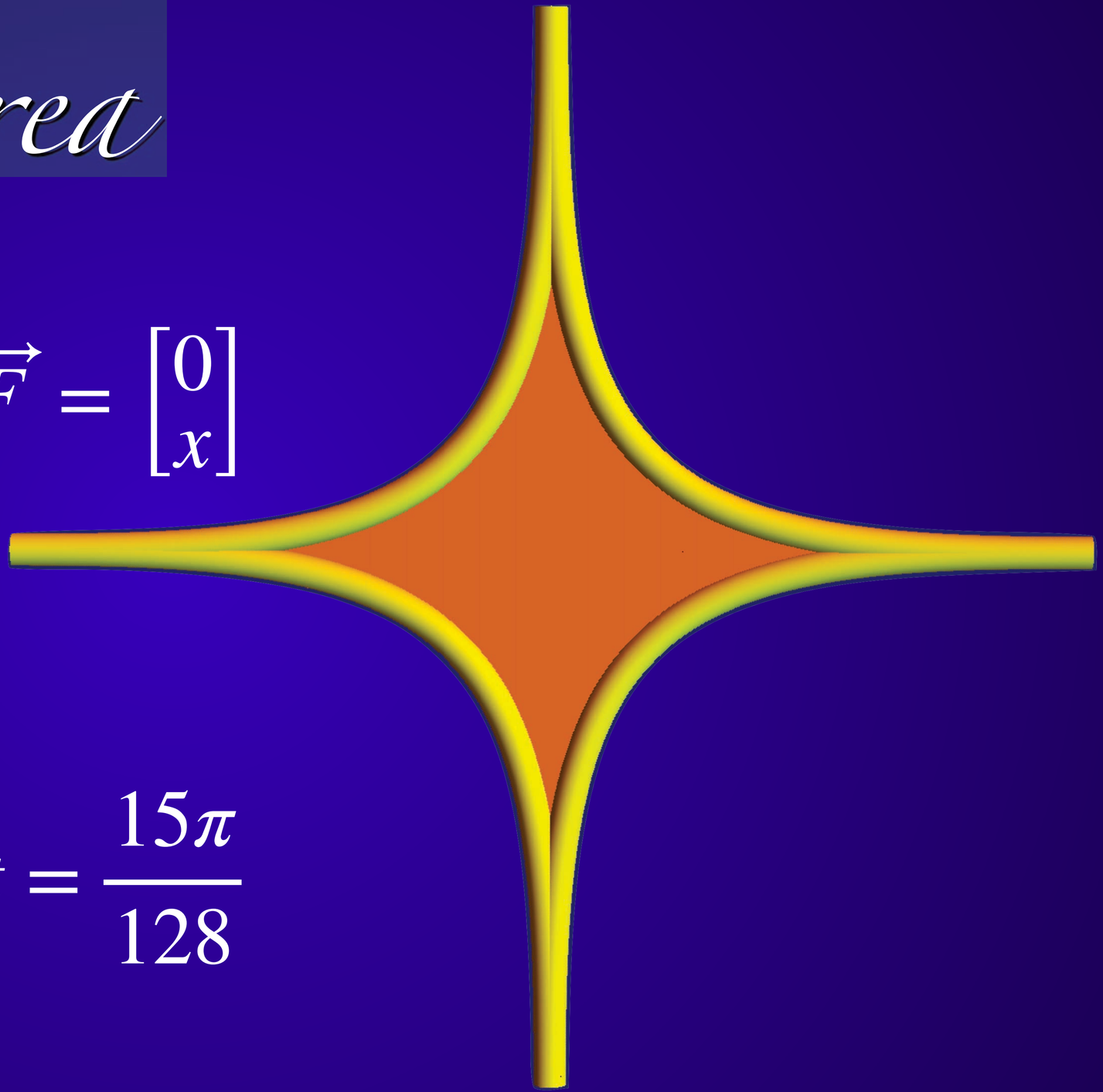
"needs background insight"

the characteristic polynomial of a symplectic matrix is palindromic

In Calculus

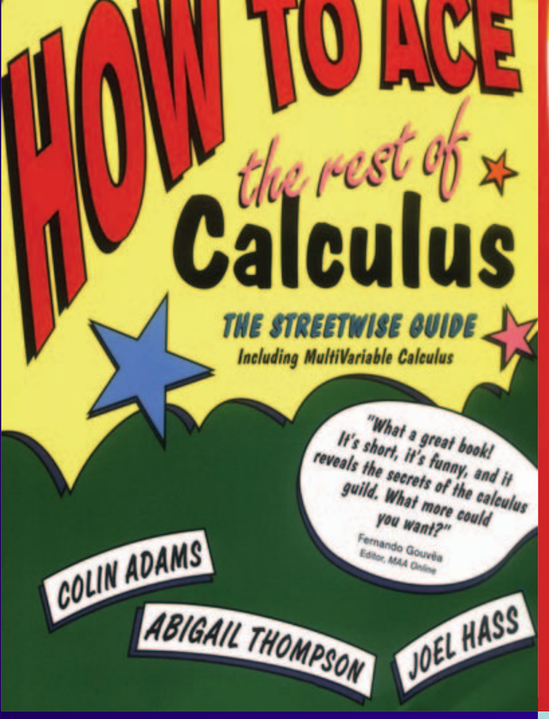
Find Area

$$\vec{r}(t) = \begin{bmatrix} \cos^5(t) \\ \sin^5(t) \end{bmatrix} \qquad \overrightarrow{F} = \begin{bmatrix} 0 \\ x \end{bmatrix}$$



$$\int_0^{2\pi} \cos^6(t) 5 \sin^4(t)\, dt = \frac{15\pi}{128}$$

Abstraction Level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Editor, MAA Online
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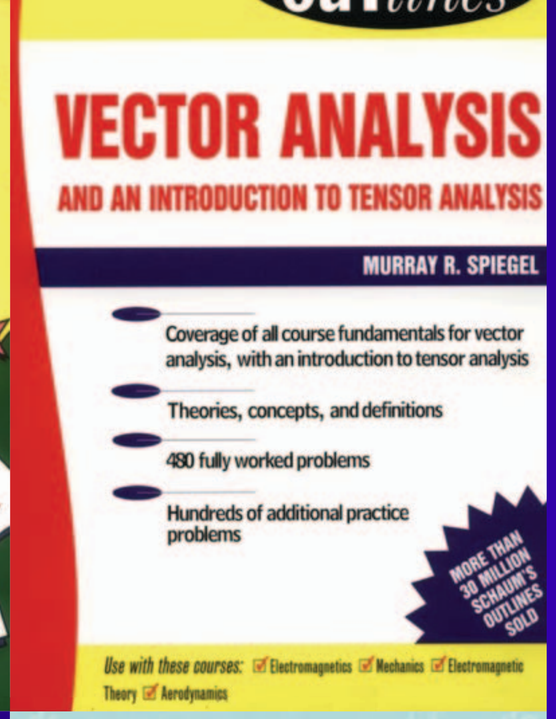
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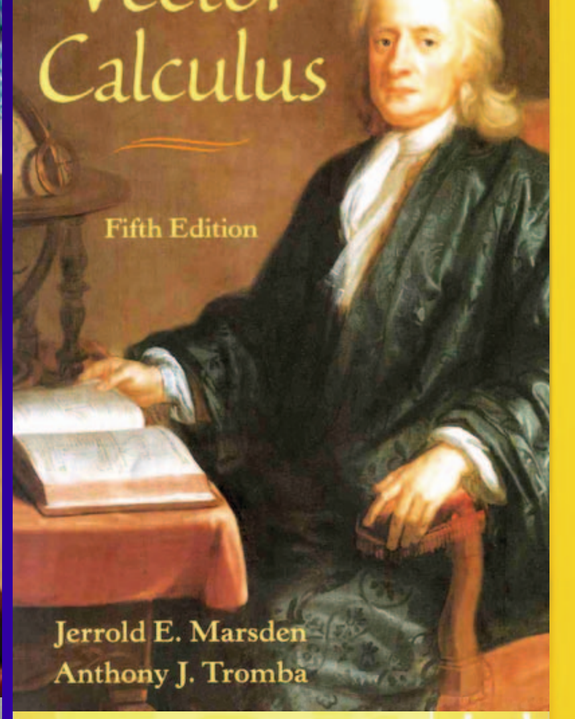
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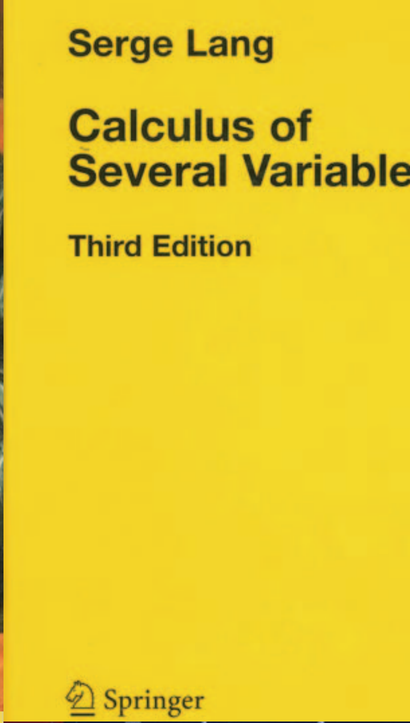
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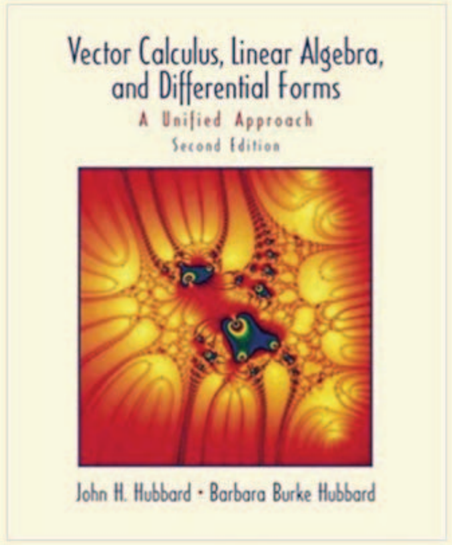
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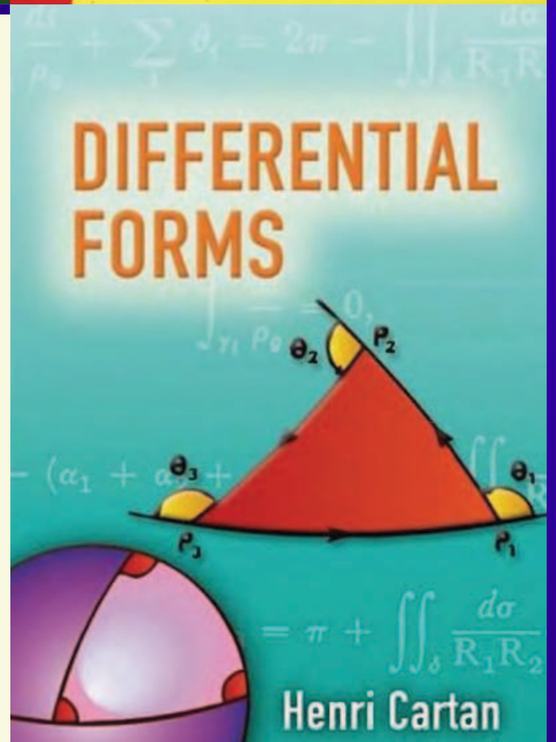
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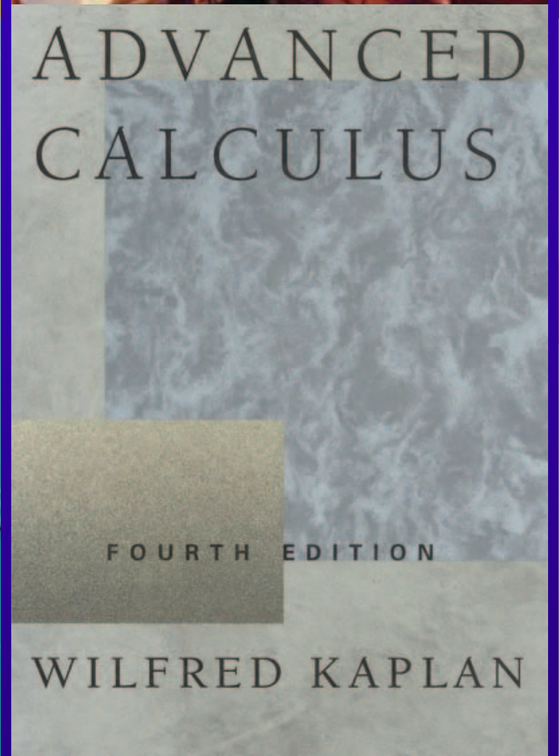
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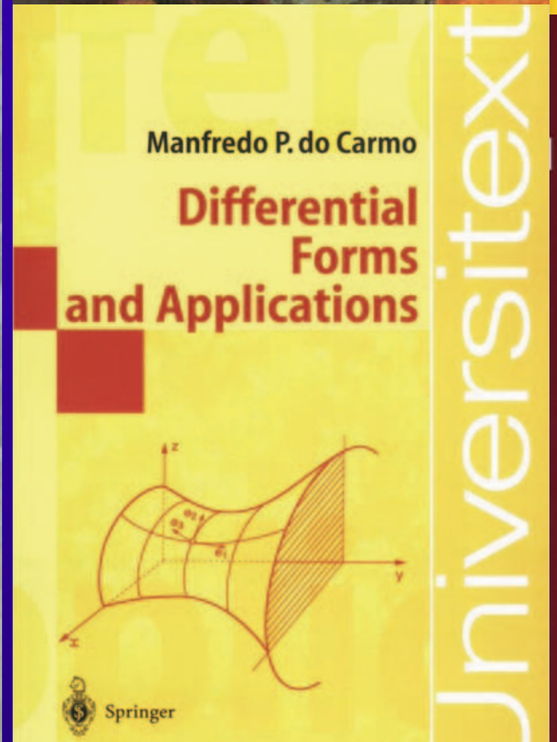
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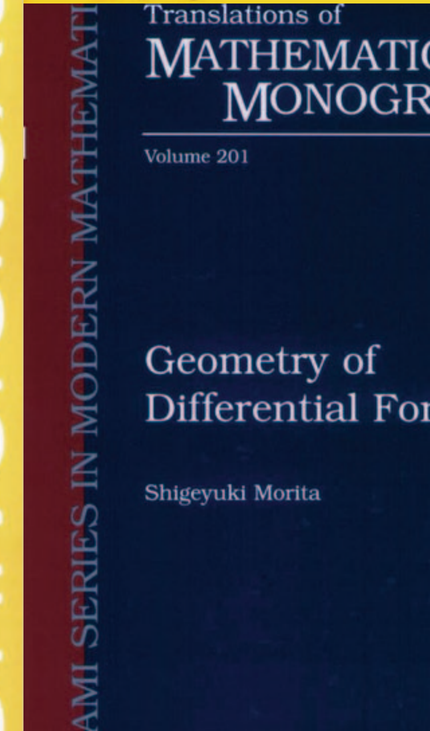
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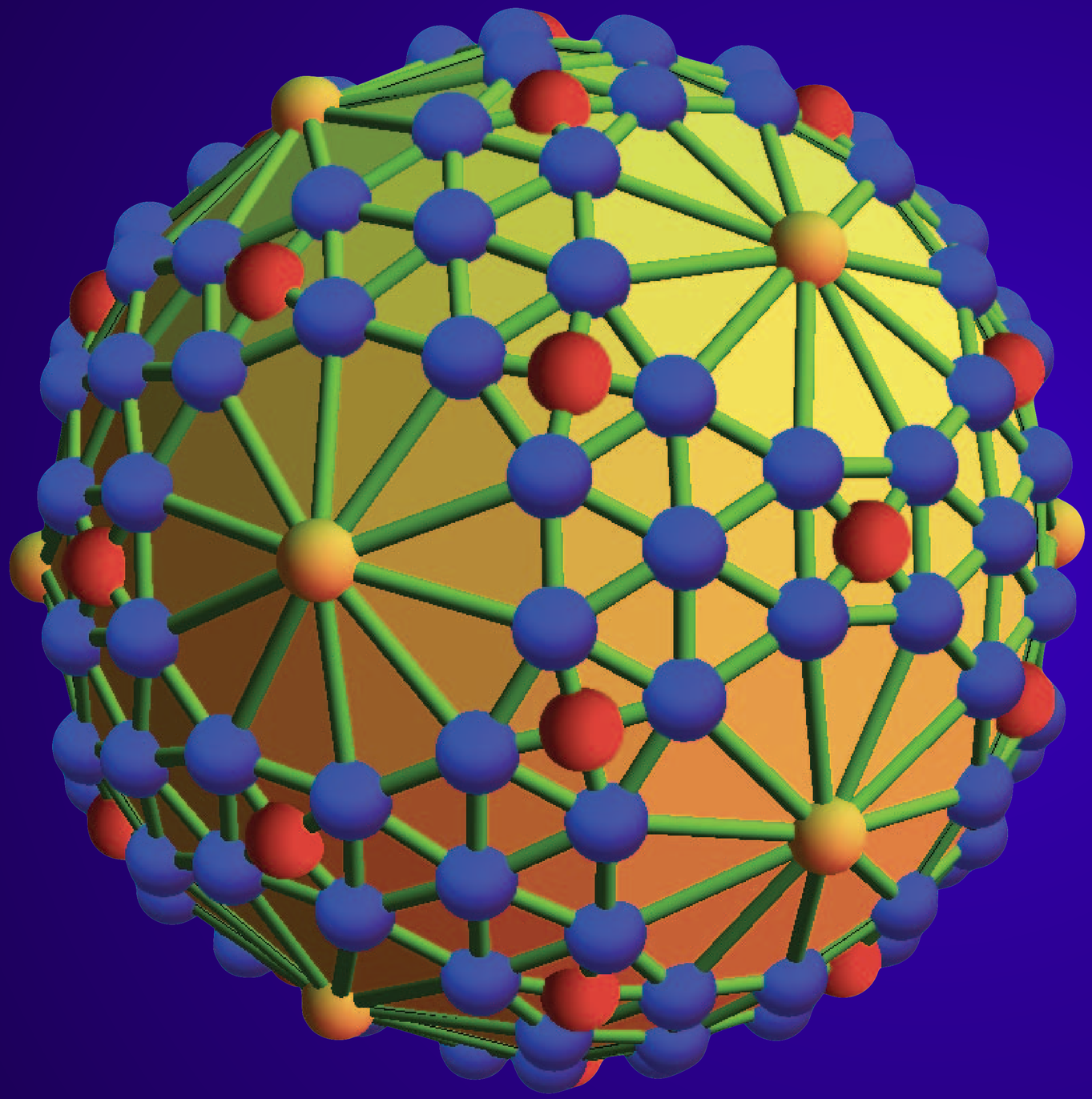
Differential For
Shigeyuki Morita

Example: Curvature



12 times 1-10/6

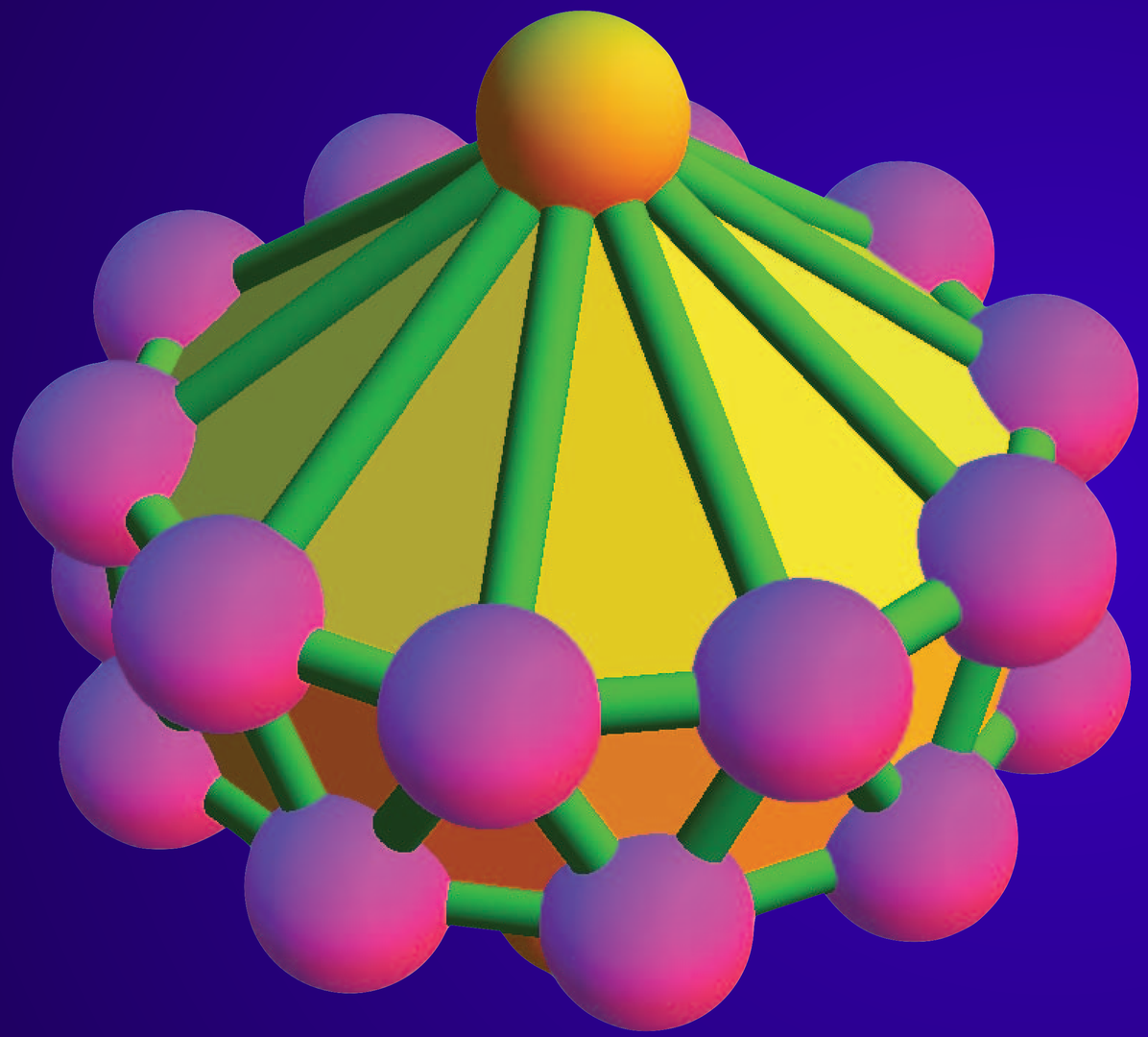
30 times 1-4/6

= 1 - d(x)/6

total

curvature

= 2

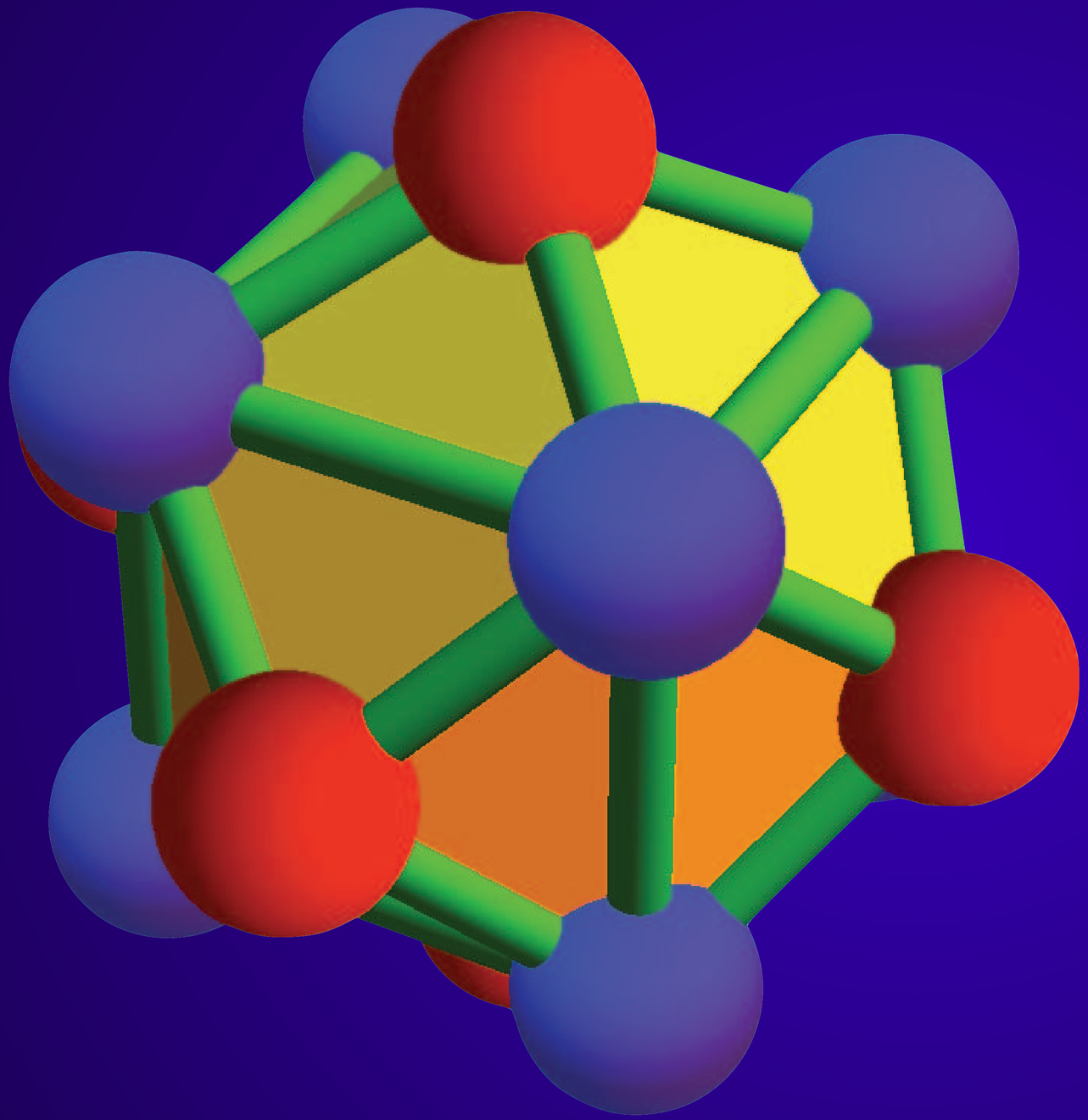


20 * 1/3 + 2 (-2/3)

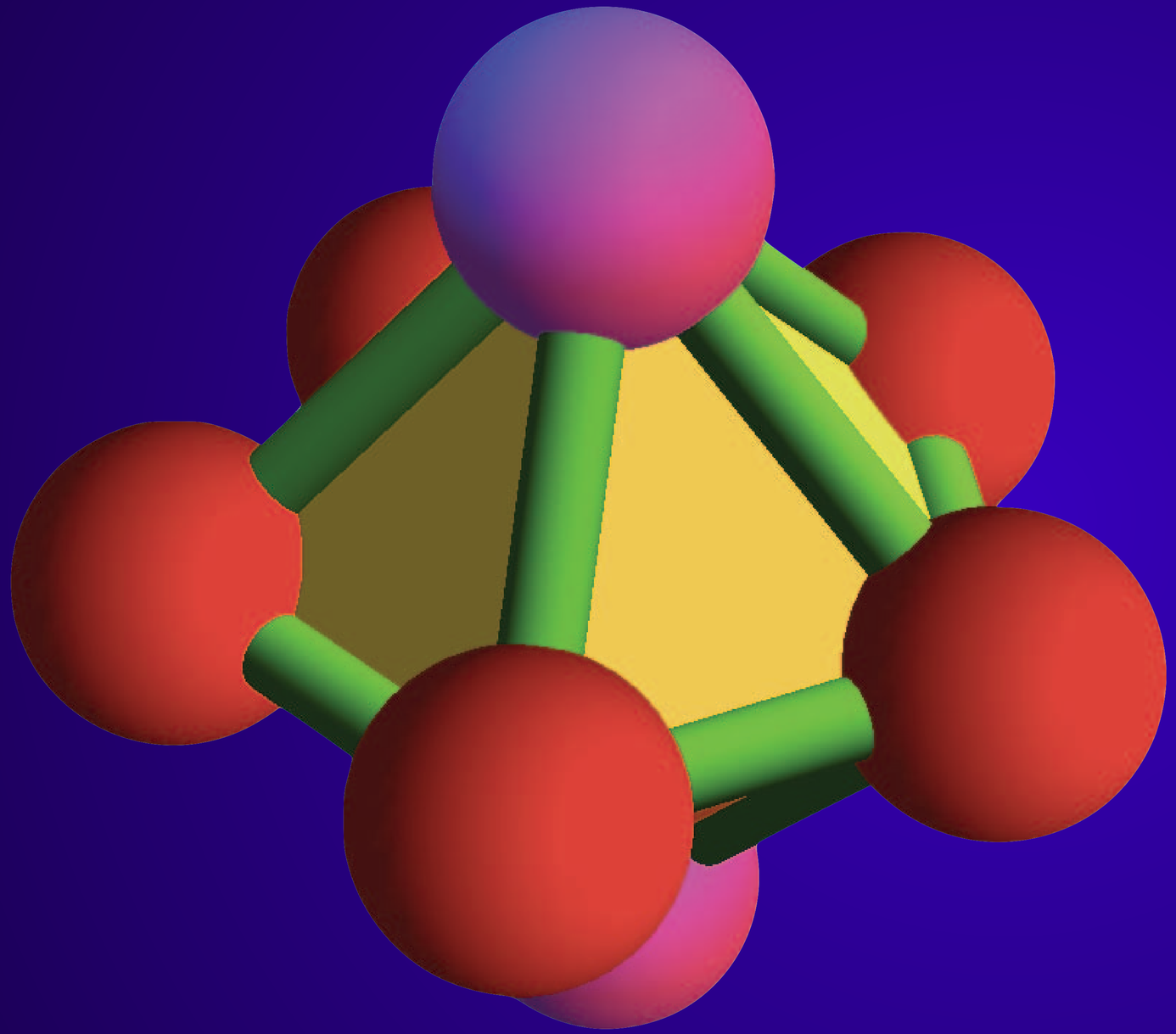
total

curvature

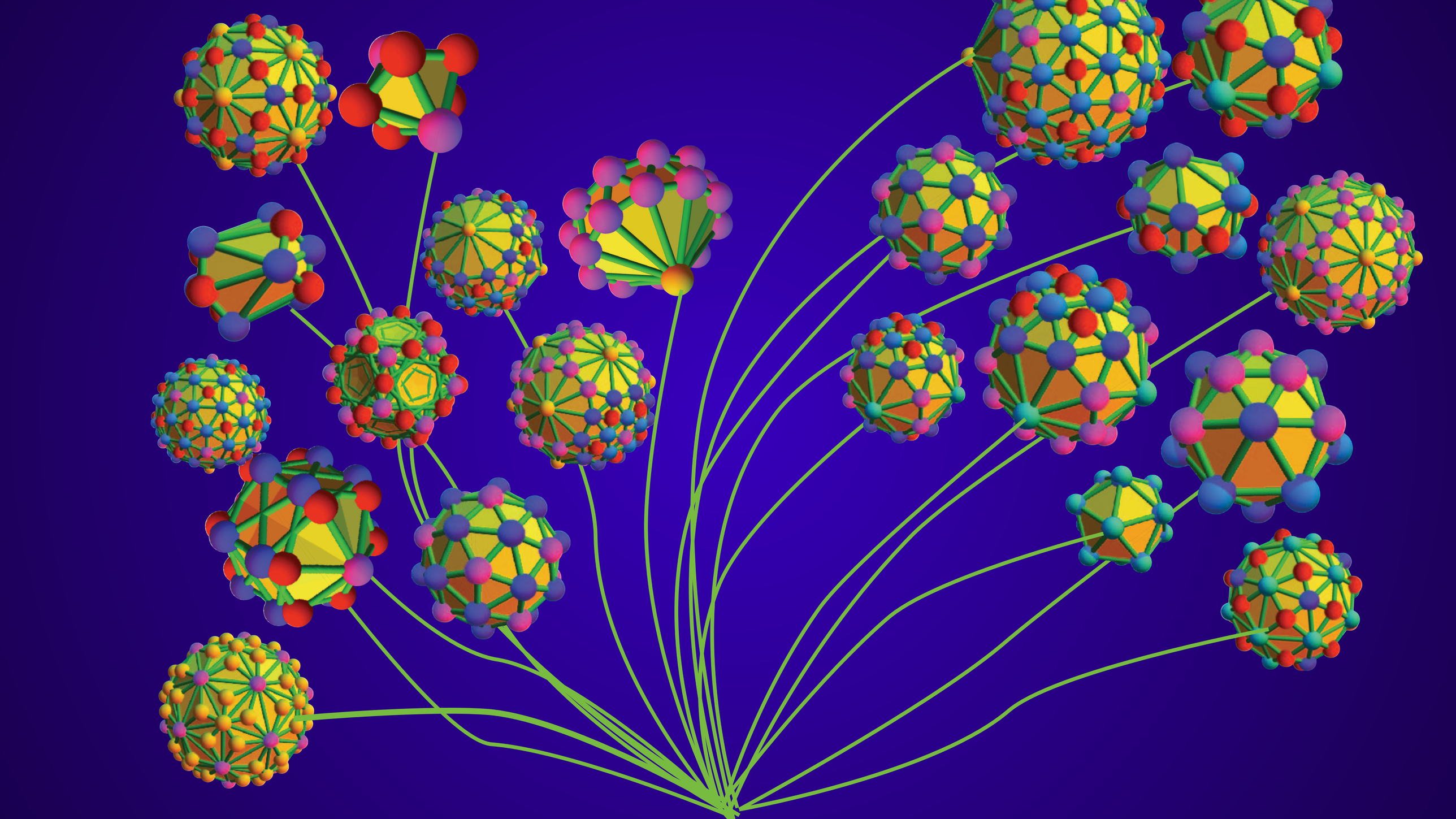
= 2



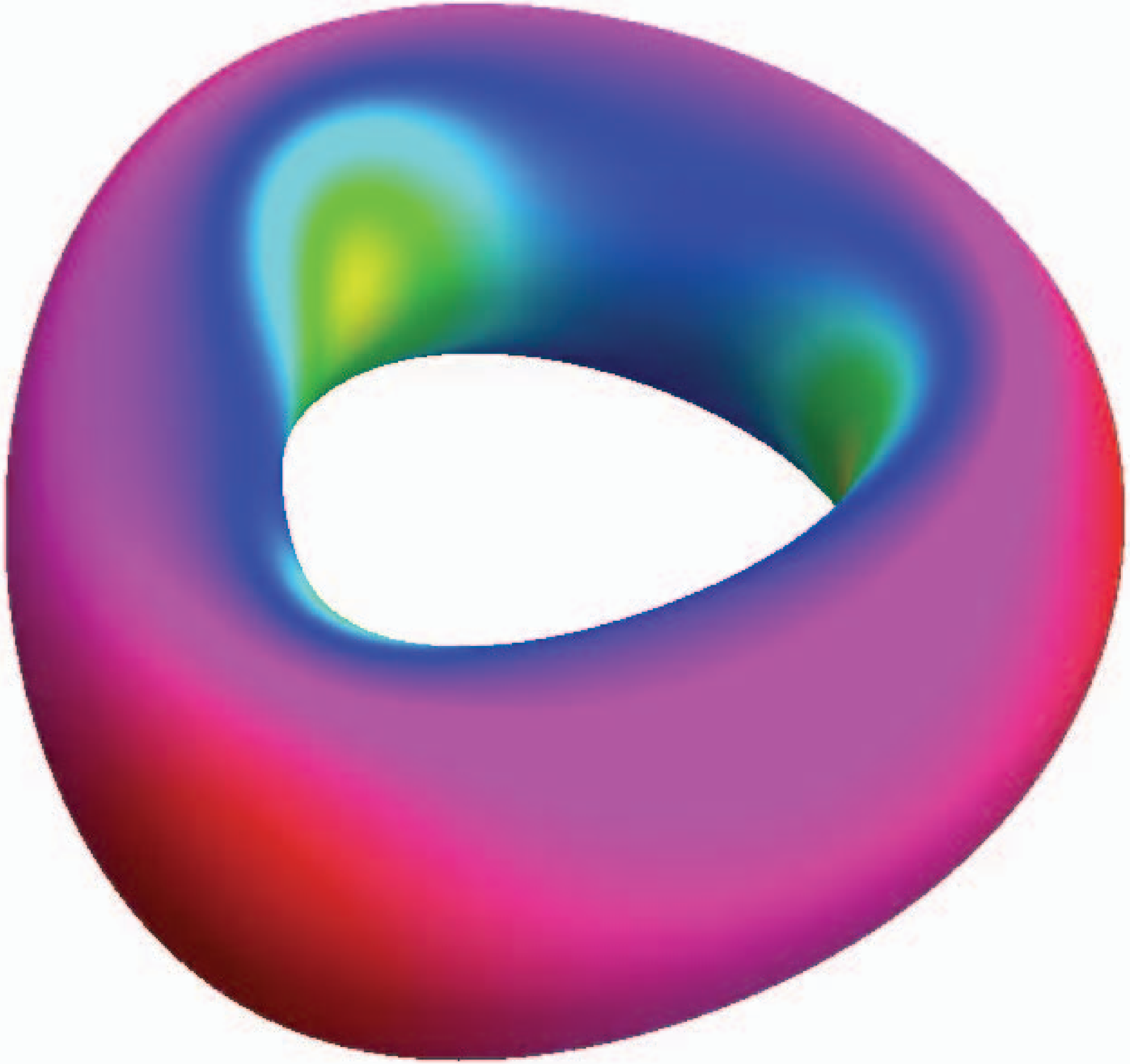
6 * 1/3



5 * 1/3 + 2*1/6

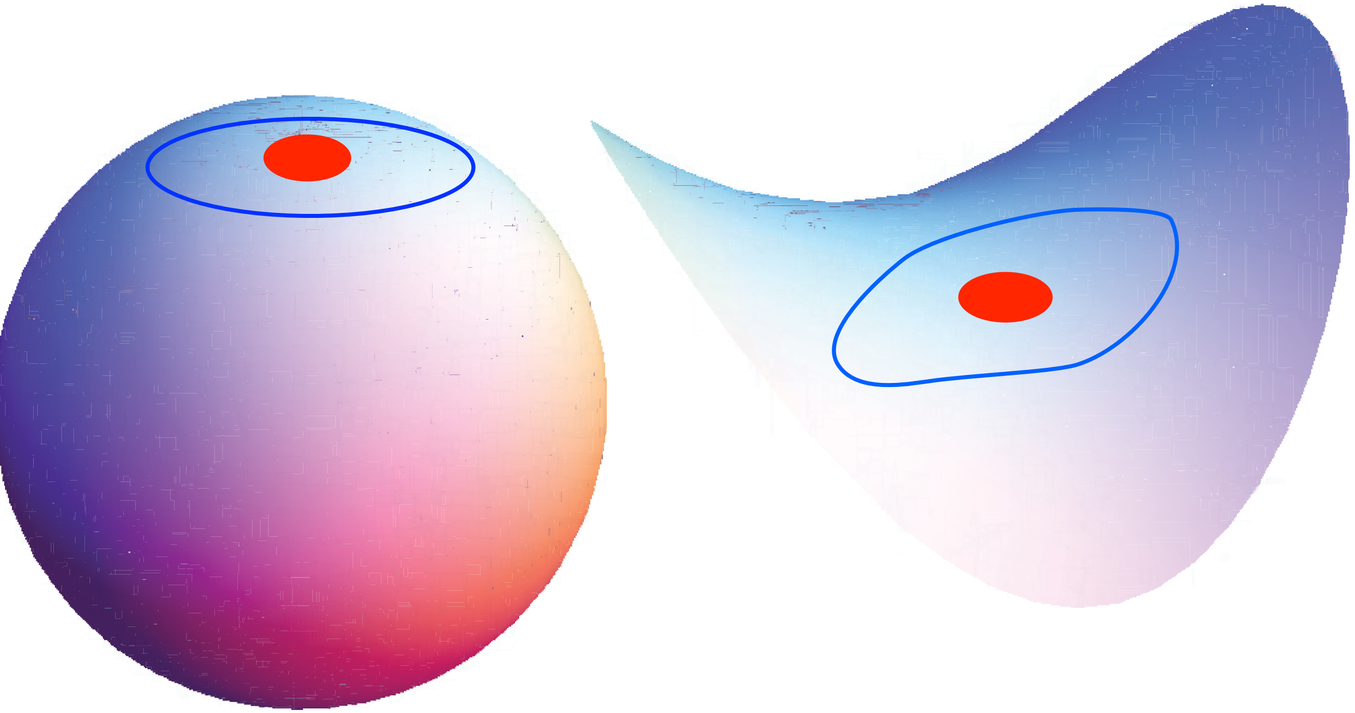


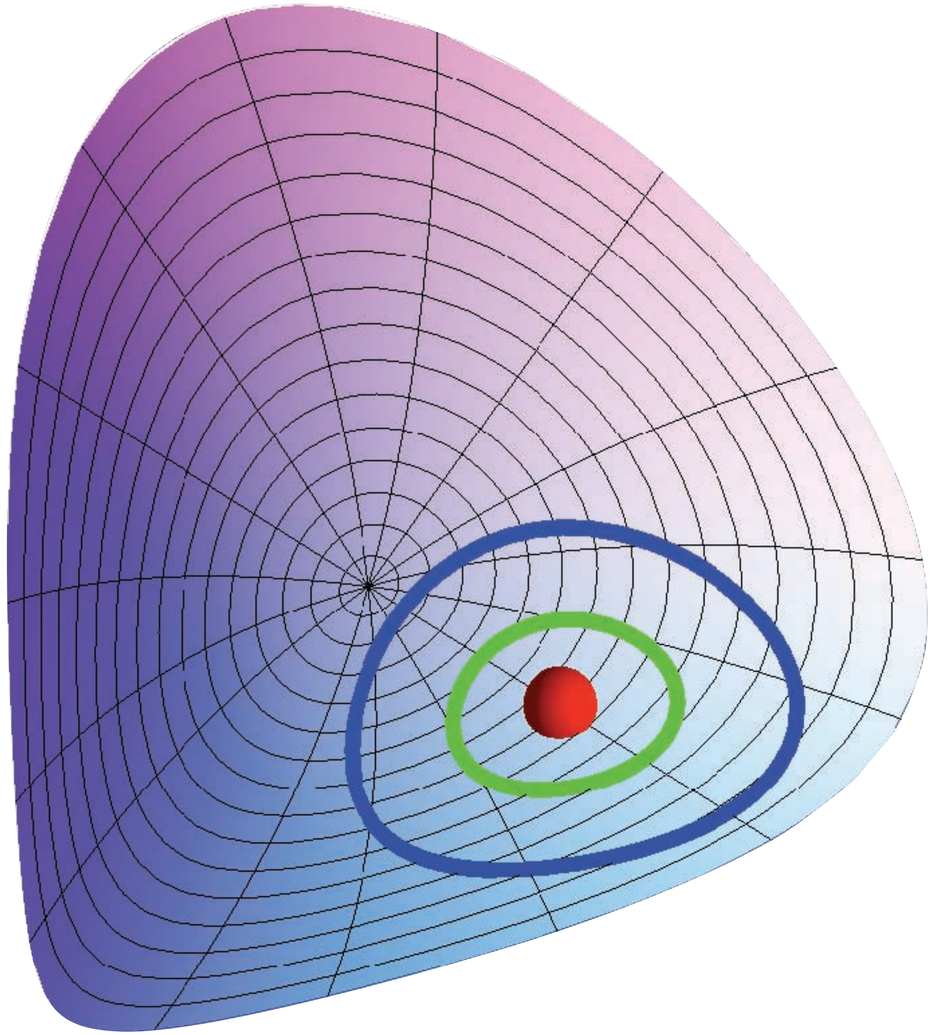
Advanced Level



K>0

K<0





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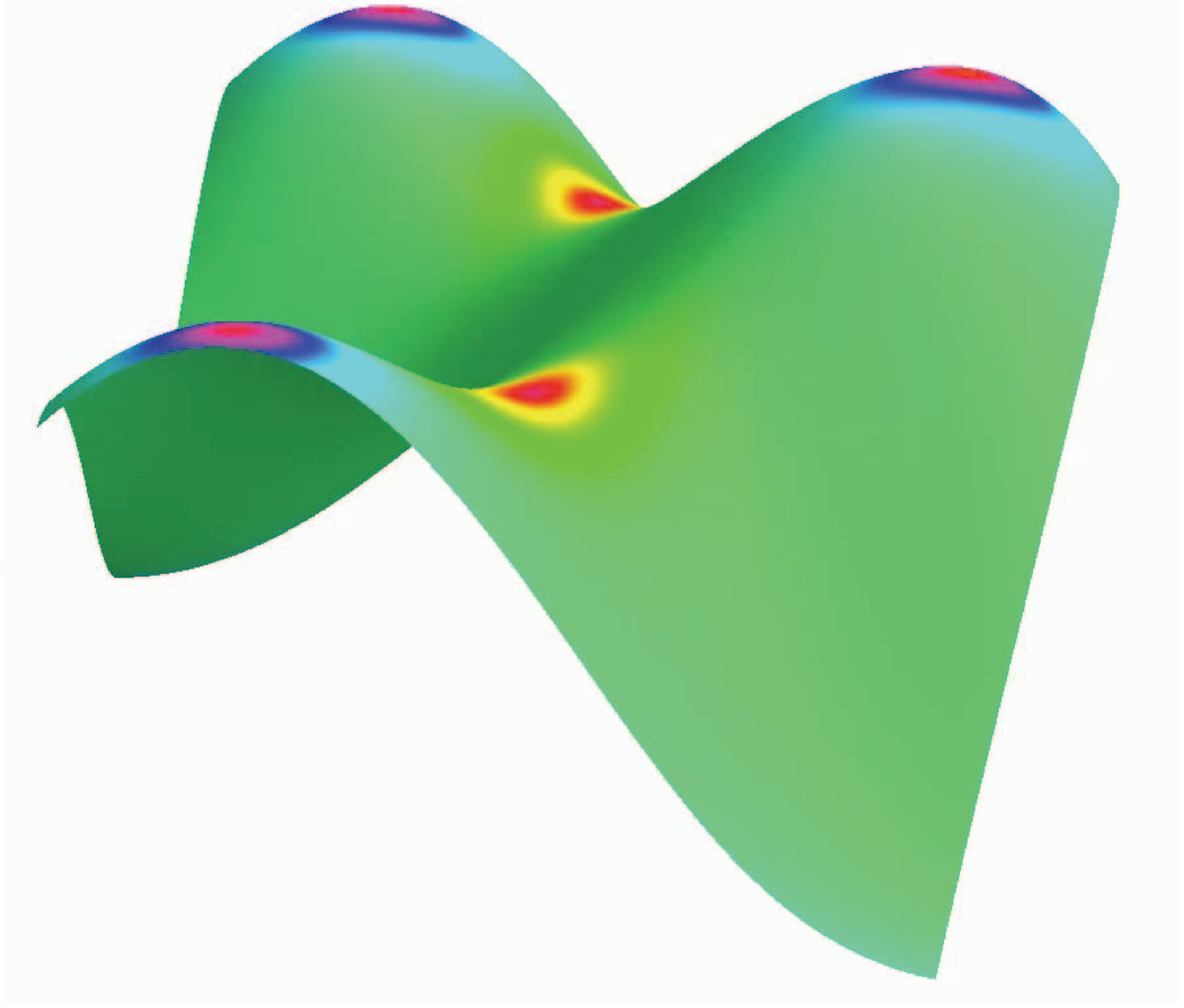
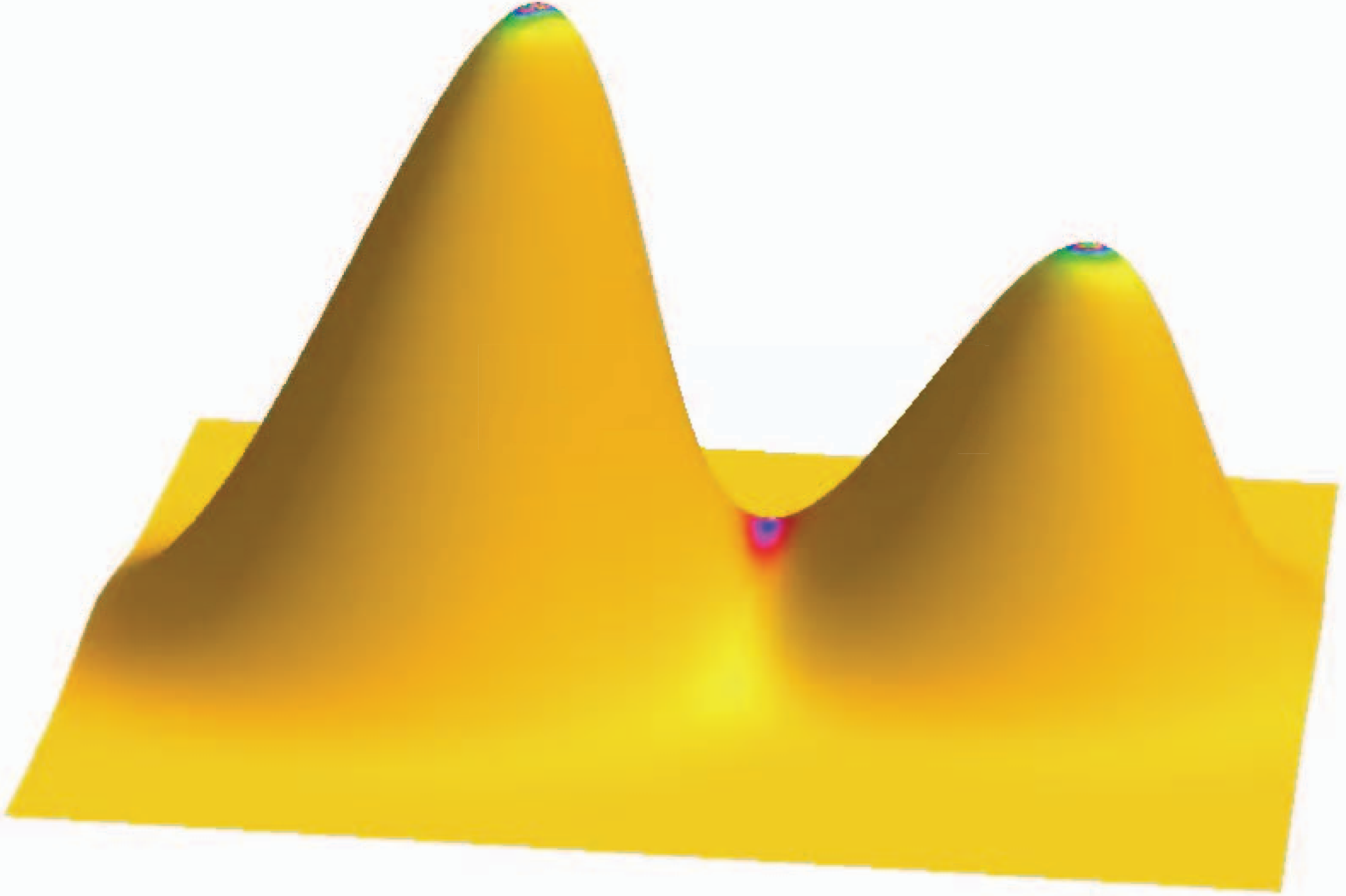




Most abstract Level

$$\langle \nabla_i X_j, X_k \rangle = \frac{1}{2}\left\{\frac{\partial}{\partial x^i} g_{jk} + \frac{\partial}{\partial x^j} g_{ik} - \frac{\partial}{\partial x^k} g_{ij}\right\}$$

(M,g) Riemannian of dimension n=2k

so

$$\nabla_i X_j = \sum \Gamma_{ij}^k X_k$$

$$\Gamma_{ij}^k = \sum_l g^{kl} \langle \nabla_i X_j, X_l \rangle$$

$$R_k{}^l{}_{ij} = \frac{\partial}{\partial x_i} \Gamma_{jk}^l - \frac{\partial}{\partial x_j} \Gamma_{ik}^l + \sum_r (\Gamma_{jk}^r \Gamma_{ir}^l - \Gamma_{ik}^r \Gamma_{jr}^l)$$

$$\int_M K(x) dV(x) = \chi(M)$$

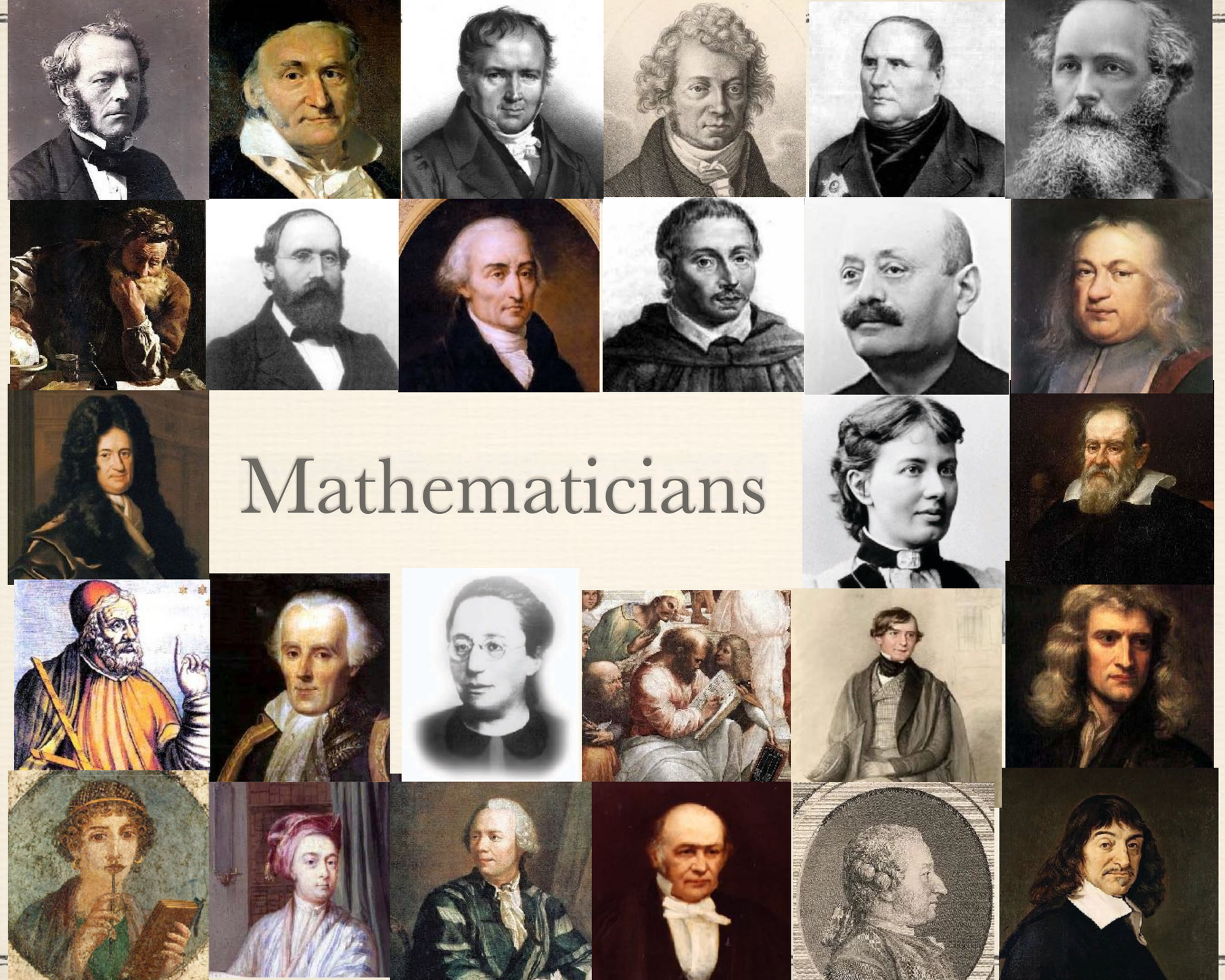
$$K(x) = \frac{(-1)^k}{(4\pi)^k k! 2^k} \sum_{\pi,\sigma} (-1)^\pi (-1)^\sigma R^{\pi(1)\pi(2)}{}_{\sigma(1)\sigma(2)} \cdots R^{\pi(n-1)\pi(n)}{}_{\sigma(n-1)\sigma(n)}$$

is a scalar (independent of coordinate systems).

formulas from Cycon-Froese-Kirsch-Simon, 1987

Example: Integral Theorems

Simple Level



Mathematicians

George Stokes

- 1819-1903

[The following Smith's Prize Exam was taken by James Clerk Maxwell at Cambridge. Question 8 is Stokes' Theorem. (Stokes was a personal friend of Maxwell.) Maxwell completed the exam tied for first.]

February, 1854.

BY GEORGE GABRIEL STOKES, ESQ. M. A.

Lucasian Professor.

1. STRAIGHT lines *AP*, *BP* pass through the fixed points *A*, *B*, and are always equally inclined to a fixed line; shew that the locus of P is a hyperbola, and find its asymptotes.

2. A number of equal vessels communicate successively with each other by small pipes, the last vessel opening into the air. The vessels being at first filled with air, a gas is gently forced at a uniform rate into the first; find the quantity of air remaining in the n^{th} vessel at the end of a given time, supposing the gas and air in each vessel at a given instant to be uniformly mixed.

3. Separate the roots of the equation

$$2x^3 - 9x^2 + 12x - 4.4 = 0,$$

and find the middle root to four places of decimals by Horner's method, or by some other.

4. Investigate a formula in Finite Differences for transforming a series the terms of

begin to move along a generating line of an elliptic cone having Q for vertex in order that consecutive tangents may ultimately intersect, but that the conditions of the problem may be impossible.

8. If X, Y, Z be functions of the rectangular co-ordinates x, y, z, dS an element of any limited surface, l, m, n the cosines of the inclinations of the normal at dS to the axes, ds an element of the bounding line, shew that

$$\iint\left\{l\left(\frac{dZ}{dy}-\frac{dY}{dx}\right)+m\left(\frac{dX}{dz}-\frac{dZ}{dx}\right)+n\left(\frac{dY}{dx}-\frac{dX}{dy}\right)\right\}dS$$
$$=\int\left(X\frac{dx}{ds}+Y\frac{dy}{ds}+Z\frac{dz}{ds}\right)ds,$$

the differential coefficients of X, Y, Z being partial, and the single integral being taken all round the perimeter of the surface.

9. Explain the geometrical relation between the curves, referred to the rectangular co-ordinates x, y, z, whose differential equations are

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R},$$

and the family of surfaces represented by the partial differential equation

From Letters to Mary Susanna Robinson 1957

You are quite right in saying that it is well not to go brooding over one's own thoughts and feelings, and in a family that is easy, but you don't know what it is to live utterly alone.

On the 31 March 1857 he wrote again expressing his feelings in rather mathematical terms:-

I too feel that I have been thinking too much of late, but in a different way, my head running on divergent series, the discontinuity of arbitrary constants, ... I often thought that you would do me good by keeping me from being too engrossed by those things.

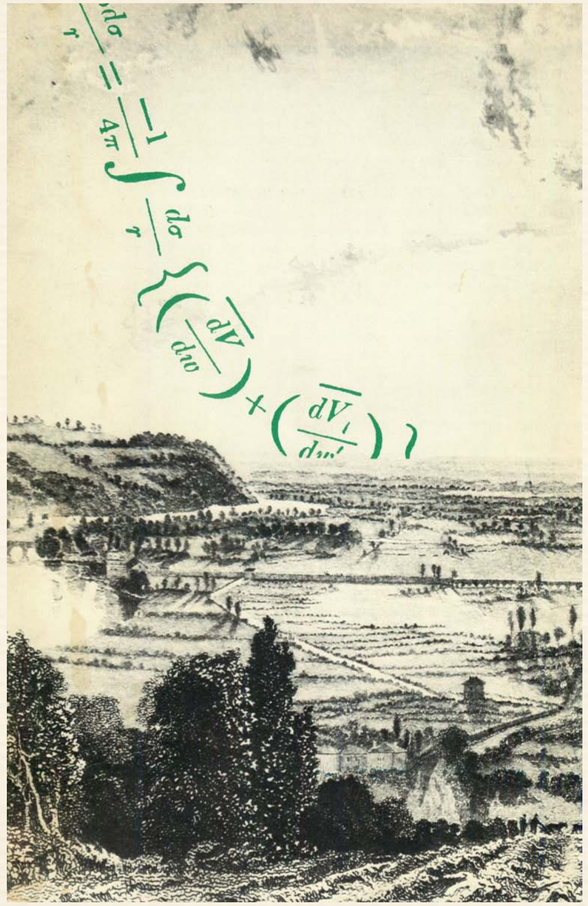
These letters clearly did not express the love that Mary hoped to find in them and when Stokes wrote her a 55 page letter about the duty he felt towards her, she came close to calling off the wedding at the last moment.

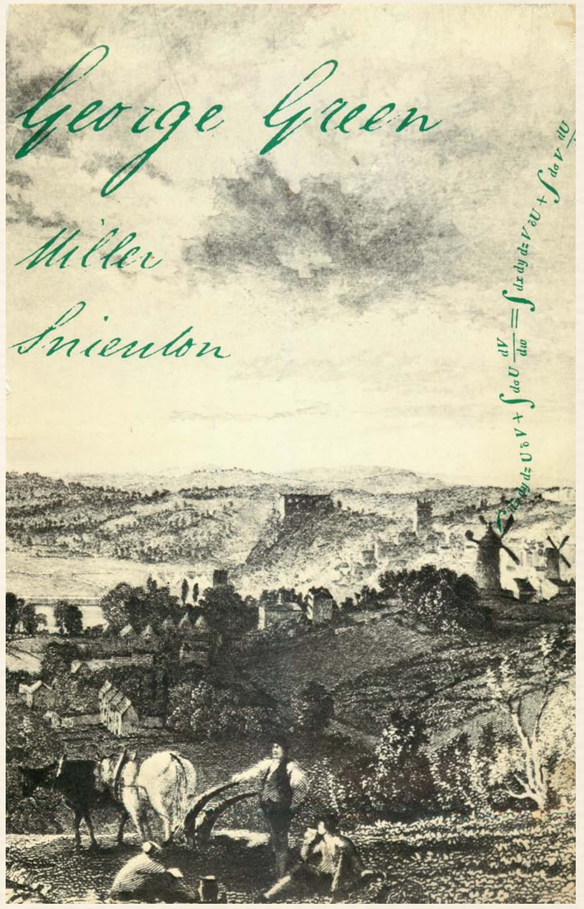
Mikael Ostogradski

- 1801-1862

George Green

- 1793-1841





John Green paid £70 a year. After a number of years John apparently relinquished direct control of the farm to his younger brother Robert and went to farm in nearby East Bridgford. However, he maintained the lease and when he made his will[5] in 1818 he stated that "my ancestors of the name of Green have been tenants to such farm for nearly five centuries".

The farm cannot have been large enough to have provided a good livelihood for all three brothers and the Mathematician's father, as the youngest of the three, was required to make his own way in the world. Shortly after his father John's death, George Green Senior, as a youth of around 15 years, left Saxondale to take up an apprenticeship in Nottingham. His family evidently sought to do him well. They apprenticed[6] him as a baker, a trade which they must have considered to have fair prospects since they were willing to pay the master the high introductory fee of 10 guineas. Masters in the framework-knitting trade, by contrast, required only a token fee of one penny at this time. The apprenticeship, for a term of 7 years commencing on 16th July 1774, was to Robert Hill, who was a Burgess of the town. The Burgesses or Freemen, a minority of the town's population, enjoyed a number of privileges, including the right to vote in Parliamentary elections and the possibility of becoming a member of the Council, the

The farmhouse on the site of the Green's family farm at Saxondale. The present house appears to be early nineteenth century.

Oakland's Mill, Sneinton - a post mill of the type replaced by the Green's tower mill. Note the tail post. The sail arrangement was the same as on the Green's Mill. Nottinghamshire Local Studies Library 1005.


MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Good Will Hunting

1777-1855 CARL FRIEDRICH GAUSS

1793-1841 GEORGE GREEN

1819-1903 GEORGE STOKES

1775-1846 JEAN MARY AMPERE

1801-1862 MIKHAEL OSTROGRADSKY

1789-1857 AUGUSTINE CAUCHY

GAUSS THEOREM 1826

FTLI 1840

GREEN THEOREM 1846

STOKES THEOREM 1854

published

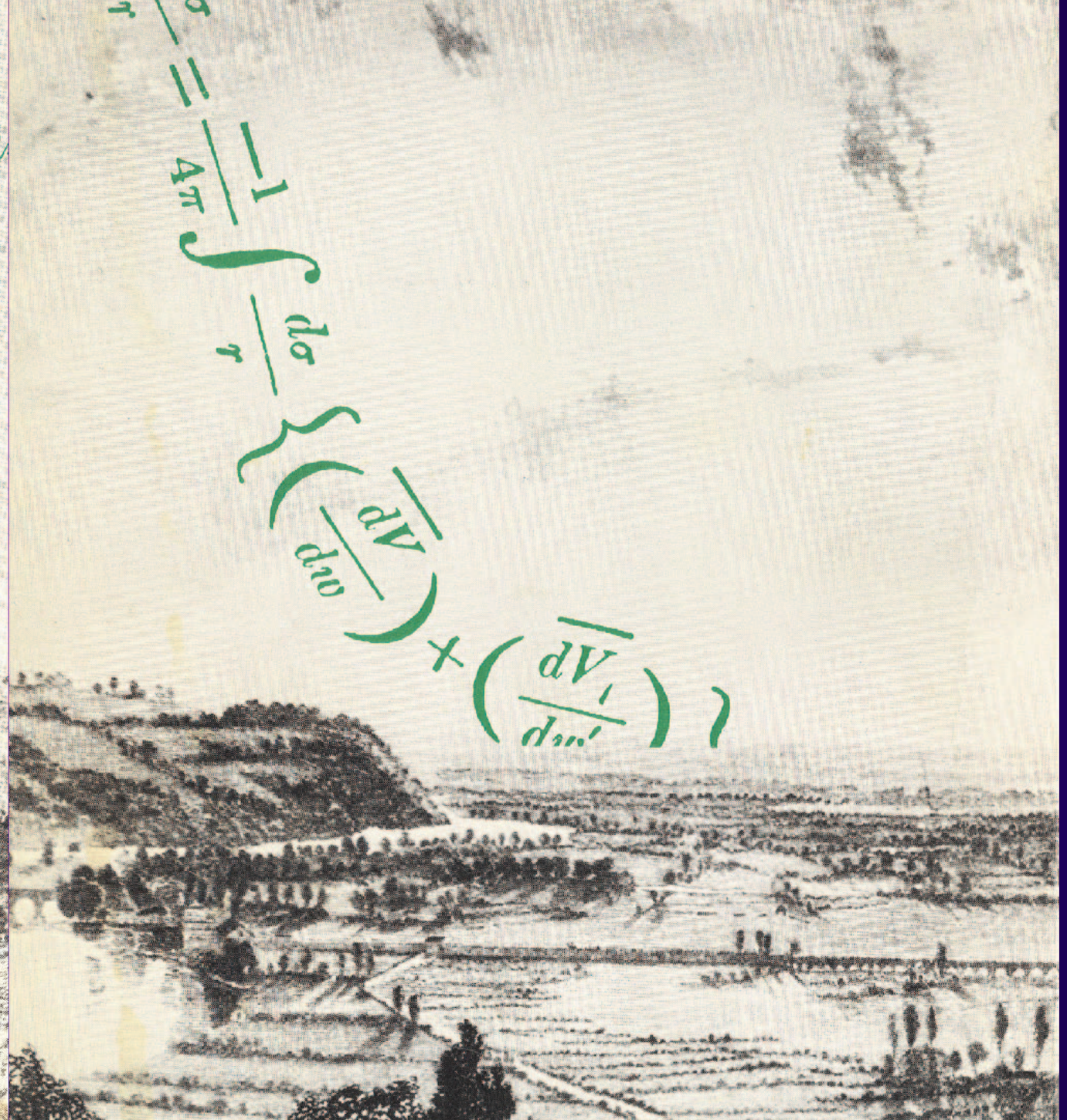
George Green

Miller

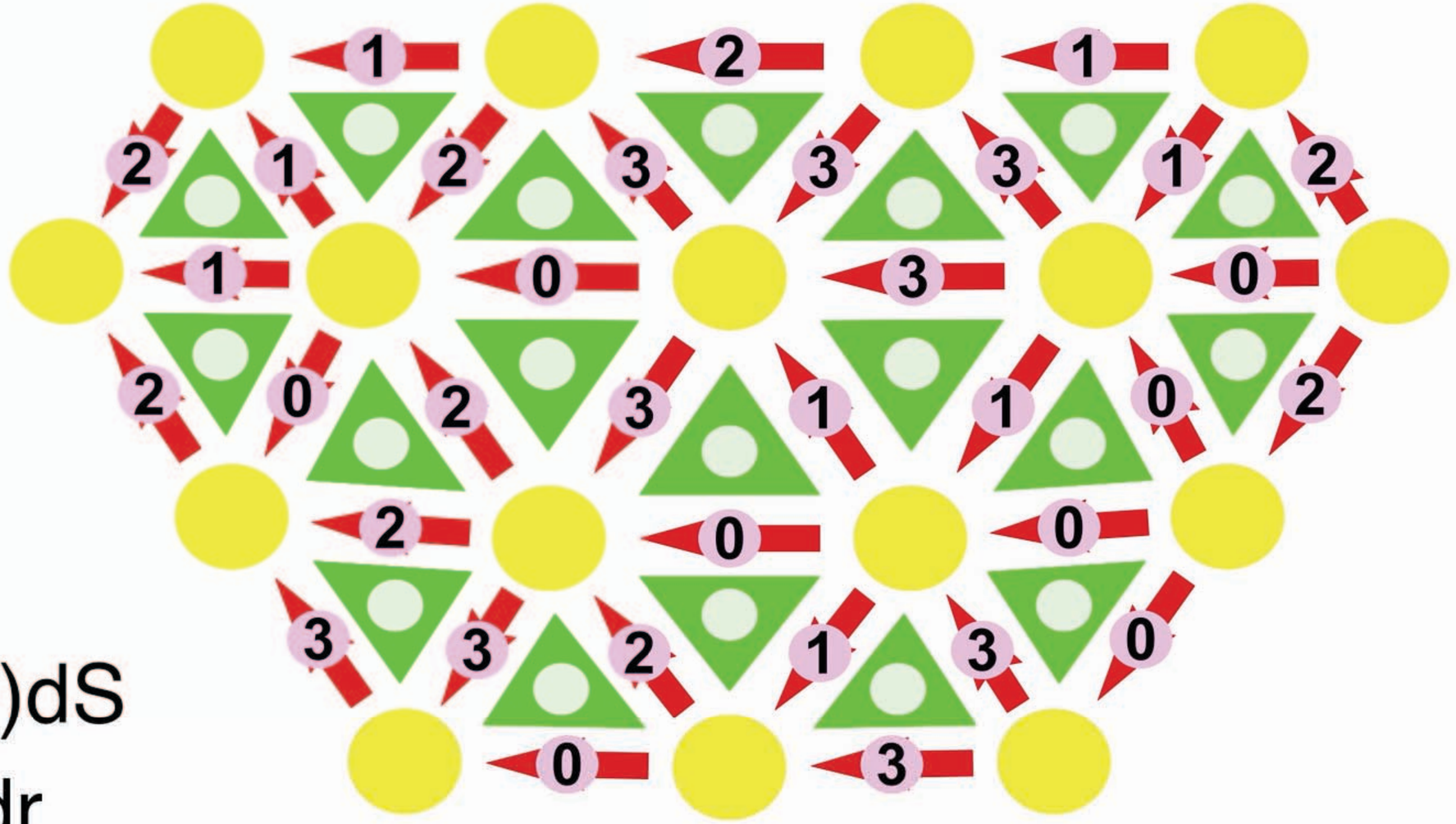
Science

$$\int dx\,dy\,dz\; U \nabla^2 V + \int d\sigma\, U \frac{dV}{dn} = \int dx\,dy\,dz\; V \nabla^2 U + \int d\sigma\, V \frac{dU}{dn}$$

$$= -\frac{1}{4\pi} \int \frac{d\sigma}{r} \left\{ \left(\frac{dV}{dn} \right) + \left(\frac{dV'}{dn'} \right) \right\}$$



Stokes Theorem



$$\iint \text{curl}(F)\,dS = \int F\,dr$$

Usual way

$$\iint_G \operatorname{curl}(\vec{F})\, dx\, dy$$

$$= \int_C \vec{F}(\vec{r}(t))\, \vec{r}'(t)\, dt$$

Advanced Level

let F be a 1-form, then

$$\int_G dF = \int_{\delta G} F$$

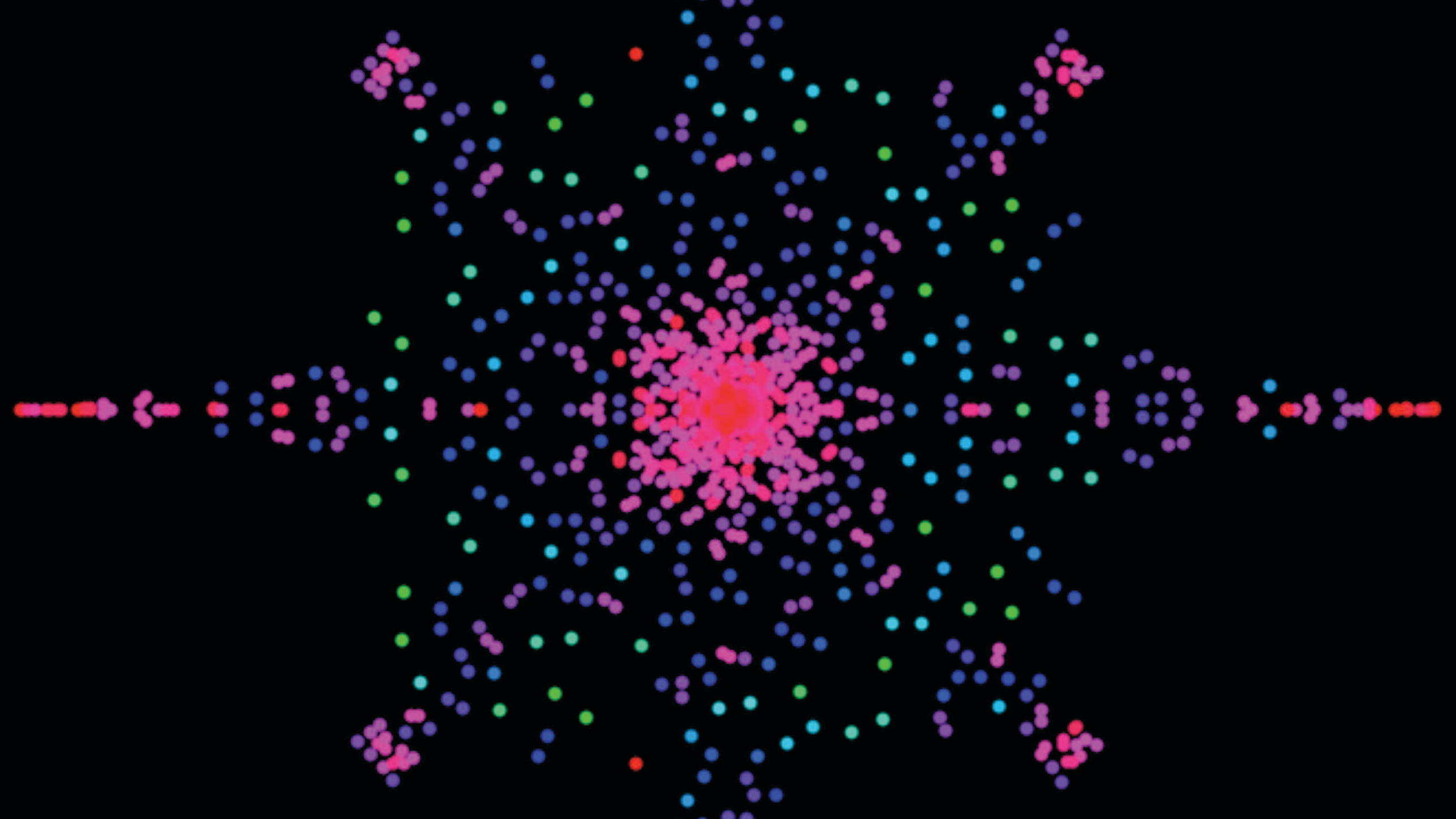
Part 3

Words of Wisdom

Teaching on an easier level
is not necessarily easier.


THE MATH MYTH
Segerman Visualizing Mathematics with 3D Printing
THE HISTORY OF STATISTICS
THE EMERGENCE OF PROBABILITY
Convex Polytopes

Happyness is achievement
minus expectation.



Be prepared to
hack your value system.

Lebowski Theorem (Joscha Bach)

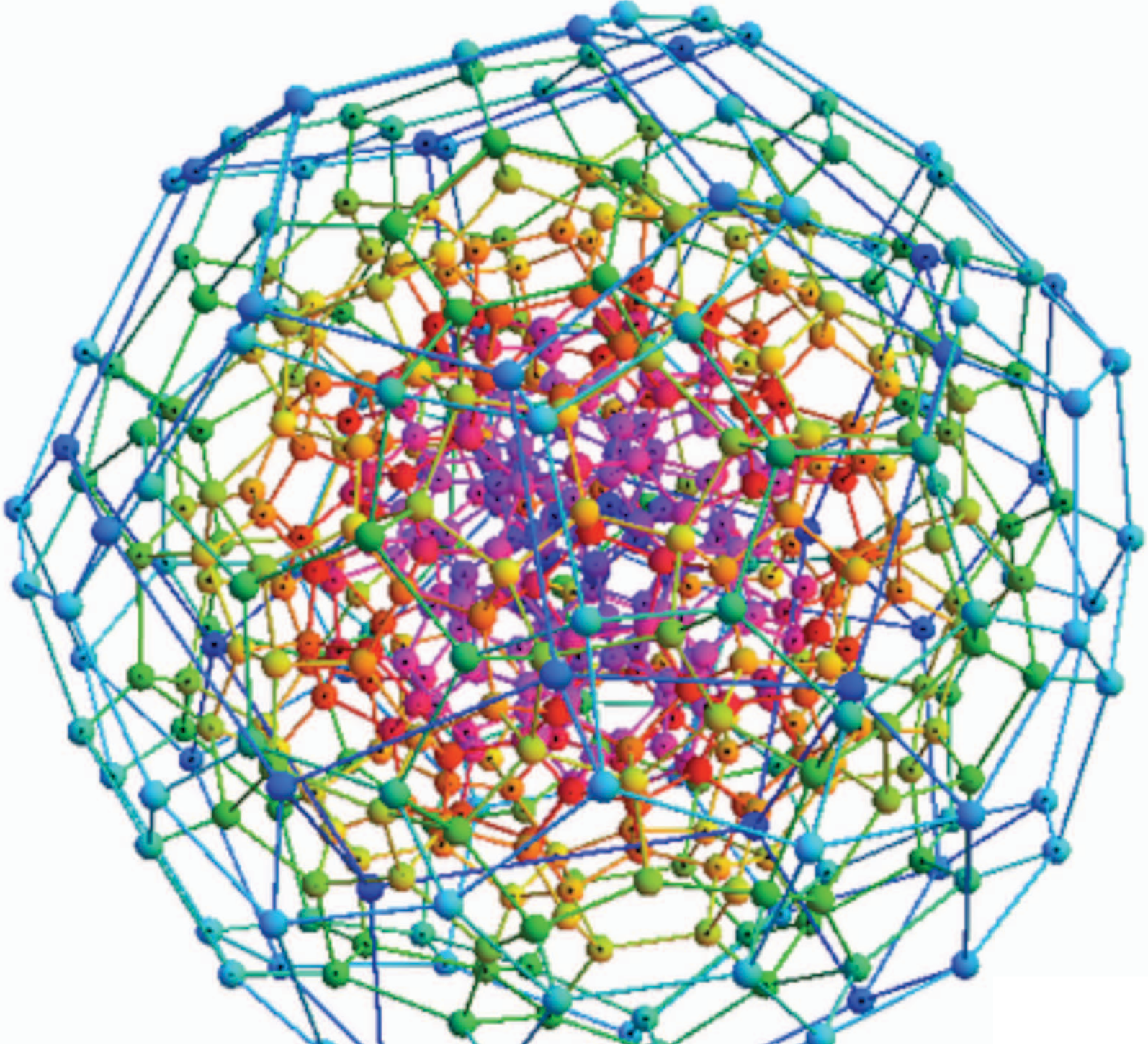
No AI will bother after hacking its own value system.

Disapointments
can be opportunities

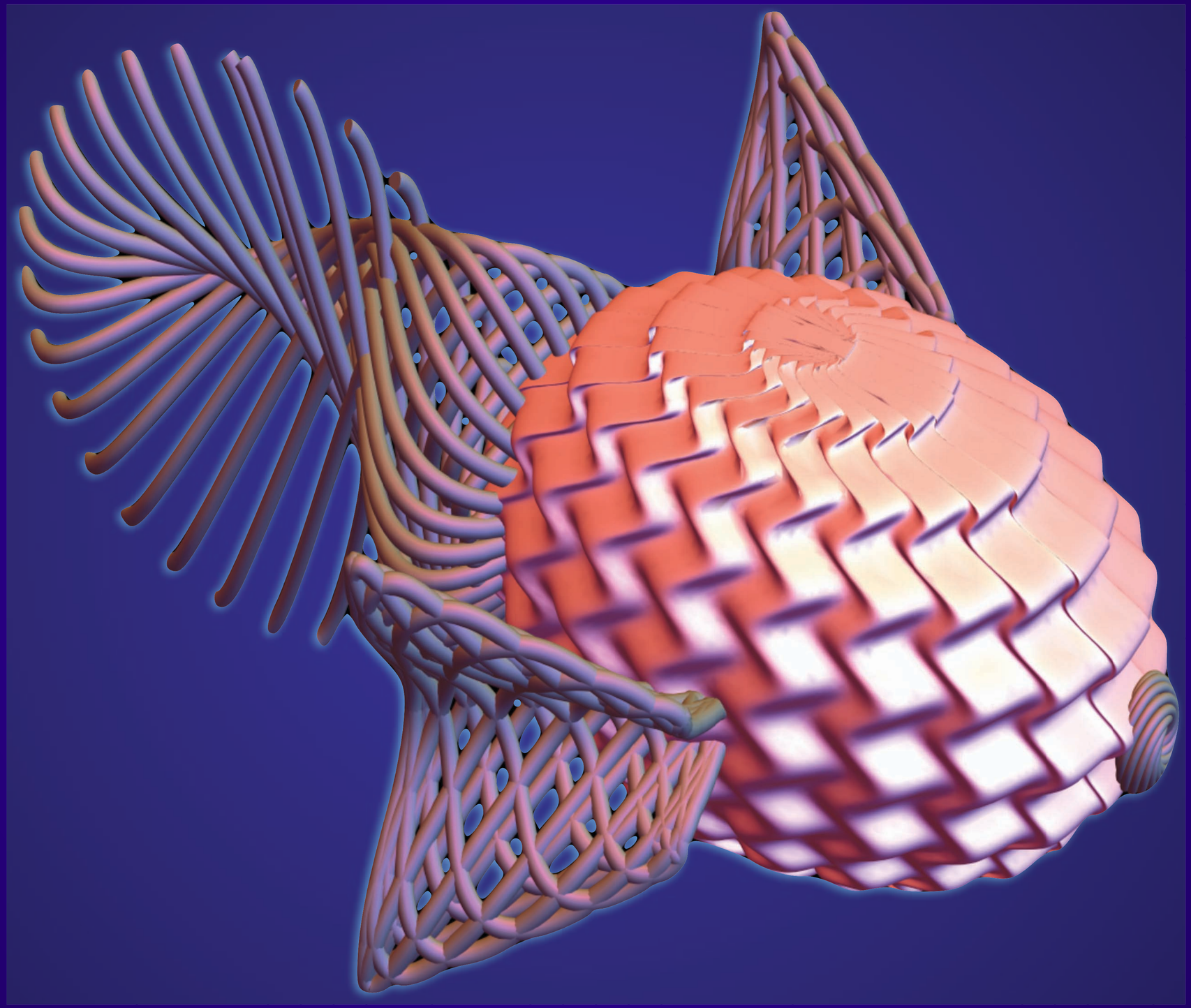
NEVER GIVE UP

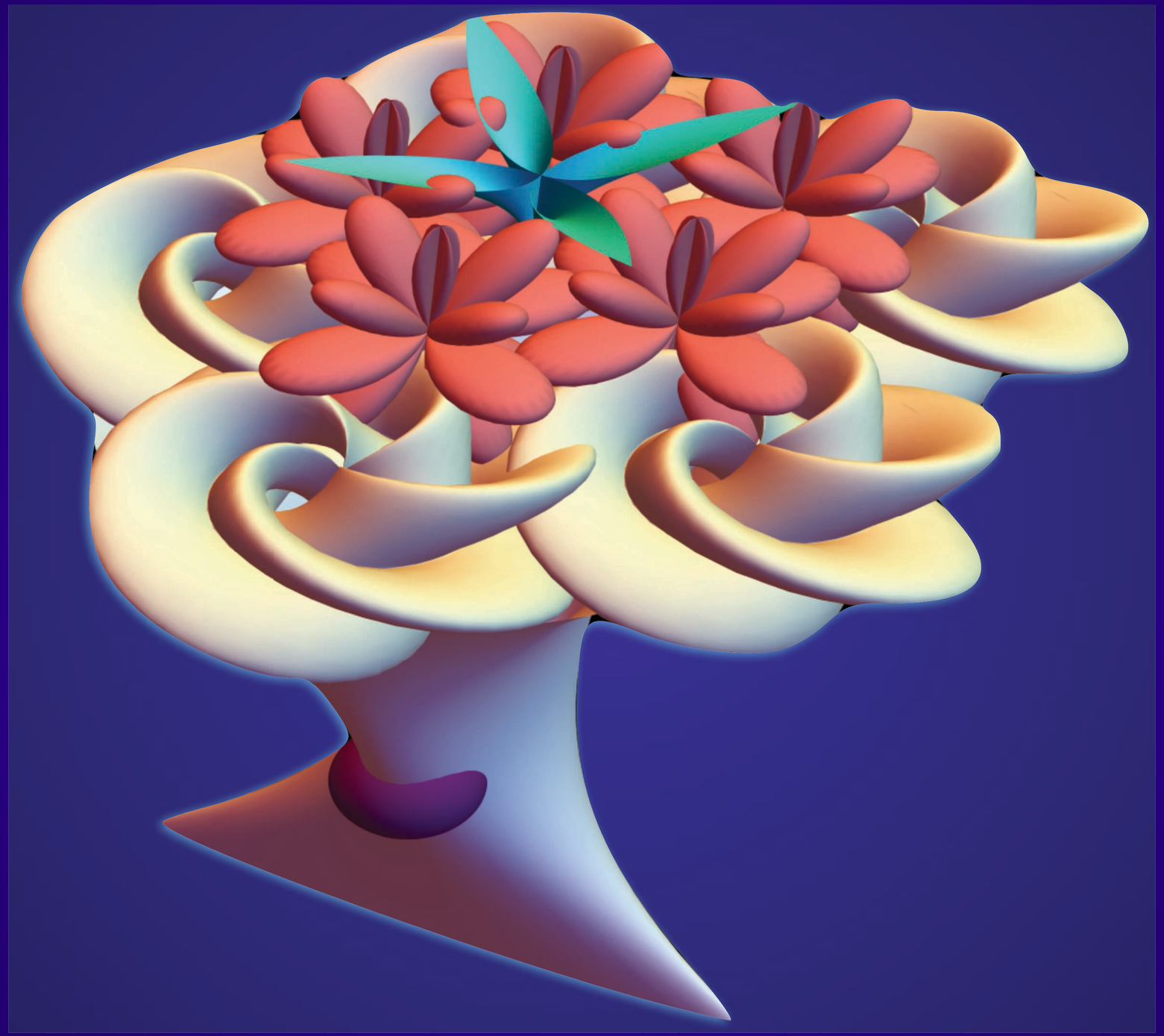
NEVER STOP TRYING TO EXCEED YOUR LIMITS. WE NEED THE ENTERTAINMENT.

Doing can be more fulfilling than managing.

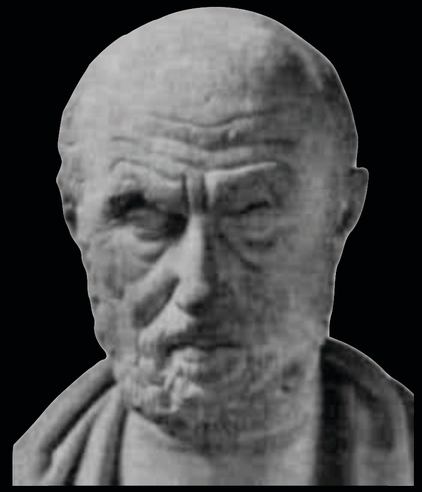


What counts is what
is the daily work.



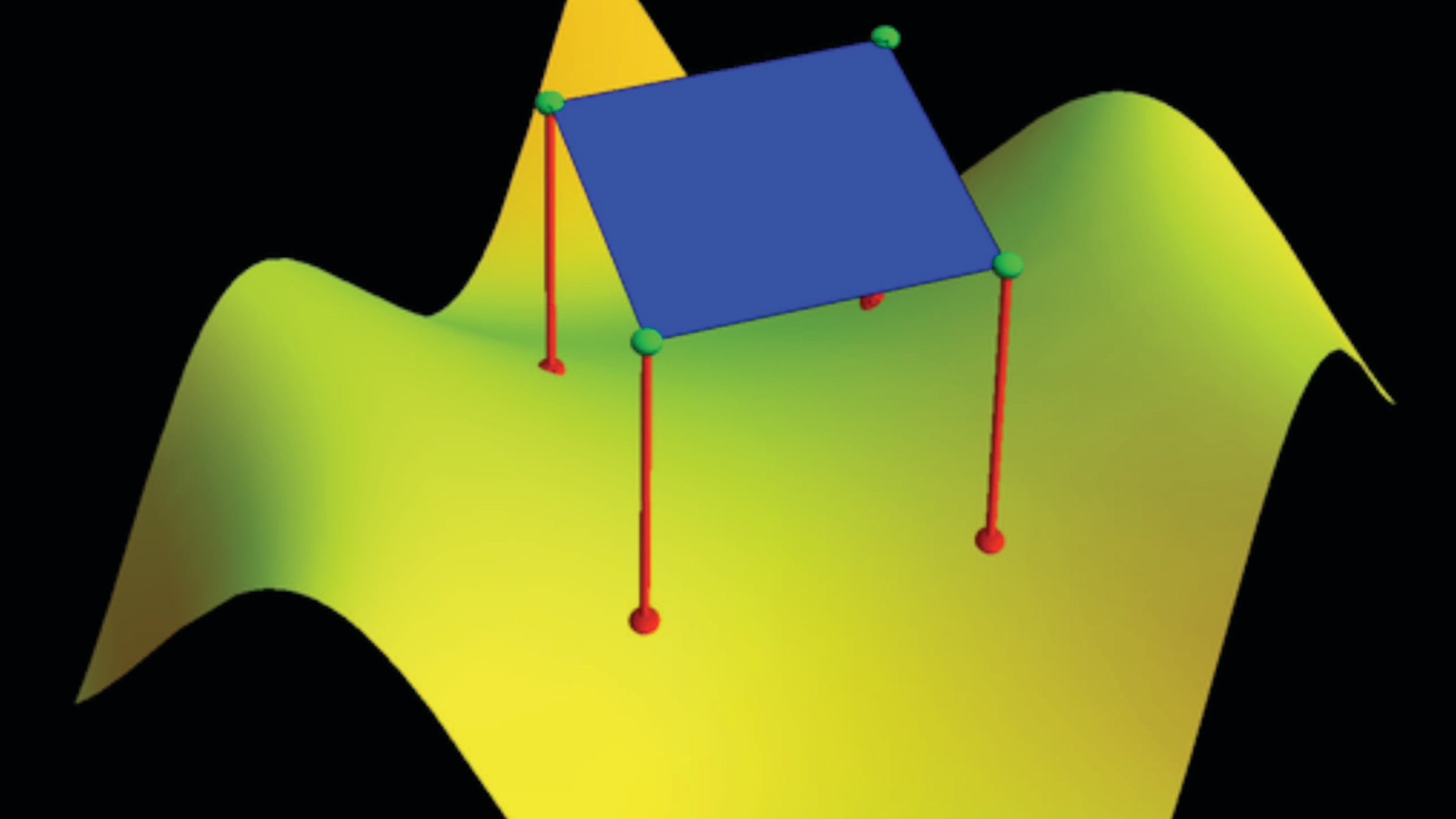


Appreciate the unspectacular parts



Hipocrates

Things change



Adjusting
difficulty can be difficult.

We live in a time
of reinvention


Talks at Google

There are opportunities
off the chart, especially
in uncharted teritory.

The End.