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(*  Mathematica program   propertyellipse.m                        *)
(*  http://www.ma.utexas.edu/~knill/Mathematica/propertyellipse.m *)
(*  Oliver Knill, December 14, 1999                               *)

(* "The product of distances from the two focal points is         *)
(* invariant for paths in the billiard flow in an ellipse."       *)

Focal1 = { Sqrt[a^2-b^2],0};               (* right focal point   *)
Focal2 = {-Sqrt[a^2-b^2],0};               (* left  focal point   *)

s[t_] :={ a*Cos[t],b*Sin[t]};              (* point on ellipse E  *)
n[t_] :={ b*Cos[t],a*Sin[t]};              (* normal vector       *)
v[t_] :={-a*Sin[t],b*Cos[t]};              (* tangent vector      *)

A0=s[t];                                   (* general point on E  *)
B1=s[t]+n[t]+r*v[t];                       (* B1 on incoming ray  *)
B2=s[t]+n[t]-r*v[t];                       (* B2 on outgoing ray  *)

Norm[A_]:=Sqrt[A[[1]]^2+A[[2]]^2];
CrossProd[A_,B_]:=A[[1]]*B[[2]]-A[[2]]*B[[1]];
Distance[P_,A_,B_]:=CrossProd[(P-A),(B-A)]/Norm[B-A];

d1   = Distance[Focal1,A0,B1];
d2   = Distance[Focal2,A0,B1];

e1   = Distance[Focal1,A0,B2];
e2   = Distance[Focal2,A0,B2];

f1   = Simplify[d1*d2];
f2   = Simplify[e1*e2];

f    = Simplify[f1-f2];
f    == 0                                  (* this should be true *)

(*  Here is the value of f1

          2    2         2      2    2
        -a  + b  (1 + 2 r ) + (a  - b ) Cos[2 t]
f1 =    ----------------------------------------       = f1
                              2
                     2 (1 + r )
*)
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Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.