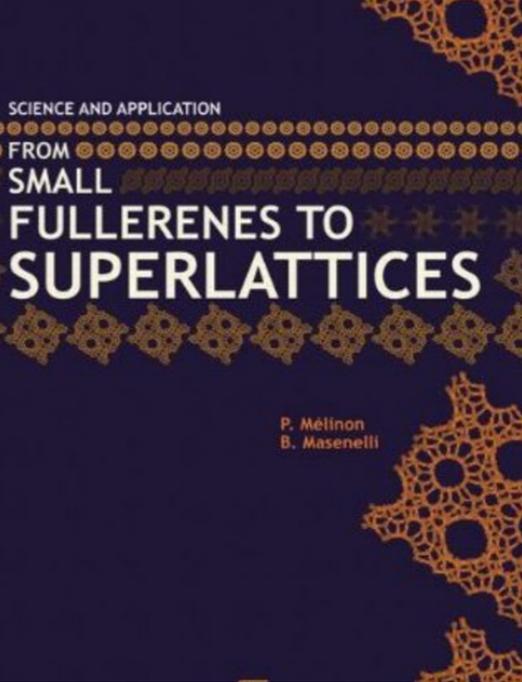
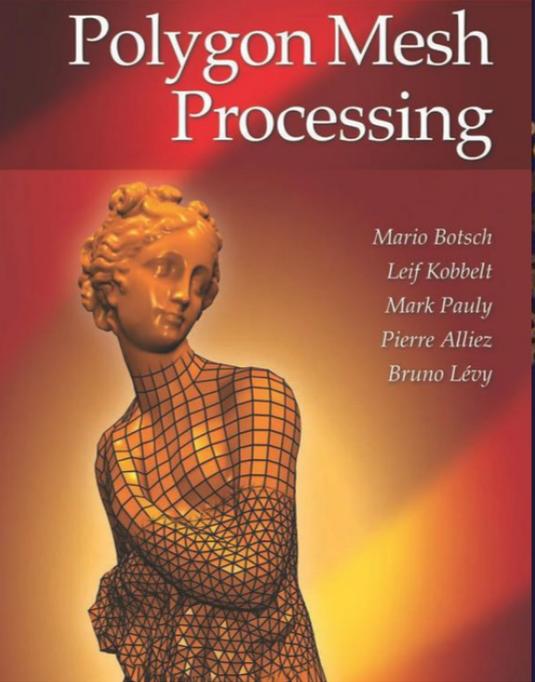
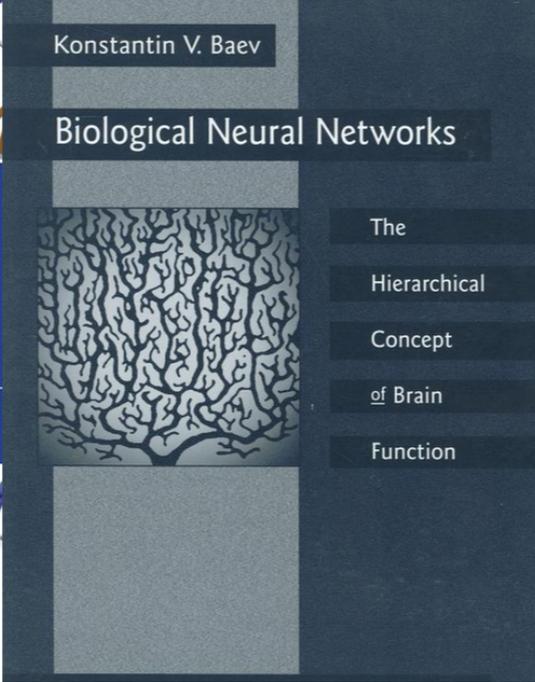
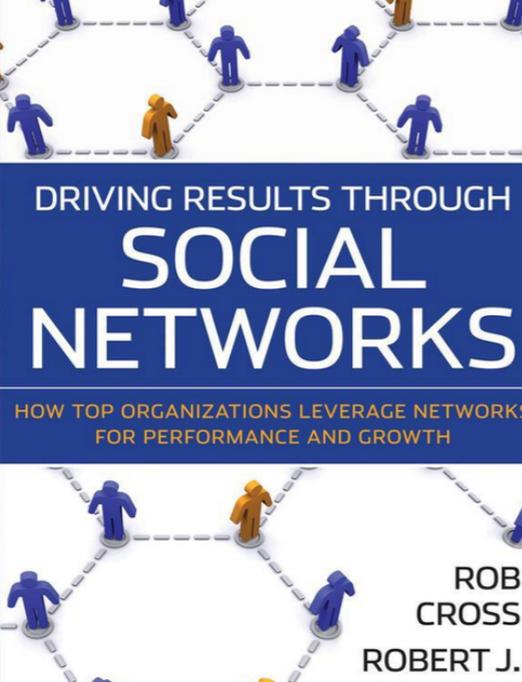
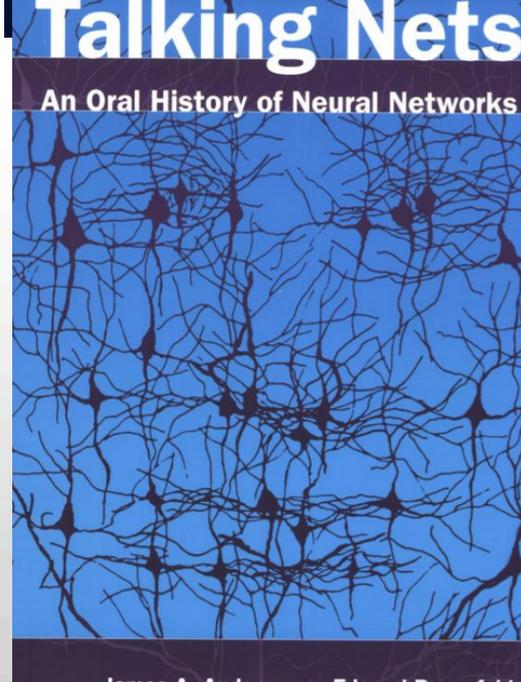
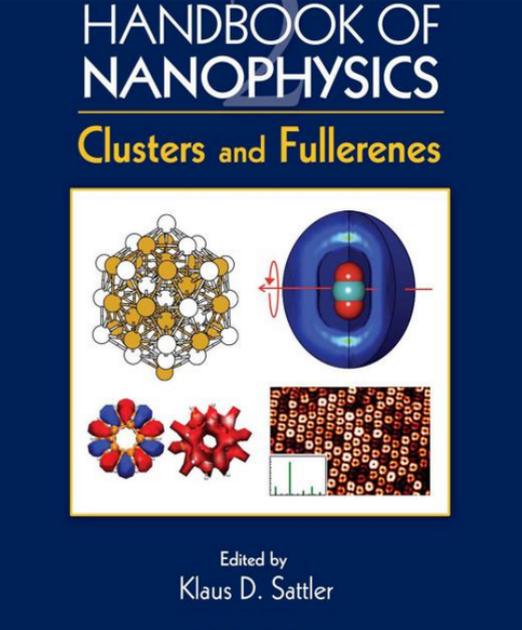
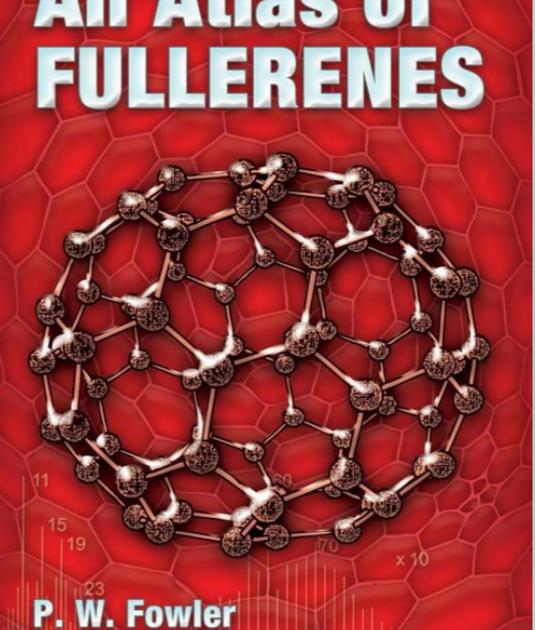
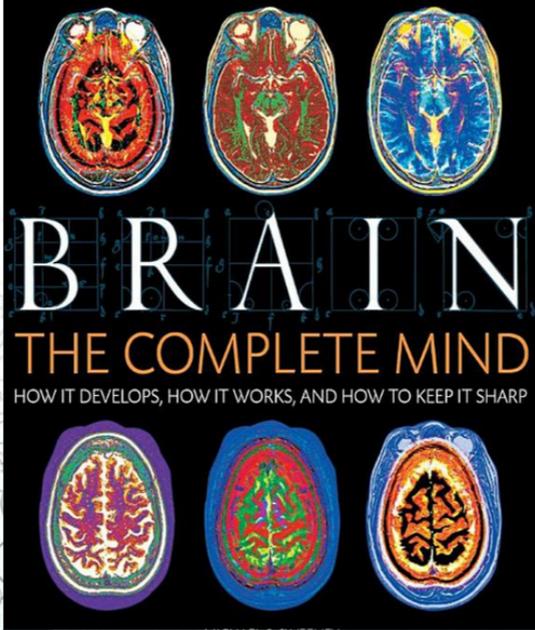
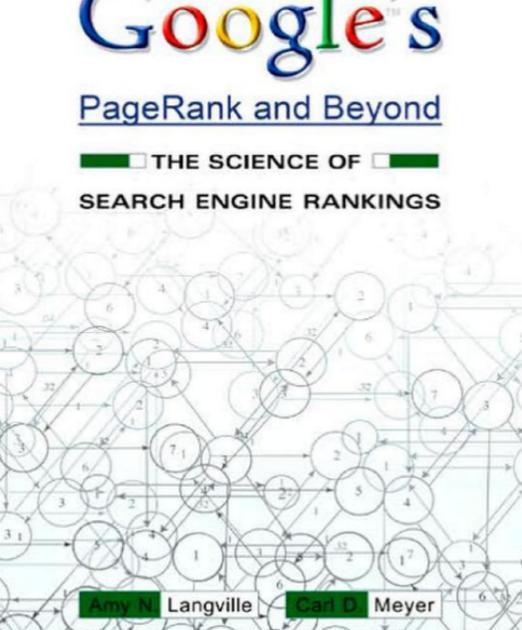
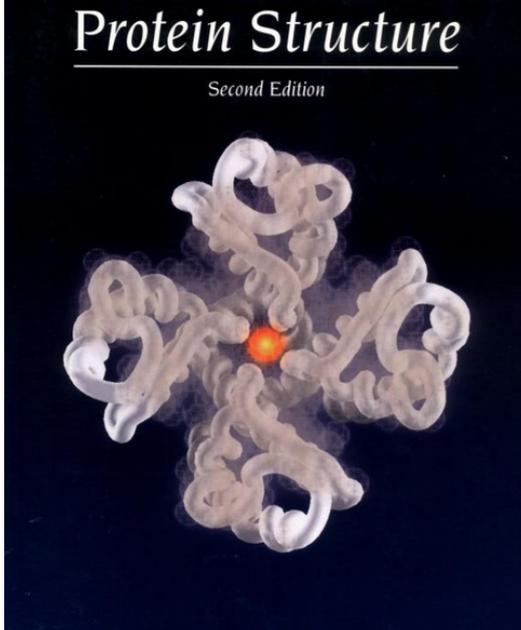
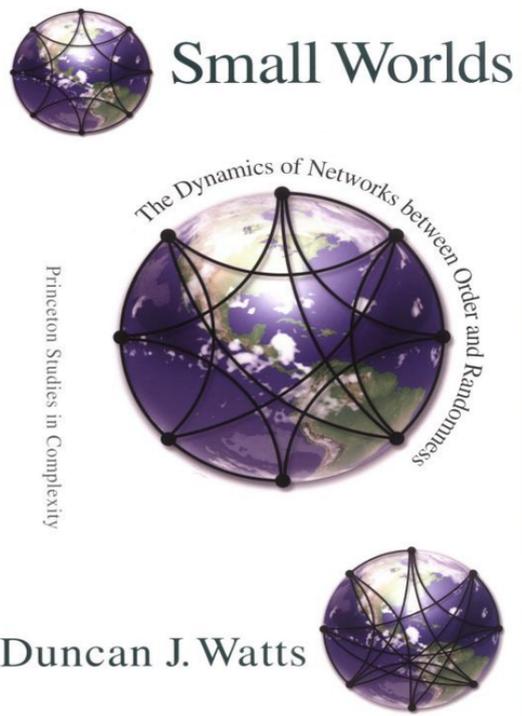
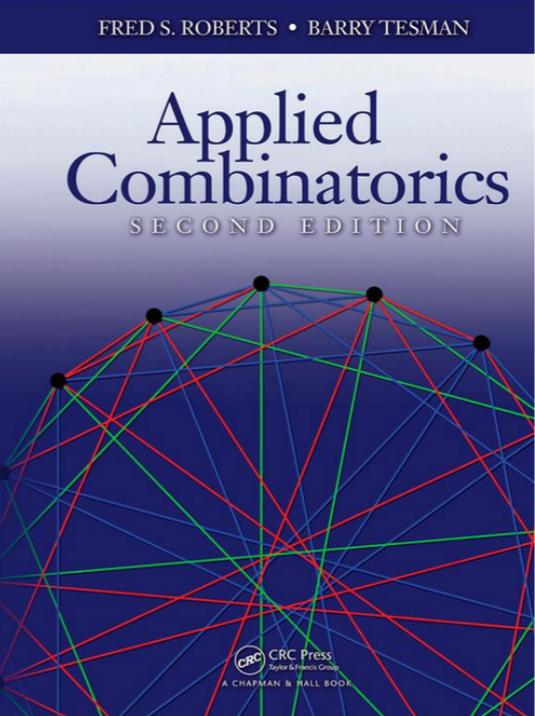
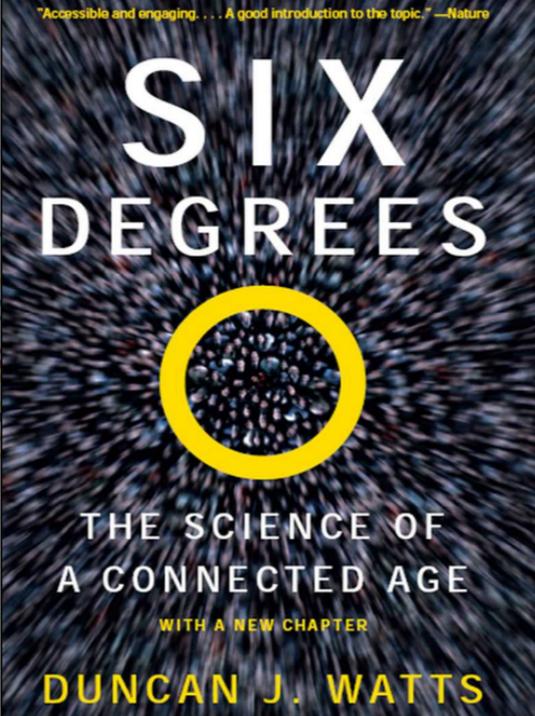
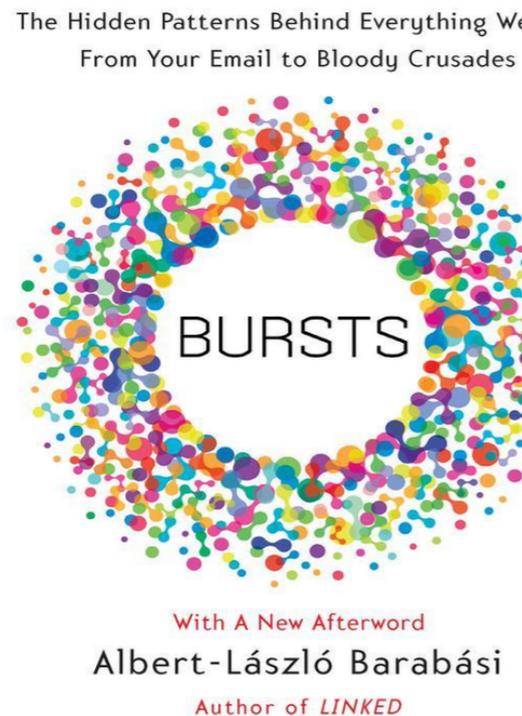
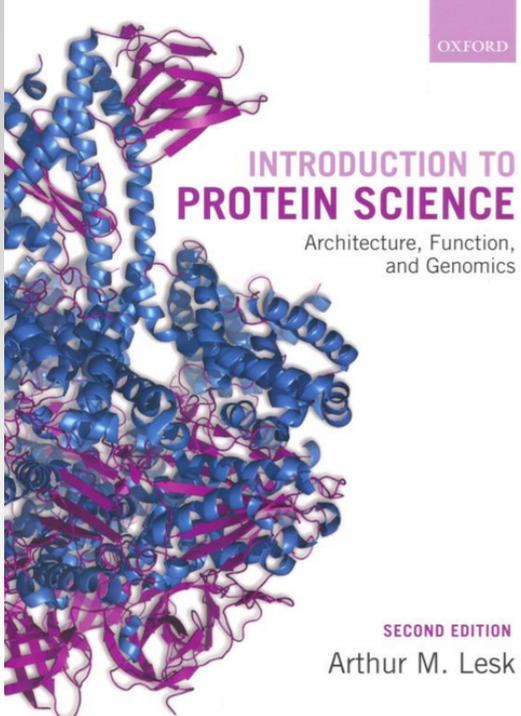


The Geometry of Networks

April 1, 2012

What is "hot" today in Math?





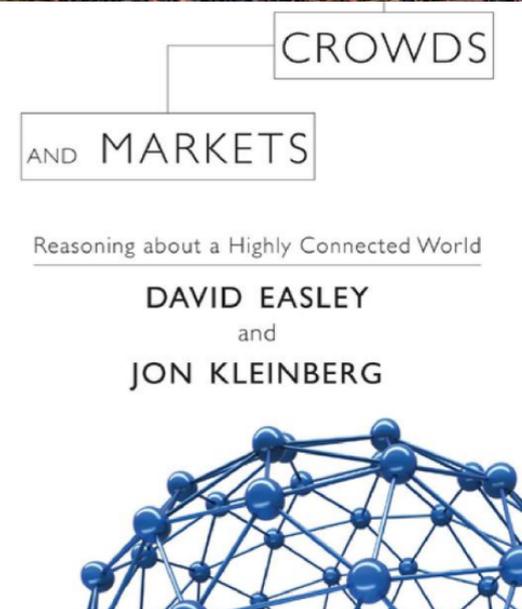
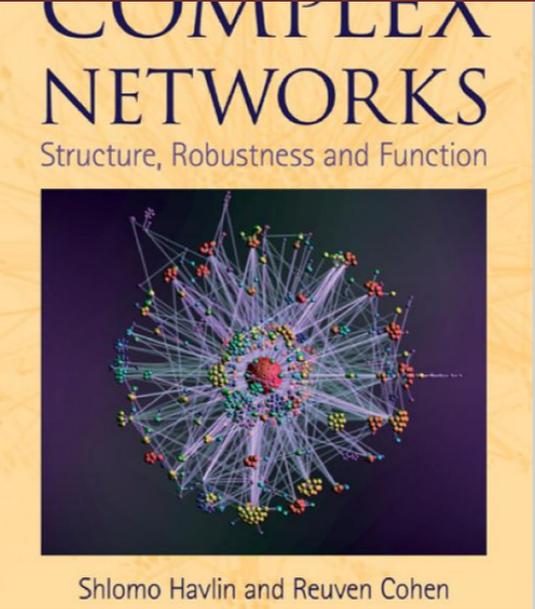
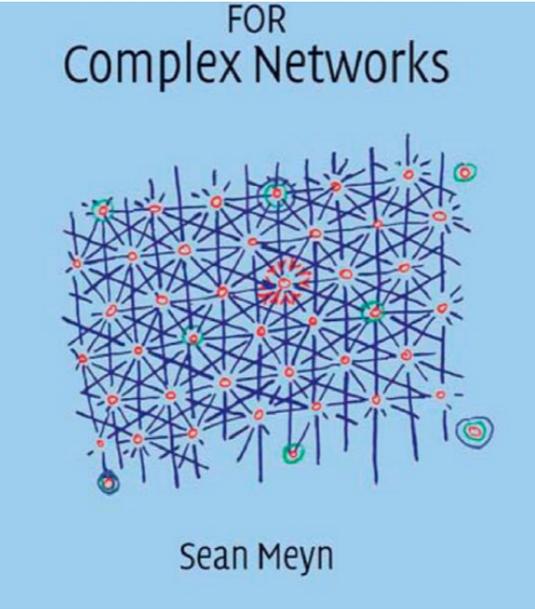
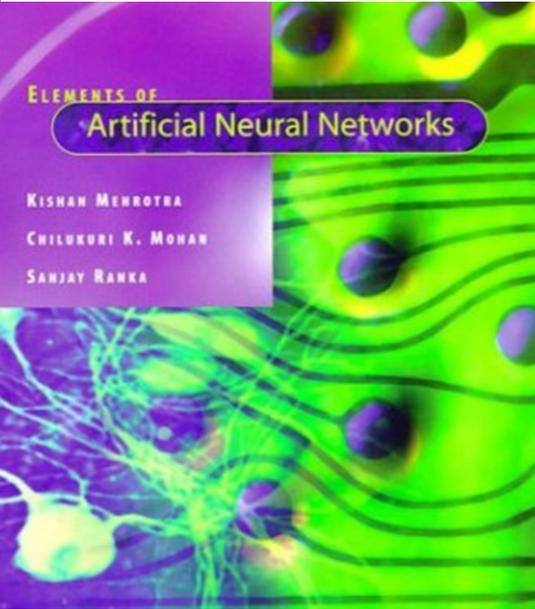
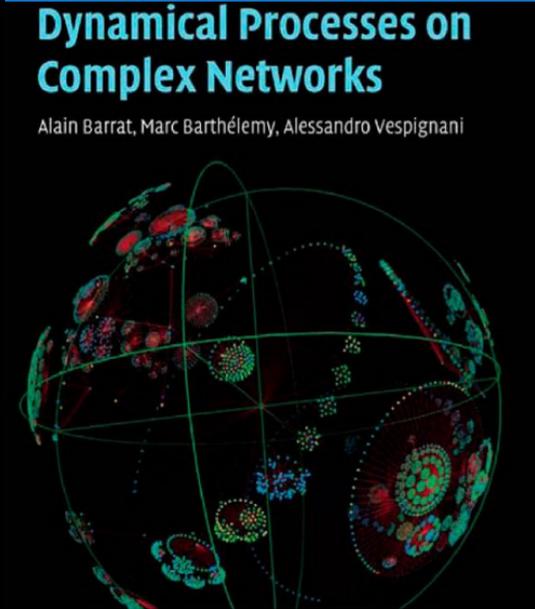
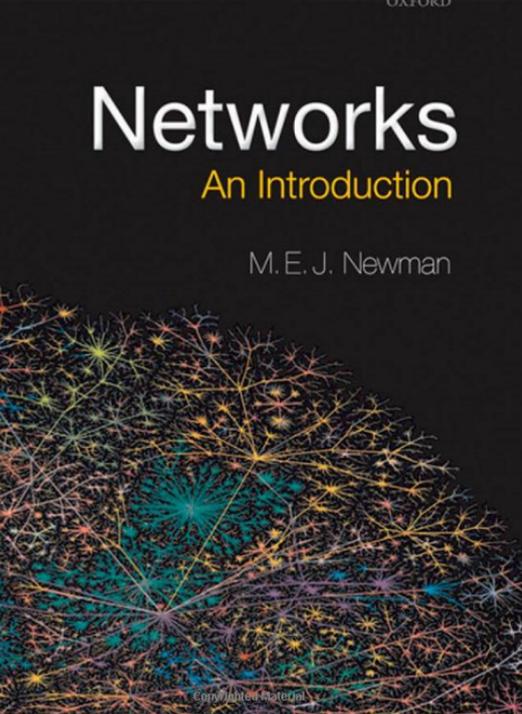
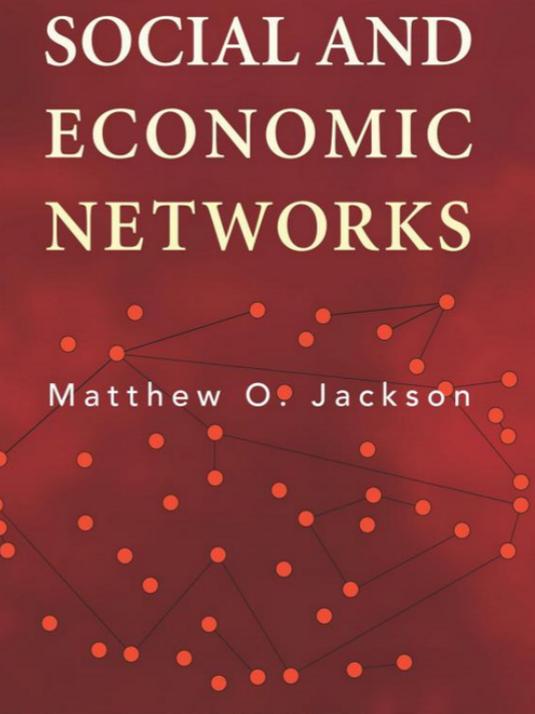
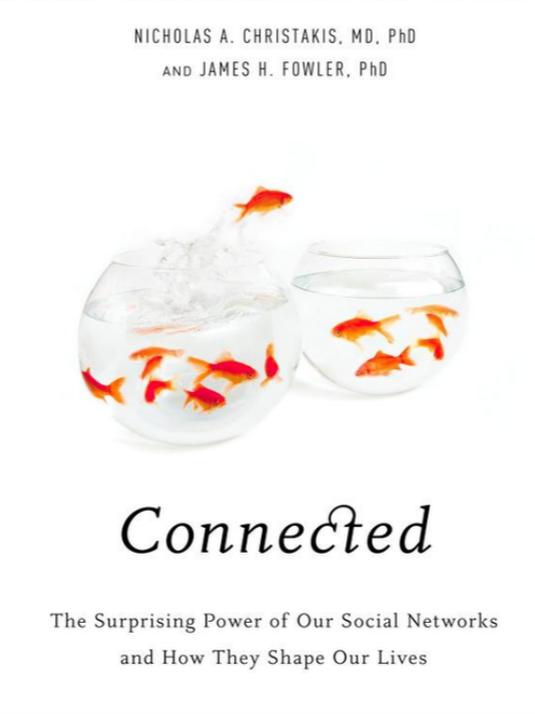
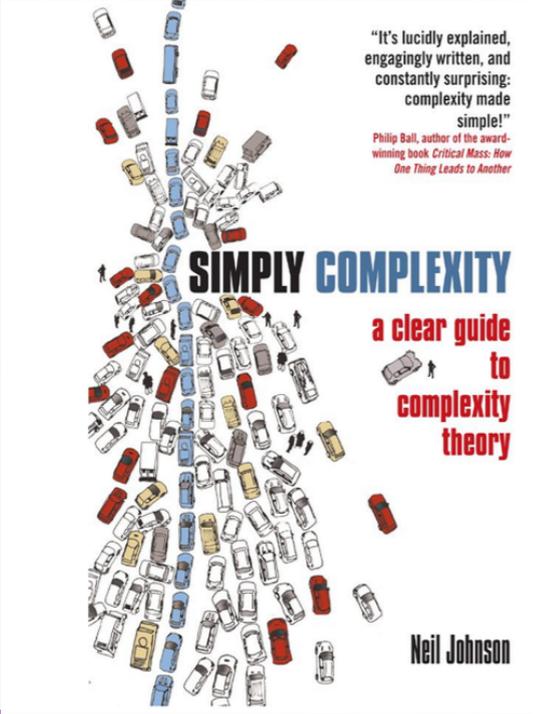
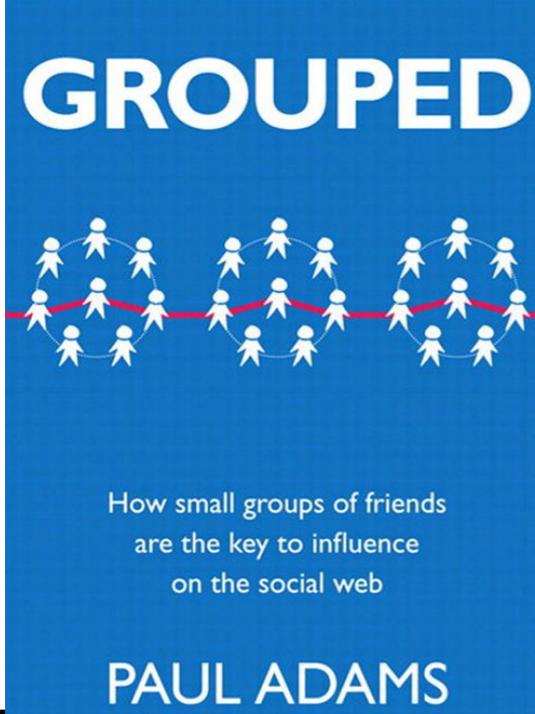
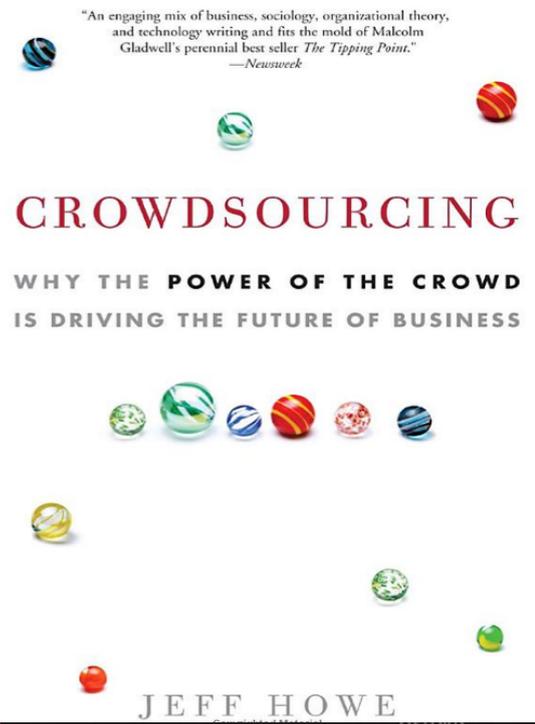
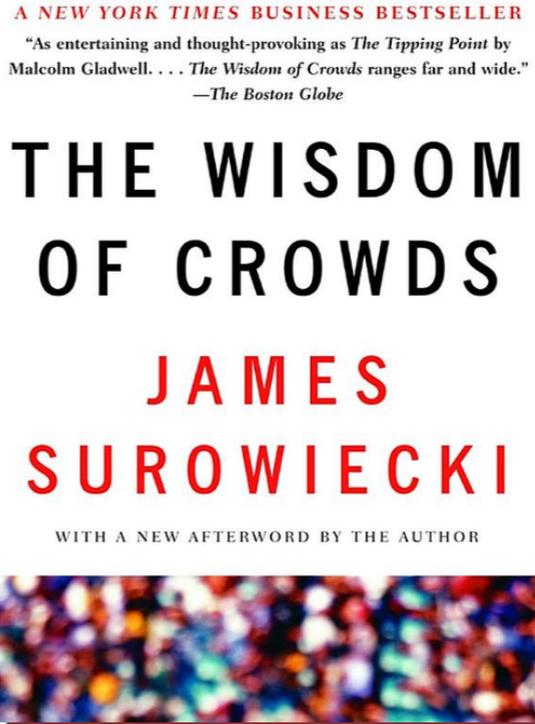
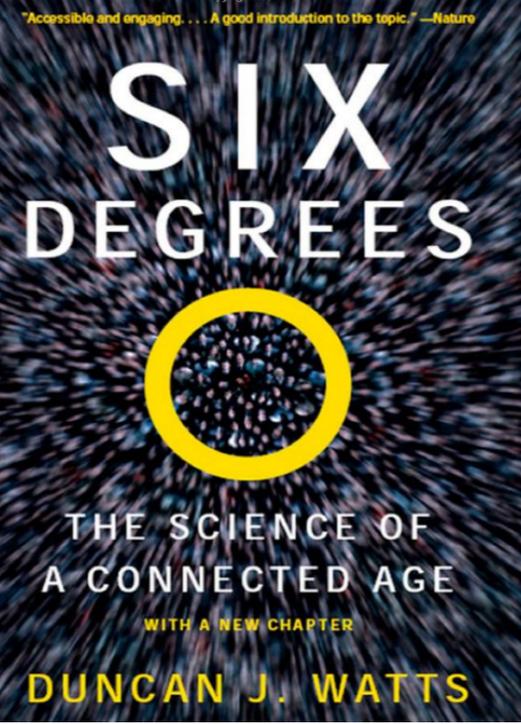
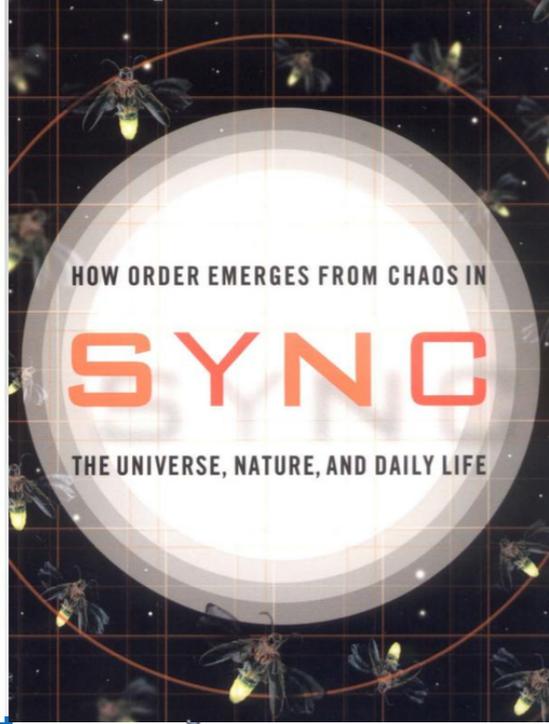
How Everything Is Connected to Everything Else and What It Means for Business, Science, and Everyday Life

Linked



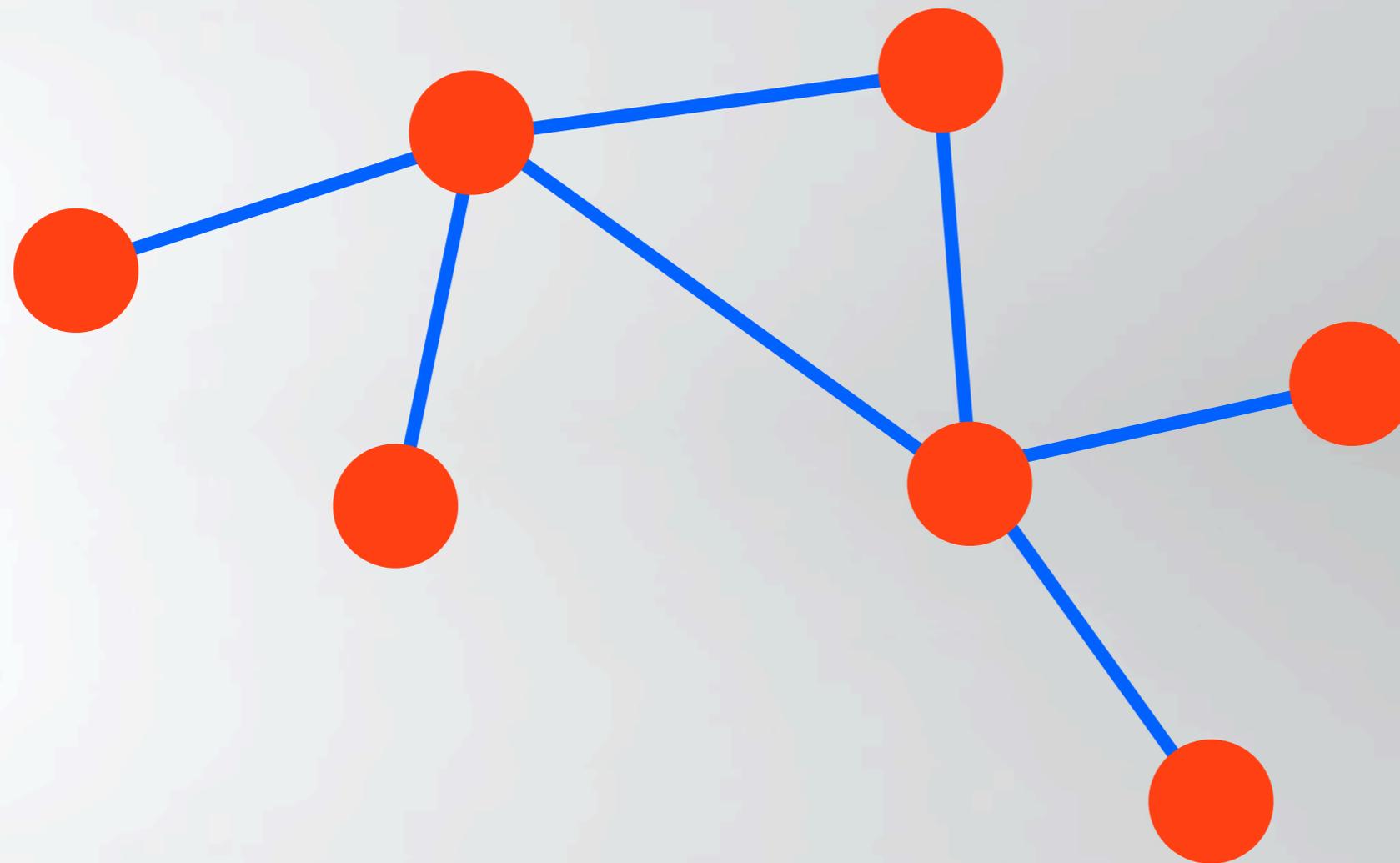
"Linked could alter the way we think about all of the networks that affect our lives." —The New York Times

Albert-László Barabási



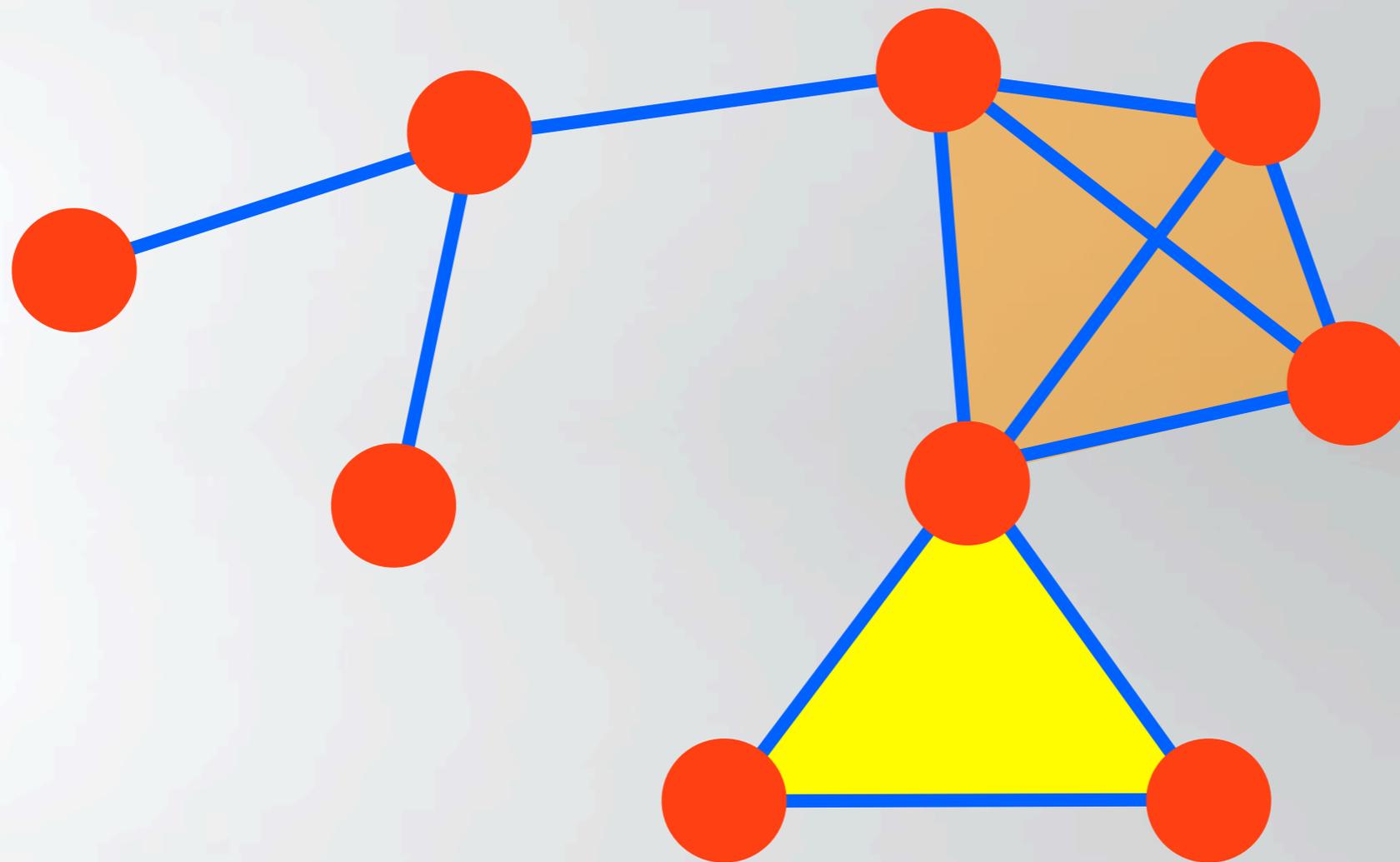
Geometry on Networks

Graphs

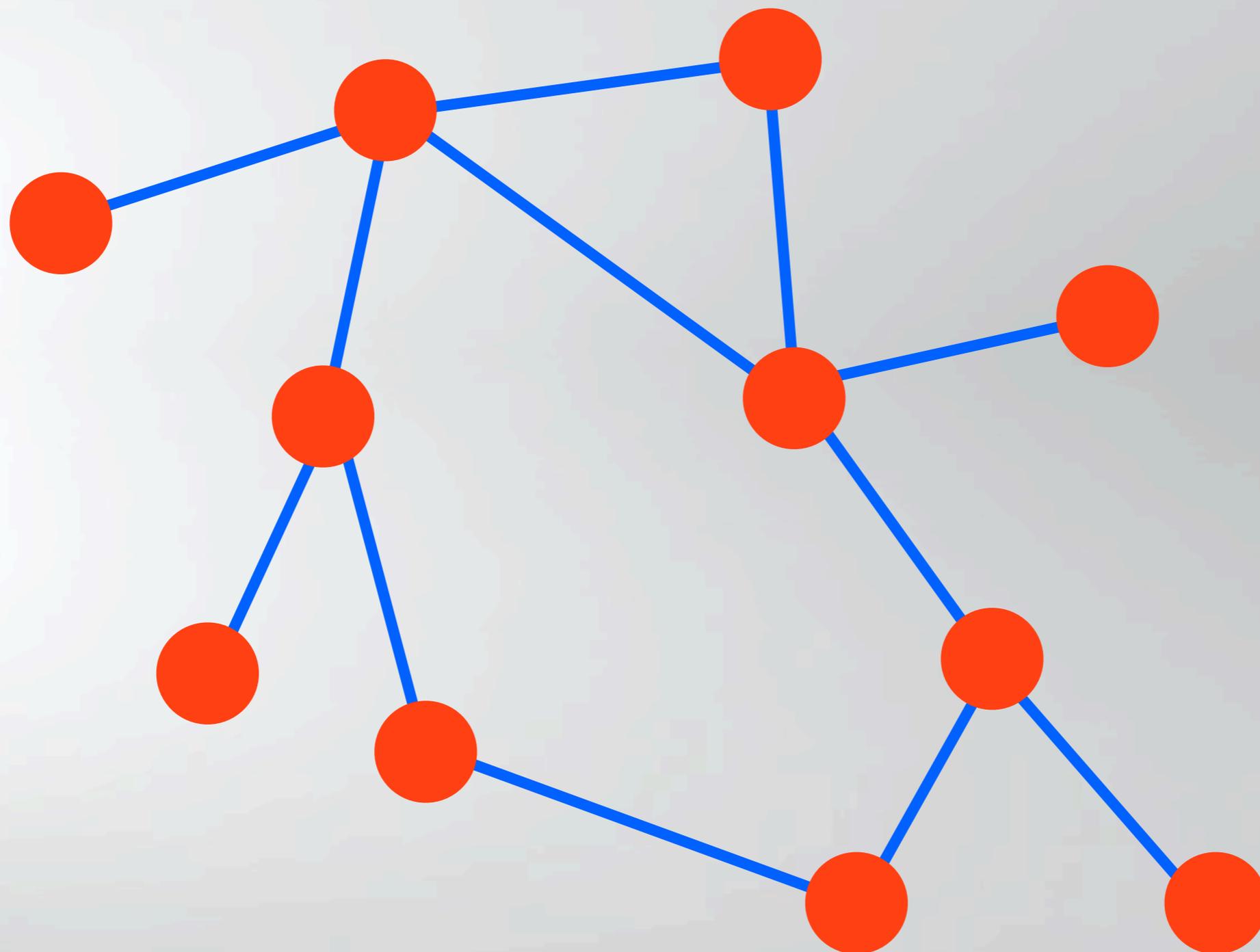


Vertices
Edges

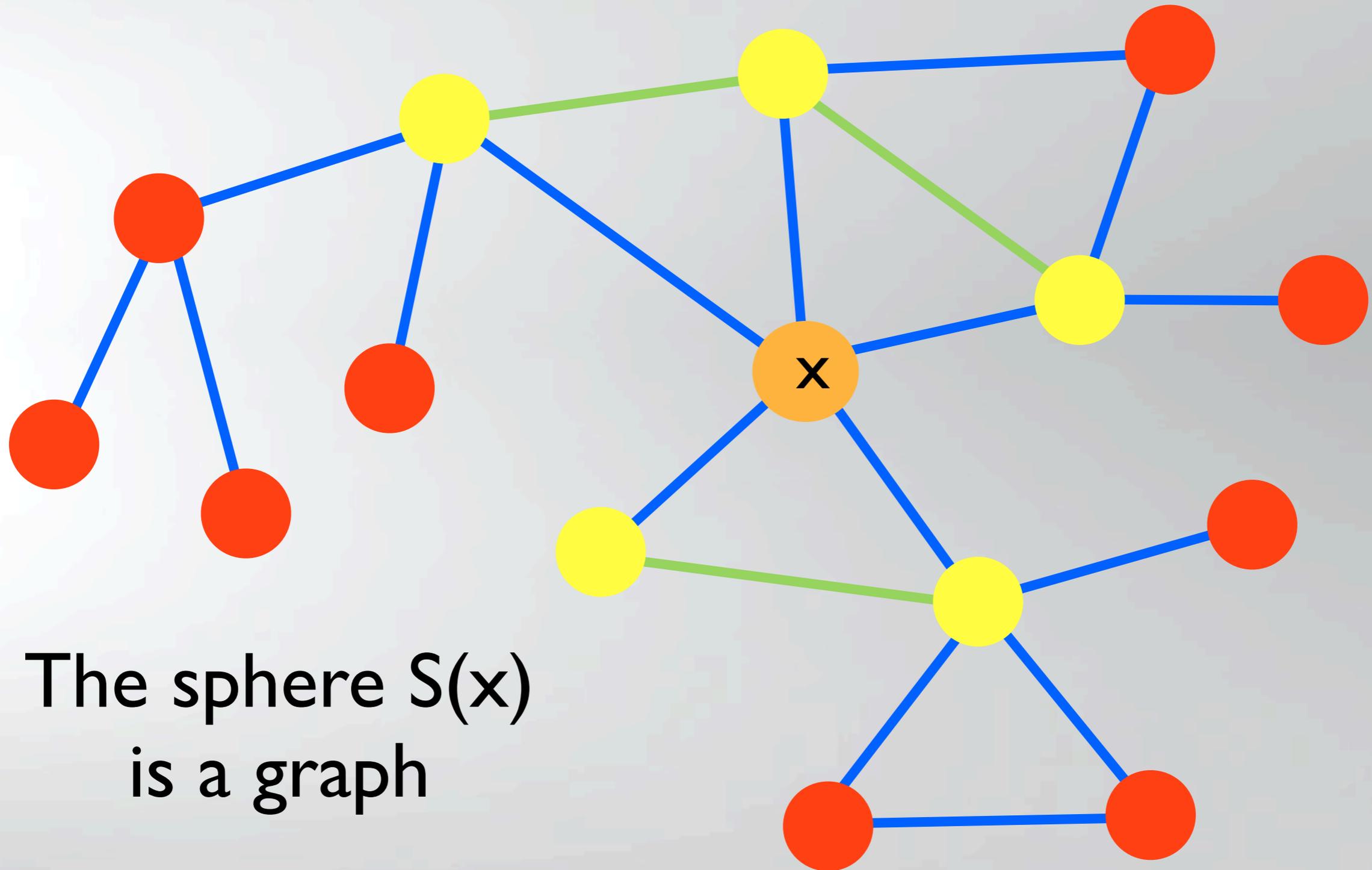
Cliques



Diameter

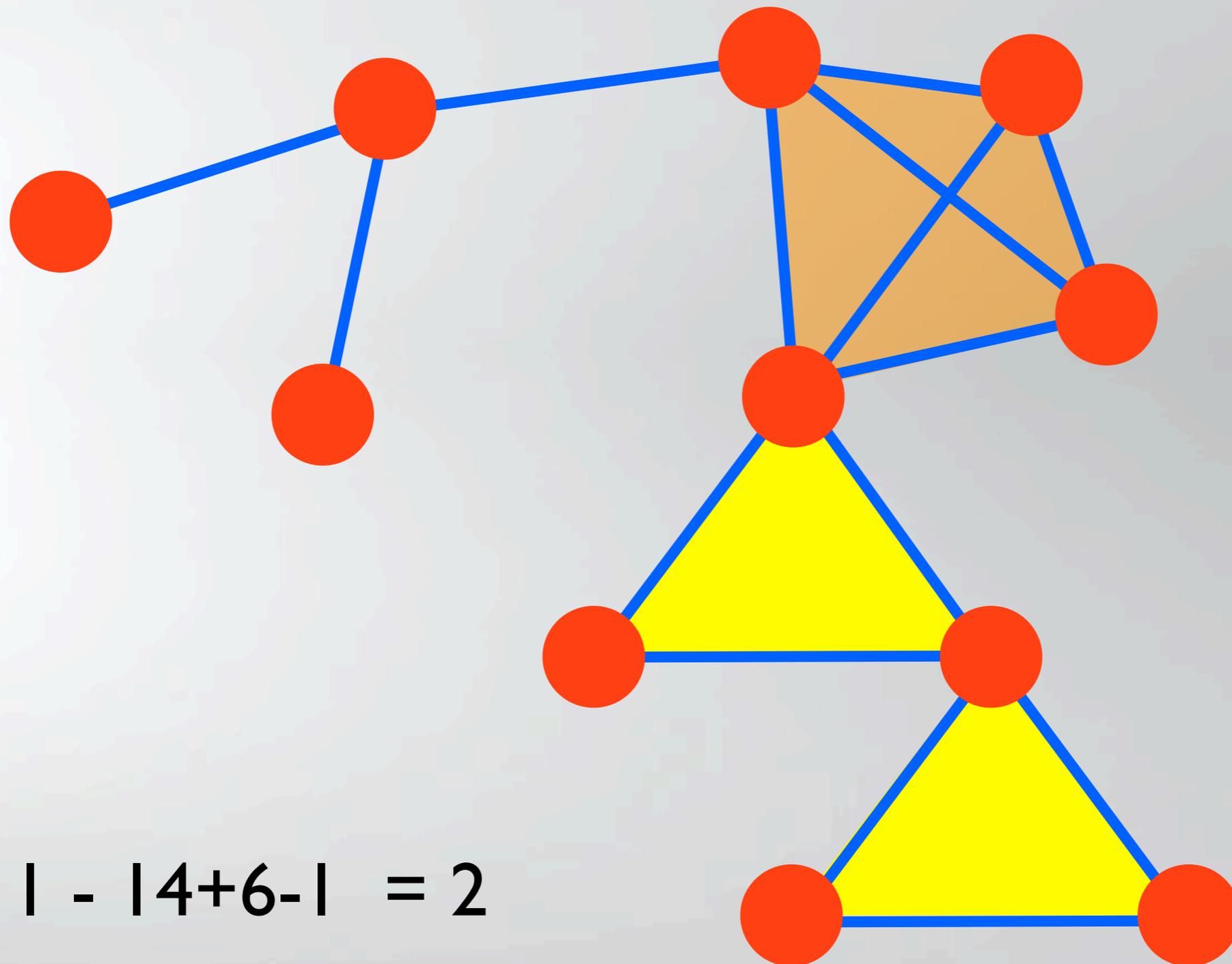


Spheres



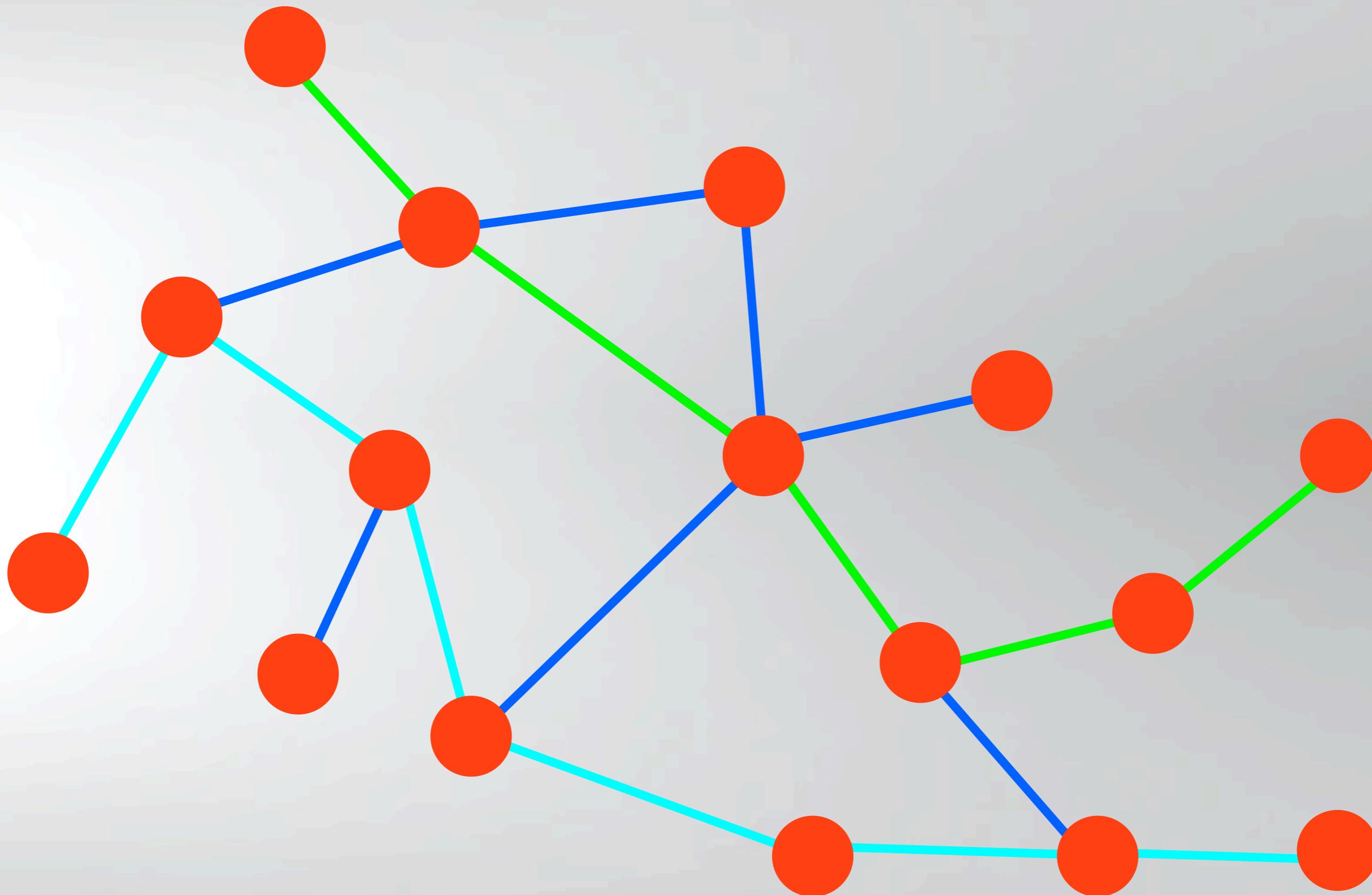
The sphere $S(x)$
is a graph

Euler Characteristic

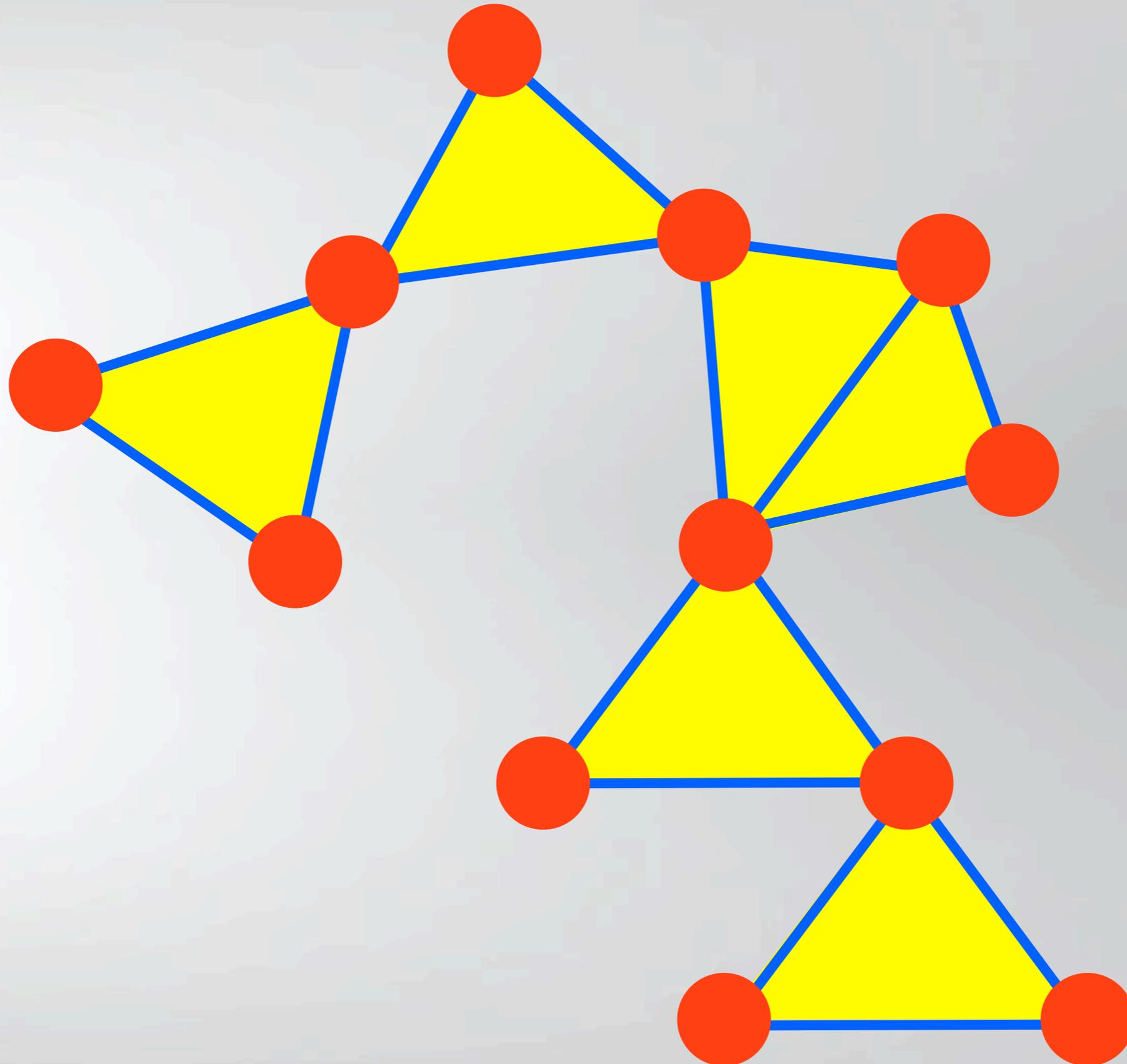


$$11 - 14 + 6 - 1 = 2$$

Parallel Lines



Area



Examples of Networks

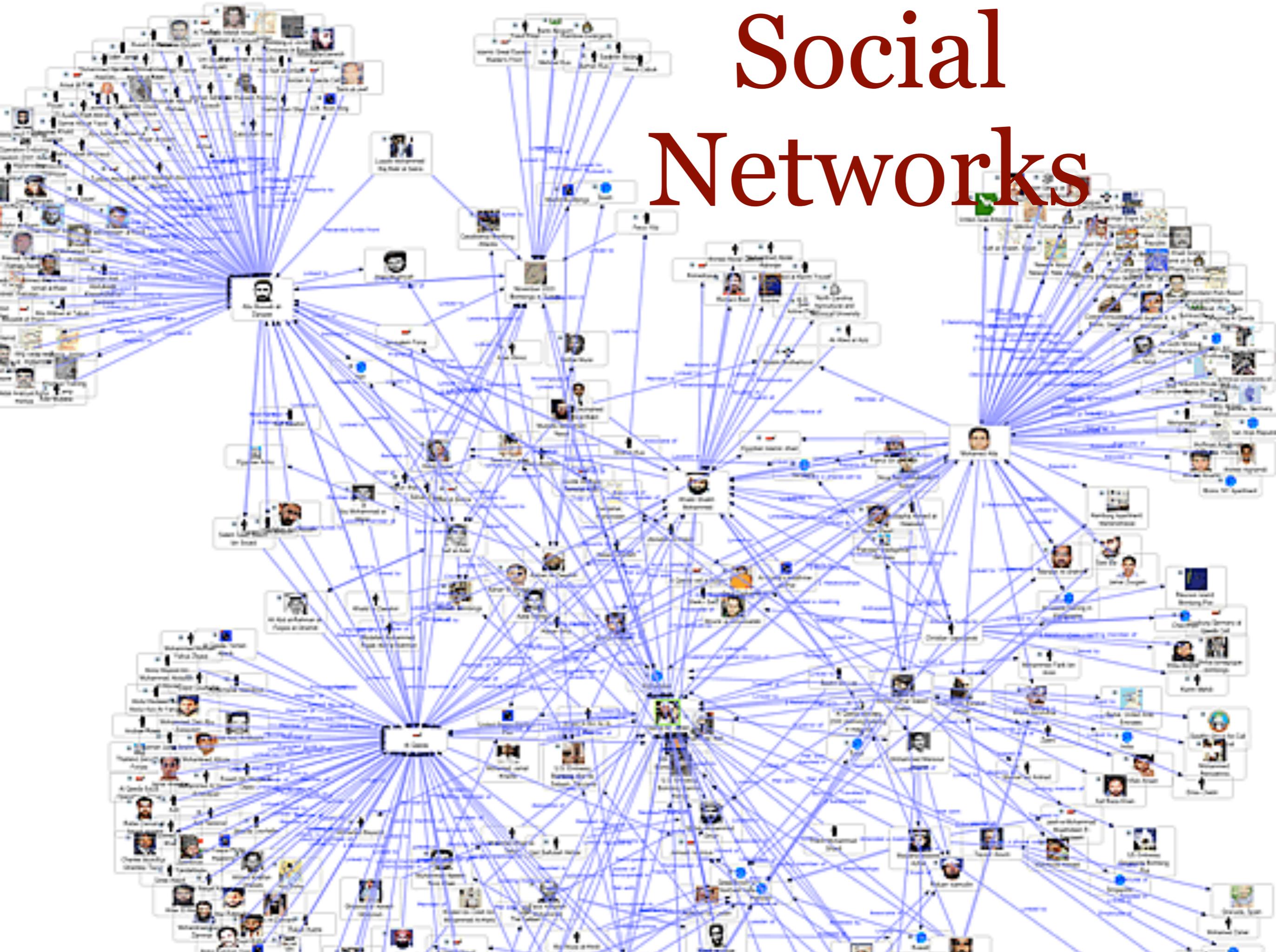


MASSACHUSETTS BAY TRANSPORTATION AUTHORITY RAPID TRANSIT/ KEY BUS ROUTES MAP



Transportation

Social Networks



Social networks



Duncan Watts



in "Six Degrees"

"In the world of networks, sociologists, economists, mathematicians, computer scientists, biologists, engineers and physicists all have something to offer each other and much to learn."

Sociology



Sociologist: Morris Zeldich

1969 Zeldich:
rhetorical
question:

Can you really
study an army in
a laboratory?

2011: Christakis:

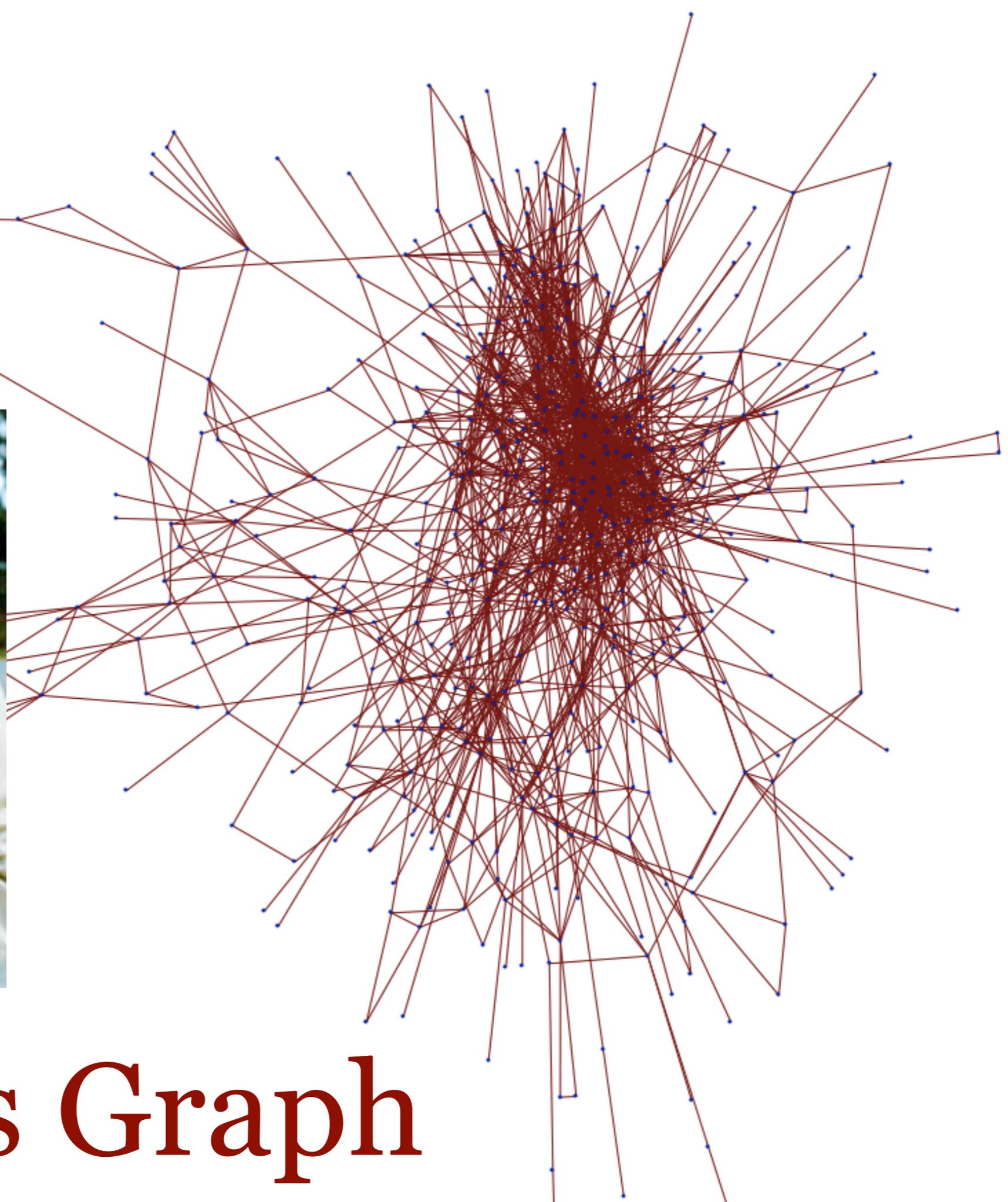
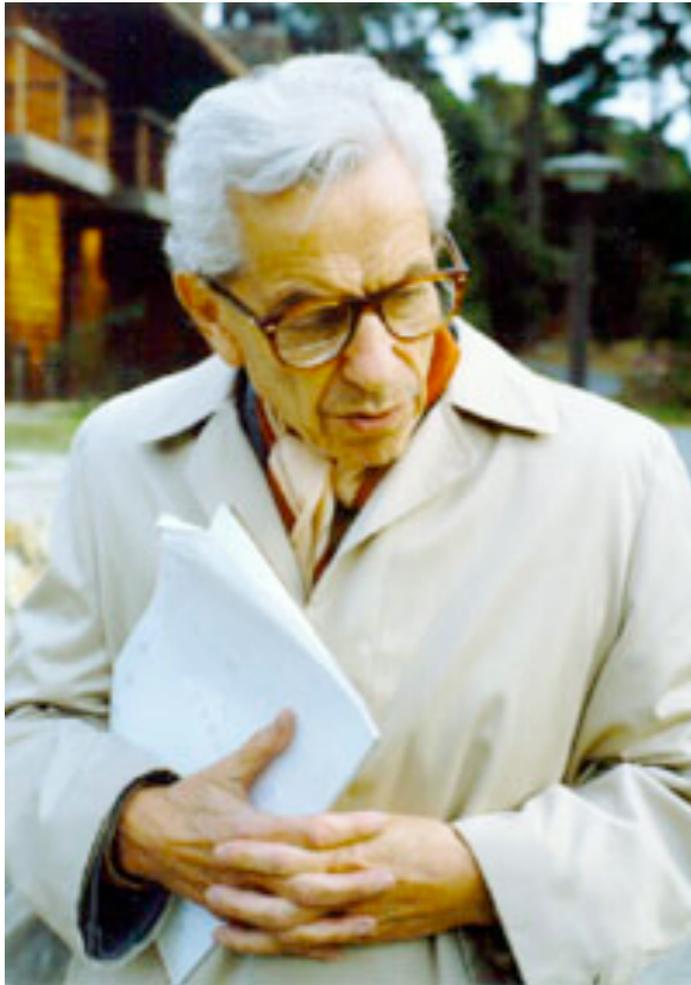
"Nearly half a
century later, the
answer appears to
be yes."

**The Trouble With Common
Sense,**

Nicholas Christakis,

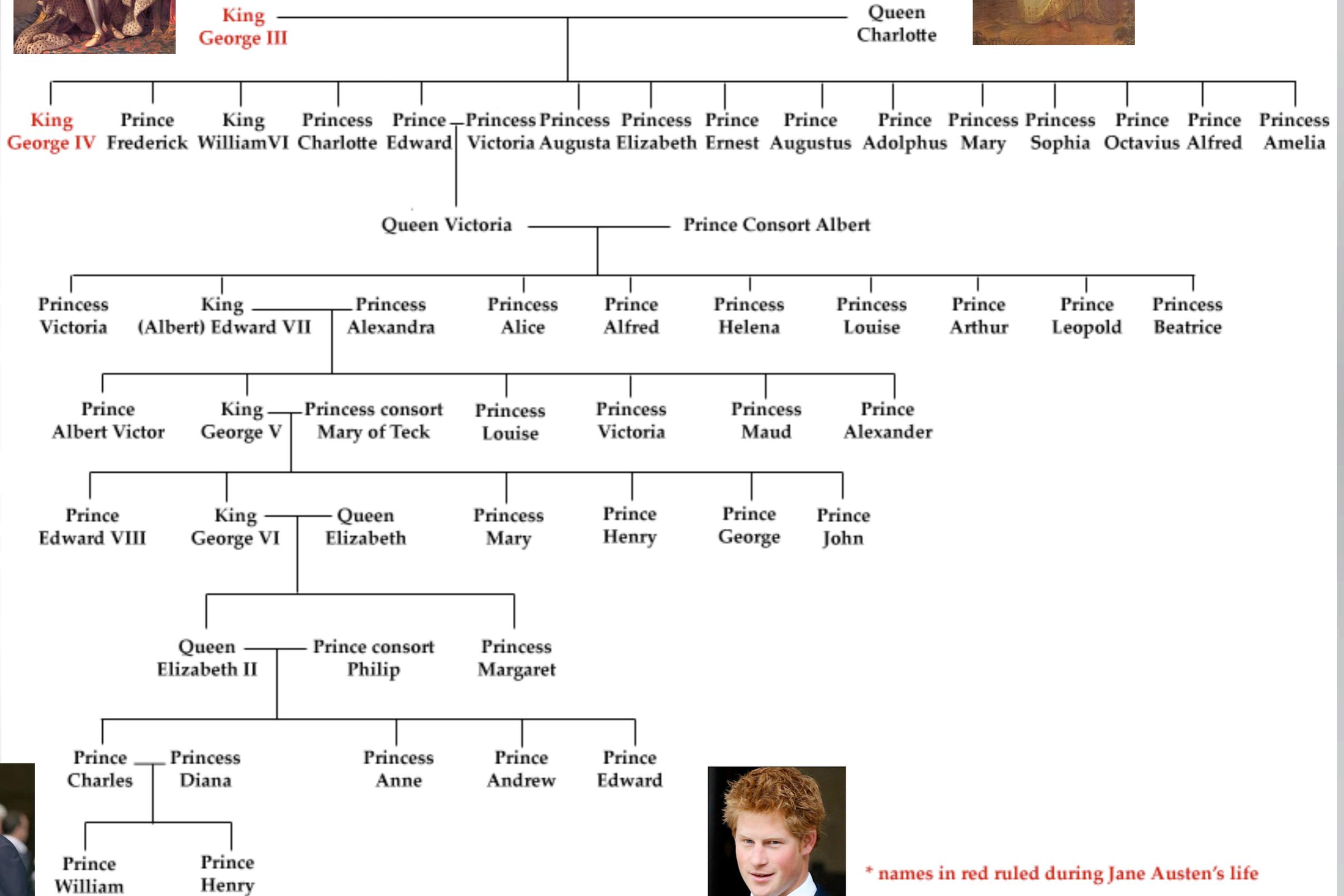
NYT June 24, 2011





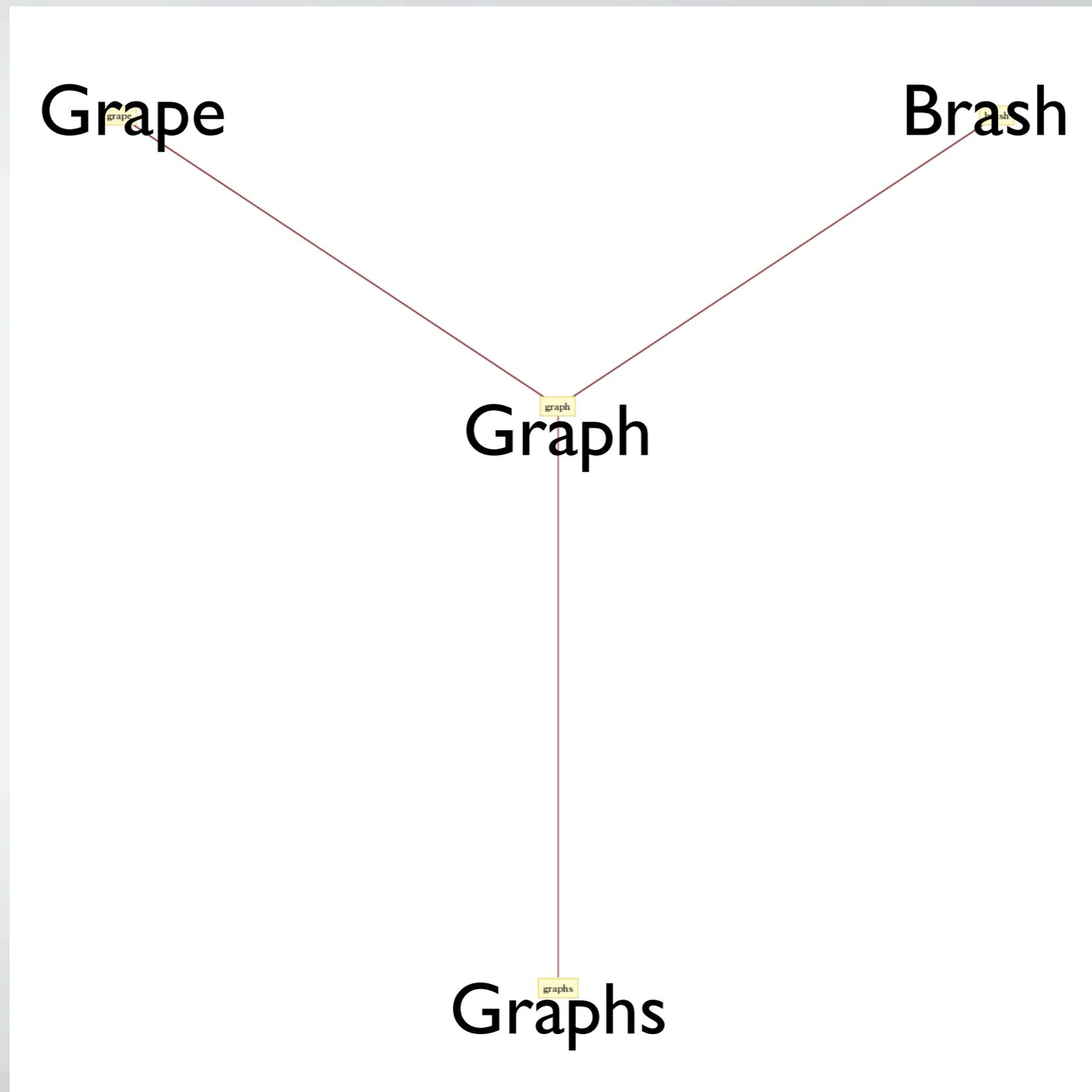
Erdős Graph

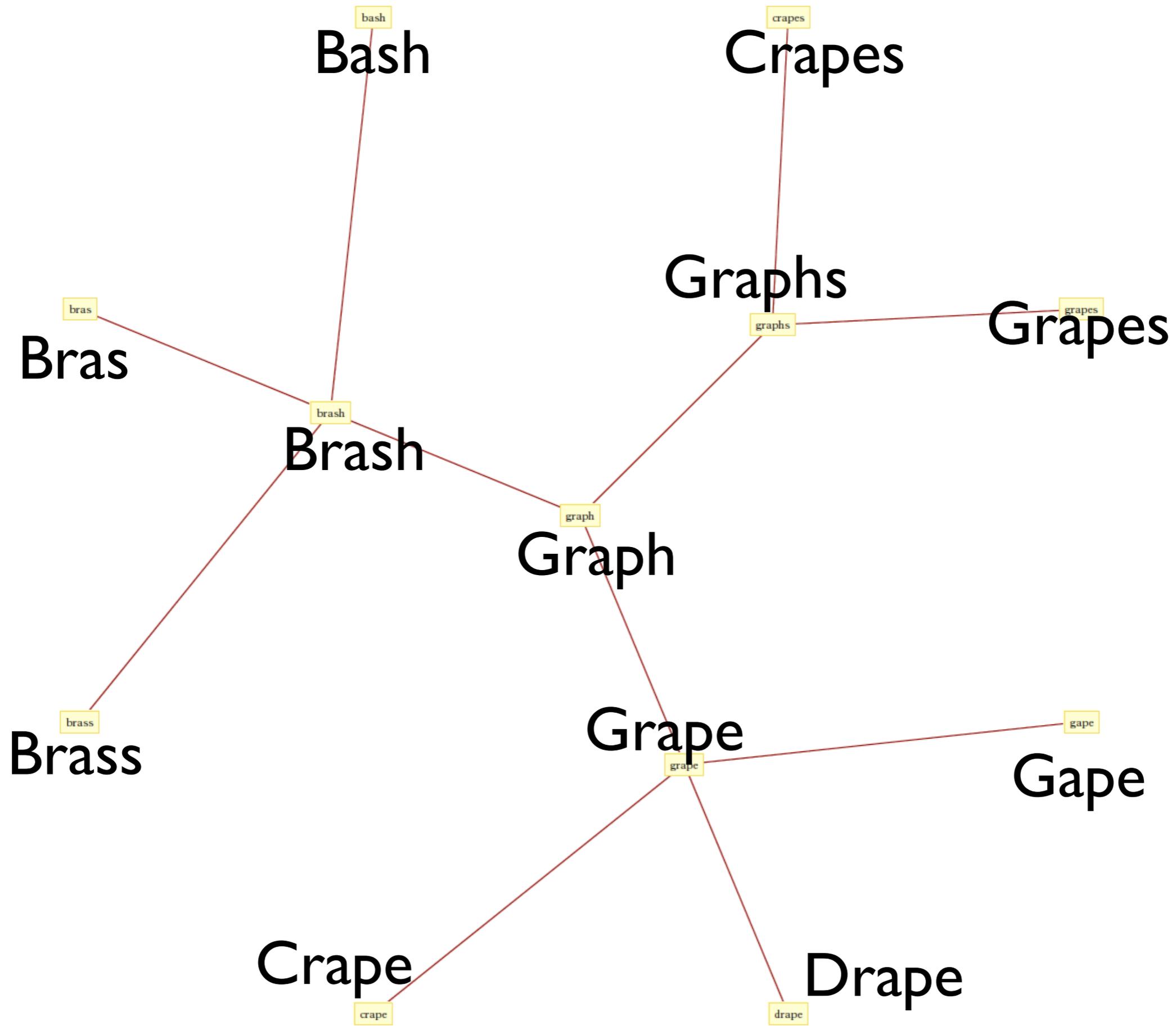
Genealogy



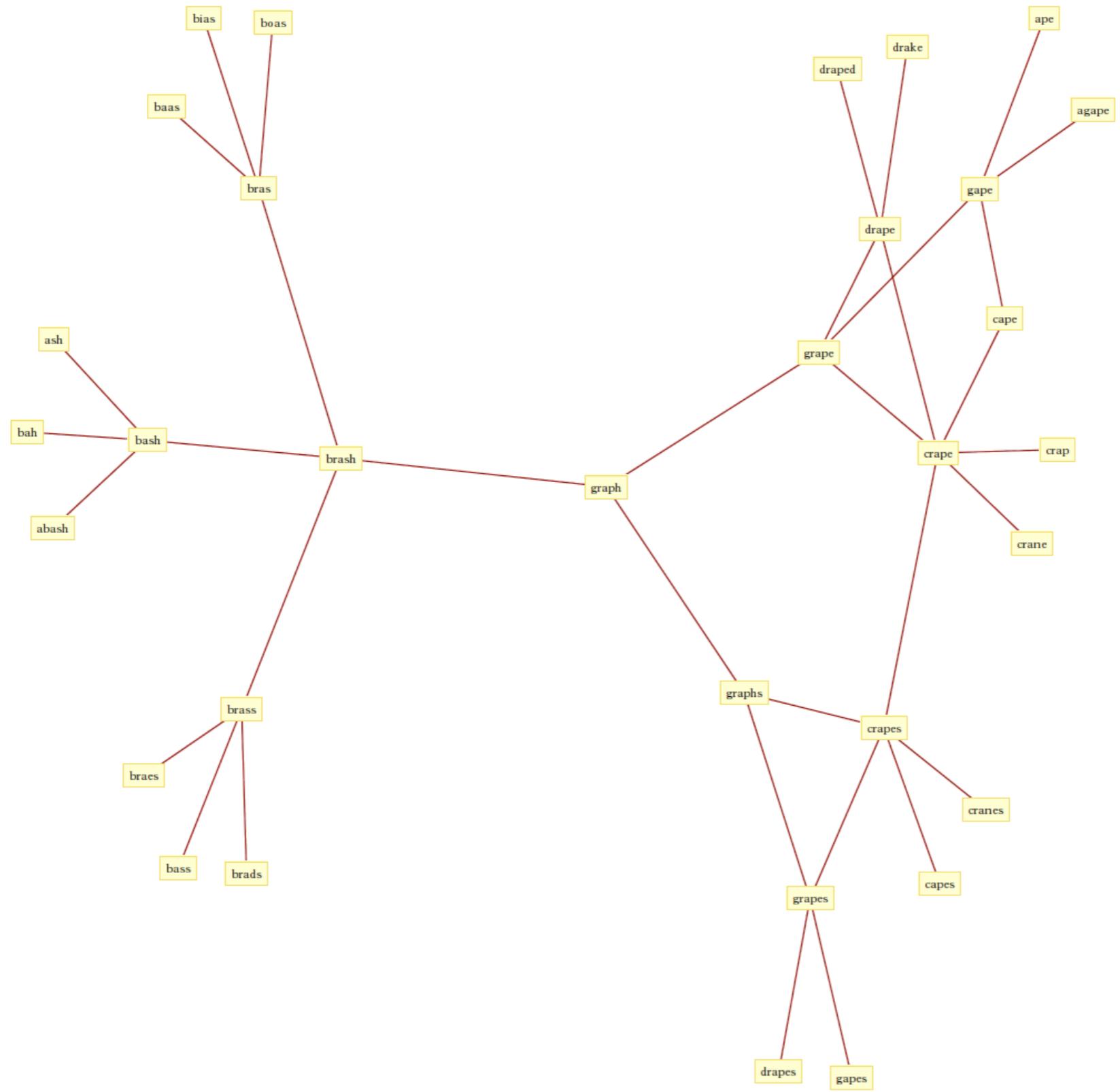
* names in red ruled during Jane Austen's life

Word changes

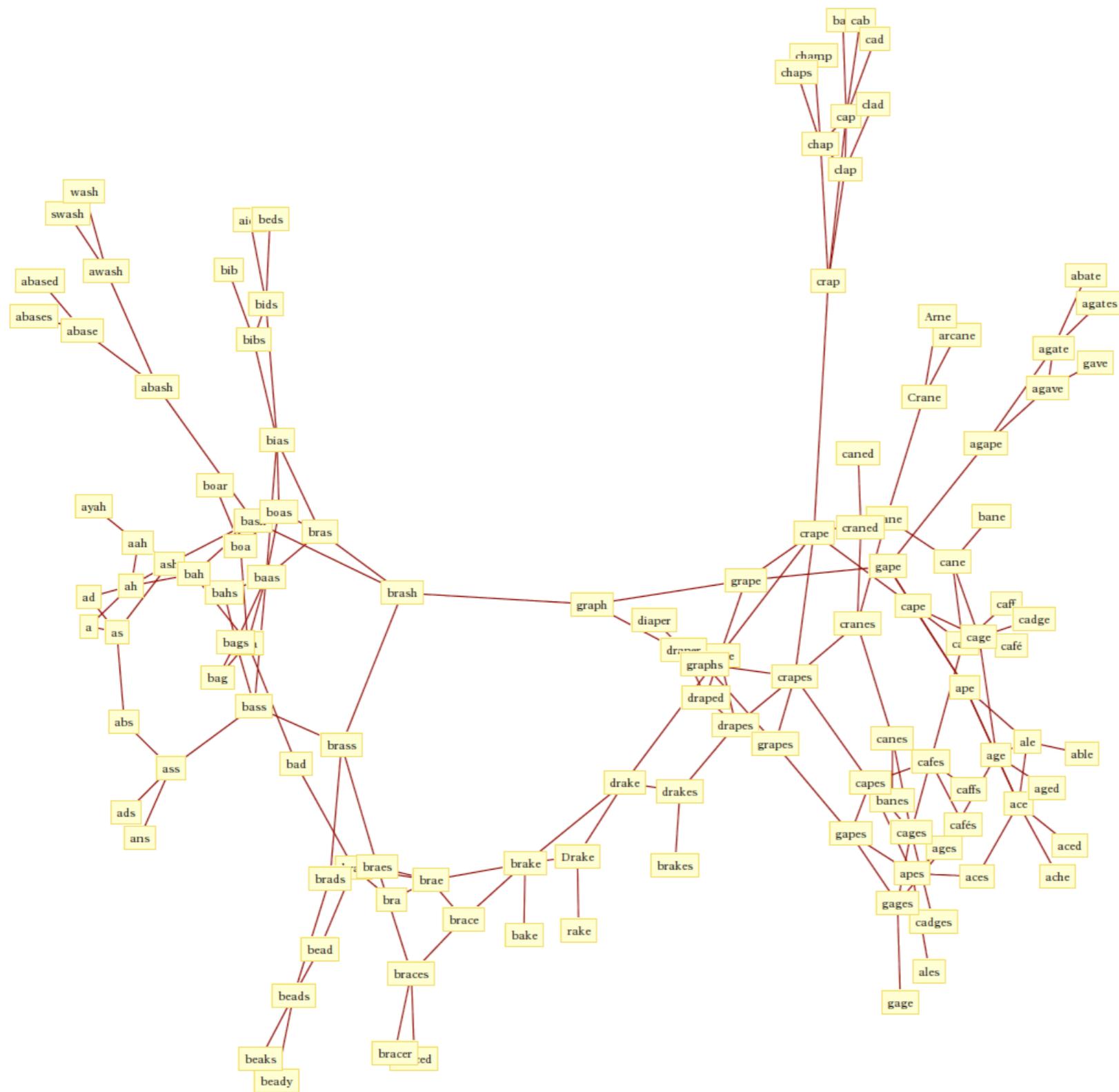




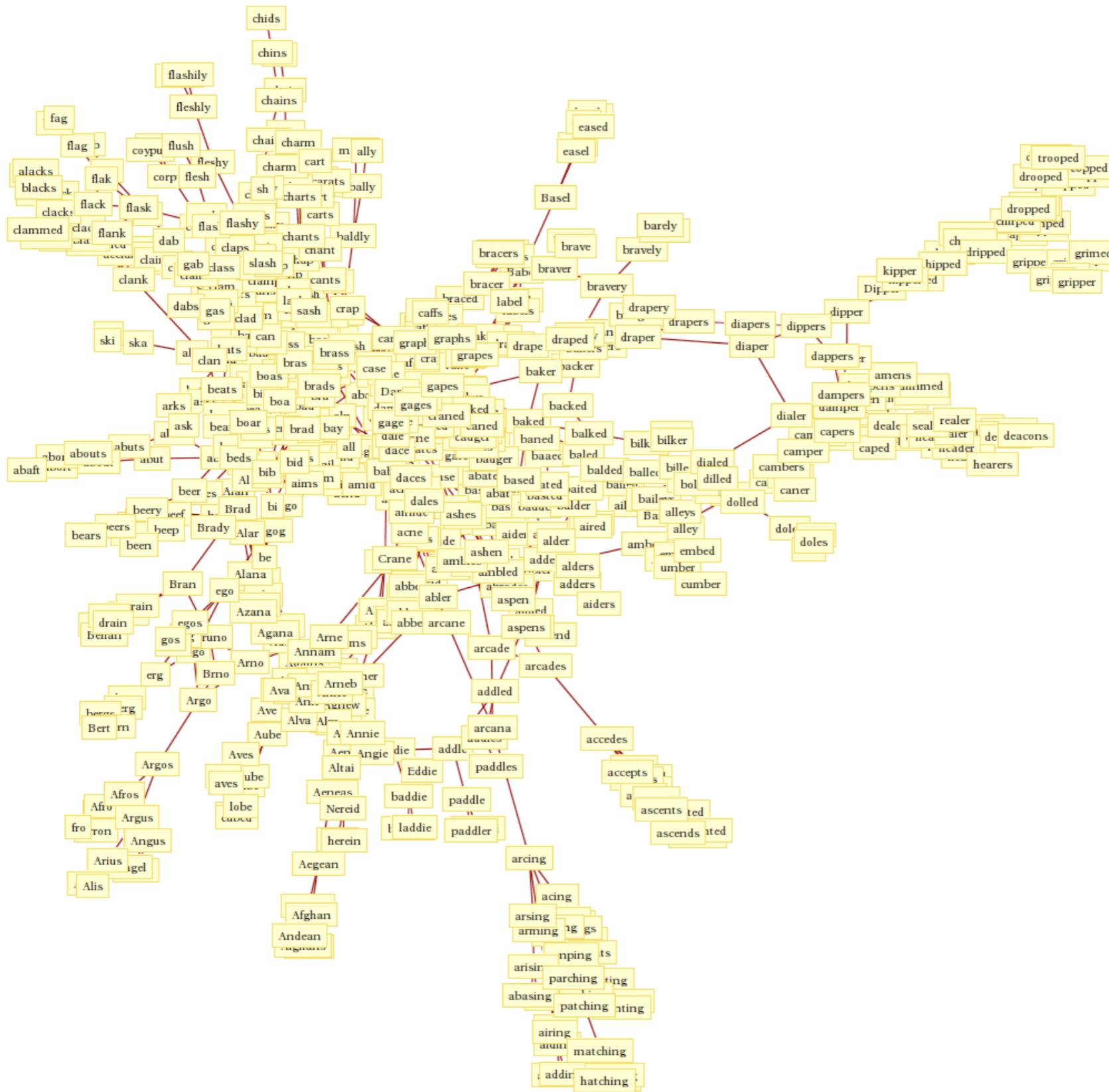
distance 2



distance 3



distance 5

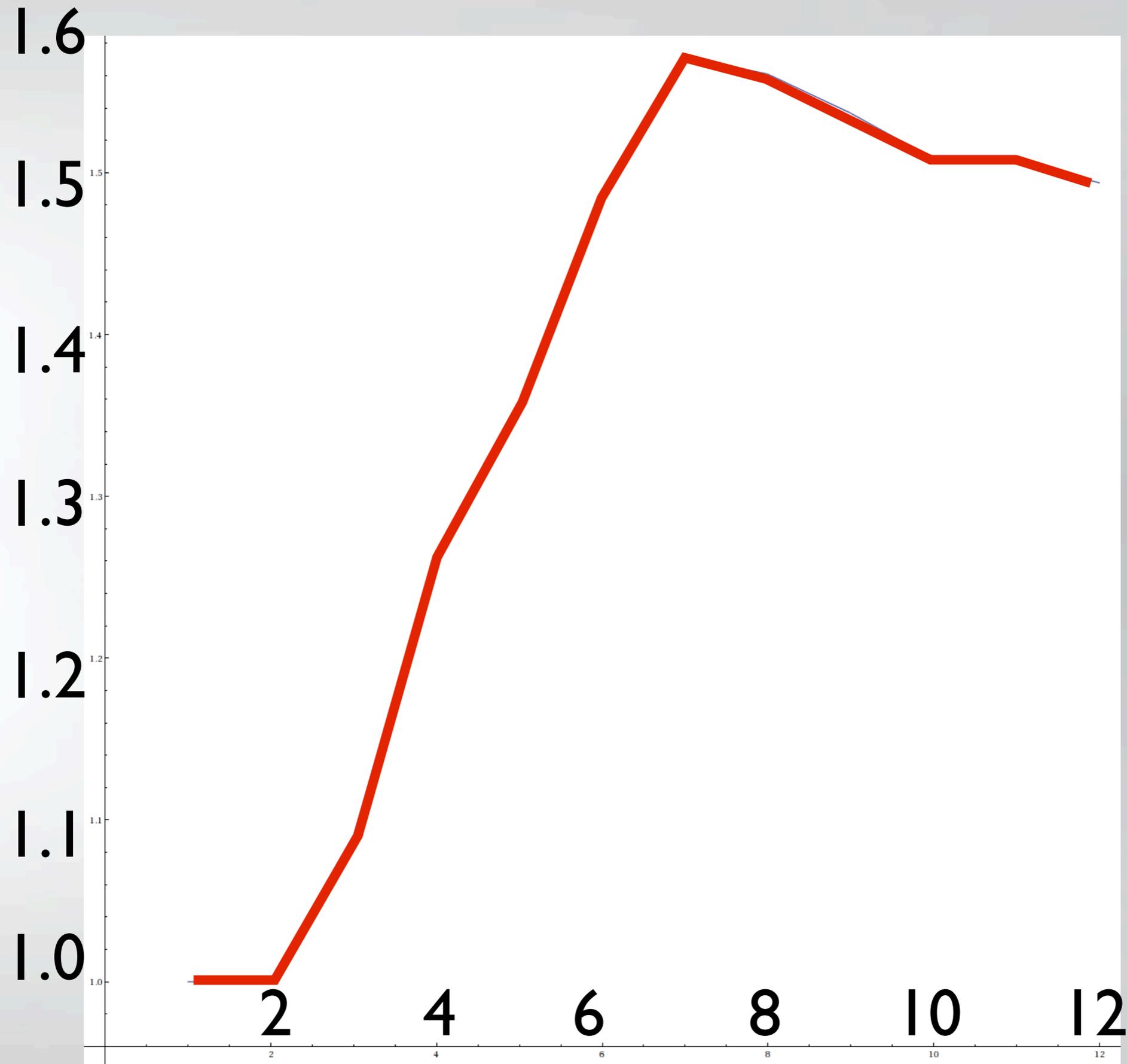


distance 11

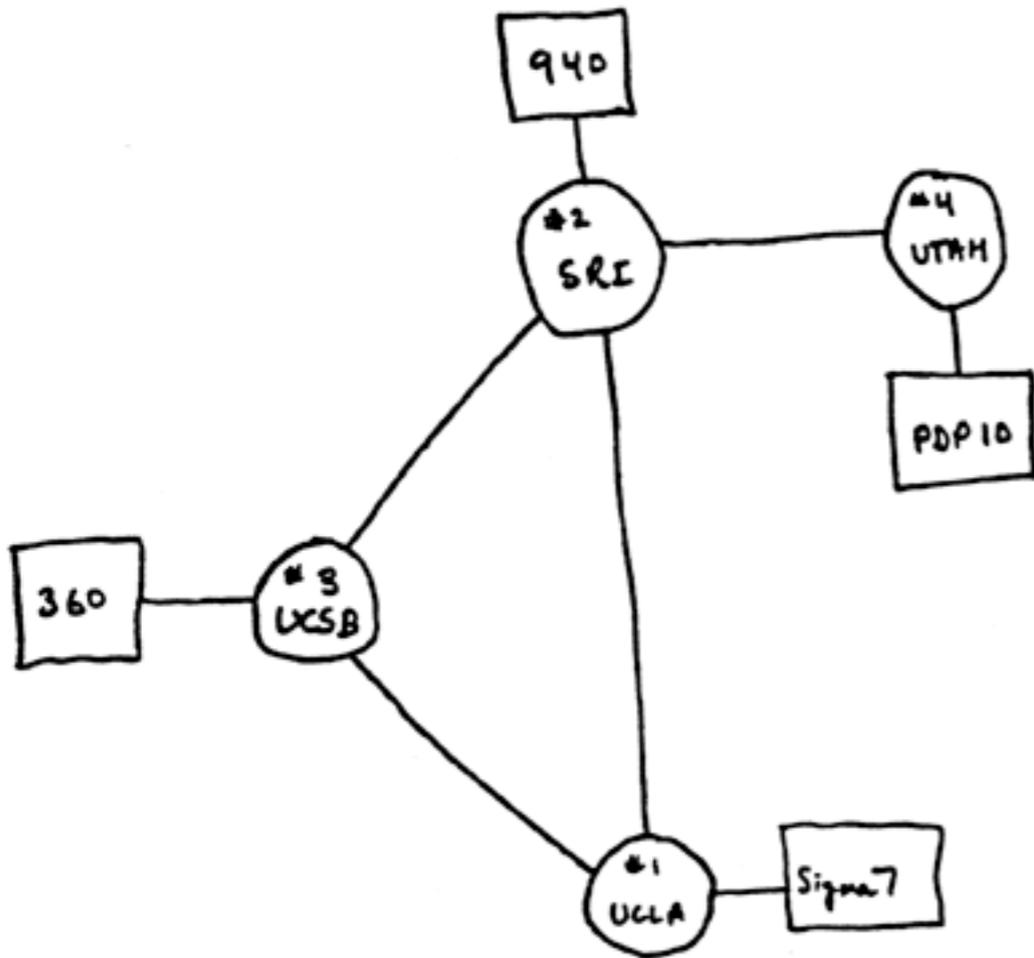


distance 12

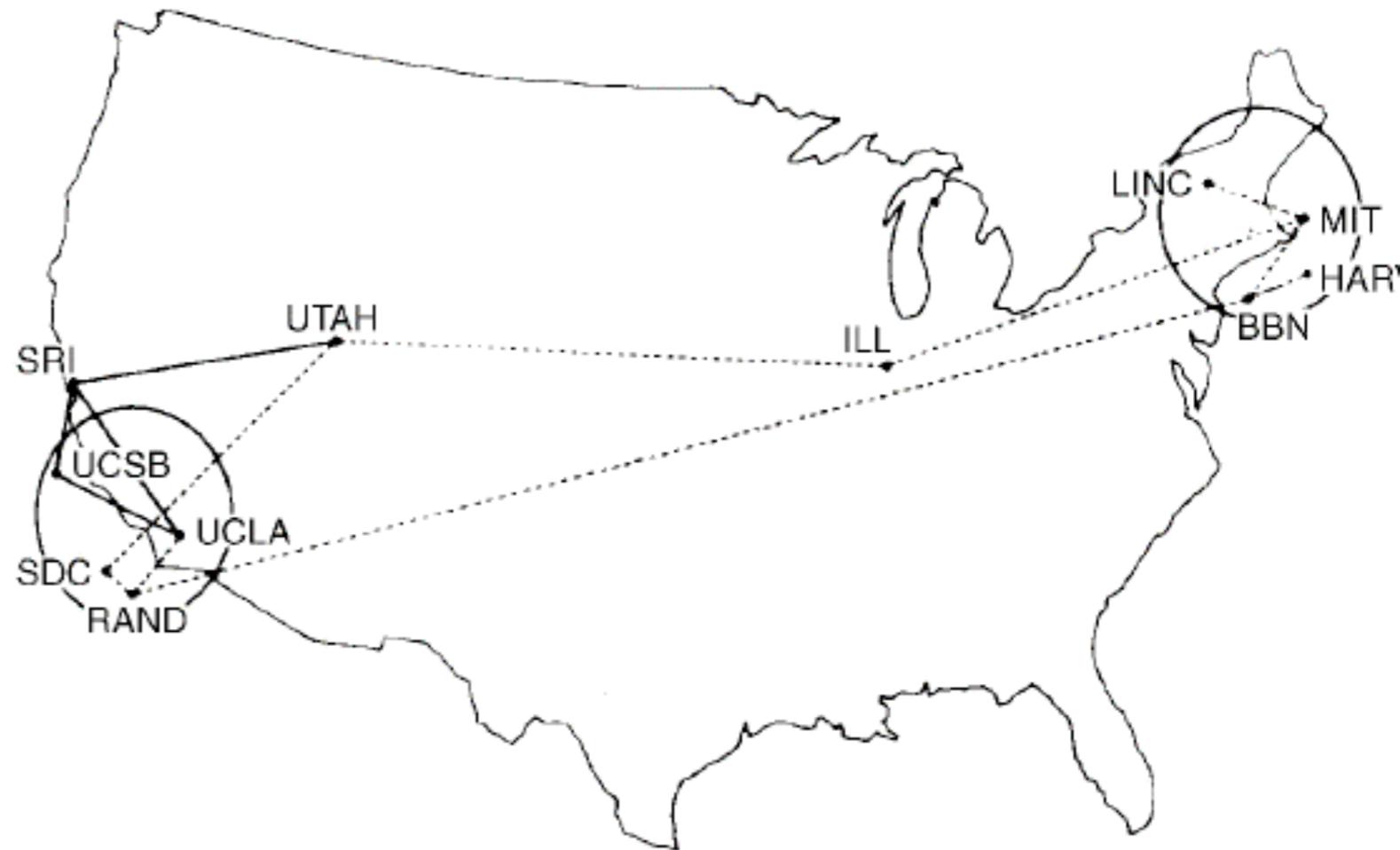
Dimension growth



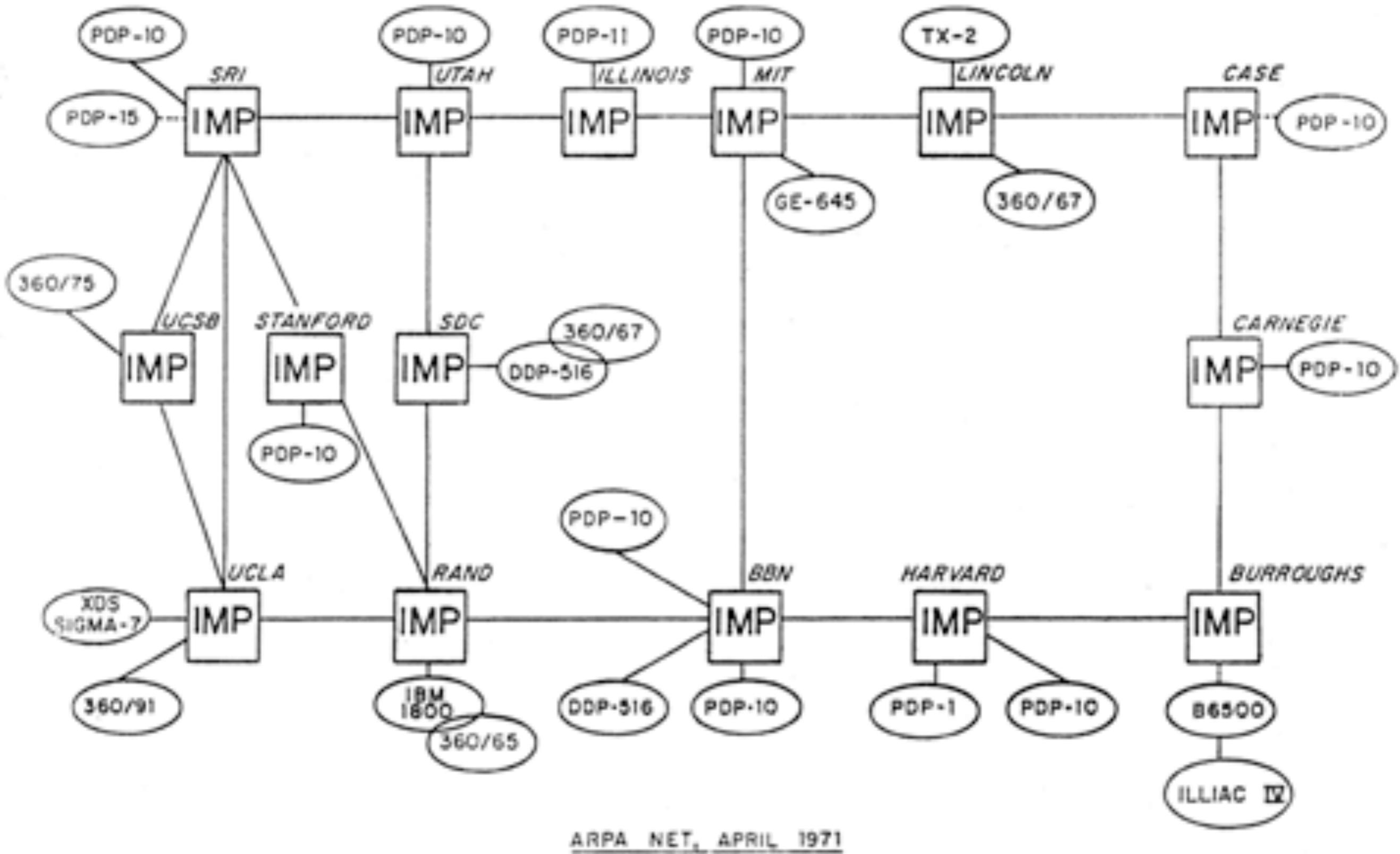
Internet



1969



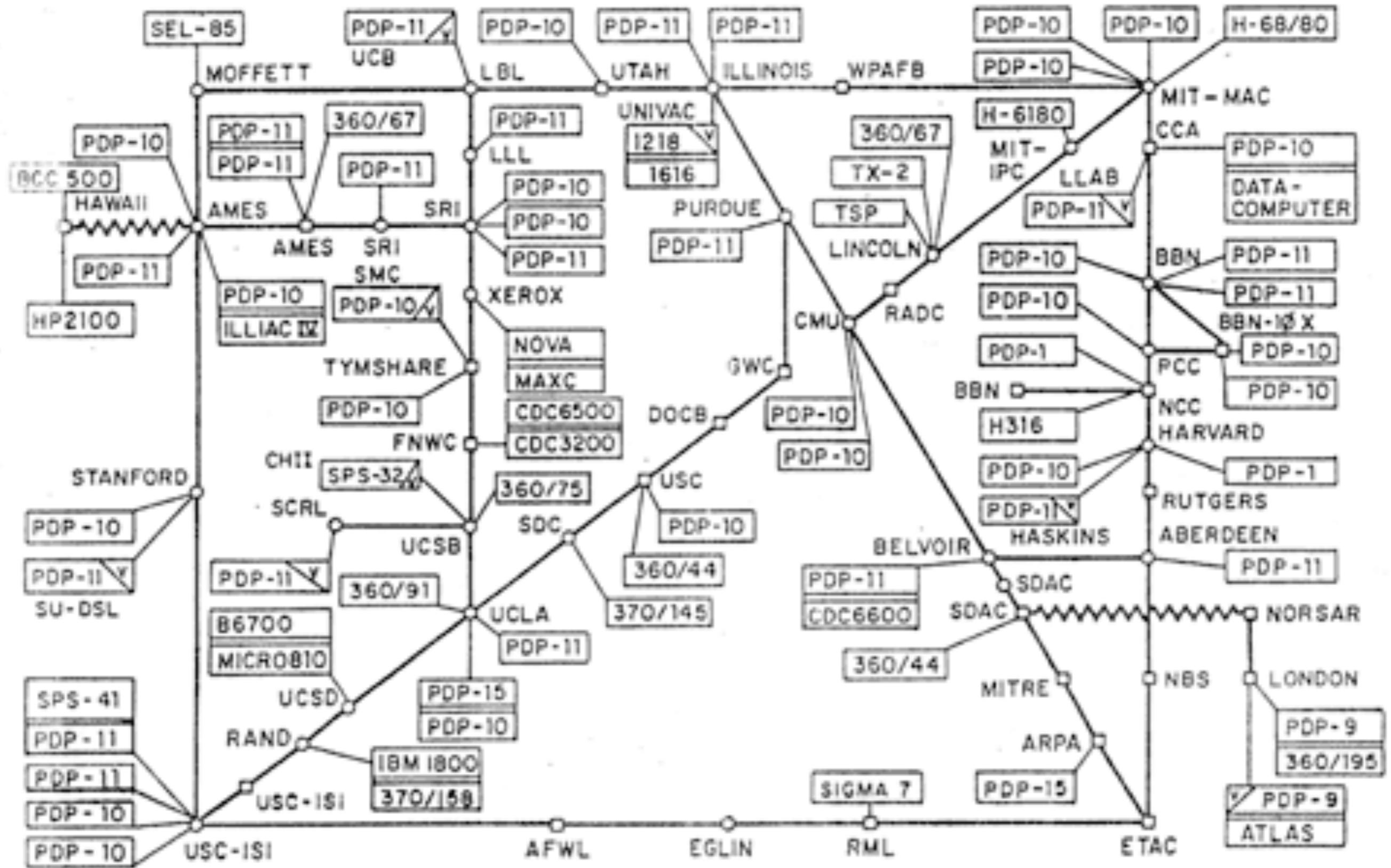
1970



1971

ARPA NETWORK, LOGICAL MAP, JANUARY 1975

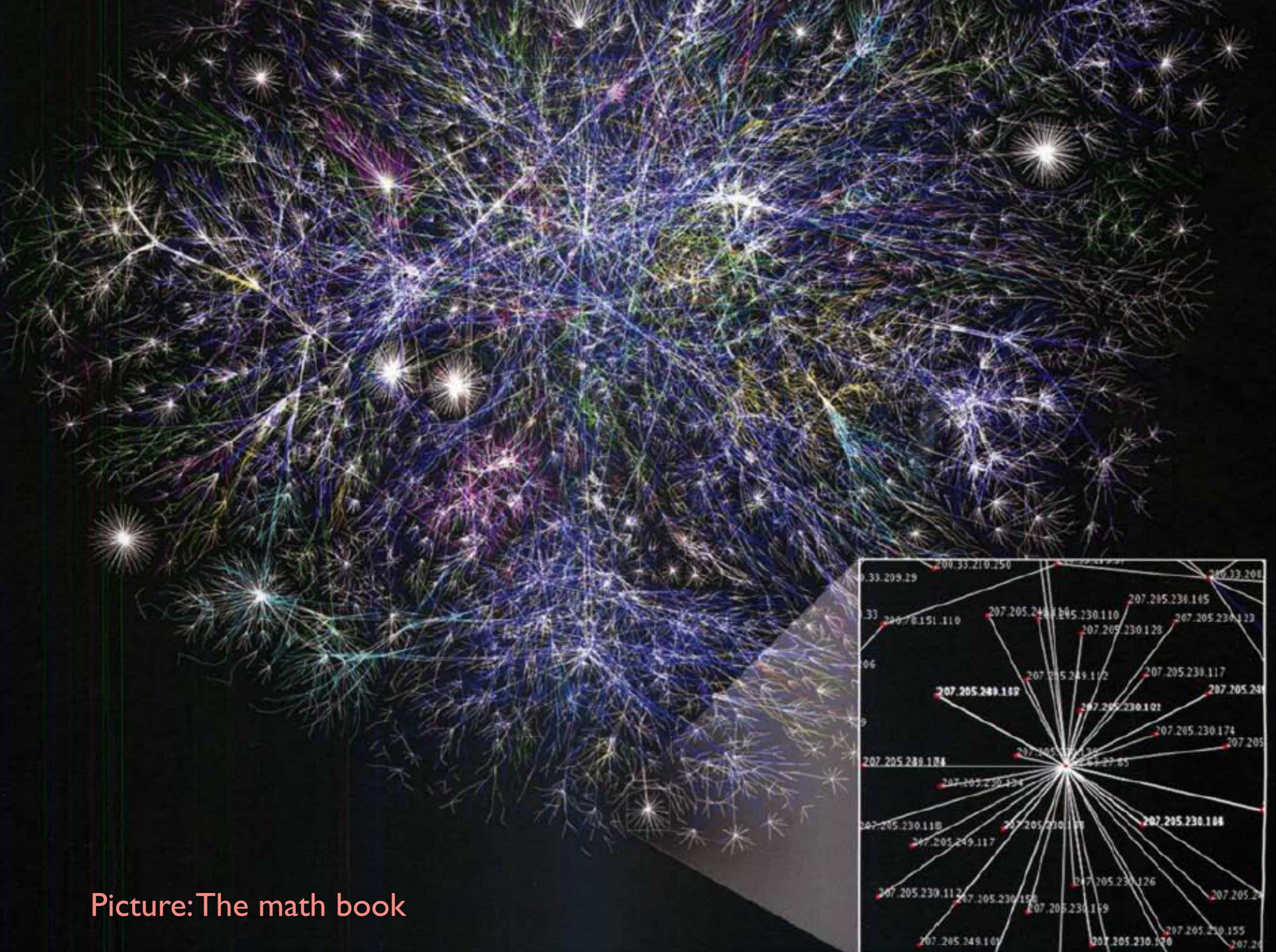
1975



○ IMP
□ TIP

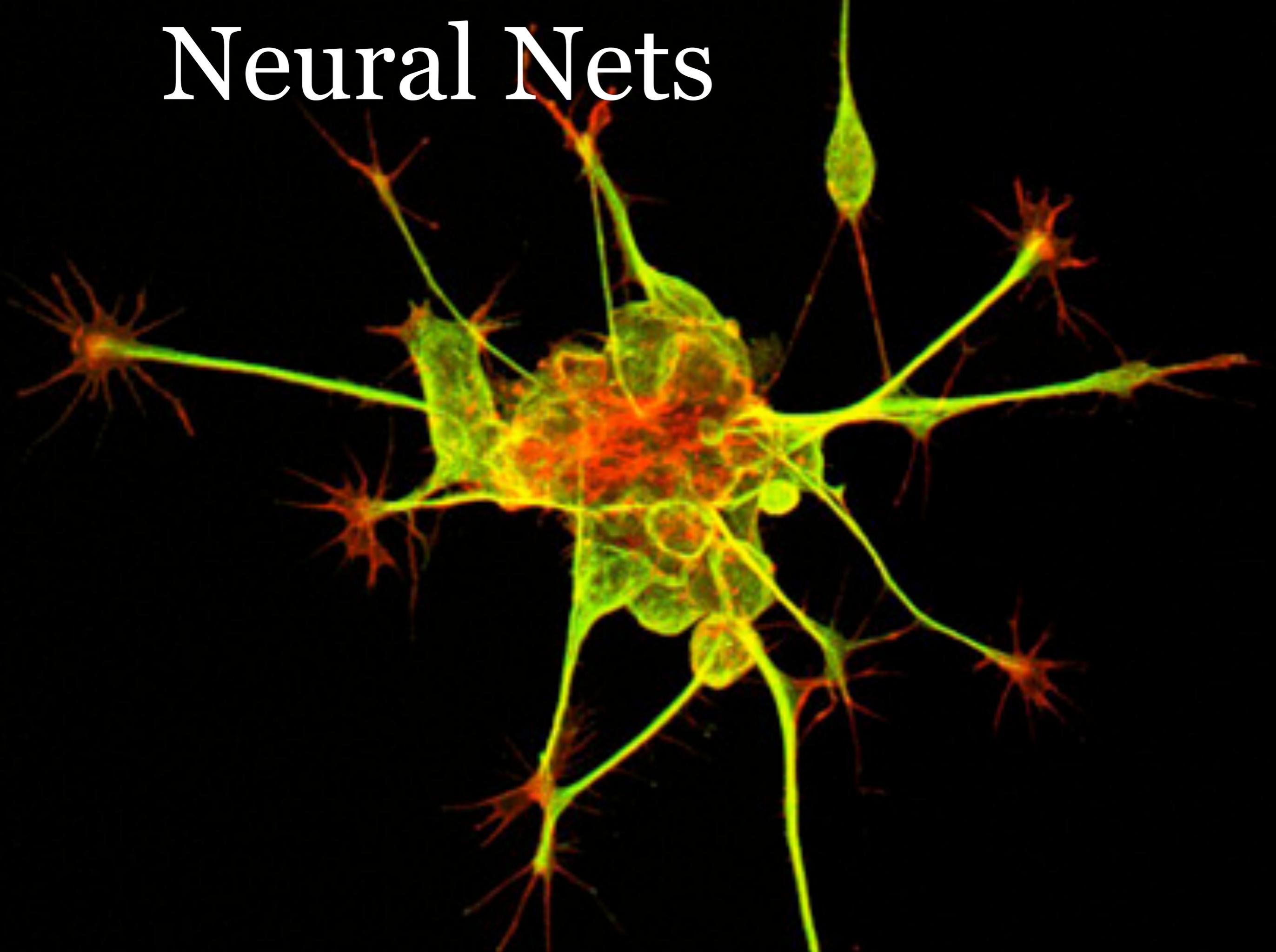


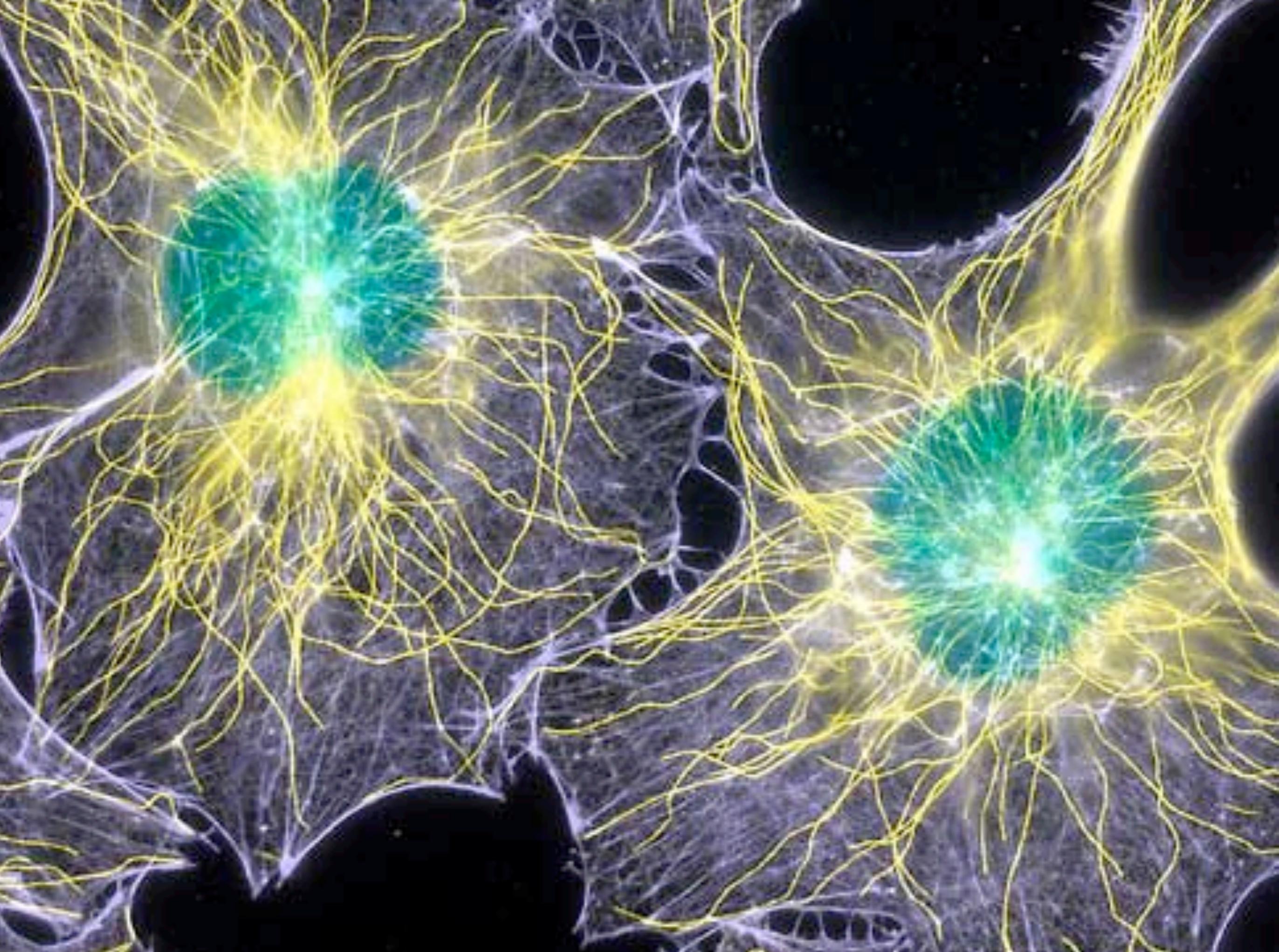
World Wide Web



Picture: The math book

Neural Nets





Brain



van Wedeen Harvard University



Quantum Spin Network



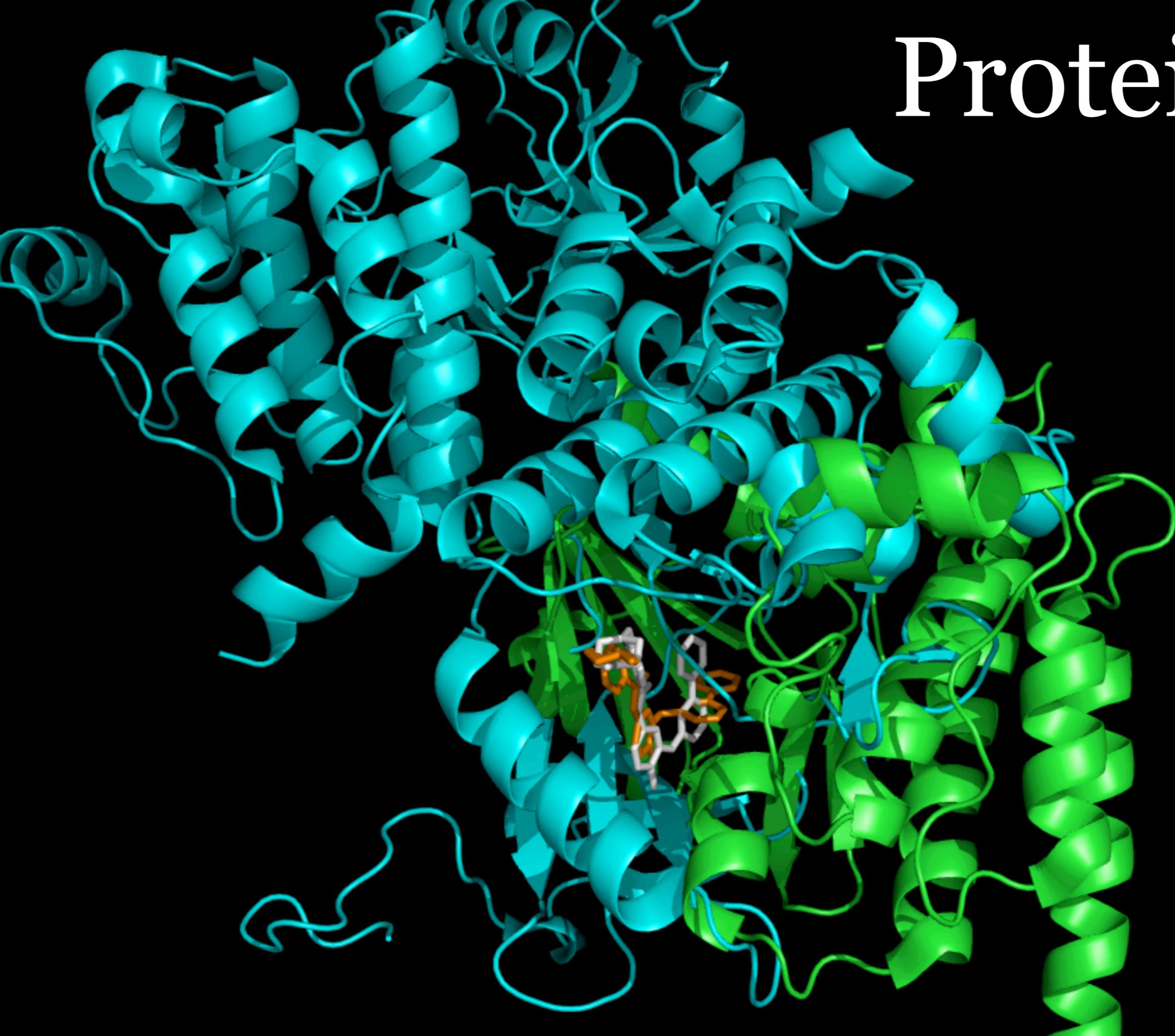
NOVA

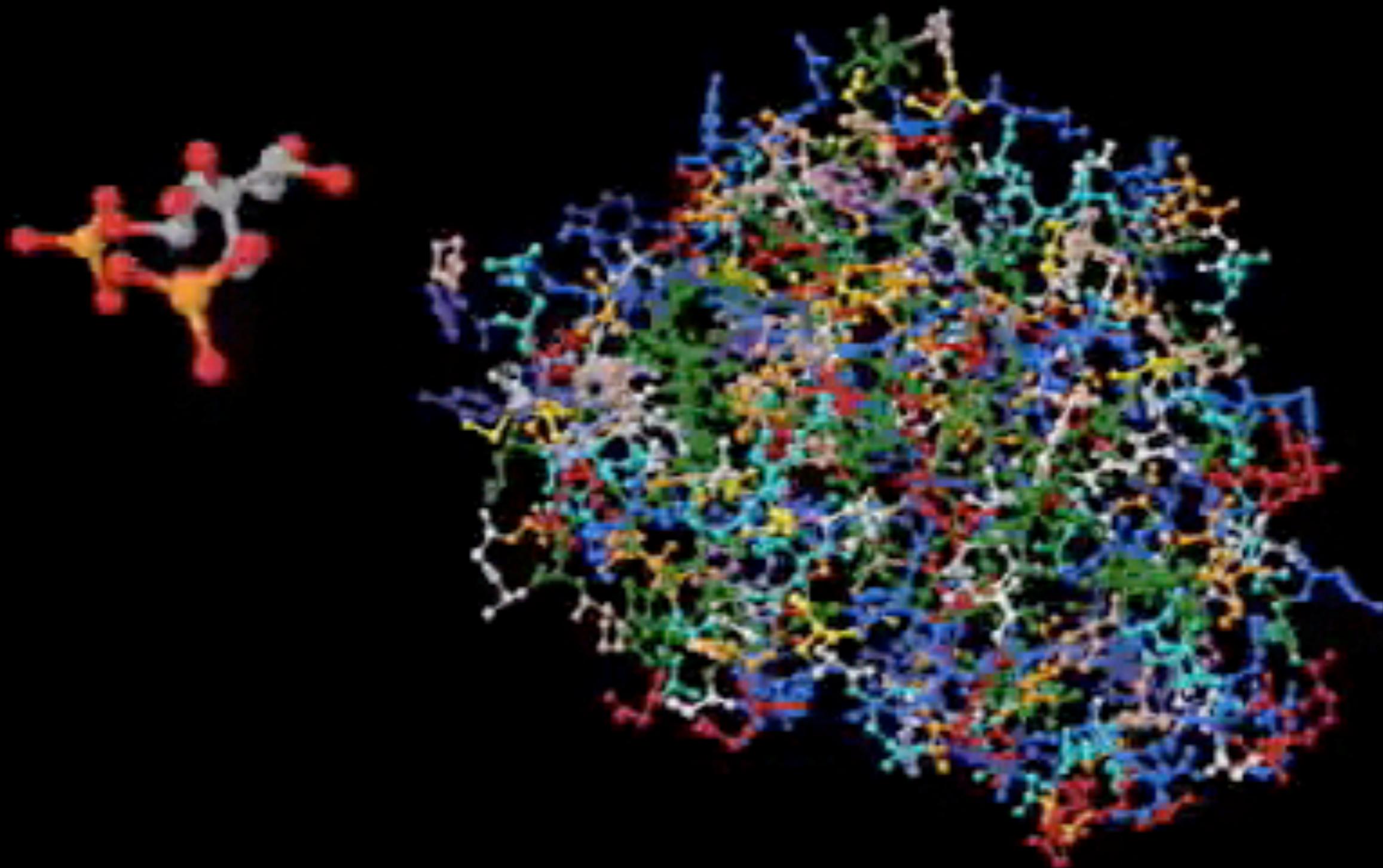
Saturday, April 7, 12

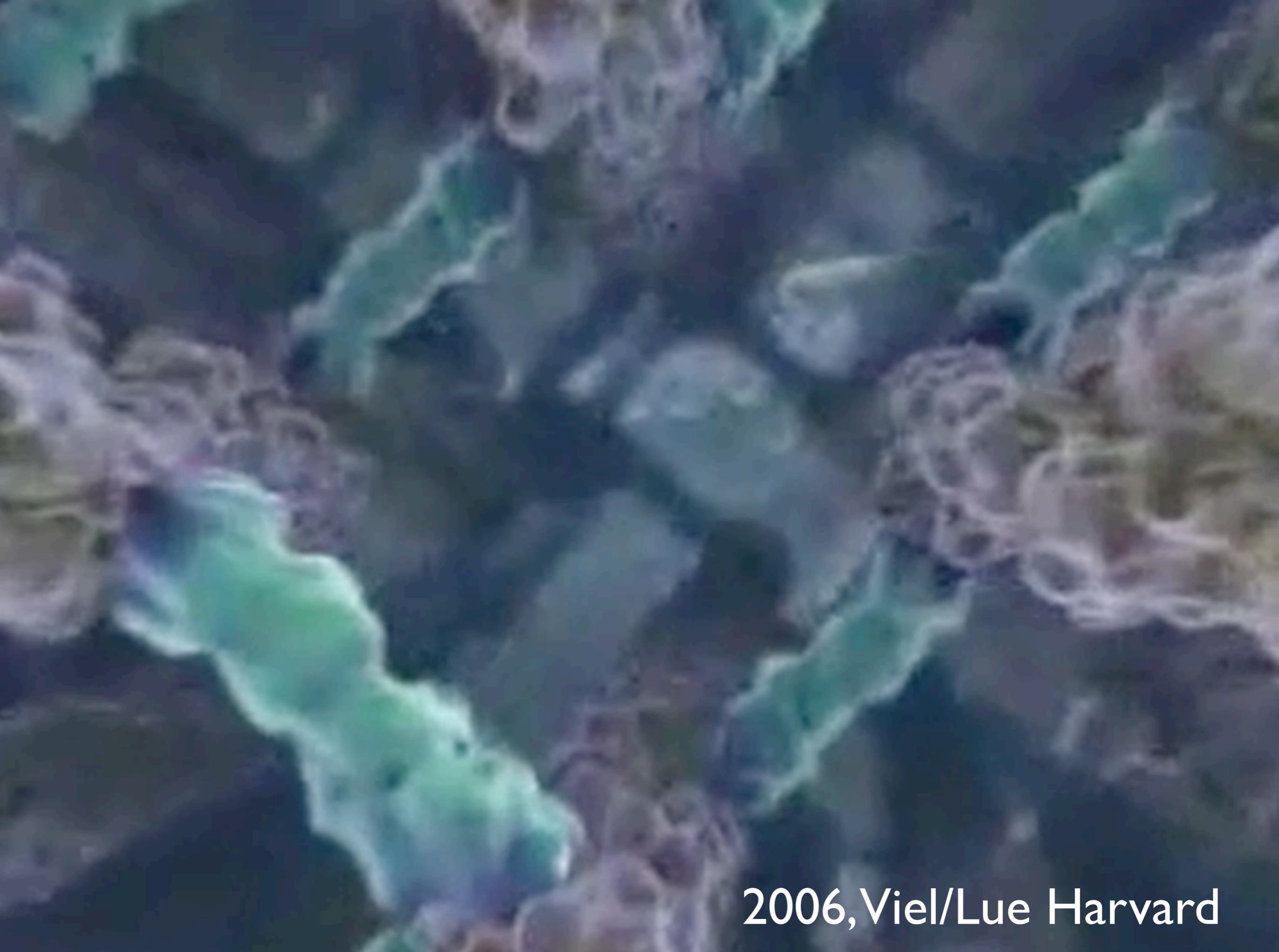
Max Planck Institut, Potsdam



Proteins

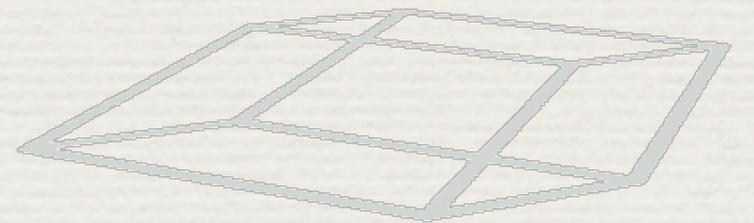
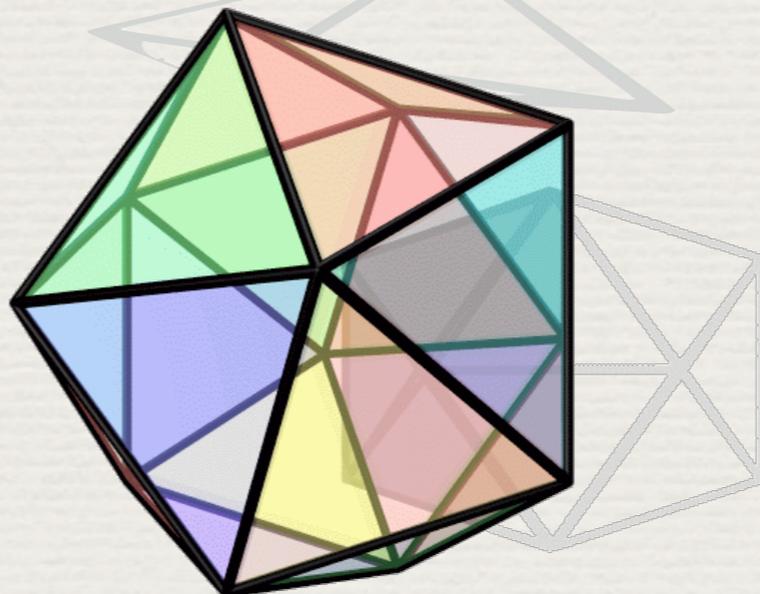
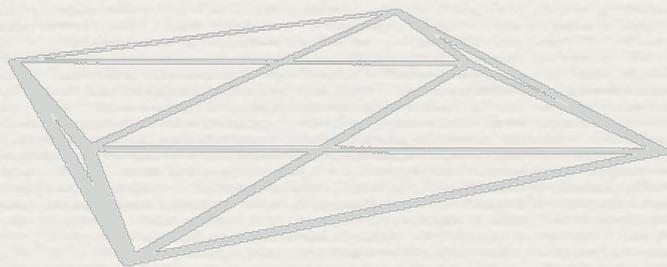
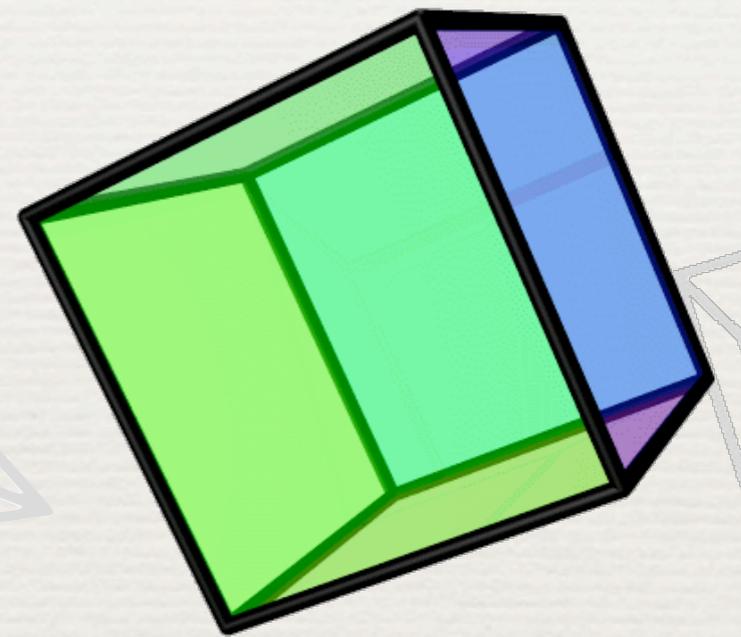
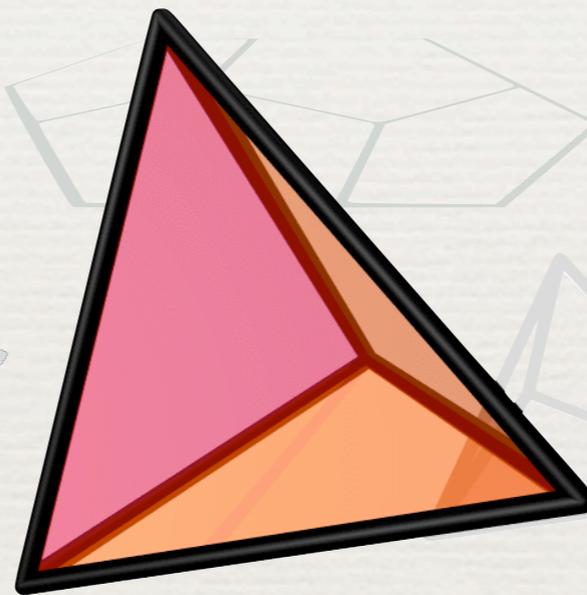
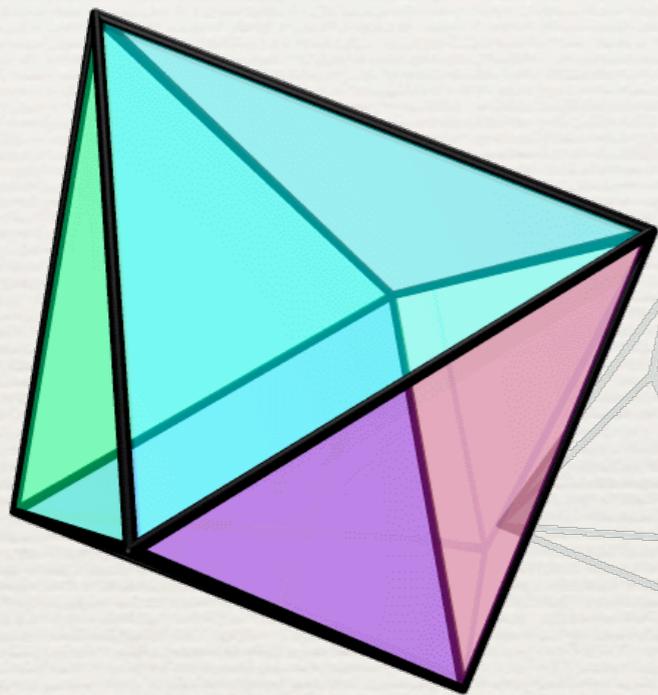
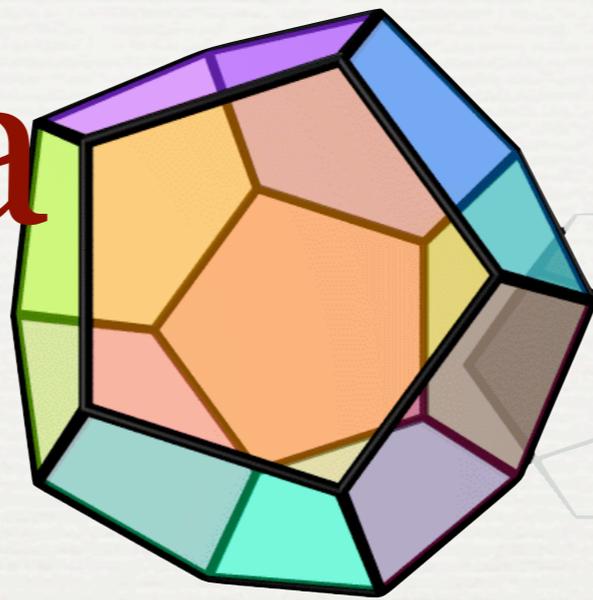




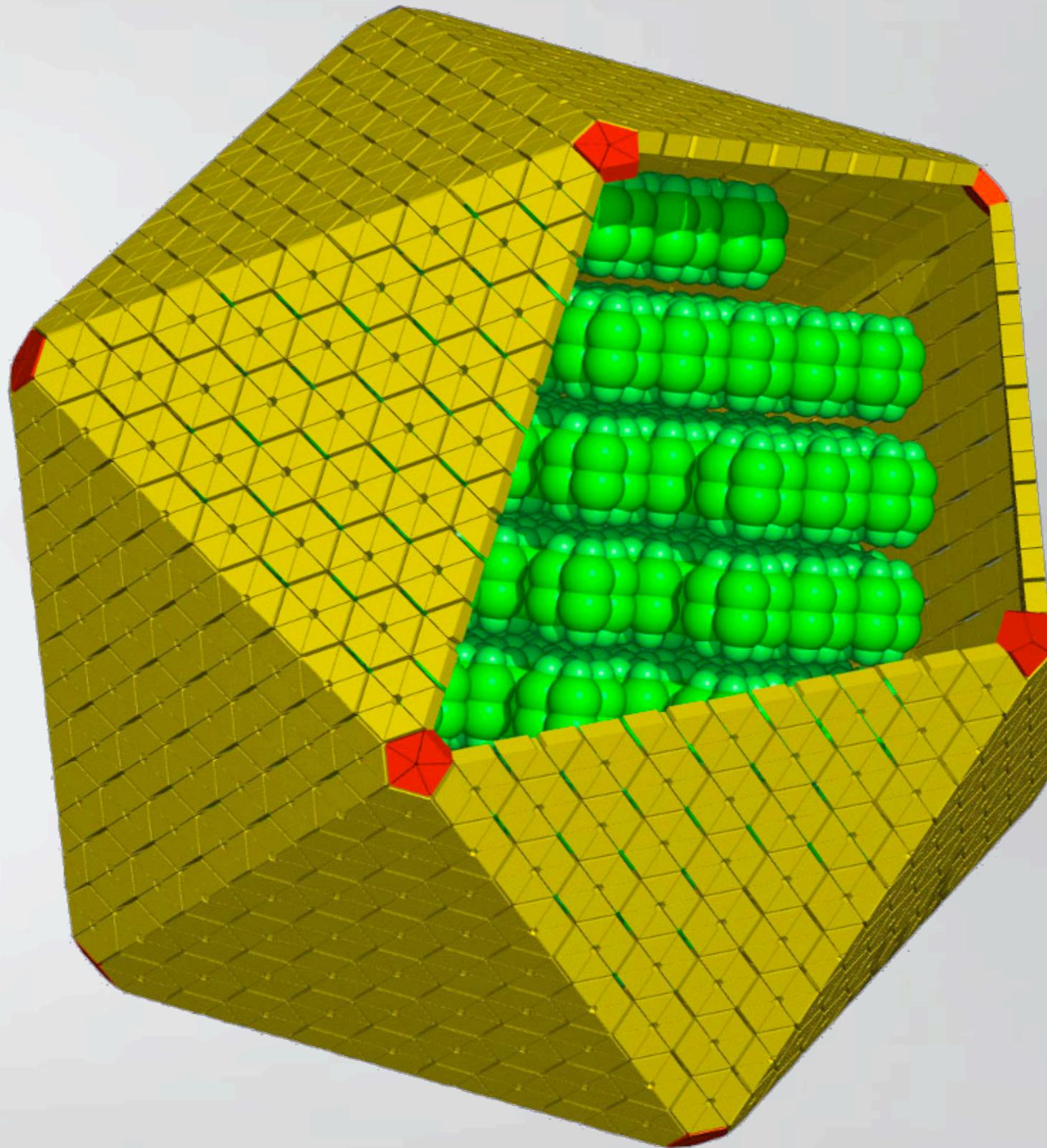


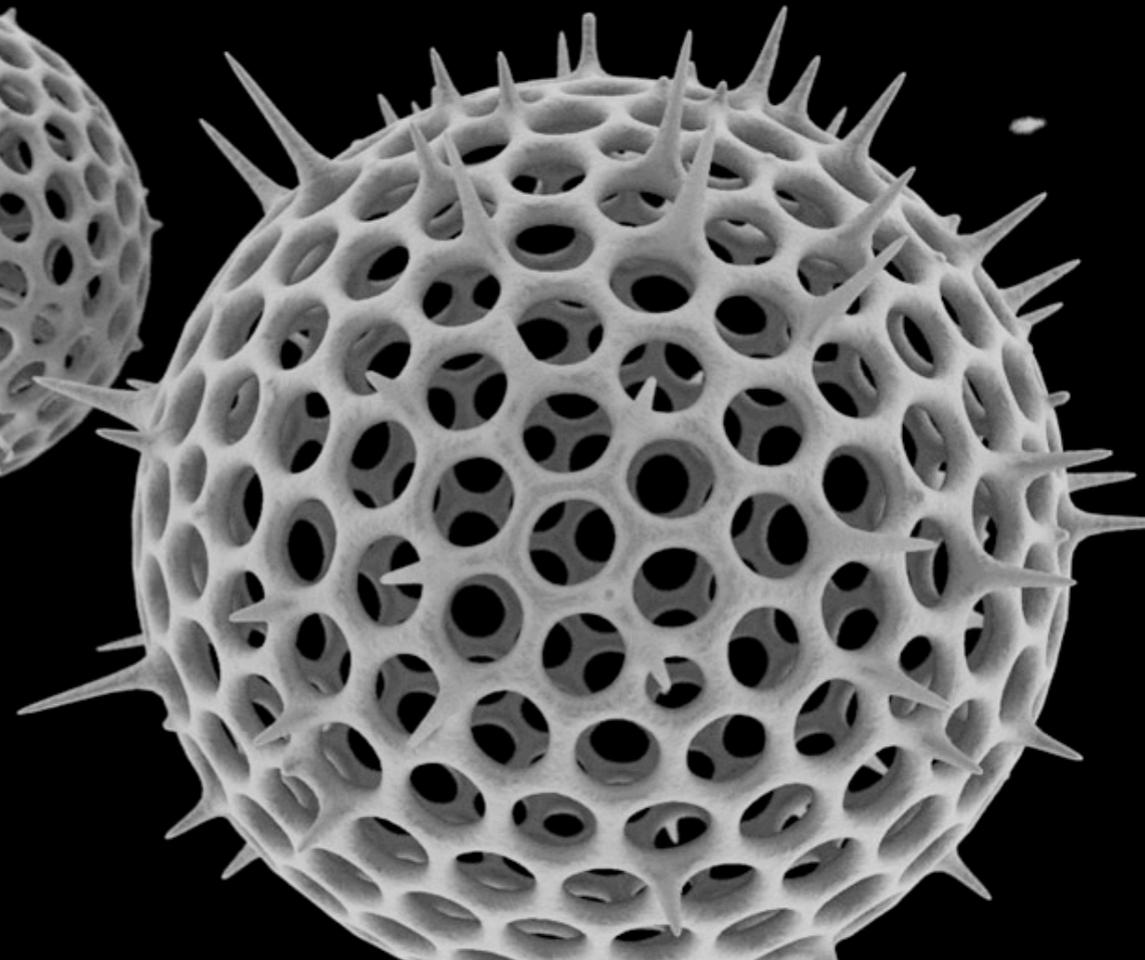
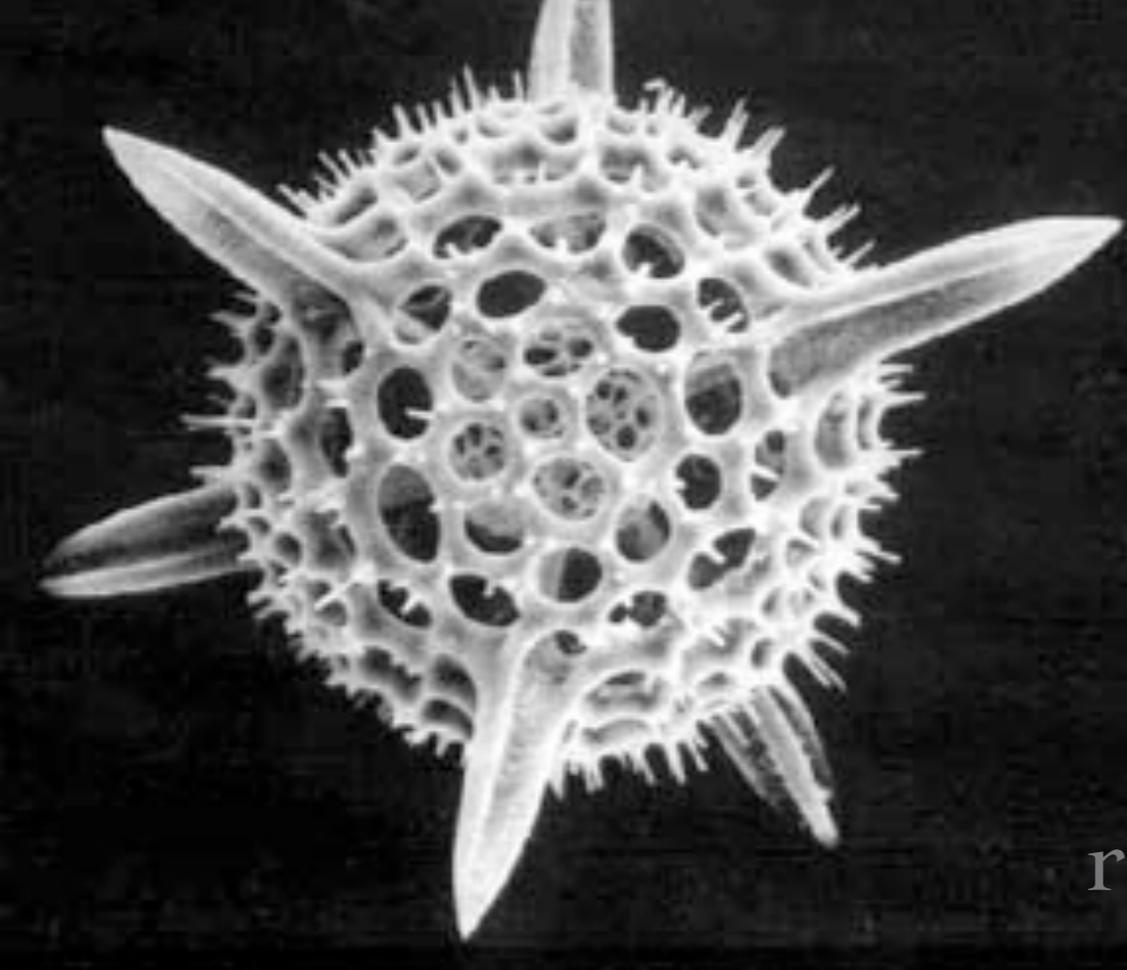
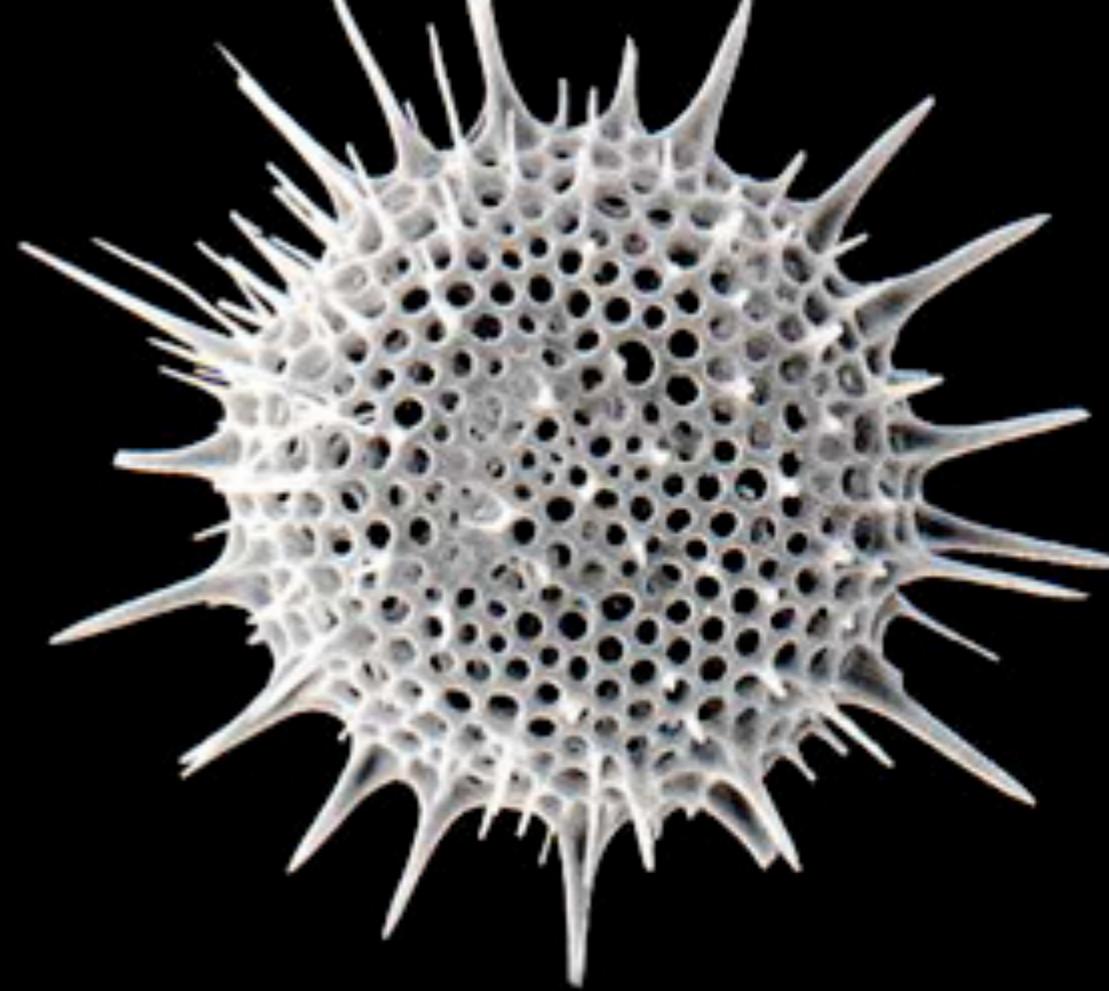
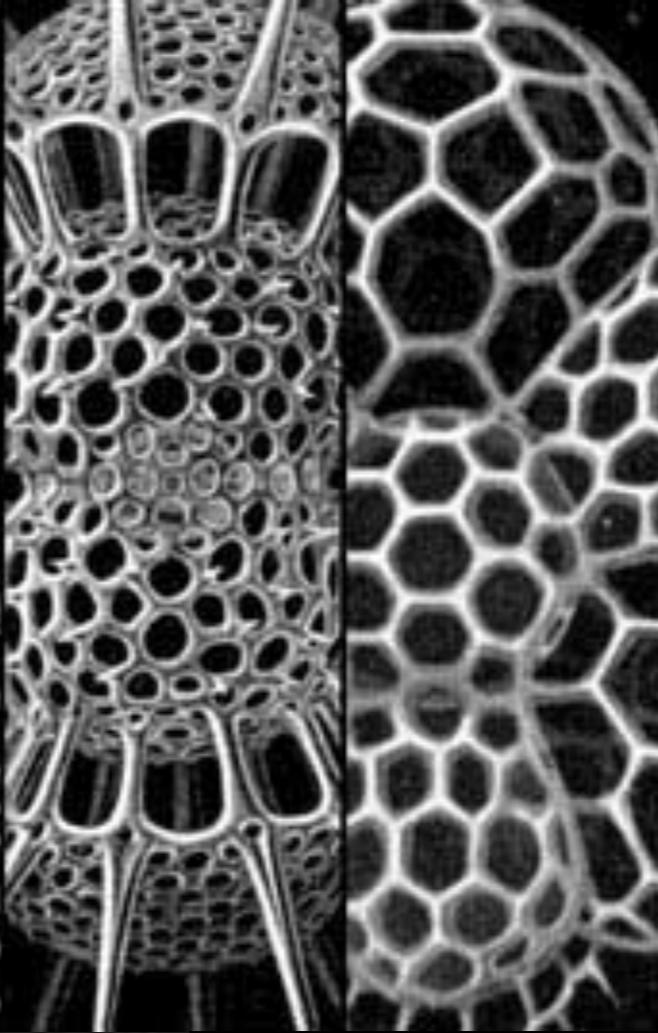
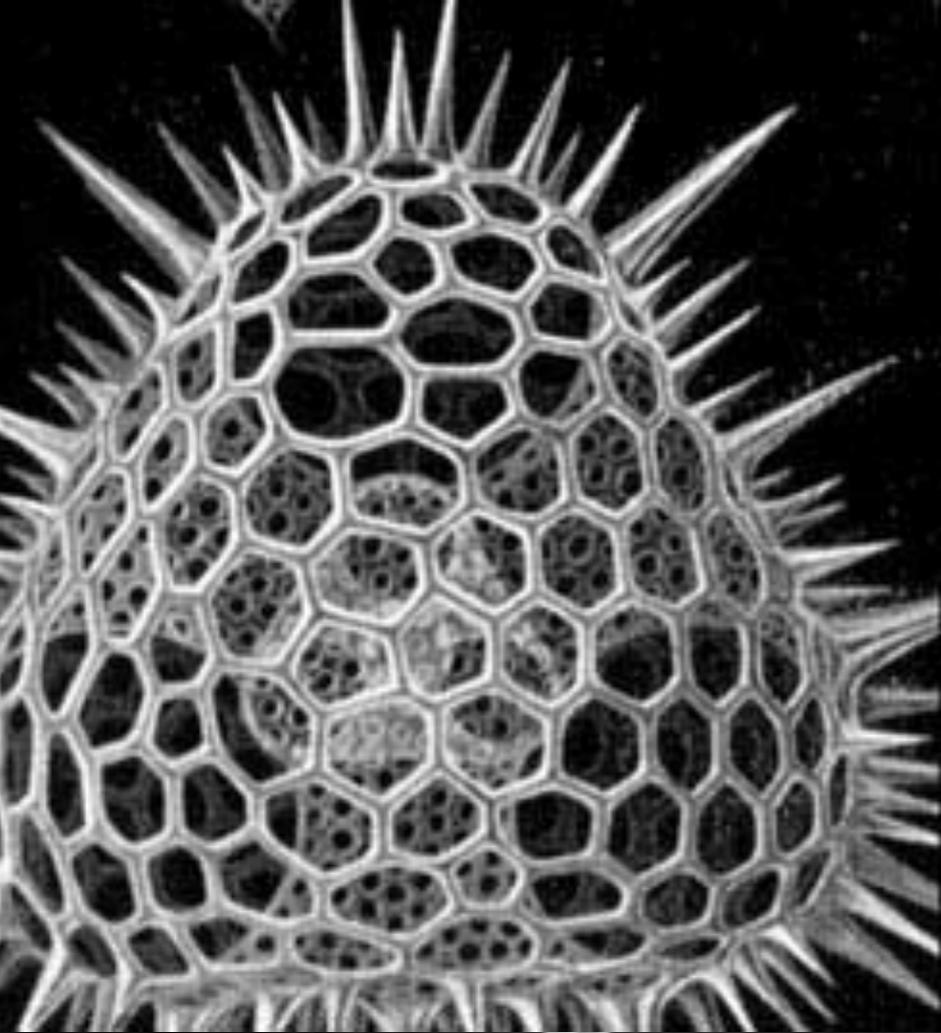
2006, Viel/Lue Harvard

Polyhedra

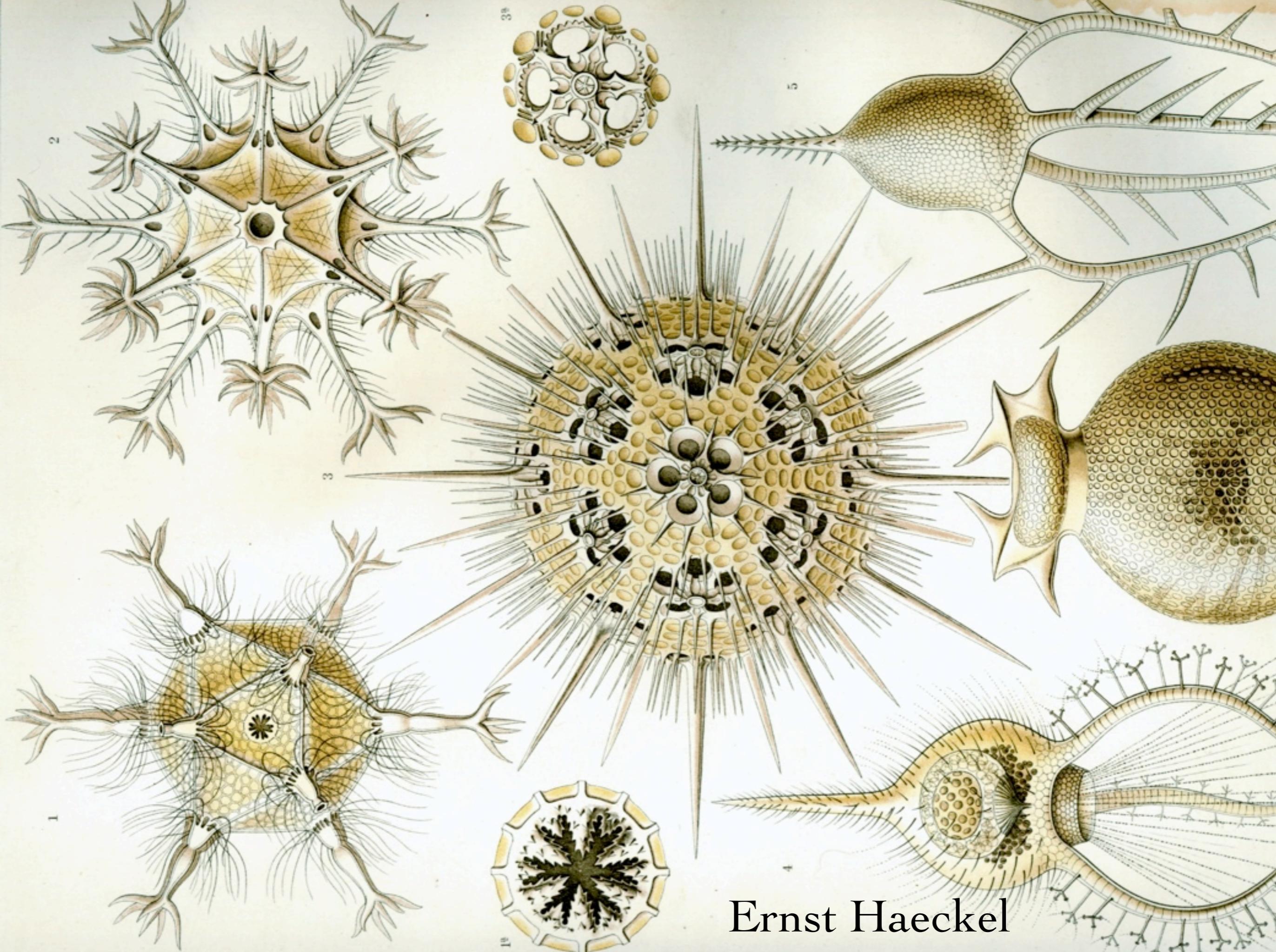


Carboxysomes



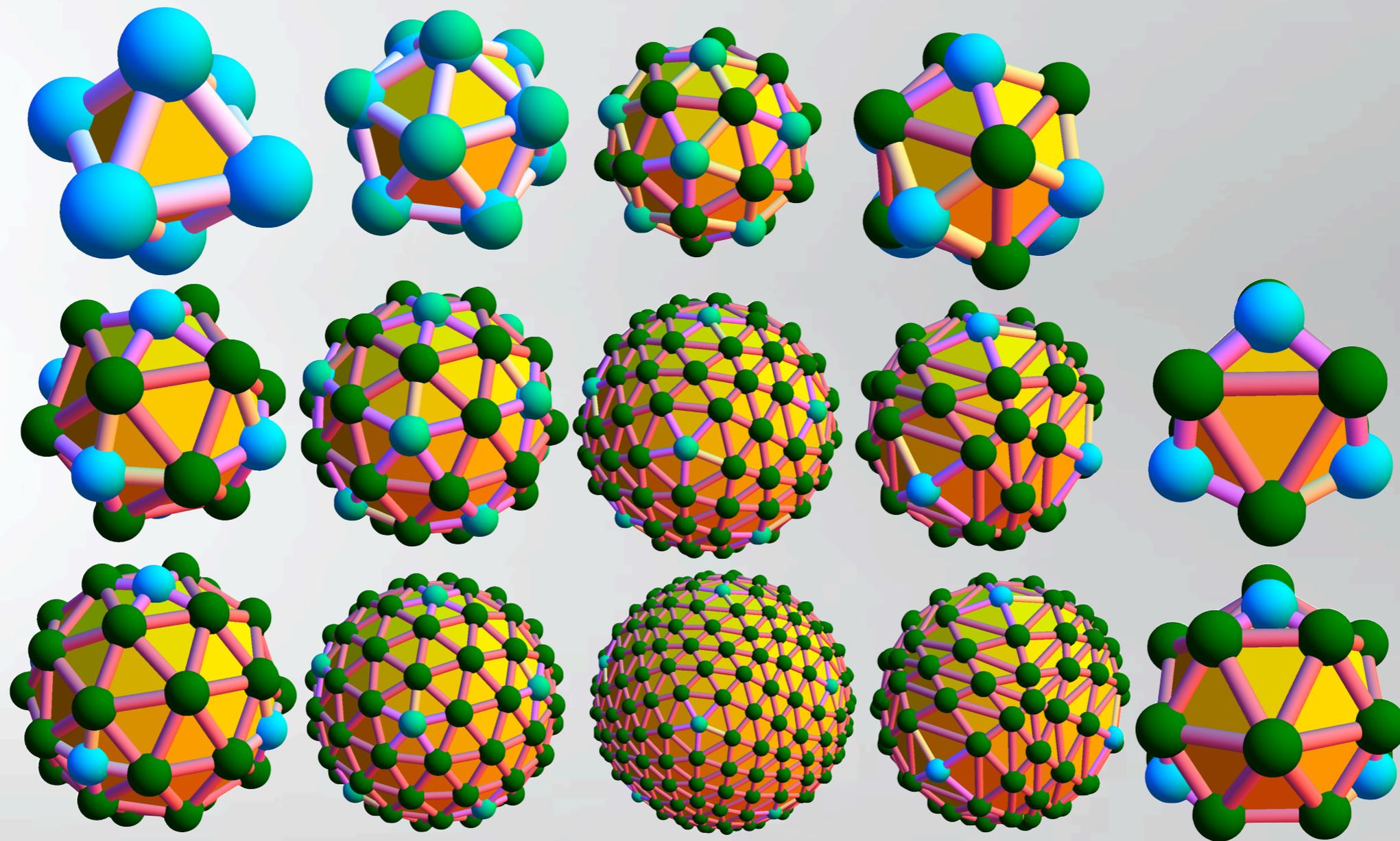


radiolarian

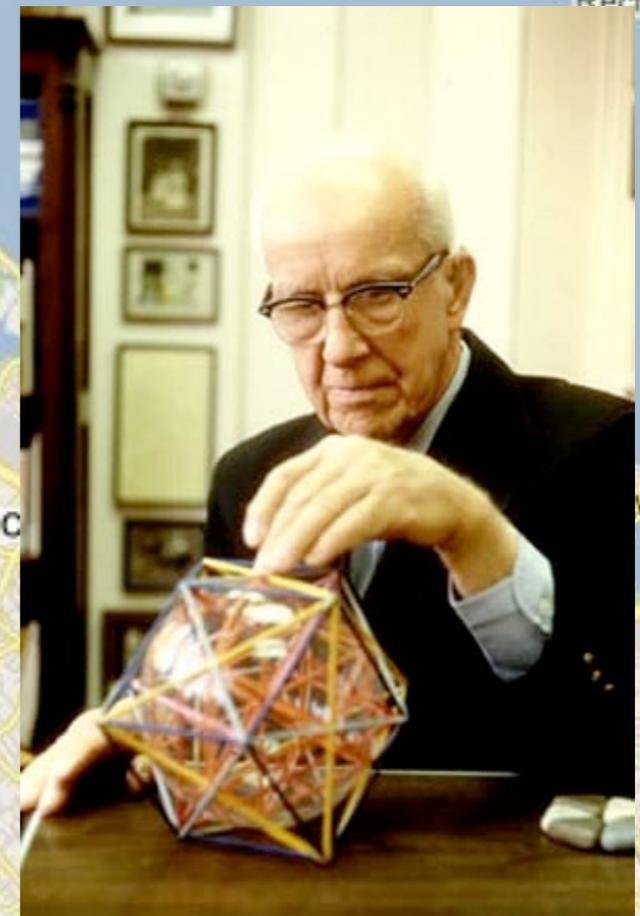
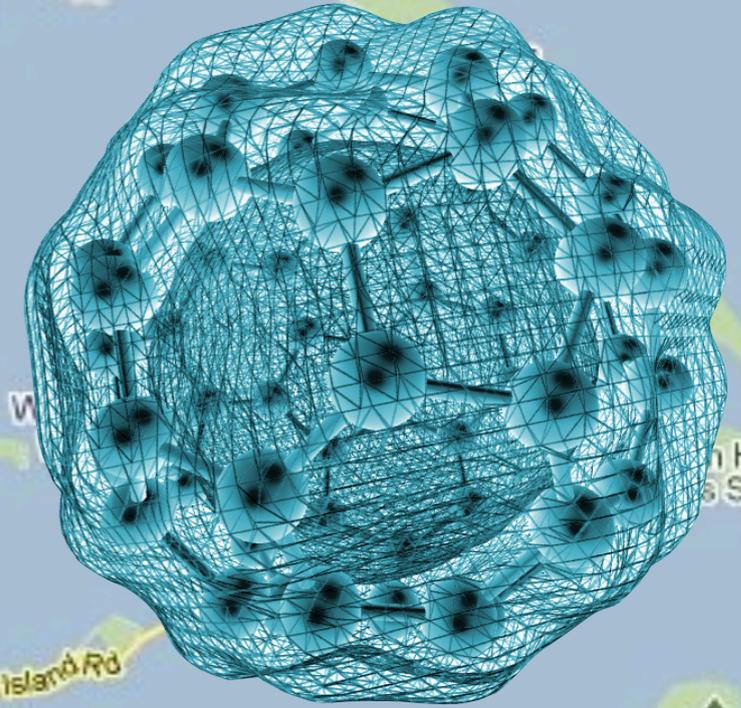


Ernst Haeckel

Fullerene



Buckminster Fuller



R. Buckminster-
Fuller 1895-1983

Arcitecture

Montreal
biosphere



Zeil Frankfurt

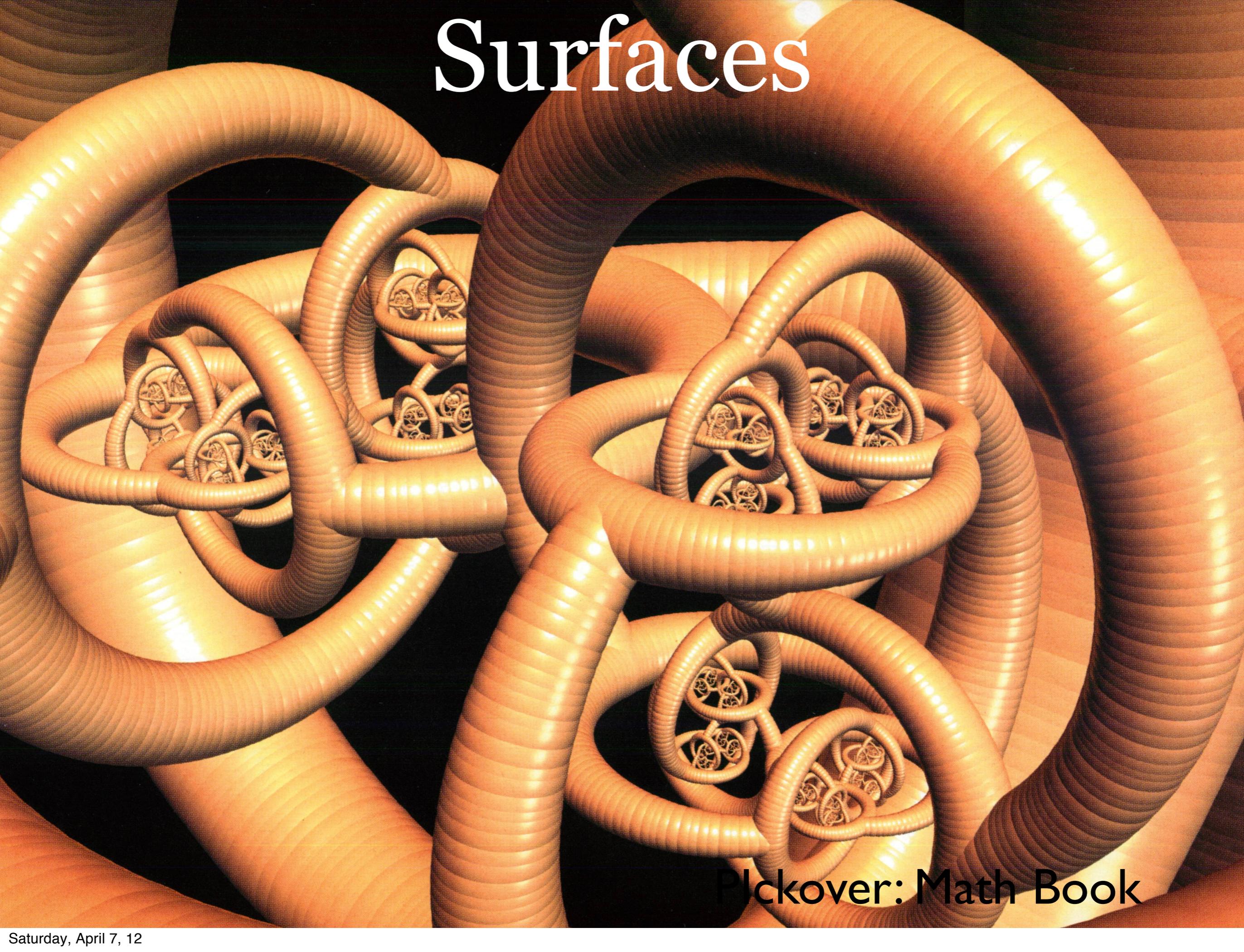




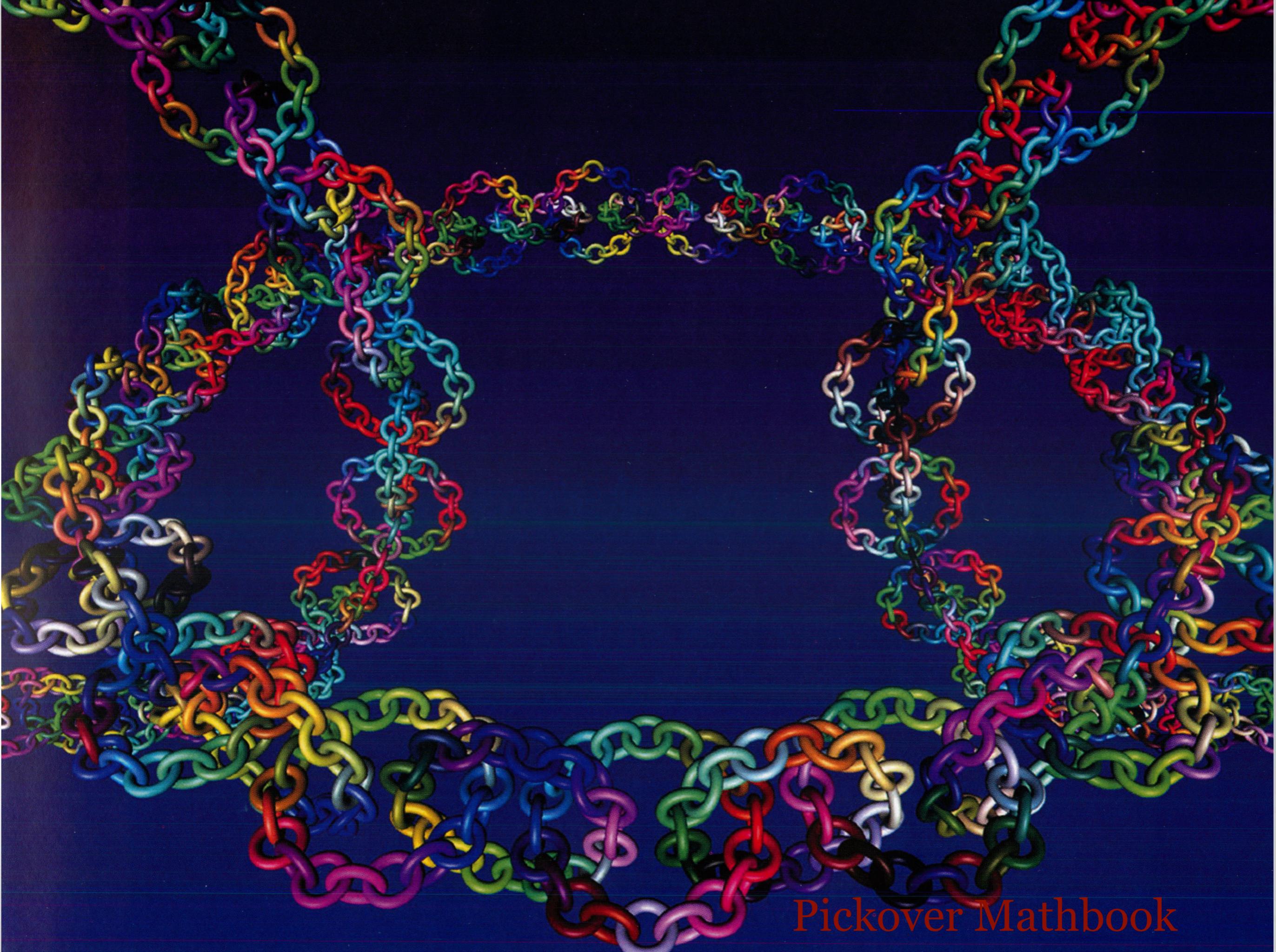


Michael Hansmeyer

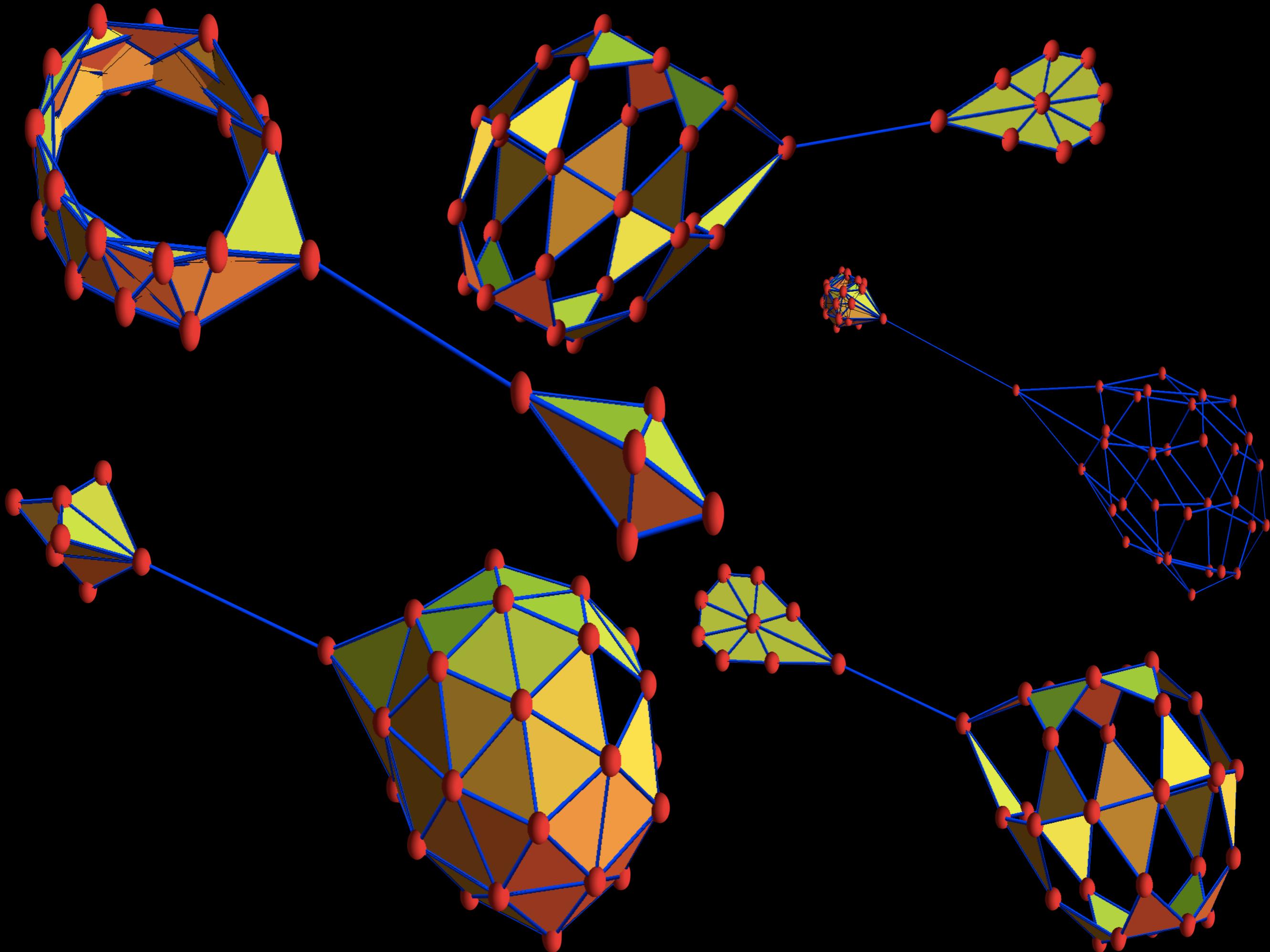
Surfaces

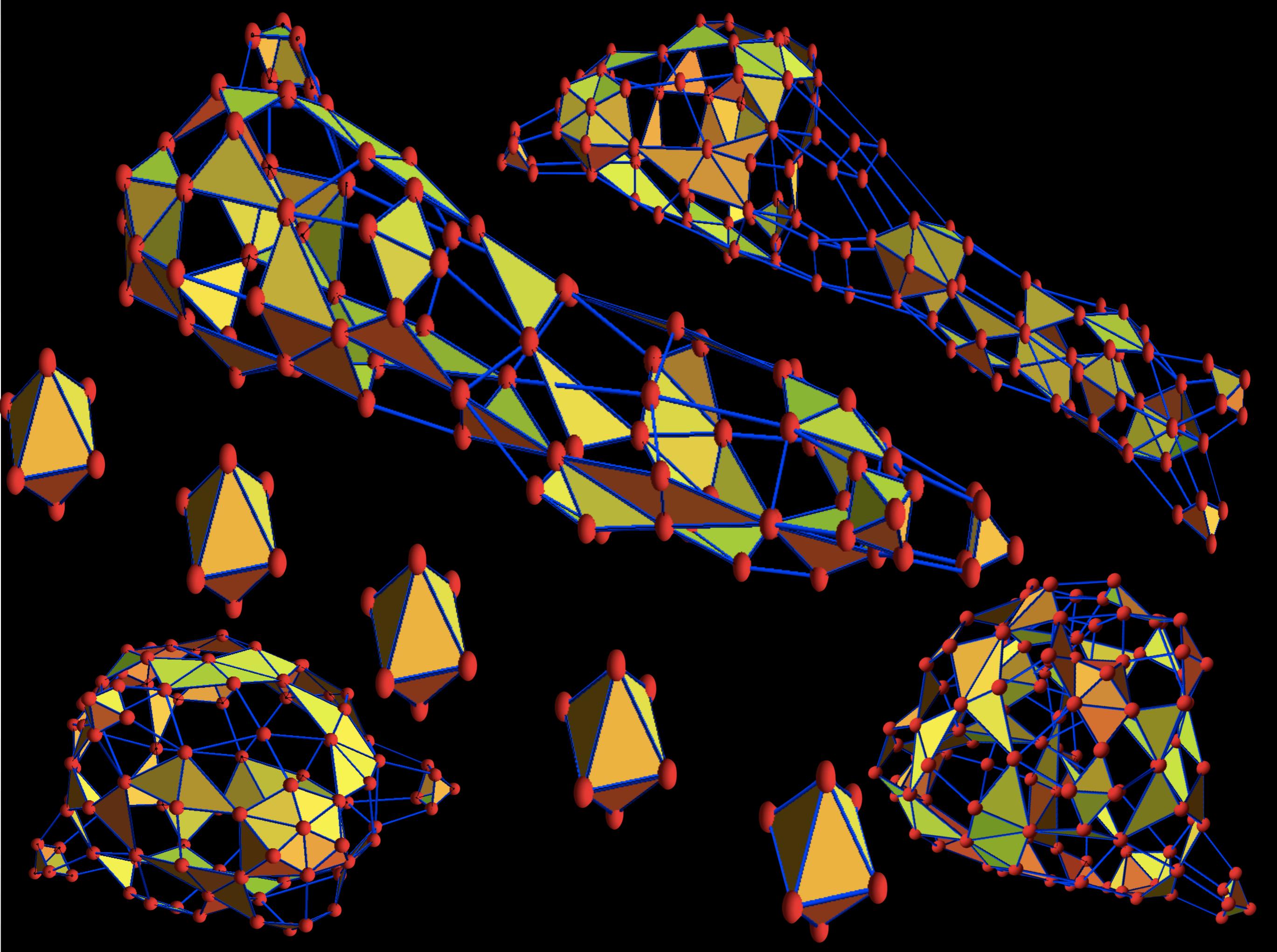


Clickover: Math Book

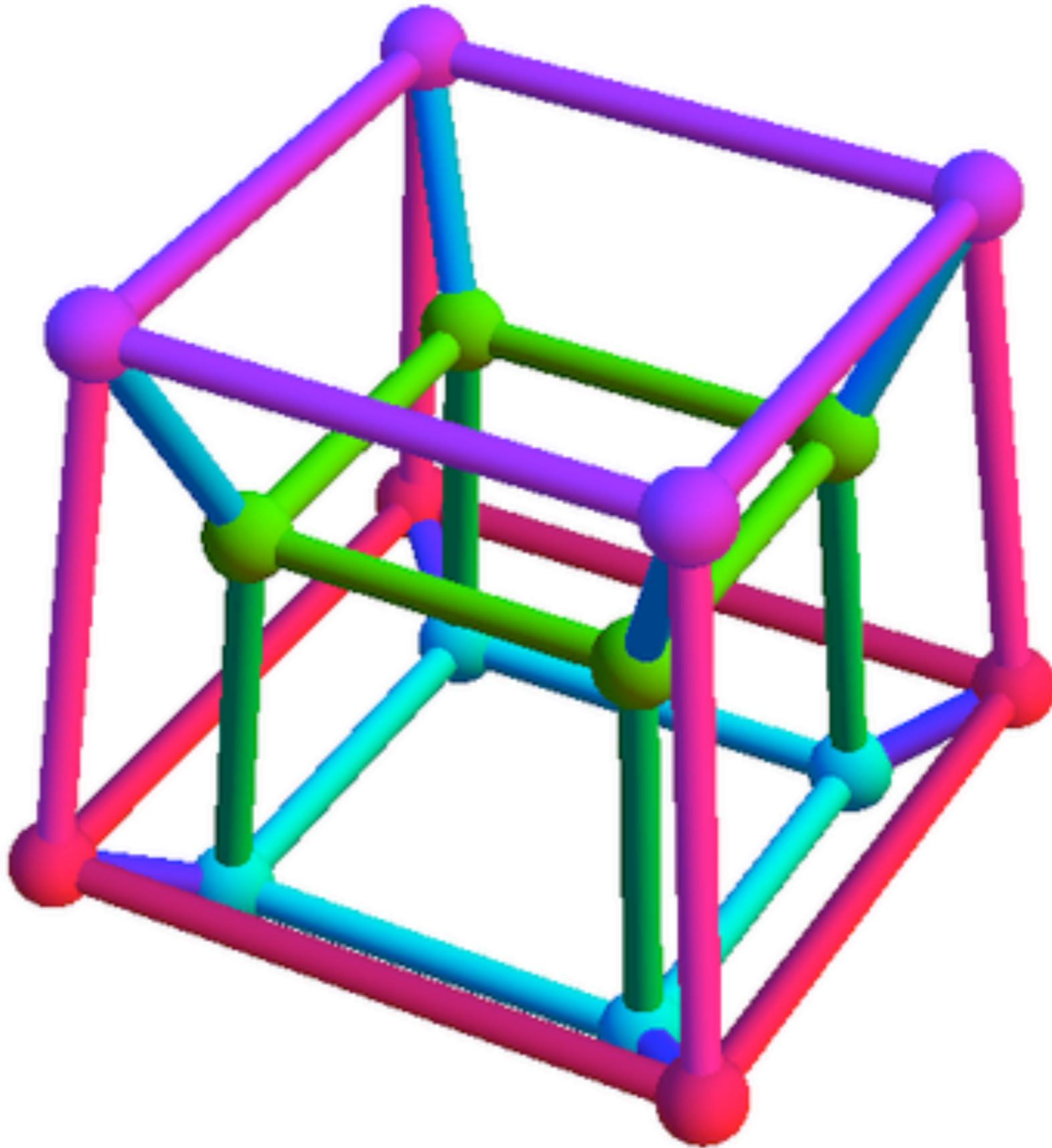


Pickover Mathbook

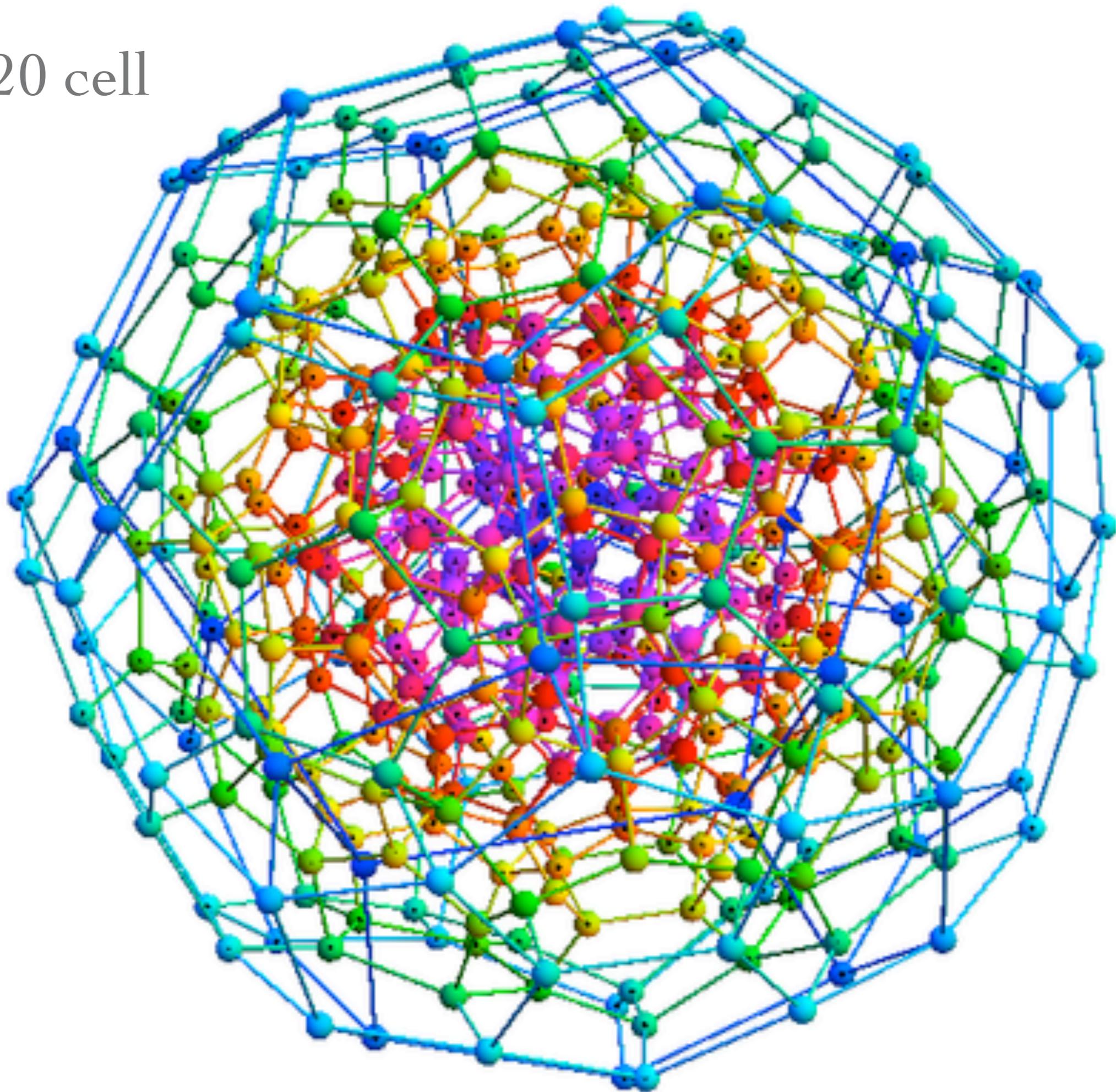


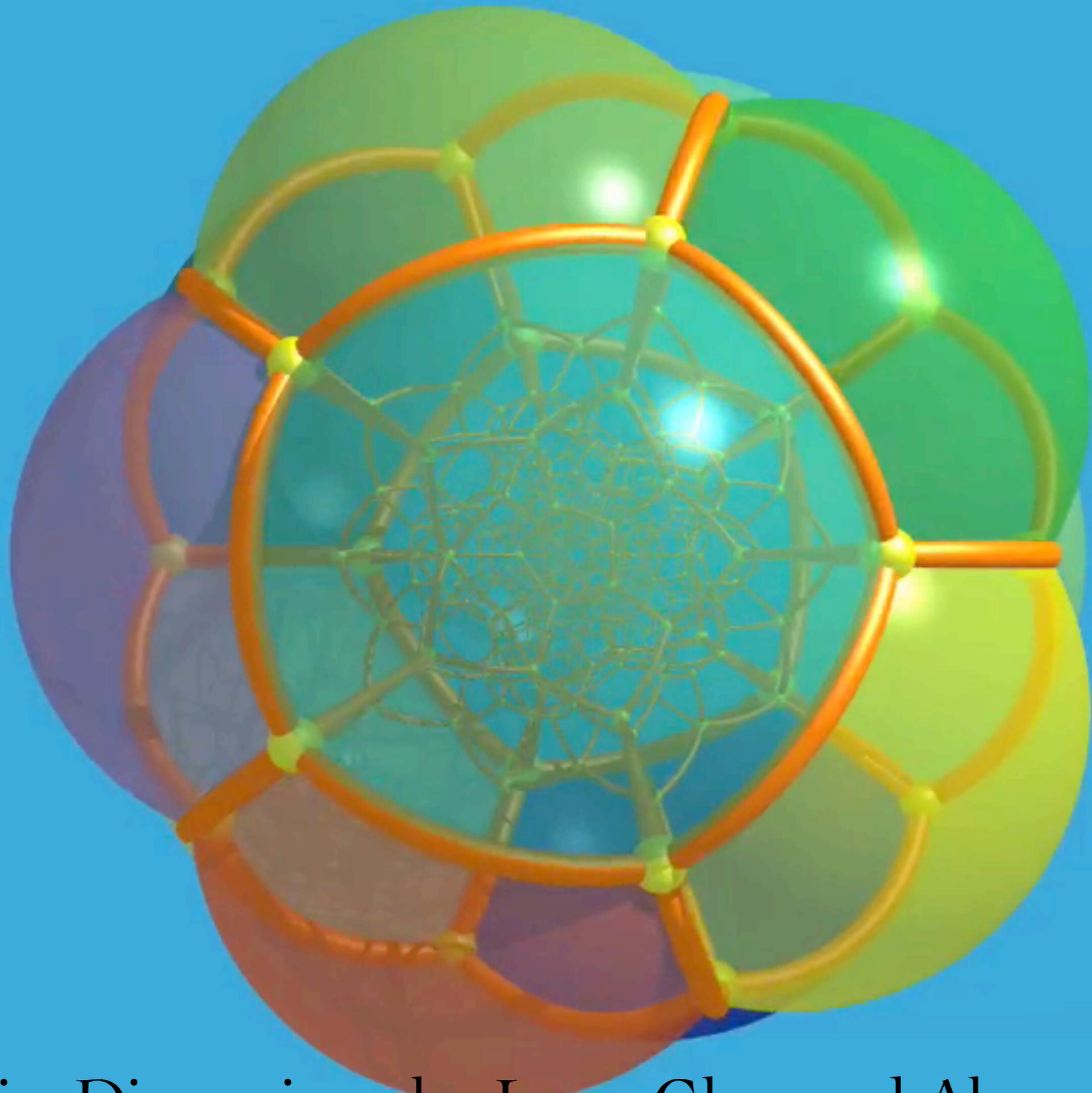


8 cell
tesseract

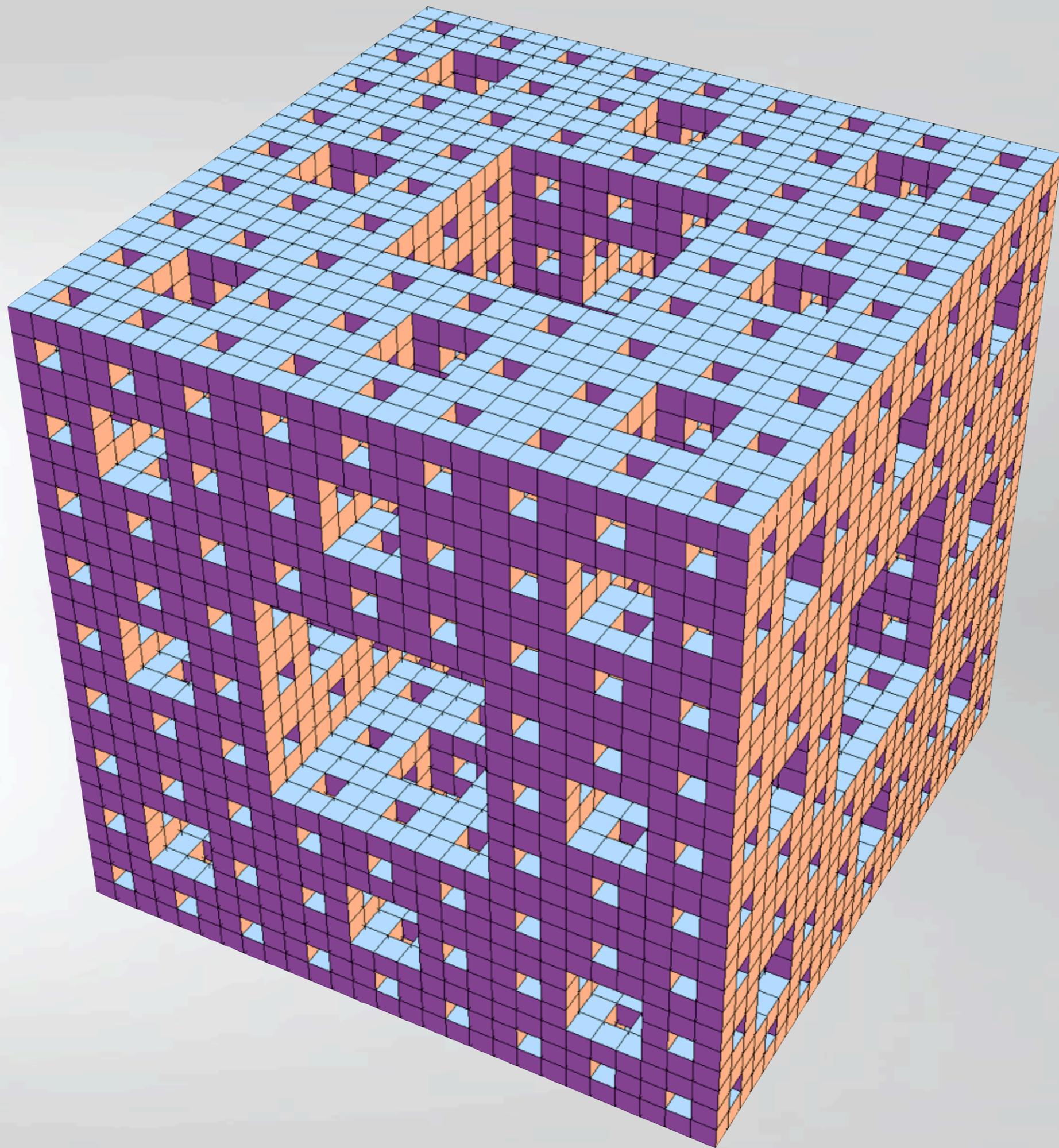


120 cell

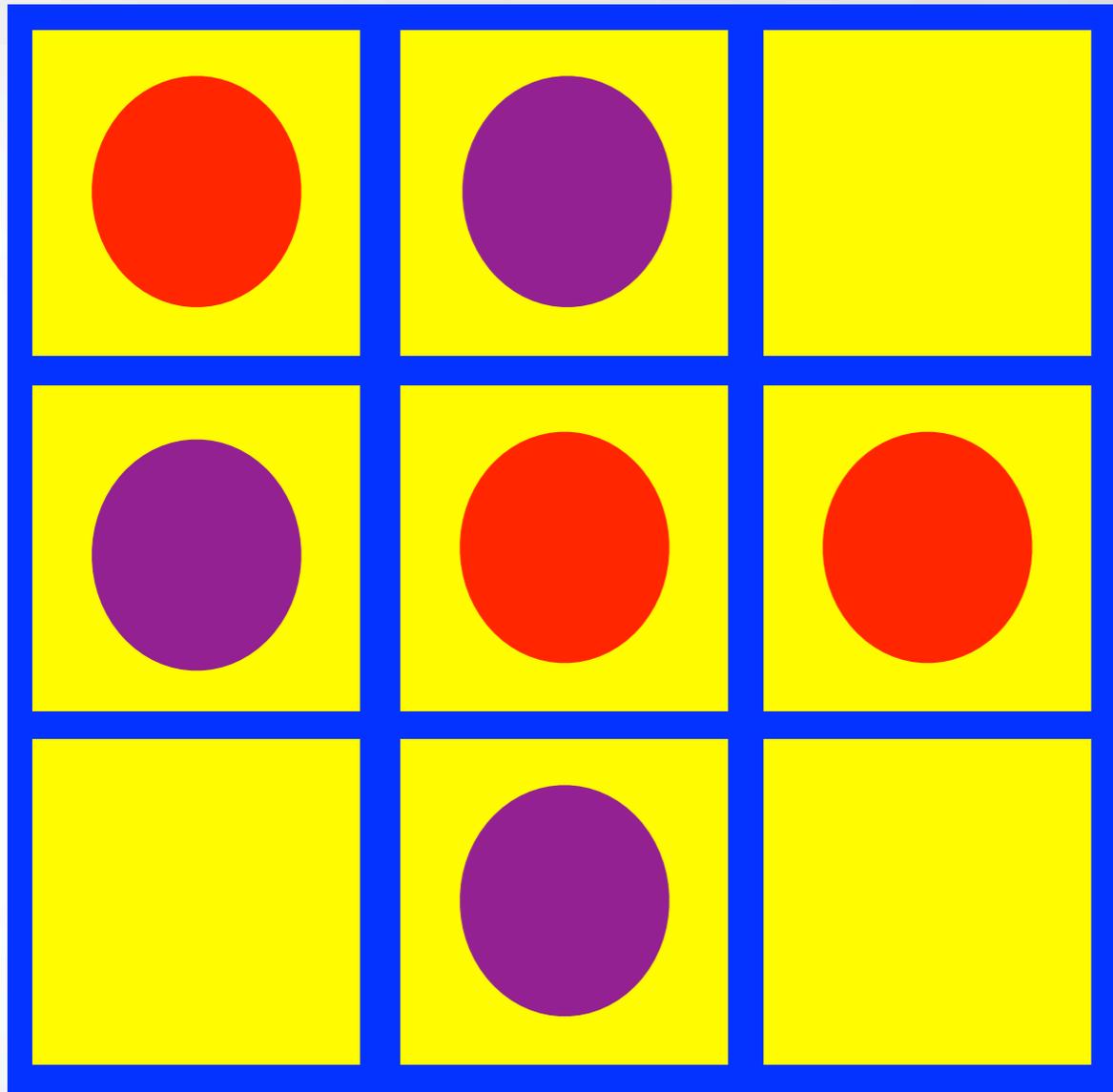




Movie: Dimension, by Leys, Ghys and Alvarez, 2008



Games



Tic Tac Toe



Chess

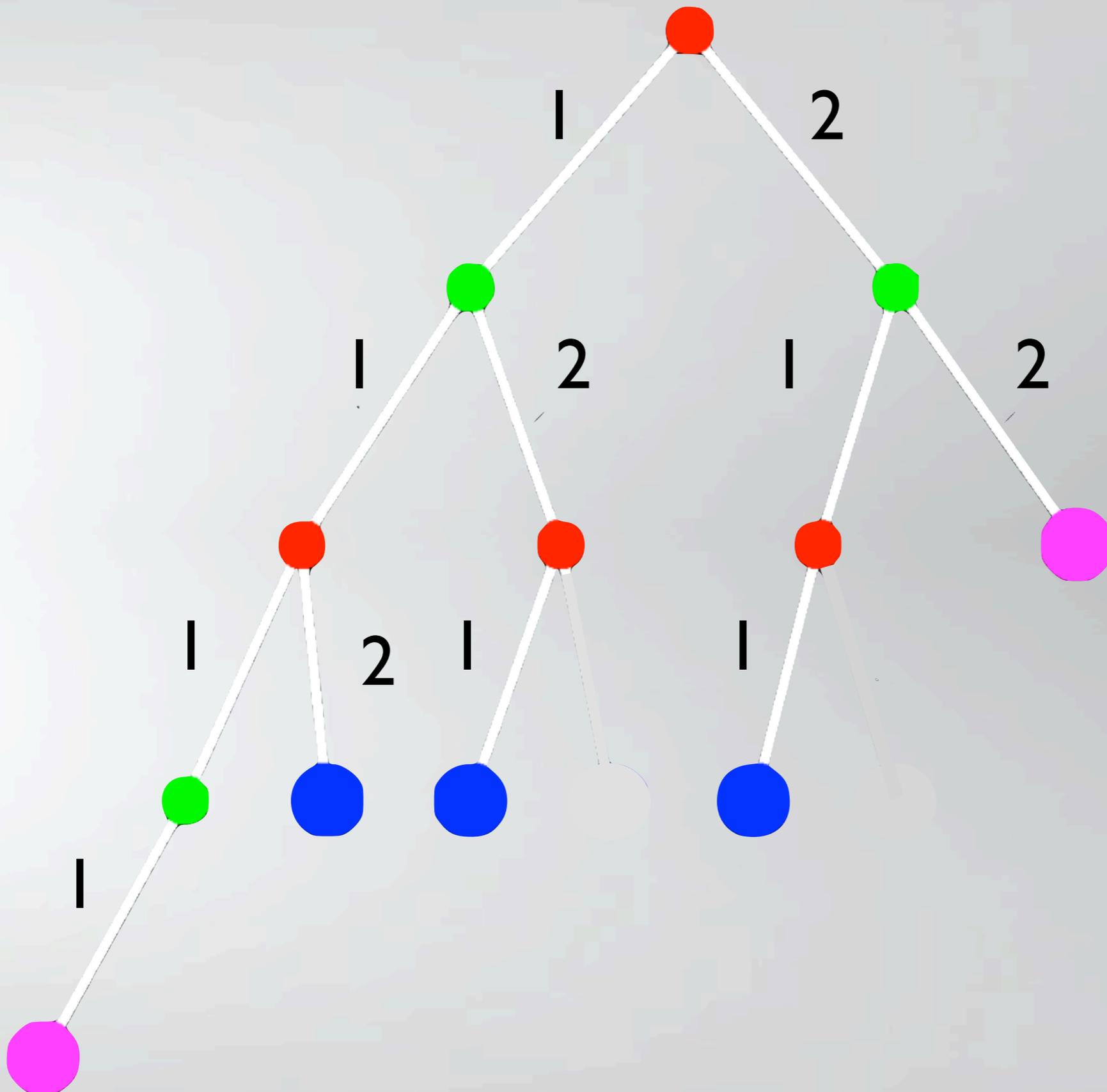
Games: 4 wins



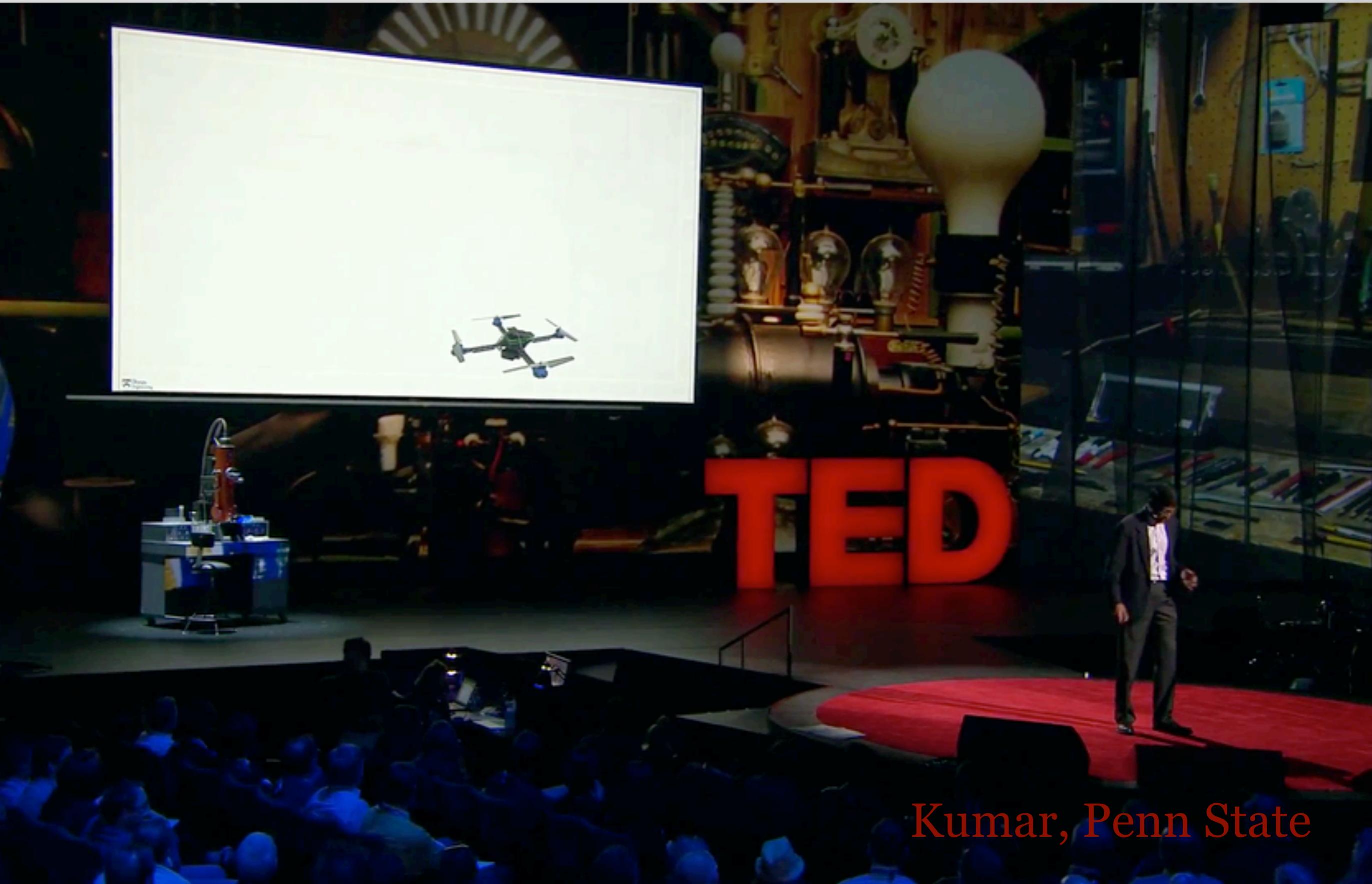
Rules: Each player picks one or two. The one who gets the last one, wins.

NIM: taking the last, loses

The graph

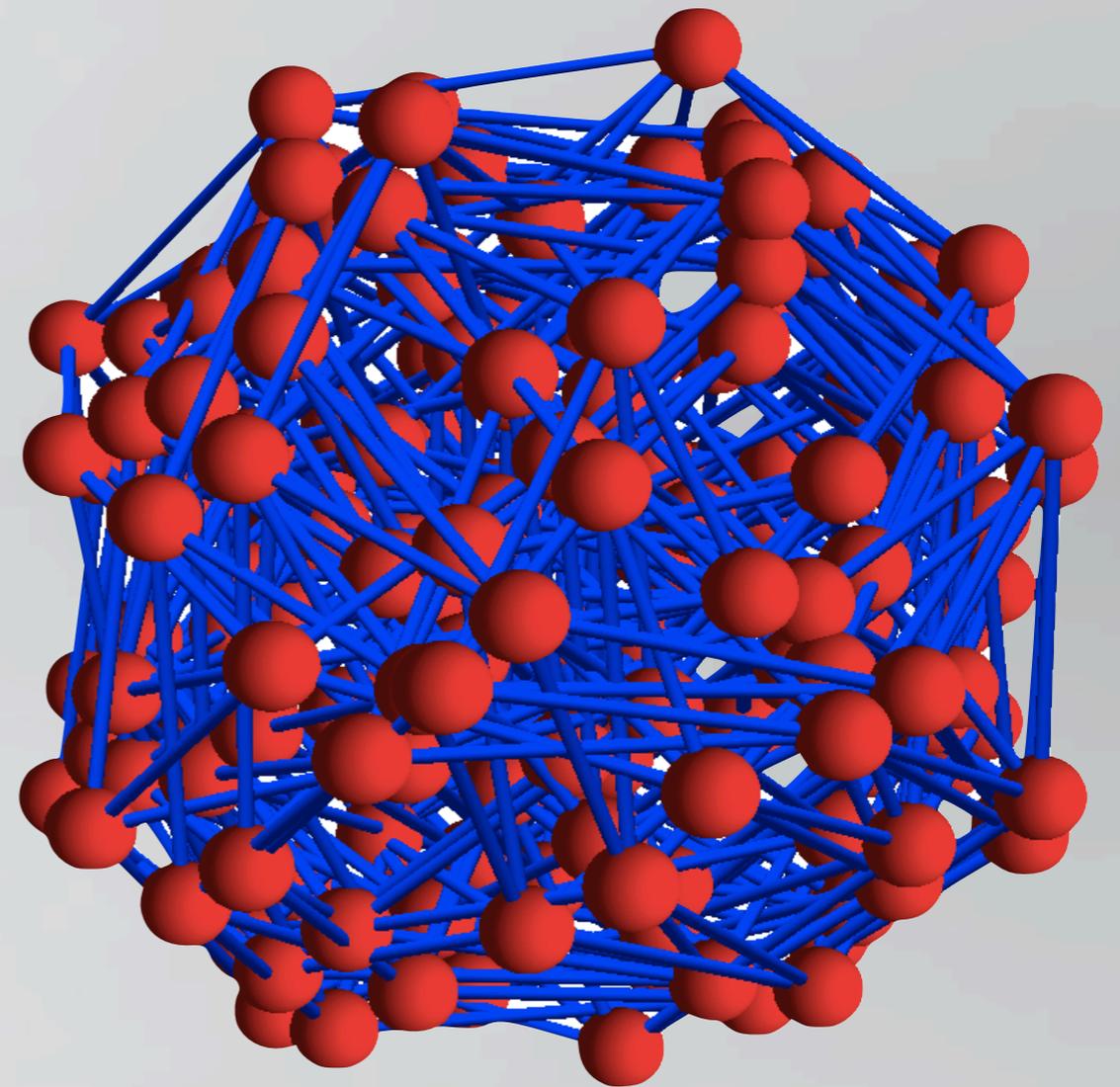
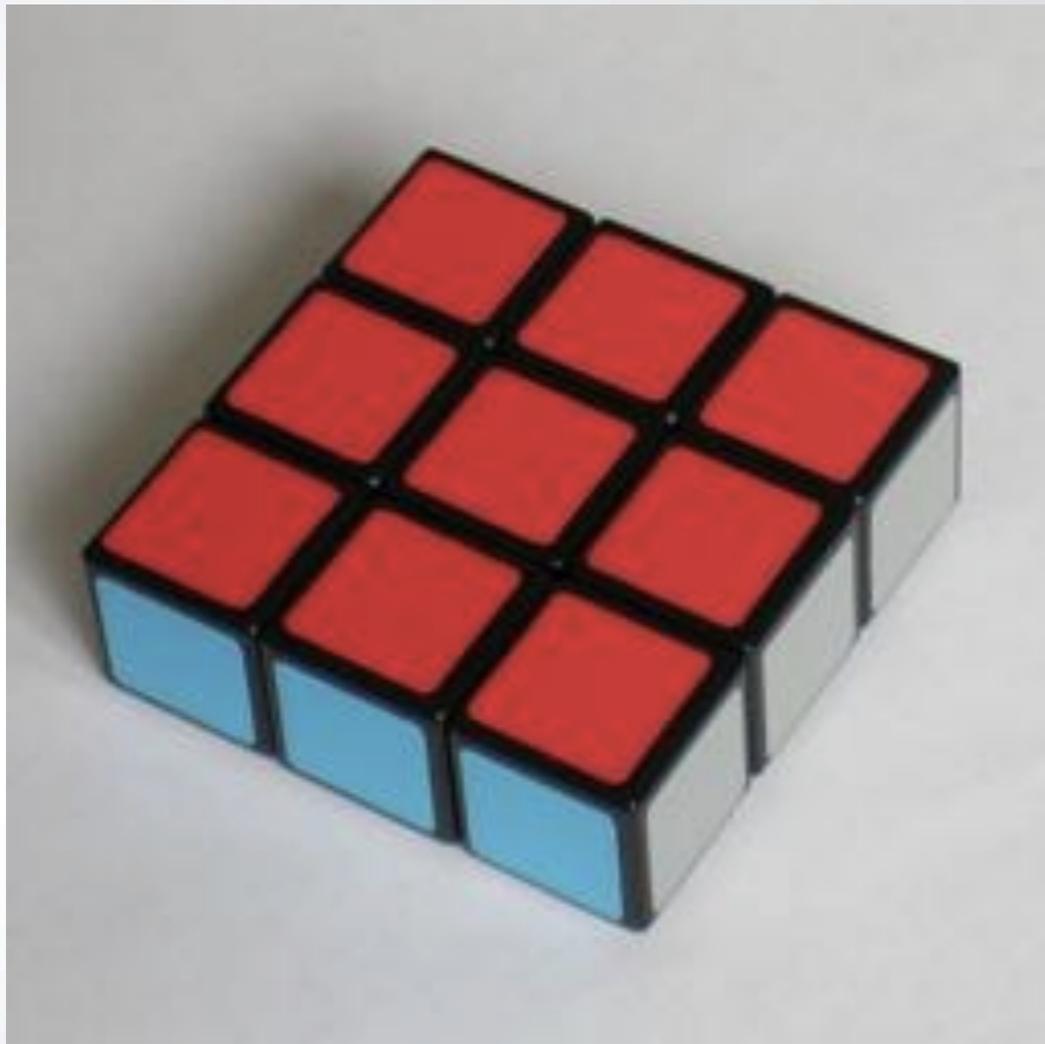


Swarm intelligence



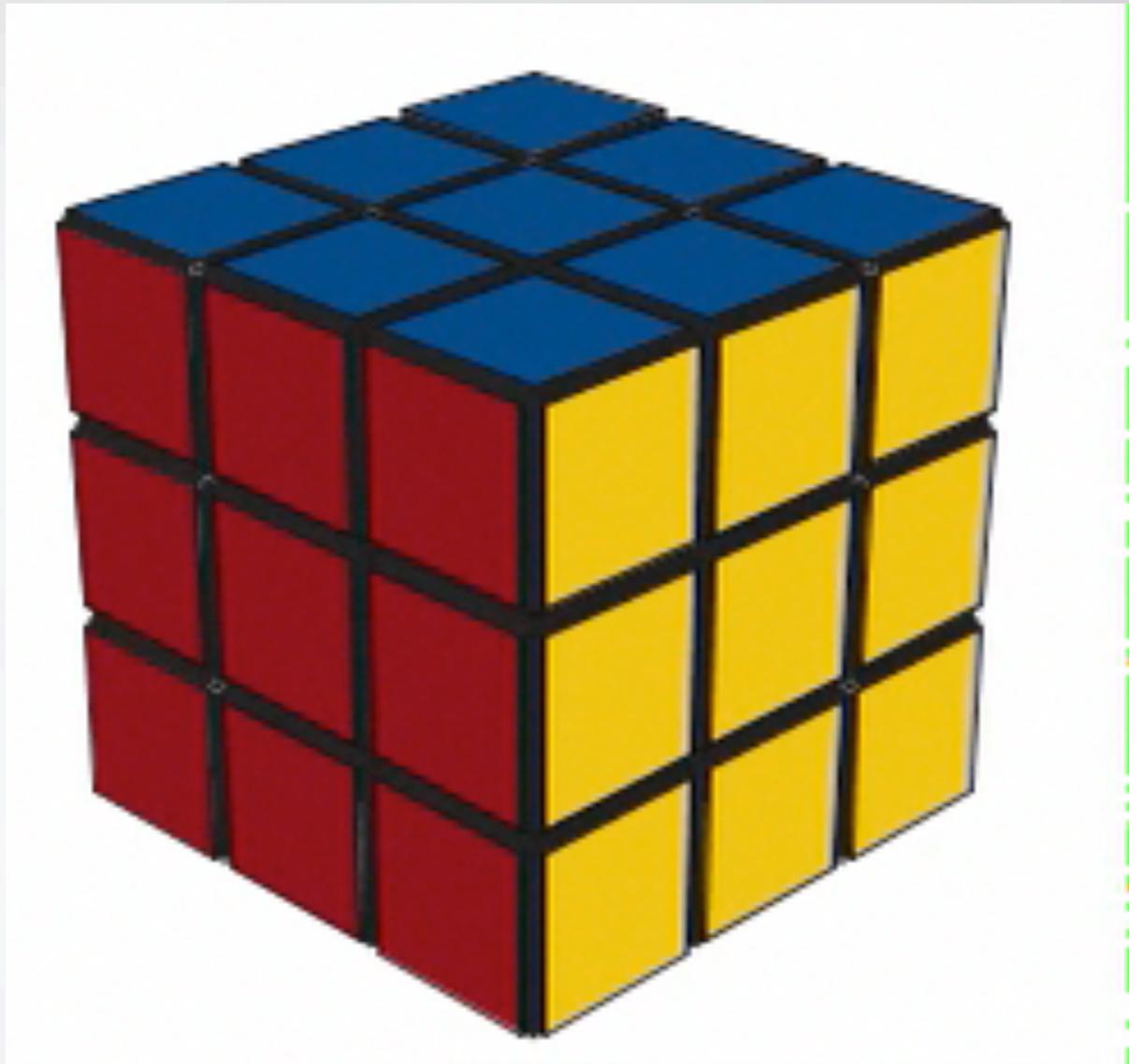
Kumar, Penn State

Puzzles



Floppy

Rubik



The diameter
of the Rubik
network is 20

Superflip

God Number (cube20.org)

What is Geometry?



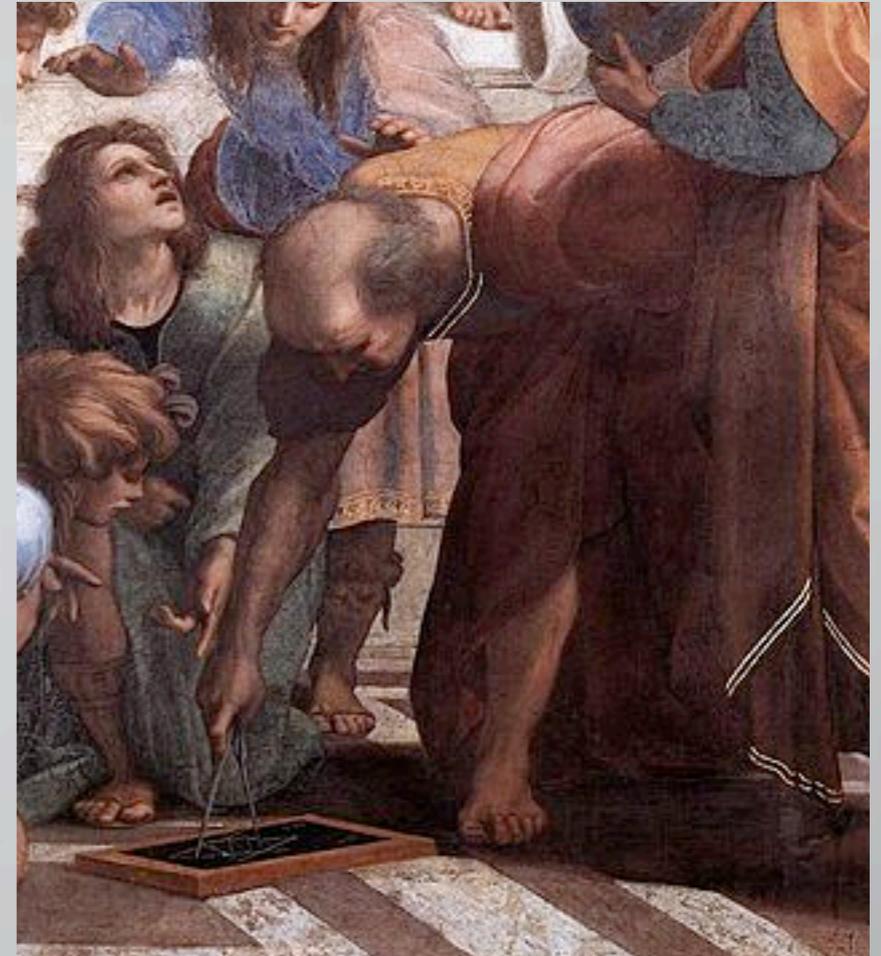
Antiquity



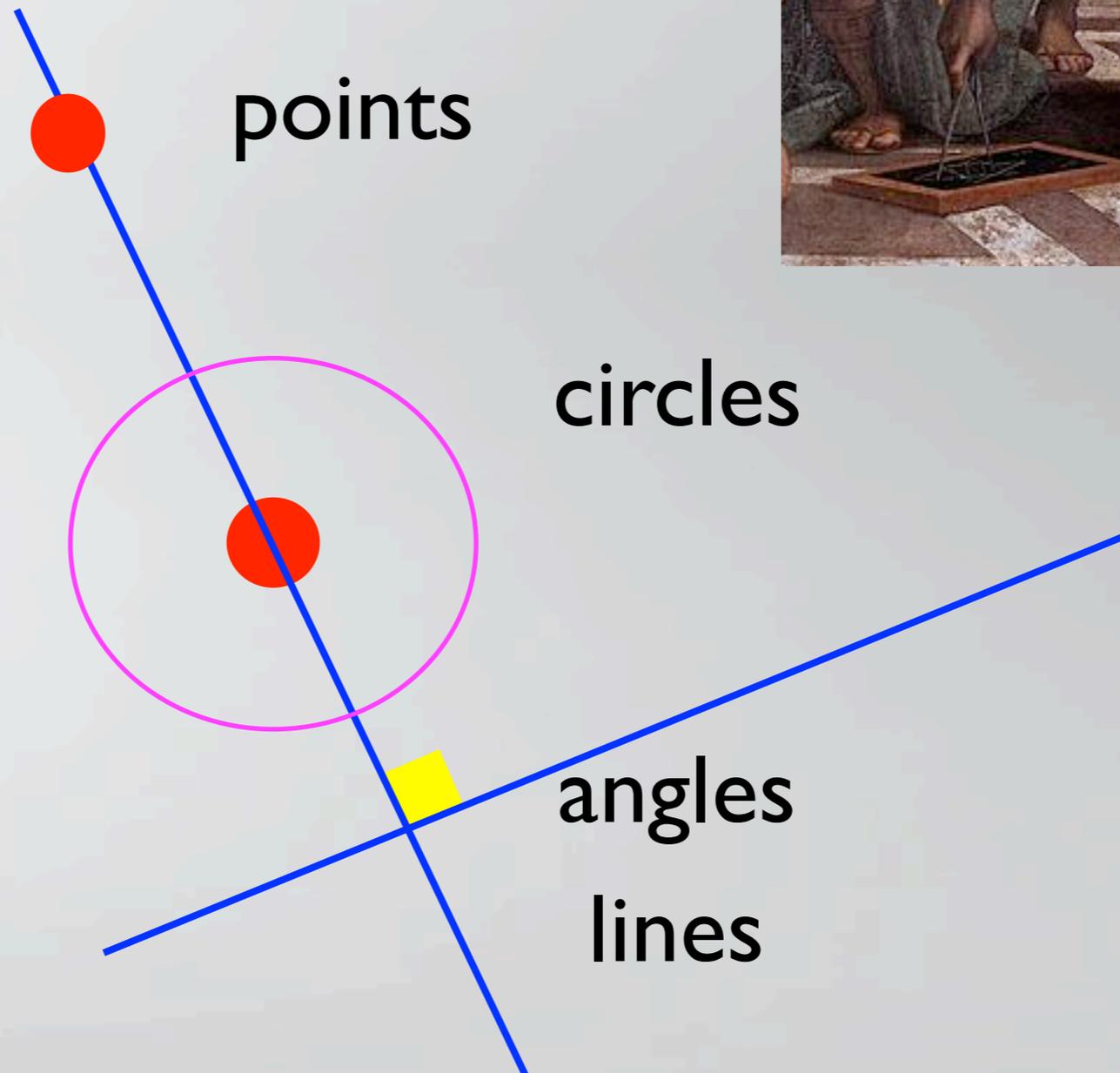
Geodesy. Measure length and area

Euclid

300 BC



Axiomatics of planimetry

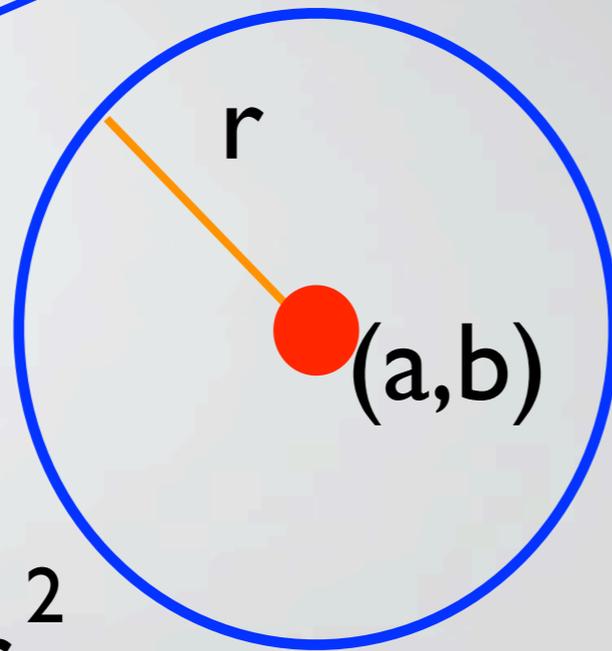


Descartes

1596-1650

Geometric objects can be defined algebraically

$$y=ax+b$$



$$(x-a)^2 + (y-b)^2 = r^2$$



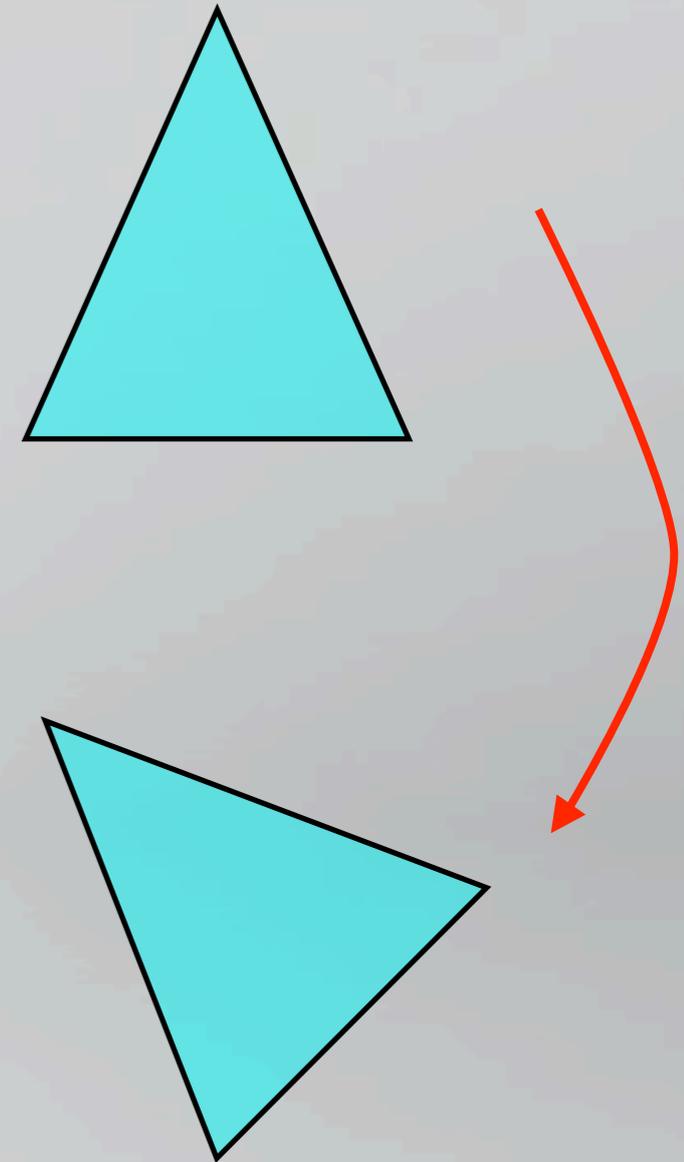
Klein 1849-1925

Geometry is defined by the group of symmetries of the Geometry.



examples:

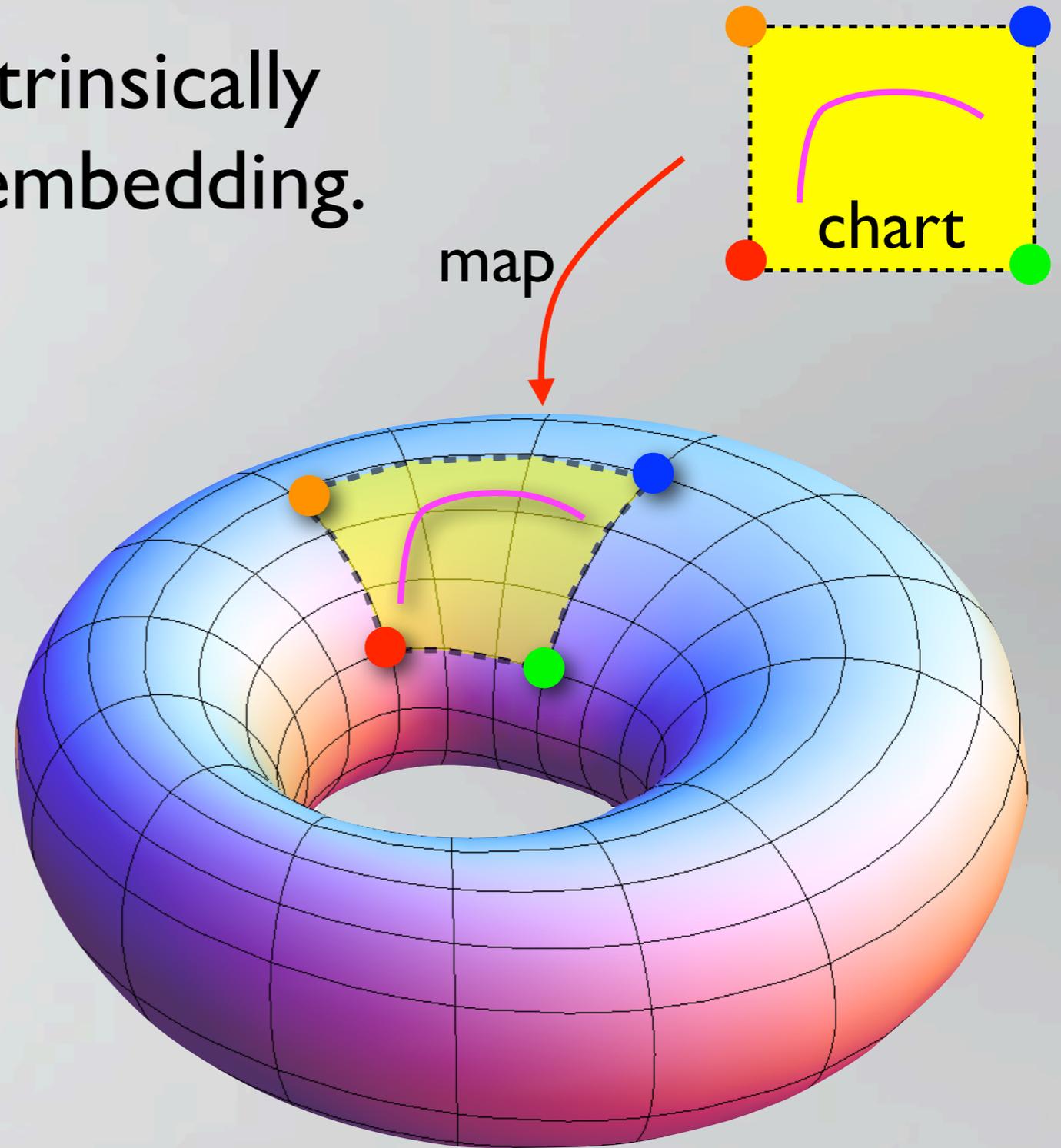
Conformal geometry
Affine geometry
Projective Geometry
Topology
Differential geometry



Riemann

1826-1866

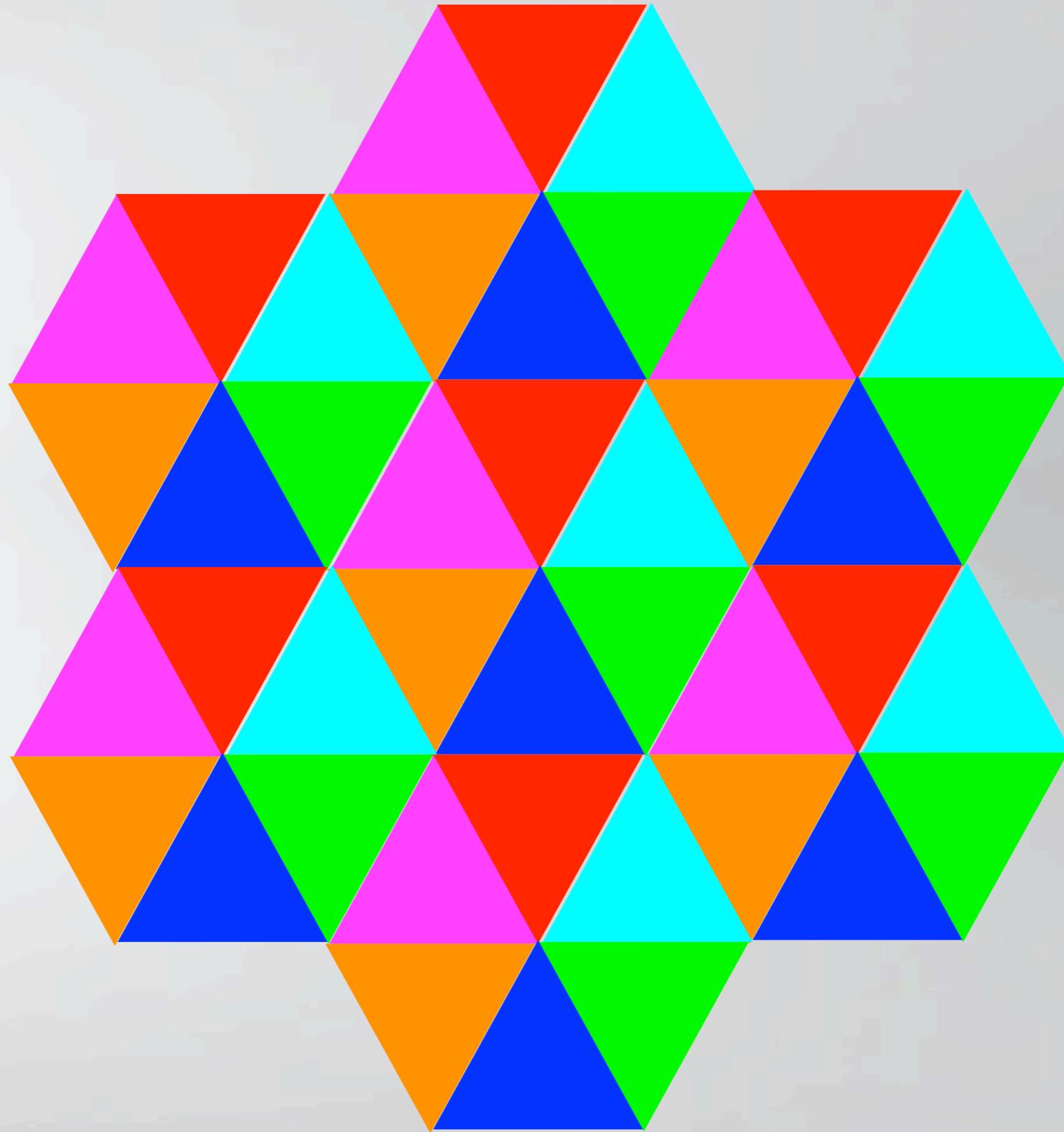
Riemannian manifold. Intrinsically defined metric without embedding.

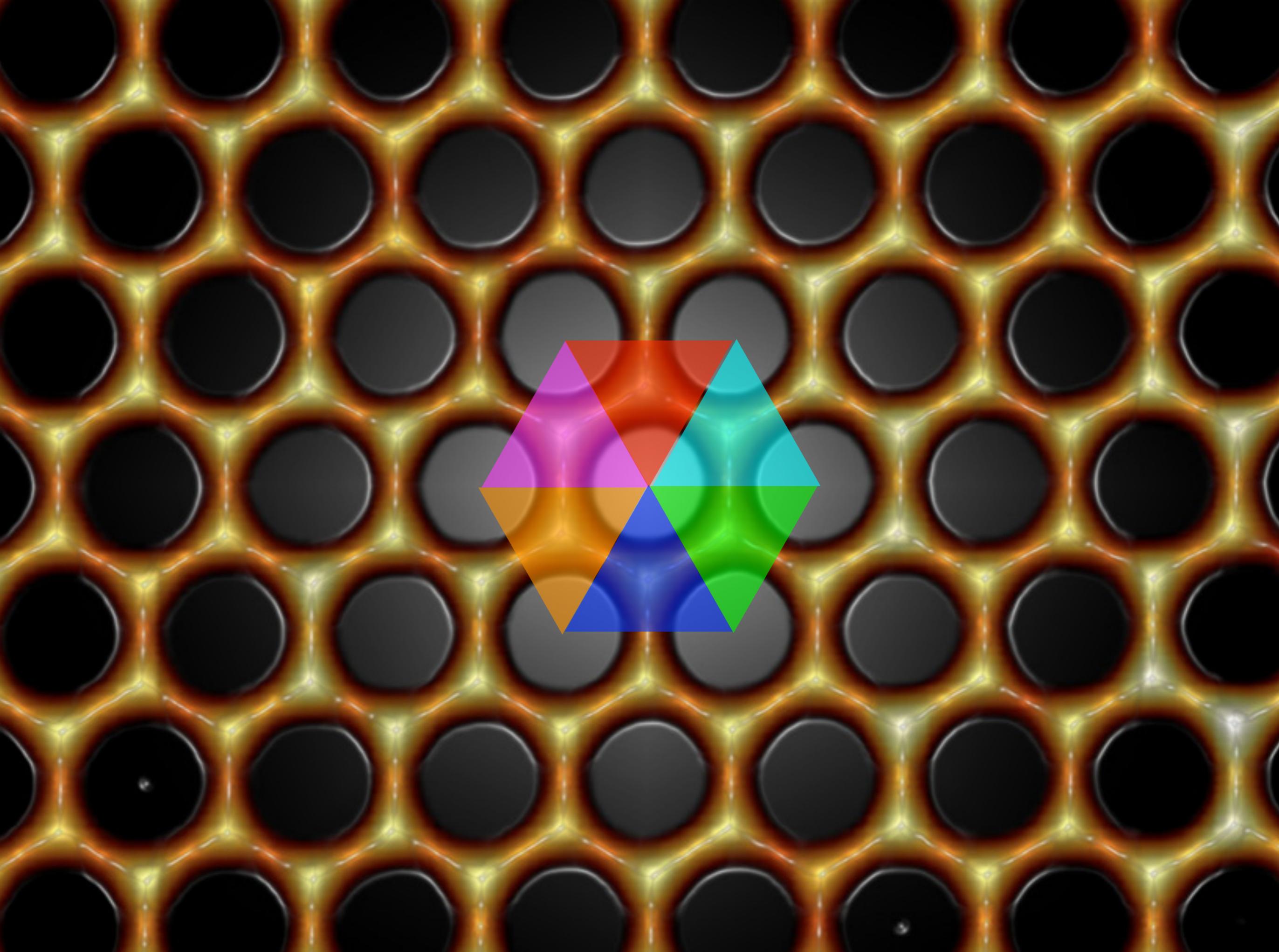


Manifold: space equipped with atlas

Some geometric questions in Network

The Triangular lattice





ADC 1 : Tip Height
Range : 100
Points : 101
Conus : 13
Angle : -16.91

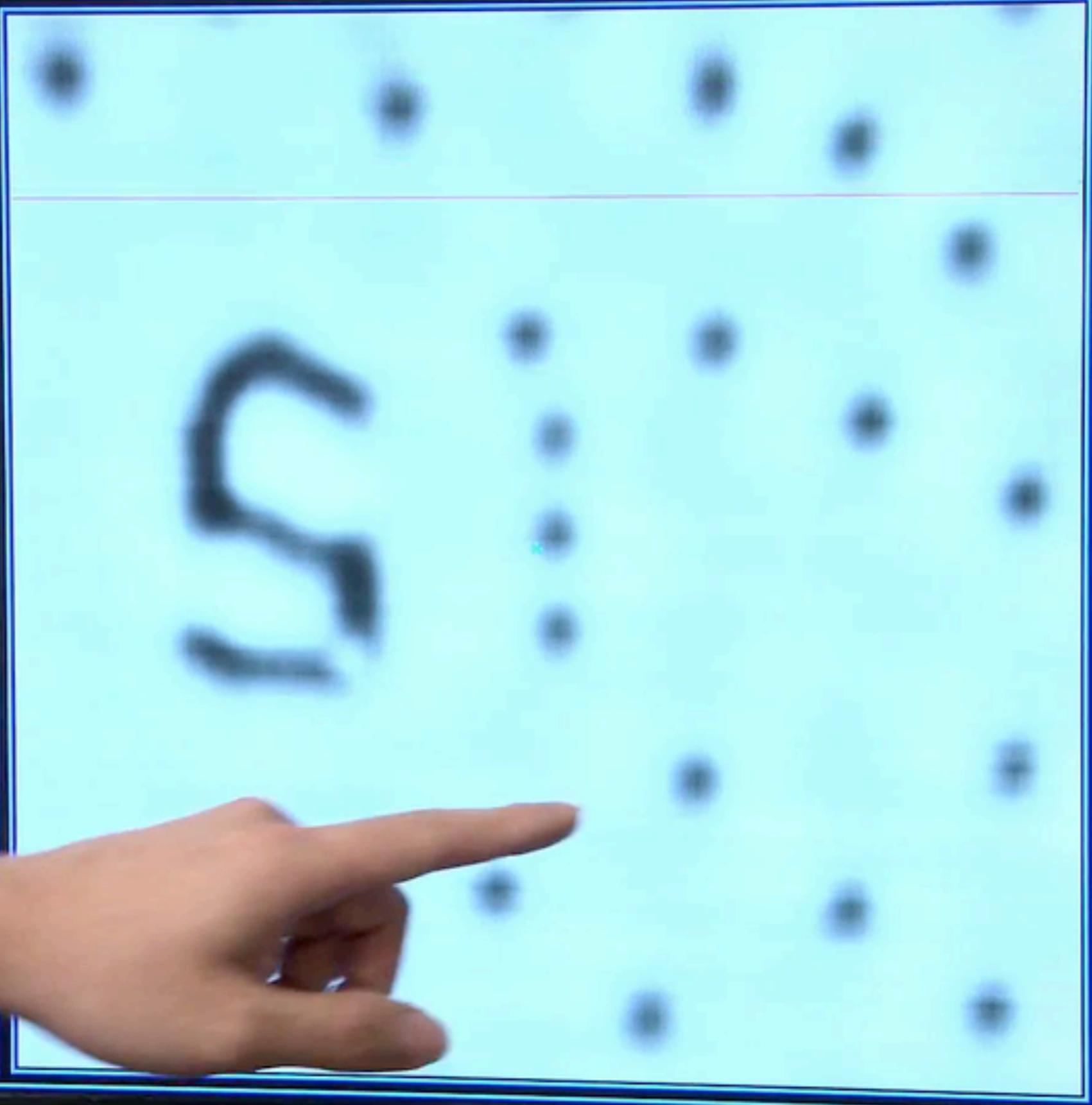
X : 593.68
Y : 1612.10
Z : 129.16

U : -1.000 E-2
I : 1.000 E-9

X DAC Gain : 1
Y DAC Gain : 1
Z DAC Gain : 1
Z Non Gain : .1
Preamp : 1ES U/n

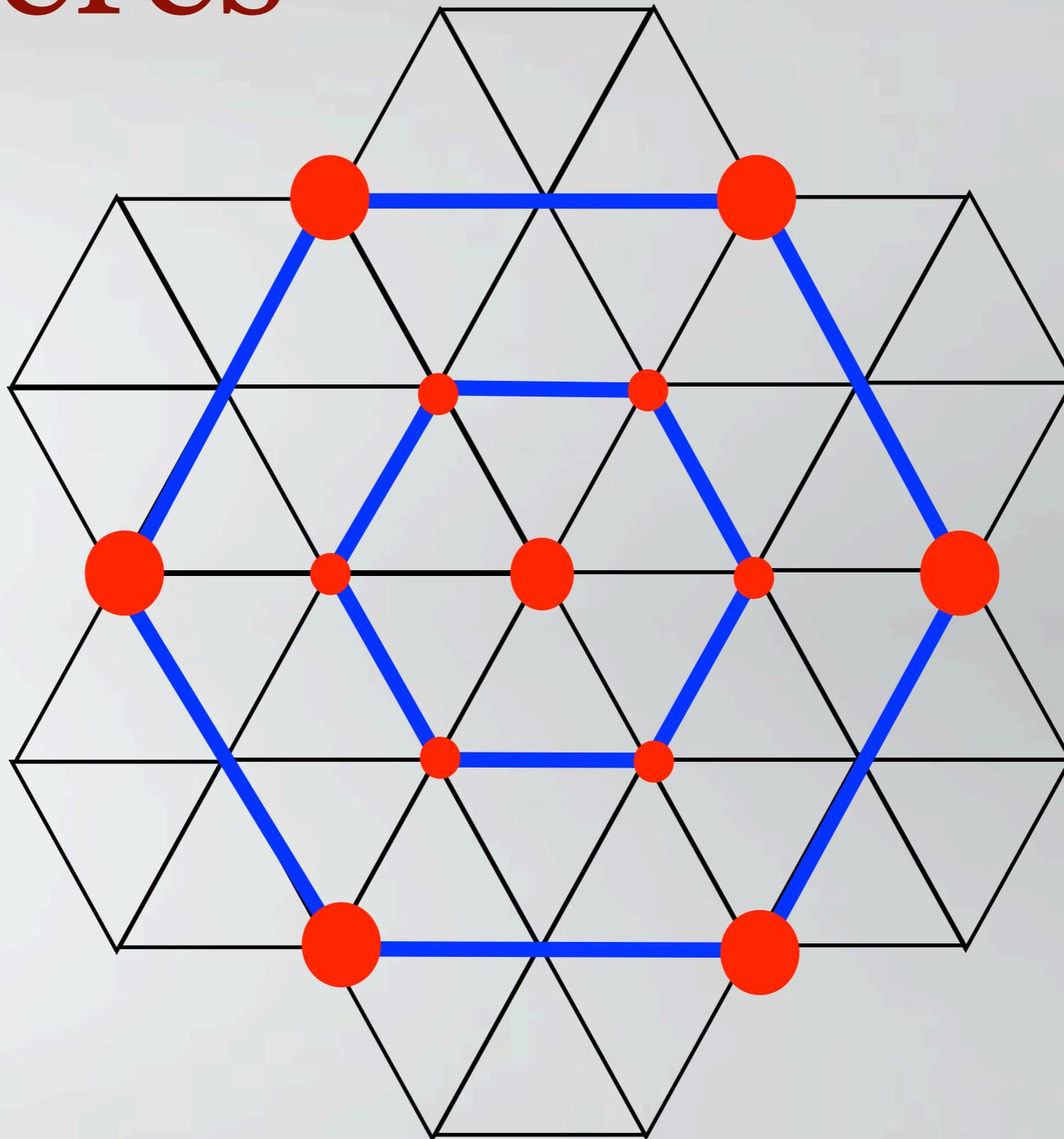
Sliding Mode

SCANNING

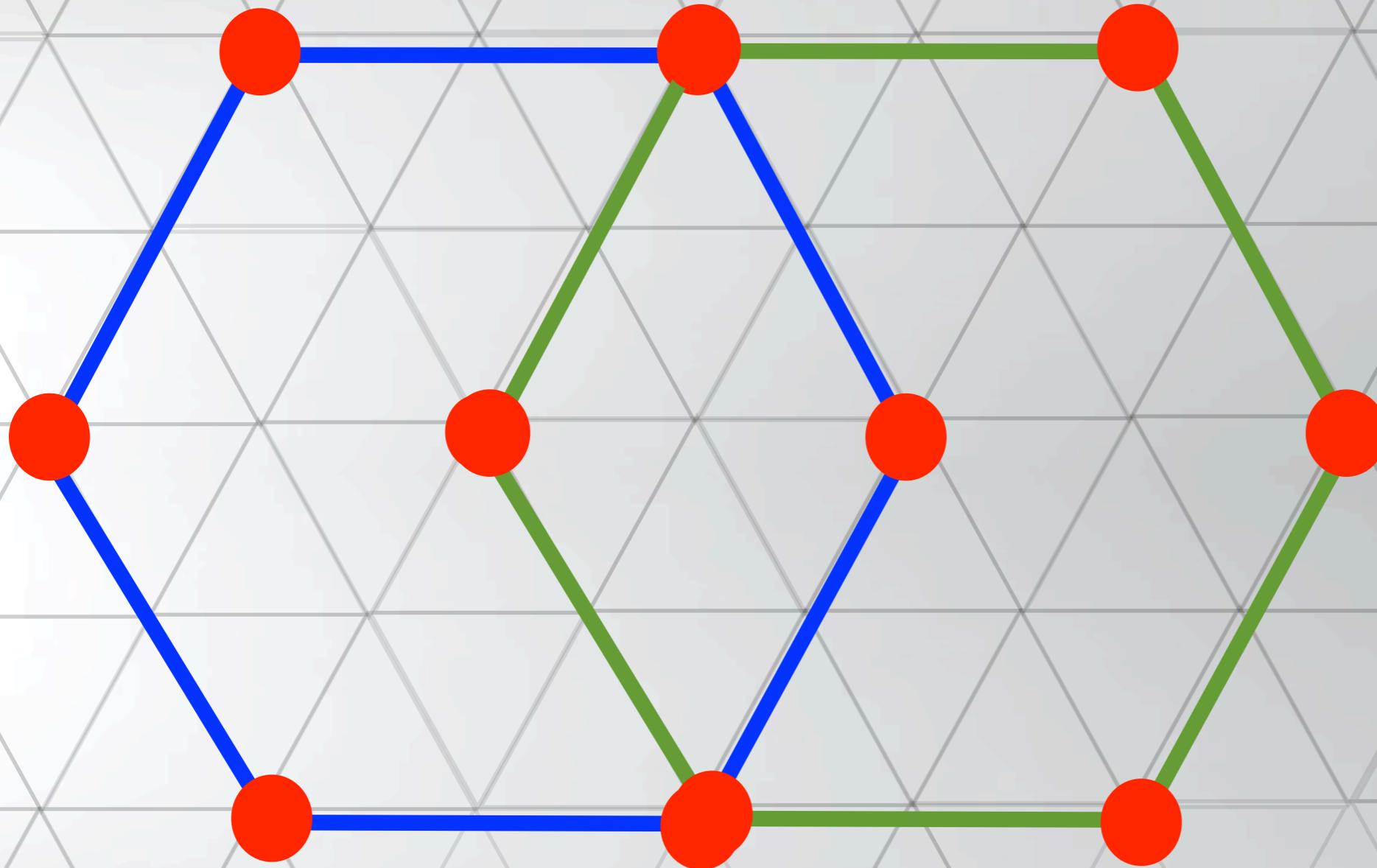


DELL

Spheres



Intersection of Spheres



Growth of Circle Length

$|S(r)|''$ positive:

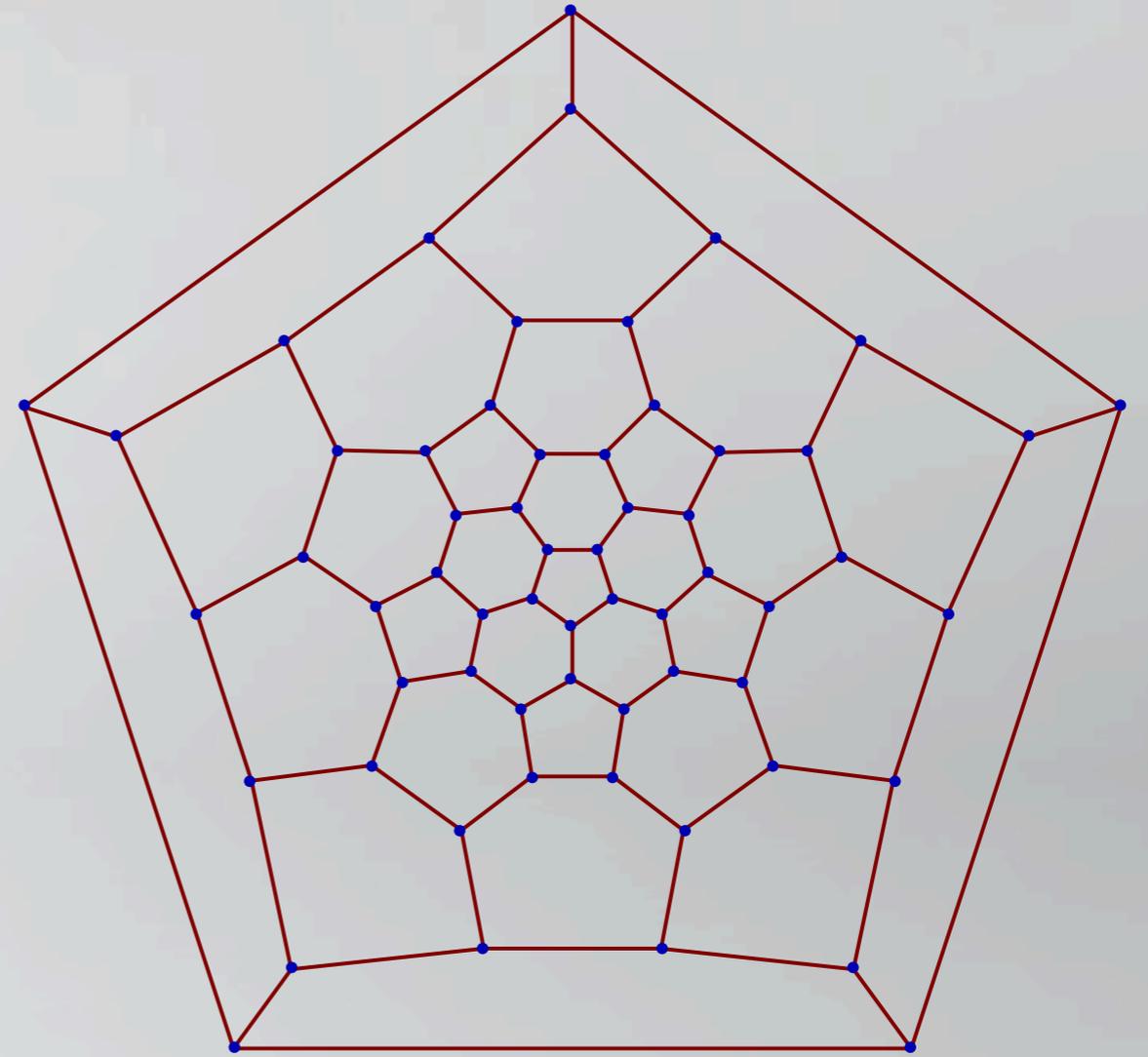
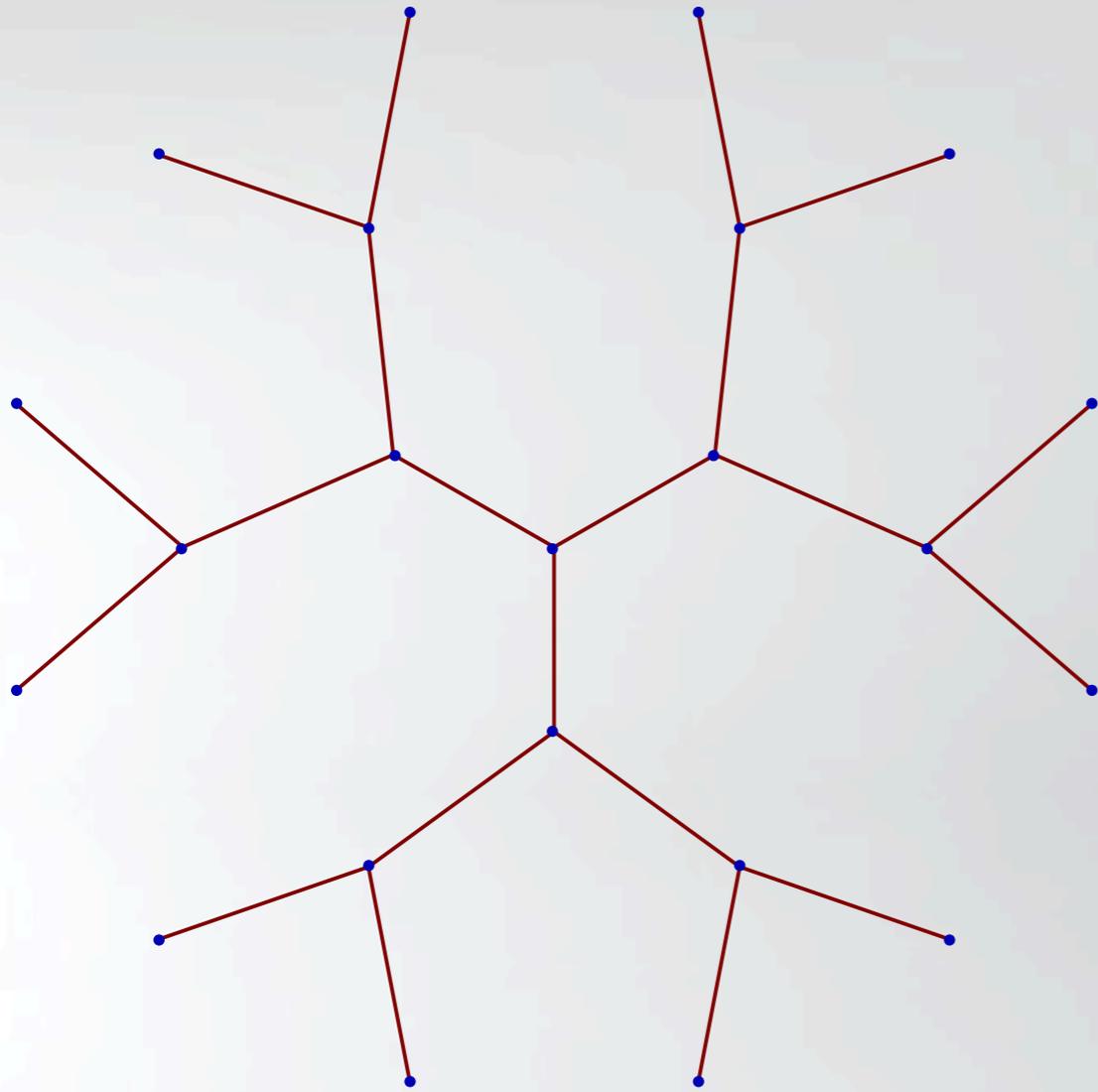
hyperbolic

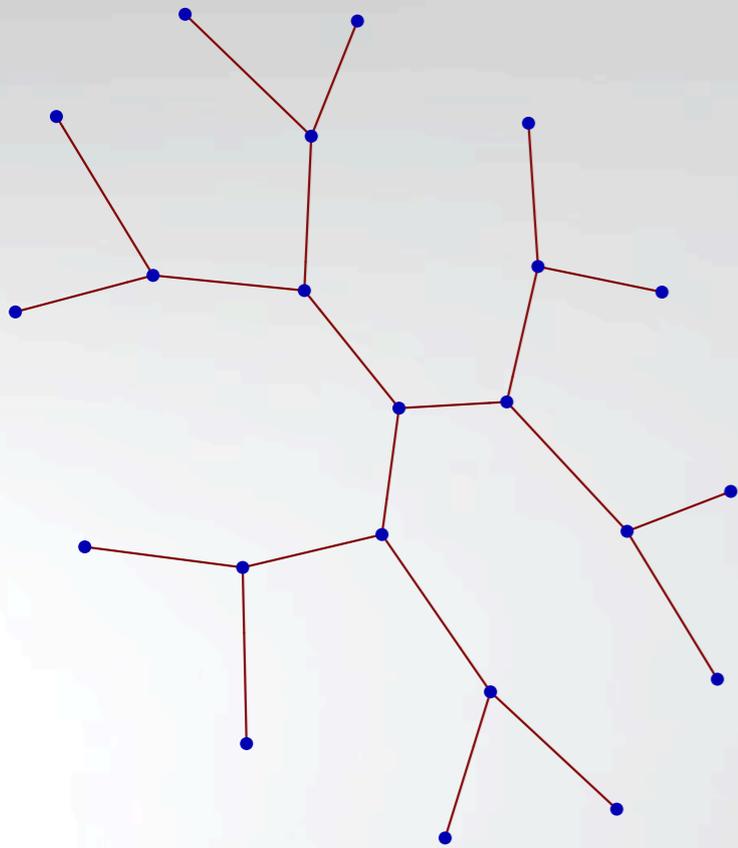
$|S(r)|'' = 0$

flat

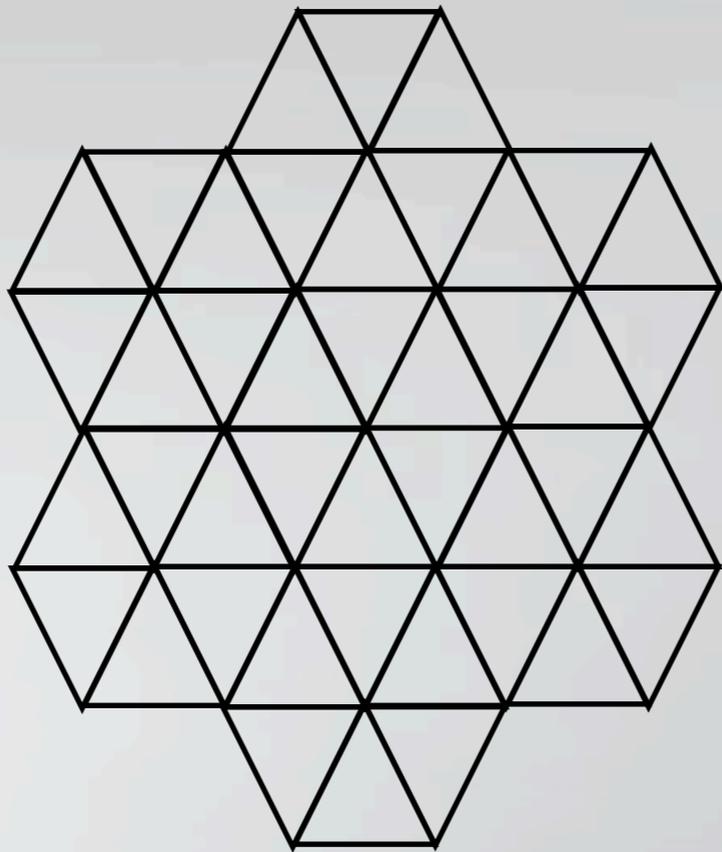
$|S(r)|''$ negative grows polynomially:

elliptic

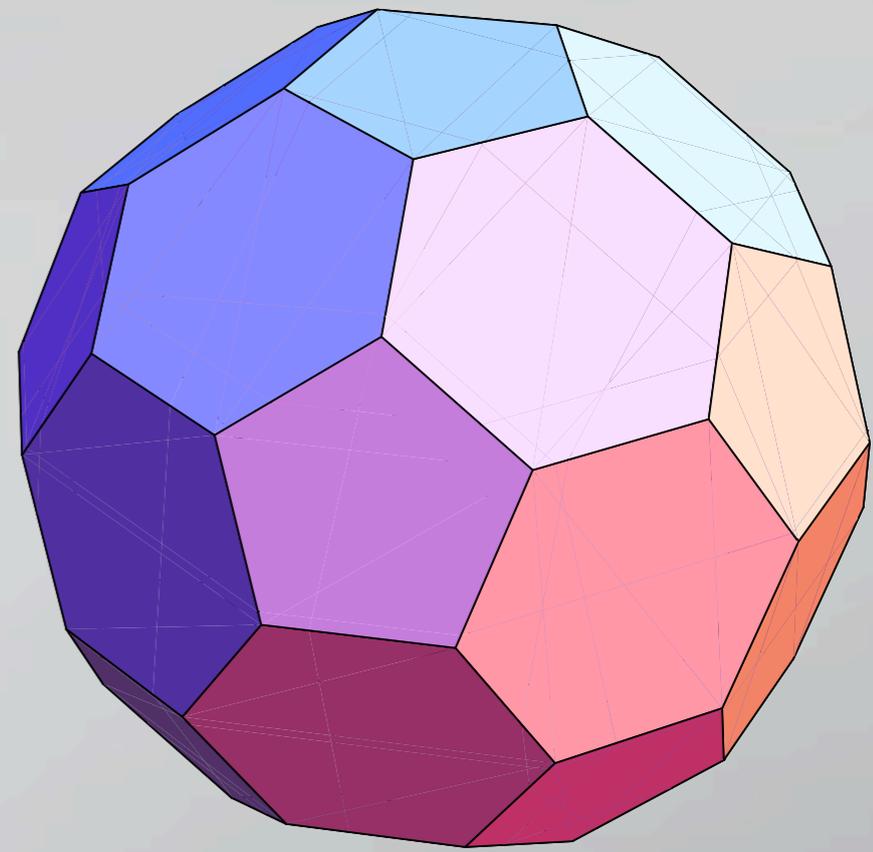




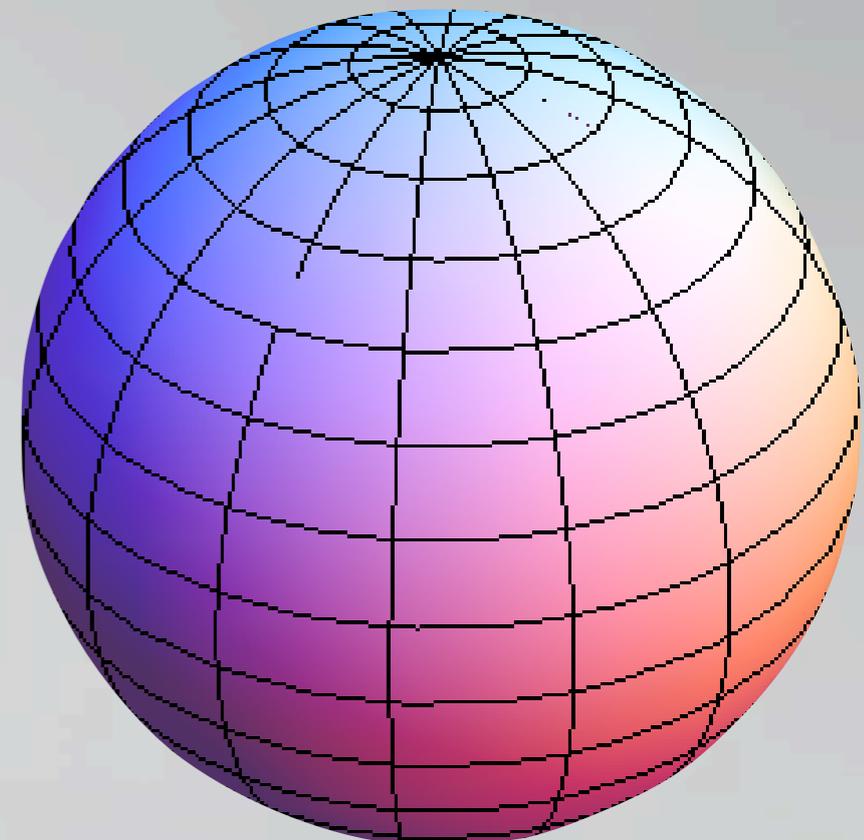
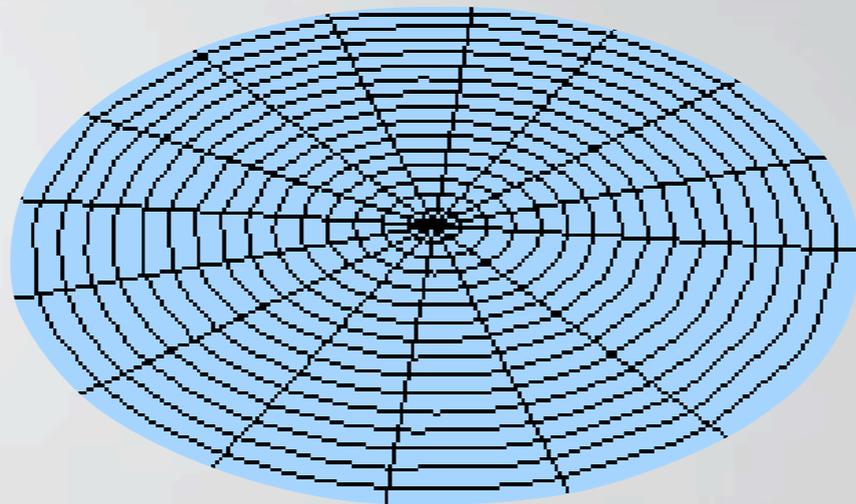
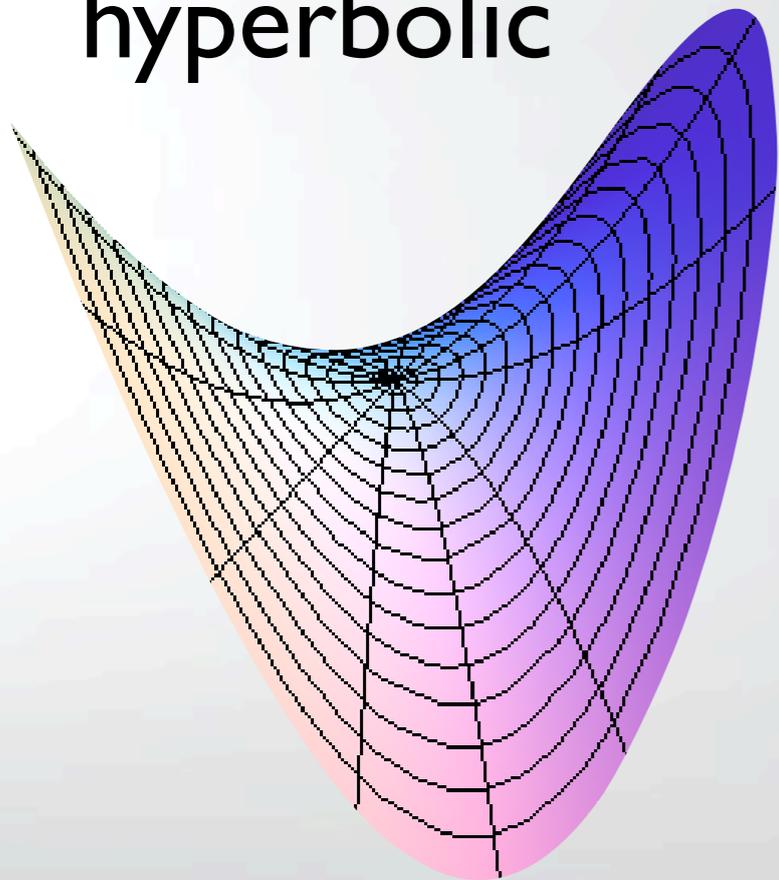
hyperbolic



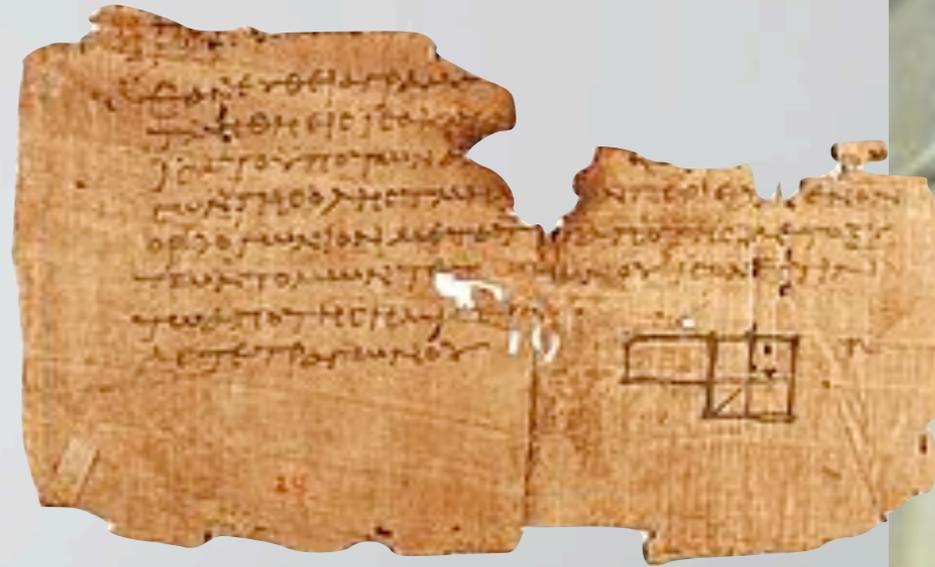
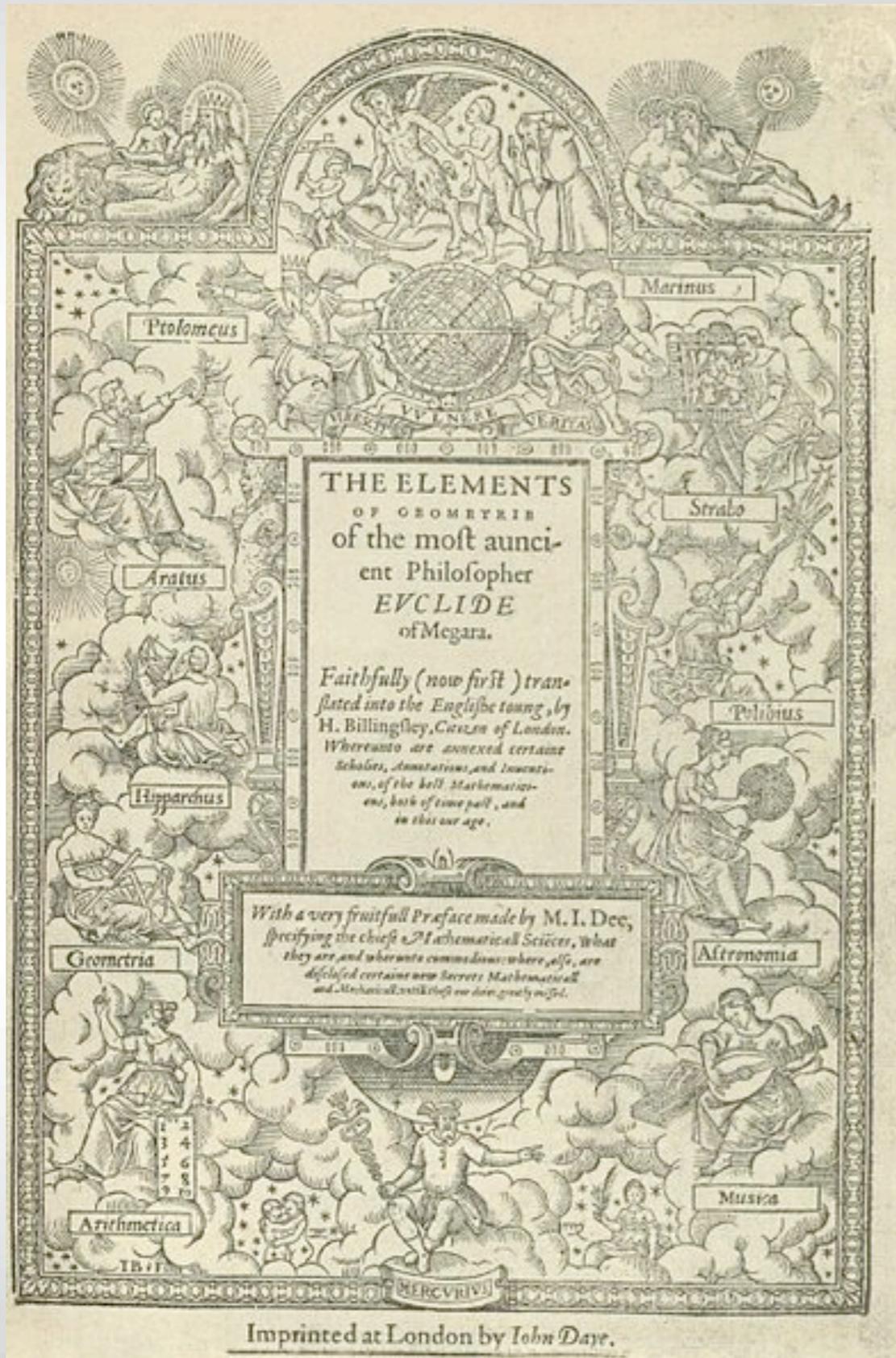
flat



elliptic



Euclids Axioms



Euclid
300BC



Euclid's Axioms of Planimetry

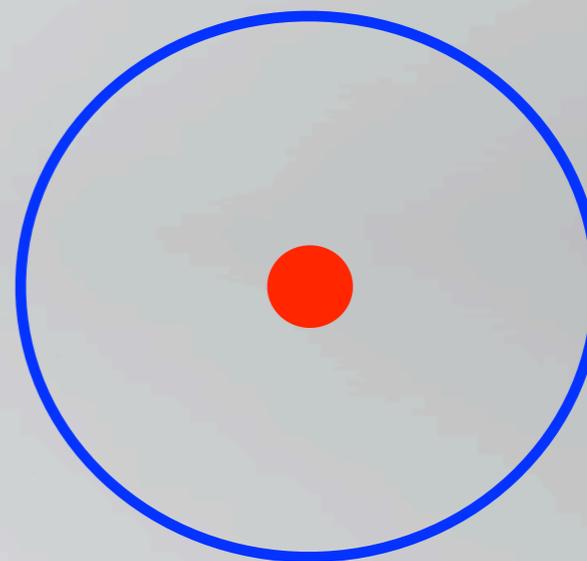
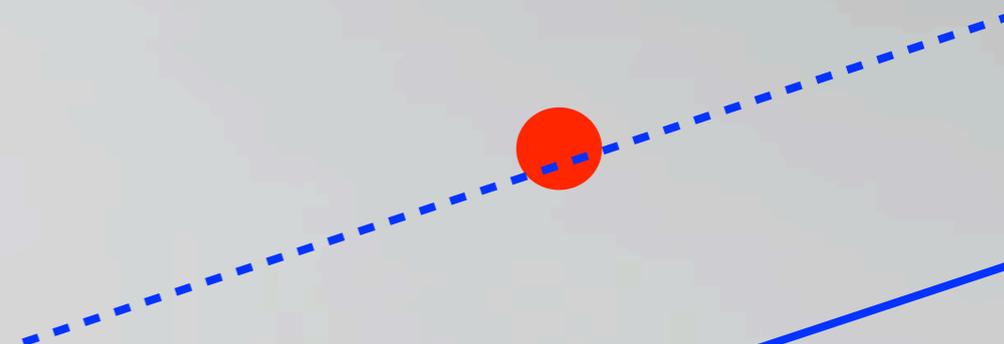
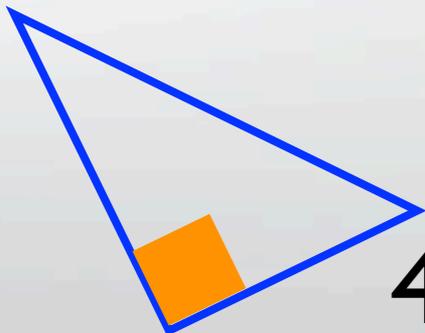
1. Line between two points

2. Extend line

3. Build Circle

4. Right angles congruent

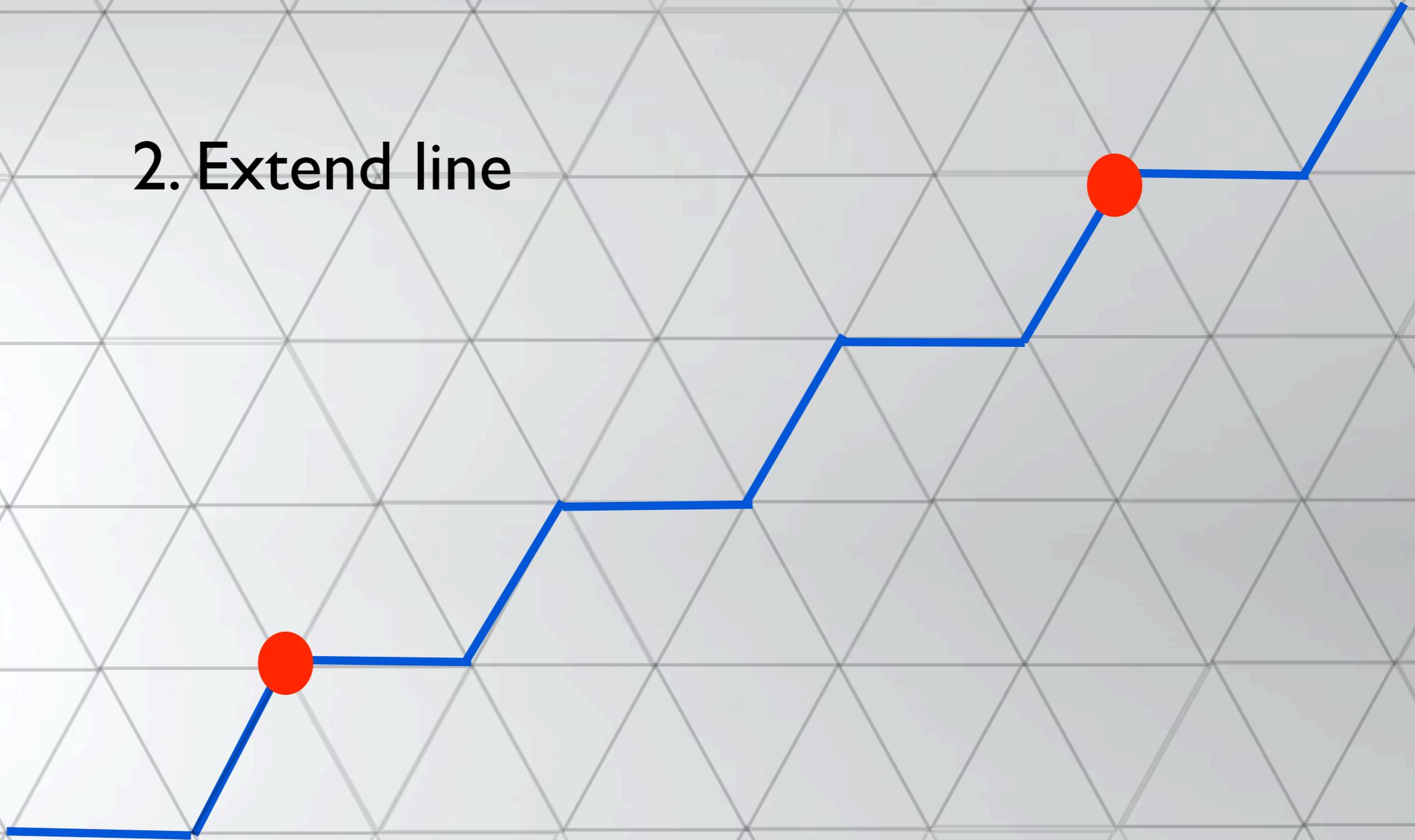
5. Parallel Axiom



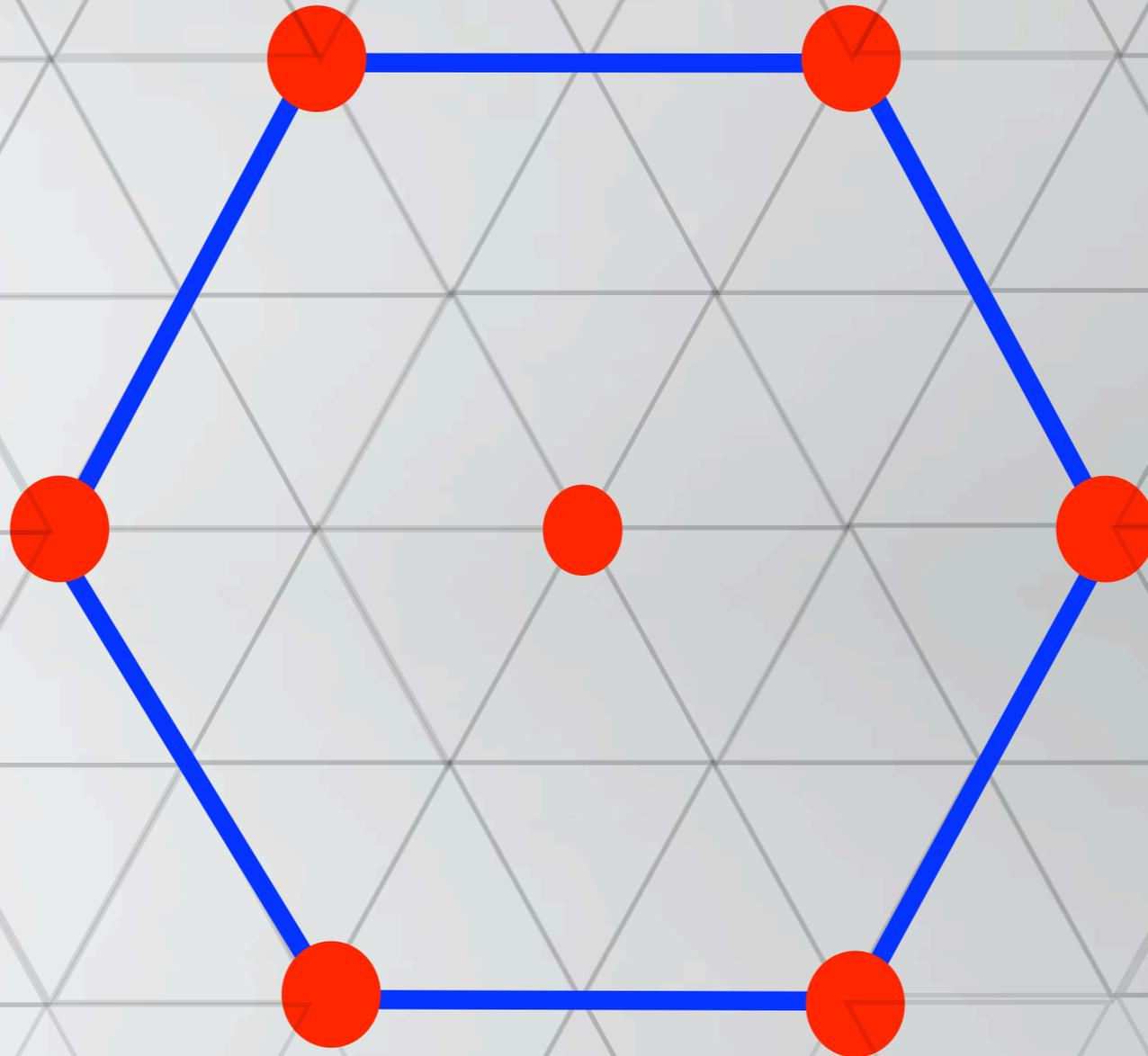
I. Line between two points



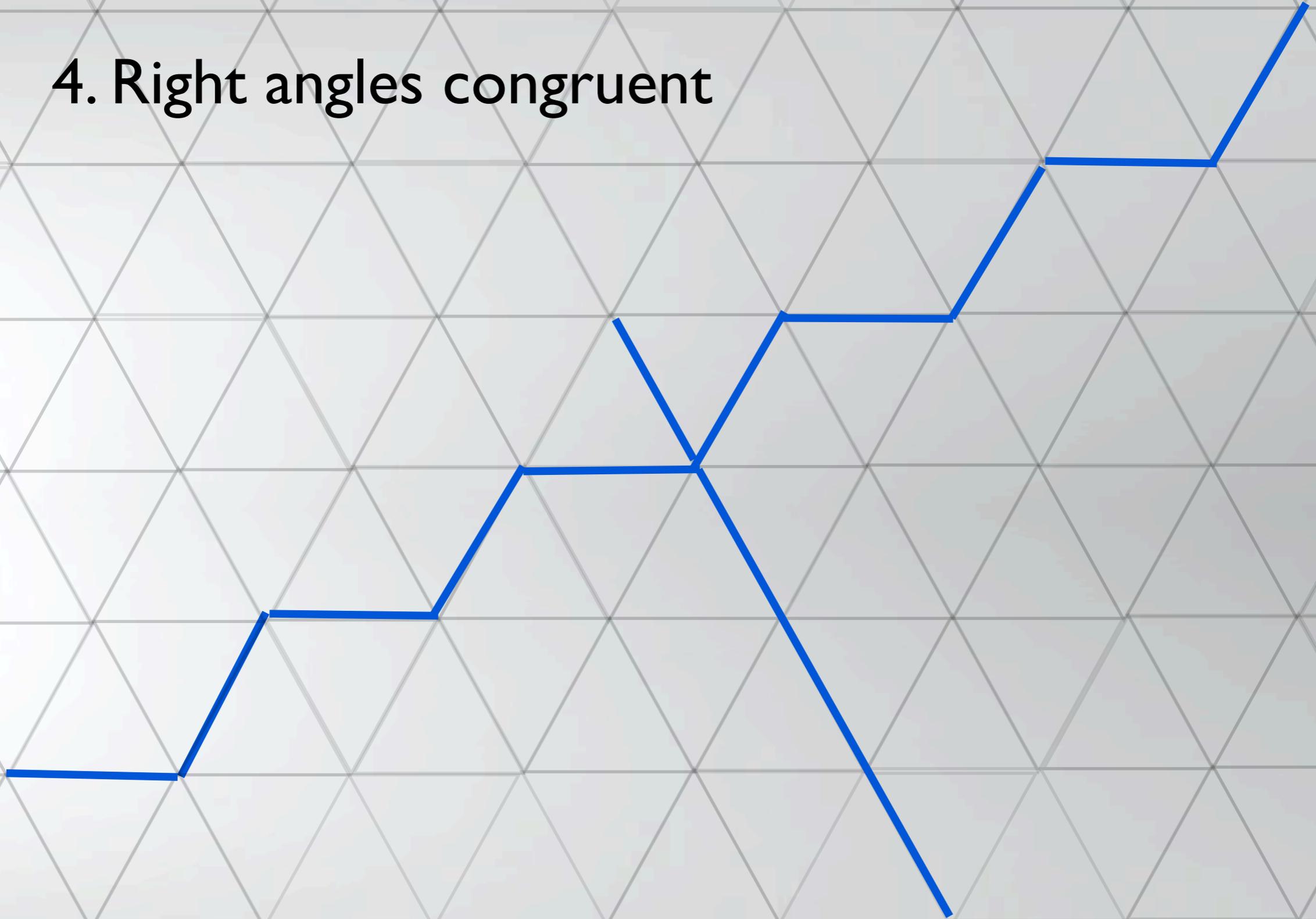
2. Extend line



3.Build Circle

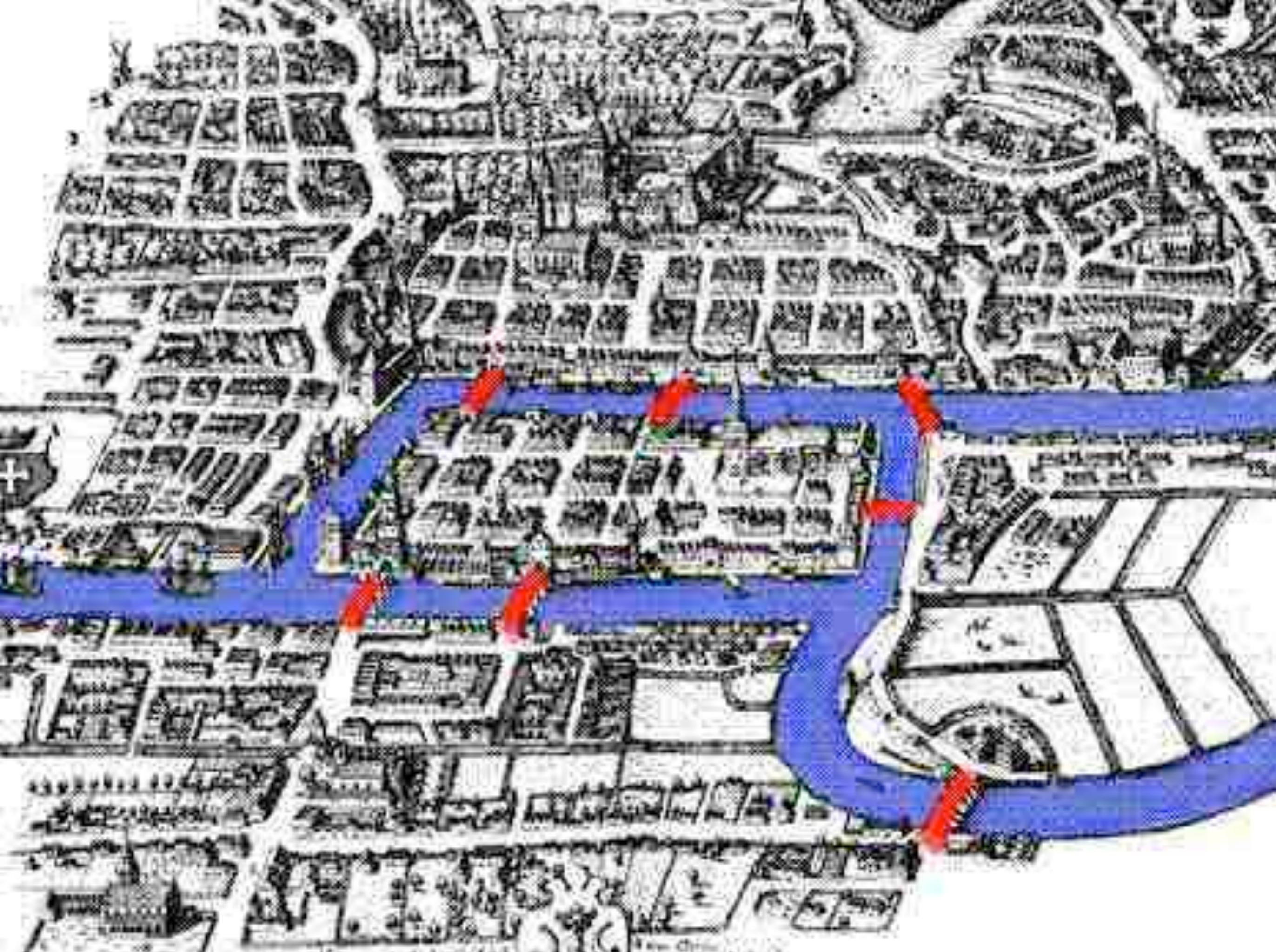


4. Right angles congruent



Birth of Topology



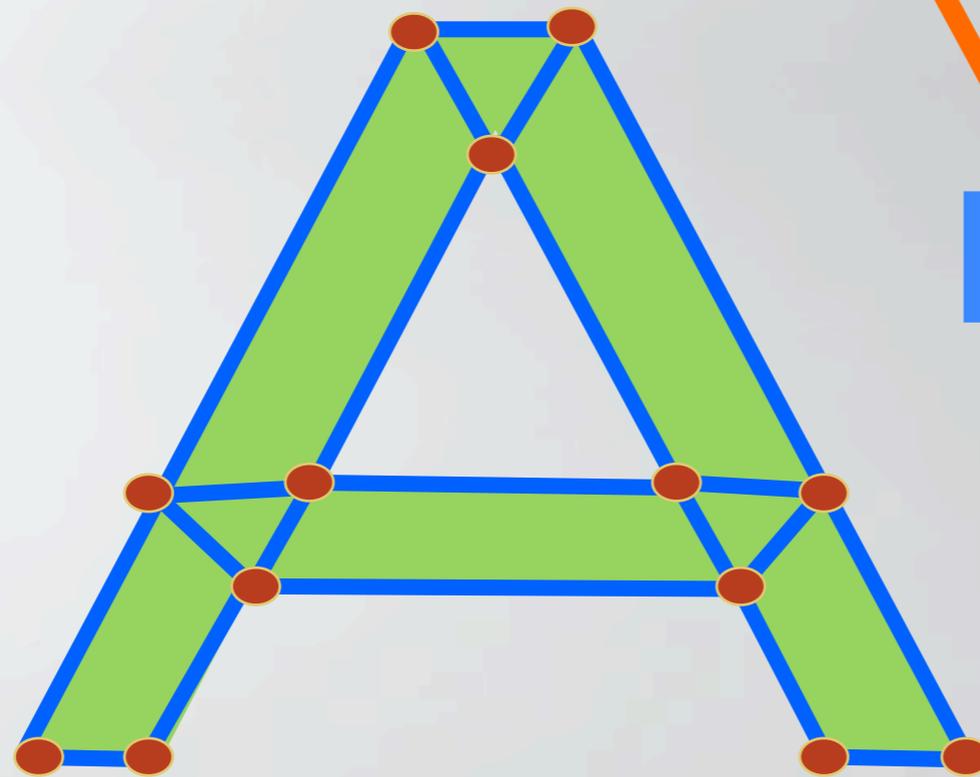




Story of Math II

Euler Characteristic

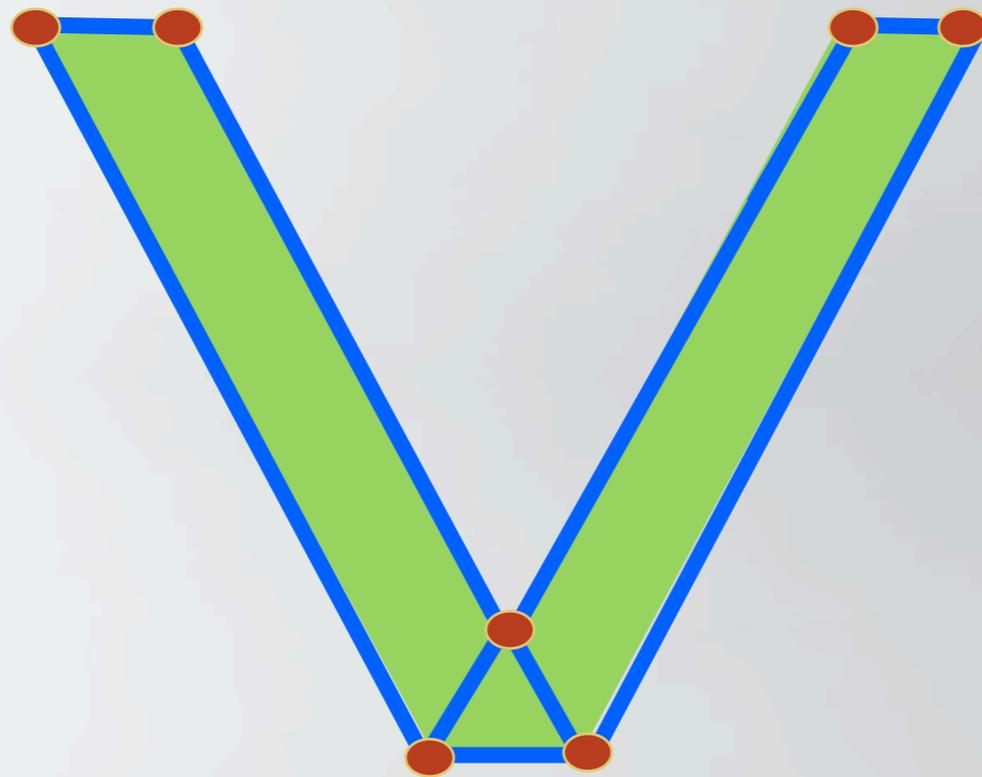
$$\chi = V - E + F$$



$$V = 13$$

$$E = 21$$

$$F = 8$$



$$V = 7$$

$$E = 9$$

$$F = 3$$

Euler's Gem $V - E + F = 2$



V

E

F

4	6	4
6	12	8
8	12	6
12	30	20
20	30	12

DAVID S. RICHESON



EULER'S GEM

THE POLYHEDRON FORMULA

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A TRUE TALE OF MATHEMATICS, MYSTICISM, AND THE QUEST TO UNDERSTAND THE UNIVERSE



DESCARTES'S SECRET NOTEBOOK

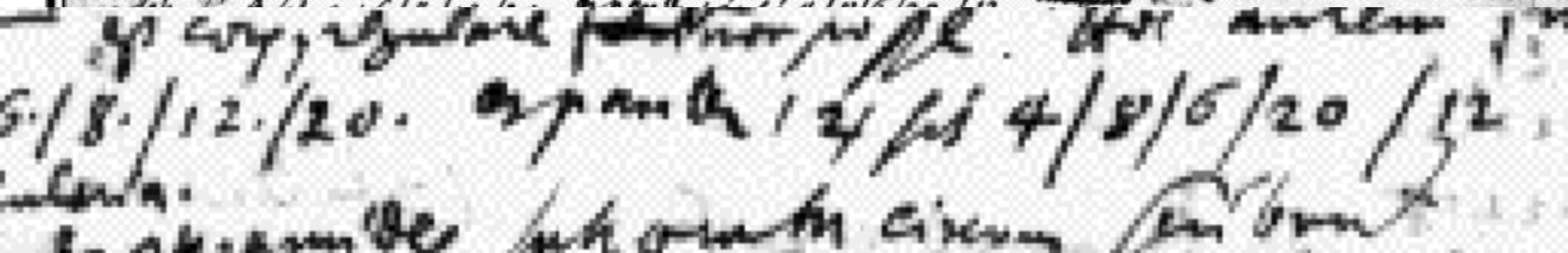
Author of *Fermat's Last Theorem*

AMIR D. ACZEL

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per unumquemque quatuordecim...
 quibus in quatuordecim...
 ad unumquemque...
 2 $\alpha = 4$ et $2 \beta = 4$ ab ut nullo...
 5.7. 10. 12. 14. 16. 18. 20.



Invenitur quod in eodem corpore...
 et per se dicitur...
 et nullum tale...
 numero faciem...
 et per se dicitur...
 120...
 faciem...
 quod...
 5 facies...
 18 anguli...
 3 quadrata...
 4 triangula...
 3 quadrata...
 3 triangula...
 3 quadrata...

Rubber Geometry



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Movie clip by Max-Planck Institut fuer Gravitationsphysik	2012

Molecules and Words from Mathematica 8

Literature

