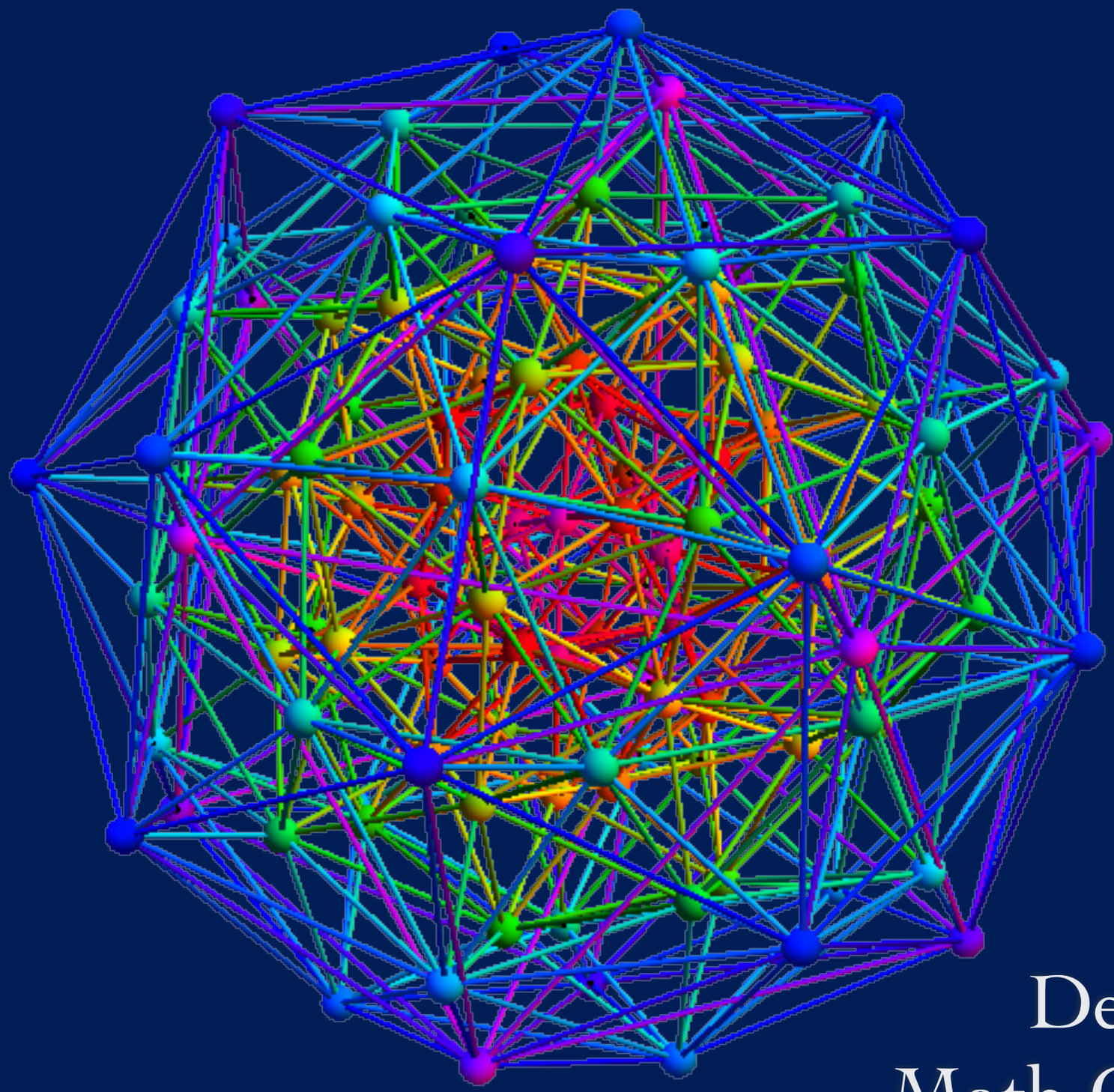
Polytopes II

Oliver Knill



December 6, 2009

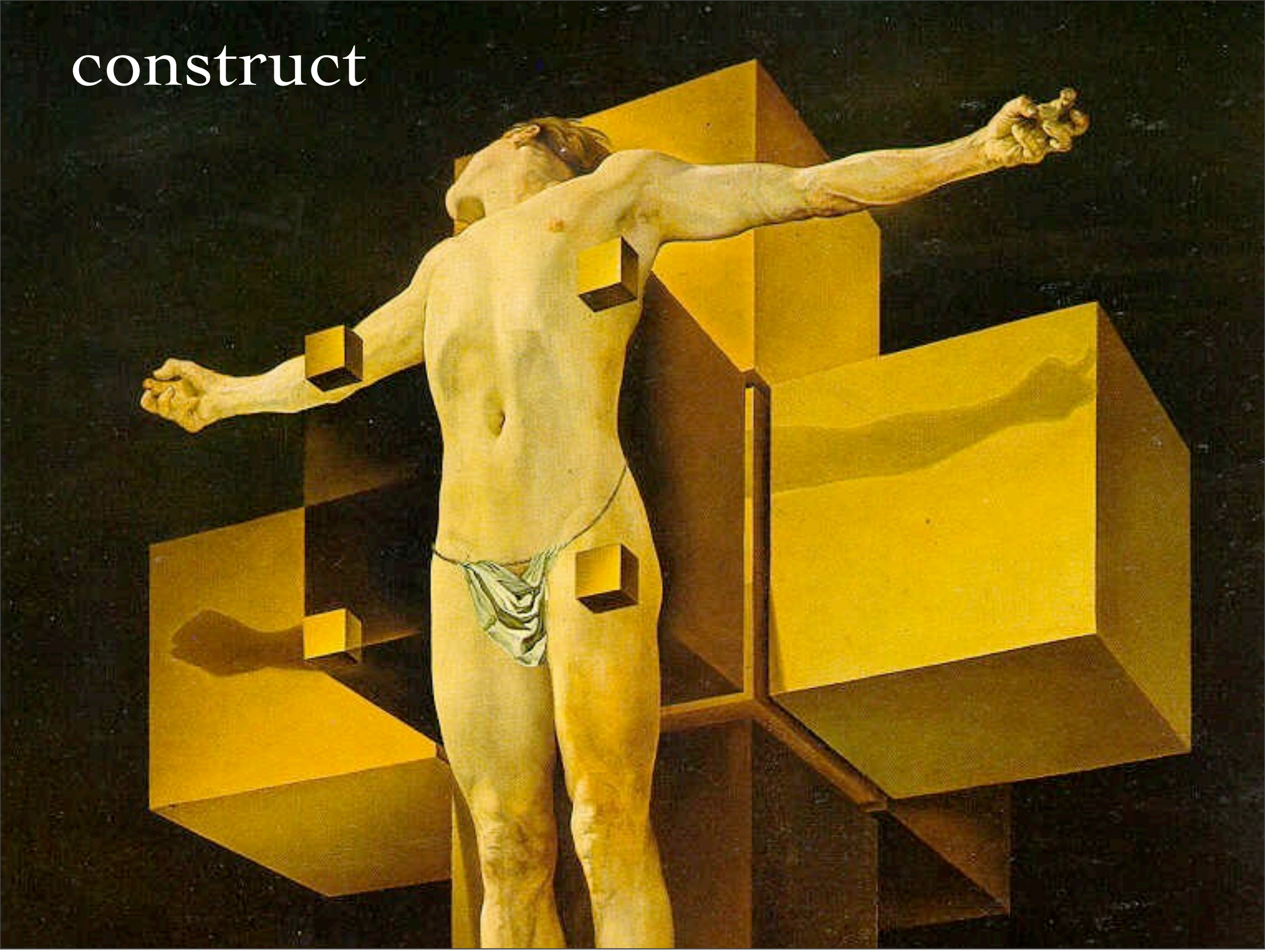
Math Circle, Northeastern

How to “see” in higher dimensions?

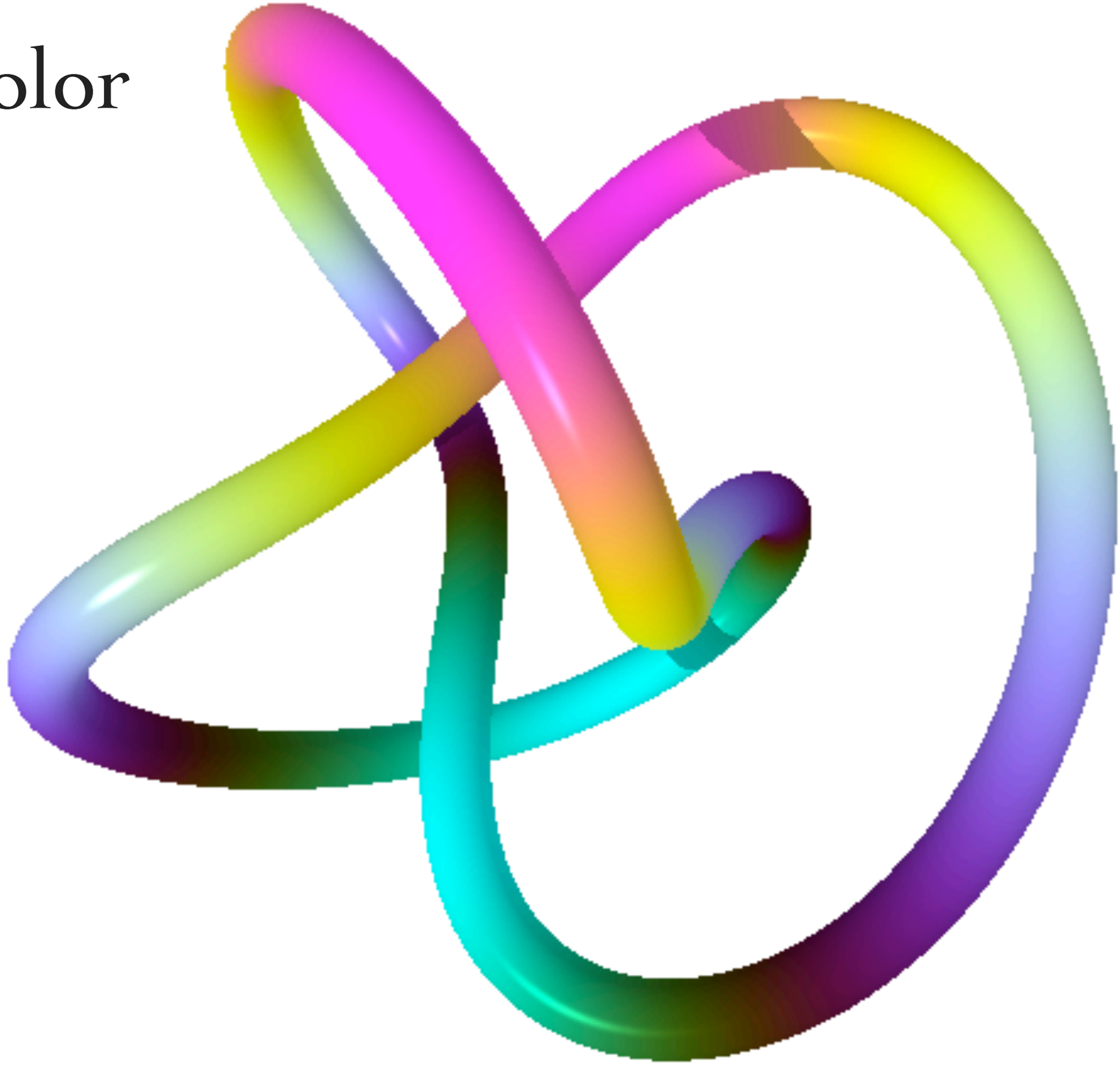
Three Ways:

- Construct and project!
- Use time or color!
- Live and die in it!

construct



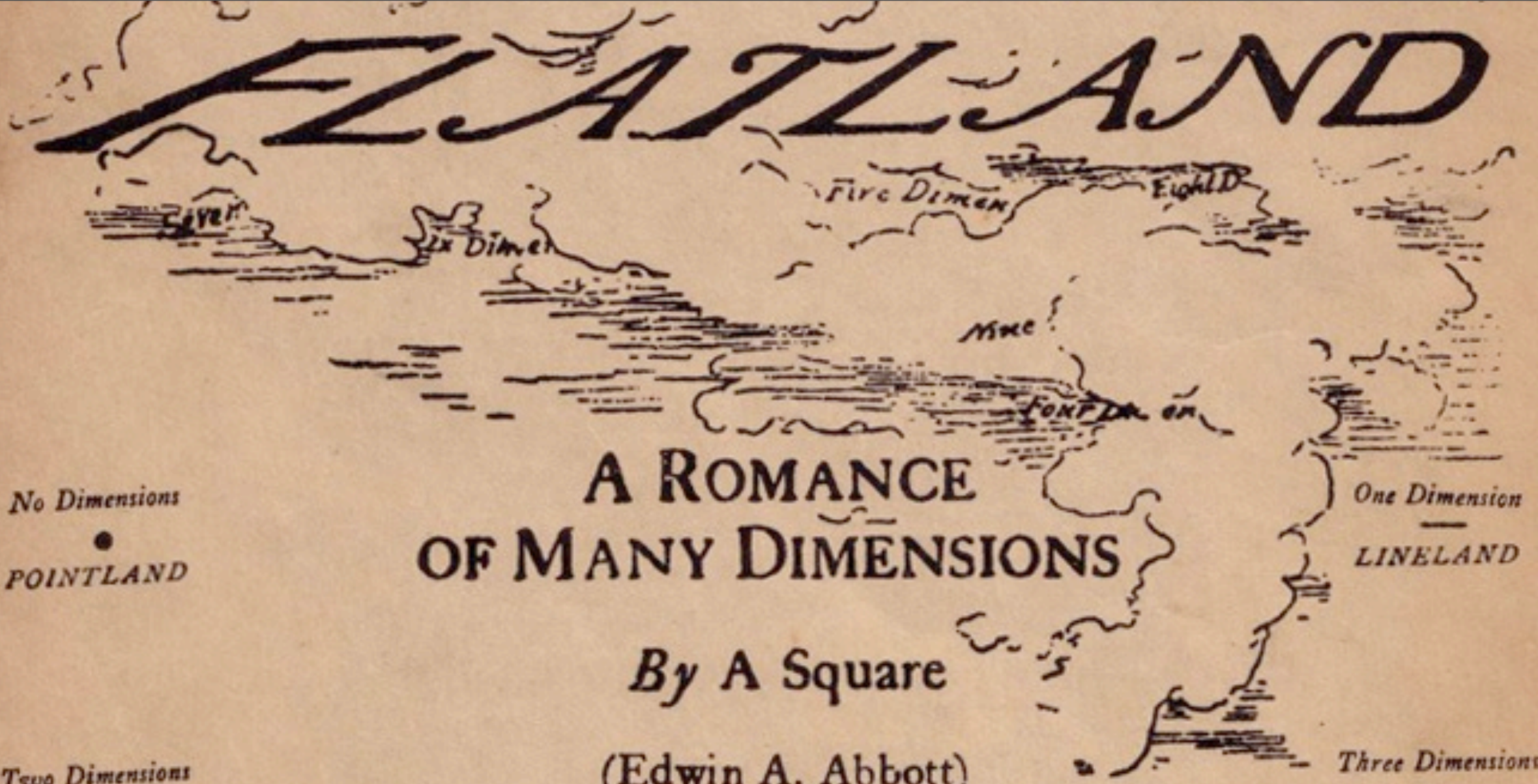
color



live and die

Cube 2, Hypercube, 2002

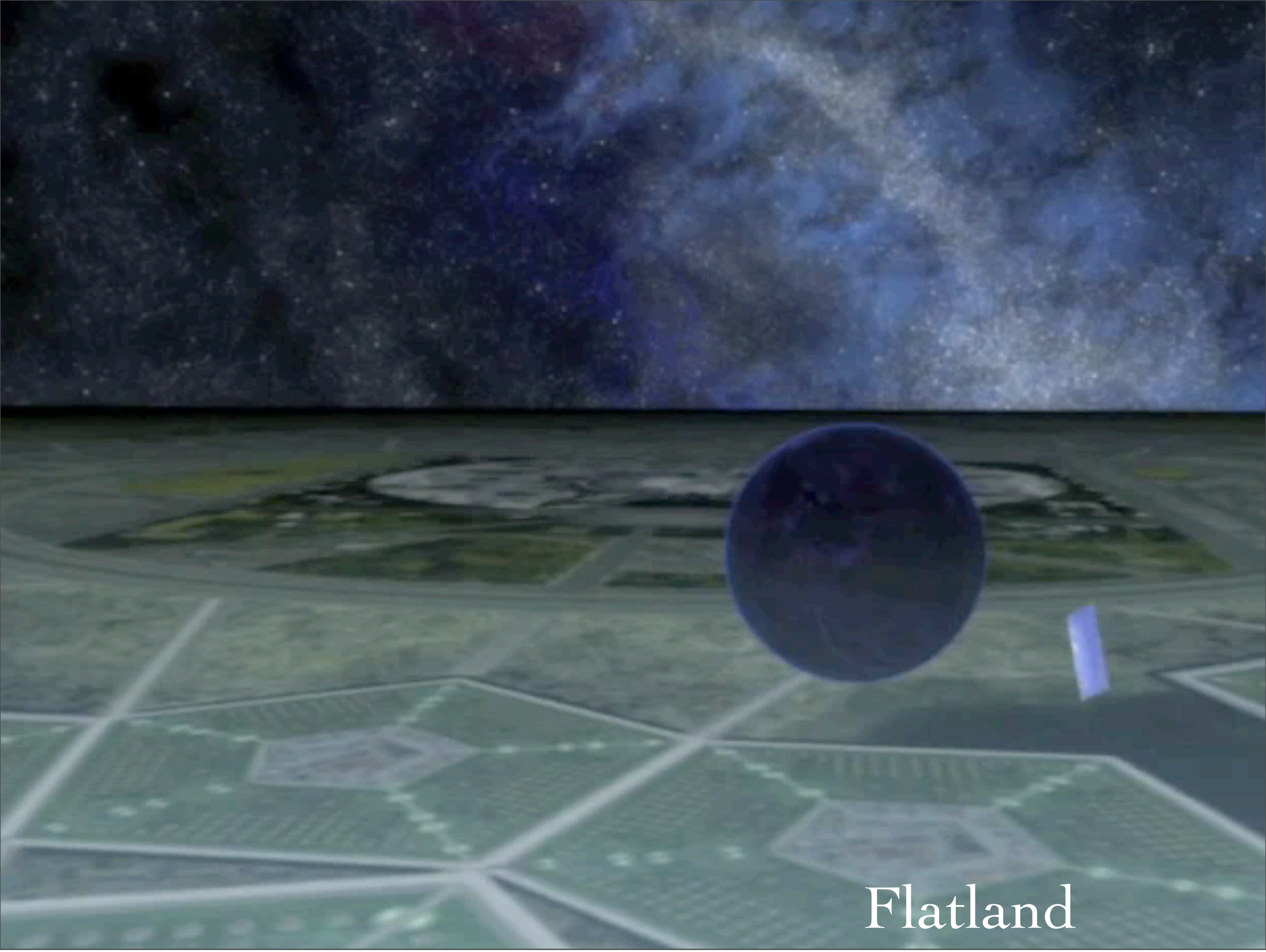
FLATLAND



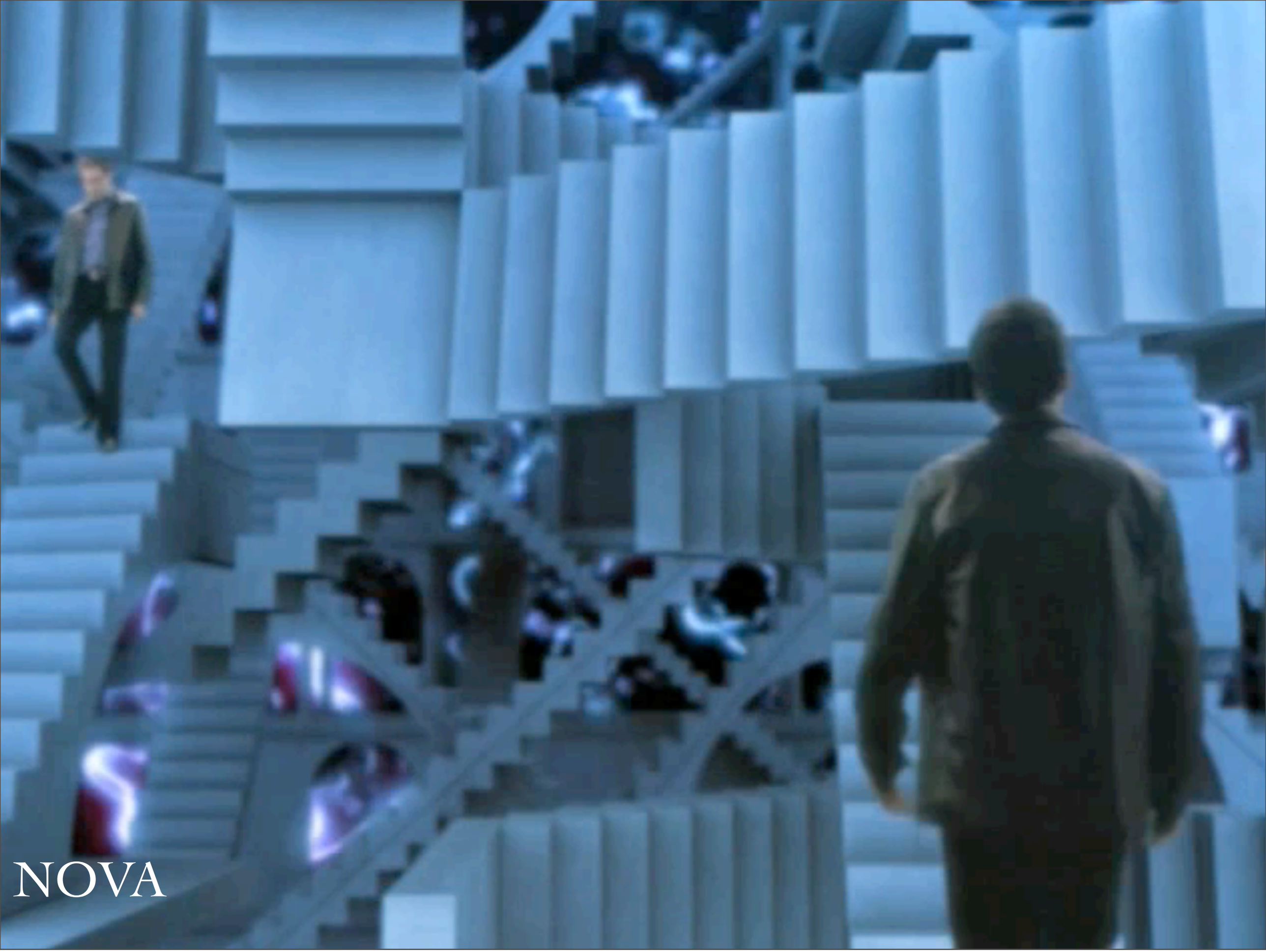
A ROMANCE OF MANY DIMENSIONS

By A Square

(Edwin A. Abbott)


Flatland

Our physical space turns out to be higher dimensional

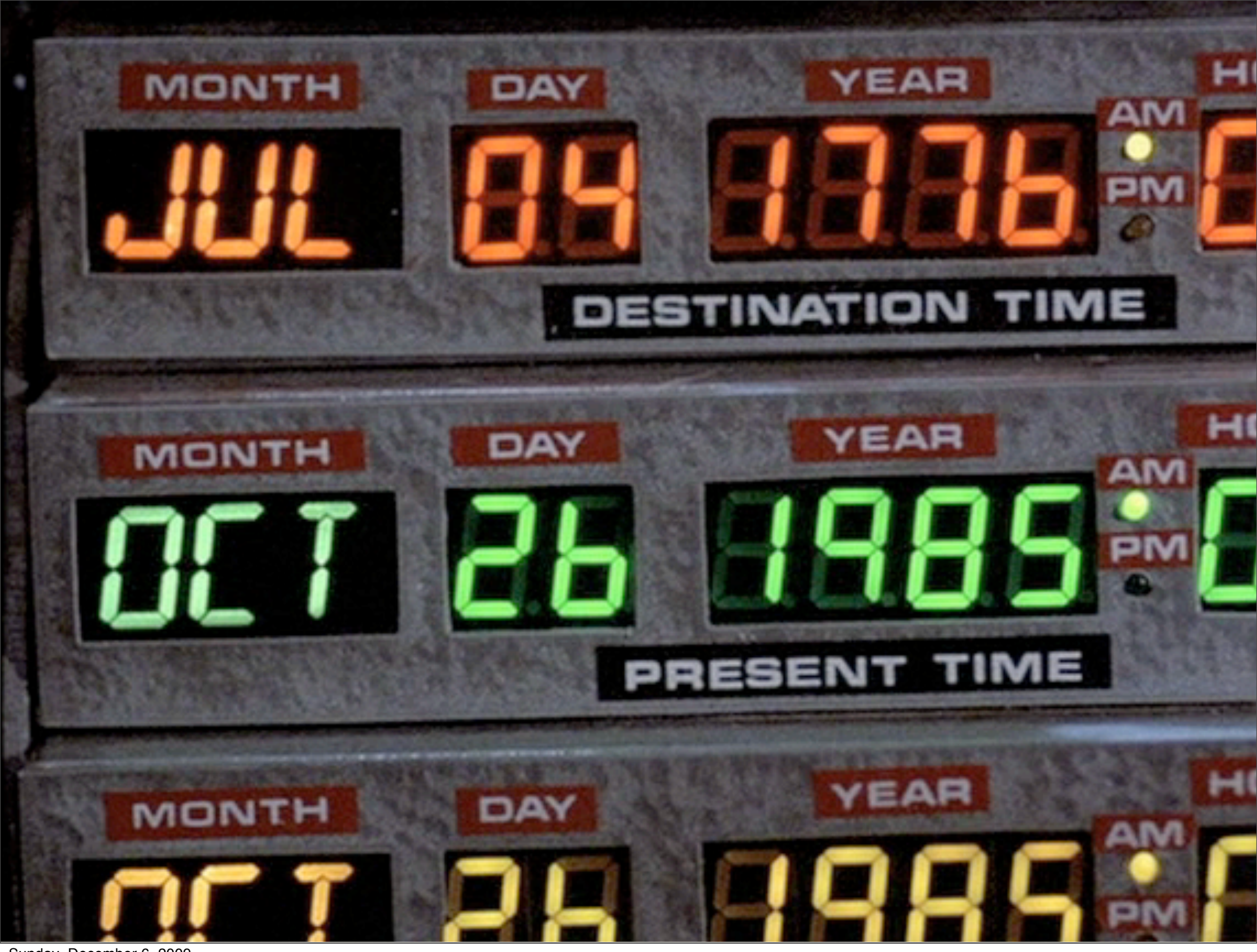

NOVA

NOVA


NOVA


NOVA

Sunday, December 6, 2009

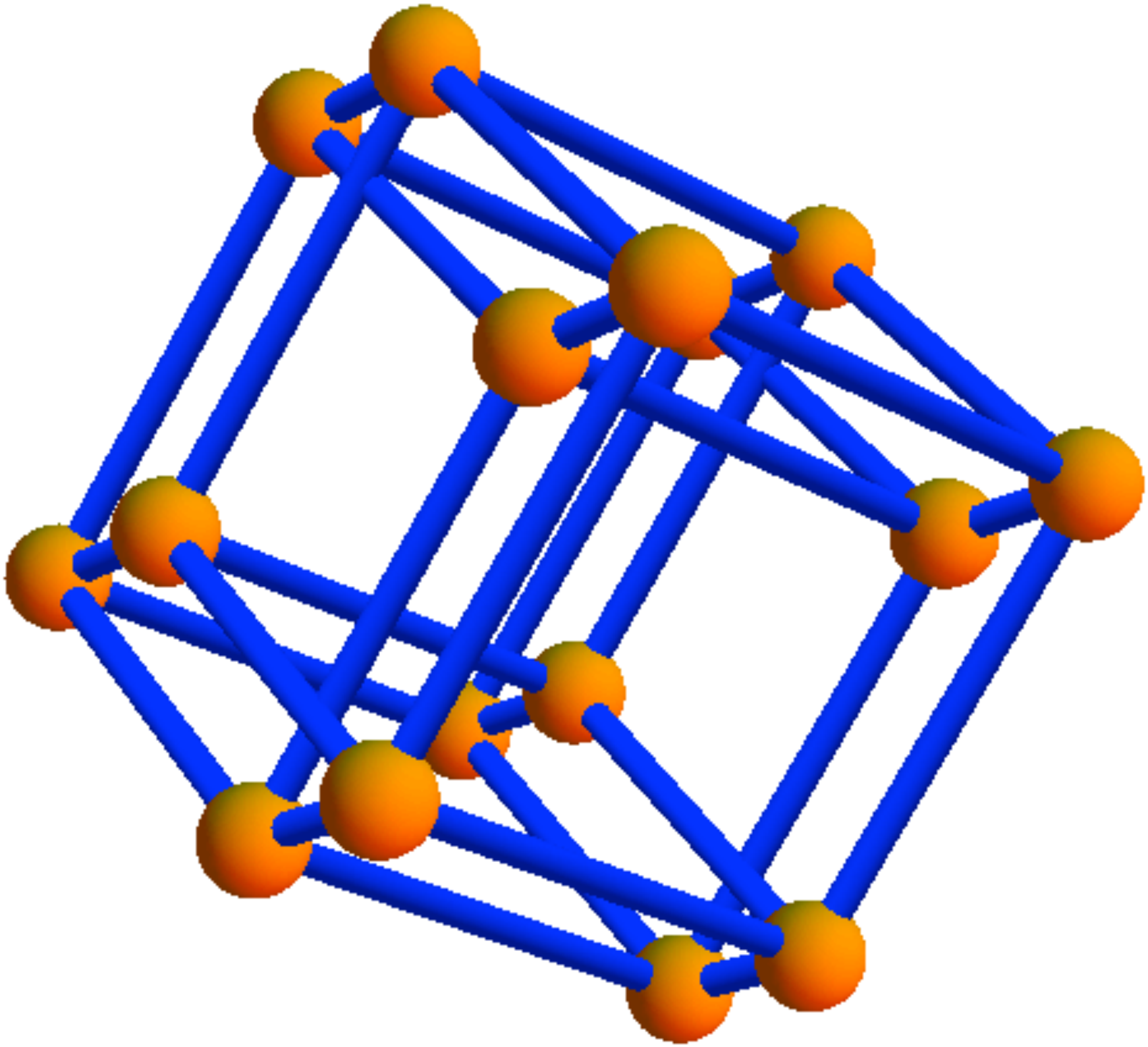

MONTH
DAY
YEAR
JUL
04
1776
AM
PM
DESTINATION TIME
MONTH
DAY
YEAR
OCT
26
1985
AM
PM
PRESENT TIME
MONTH
DAY
YEAR
OCT
26
1985
AM
PM

Polychera

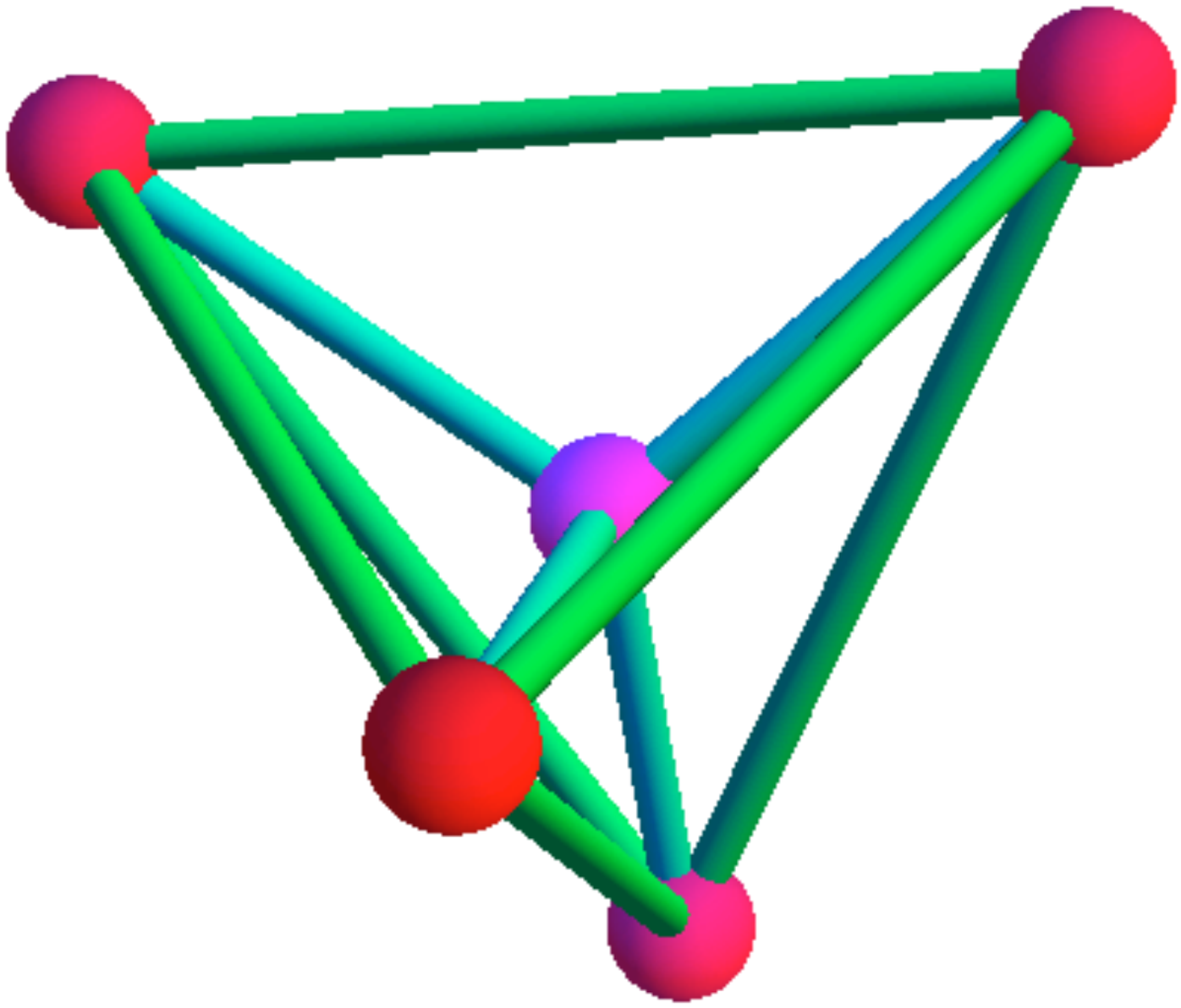
Theorem of Schläfli

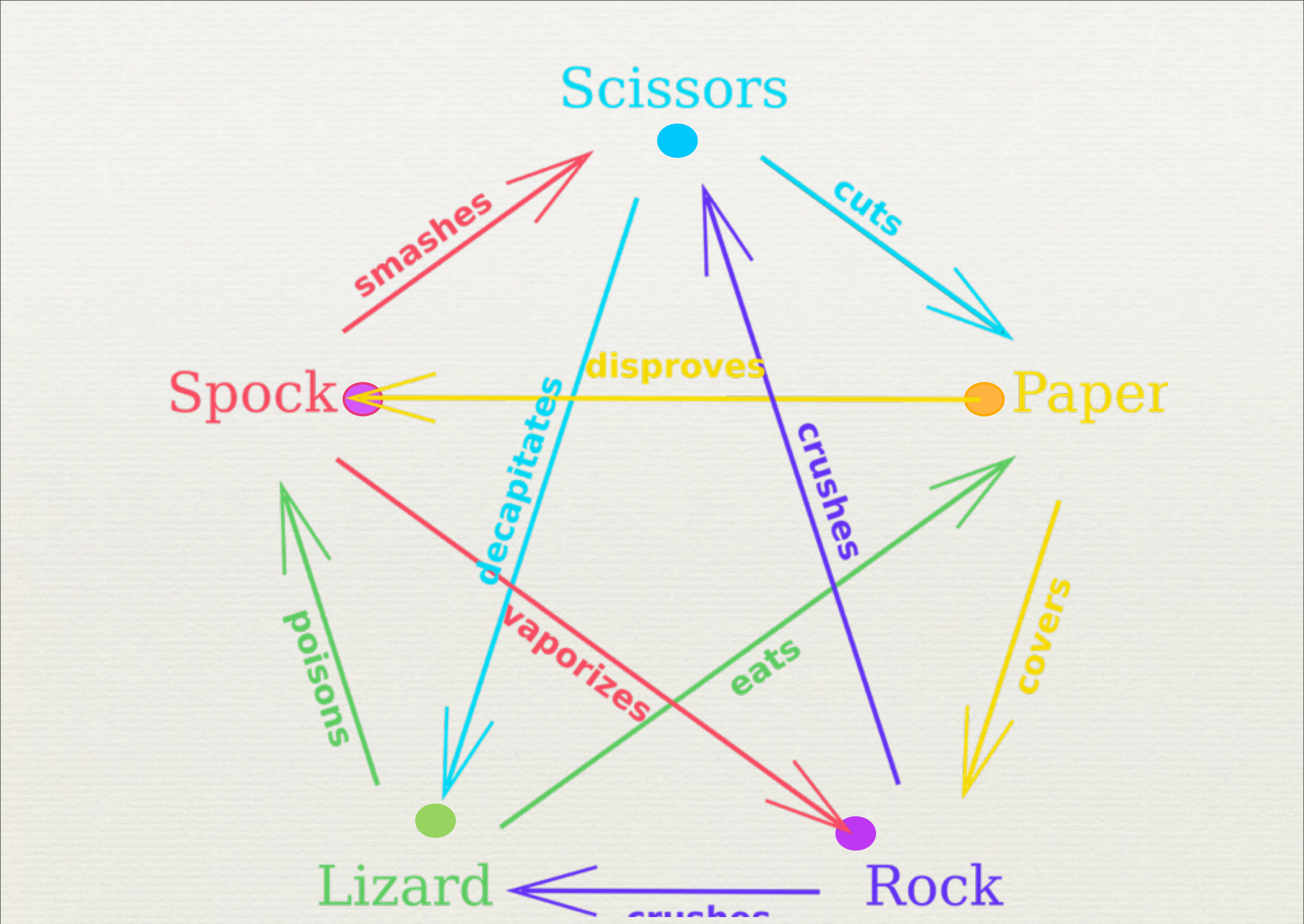
There are 6 platonic solids in 4D, the simplex, hypercube, crosspolytope, 24cell, 120 cell and 600 cell.

we first look at them by projecting orthogonally onto 3D space:

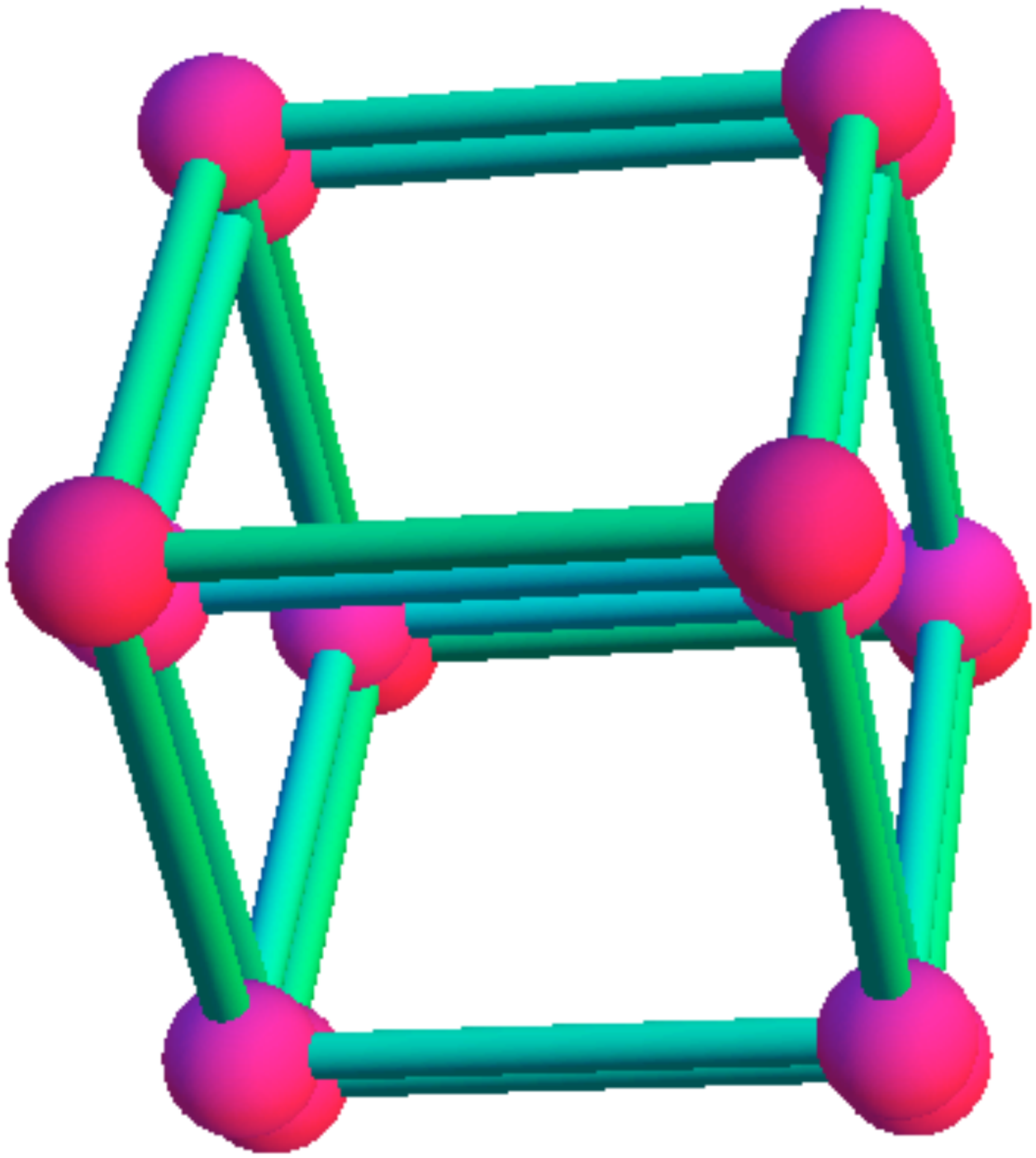


Polychora
5 cell, pentatope

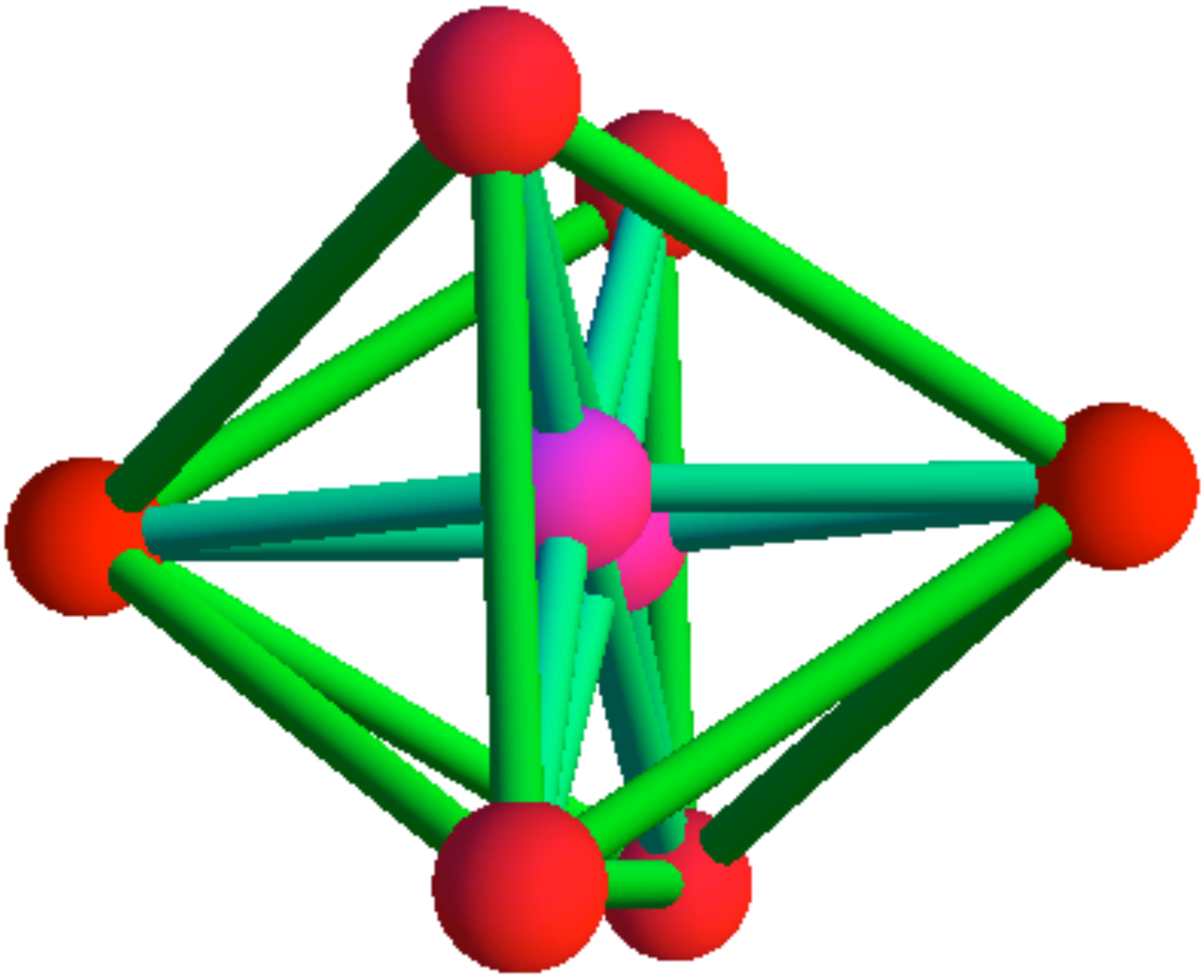



Scissors
smashes
cuts
Spock
disproves
Paper
decapitates
crushes
poisons
vaporizes
eats
covers
Lizard
Rock

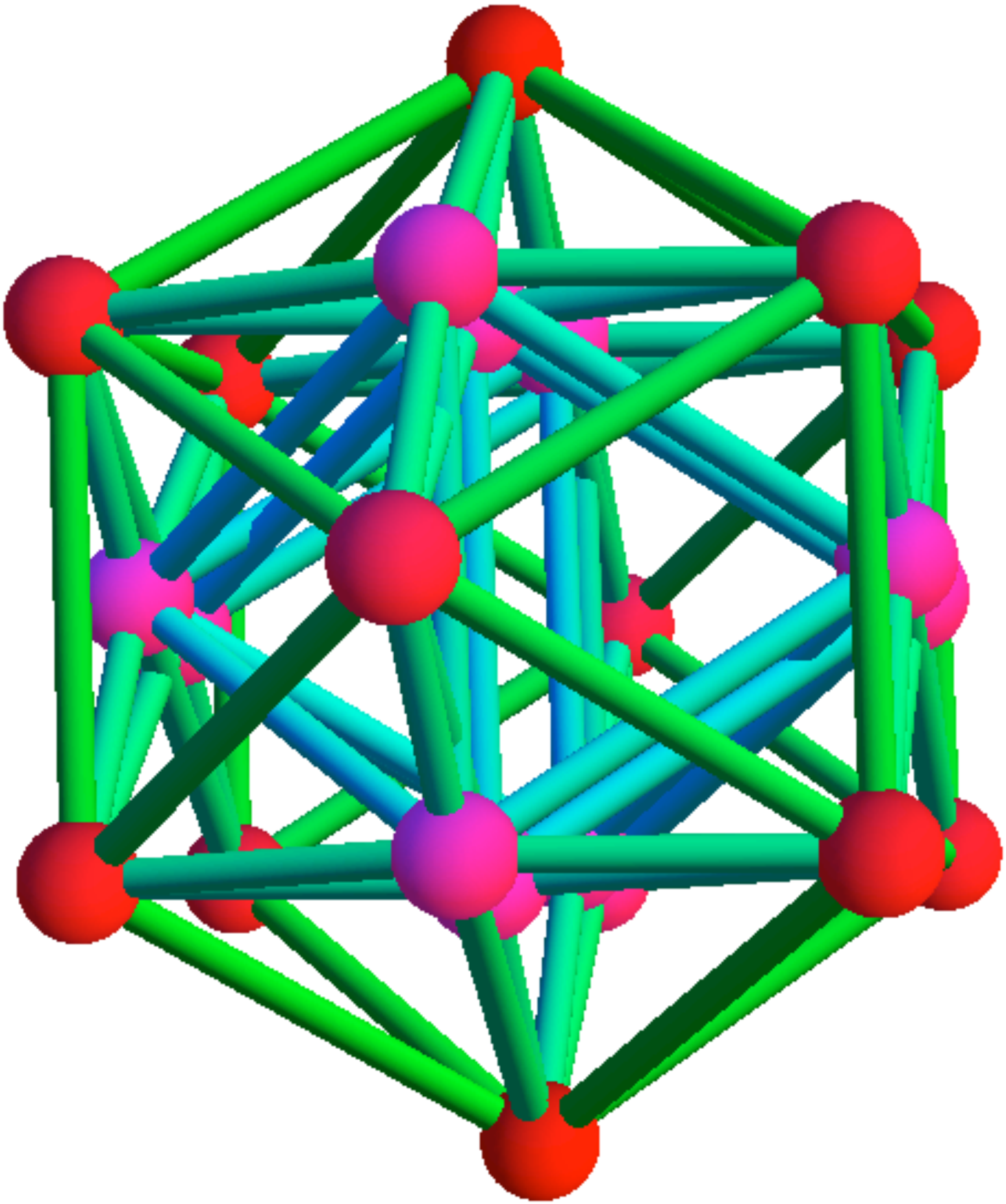
Polychora
8 cell
tesseract



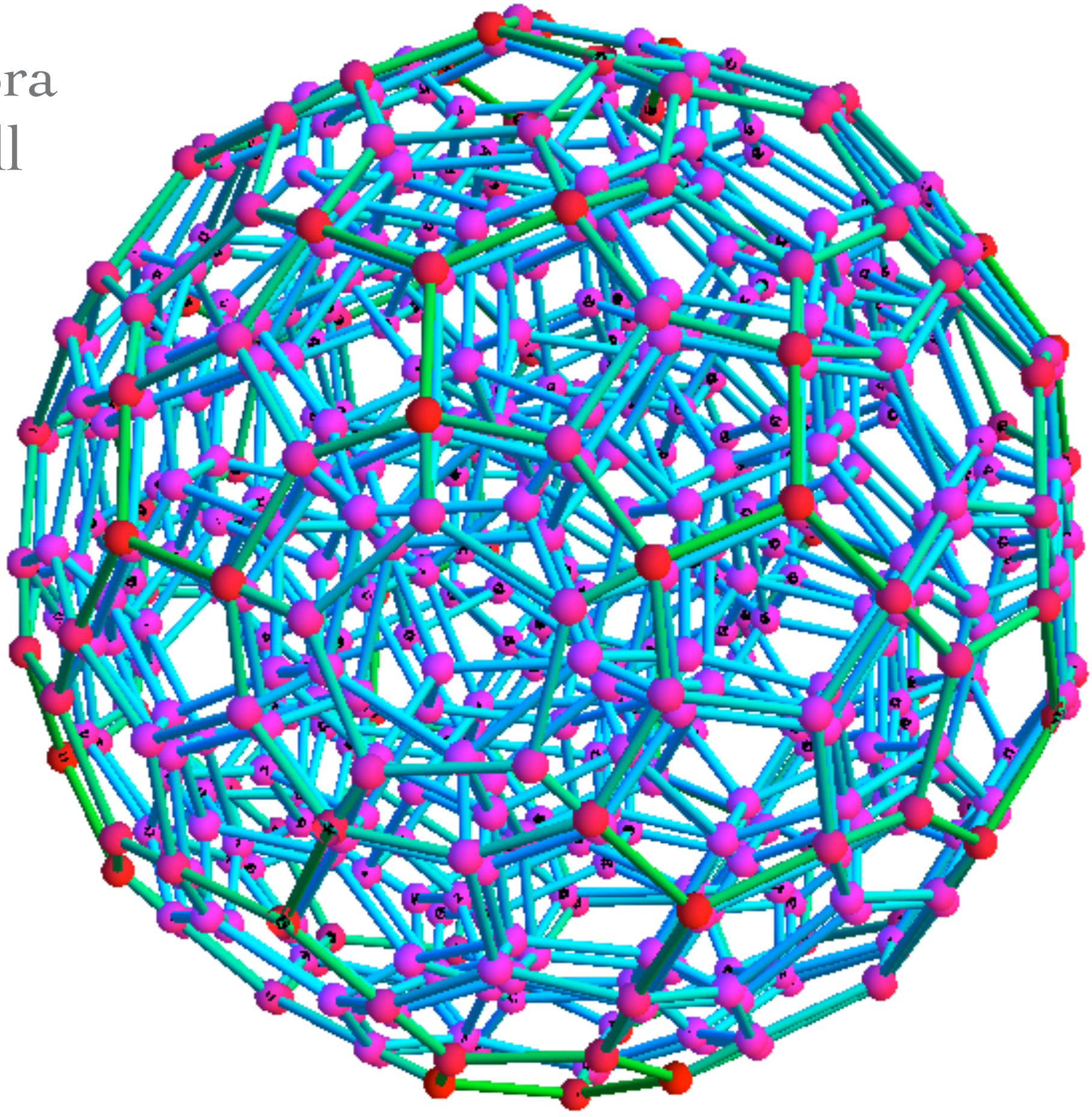
Polychora
16 cell



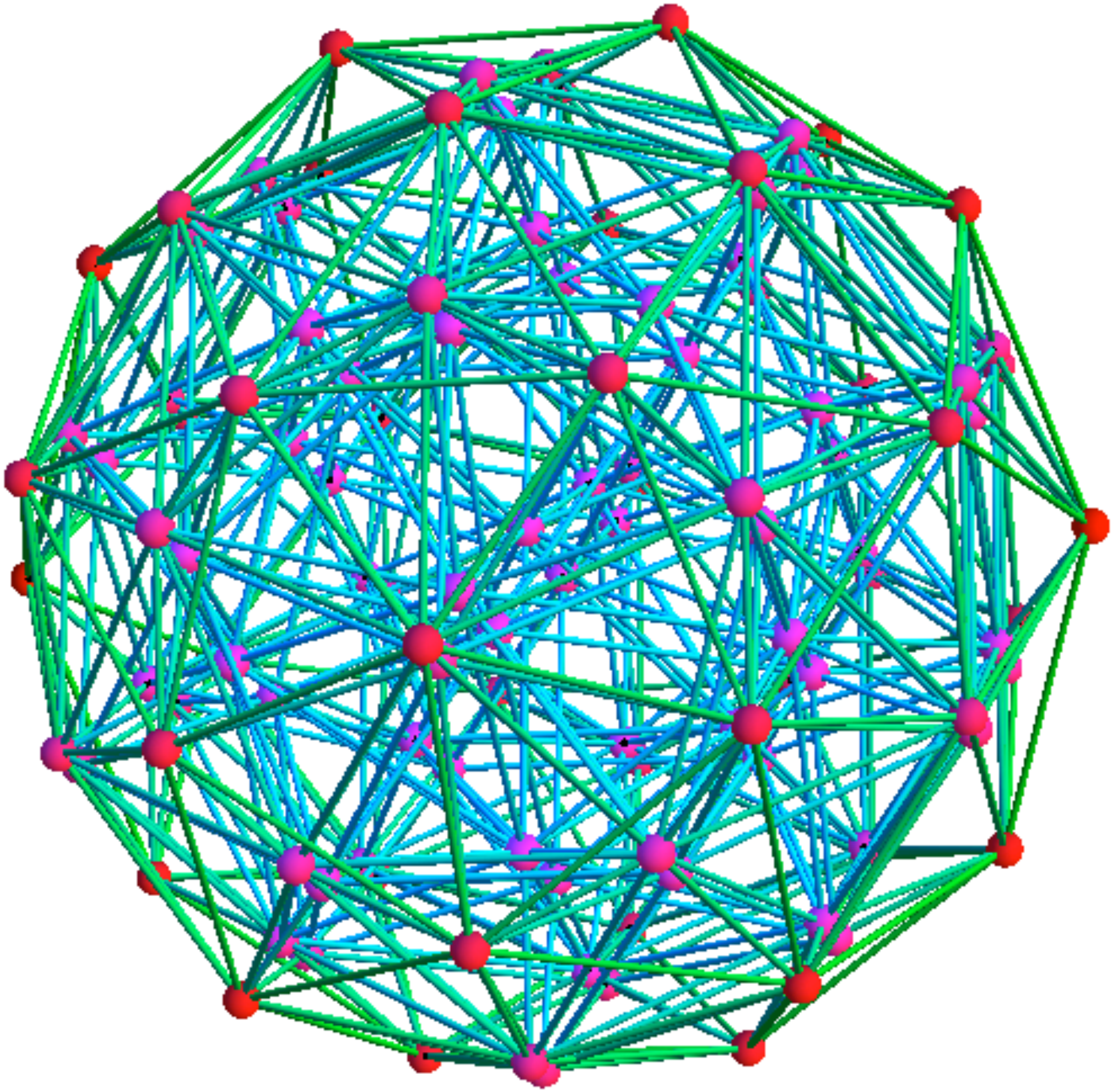
24 cell



Polychora
120 cell



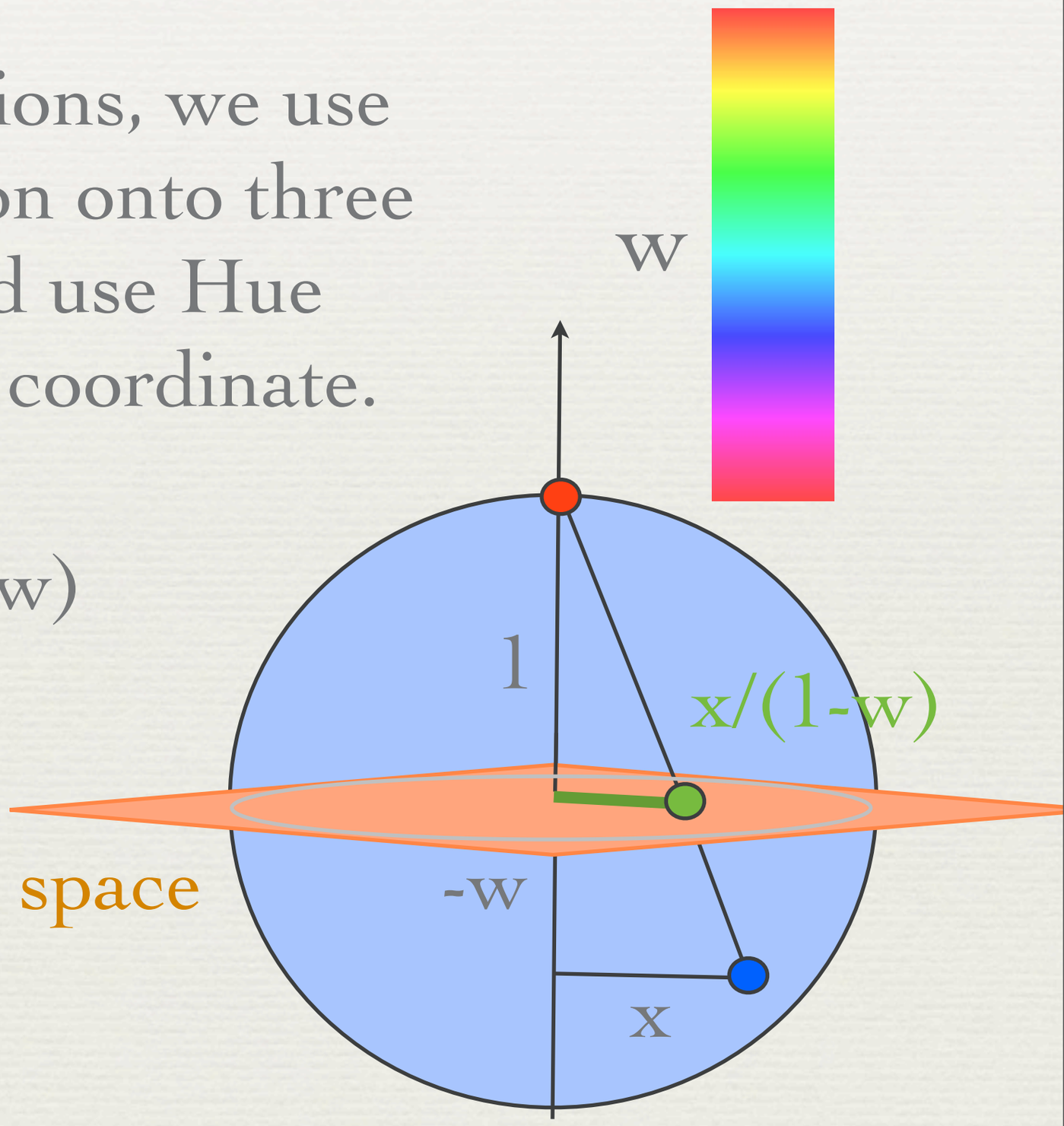
Polychora
600 cell



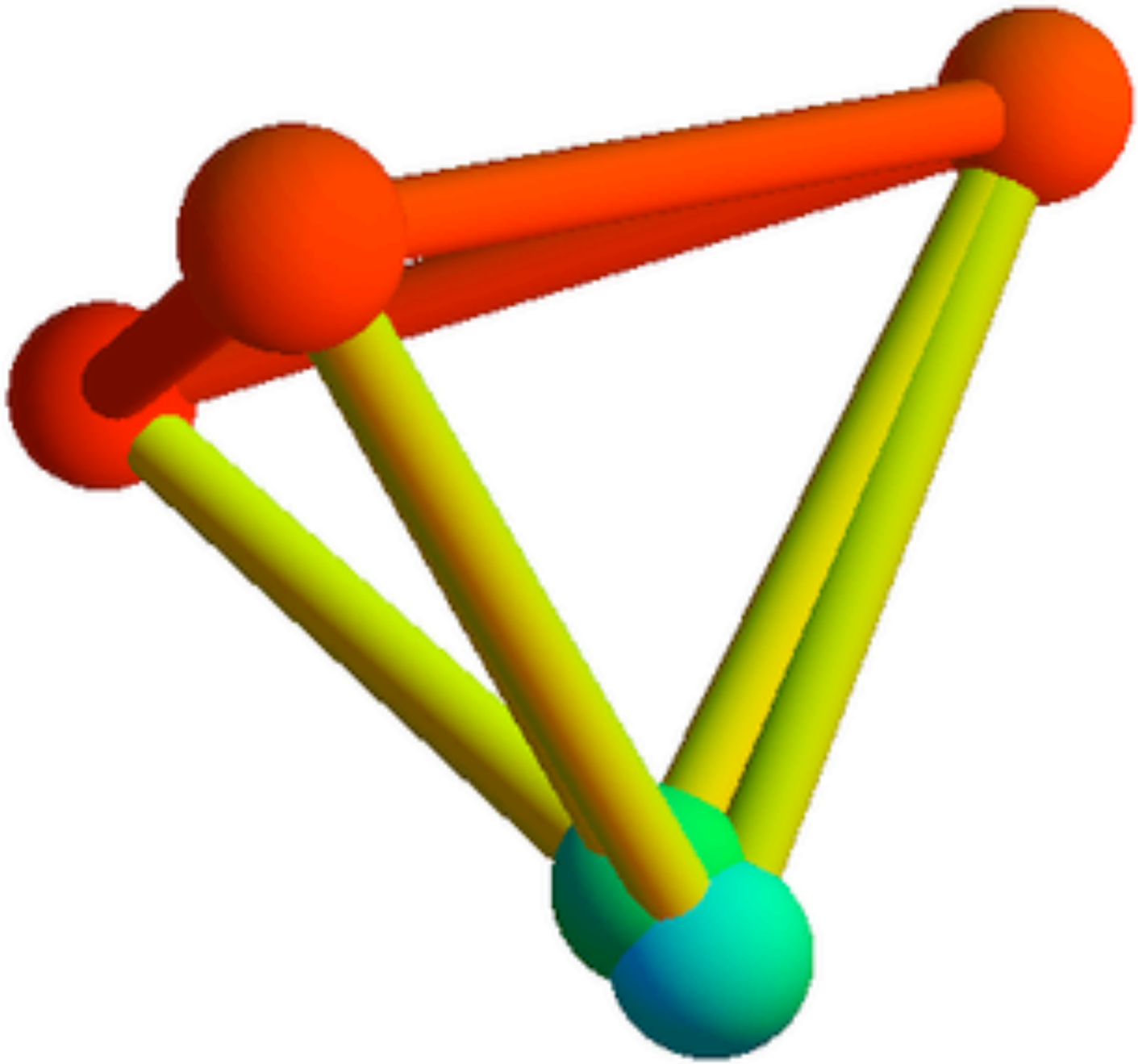
Stereographic projection

In the following animations, we use a stereographic projection onto three dimensional space and use Hue color to encode the 4th coordinate.

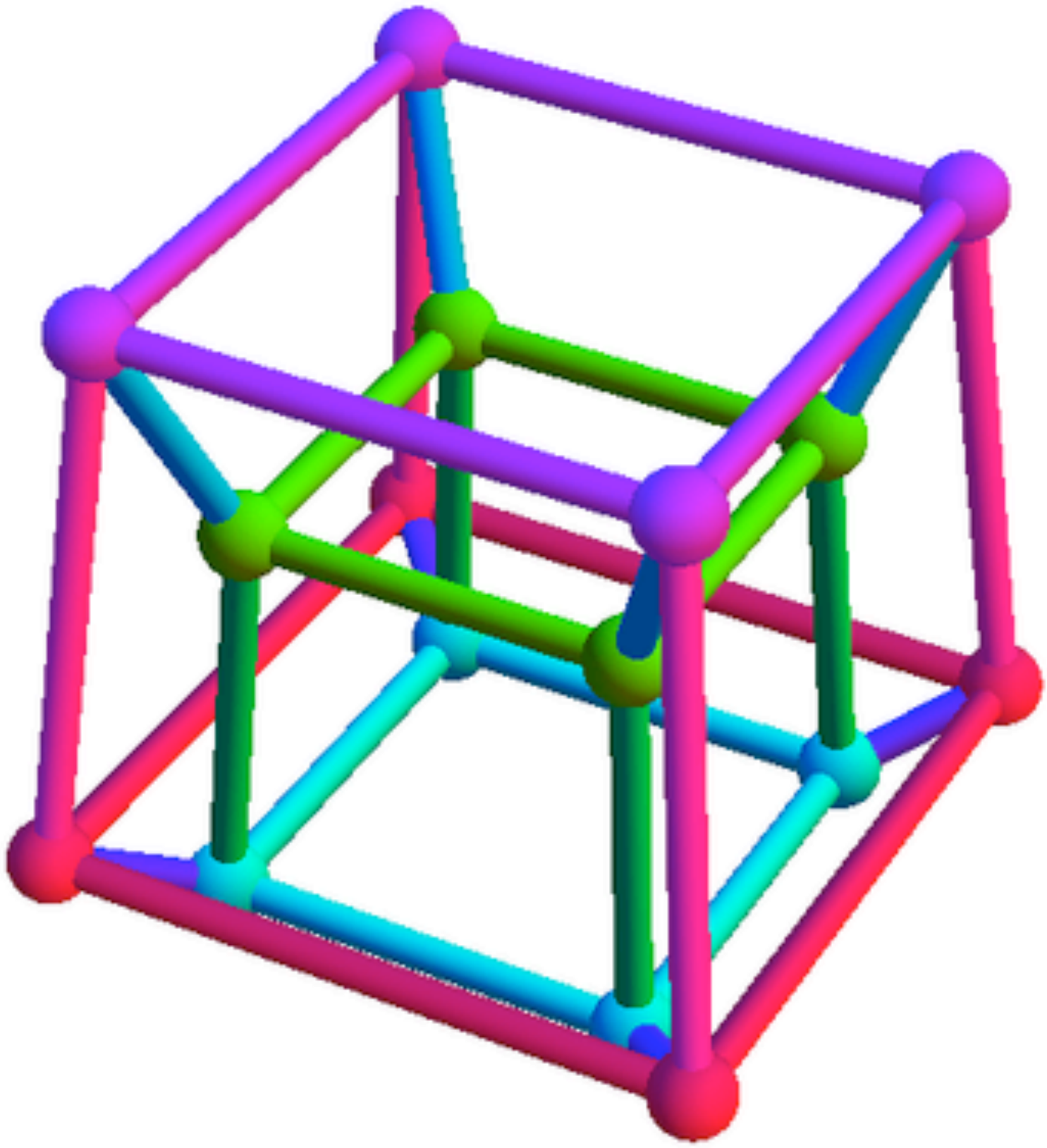
$$P(x,y,z,w) = (x/(1-w),y/(1-w),z/(1-w)$$



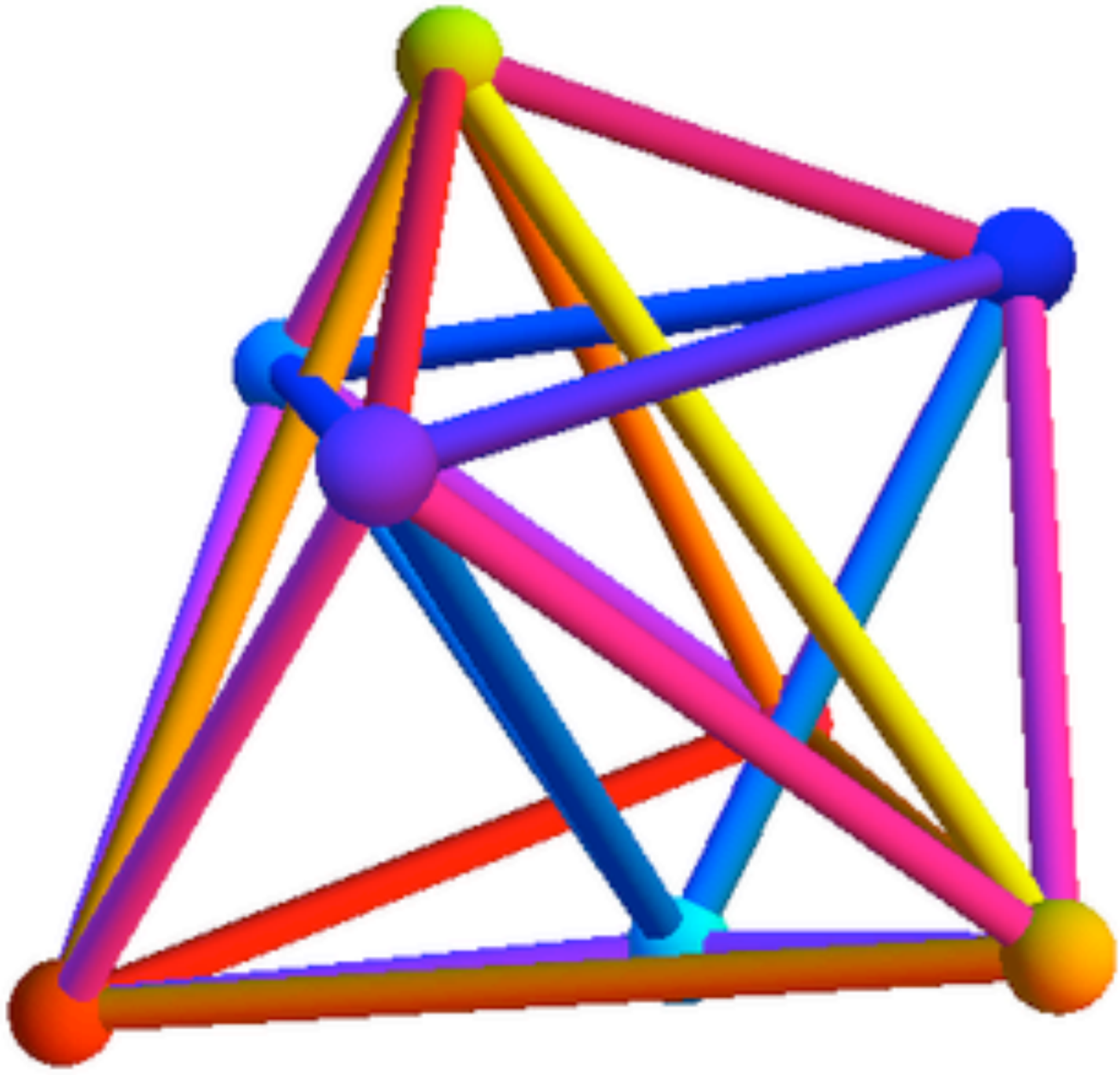
5 cell



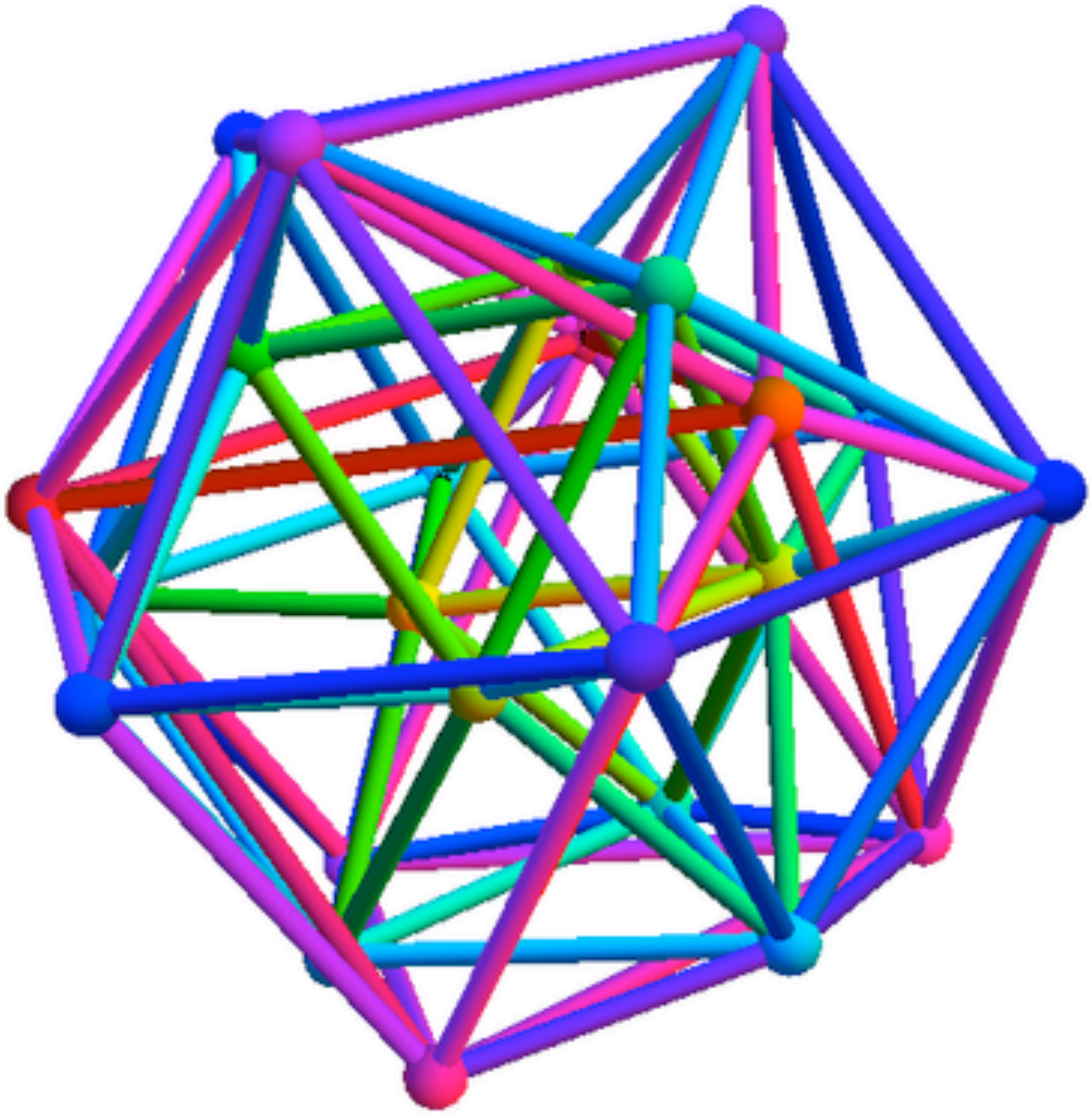
8 cell
tesseract



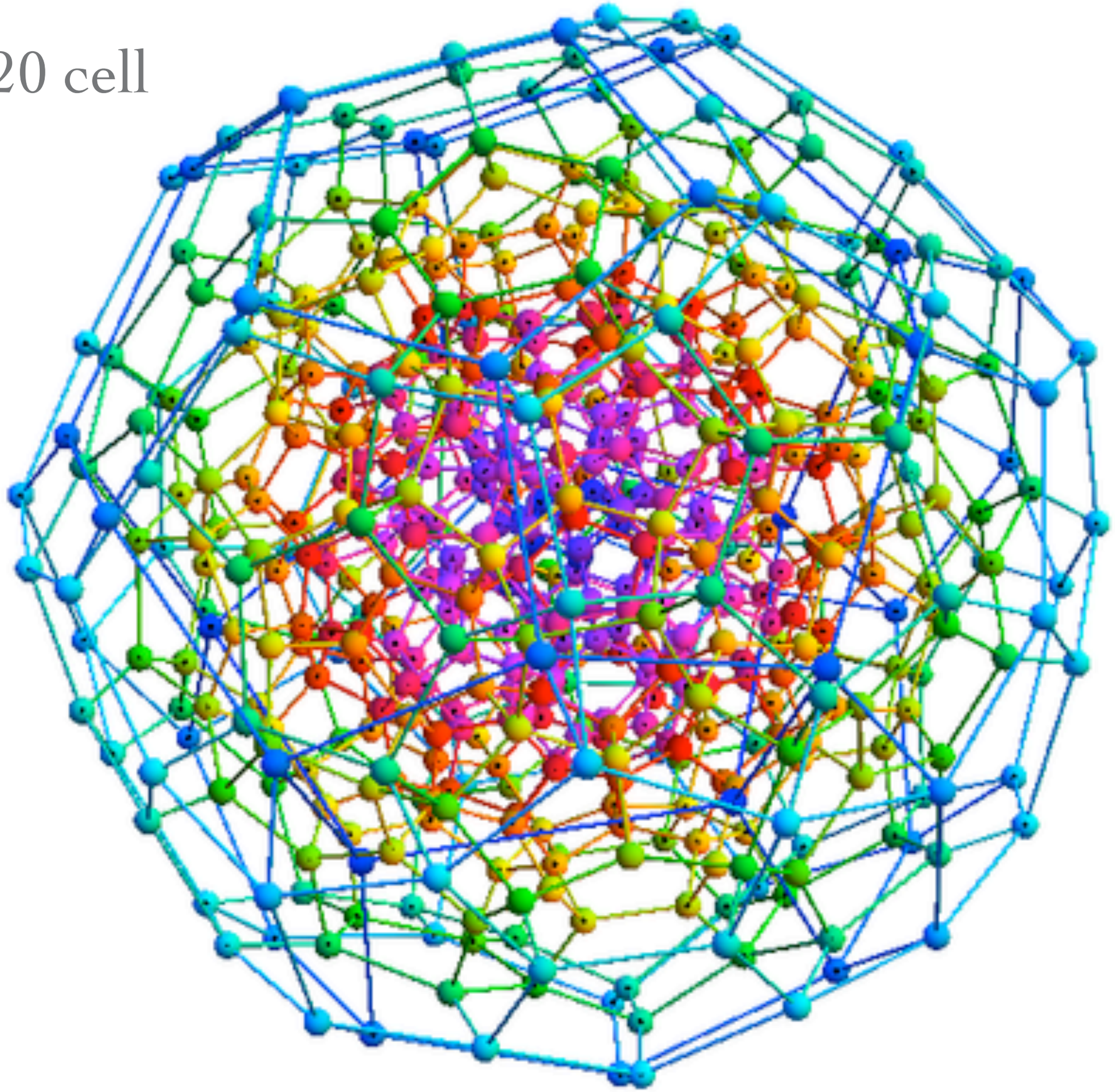
16 cell



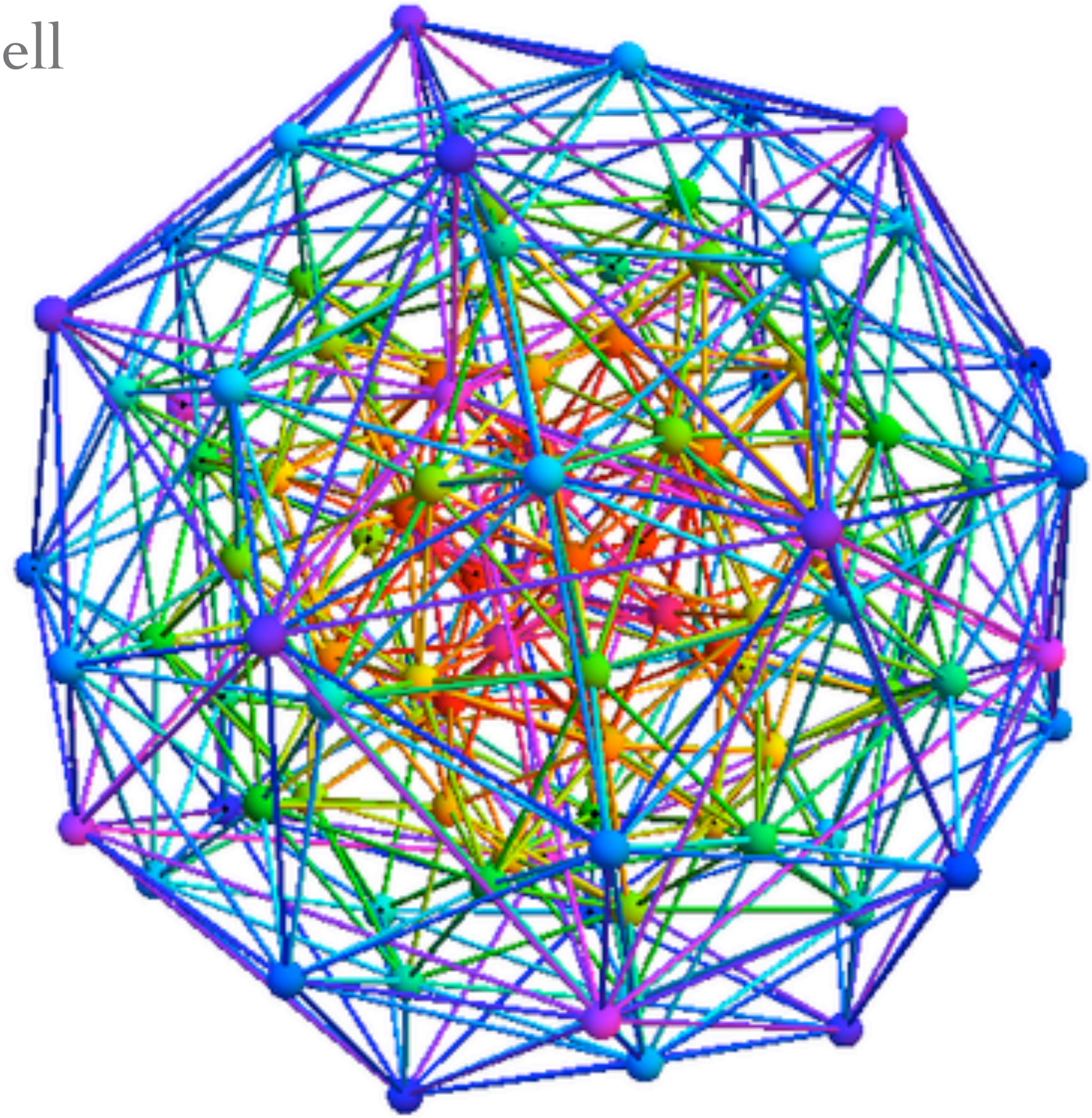
24 cell



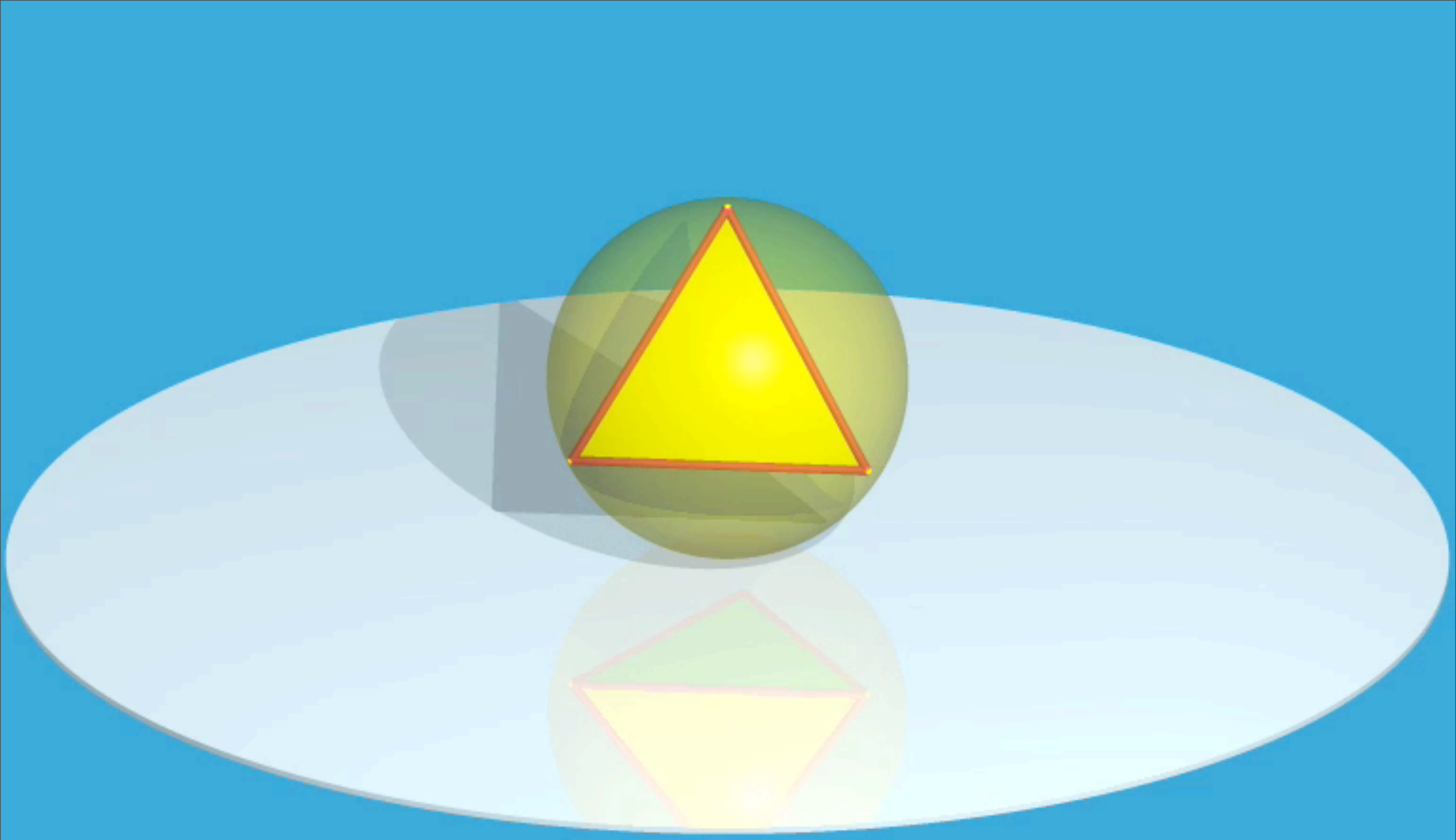
120 cell



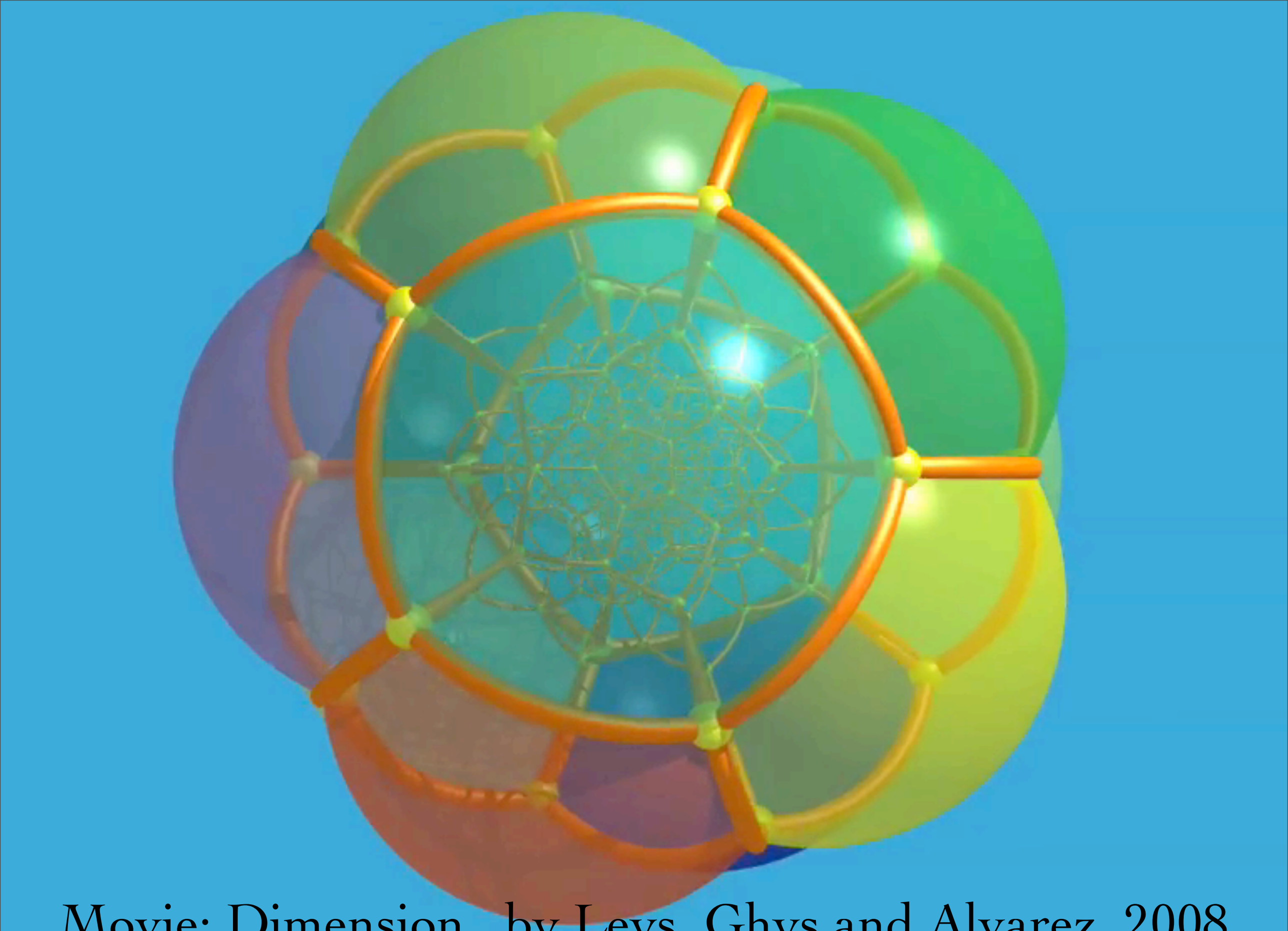
600 cell



Gluing onto the sphere



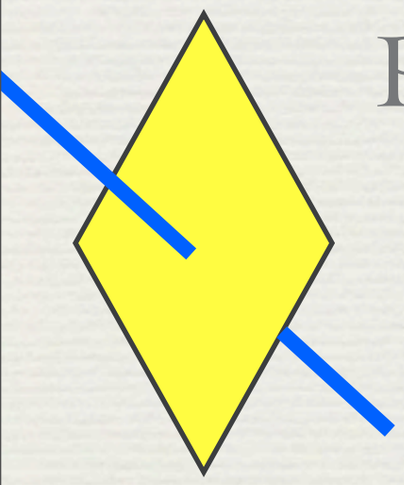
Movie: Dimension, by Leys, Ghys and Alvarez, 2008



Movie: Dimension, by Leys, Ghys and Alvarez, 2008

Schläfli's Gem

$$v-e+f-s=0$$

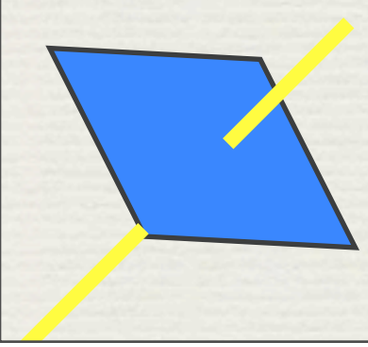


Proof: if the right hand side is constant, then it has to be 0 by duality!

$$v-e+f-s$$

$\updownarrow$ duality changes sign

$$s-f+e-v$$

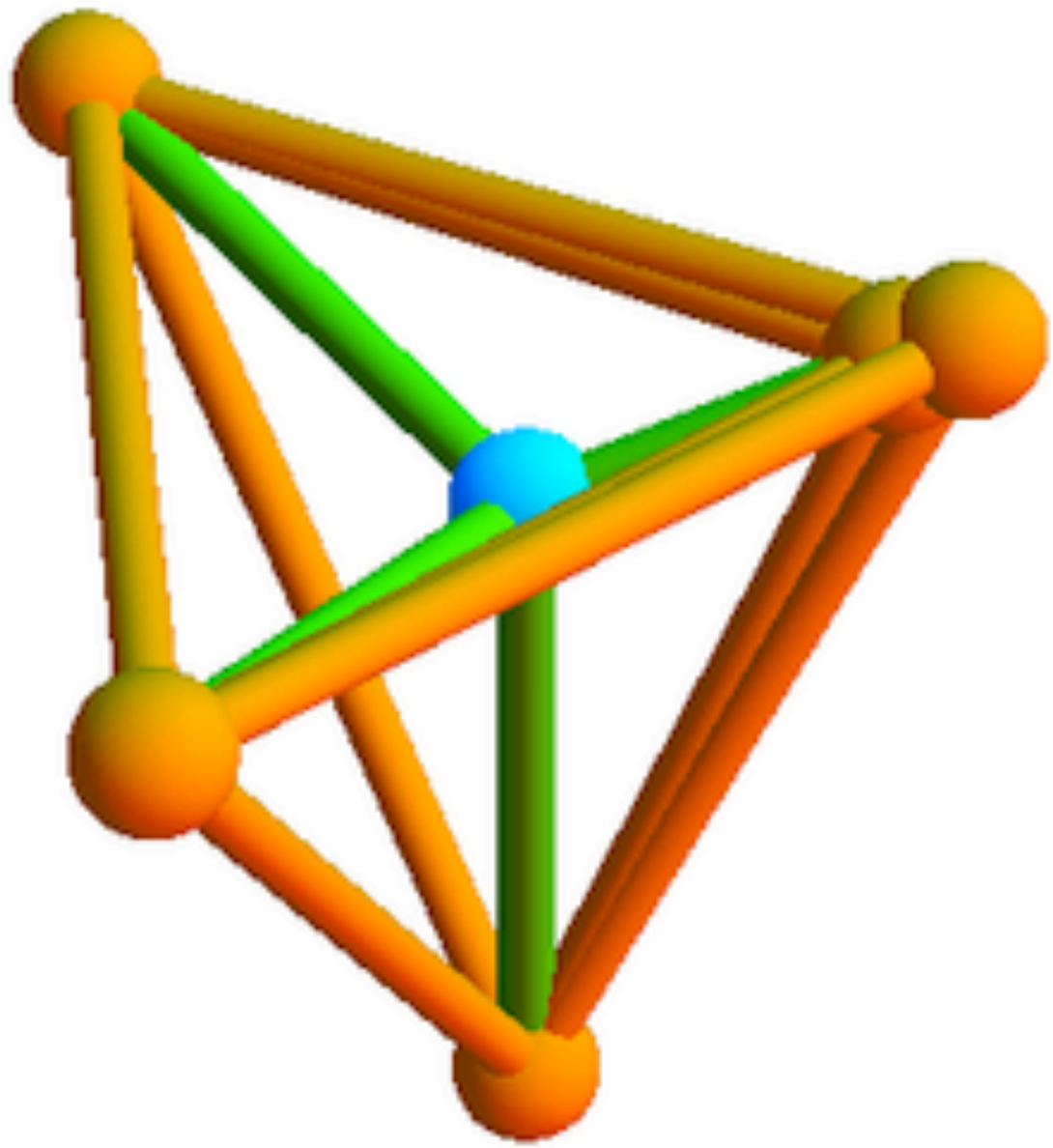


Theorem of Schläfli

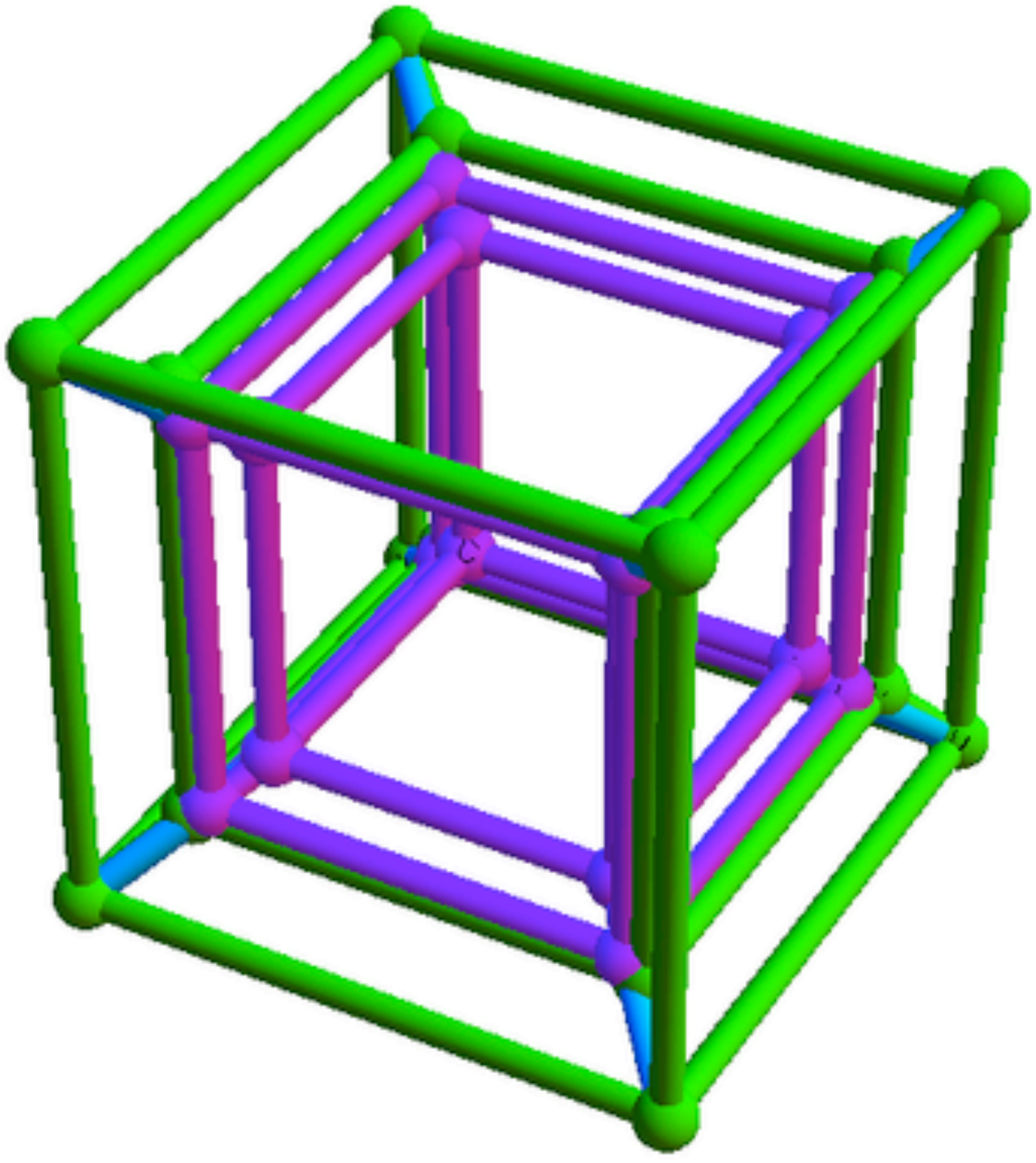
In dimensions 5 and higher, there are 3 platonic solids: the simplex, hypercube and cross polytop.

Polyterons

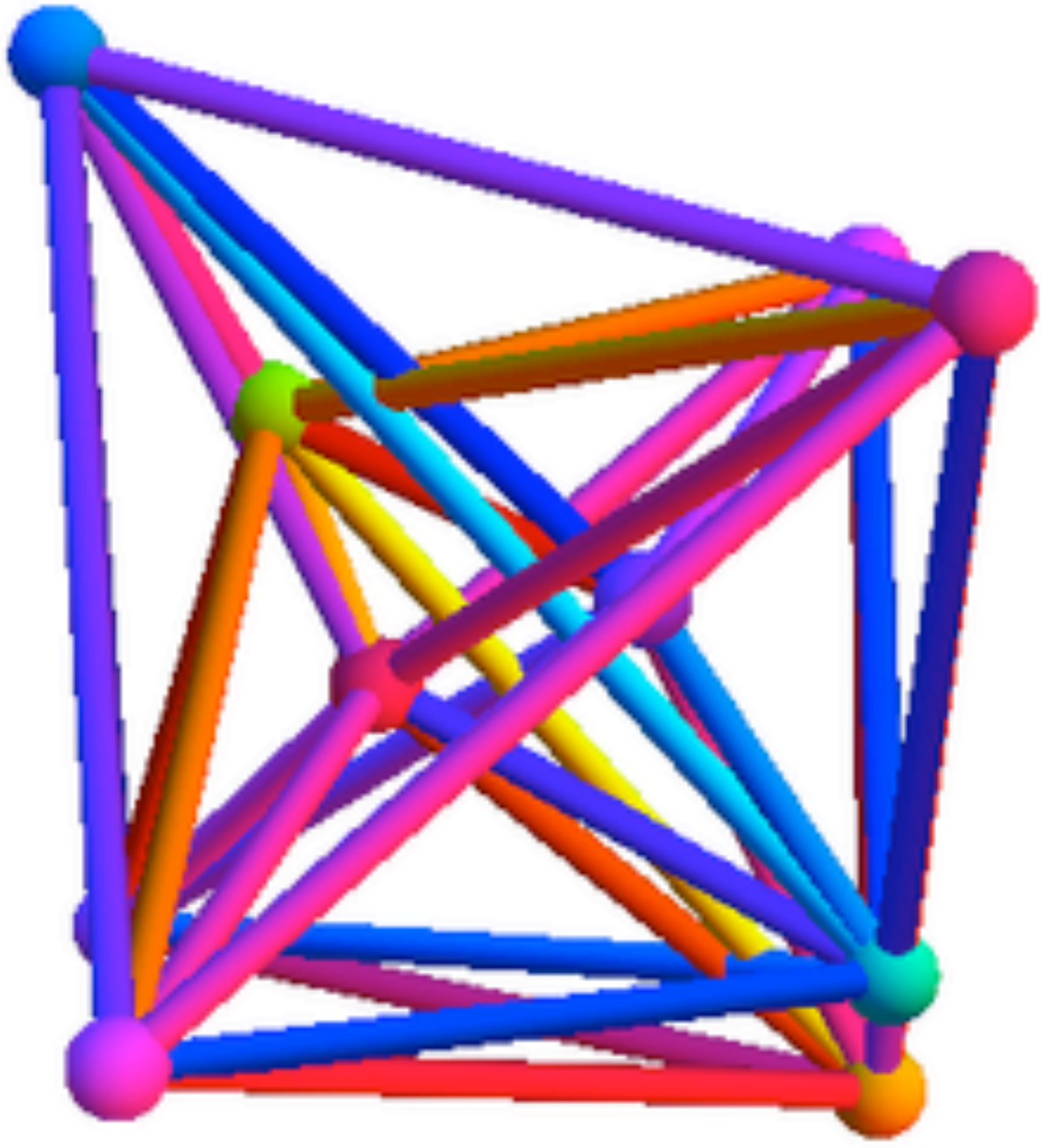
Polyteron
6 cell



Polyteron hypercube



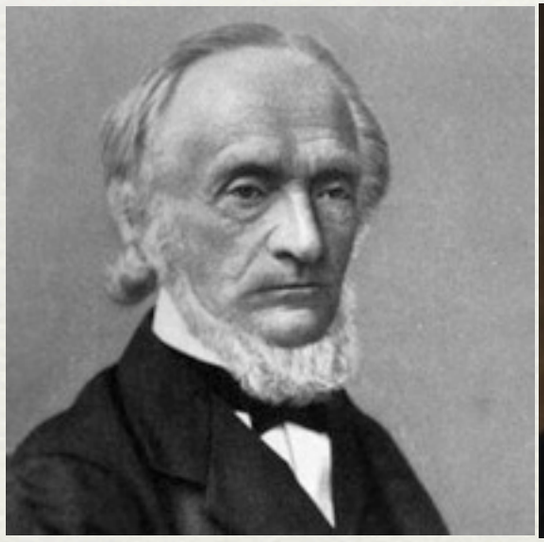
Polyteron crosspolytop

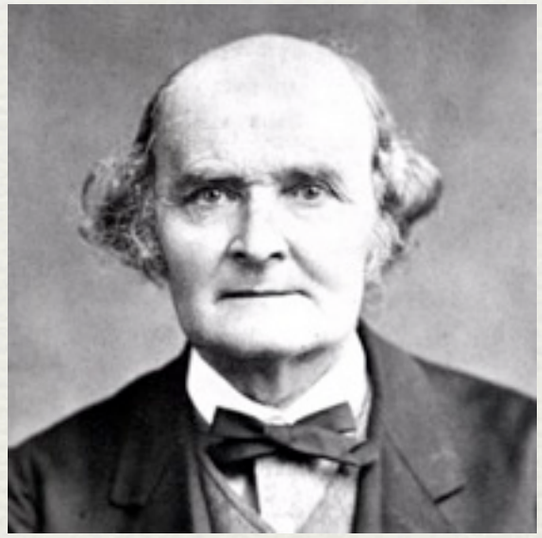


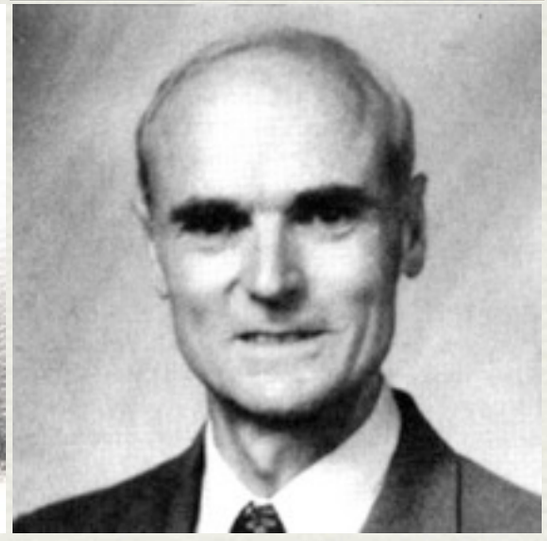
Mathematicians



















400 200 1600 1800 2000

300 100 0 100 1500 1700 1900

Niedergrasswil

Grasswil

Grasswil, Seeberg, Switzerland

Obergrasswil

Spiegelberg

The prince of Polytopia: Ludwig Schläfli

Sunday, December 6, 2009

St Lukes

Cork, Co. Cork, Ireland

11–12 Pembroke St, Cork, Co. Cork, Ireland

The princess of Polytopia: Alicia Boole Stott

Credits

- For the polytope animation with Mathematica music by Tchaikovsky: Swan Lake Suite, by Albert Lizzio: London Festival orchestra.
- 5D polytera generated with Mathematica were illustrated with music of Juno reactor, Wardogs (Labyrinth) and Tanta Pena (Gods and monsters), Guardian Angles (Labyrinth).
- Most objects were generated with Mathematica 7 or Povray.
- More credits and literature are on my lecture notes for this talk.
- Movie clip on stereographic projection by movie “dimensions” 2008
- Movie clip “platonic solid rock” by Dan Radin (DVD)
- Movie clip “Cube” and “Hypercube”, “Phantom Tollbooth”
- Movie clip “Flatland”