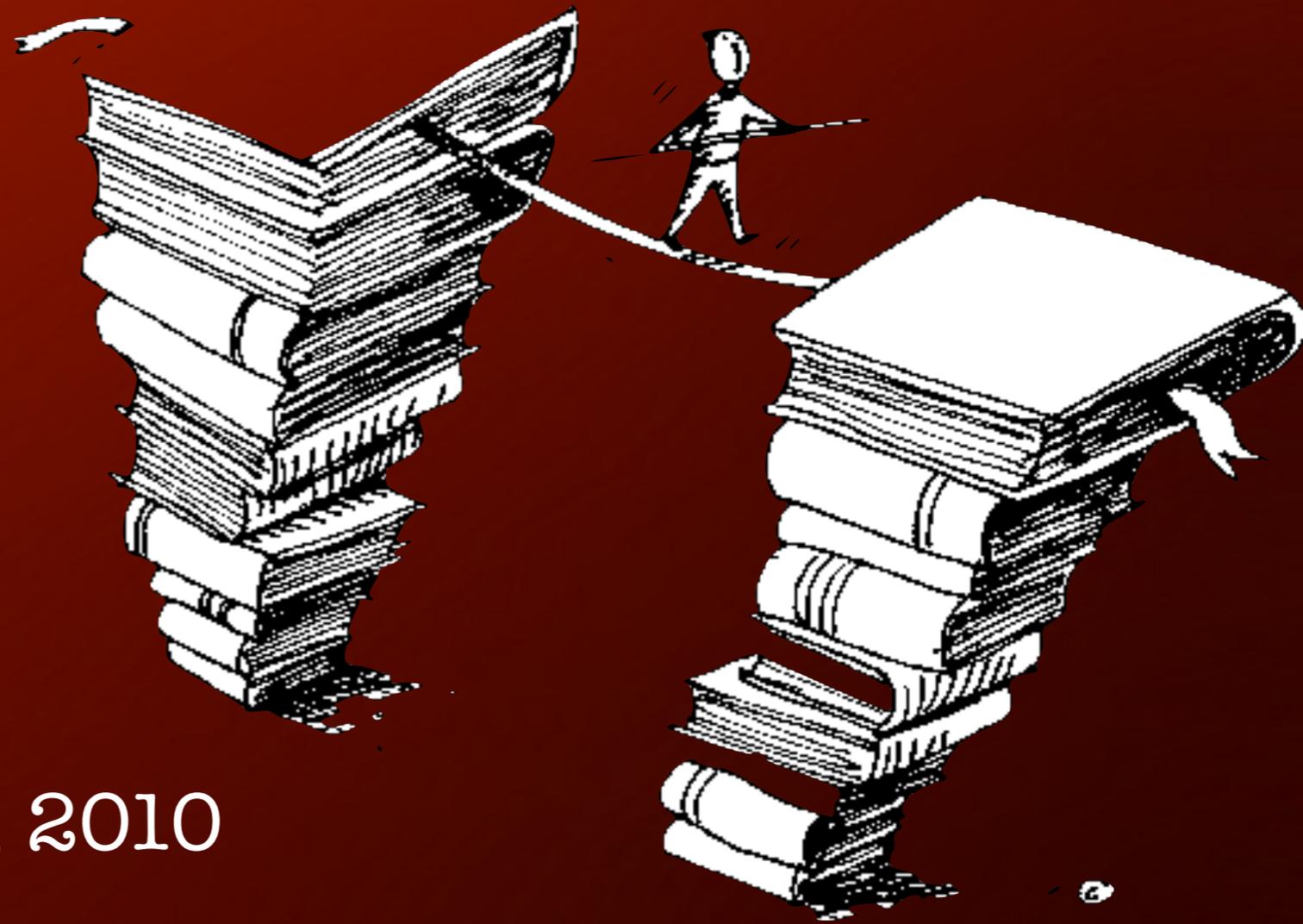


# Balancing Risks in Research and Teaching



Oliver Knill

September 30, Dunster, 2010

This talk is a risk itself:

I'm getting too personal,  
sound whimsical,

bragging, bitter or naïve.

But this is the topic and I  
consider this part as an  
experiment in taking  
risks.

# Lets jump in!



DIVX  
DIGITAL VIDEO DISC

# Outline

Having been in the position to balance risks of teaching from the junior faculty position perspective for a long time, I might have something to say. The talk is divided into two parts:

- Part I: Risks in Research
- Part II: Risks in Teaching

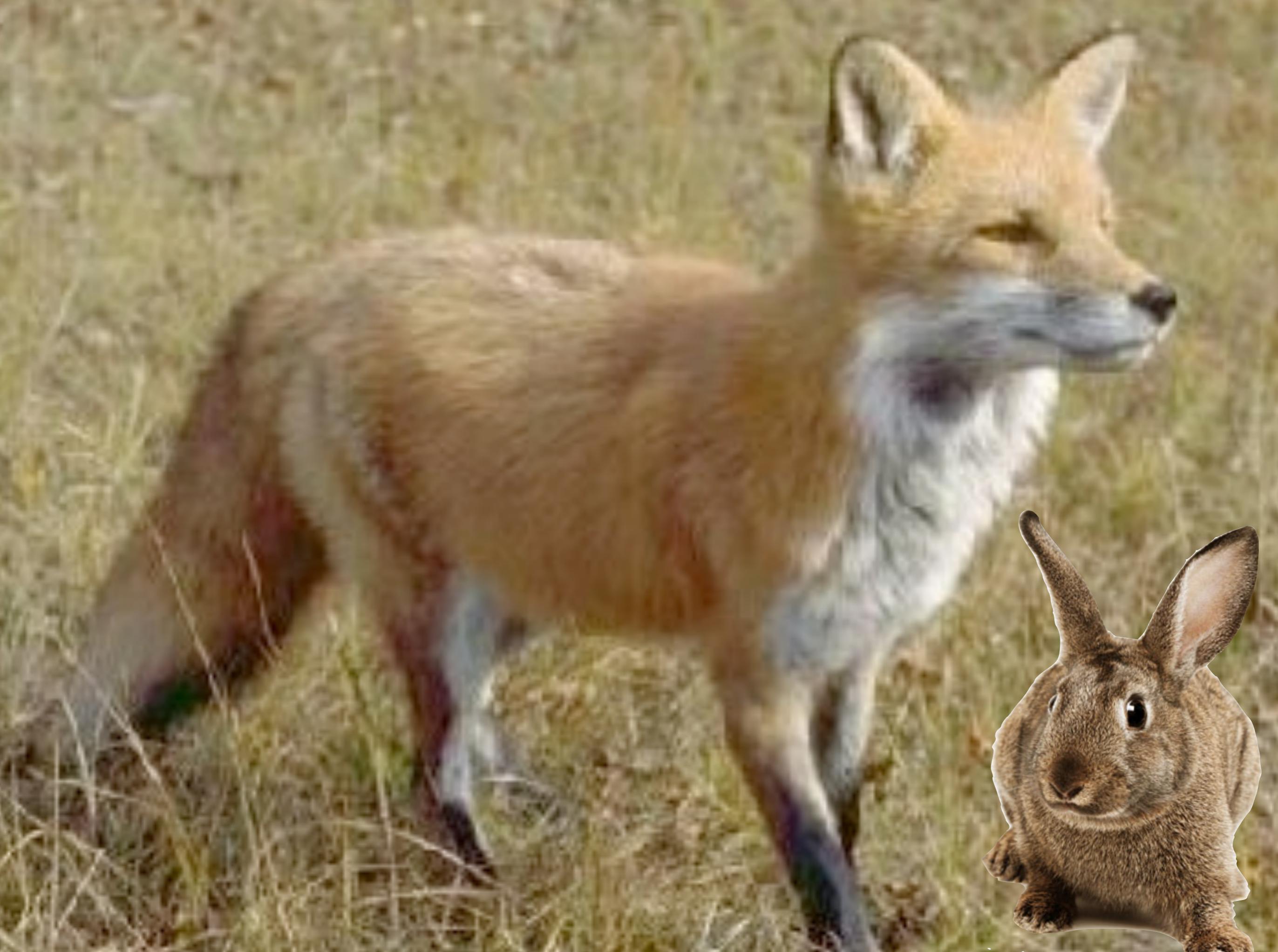
# Part I Research

# Part I: Research

- Advisor
- Letter writers
- Topic
- Independence?

# The Advisor









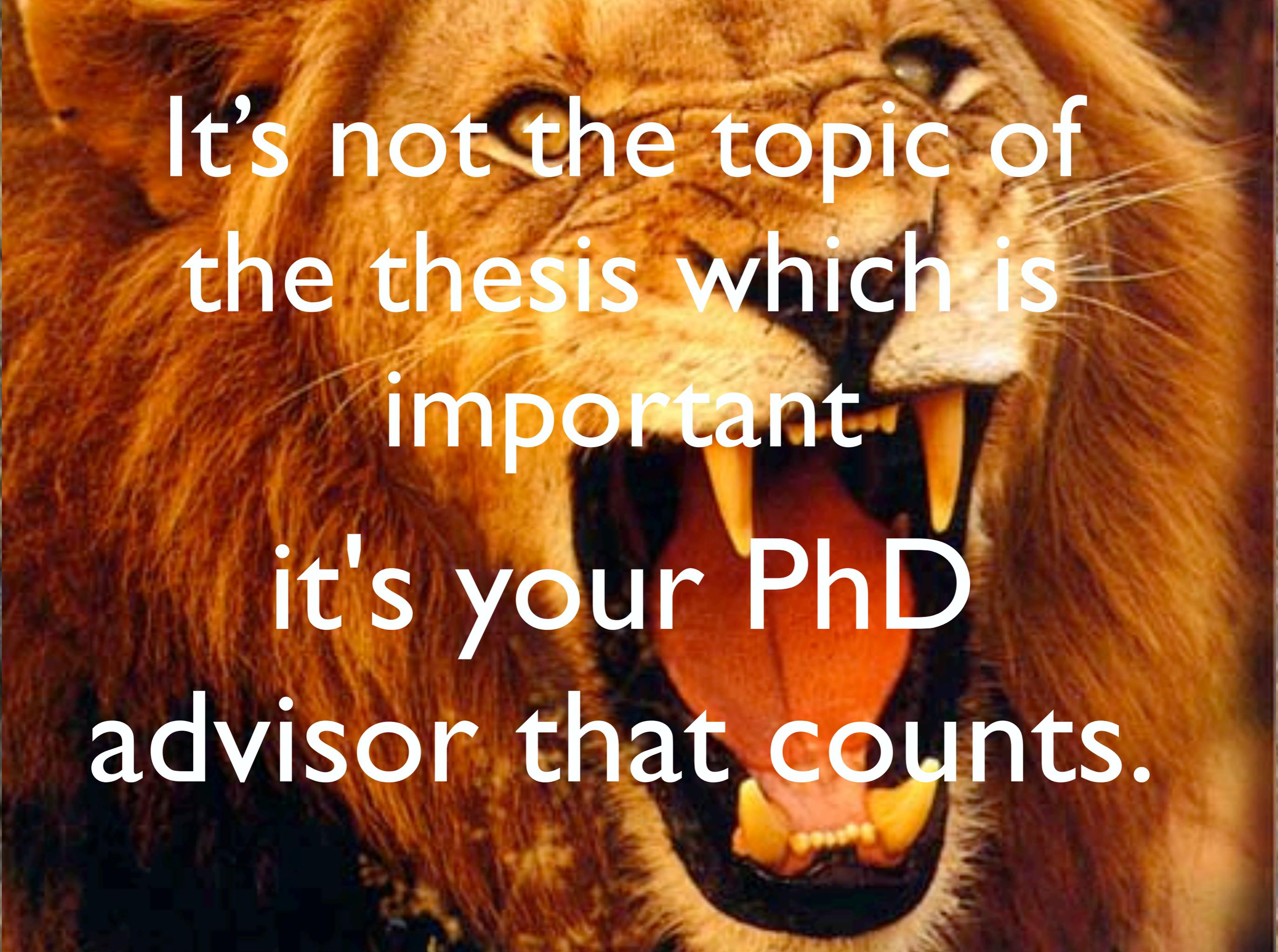






.....

What is the Moral of the  
story?



It's not the topic of  
the thesis which is  
important  
it's your PhD  
advisor that counts.

# MERRIE MELODIES

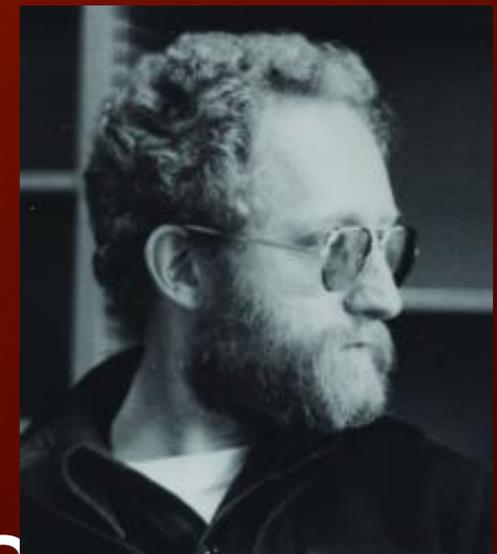
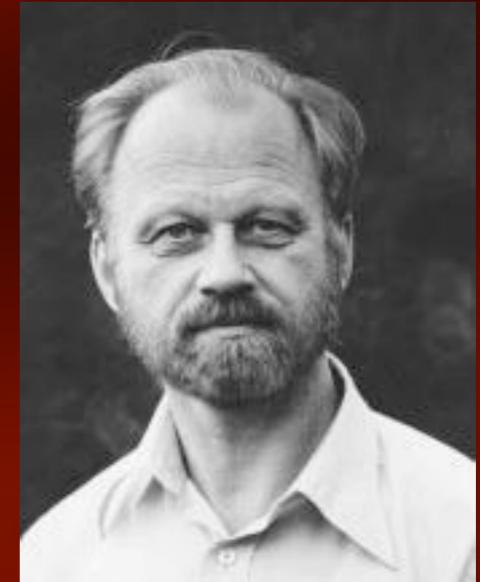
a WARNER BROS. CARTOON

TECHNICOLOR

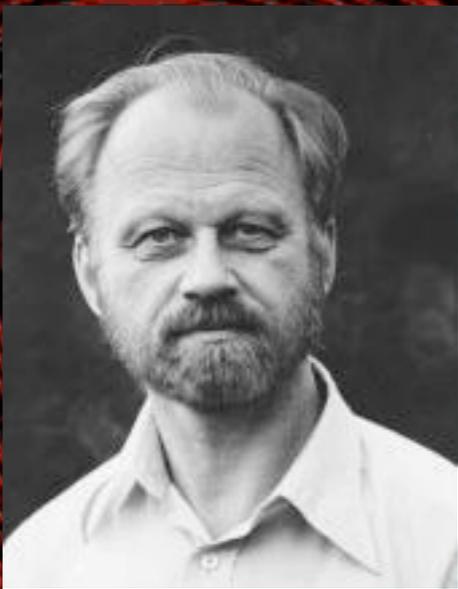


# My Advisors

- Juergen Moser  
Undergraduate advisor
- Oscar Lanford Graduate  
advisor
- Barry Simon, my postdoc  
advisor



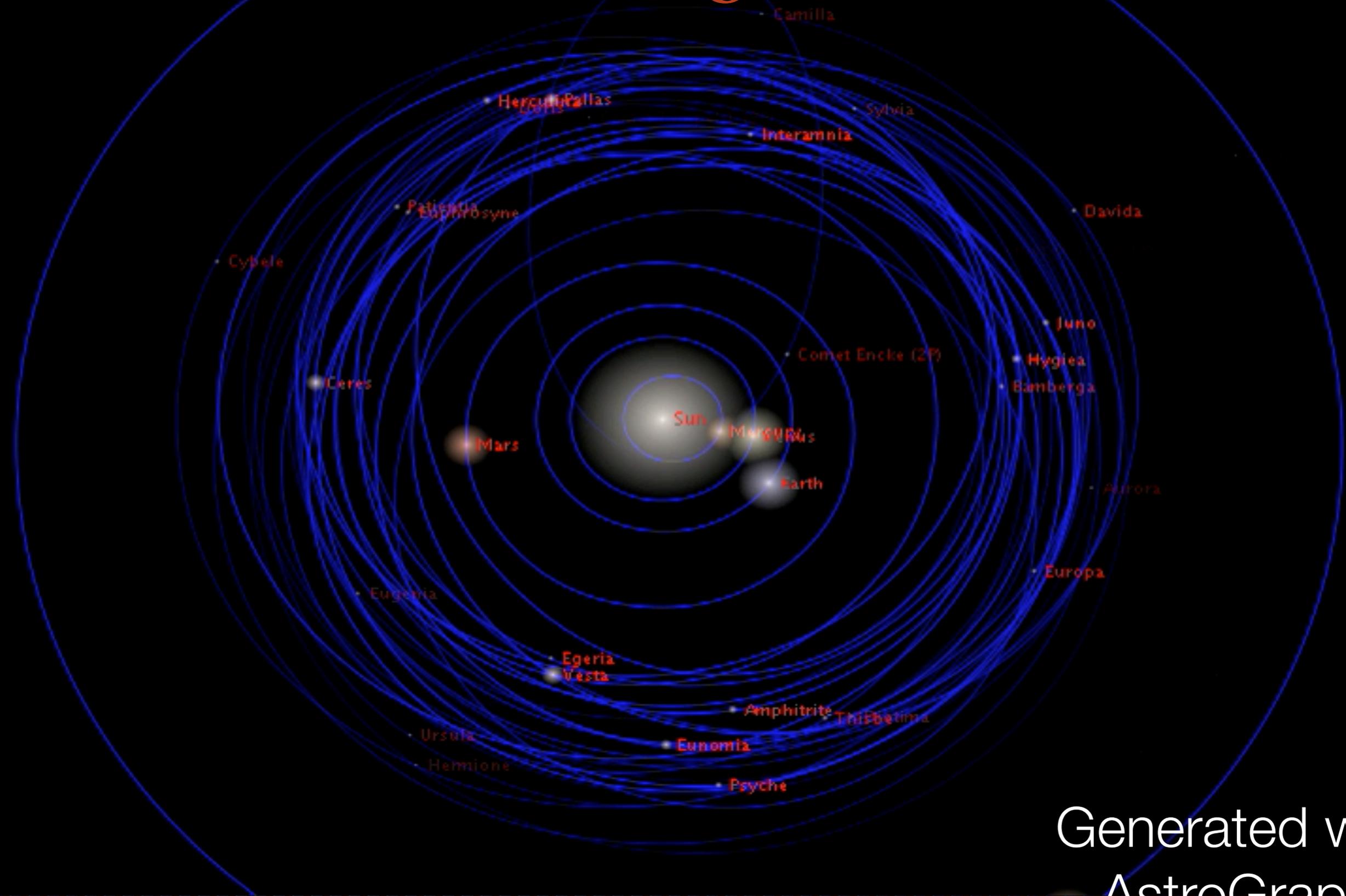
Three Lions!



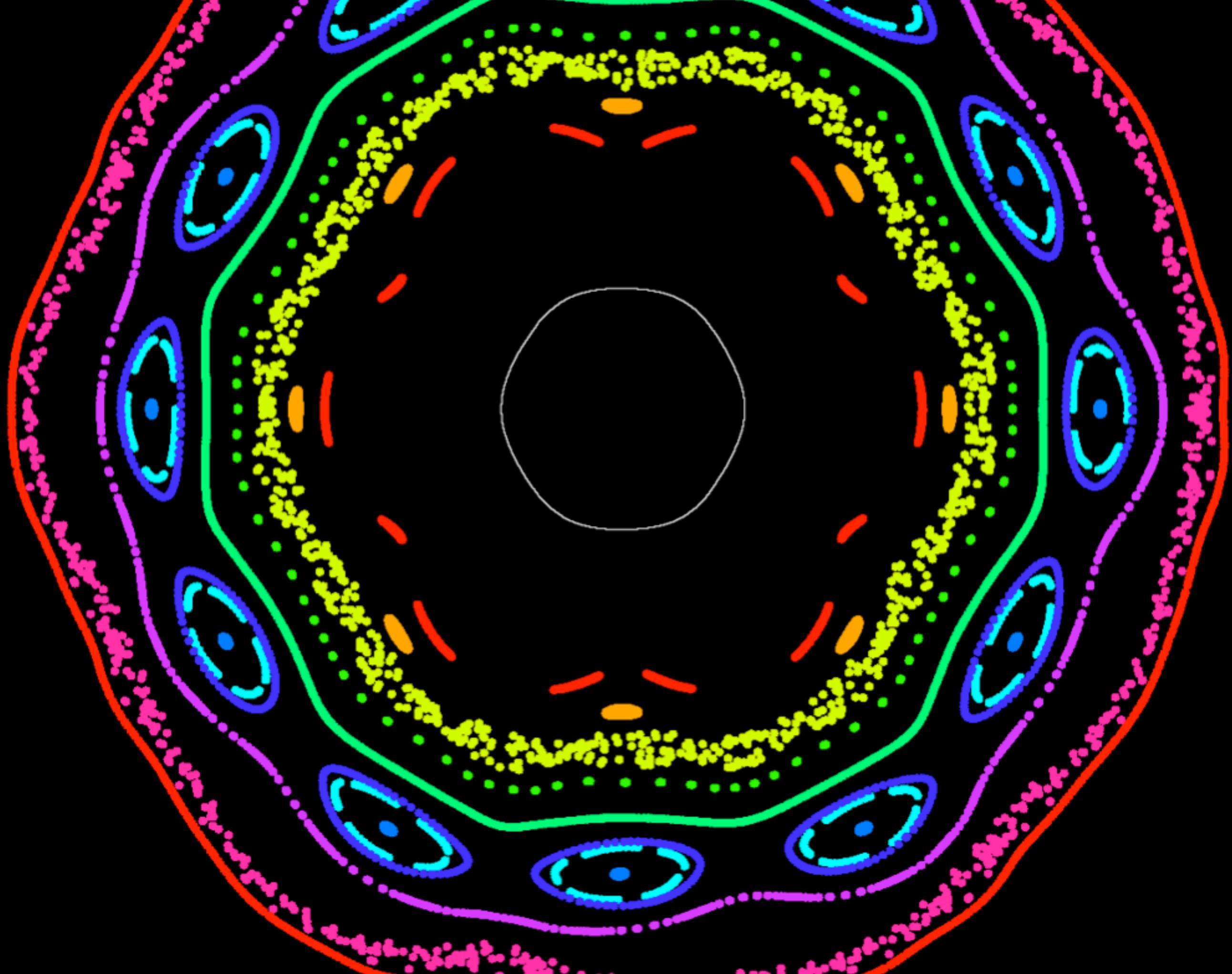
$$T \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$\begin{bmatrix} c \sin(x) + 2x - y \\ x \end{bmatrix}$$

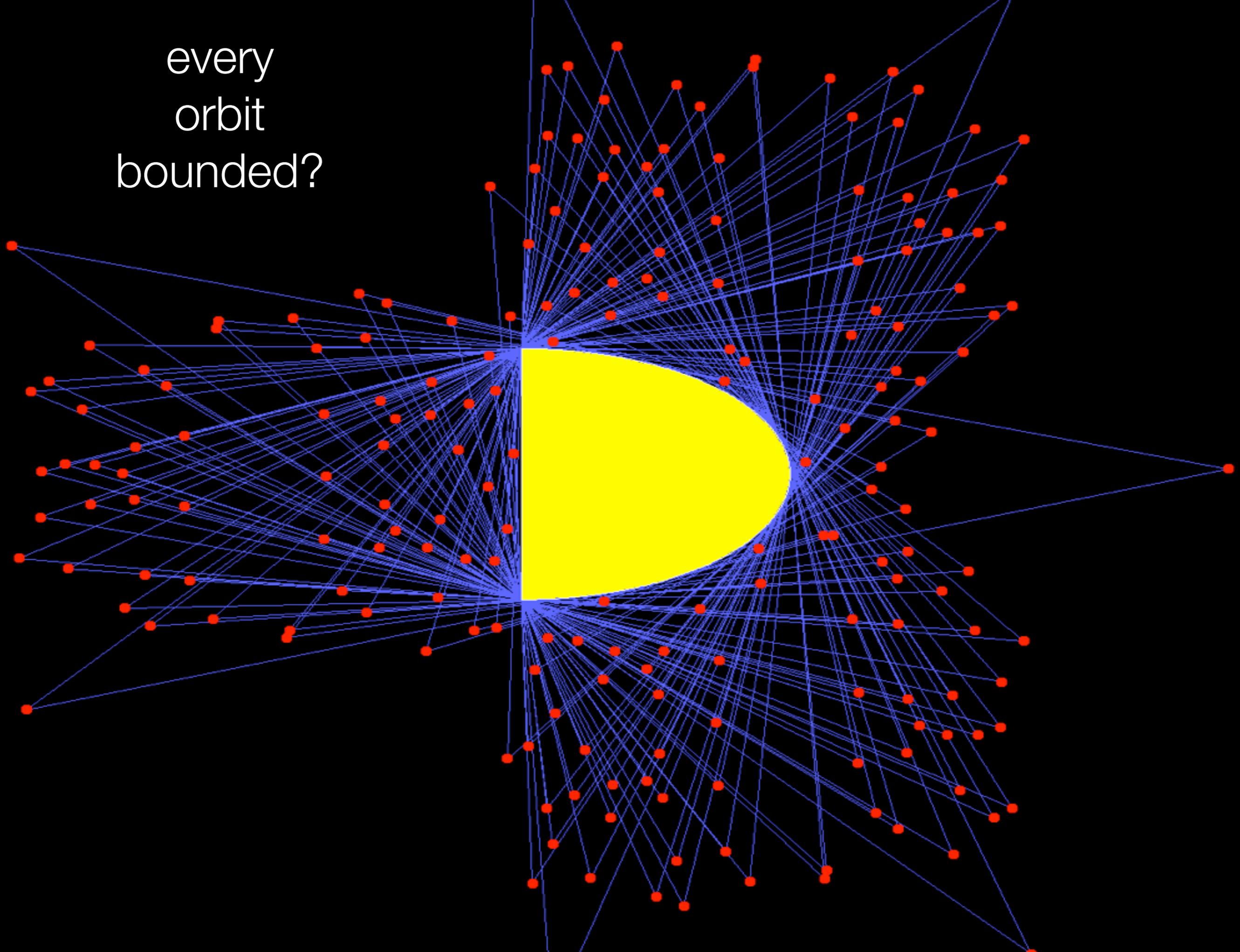
# Planetary Motion

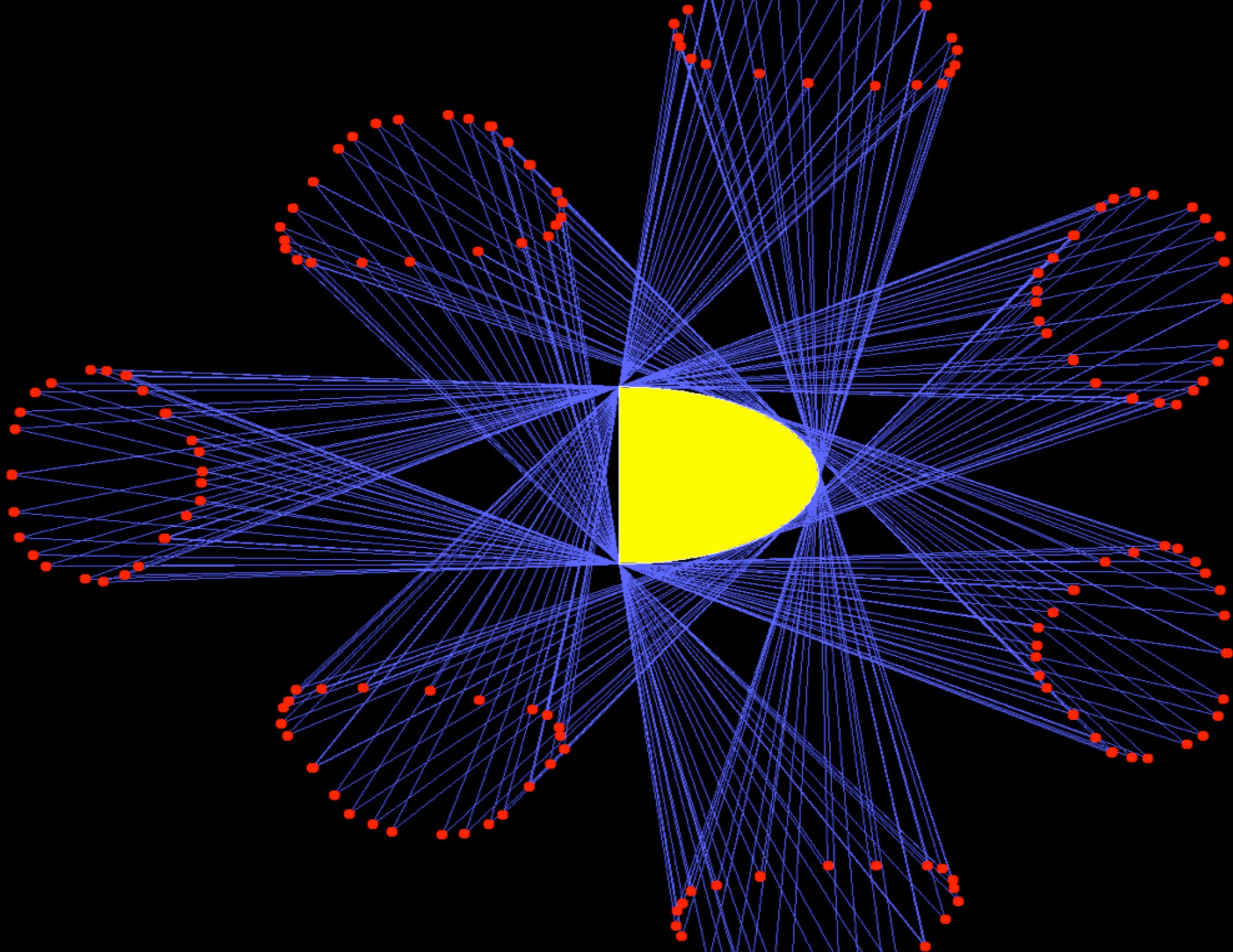


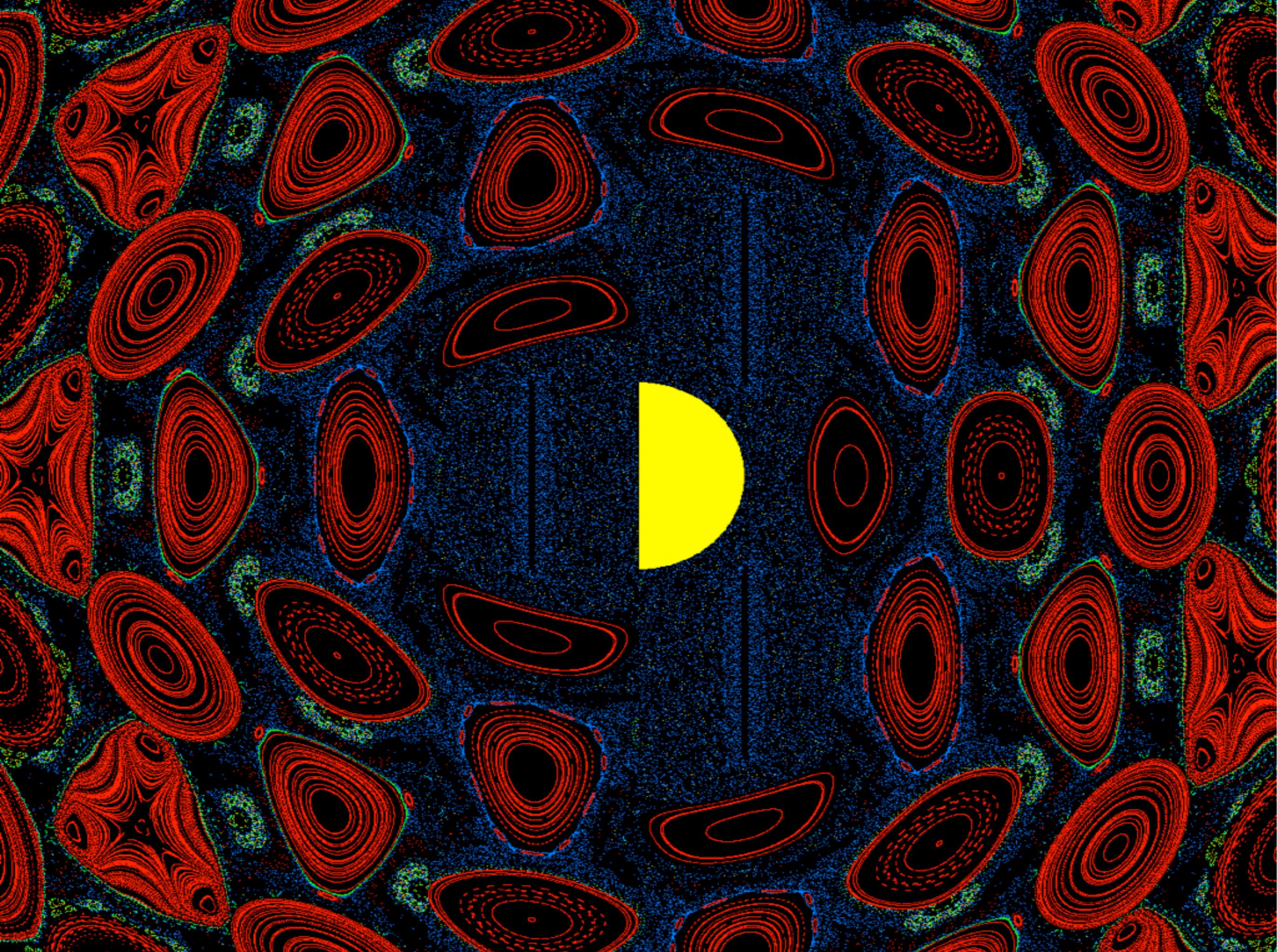
Generated with  
AstroGraph

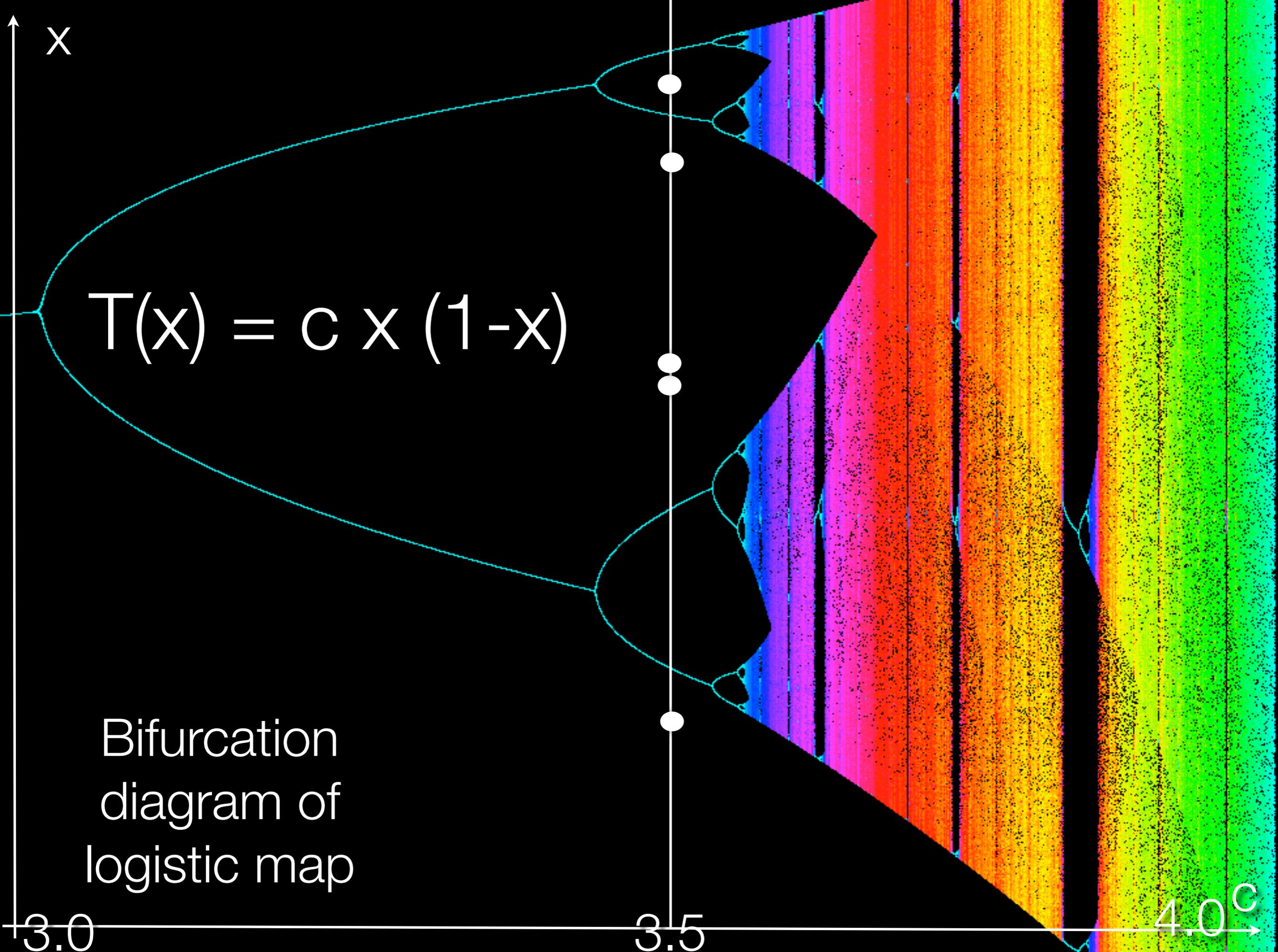


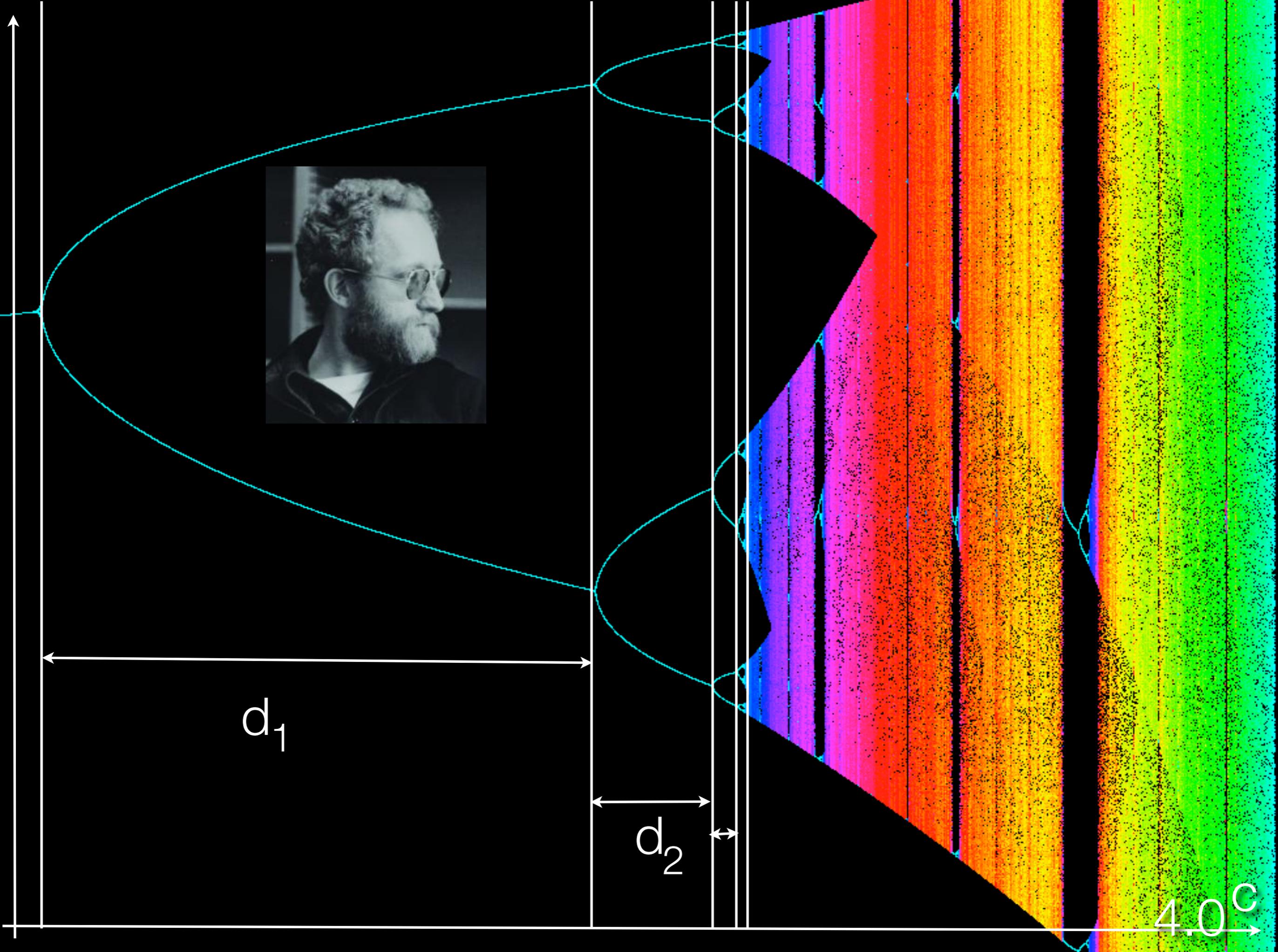
every  
orbit  
bounded?

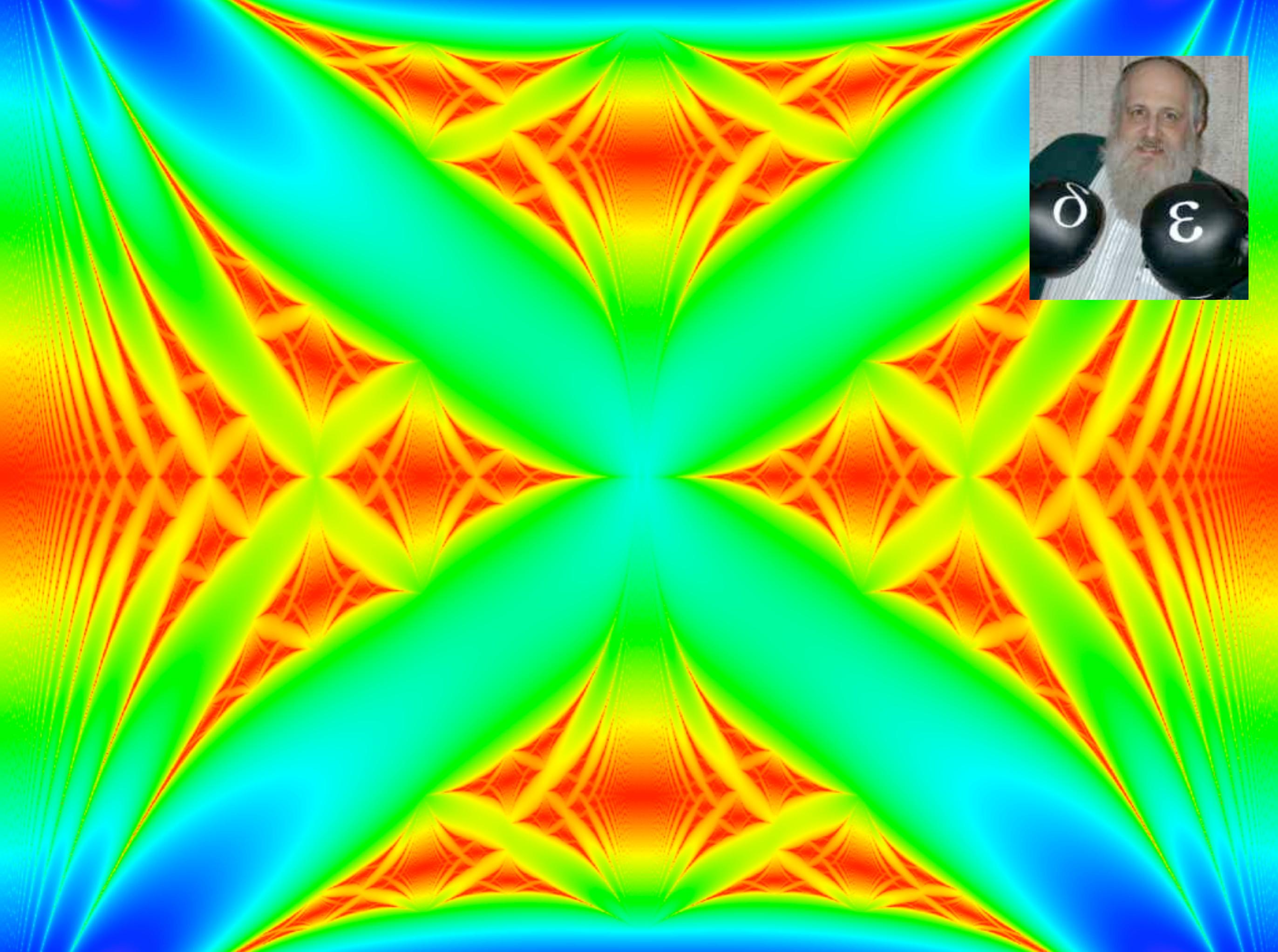


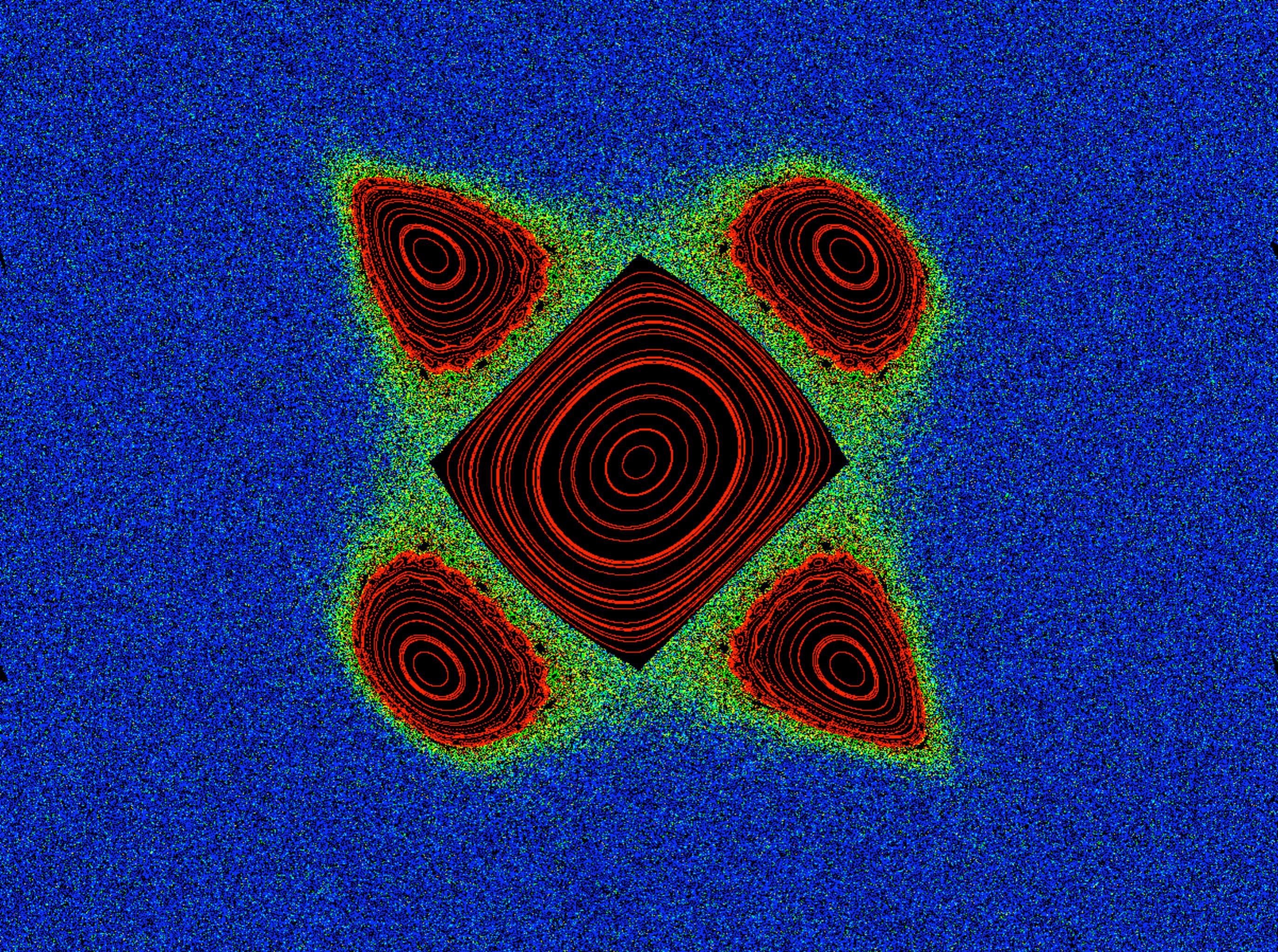


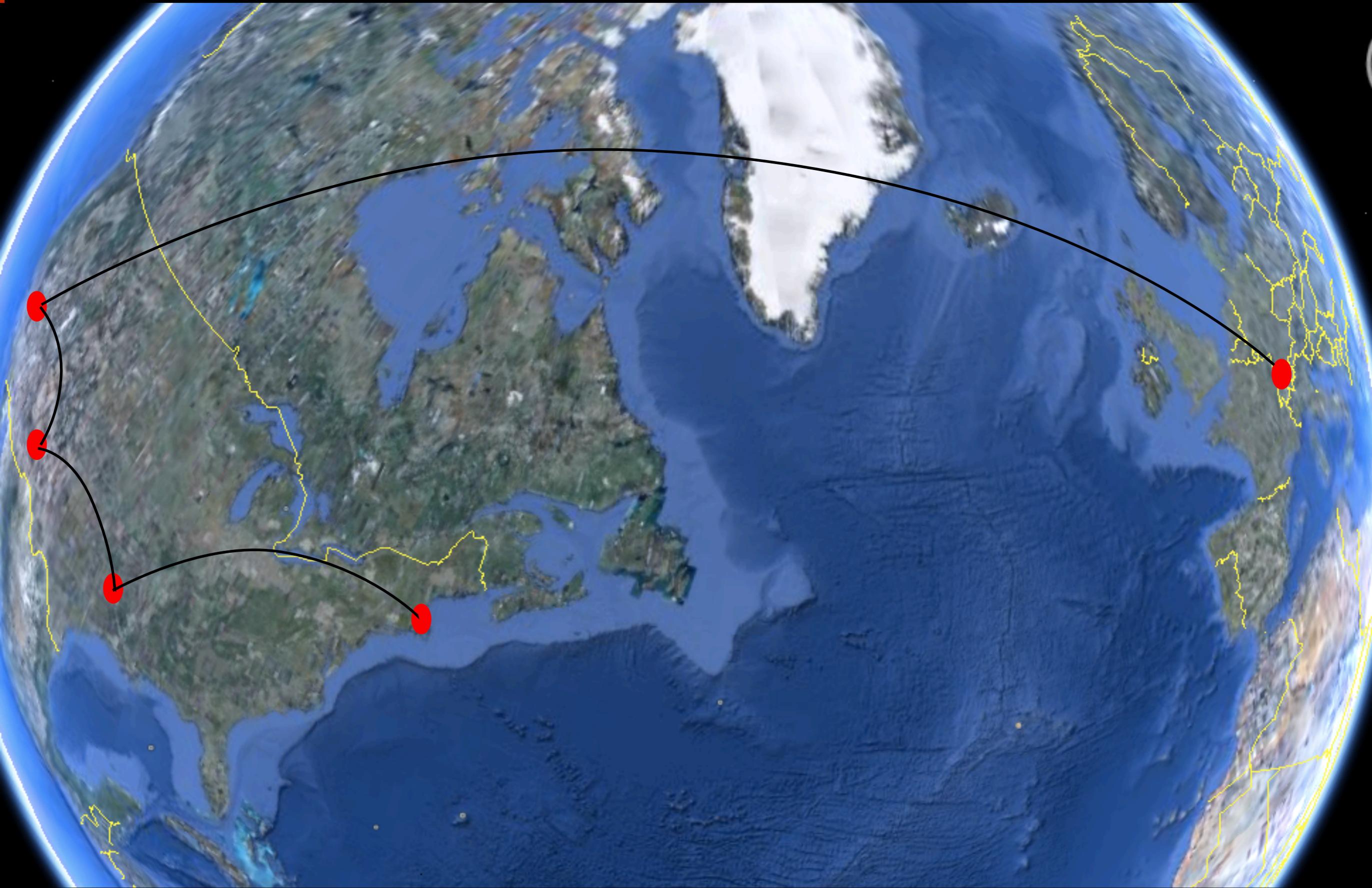












# Letter Writers

# This is a crucial thing

- several letters needed
- no control what is written
- one bad letter can be devastating

Dear Search Committee Chair,

I am writing this letter for Mr. Still Student who has applied for a position in your department. I should start by saying that I cannot recommend him too highly. In fact, there is no other student with whom I can adequately compare him, and I am sure that the amount of mathematics he knows will surprise you.

His dissertation is the sort of work you don't expect to see these days. It definitely demonstrates his complete capabilities.

In closing, let me say that you will be fortunate if you can get him to work for you.

Sincerely,

A. D. Advisor (Prof.)

-- from MAA Focus Newsletter

# Letter writers

## A Primer of Mathematical Writing

Being a Disquisition on Having Your  
Ideas Recorded, Typeset, Published,  
Read, and Appreciated

Steven G. Krantz



American Mathematical Society

Copyright © 2000

You have asked for my opinion on the tenure, and promotion to Associate Professor, of Dr. Aloysius K. Foofnar. Dr. Foofnar is now six years from the Ph.D., and in that time has produced nothing but some rotten teaching evaluations and a letter to the editor of the Two-Year College Math Journal. Based on that track record, my opinion is that he is worthy of neither tenure nor of promotion.

# My own experience

- Grants
- Postdocs
- Green card

# Research Topic

# Wish List

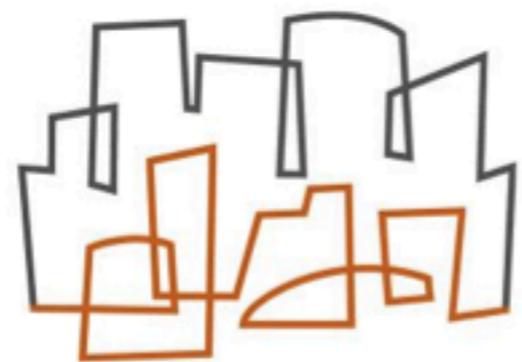
- Needs to be fresh
- But not too fresh
- Not too crowded
- Not too ambitious

# Risks?

- Follow your dreams?
- Try hard problems?
- Work independently?
- Be ambitious?

# MIT Seminar Dangerous Ideas

<http://www.ai.mit.edu/lab/dangerous-ideas/>



**CSAIL**

Seminar on

**Dangerous Ideas**

# Five Questions:

Why do I need to **fear** your work?

Why should I **enjoy** your work ?

What can I tell your **mother** from it?

What's your **coolest** discovery ?

What is the most **recent** discovery?

Fear?

IT HAS BEEN SAID THAT THE FIRST WORLD WAR WAS THE **CHEMISTS' WAR** BECAUSE MUSTARD GAS AND CHLORINE WERE EMPLOYED FOR THE FIRST TIME, AND THAT THE SECOND WORLD WAR WAS THE **PHYSICISTS' WAR**, BECAUSE THE ATOM BOMB WAS DETONATED. SIMILARLY, IT HAS BEEN ARGUED THAT THE THIRD WORLD WAR WOULD BE THE **MATHEMATICIANS' WAR**, BECAUSE MATHEMATICS WILL HAVE CONTROL OVER THE NEXT GREAT WEAPON OF WAR - INFORMATION.



# Cryptoanalysis

IT STARTED A  
LONG TIME AGO:

- ▣ AL KINDI (801-873) first known recorded explanation of cryptanalysis
- ▣ ALTBASH (HEBREW)  
CESAR (ROMAN)

UNTIL TODAY: THE  
NSA IS THE  
LARGEST  
EMPLOYER OF  
MATHEMATICIANS



▣ AL KINDI

# Example I: Babington Plot

Handwritten text in a cypher script, likely a cipher key or a message from the Babington Plot.

Handwritten text in a cypher script, including a key and a message. The key lists letters and their corresponding cypher symbols. The message is signed by Anthony Babington.

English text at the bottom of the page:

English text is by Anthony Babington, by which only I have written  
 such an answer of Colob, or received thereof from you.

Anthony Babington

Acknowledged & subscribed by Babington  
 prime Sept. 1586. in presence of Edwarde Barber.

Plaine	Cypher	Plaine	Cypher	Plaine	Cypher	Plaine	Cypher
A	1	B	2	C	3	D	4
E	5	F	6	G	7	H	8
I	9	K	10	L	11	M	12
N	13	O	14	P	15	Q	16
R	17	S	18	T	19	U	20
V	21	X	22	Y	23	Z	24
a	25	b	26	c	27	d	28
e	29	f	30	g	31	h	32
i	33	k	34	l	35	m	36
n	37	o	38	p	39	q	40
r	41	s	42	t	43	u	44
v	45	x	46	y	47	z	48
aa	49	bb	50	cc	51	dd	52
ee	53	ff	54	gg	55	hh	56
ii	57	kk	58	ll	59	mm	60
nn	61	oo	62	pp	63	qq	64
rr	65	ss	66	tt	67	uu	68
vv	69	xx	70	yy	71	zz	72
aa	73	bb	74	cc	75	dd	76
ee	77	ff	78	gg	79	hh	80
ii	81	kk	82	ll	83	mm	84
nn	85	oo	86	pp	87	qq	88
rr	89	ss	90	tt	91	uu	92
vv	93	xx	94	yy	95	zz	96
aa	97	bb	98	cc	99	dd	100



CYPHER

BABINGTON PLOT 1585

MARY I QUEEN OF SCOTS  
 EXECUTED 1587

# Example 2: Zimmermann Telegram

MAILED  
 Director 1-8-58  
 ...son, State Dept.  
 ... A. Eckhoff ...  
 Oct. 27, 1917

**TELEGRAM RECEIVED.**

FROM 2nd from London # 5747.

"We intend to begin on the first of February unrestricted submarine warfare. We shall endeavor in spite of this to keep the United States of America neutral. In the event of this not succeeding, we make Mexico a proposal of alliance on the following basis: make war together, make peace together, generous financial support and an understanding on our part that Mexico is to reconquer the lost territory in Texas, New Mexico, and Arizona. The settlement in detail is left to you. You will inform the President of the above most secretly as soon as the outbreak of war with the United States of America is certain and add the suggestion that he should, on his own initiative, ~~invite~~ <sup>invite</sup> Japan to immediate adherence and at the same time mediate between Japan and ourselves. Please call the President's attention to the fact that the ruthless employment of our submarines now offers the prospect of compelling England in a few months to make peace." Signed, ZIMMERMANN.

**WESTERN UNION TELEGRAM**

Send the following telegram, subject to the terms on back hereof, which are hereby agreed to

via Galveston

JAN 16 1917

GERMAN LEGATION  
 MEXICO CITY

130	13042	13401	8501	115	3528	416	17214	8491	11310
18147	18222	21580	10847	11518	23677	13805	3494	14936	
98092	5905	11311	10392	10371	0302	21290	5181	39895	
23571	17504	11289	18278	18101	0317	0228	17894	4473	
23224	22200	19452	21589	87893	5589	13918	8958	12137	
1333	4725	4458	5905	17166	15851	4458	17149	14471	8706
13850	12224	8929	14991	7382	15857	67893	14218	56477	
5870	17553	67893	5870	5454	16102	15217	22801	17138	
11001	17388	7140	23838	18222	6710	14331	15021	23845	
3100	23552	22096	21604	4797	9497	22464	20855	4377	
23410	18140	22260	5905	13347	20420	39889	13732	20007	
8020	5078	18507	52222	1340	22049	13339	11285	22295	
10439	14814	4178	8992	8784	7632	7357	8928	52262	11267
21100	21272	9346	9559	22464	15874	18502	18500	15857	
2180	5378	7381	98092	15127	13486	9380	9220	76036	14219
6144	2831	17920	11347	12142	11264	7607	7768	15099	9110
10482	97556	3589	3070						

Charge German Embassy

Sent from Germany to Washington to Mexico January 16, 1917

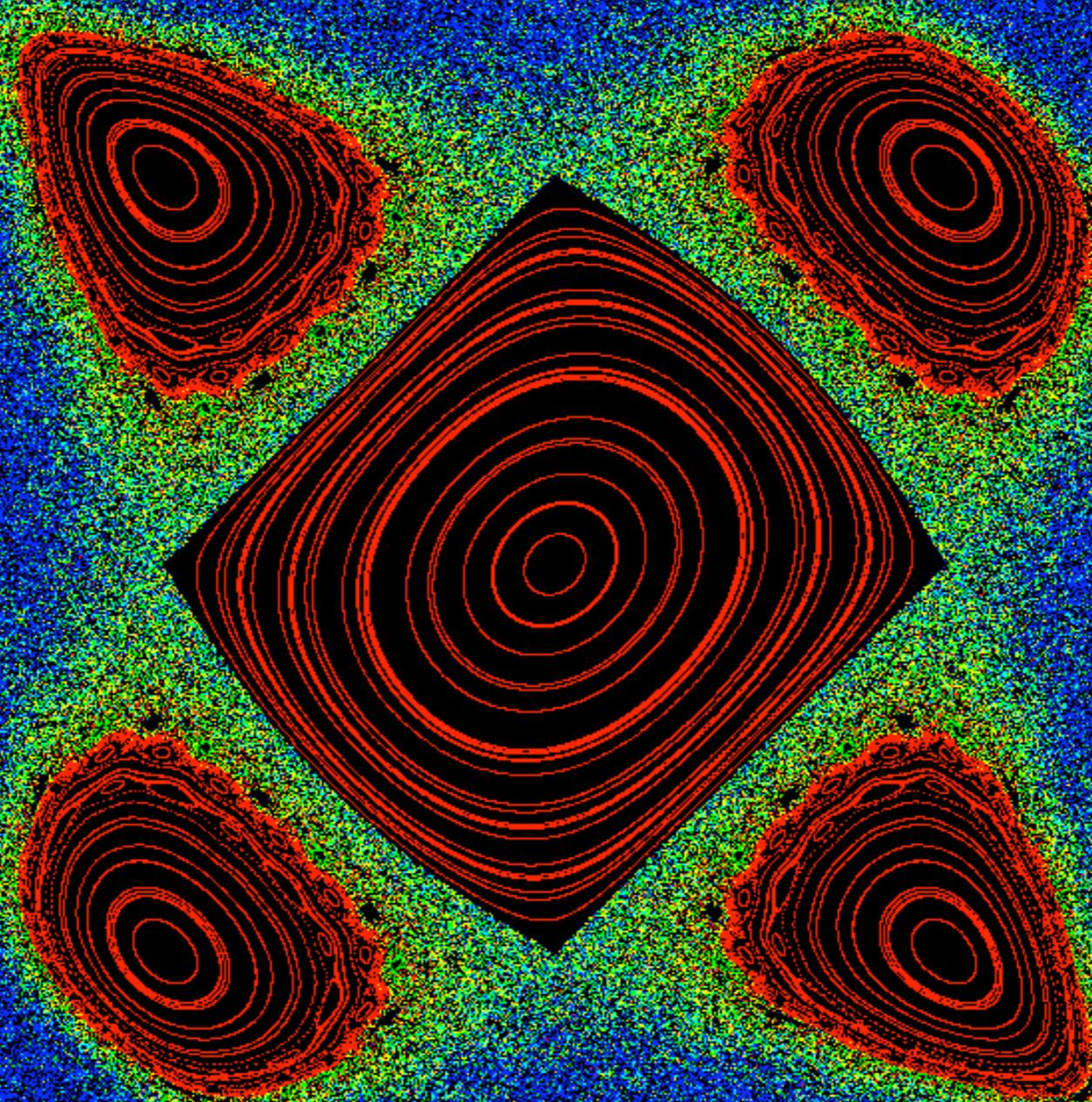
Enjoy?

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c \sin(x) + 2x - y \\ x \end{bmatrix} \quad \text{defines } x_n$$

$$A(x_n) = \begin{bmatrix} c \cos(x_n) + 2 & -1 \\ 1 & 0 \end{bmatrix}$$

Does the  
product of  
such matrices  
grow  
exponentially?

Simplicity!



Tell Mom?

May 18, 2010

# OMNIVIS: 3D Space and Camera Path Reconstruction for Omnidirectional Vision



Oliver Knill  
Director



Jeff  
Parker  
Research  
Advisor



Jose L.  
Ramirez  
Master in IT

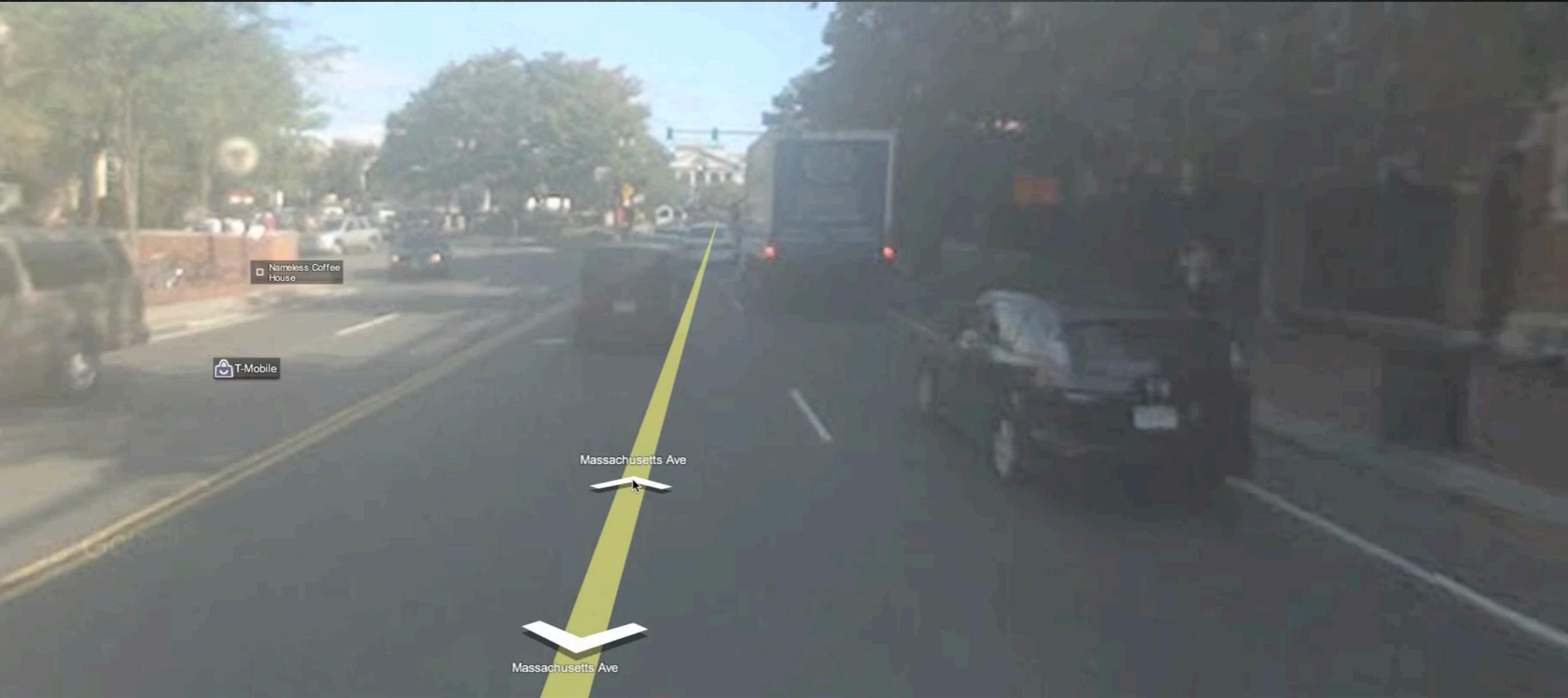


Exhibit

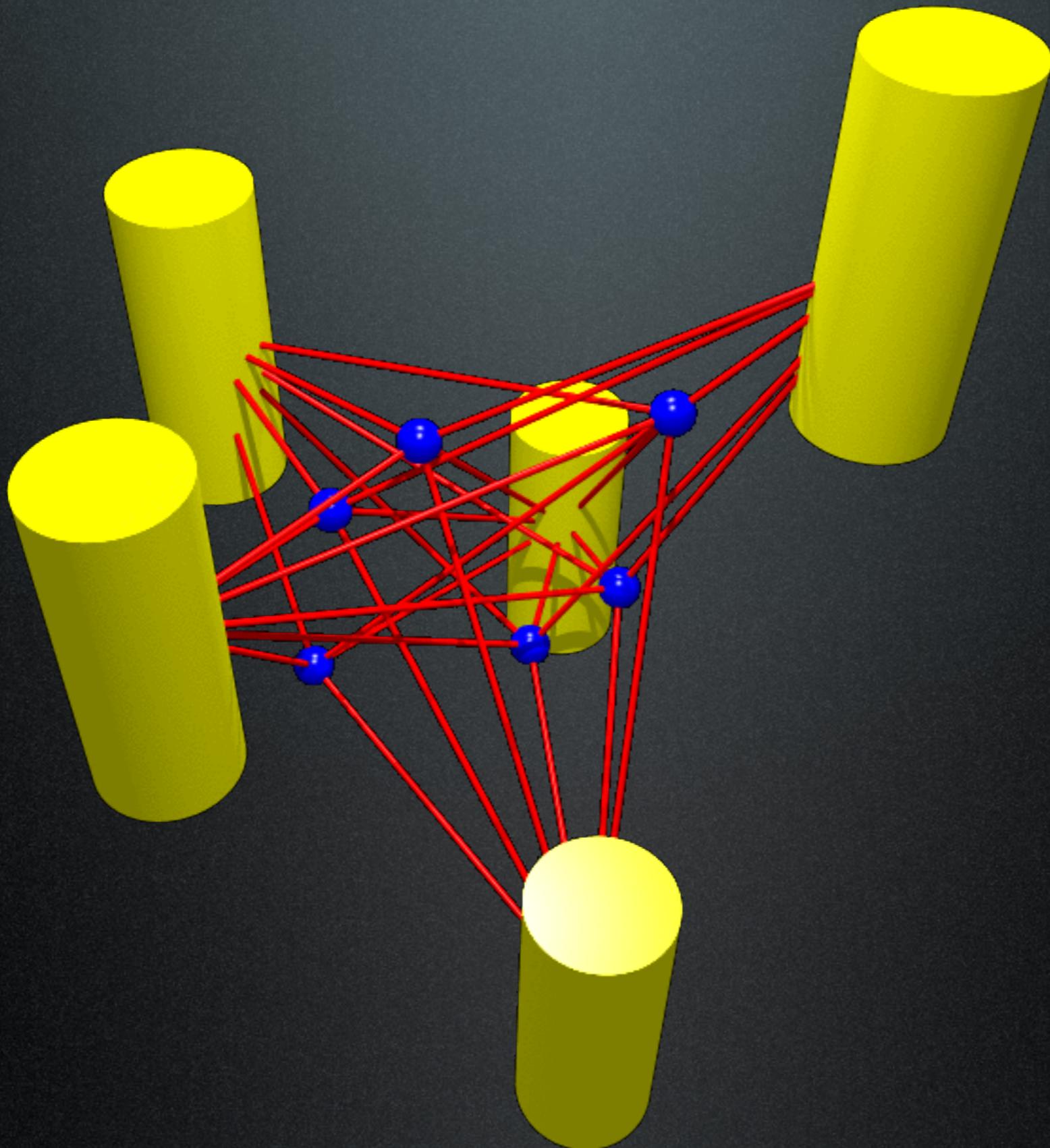
<http://math.harvard.edu/~knill/3dscan2/papers/index.html>

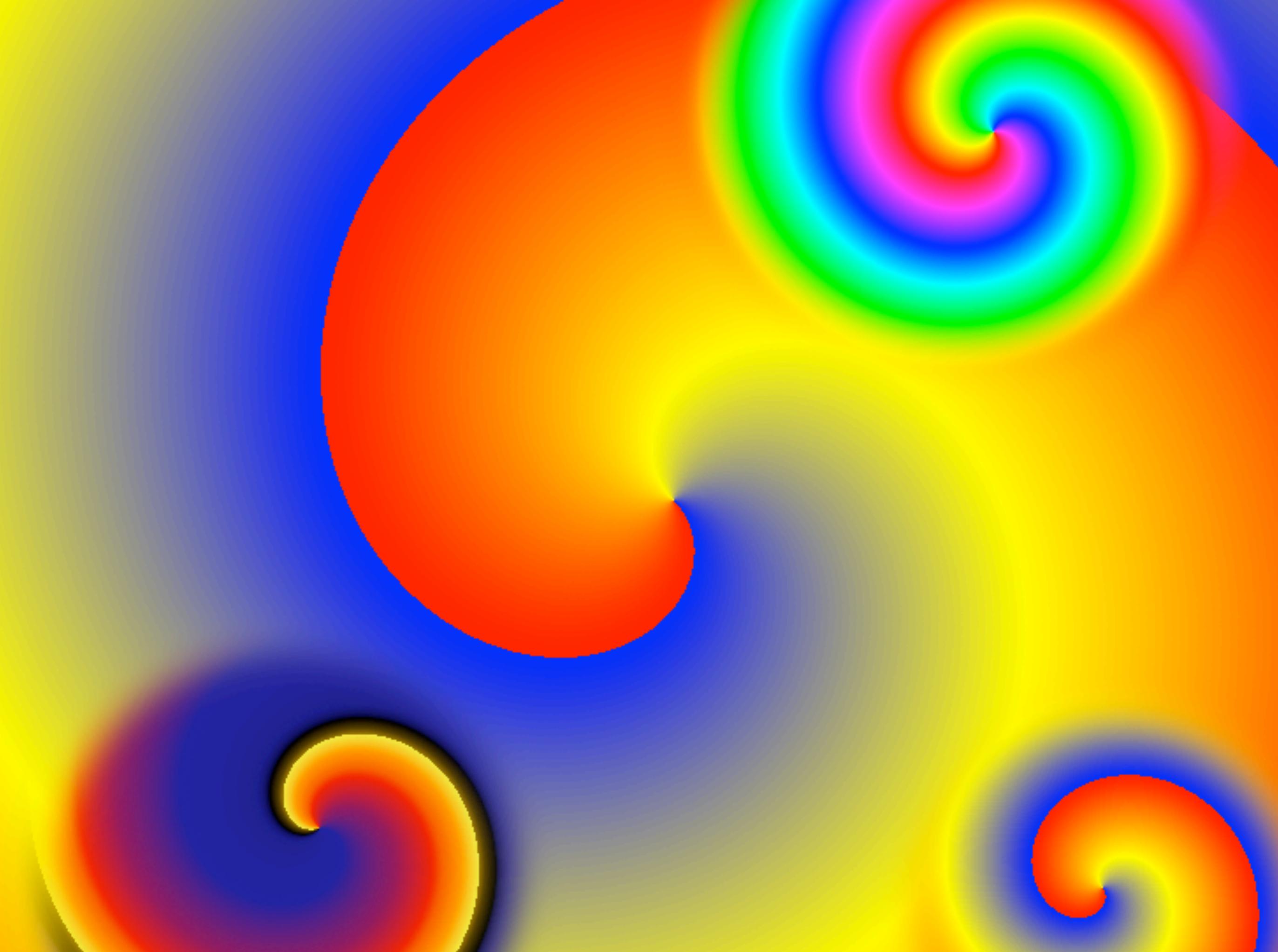


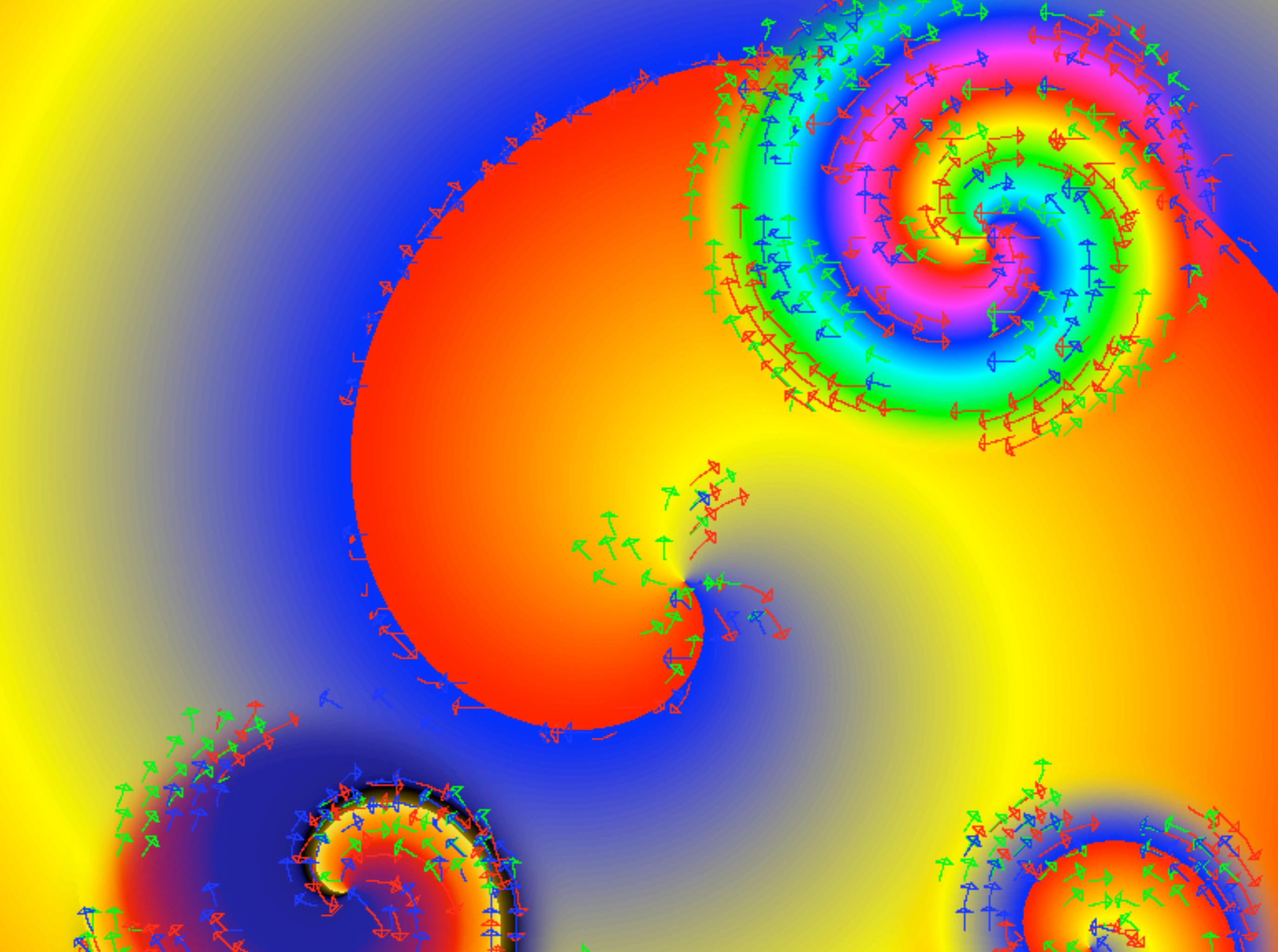
# Google Street View Video - Harvard

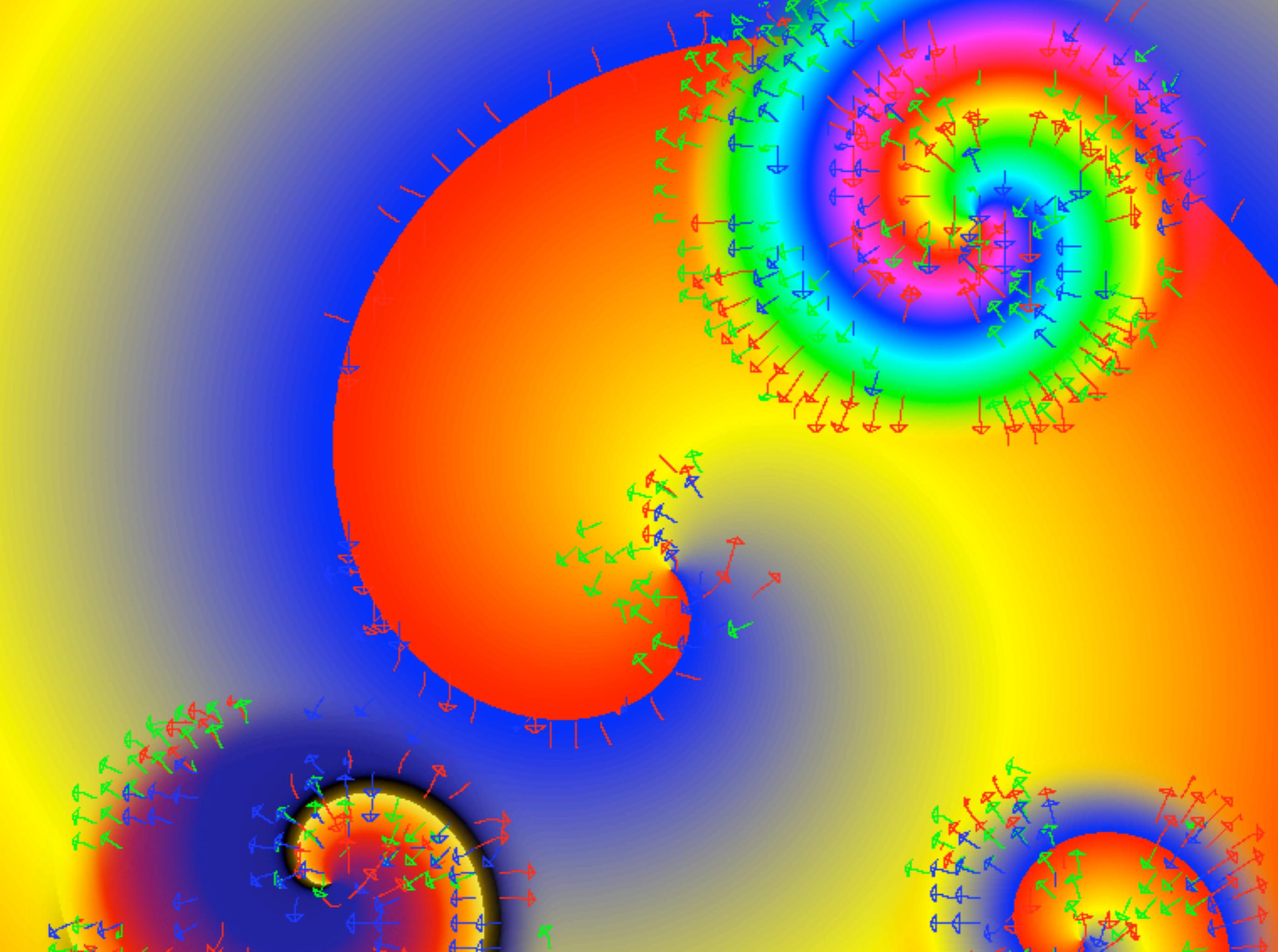


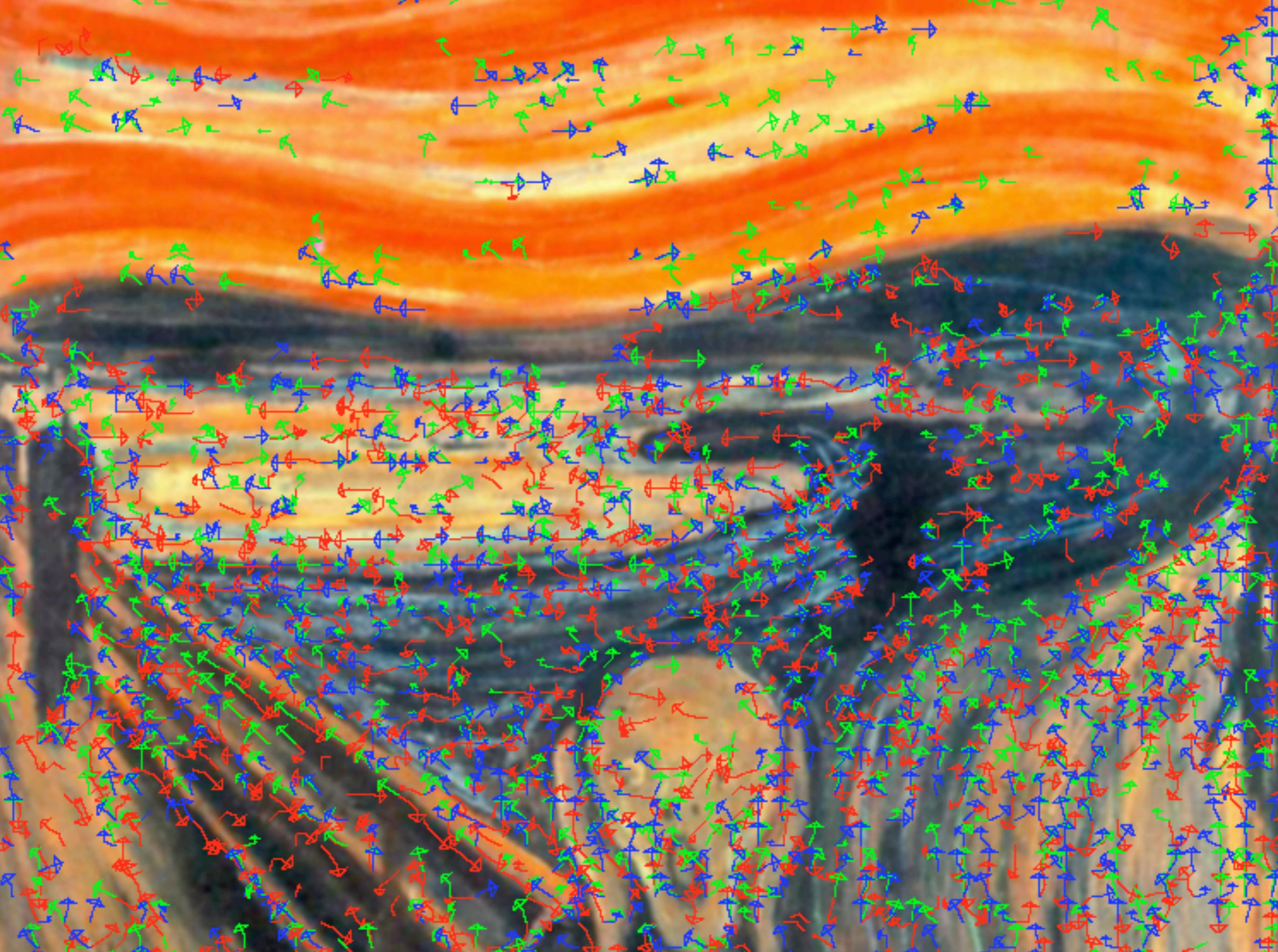
# Structure from Motion













tracing level curves

done with our code

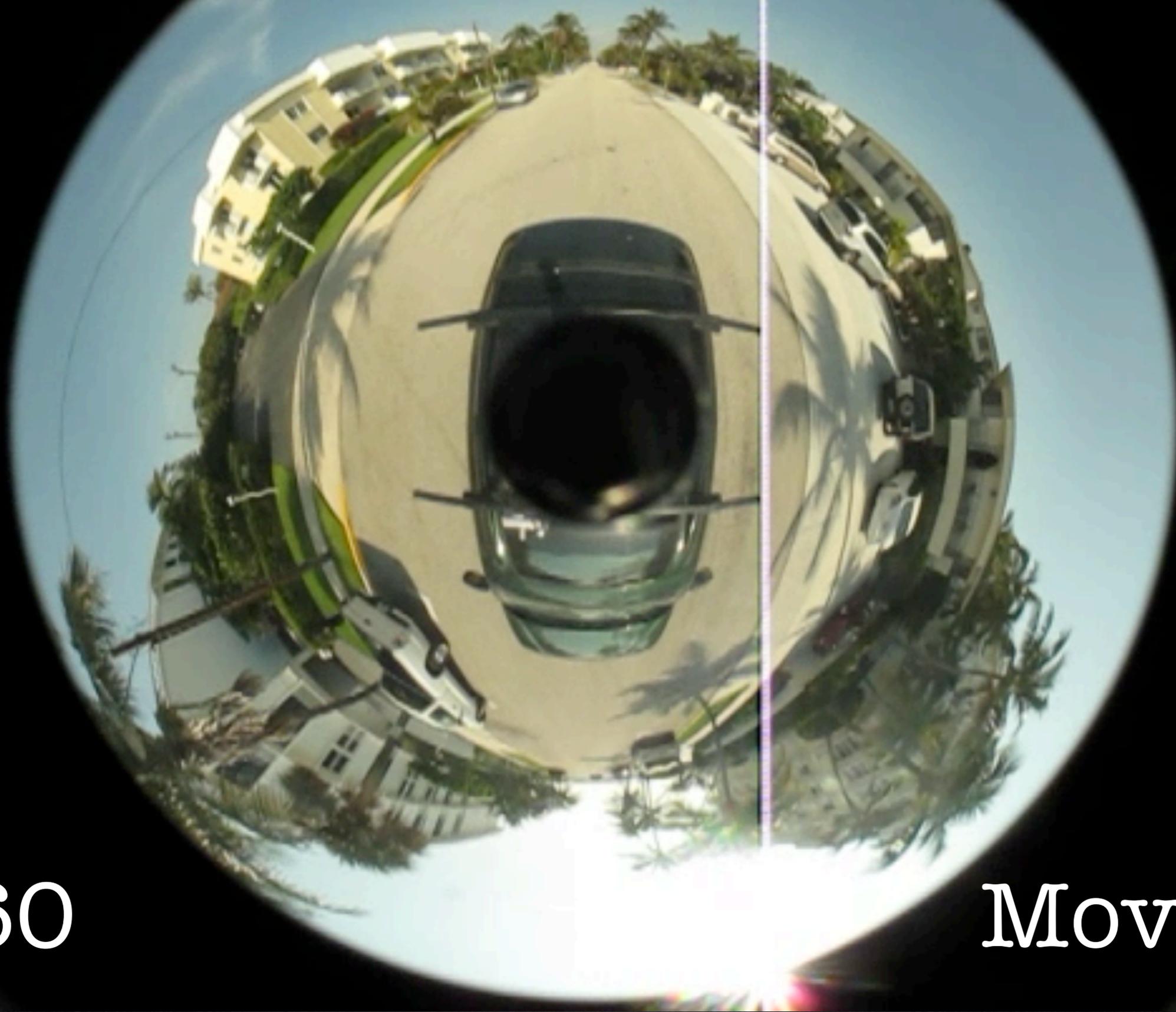




tracing level curves of a picture

Florida

Original



360

Movie



Florida Scene, Gaussian Smoothing

# Florida Scene, Curvature



# Florida Scene, Gradient



# Florida Scene, Tracking

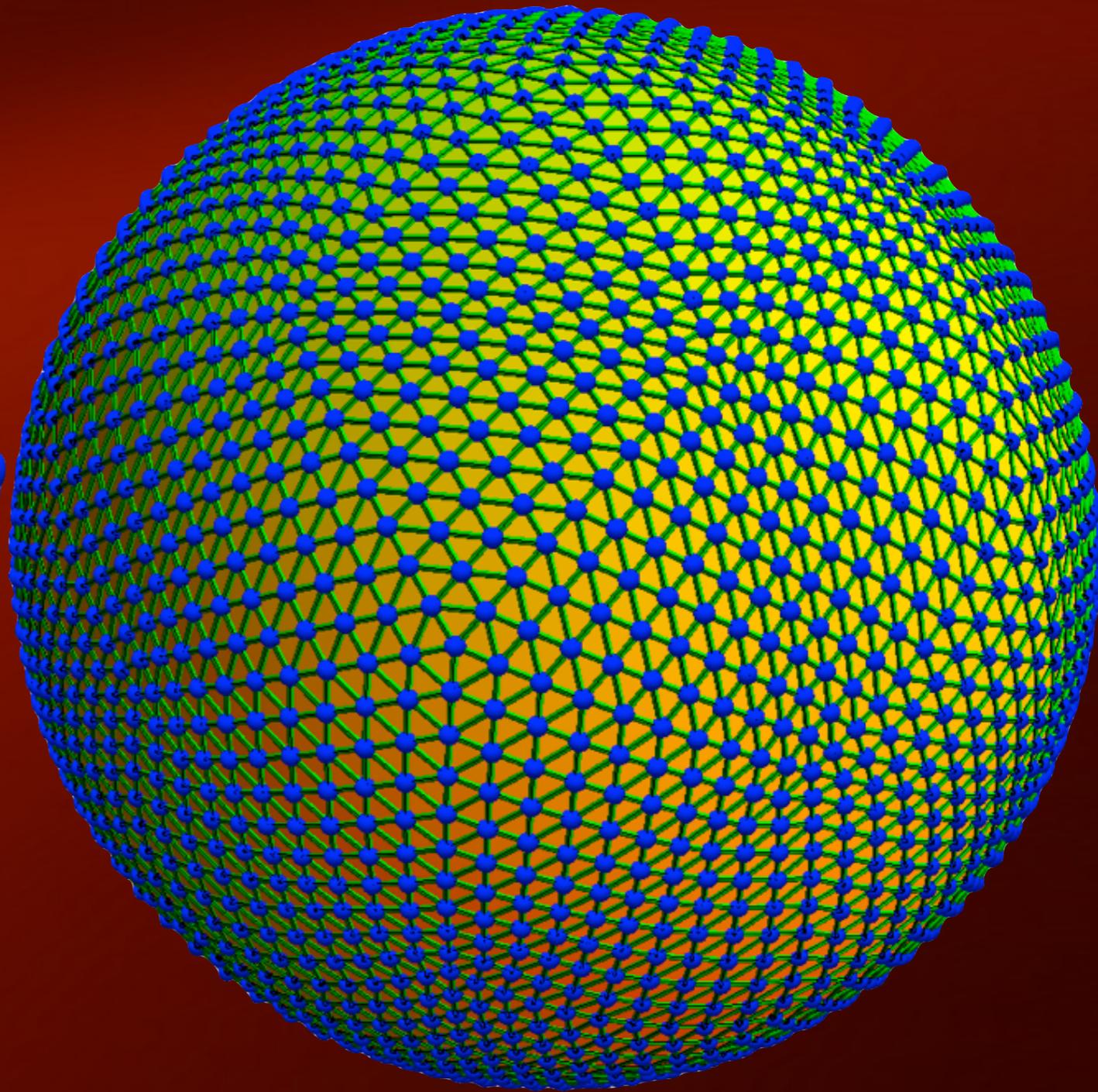
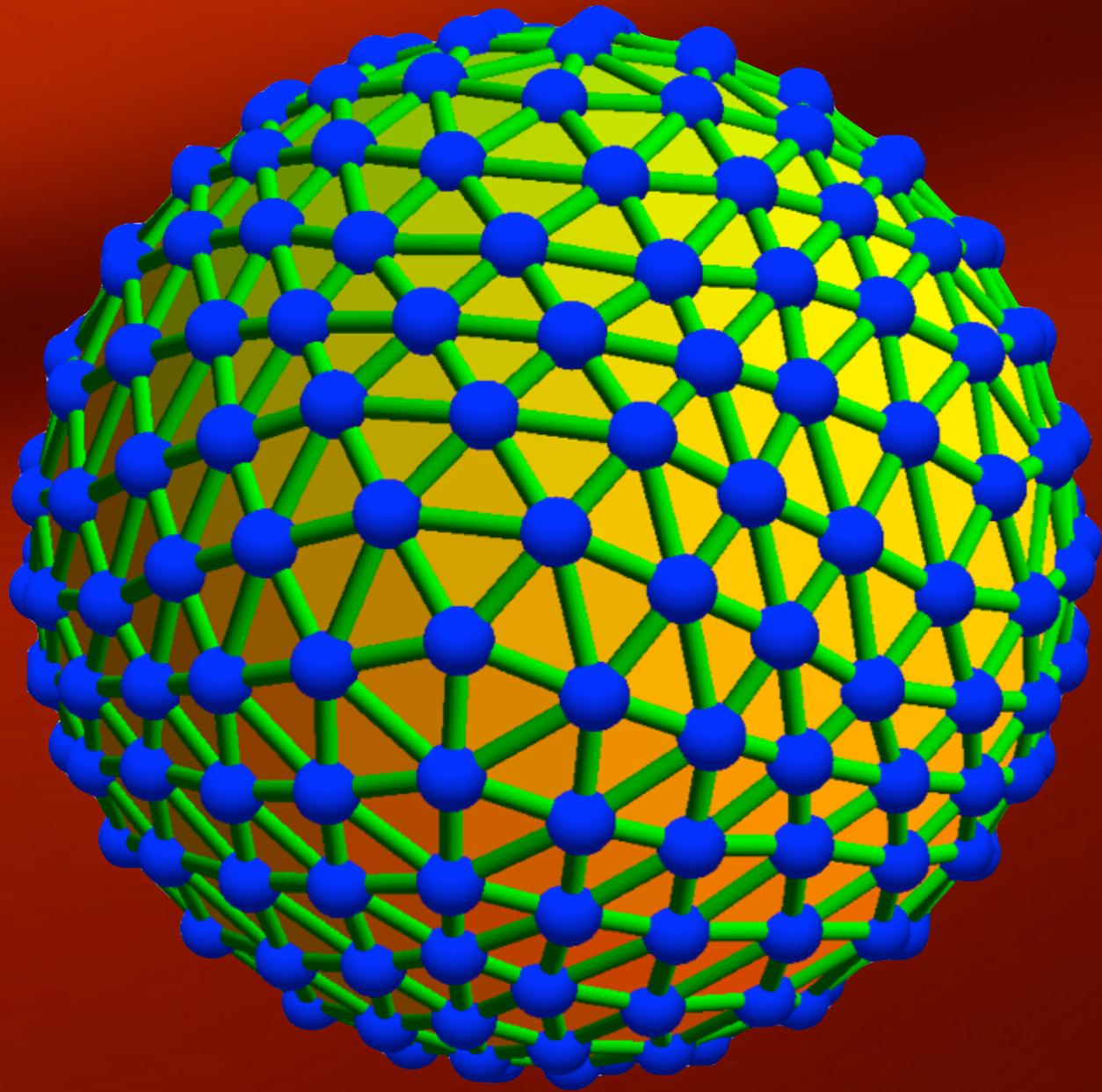


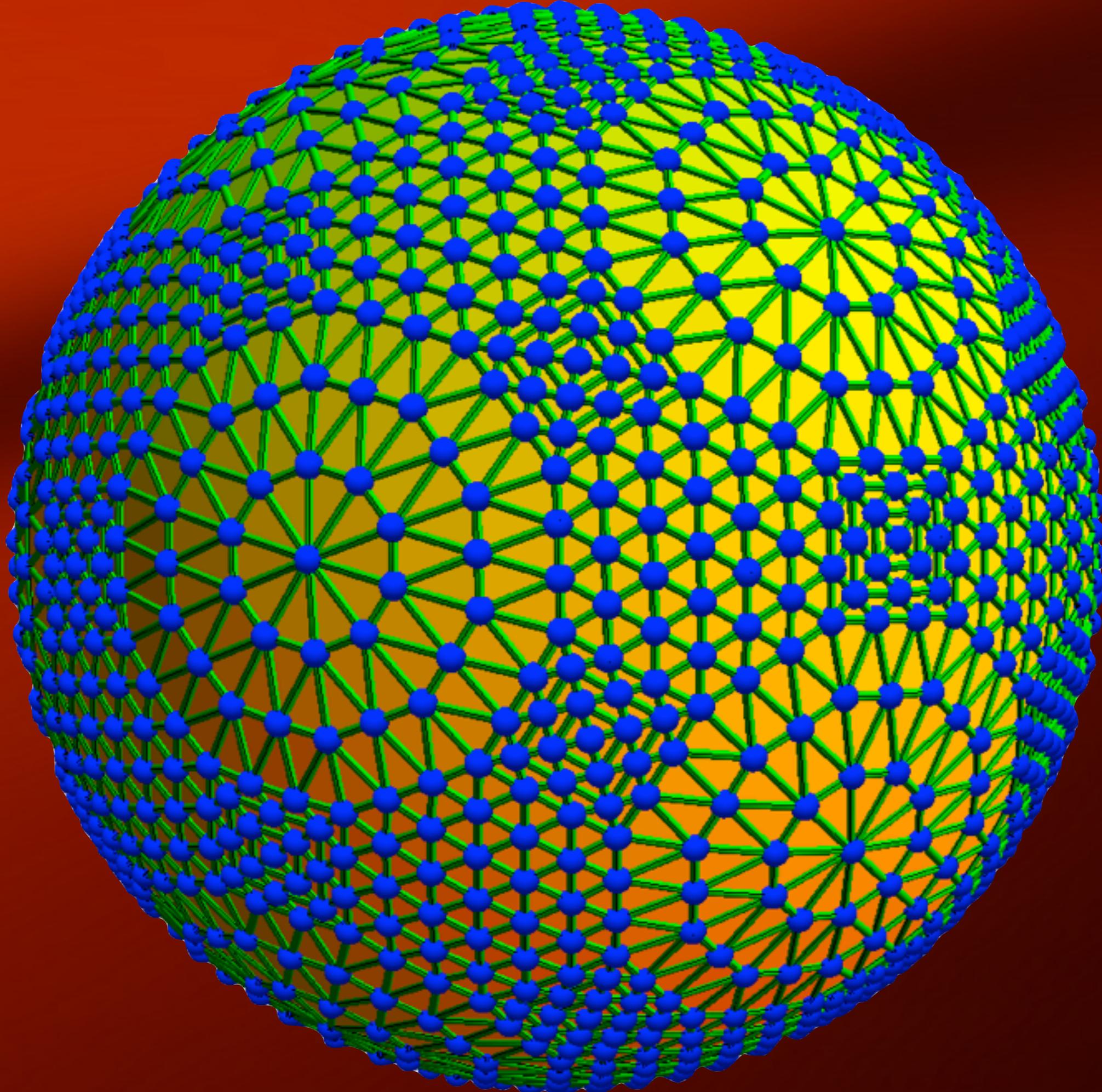
# Florida, different Smoothing

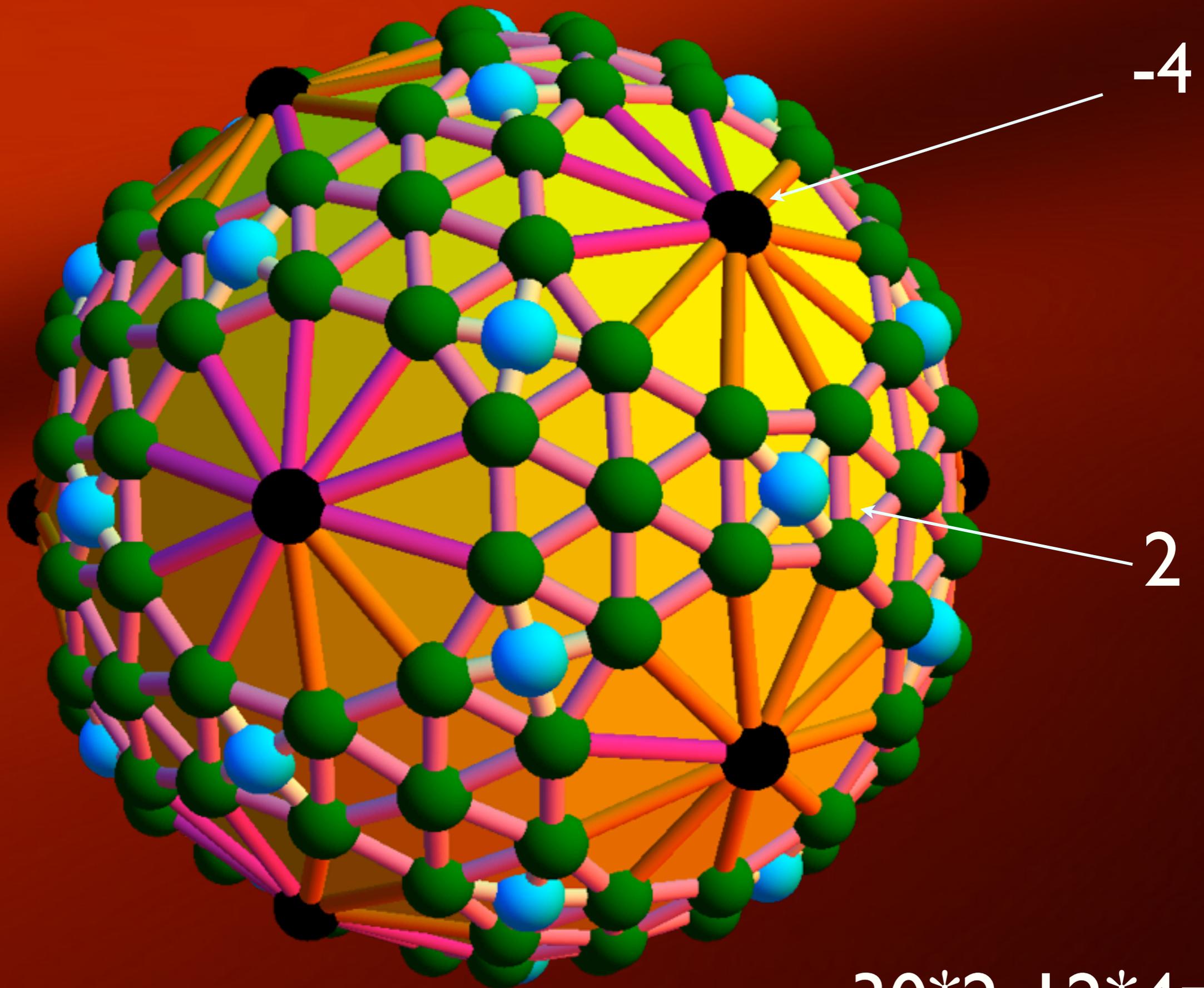


# Coollest Discovery

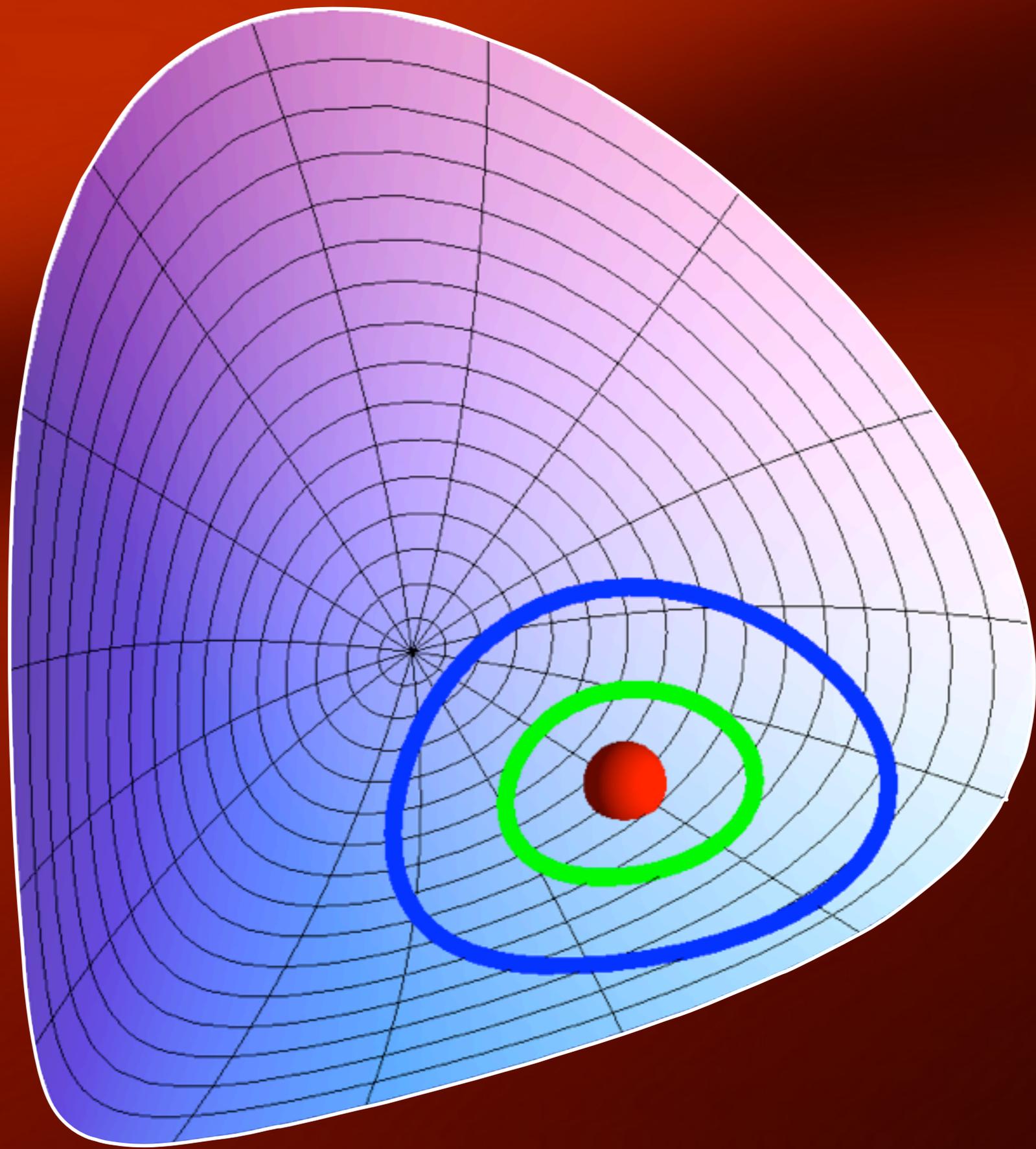
# Graphs

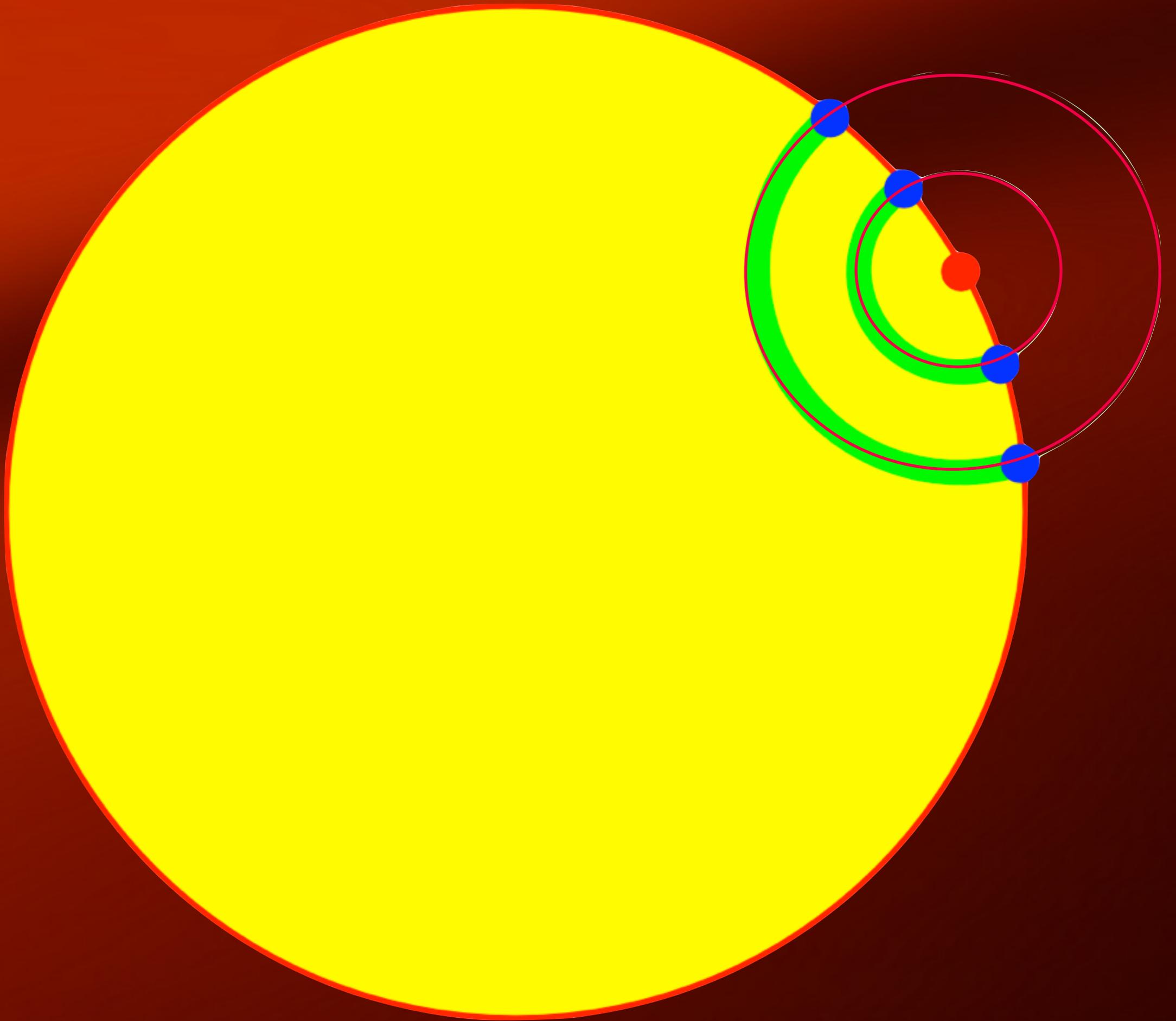


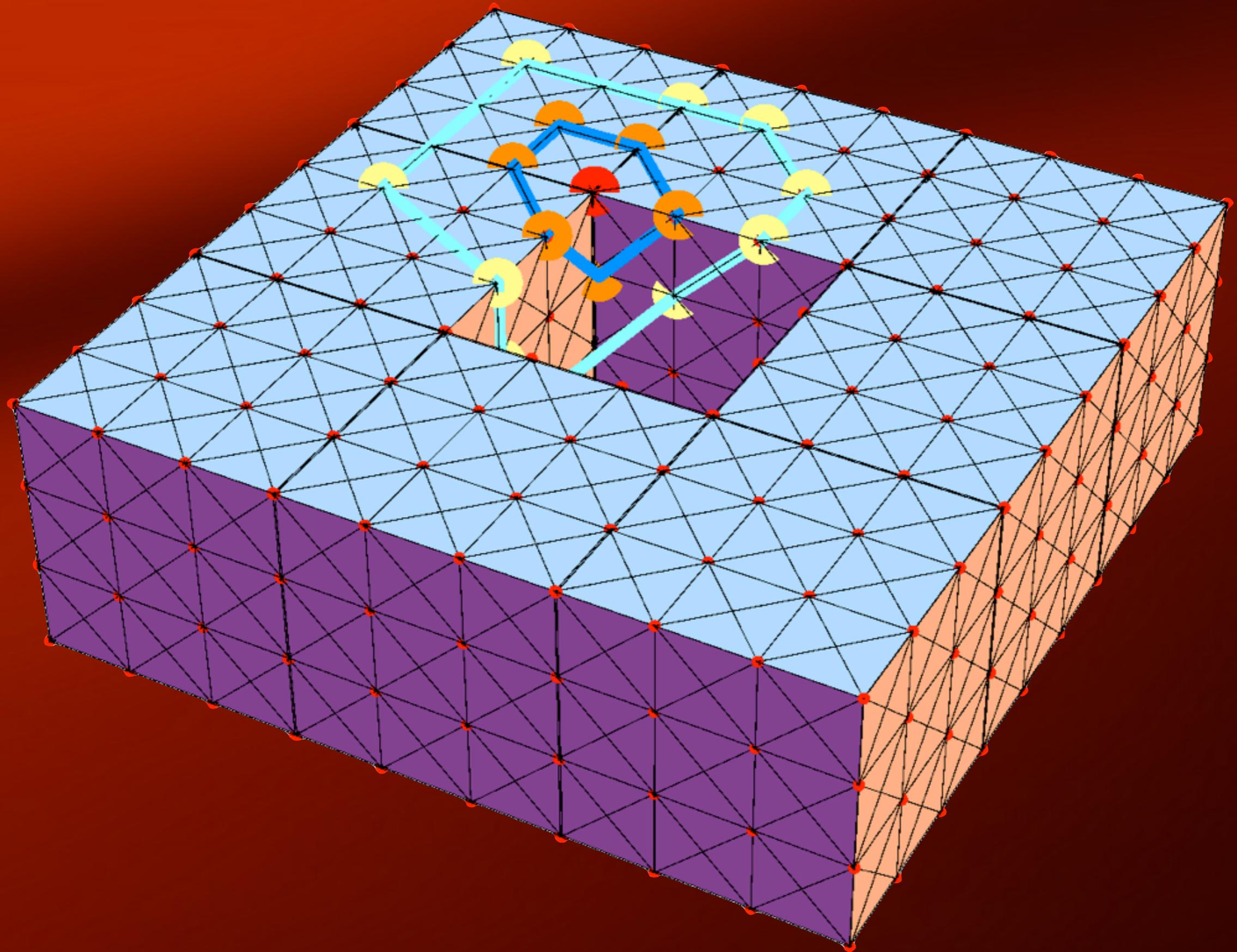




$$30 \cdot 2 - 12 \cdot 4 = 12$$

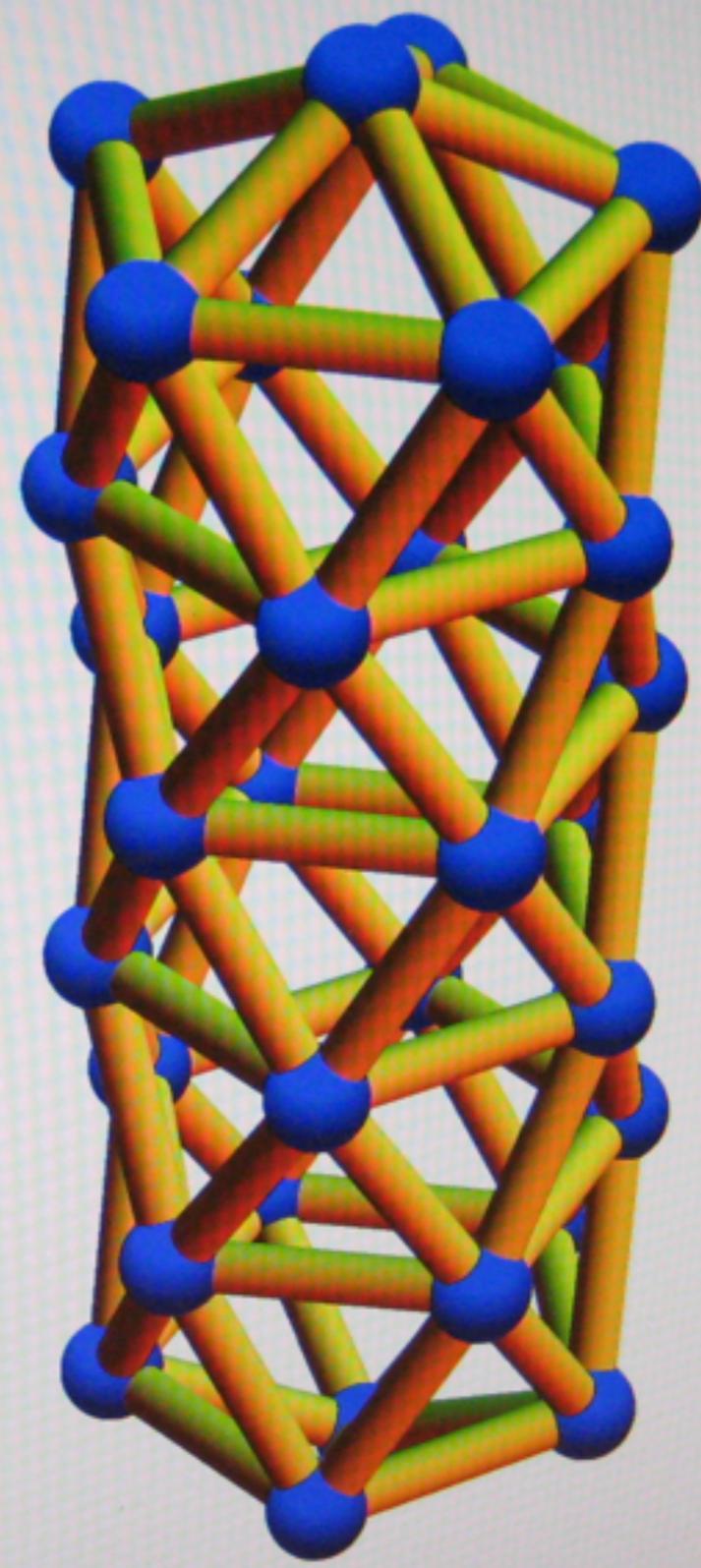








```
Out[32]= {16385, 16386, 16387, 16388}
In[33]:= Triangles[{16485,16387}]
Out[33]= {}
In[34]:= InteriorP
Out[34]= {1, 17, 65, 66, 67, 68, 257, 258, 259, 260, 1000,
> 1026, 4099, 4100, 16385, 16386, 16387, 16388}
```

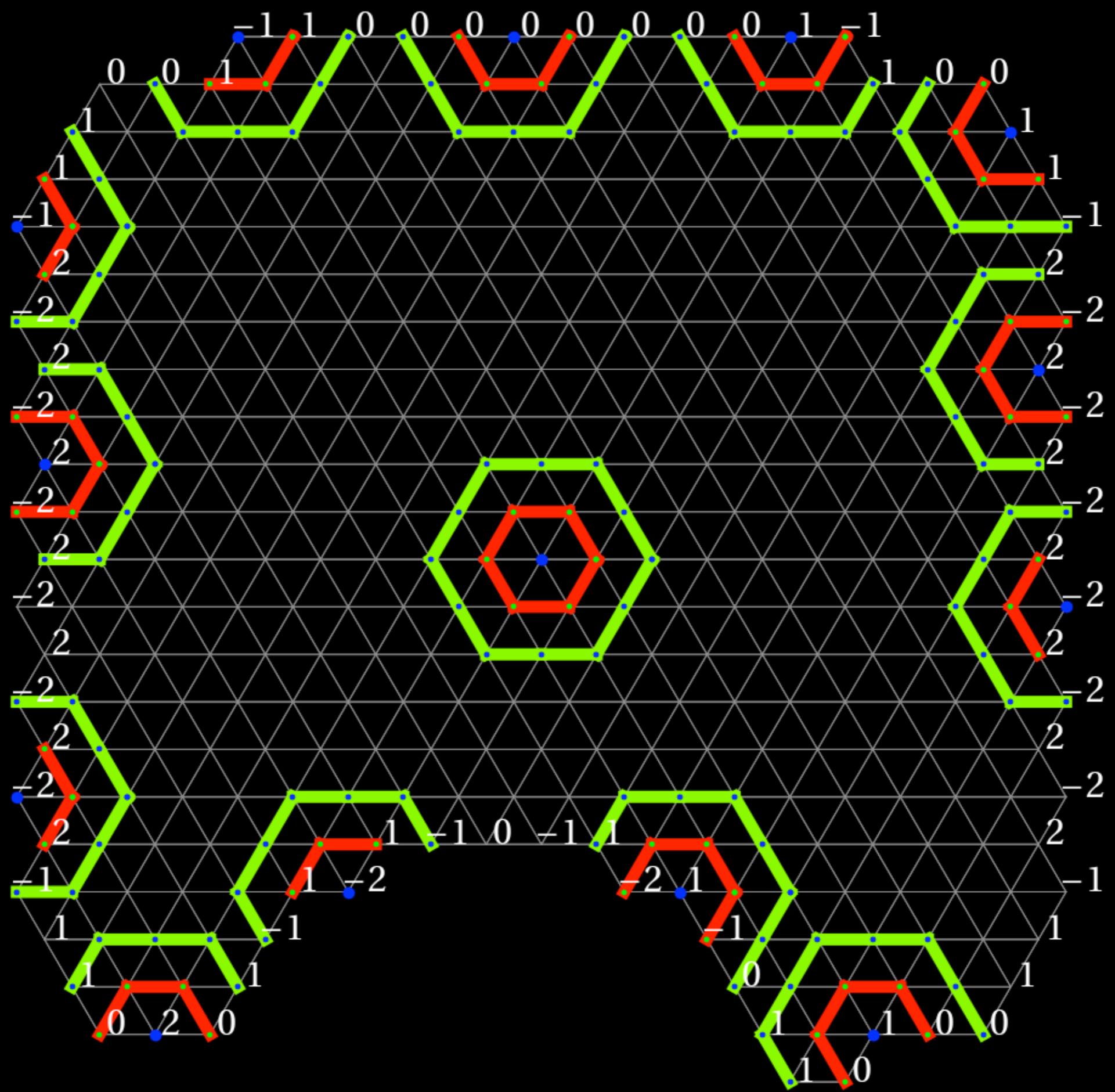


```
25), {16388, 16385},
257, 1025},
5), {19, 67},
, {4100, 16388},
257, 1028},
5), {19, 66},
, {4100, 16387},
), {1025, 18},
), {16388, 260},
), {1026, 19},
), {16388, 257})
4100.}, {1000., 65.},
7.}, {260., 257.},
4098.}, {18., 19.},
260.}, {4099., 4100.},
5.}, {1028., 1025.},
385., 16386.},
., 68.},
28., 1025.},
0., 16388.},
66.}, {258., 1025.},
., 16387.}, {20., 68.},
1027.},
., 16388.}, {18., 65.},
1026.},
., 16387.},
5., 257.},
7., 259.},
8., 257.},
8., 259.},
8., 257.})
259, 260, 1000, 1025,
16386, 16387, 16388)
```

```
cylinder.ps cylinder5.m cylinder6.
cylinder4.m cylinder5.png cylinder6.
cylinder4.png cylinder5.m cylinder6.
r5.png
```

DELL

10:12 PM



```
in[100]->SSSO = ss[0];  
  
in[101]->TF1 = {{1, 0}, {0, 1}, {-1, 1}, {-1, 0}, {0, -1}, {1, -1}}; (*steepest spaces*)  
connection[{{p_, e_}}] := Module[{}, If[BoundaryPoint[p + e], -e, e]]  
ExpMap[{{p_, e_}}] := {p + e, connection[{{p + e, e}}]}  
{p0, t0} = {{1, 1}, {1, -1}};  
orbit = NestList[ExpMap, {p0, t0}, 10];  
Orbit =  
  Show[  
    Graphics[  
      {Thickness[0.05], RGBColor[0, 0, 1],  
        Table[Point[r[orbit][[k, 1]]], {k, Length[orbit]}],  
        {PointSize[0.01], RGBColor[0.4, 0.4, 1],  
          Line[Table[r[orbit][[k, 1]], {k, Length[orbit]}]}]}];  
  Show[SSSO, Orbit]
```



Out[101]->

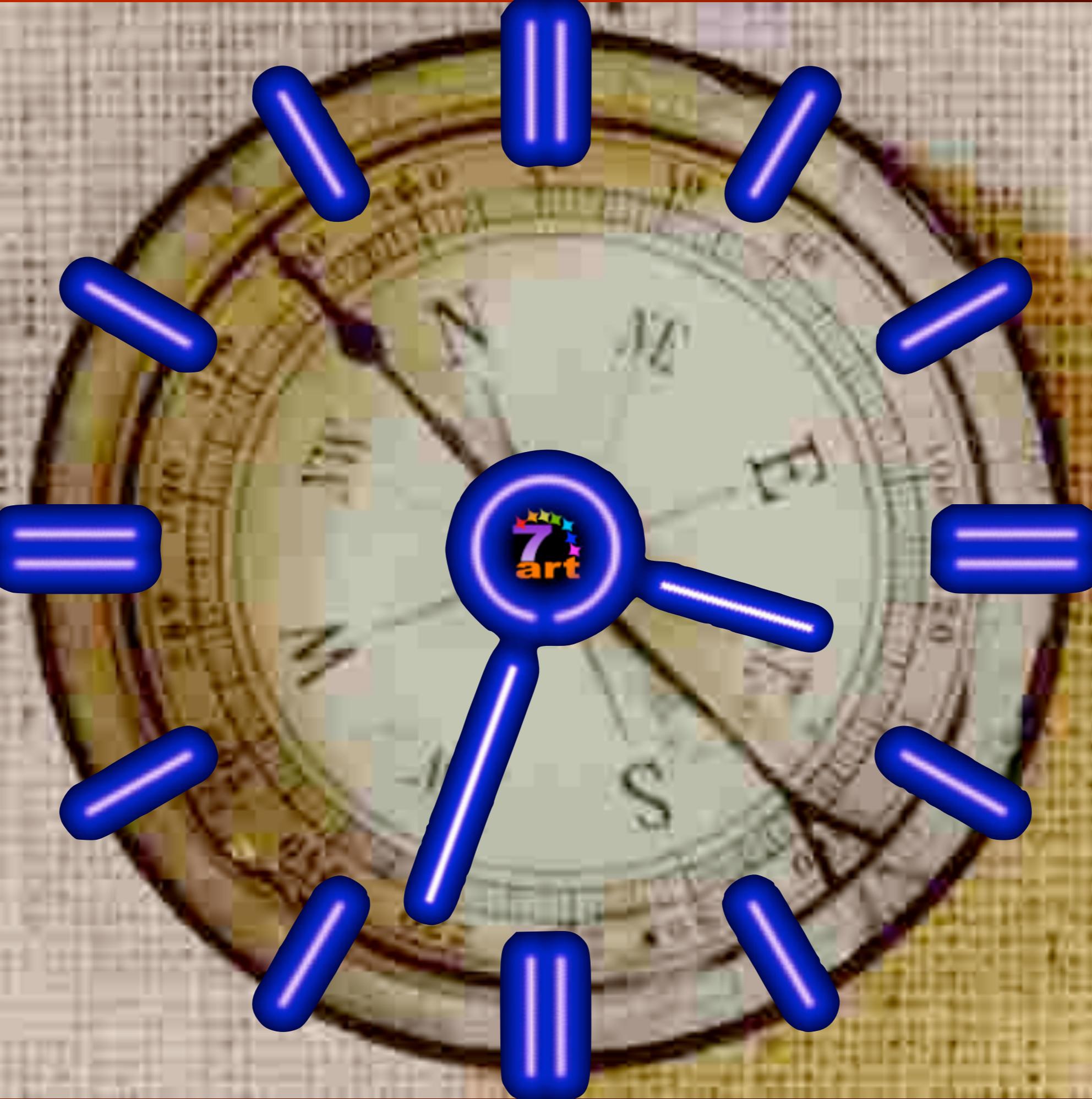


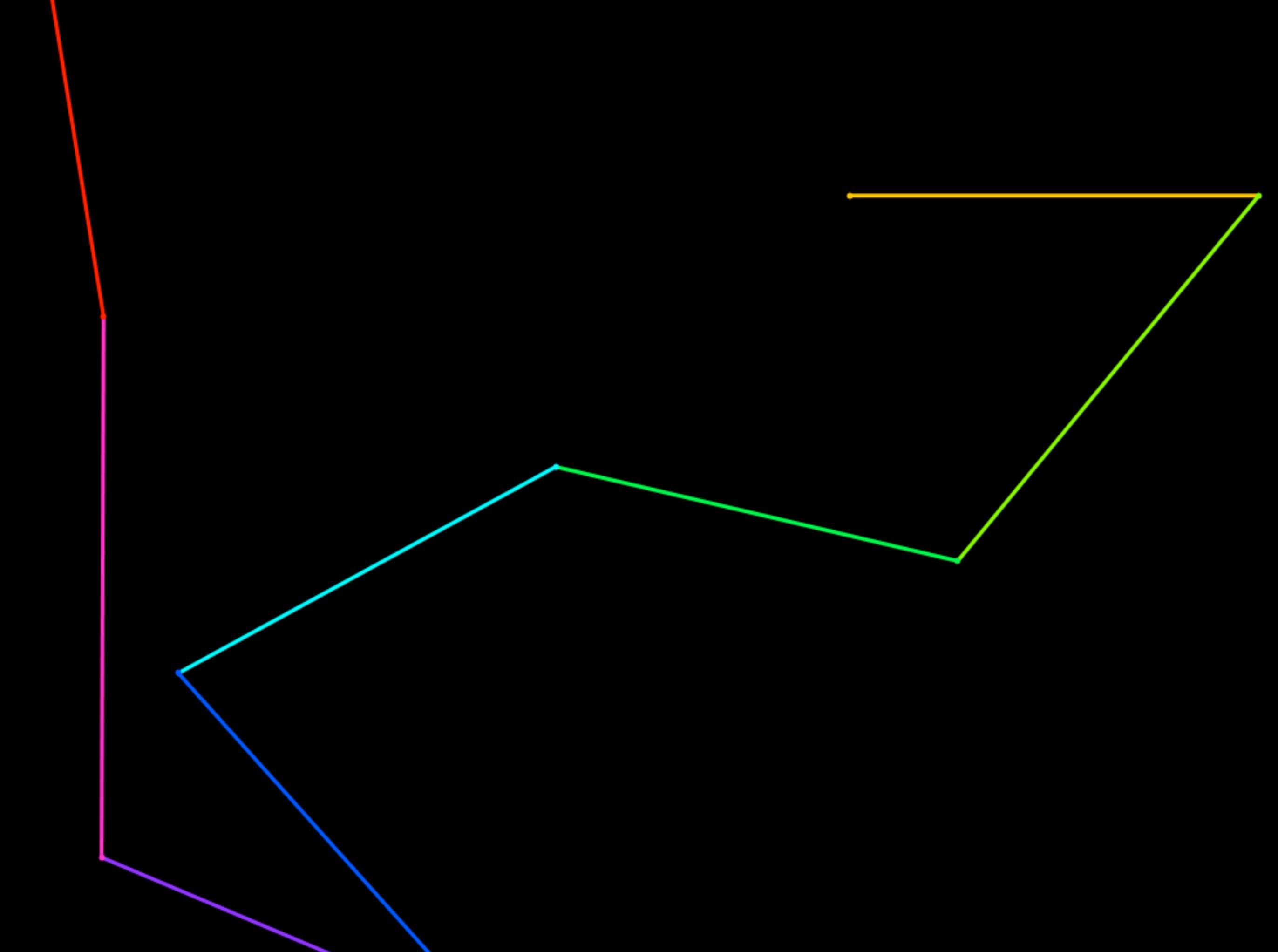
Most recent  
problem

# Spectra

$$A_{nm} = \cos(n^2 m a + m \beta)$$





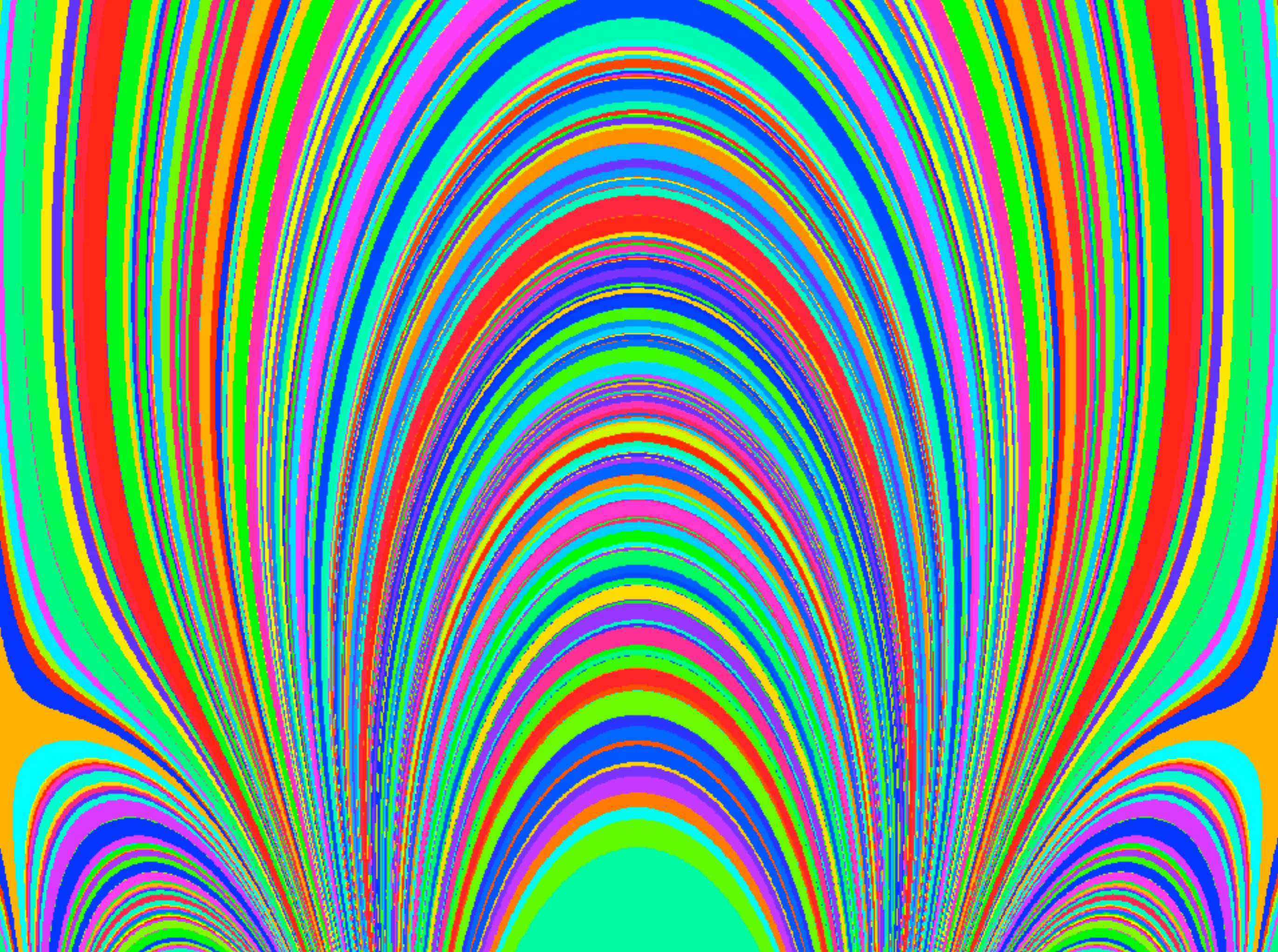


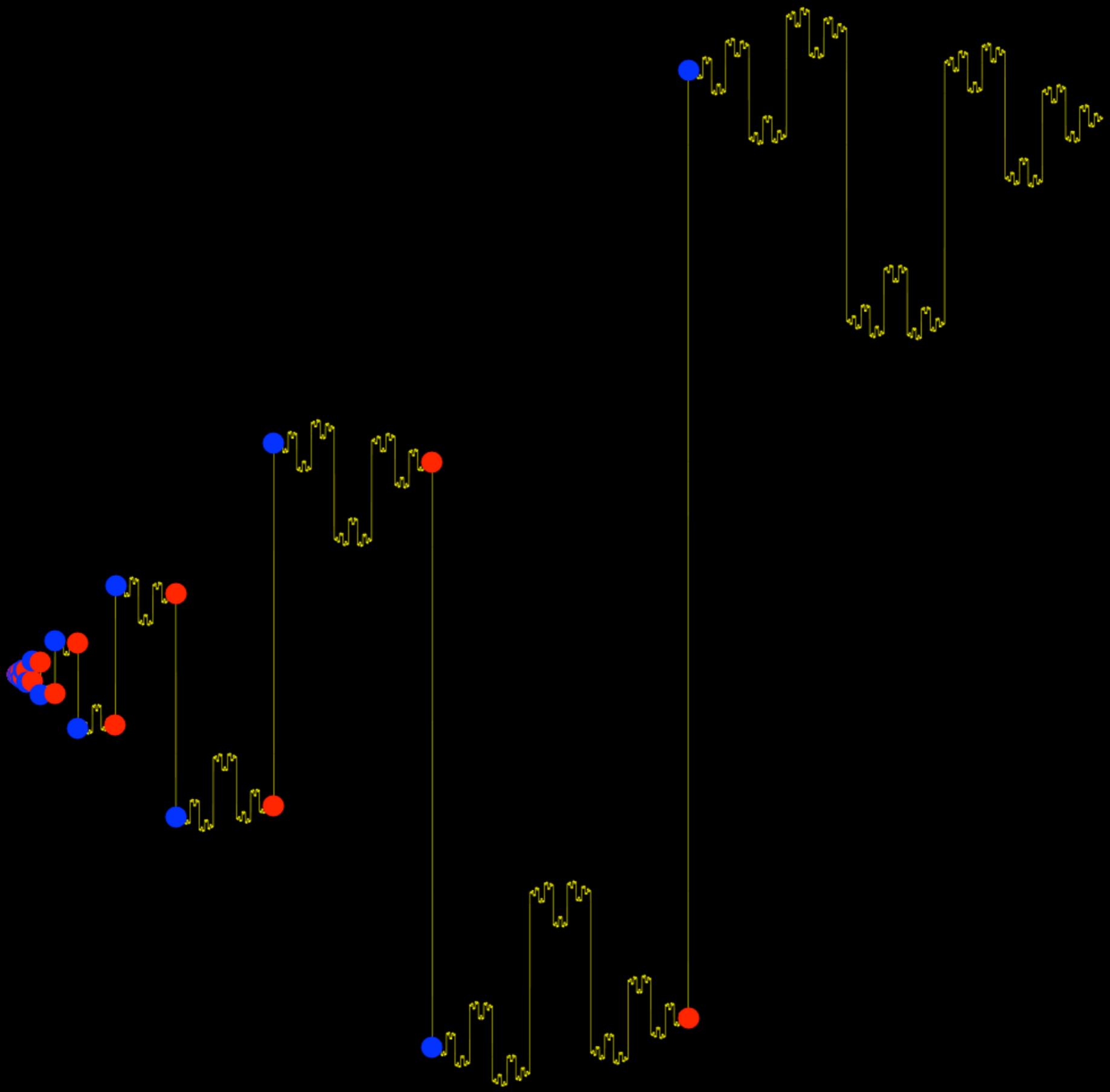
# Curlicues

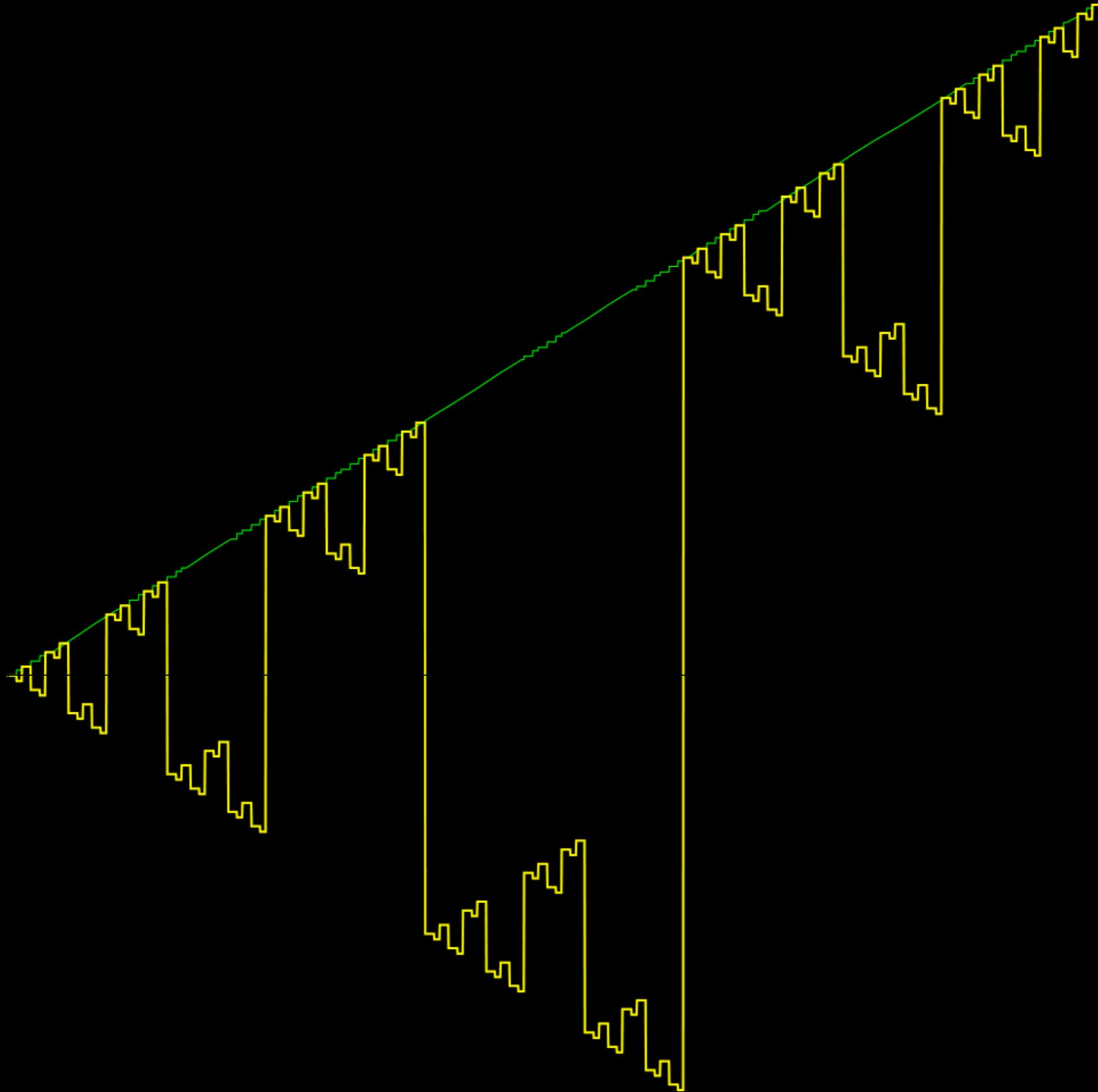
---

# Theta function

$$f(a) = \sum_{n=1} e^{i n^2 a}$$







# More about risks:

- Unfortunately, most failures in research are not documented, except if it is connected with success.

# Stanford Grad School of Business

“If you want to learn the secrets of success, it seems perfectly reasonable to study successful people and organizations. But the research of Jerker Denrell, an associate professor of organizational behavior, suggests that studying successes without also looking at failures tends to create a misleading — if not entirely wrong — picture of what it takes to succeed.”

# Example: Cantor

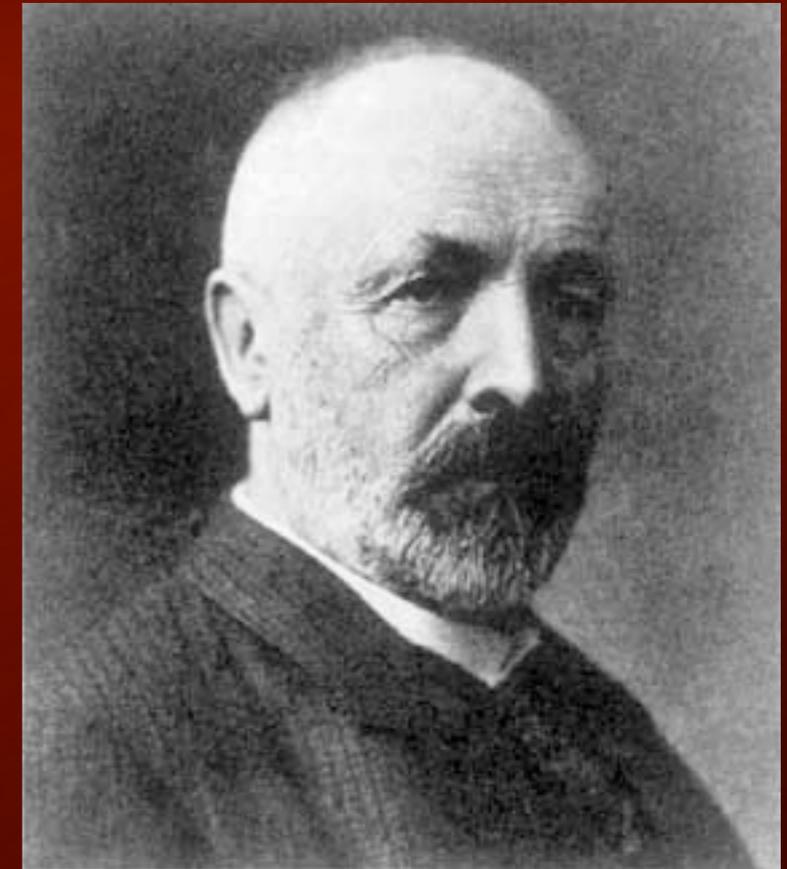
Risky research Area:

Transfinite numbers

Foundation of mathematics

Cardinality

Fractals



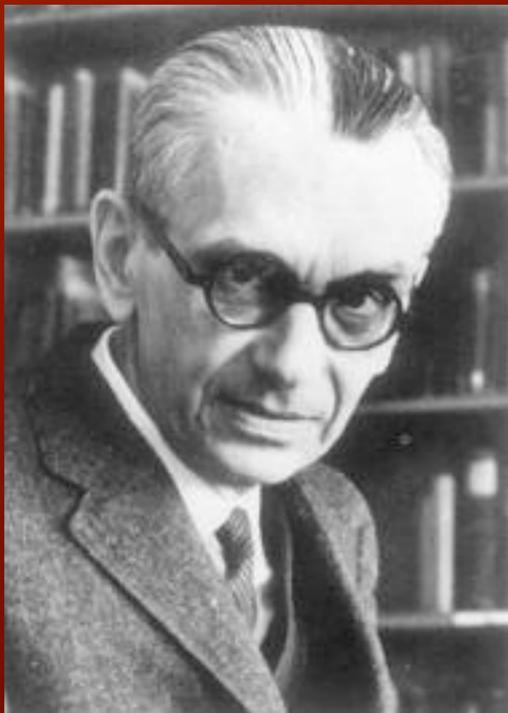
Under Attack by traditional  
mathemematics: Weierstrass,  
Kronecker

Cantor had several nervous breakdowns  
and died 1918 in a mental institution in  
Halle an der Saale in Germany

Apropos:

# The Black Theorem

whoever finds it, becomes mad.



Most logicians  
however remained  
sane

# Other risks: Germaine

1776 – 1831

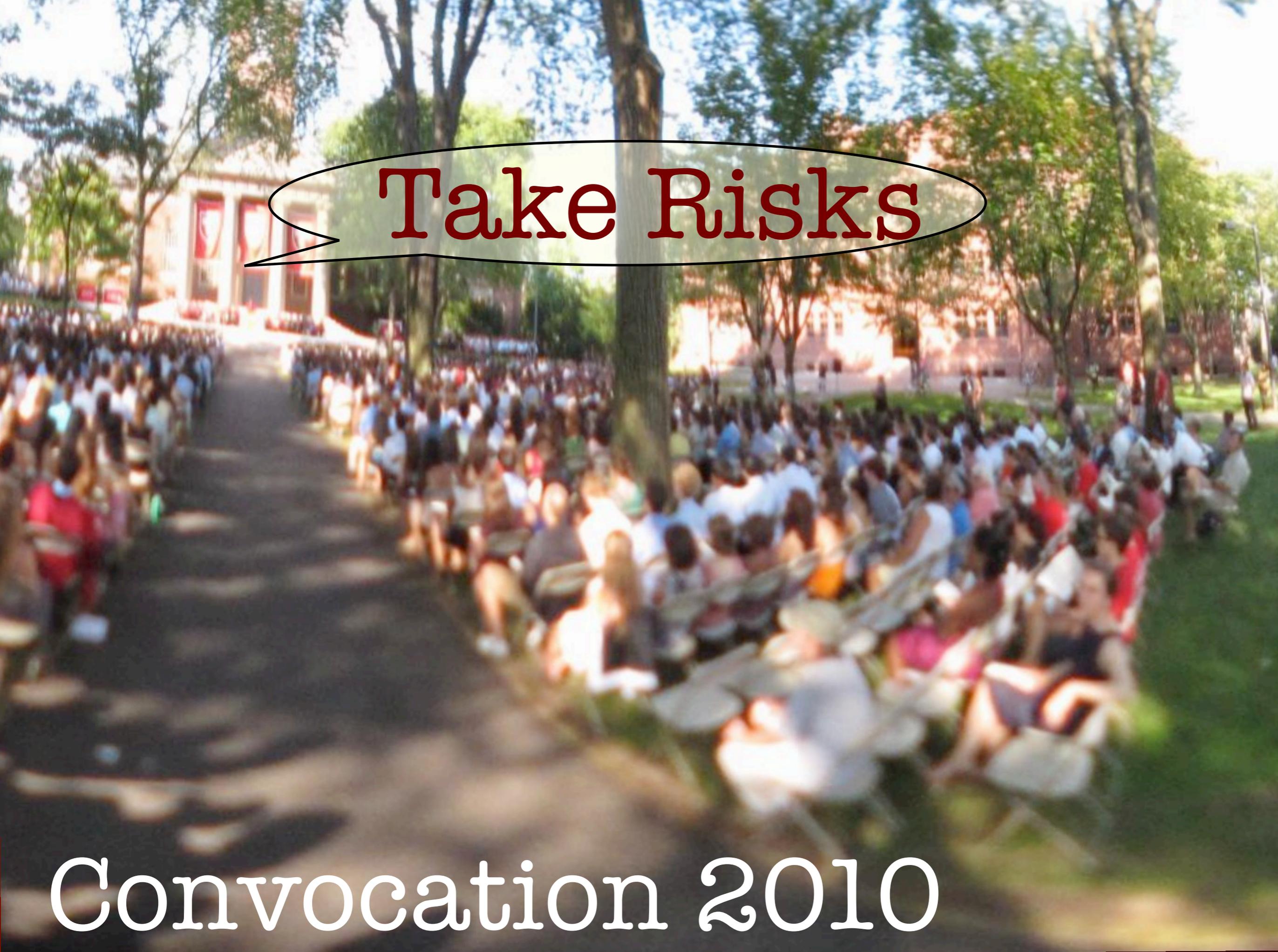
Risk as a women to go into Math.

Her parents forbid her to do math  
Was as a pseudonym M. Le Blanc  
at the Ecole Polytechnique since  
women were not allowed to  
attend.



Mathematicians like Lagrange, Gauss  
supported her.

Part II  
Teaching

A large crowd of people is gathered in a park-like setting, sitting on the ground. In the background, there is a large, multi-story building with a classical architectural style, featuring columns and a red facade. The scene is outdoors with many trees and a clear sky. A speech bubble graphic is overlaid on the image, containing the text "Take Risks".

Take Risks

Convocation 2010

# II) In Teaching

- Technology
- Copy right
- Lecture
- Presentations
- Dogmas

# Technology

- Benefits and risks of technology in the classroom

# Pitfalls in teaching and learning calculus

Oliver Knill

5/20/2005, working draft

## Abstract

Students studying calculus often make the same mistakes. Similarly, teachers teaching calculus have patterns of mistakes. It is much easier to tell how things should **not** be done than how things should be done. I collect in this document some "no-go's". This is a living document and is expected to be updated and adapted in the future.

## General teacher mistakes

**Lack of preparation.** Even routine calculus problems need to be rehearsed.

**Examples:** a teacher takes a random 3x3 matrix and goes onto the task to find the eigenvalues of this matrix. Row reduction leads to mess, the characteristic polynomial has no apparent root.

**Improvisation.** Rely on improvisation skills for lectures.

**Example:** In a multivariable calculus course, an arc length computation which does not work like for a Lissajou curve  $\vec{r}(t) = (\cos(5t), \sin(7t))$ .

**Difficulty.** Assign too difficult problems as homework.

**Example:** Find the polytop with 12 edges with fixed surface area and maximal volume using Lagrange extremization.

**Political correctness** Use politically incorrect problems or jokes.

**Example:** show slides with sexist joke. Also politically biased statements can be tricky. There are students of almost any political orientation in the classroom.

**Lecture too long.** The flow of lectures at the college is interrupted. Students come too late to the next class. Everybody has to wait.

**Example:** the teacher discusses with students after class in the class room, while students of the next class have to wait. The next class gets delayed.

**Too late.** Come too late to class. After cleaning the black board, collecting homework.

**Example:** At Harvard, classes start 7 minutes after the hour.

**No interaction.** Have no interaction or activities during class. Students turn off.

**Example:** Tuesday-Thursday calculus classes at Harvard are 90 minutes. This is a long time to keep the attention span.

**Wrong lecture.** Cover an other topic than the other sections. The students can not do the homework and later do not know the material for the course wide exam.

**Example:** Basic definitions which are necessary for the homework are not introduced.

**Outfit.** Improper dressing can be a distraction. While in educational environments, almost everything goes, there are limits, like avoiding coming barefoot. Rule of thumb: have at least a shirt on ...

**Untested technology.** The use of technology unfamiliar to the teacher can eat away precious classroom time.

**Misuse of technology.** Each technology has its own advantages. It is often difficult to transfer one to an other.

**Exceptions.** Focus too early on exceptions.

**Uninspiring.** Showing lack of preparation and enthusiasm. Why should people at all come to class?

**Ignore student questions.** A teacher brushes over a student question. Better: possibly repeat the question and answer carefully. If the question is not clear, restate it. If the answer is not known, refer to the next hour, or give it as a challenge to the class.

**Mumble.** Instead of using loud and clear voice and speaking to the class, the teacher mumbles to the blackboard.

**Giving away solutions.** Providing full solutions to problems instead of hints to problems. the solution. Instead, one can give hints, still leaving the task of solving the problem to the student.

**Example:** For male teachers: check trousers after the bathroom. The range of acceptable clothing or hairstyle certainly also depends on the personality of the teacher and students are quite tolerant. But clothing issues should not become a distraction or an embarrassment.

**Example:** it is annoying to watch somebody fiddling with the computer, the overhead projector or trying to fix a line of Mathematica code in front of the class.

**Examples:** 1) make slides by scanning a handwritten paper and show it with the overhead projector. 2) take a PDF version of a text and pull it into a power point presentation 3) use the overhead projector to sketch a figure.

**Example:** "the infimum of a set is not always smaller than the supremum of a set". State exceptional examples too early: like examples, where  $f_{xy} \neq f_{yx}$ . For nonmathematicians, such counter examples may be amusing but have also a pathological touch.

**Example:** A teacher copies his notes word by word onto the blackboard. Worse: a teacher copies problems from the book directly onto the blackboard.

**Example:** Student: "Could you please explain the last step again". Teacher: "I'm sorry, we do not have time for that." or "You should have payed more attention to what I just sayd".

**Example:** A long computation is performed on the blackboard. The class sees only the back of the teacher. Worse, the teachers body covers the written part.

**Example:** Compute  $\int_0^{2\pi} \sin^4(x) dx$ . Hint 1: Use  $\sin^2(x) - 1 = \cos^2(x)$  Hint 2: use this only for one  $\sin^2$  and leave the other  $\sin^2$ . Hint 3: use then a trigonometric identity.

## Mistakes by Course-head

**Lack of guidance.** Syllabus, homework problems, homework solutions, practice problems, quizzes are not available ahead of time.

**Too much guidance.** Too much advise like detailed prescribed lecture scripts can kill the innovation of the individual teacher. Some feel just becoming a narrator of a script. Sharing advise, tricks and experience is good but teaching is also a creative act.

**Distributing the responsibility.** Organization tasks can be shared with the section leaders but ultimately, the course head has the responsibility.

**Use of CAs.** CAs have about 15 hours per week. Some volunteer to do more and are happy to share more responsibility.

**Change the syllabus.** In service courses like calculus, there are some things which have to be taught and which other departments require to have included. While there is some variation of the syllabus possible, it should be fixed before the semester starts. The syllabus should have flexibility in the time frame like one or two hours to "catch up".

**Examples:** The homework assignment is only fixed a day before class. Because a lecture has to be tailored to make the homework useful, the section leaders can not prepared early enough.

**Examples:** Individual lecture-plans make often only sense for the teacher who prepares the lecture. They often do not mention things which are obvious. The most useless "lecture plan" is a list of the section titles which are covered in the book. Better: point out possible pitfalls, student questions, common misconceptions.

**Examples:** A section leader is assigned to proctor a midterm and does not show up. Because midterms can not be repeated easily (schedule, holidays, conflicts, lecture hall reservation), this is a disaster.

**Examples:** CAs have to grade midterms or even the final exam. CAs have to take their own midterms and exams, usually just at the time of the calculus midterms or exams. There also seems to be a Harvard policy that undergraduates are not allowed to grade exams. This makes sense because some CAs will have to grade the exams of their own friends.

**Examples:** Some section falls behind and all other sections adapt to the pace of the slower section. It can happen that classroom time is spent with valuable discussion time and not everything can be covered. In such a case, one has to ask the students to read the rest in the book. It is better to anticipate that the hour does not suffice for all the material, reduce the complexity of the examples or ask students to preread.

## Mathematical teacher mistakes

**Ambiguous notation.** Use ambiguous notation.

**Uncommon notation.** Use uncommon notation.

**Example:** let  $\mathbf{v}$  and  $\mathbf{w}$  be vectors in space. Define the parametrized surface  $r(u, v) = u\mathbf{v} + v\mathbf{w}$ . Even so the difference between vectors and real numbers is done by taking bold face letter, such features can get lost on the blackboard.

**Example:** let  $\alpha, \beta, \gamma$  be points in the plane and let  $P, Q, R$  denote the line segments connecting them.

**Unusual twists.** Unusual orientation of coordinate system.

**Incorrect definitions.** Give incorrect definitions of mathematical concepts.

**Overkill in rigor.** Adapt the rigor to the class.

**Too difficult problems** Give too difficult problems without a hint.

**Incorrect theorems** State incorrect theorems.

## Course assistant mistakes

**Not coming to class.** It is not only to see what material is covered, the CA should also get a feel, what how students cope.

**Not grading in time.** Bringing homework back too late is the biggest turn off.

**Example:** use a left handed orientation  $e_1, e_3, e_2$ .

**Example:** like a vector is an object with length and orientation.

**Example:** a  $\epsilon - \delta$  definition of continuity is ok for mathematicians and physics, but not appreciated by biochem majors for example. Eulers definition  $x$  close to  $y$ , then  $f(x)$  close to  $f(y)$  is more intuitive.

**Example:** Calculate  $\int_0^{\pi/2} \log(\sin(x)) dx$ . There is an easy solution to this problem, but it is not easy to find. 99 percent of all students would get very frustrated when assigned this problem.

**Example:** all extrema of  $f(x, y)$  under the constraint  $g(x, y) = c$  are obtained at points, where  $\nabla f = \lambda \nabla g$ . (This is incorrectly stated as a theorem in many text books.)

**Example:** Real life example: A course assistant worked for the second year. Feeling he knows the material enough, fails to come to lectures. He got fired.

**Example:** Real life and worst case scenario: a course assistant does not grade for several weeks and is out of town. The papers pile up. It becomes impossible for one person to do it. The teaching fellows spend an afternoon grading the papers.

## General student mistakes

**Do not come to lectures.**

**Do not speak up.** If something is not clear.

**Copy homework from someone else.**

It is possible to learn mathematics by reading the book. However, the brain does not keep the information as reliably. Having seen and heard the material makes it stick better. In a time, where distraction (phone, Internet, friends etc.) is tempting, it needs stamina to make up the learning time away from class.

Most of the time, if something is not clear to one student, it is not clear to the entire class.

Mathematics is doing. The process of struggling with a homework problem is important too.

Come unprepared to class.

The most effective use of class room time is to have already partly absorbed the material before hand.

Cheat during exams.

Sometimes, this happens unconsciously. A student uses a calculator in an exam, where none are allowed.

## Mathematical student mistakes

Improper use of substitution.

**Example:**  $\int_0^{3\pi/4} \sin^2(x) \cos(x) dx = \int_0^{\sin 3\pi/4} u^2 du = \int_0^{1/\sqrt{2}} u^2 du$ , where  $u = \sin(x), du = \cos(x) dx$ .

Improper handling of indefinite integrals.

**Example:**  $\int_{-\pi}^{\pi} 1/x dx = 0$  so that  $\int_0^{\pi} 1/x dx = 0$ .

Matrix algebra issues:

$$(A + B)^2 = A^2 + 2AB + B^2.$$

Wrong linearity of determinant:

$$\det(2A) = 2\det(A).$$

**Too linear** Inappropriate linearity:

$$\sqrt{x+y} = \sqrt{x} + \sqrt{y}, e^{x+y} = e^x + e^y, (x+y)^2 = x^2 + y^2. 1/(x+y) = 1/x + 1/y.$$

**Too commutative** Inappropriate commutativity of composition:

$$\log \sqrt{x} = \sqrt{\log(x)}, \sin(5x) = 5 \sin(x).$$

**A sin.** A student "sin" as a joke.

$$\frac{\sin x}{x} = \sin.$$

Inappropriate cancellations.

$$\frac{((x+1) + 5x^2)/(x+1)}{\frac{3+x}{x}} = \frac{1 + 5x^2/(x+1)}{3 + x/x} = \frac{3 + x/x}{3 + 1} = 4.$$

Unclear brackets.

**Example:**  $3x/5 = 3/5x = 3/5x = 3/(5x).$

Unit confusion.

**Example:**  $1\$ = 100c = (10c)^2 = (\$0.1\$)^2 = 0.01\$ = 1c.$  (source: Eric Schechter, 2004)

Division by zero.

**Example:**  $x(x^2 - 1) = x$  implies  $x^2 = 2$  and so  $x = \pm\sqrt{2}$ .

Square root errors

**A famous example:**  $a = b$  implies  $a^2 = ab$  so that  $a^2 + a^2 = a^2 + ab$  or  $2a^2 = a^2 + ab$ . The manipulation  $a^2 - 2ab = a^2 + ab - 2ab$  gives  $2a^2 - 2ab = a^2 - ab$ . Writing this as  $2(a^2 - ab) = 1(a^2 - ab)$  and canceling  $(a^2 - ab)$  from both sides gives  $1 = 2$ .

Signs in inequalities.

**Example:**  $-a < -3$  implies  $a < 3$ .

Misconception about constants.

**Example:**  $\frac{d}{dx} x^x = x x^{x-1}.$

Integration by parts. Forget one integral.

**Example:**  $\int x \cos(x) dx = x \sin(x) - \sin(x).$

Polar coordinates. The inversion formula for polar coordinates is not complete.

**Example:** Given the point  $(-1, -1)$ . Find the polar coordinates. We have  $r = \sqrt{2}$  and  $\theta = \arctan(-1/-1) = \arctan(1) = \pi/4$ .

An SAT pitfall. True facts for integers are extrapolated to all real numbers.

**Example:** This is a common trap for SAT exams: because  $0 \leq 0^2, 1 \leq 1^2, 2 \leq 2^2, 3 \leq 3^2$  one concludes  $x \leq x^2$ .

Sloppiness about signs. Especially in connection with the chain rule.

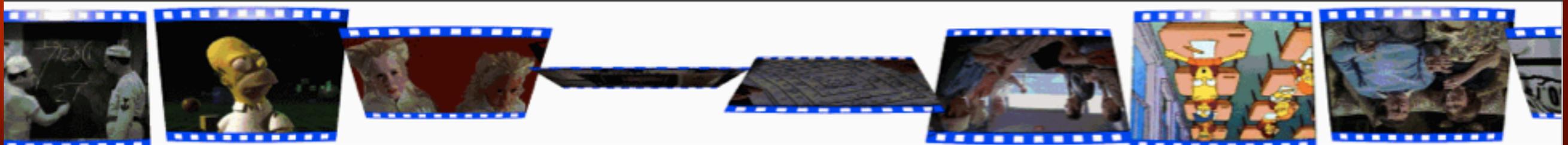
**Example:**  $\int dx/(1-x) = \log|1-x| + C.$

## Literature:

- Eric Schechter, "The most common errors in undergraduate mathematics education" <http://atlas.math.vanderbilt.edu/schectex/commerrs>
- <http://tutorial.math.lamar.edu/AllBrowsers/CommonErrors/CalculusErrors.asp>
- Andy Engelward, "do's and don'ts for Classroom style", talk at Bok Center Fall Teaching Orientation Sept. 14th, 2004

Copy right

# Copy right



## Mathematics in Movies

This is a collection of movie clips in which Mathematics appears. I'm collecting DVDs and VHS tapes of such movies. This is a working document to be extended over time. I started this page during spring break 2006. See also the page "[Begin of lectures in college teaching](#)" and "[End of lectures in college teaching](#)" and "[Examples of good talks](#)". To see the movies larger, to see it on the iphone or include it in a presentation, chose the quicktime ipod version, which are files with .m4v extension. ([Media RSS link](#)).

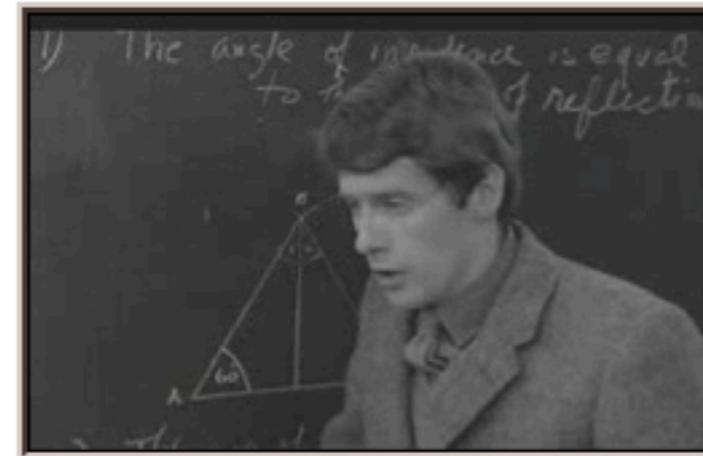
Date: March  
2006 - August  
2010  
by:  
[Oliver  
Knill](#)  
Department of  
Mathematics  
Harvard  
University



### In July [IMDb link]

Theoretical and applied computation of a free fall problem (Thanks to Andrew Ranicki for the suggestion)  
2000

Play the [flash version \(.swf\)](#), or watch the [quicktime file \(.m4v\)](#)



### The Knack ... and How to Get It [IMDb link]

The angle of incidence is equal to the angle of reflection (Thanks to Andrew Ranicki for the suggestion)  
1965

Play the [flash version \(.swf\)](#), or watch the [quicktime file \(.m4v\)](#)

# Lecture

# Level of Teaching

next slides recycled from  
a talk of mine in

ICTCM Feb 15-18, 2006

# Choice

The paradox of choice amplifies traditional dilemmas a curriculum has, also without technology. This happens on many levels, here are four:

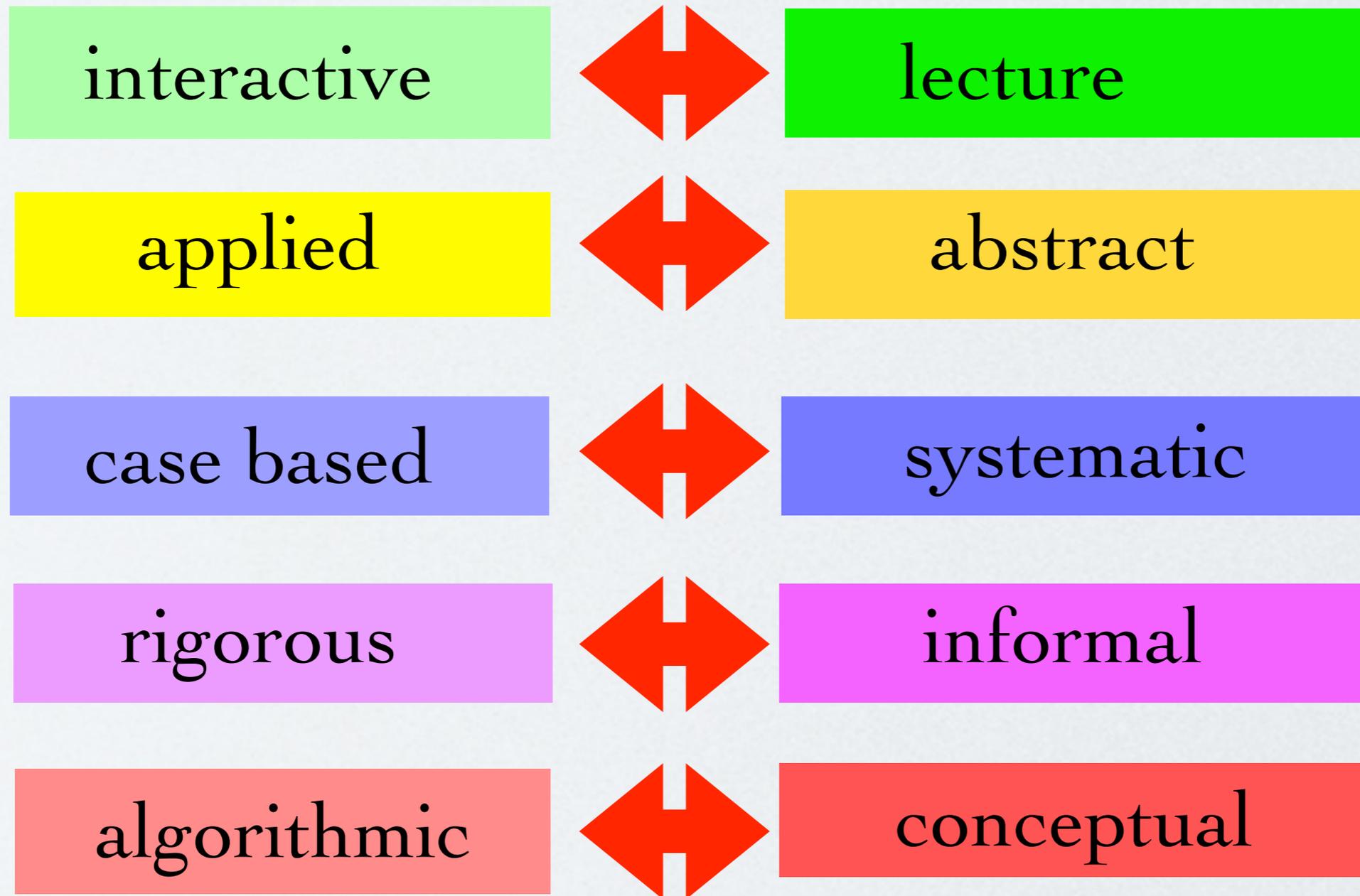
I PEDAGOGICAL LEVEL

II PRESENTATION LEVEL

III OBJECTIVE LEVEL

IV TOOL LEVEL

# Pedagogical Level



# Presentation level

increasing level of difficulty

Discovery or inquiry based

Discussion or debate

Workshop or lab

Interactive lecture

Blackboard lecture

Powerpoint presentation

# Objective Level

Visual  
Geometric

Formal  
Algebraic

Artistic  
Entertaining

Social  
Collaborative

Numerical  
Algorithmic

Application  
Practical

Humour  
Fun

Historical  
Cultural

Challenge  
Discovery

Inquiry  
Exploratory

# Tool Level



Computer algebra systems



Personal response systems



Online tests



Video podcasts



Videos



Teacher Wikis



Interactive online tools



Powerpoint lectures



Games



Demonstrations



Email support



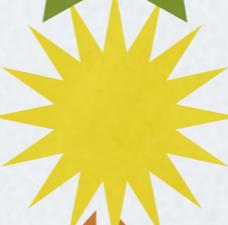
Instant messaging



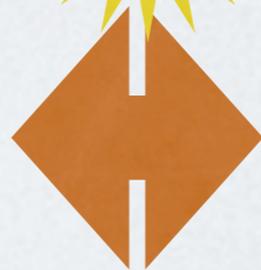
Chat bots



Course Blogs

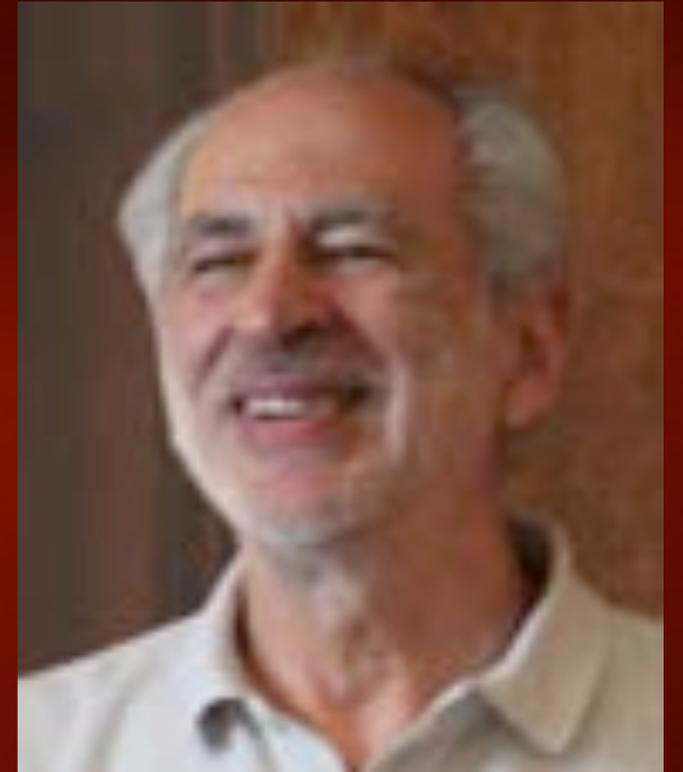


Projects



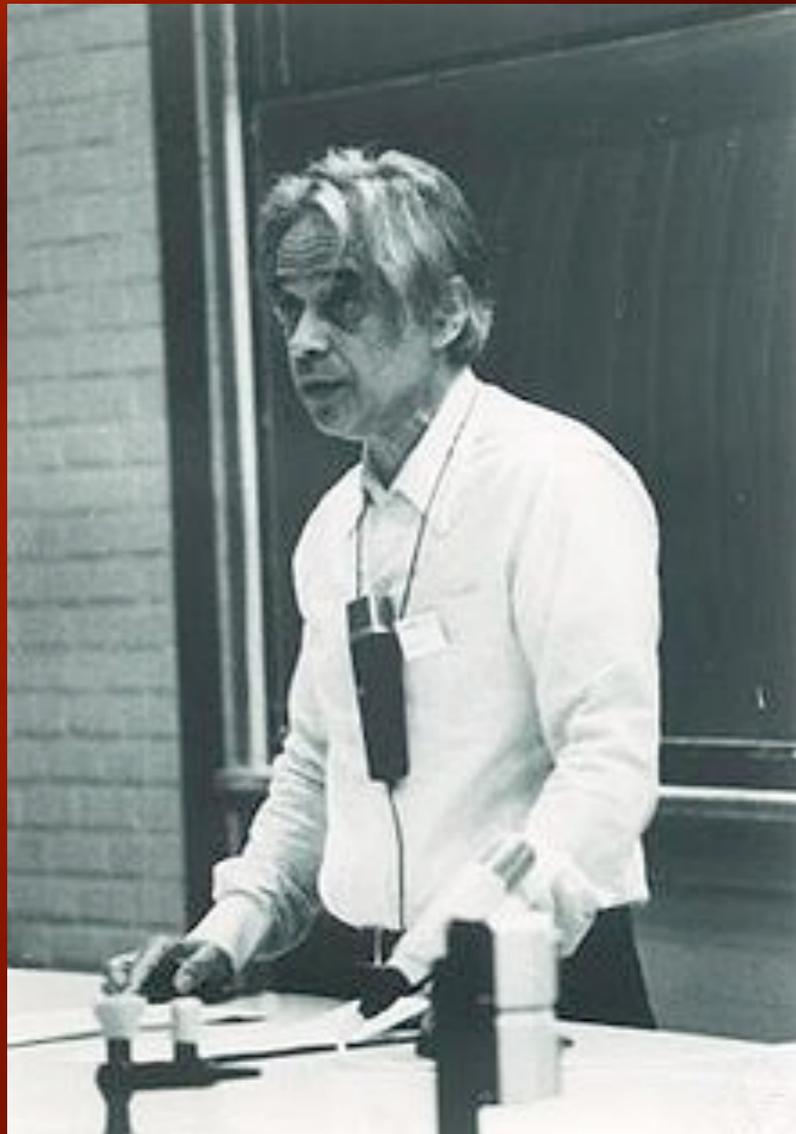
Group work

# An example by a local logician



- Gerald Sacks:
- in the middle of each class, a previously selected student would have to get up and tell a joke. The joke had to be short, funny, and inoffensive

# An episode told about teacher of mine:

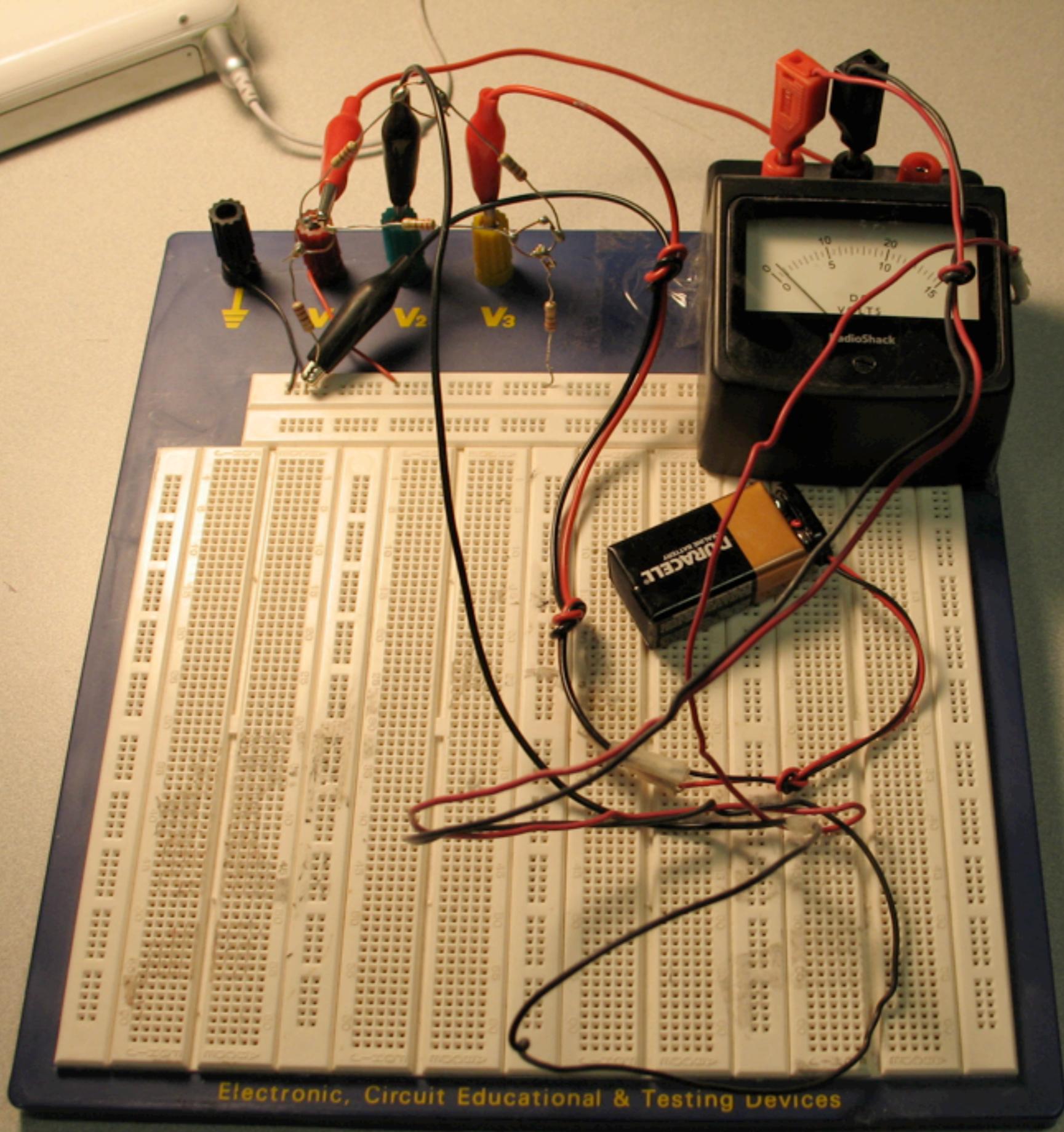


in 1982



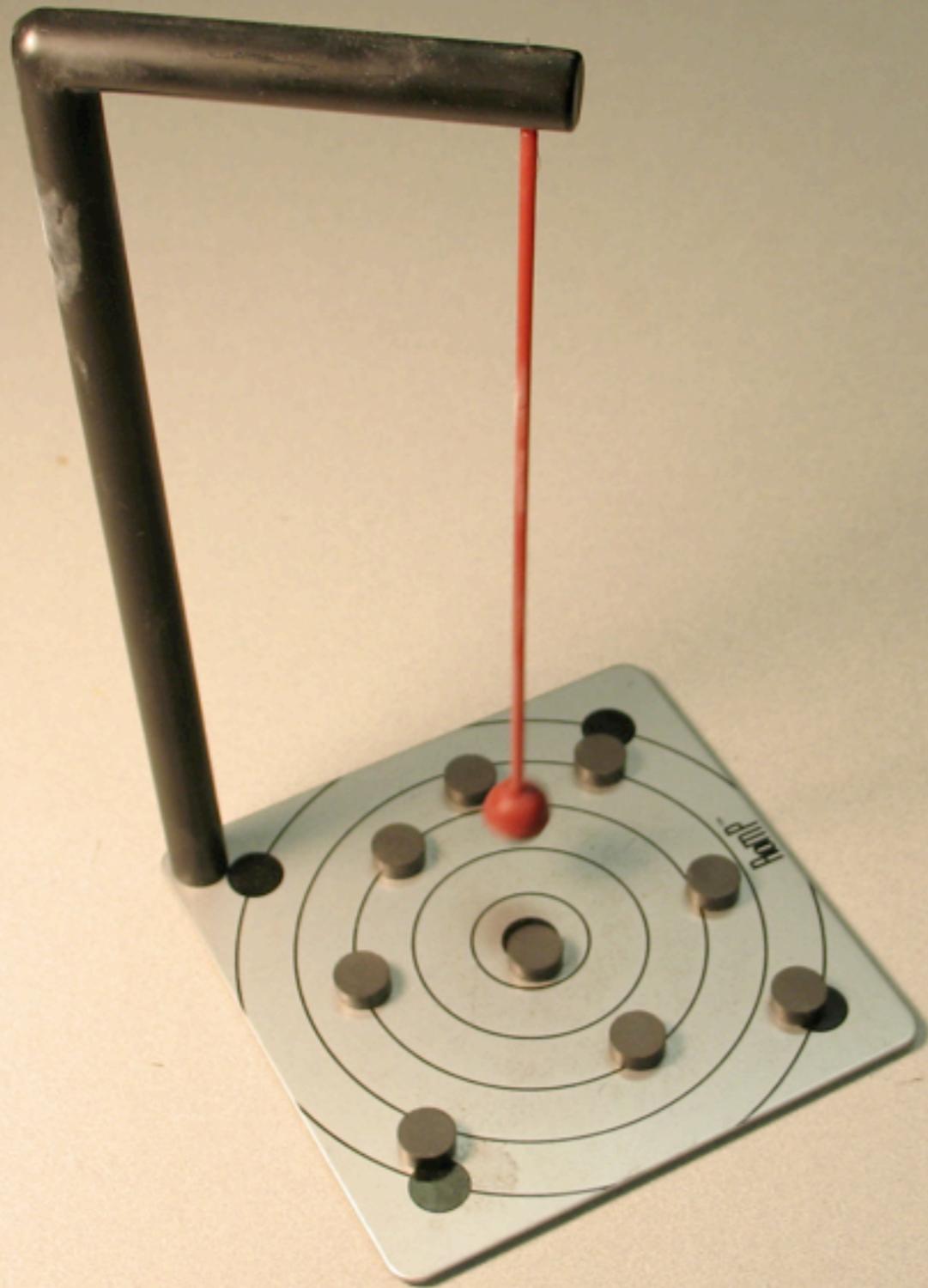
90 years

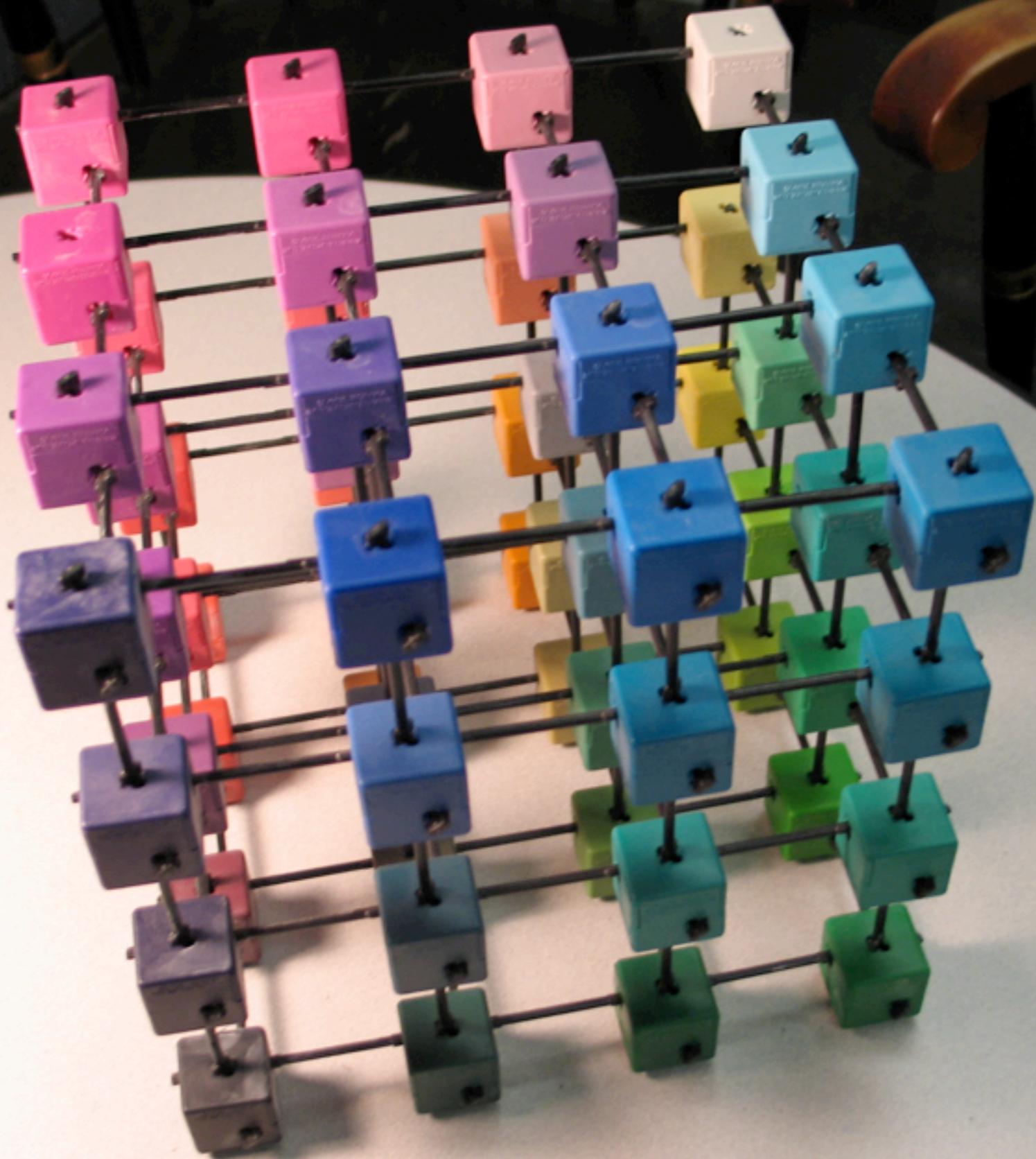
# Demonstrations



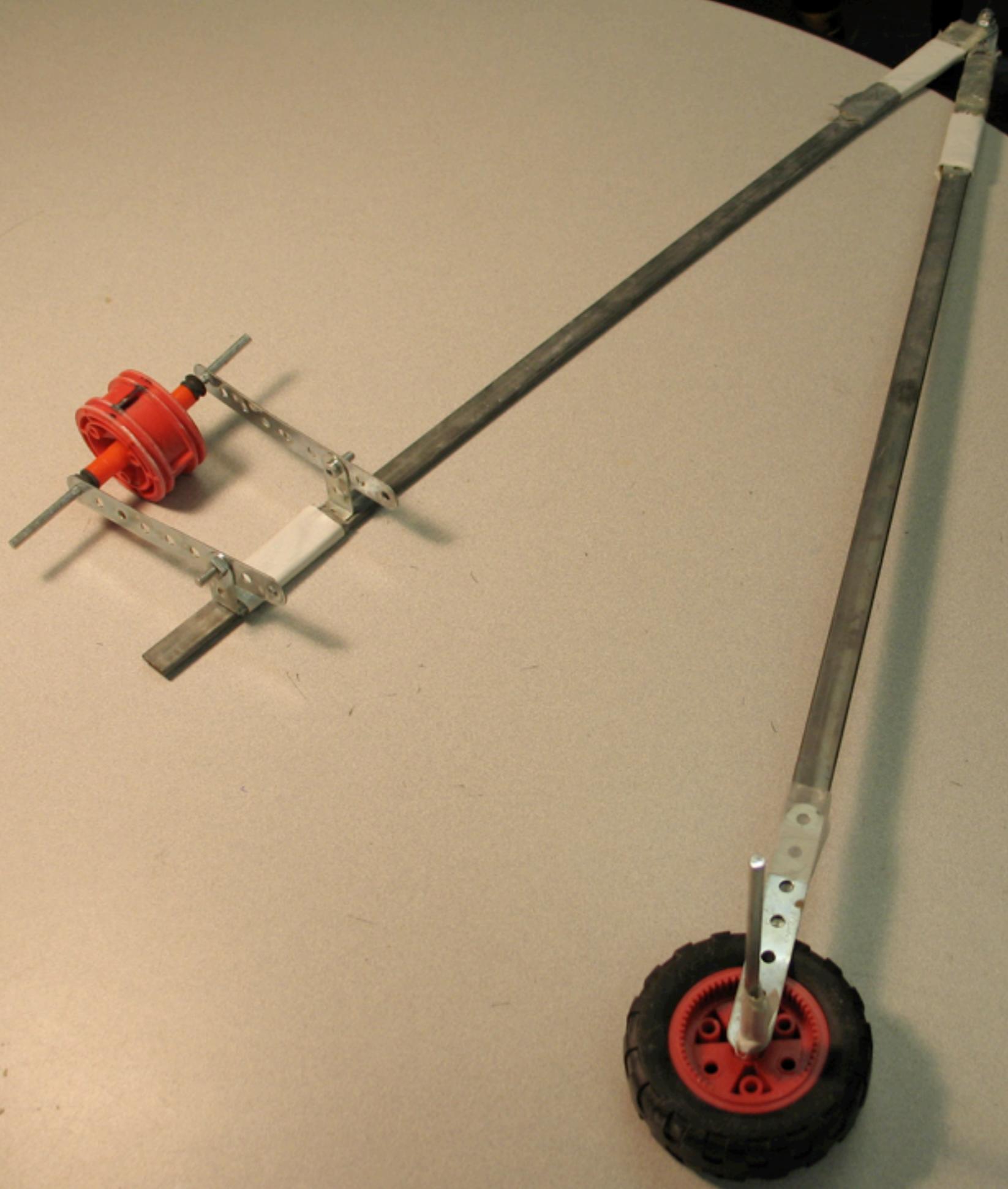
Electronic, Circuit Educational & Testing Devices













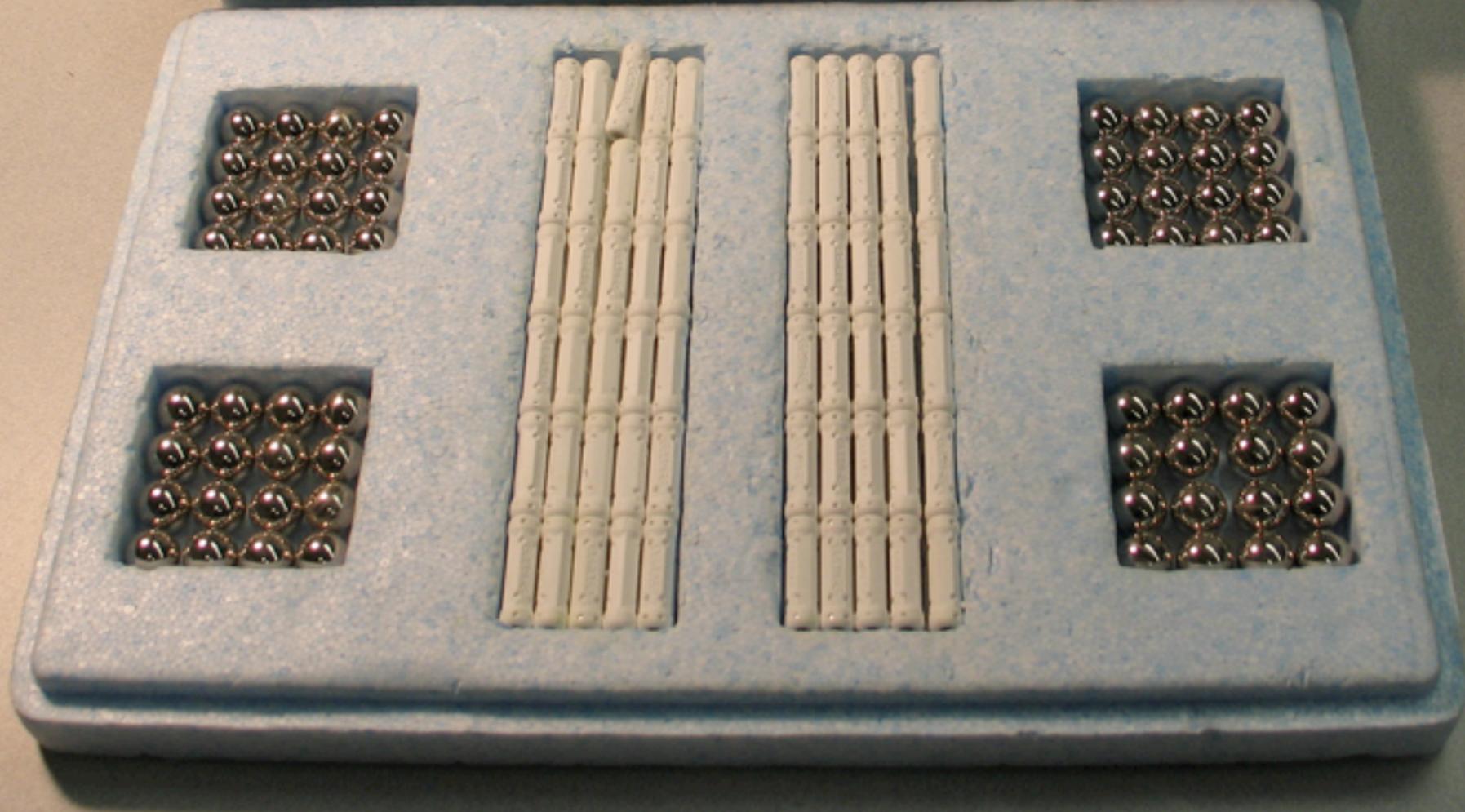
$\operatorname{arcsinh}(x)$

BICYCLE

$\int \frac{1}{\sqrt{1-x^2}} dx$

PRESS DOWN TO OPERATE

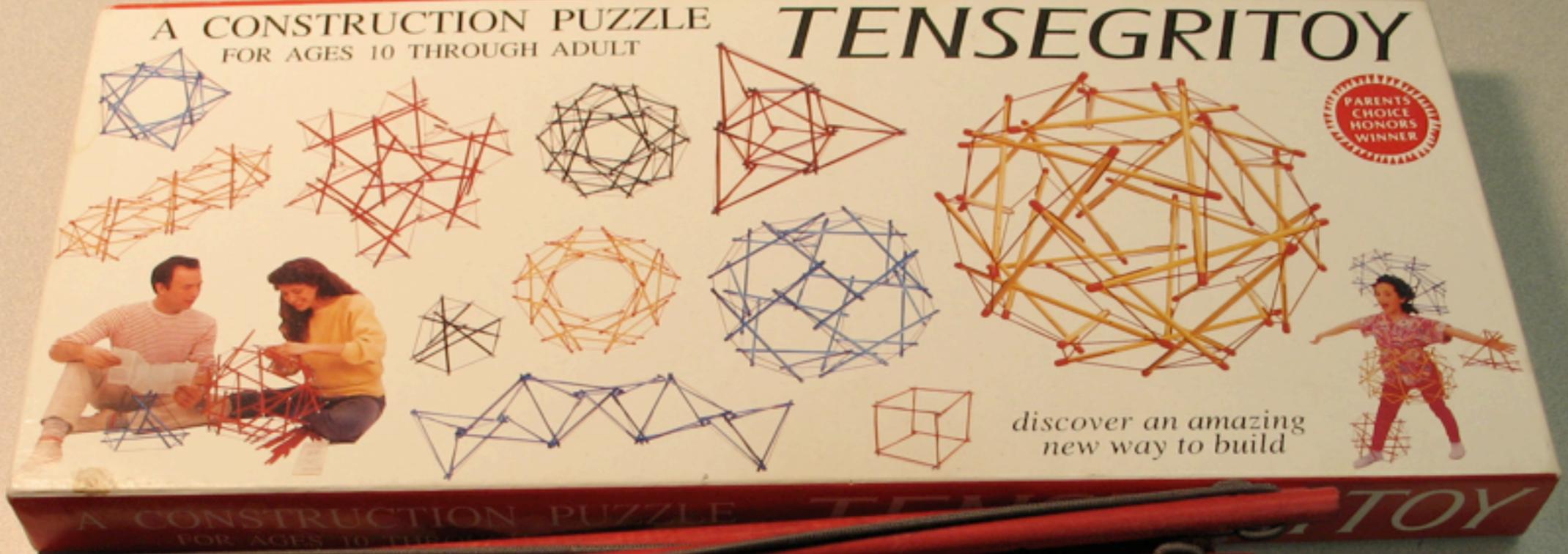




A CONSTRUCTION PUZZLE  
FOR AGES 10 THROUGH ADULT

# TENSEGRITOY

PARENTS  
CHOICE  
HONORS  
WINNER



*discover an amazing  
new way to build*

A CONSTRUCTION PUZZLE  
FOR AGES 10 THROUGH ADULT

TENSEGRITOY



# Multimedia

Here is a particularly risky part of a presentation.

Apropos integrals:

From 2006-2008, the  
first  
search entry for  
“Calculus” on  
google videos was

# Calculus:



Censored

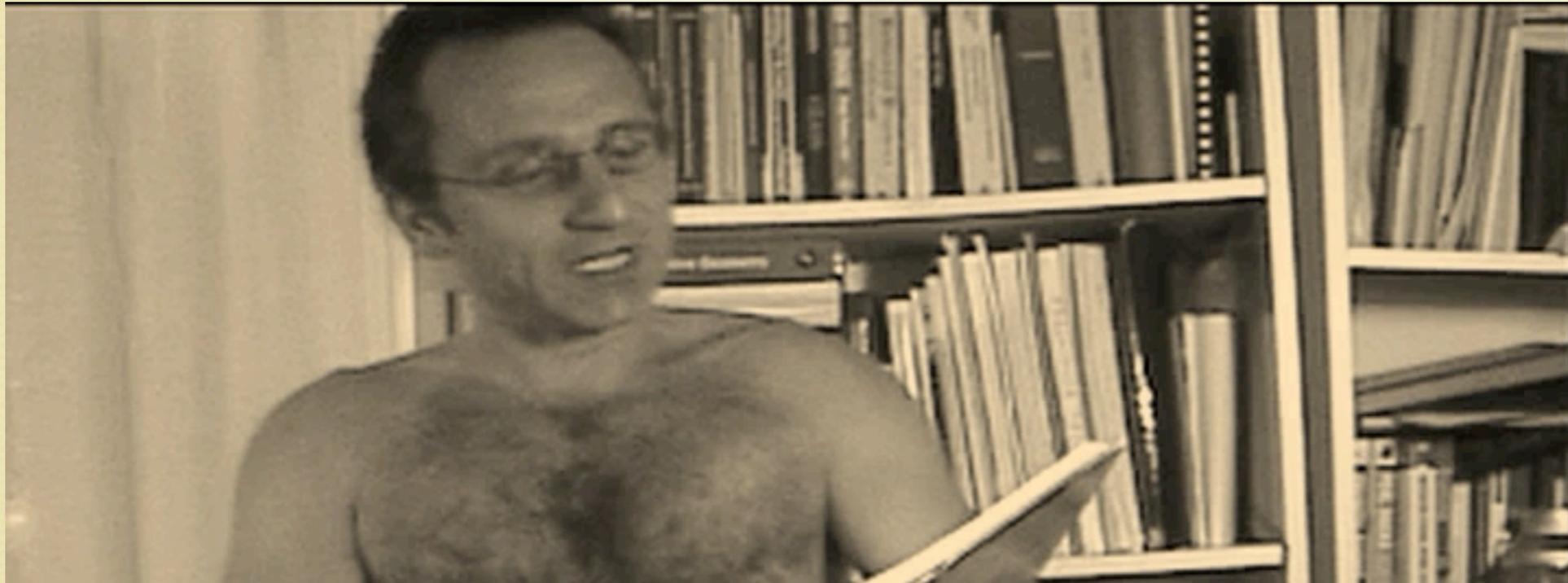
so, in 2006, I had a  
business idea:

**Bikini Calculus for  
the other gender:**

What is better  
than calculus **WITH**  
bikini. It is calculus  
**WITHOUT** bikini!

Here is part of  
the legendary test shot 2006

# WITHOUT Bikini Calculus



Censored

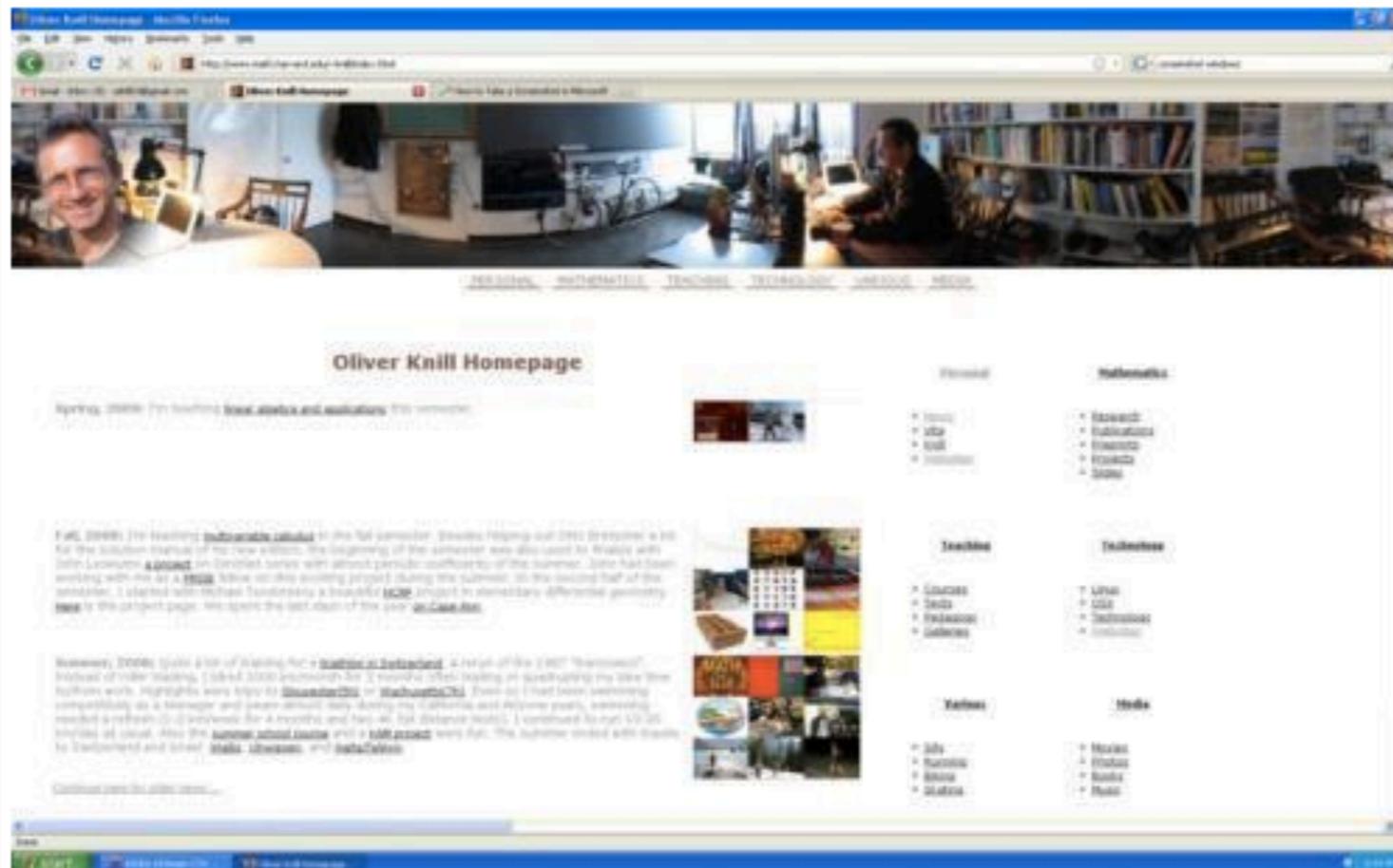
# Lecture

## Profs Online: From FYI to TMI

In an effort to reach students in an informal setting, some professors get personal

By LAURA C. SCHAFFER, CONTRIBUTING WRITER

Published: Wednesday, February 25, 2009



COURTESY JULIA B. HORNIG

Bookmark Oliver R. Knill's blog for all of your daily nipple flux and rollerblading video needs.

# Last Word

- Take risks. If things fail one can get a lot out of it.

# Famous Last word

- Be aware that the previous statement could be a famous last word.

# Sources and Credit:

- Theoni Pappas: Mathematical Scandals
- Krantz, A primer on mathematical writing
- Peter Wolf Breakthroughs in Mathematics
- Jose Ramirez, Work on Computer vision

The End