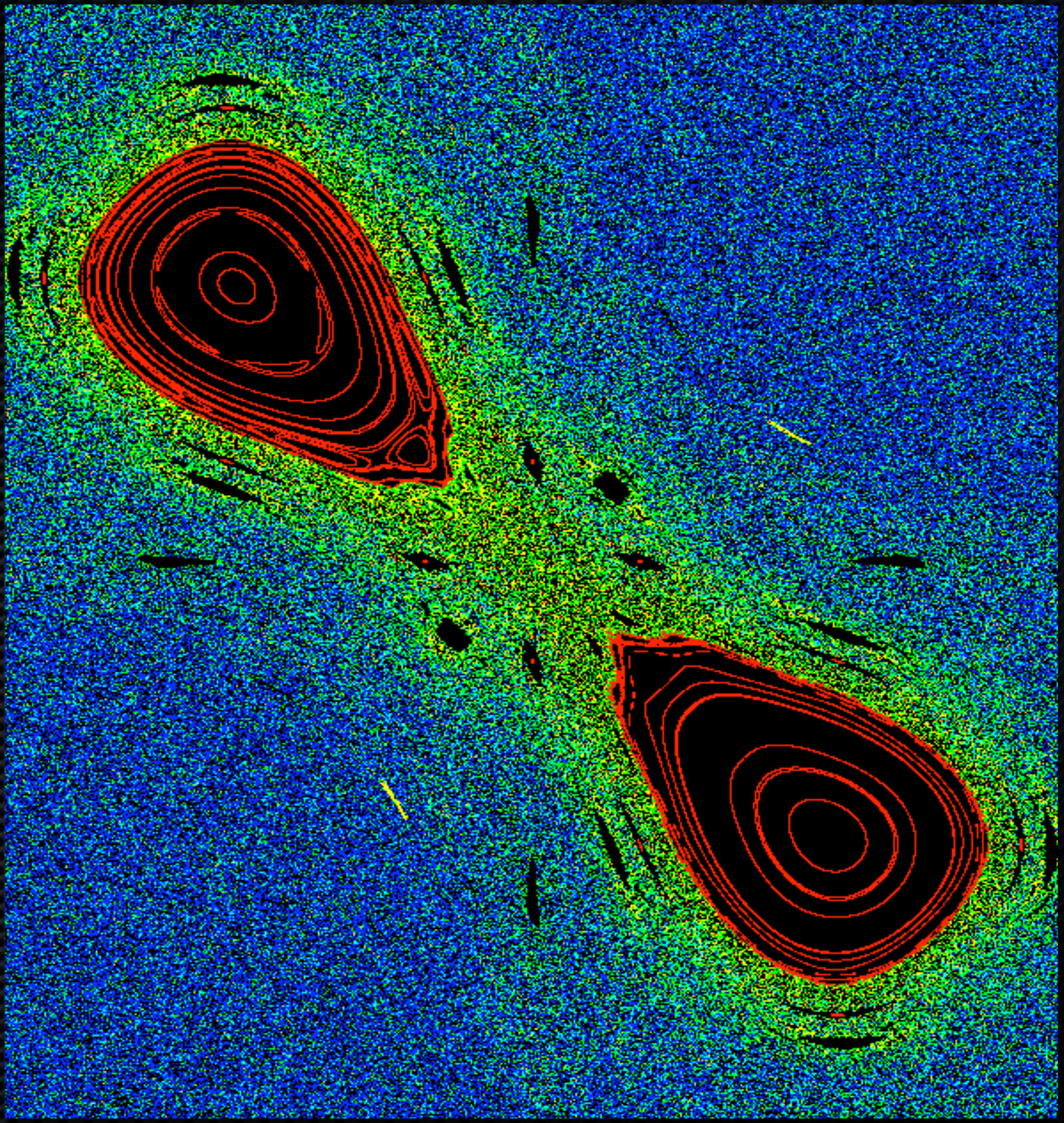
Where Chaos theory meets other disciplines



Oliver Knill

February 7, 2008

Part I: Examples of dynamical systems which exhibit chaos

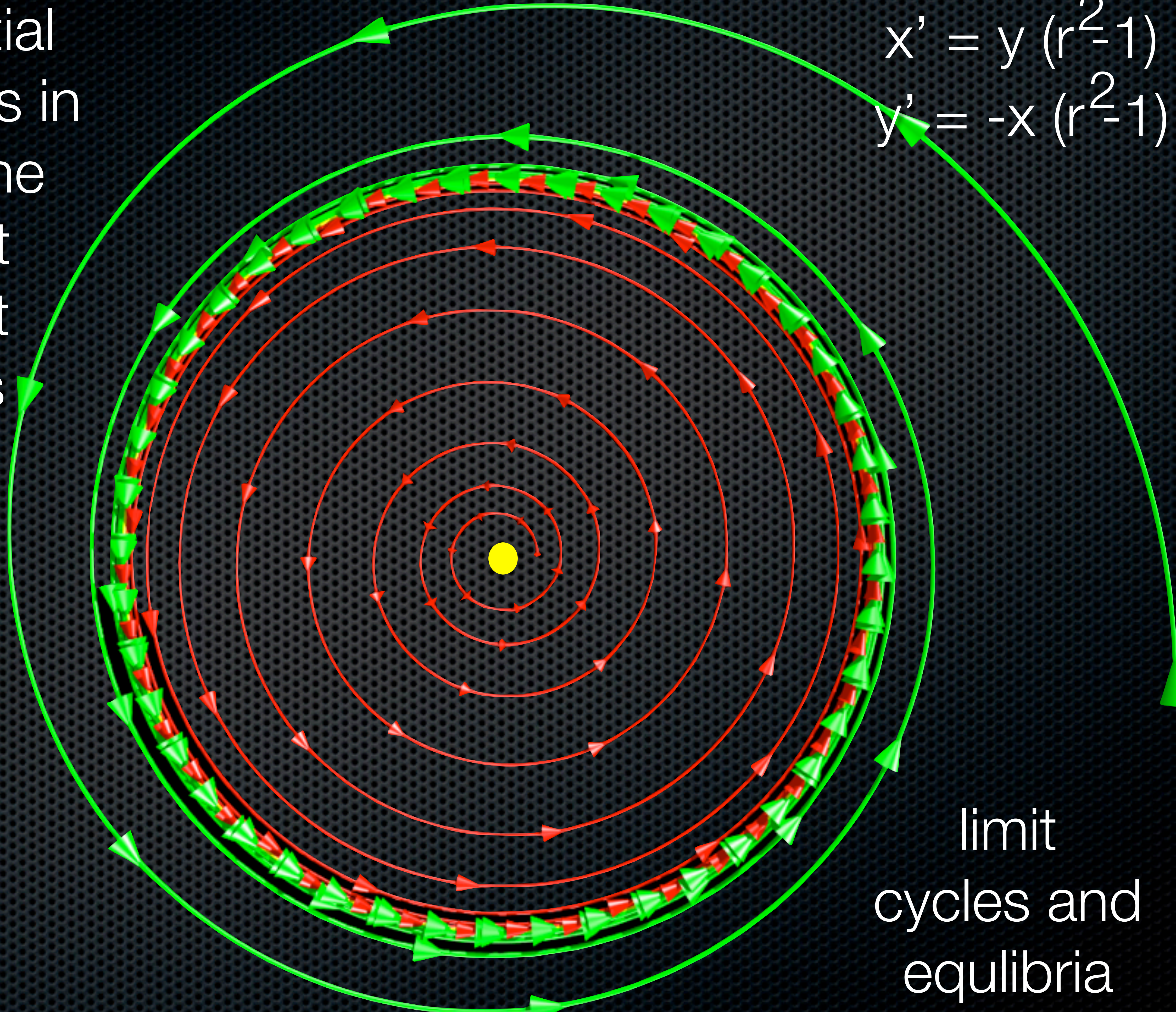
hall of fame

- the Lorentz system
- The logistic map
- The ABC flow
- The Standard map
- The quadratic map
- The Newton method
- Billiard maps

In a second part, we look closer at connections to other fields

- probability theory
- geometry
- astronomy
- sociology
- disaster prevention
- music composition
- poetry
- number theory
- chemistry
- mechanics
- christallograpy
- cryptology
- choreography

differential equations in the plane do not exhibit chaos



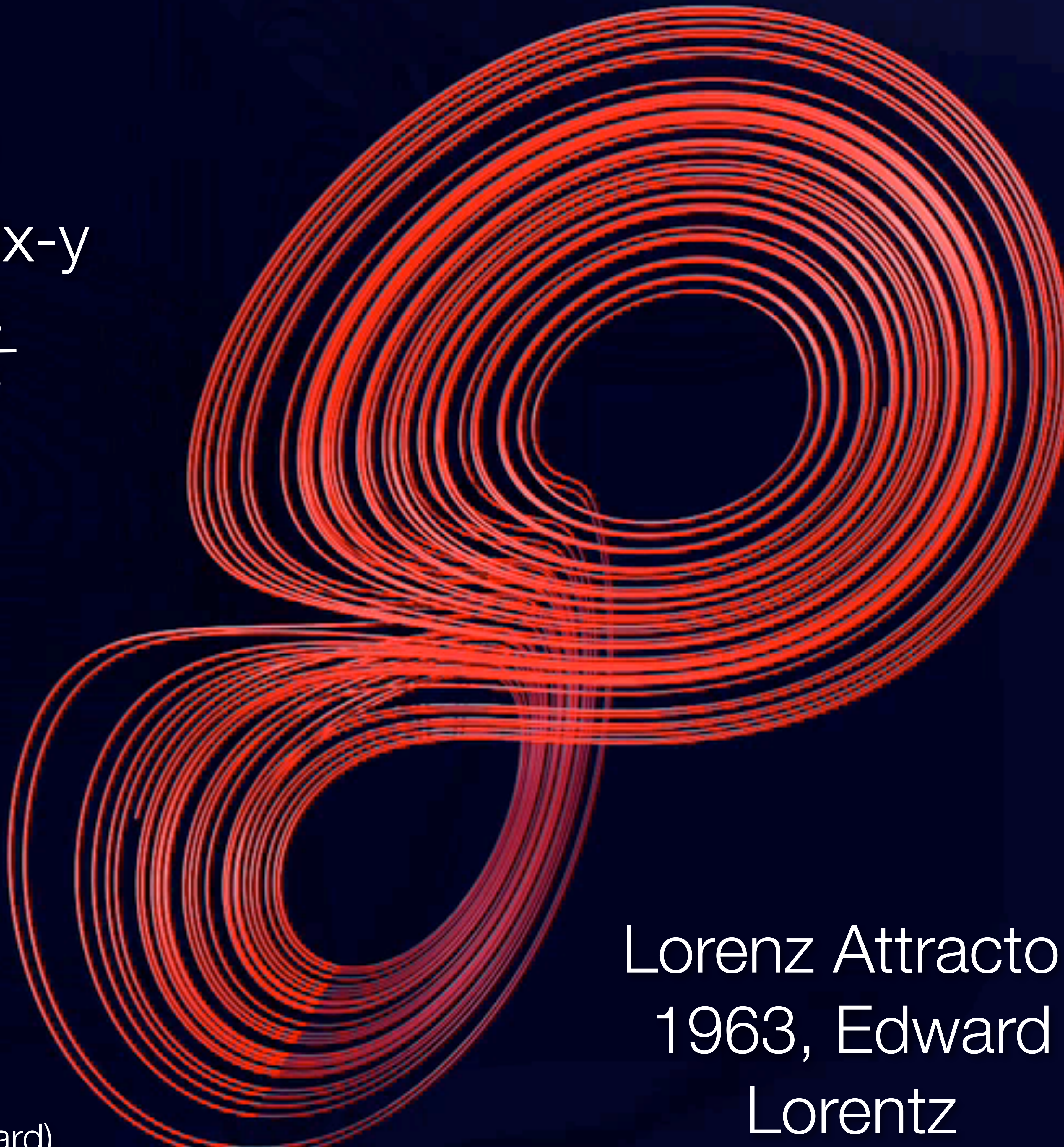
$$x' = y\,(r^2-1)$$
$$y' = -x\,(r^2-1)$$

limit cycles and equlibria

$$\dot{x} = 10(y-x)$$

$$\dot{y} = -xz+28x-y$$

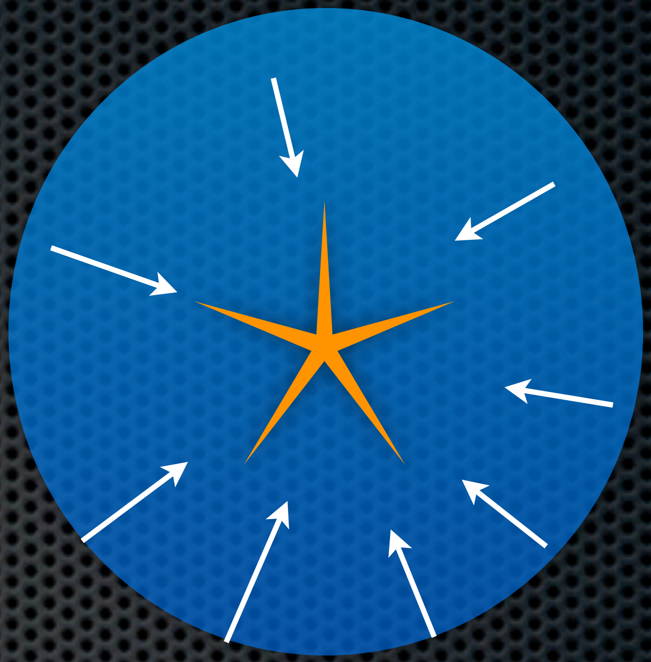
$$\dot{z} = xy - z\frac{8}{3}$$



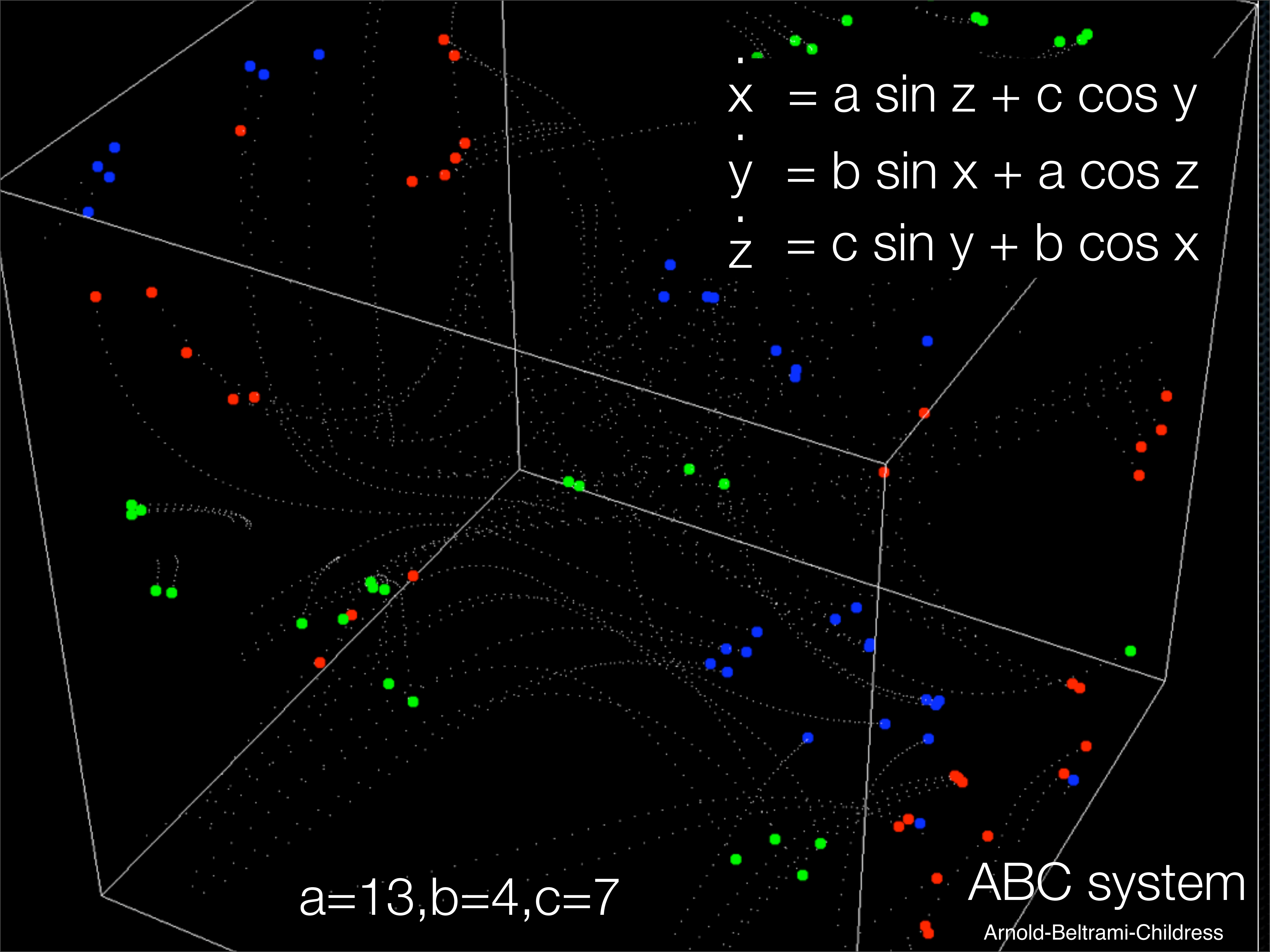
(studied math at Harvard)

Lorenz Attractor
1963, Edward
Lorentz

The Lorentz flow is dissipative, the attractor has zero volume. The attractor is a prototype of a “strange attractor”.



Lets look at an example of a volume preserving dynamics, which also exhibits “random motion”.

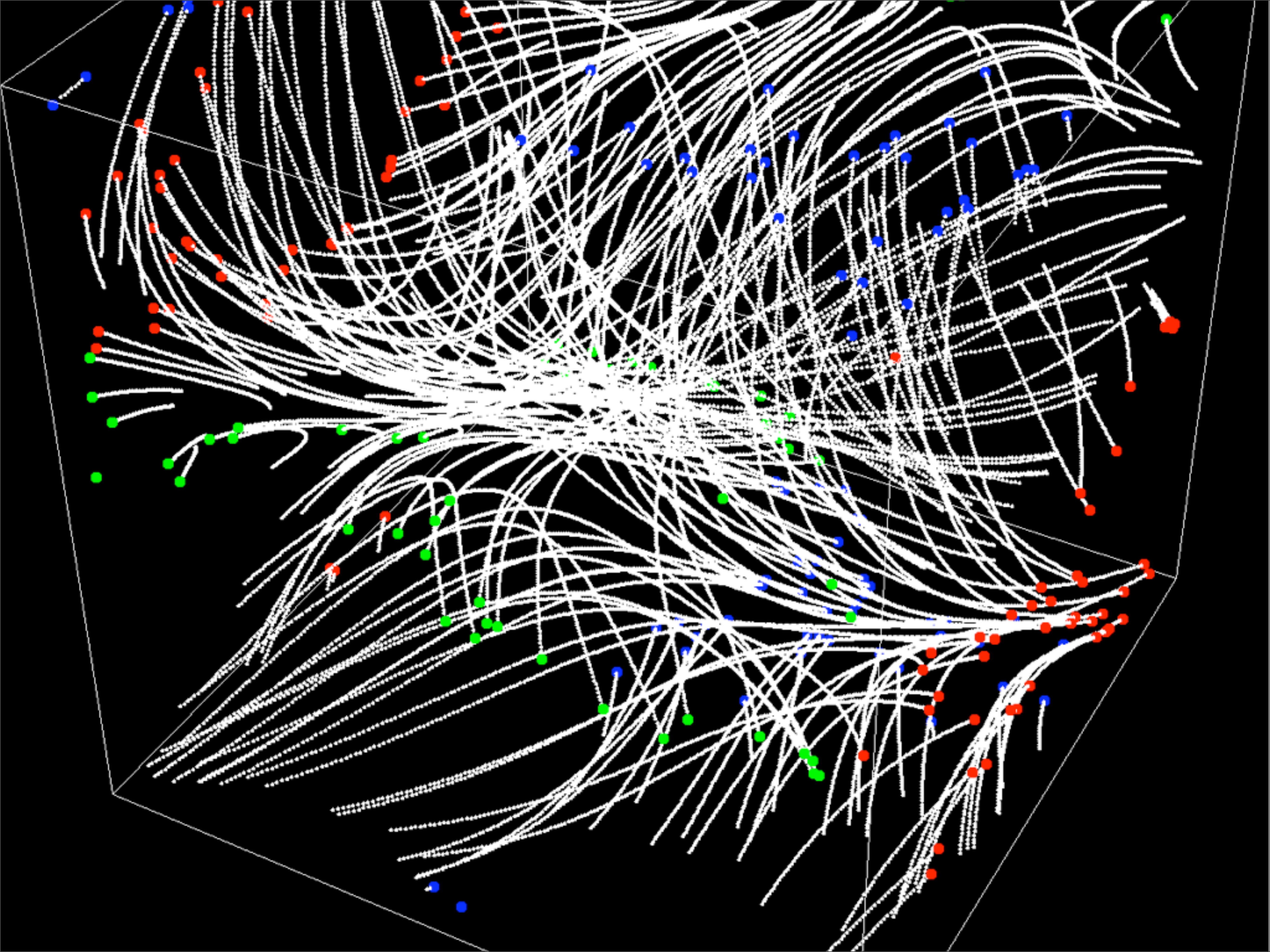


$$\dot{x} = a \sin z + c \cos y$$
$$\dot{y} = b \sin x + a \cos z$$
$$\dot{z} = c \sin y + b \cos x$$

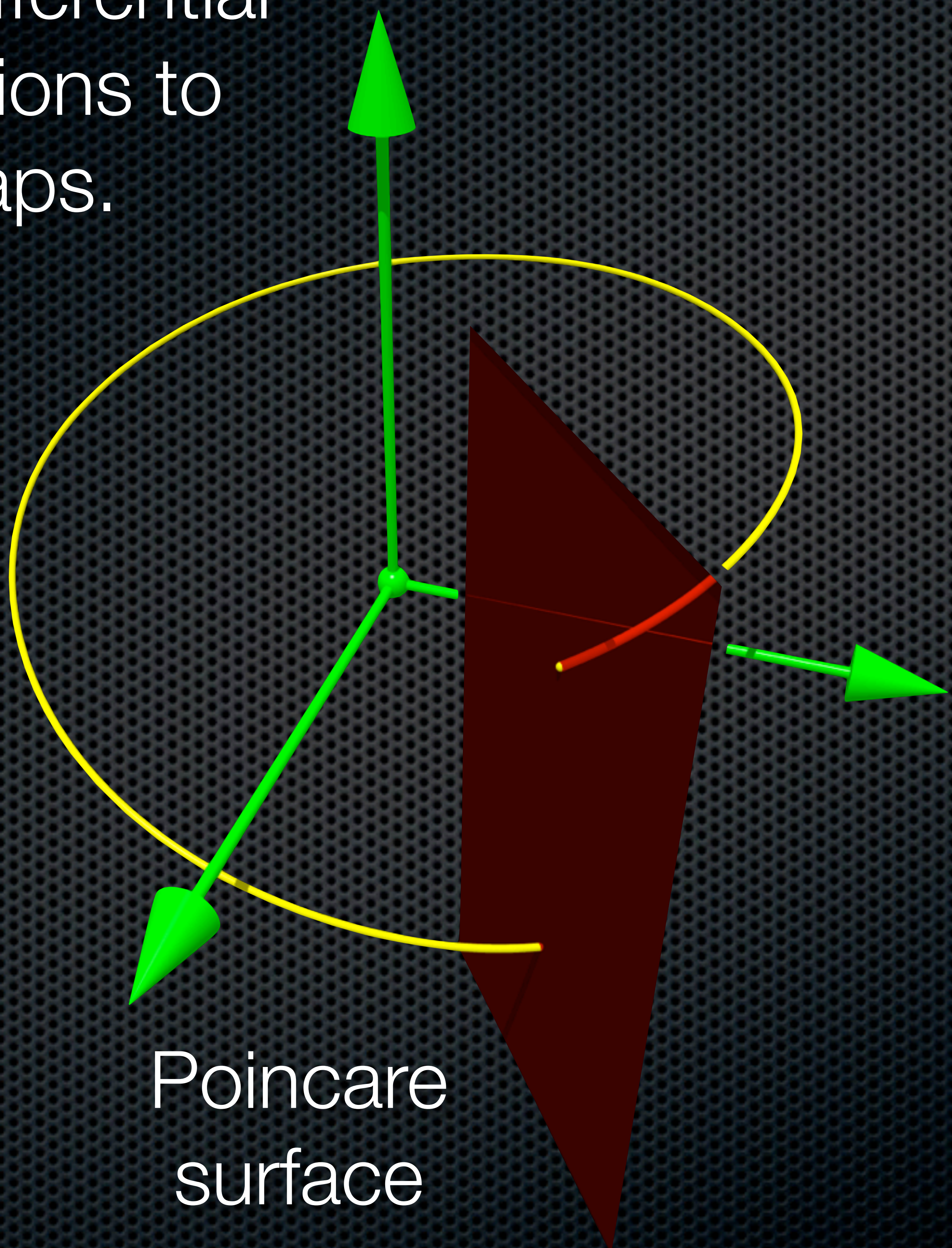
a=13,b=4,c=7

ABC system

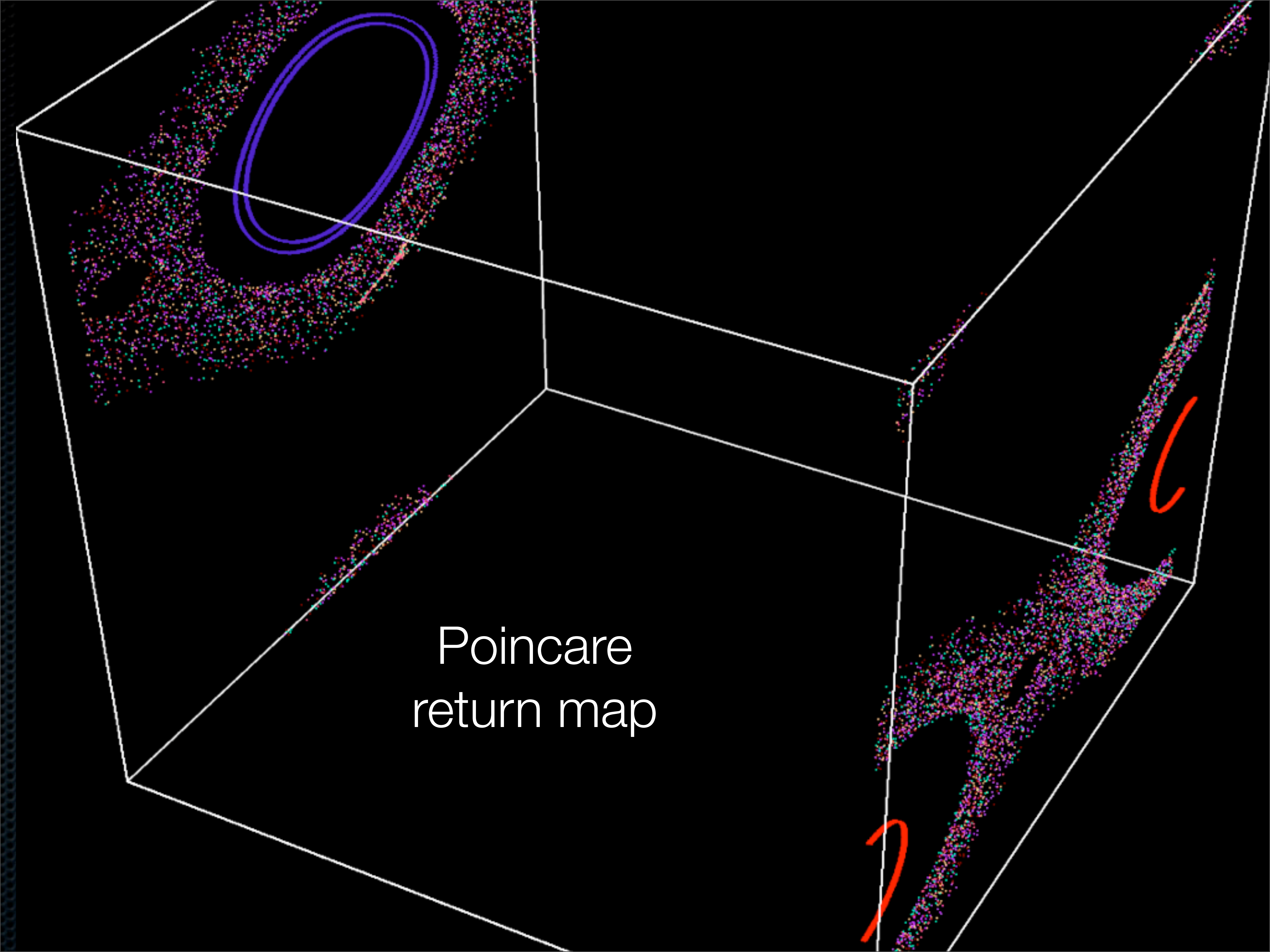
Arnold-Beltrami-Childress



from differential equations to maps.



The return map of the ABC flow is an area preserving map on the two dimensional torus


Poincare
return map

This motivates the study of area preserving maps in general. The simplest examples are maps of the form

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f(x)-y \\ x \end{bmatrix}$$

$$x_{n+1} + x_{n-1} = f(x_n)$$

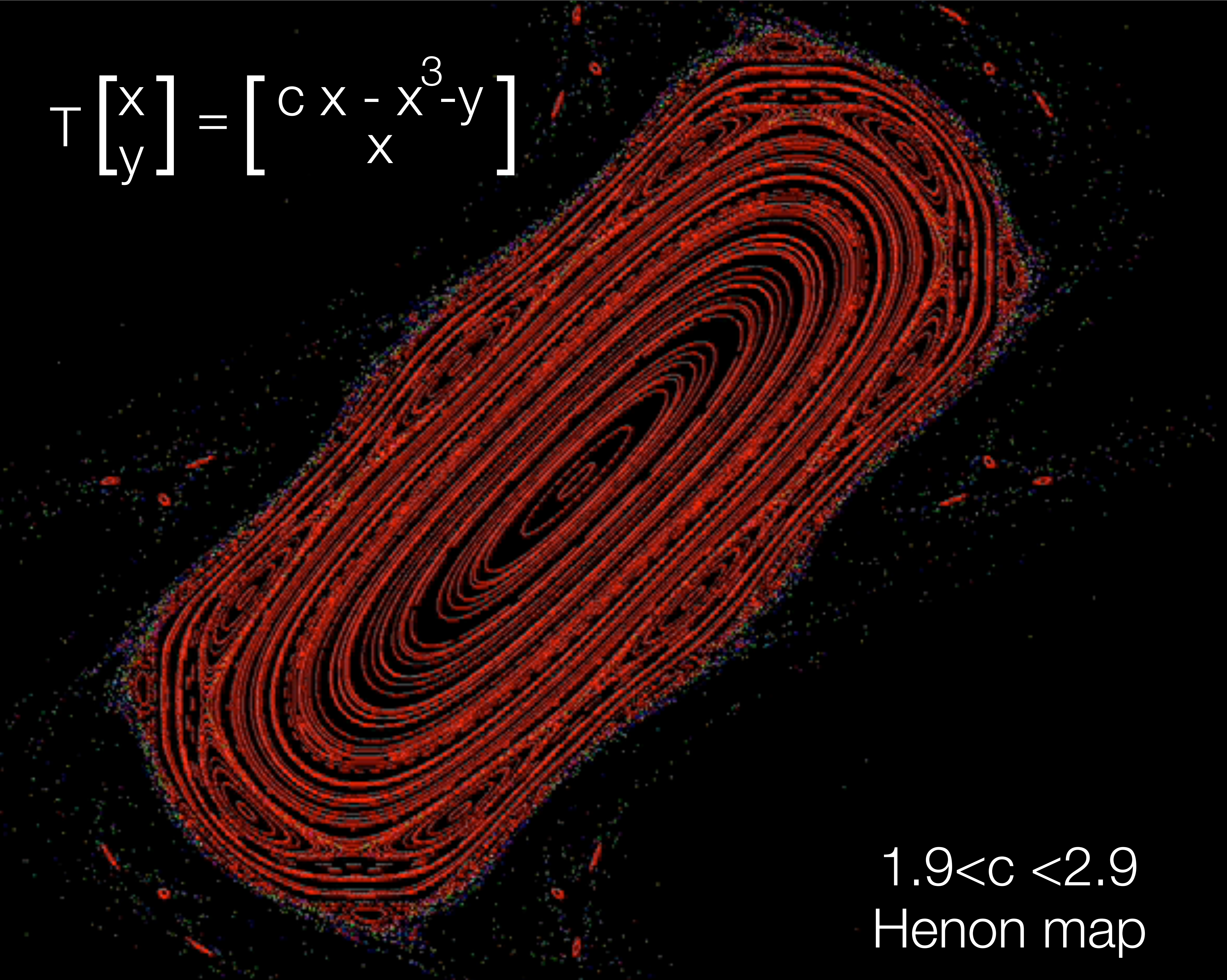
Michel Henon

f(x) polynomial: Henon type maps

f(x) trigonometric polynomial: Standard type maps

Chirikov, Taylor, Frenkel Kontorova

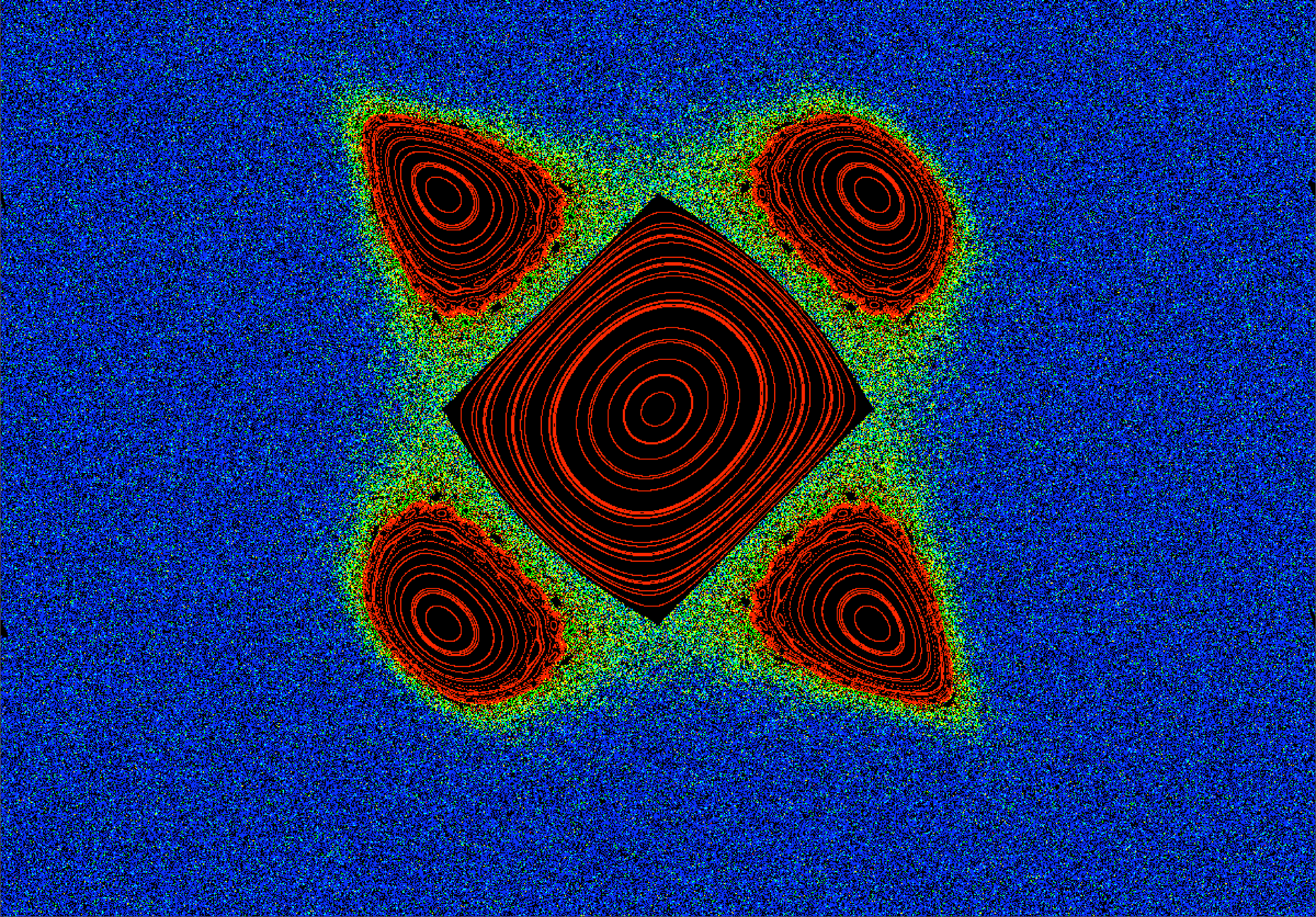
$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c\,x - x^3 - y \\ x \end{bmatrix}$$

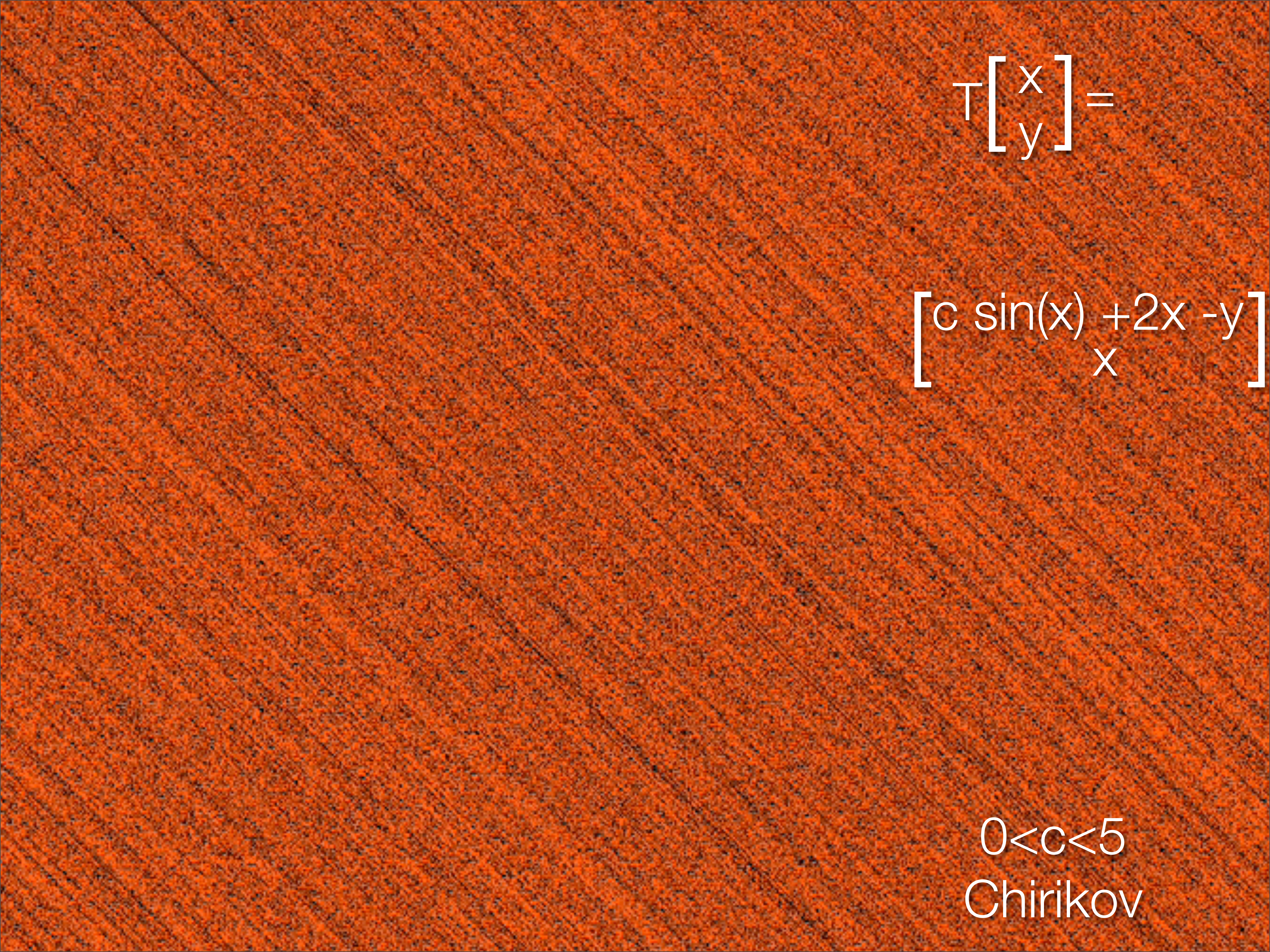


$1.9 < c < 2.9$

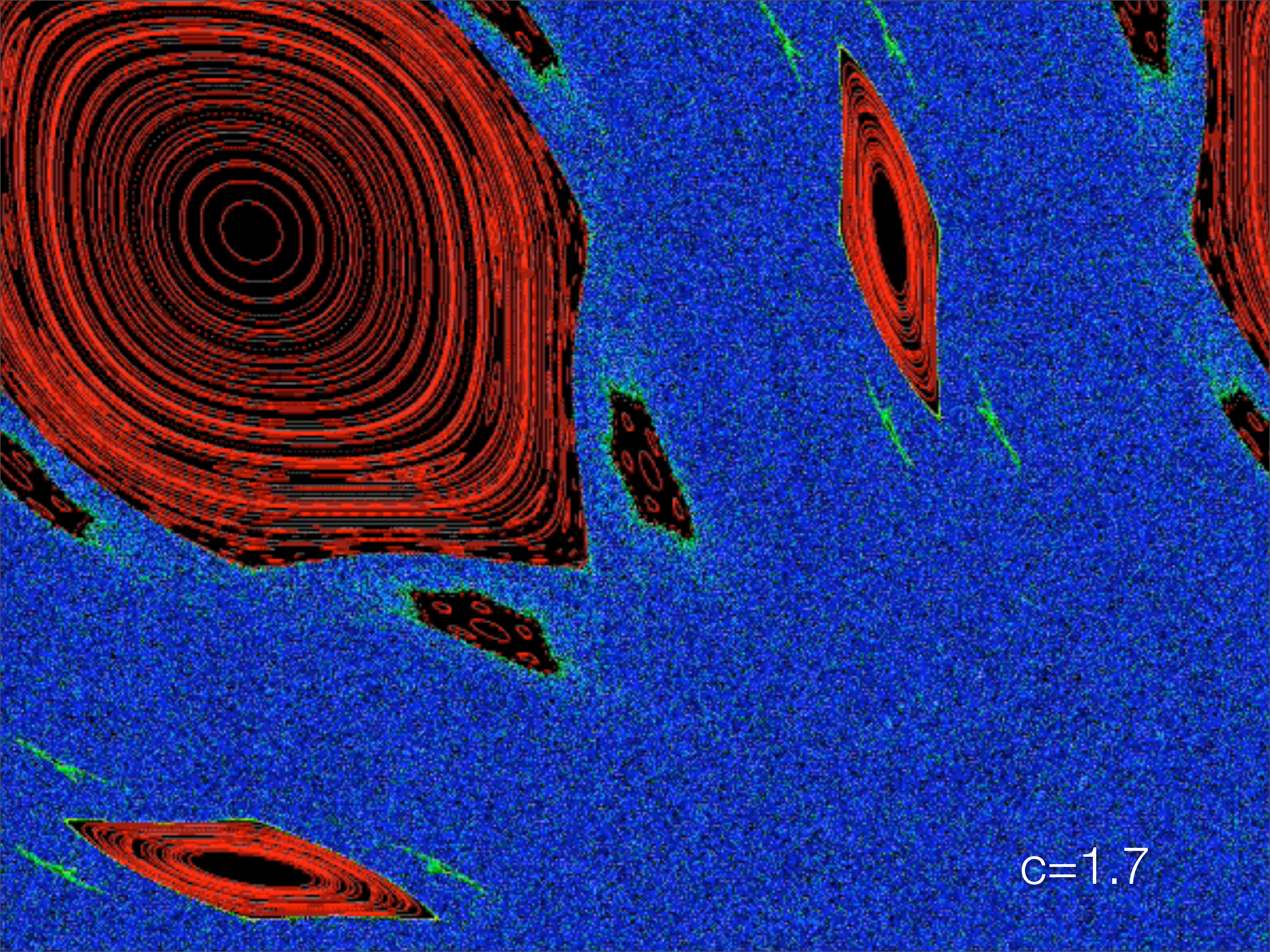
Henon map

Chirikov type maps.





$T\begin{bmatrix} x \\ y \end{bmatrix} =$
$\begin{bmatrix} c\sin(x) + 2x - y \\ x \end{bmatrix}$
0<c<5
Chirikov


c=1.7

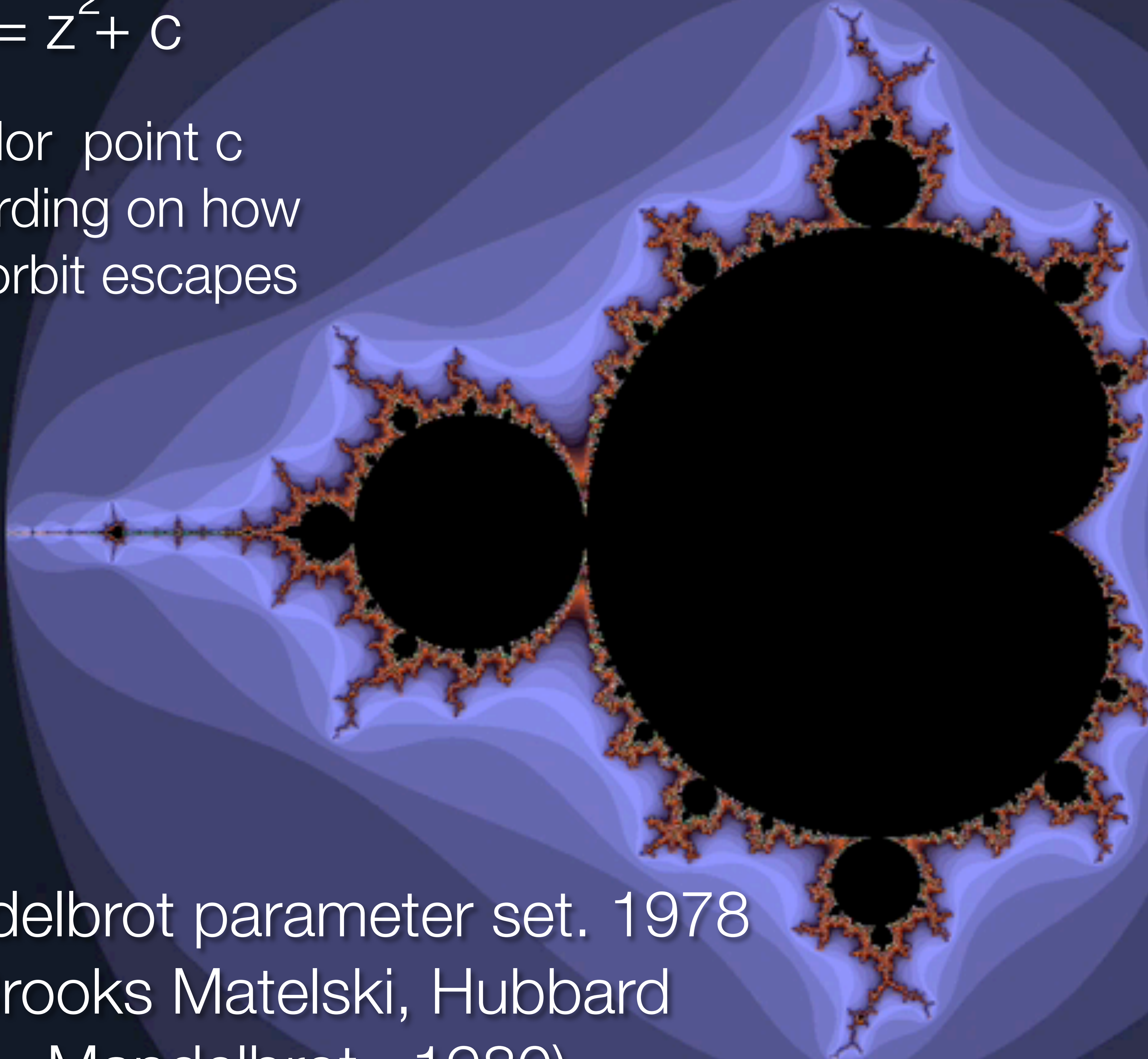
One can not talk about maps without mentioning complex maps like $T(z) = z^2 + c$, where $c = u + i v$ is a complex parameter. One can rewrite this without complex numbers as

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2 - y^2 + u \\ 2\,x\,y + v \end{bmatrix}$$

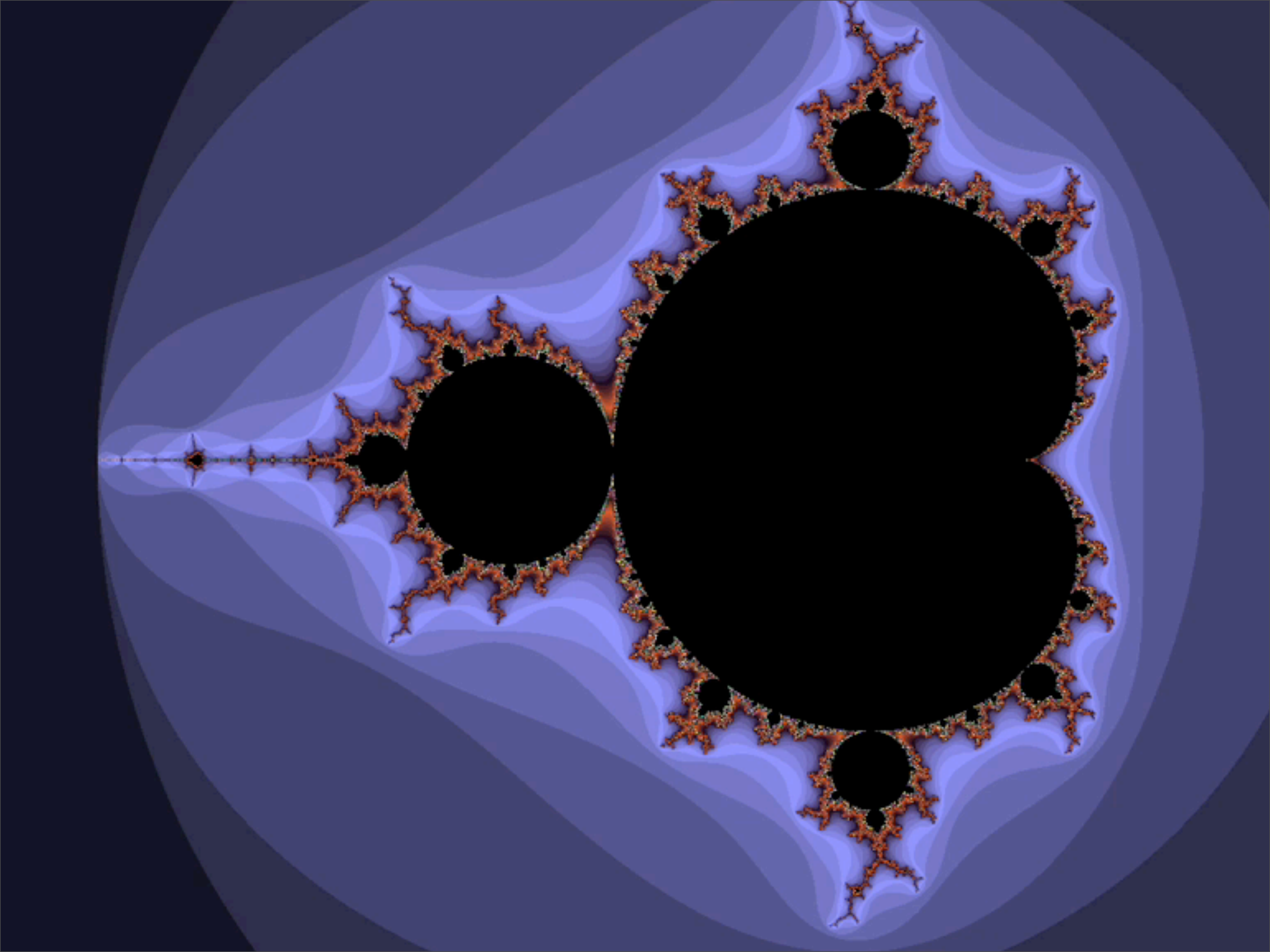
One can look for given (u,v), how fast (0,0),T(0,0), T(T(0,0)) escapes to infinity. If the orbit stays bounded, one gets the Mandelbrot set. (Brooks,Matelski,Mandelbrot, Douady Hubbard)

$T(z) = z^2 + c$

color point c
according on how
fast orbit escapes

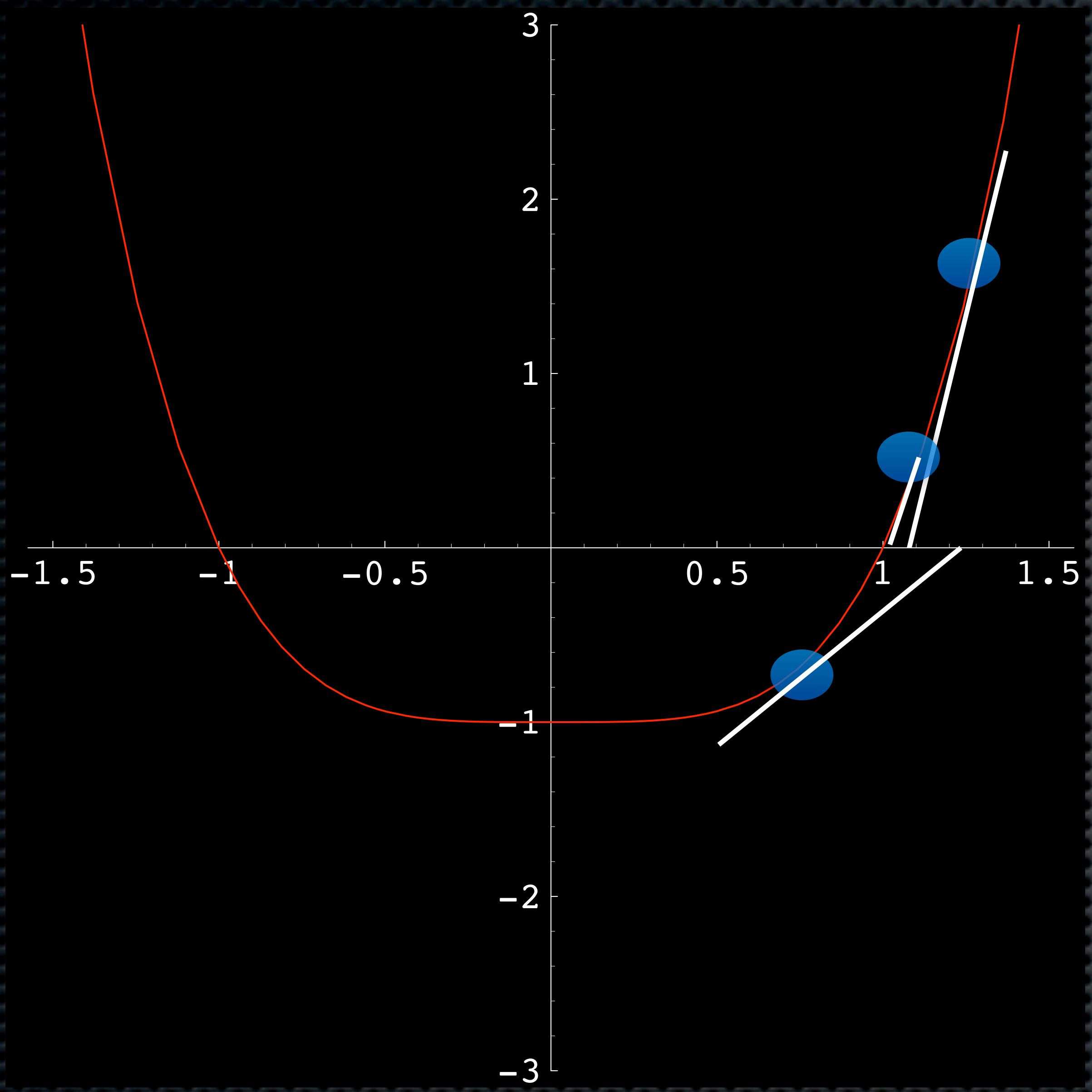


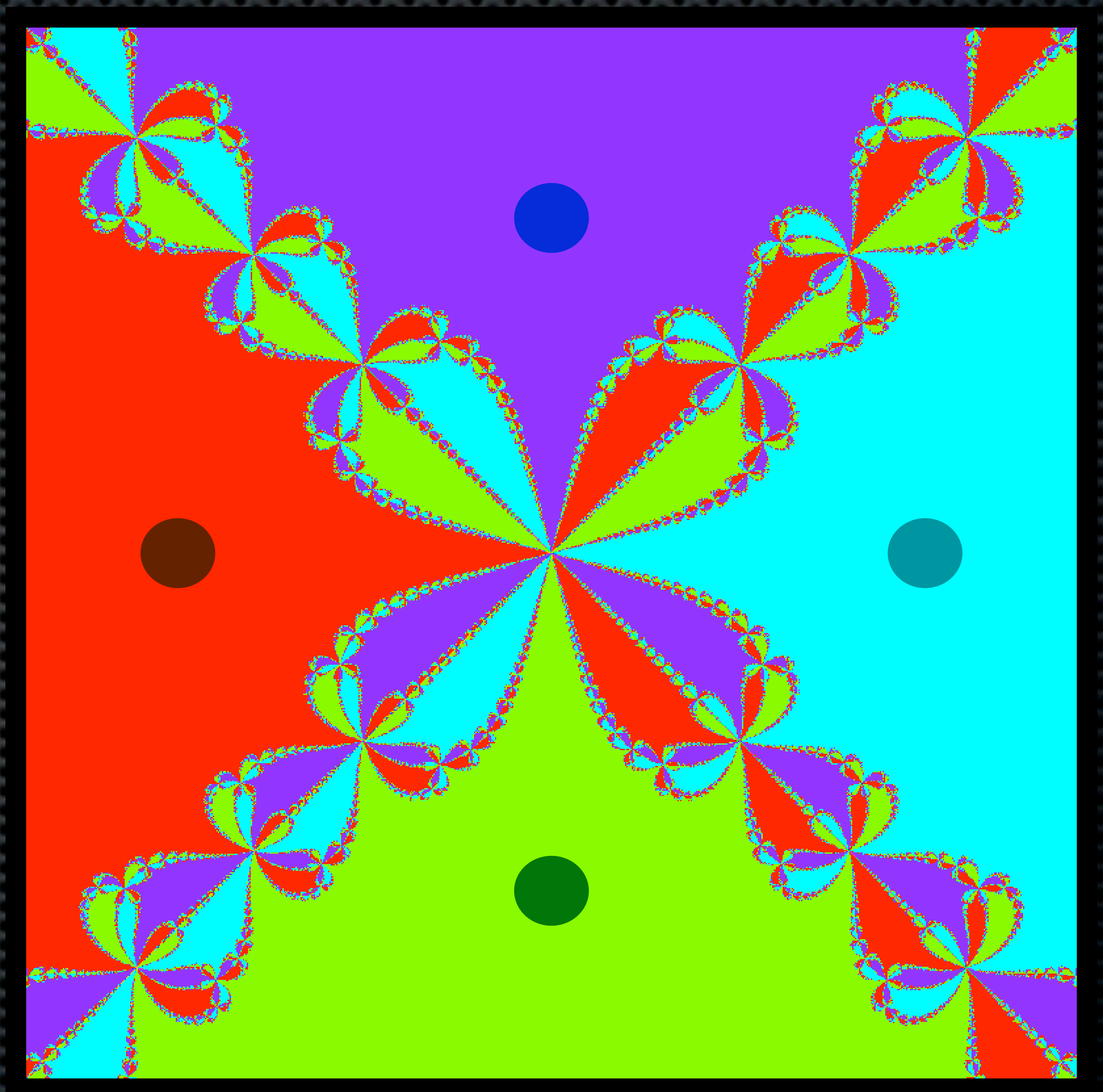
Mandelbrot parameter set. 1978
(Brooks Matelski, Hubbard
Mandelbrot, 1980)



Historically, complex dynamics has been studied much earlier by French mathematicians. Systematically by Julia and Fatou. A particular system, the Newton method has been of interest already to Cayley in the 18’th century.

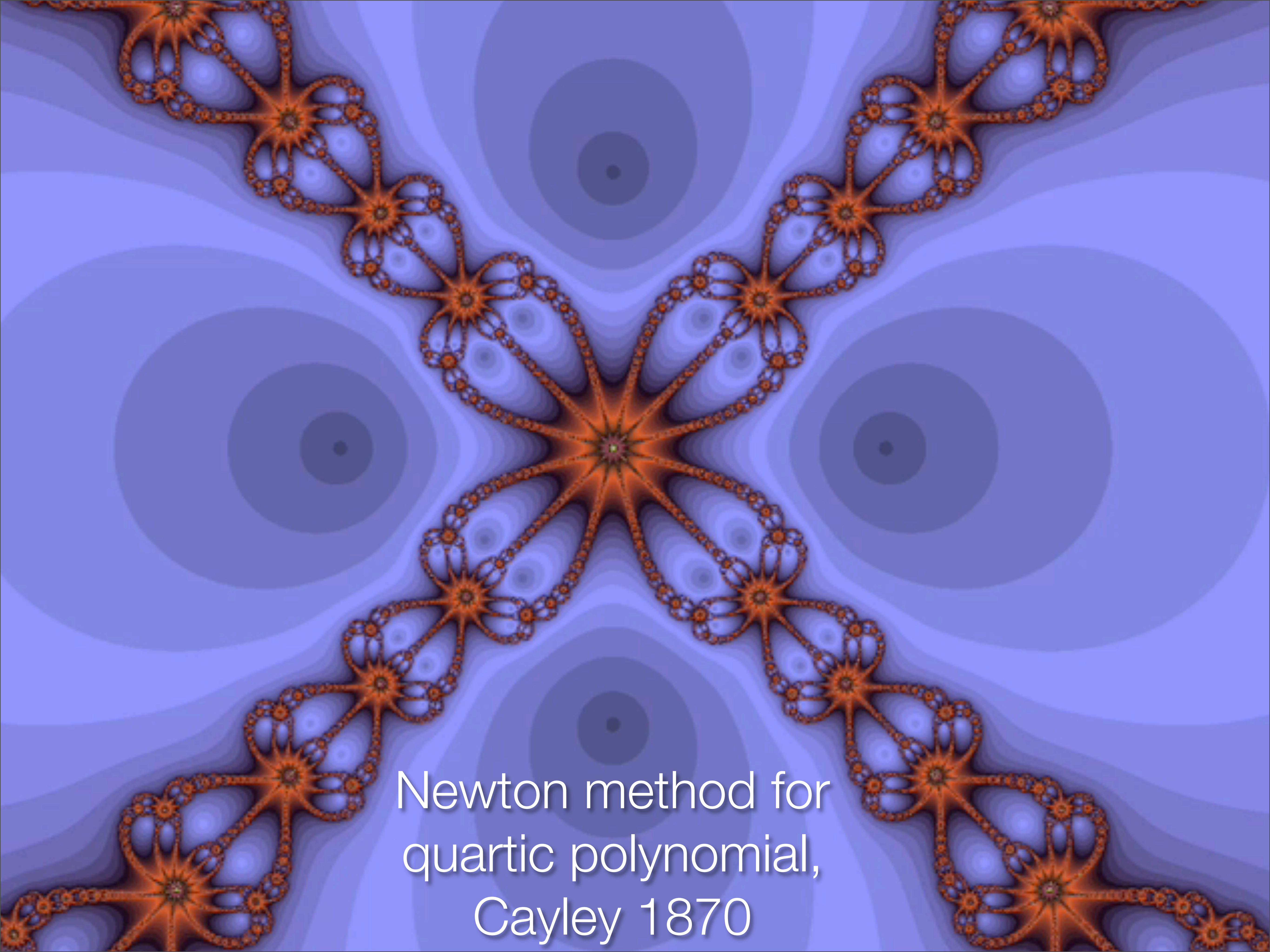
$T(z) = z - f(z)/f'(z)$ for $f(z) = z^4 - 1$





on the real axes

in the complex plane

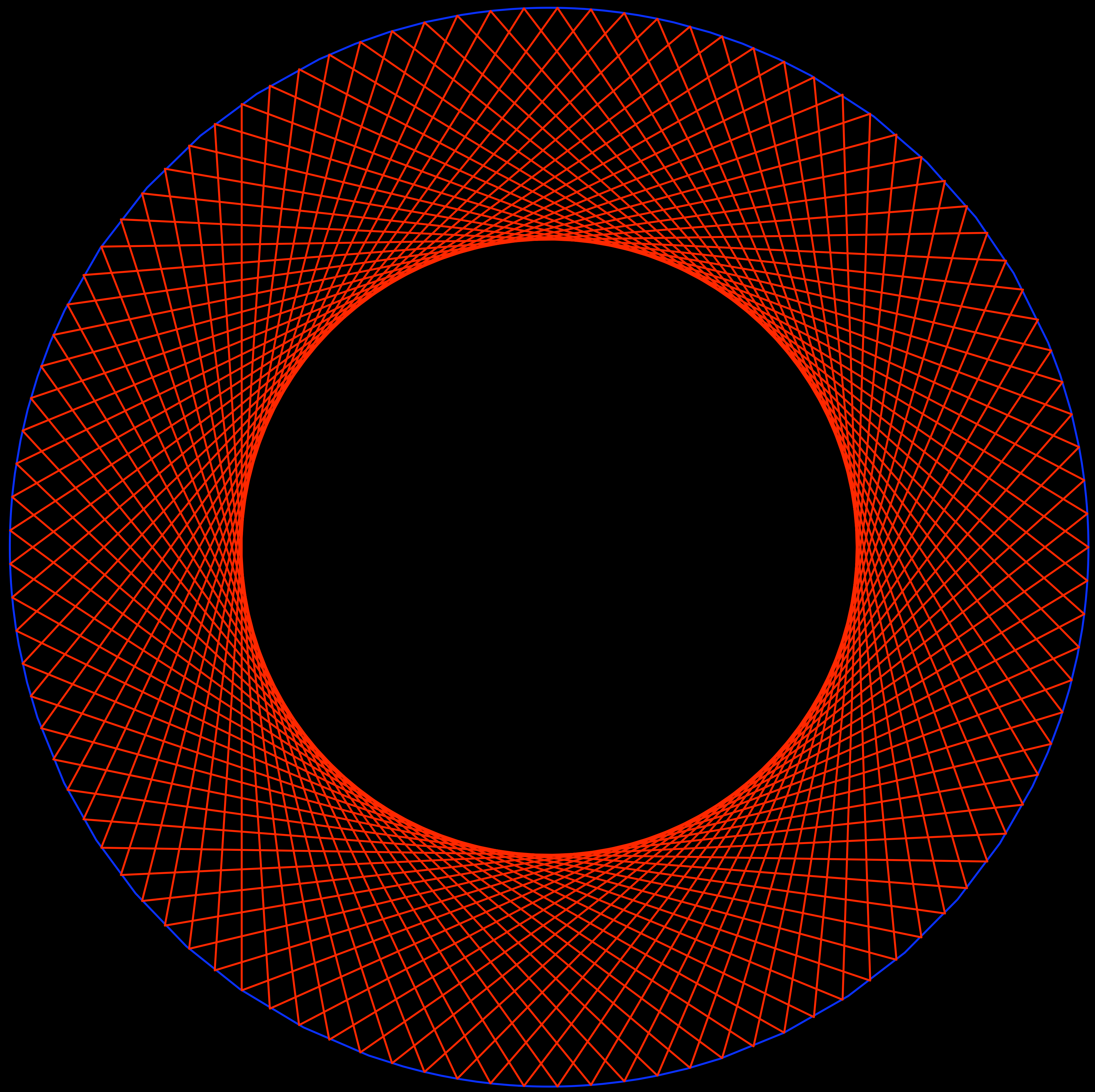


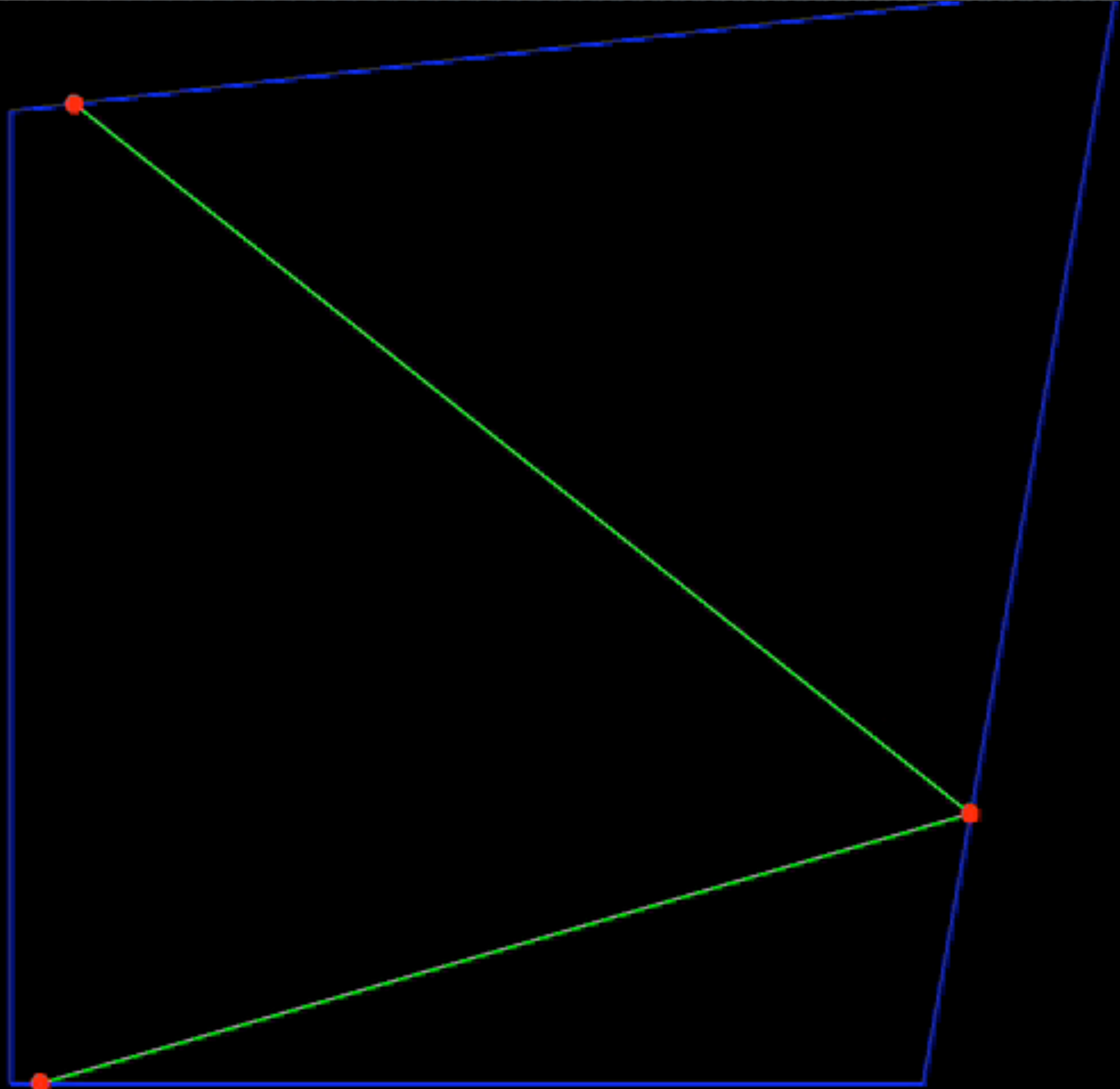
Newton method for quartic polynomial, Cayley 1870

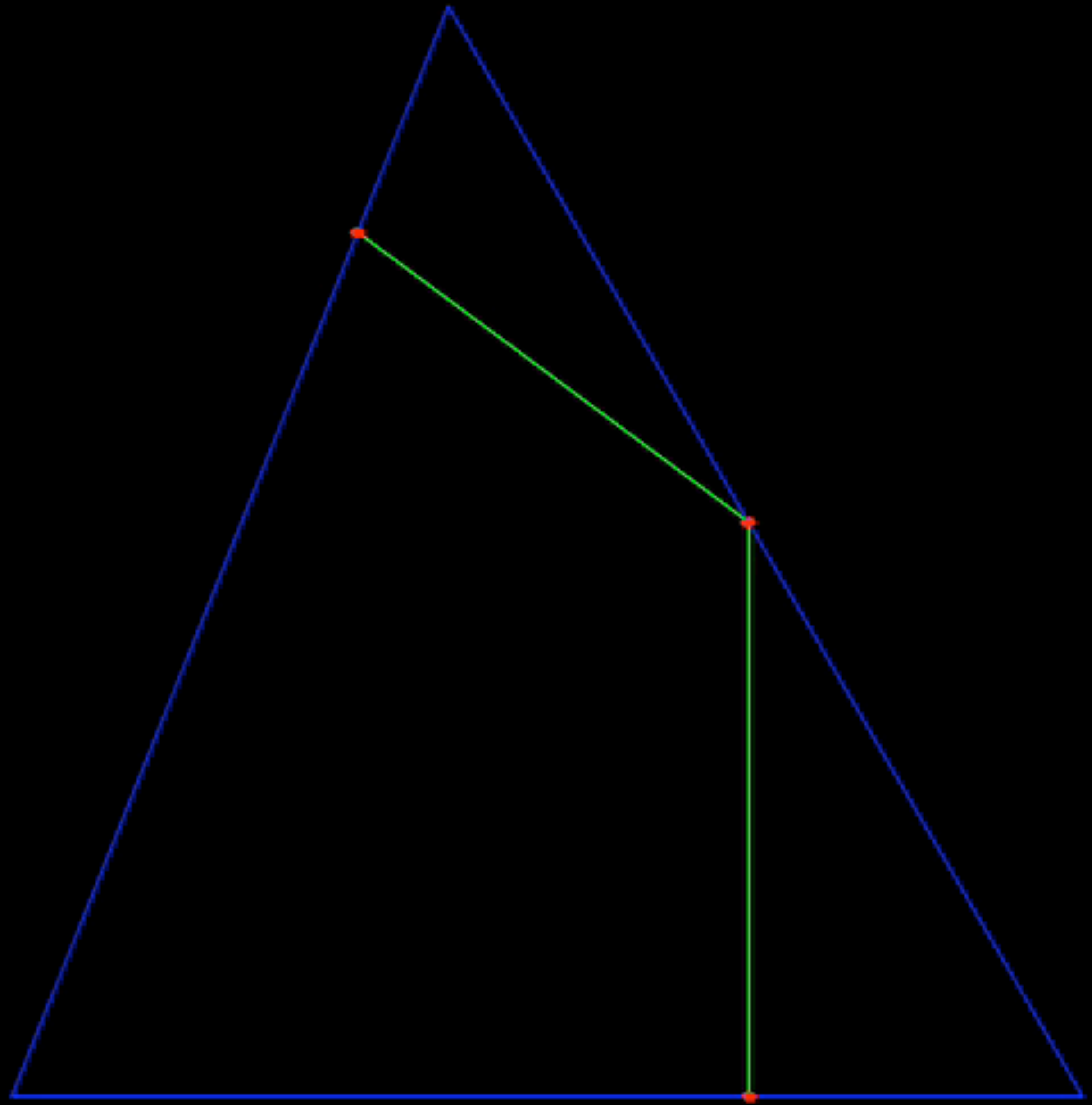
Billiards

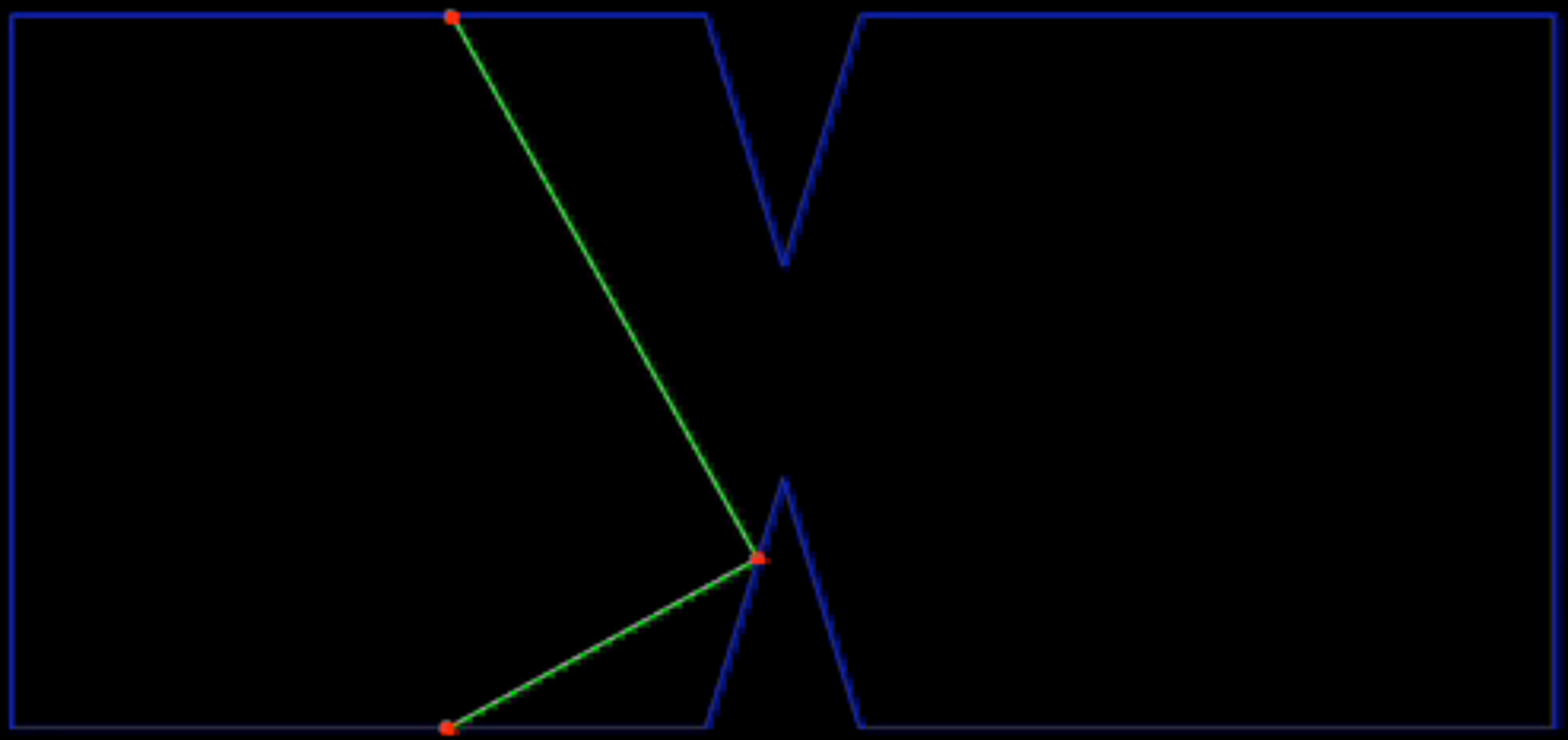
historicallly one of the earliest encounters with Chaos are billiards.

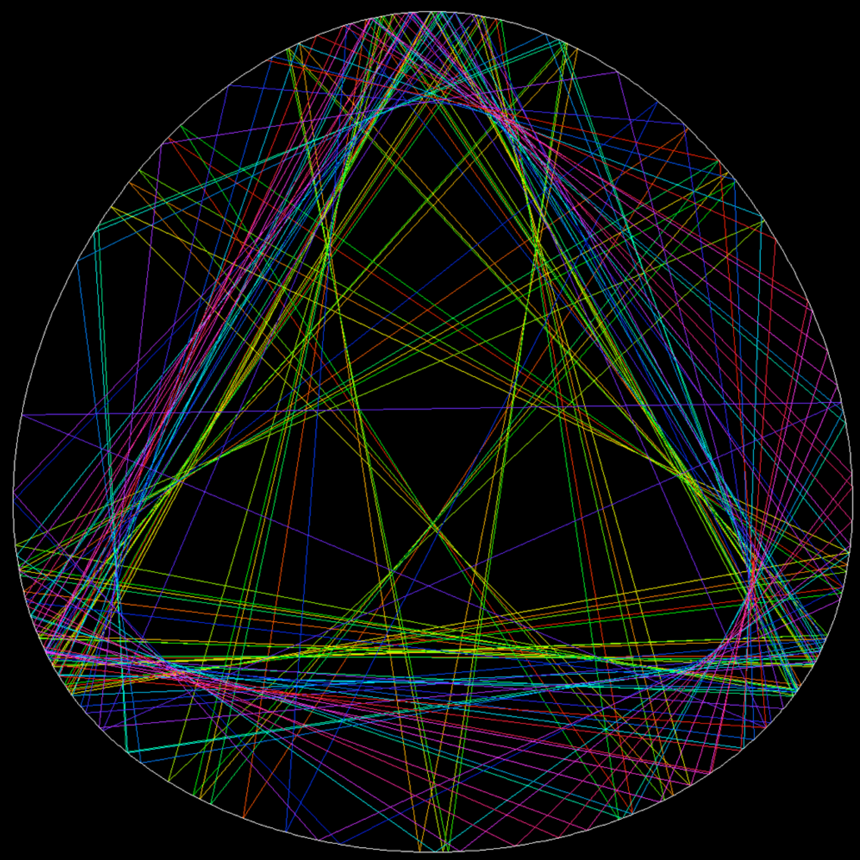
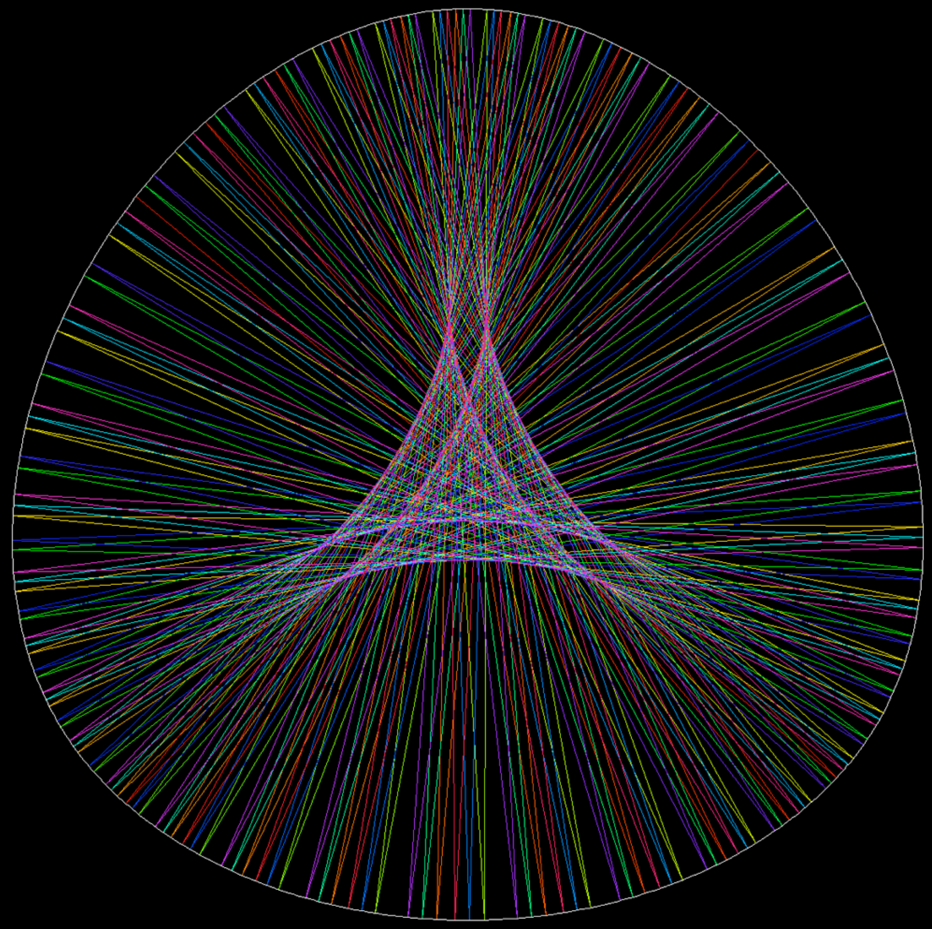
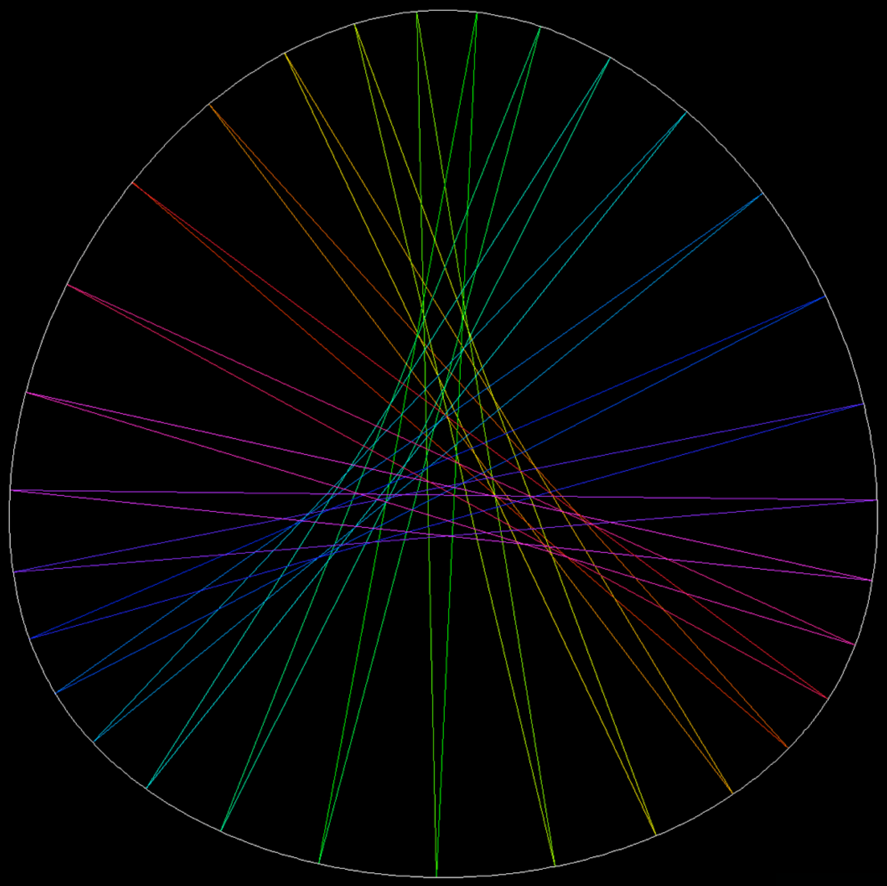
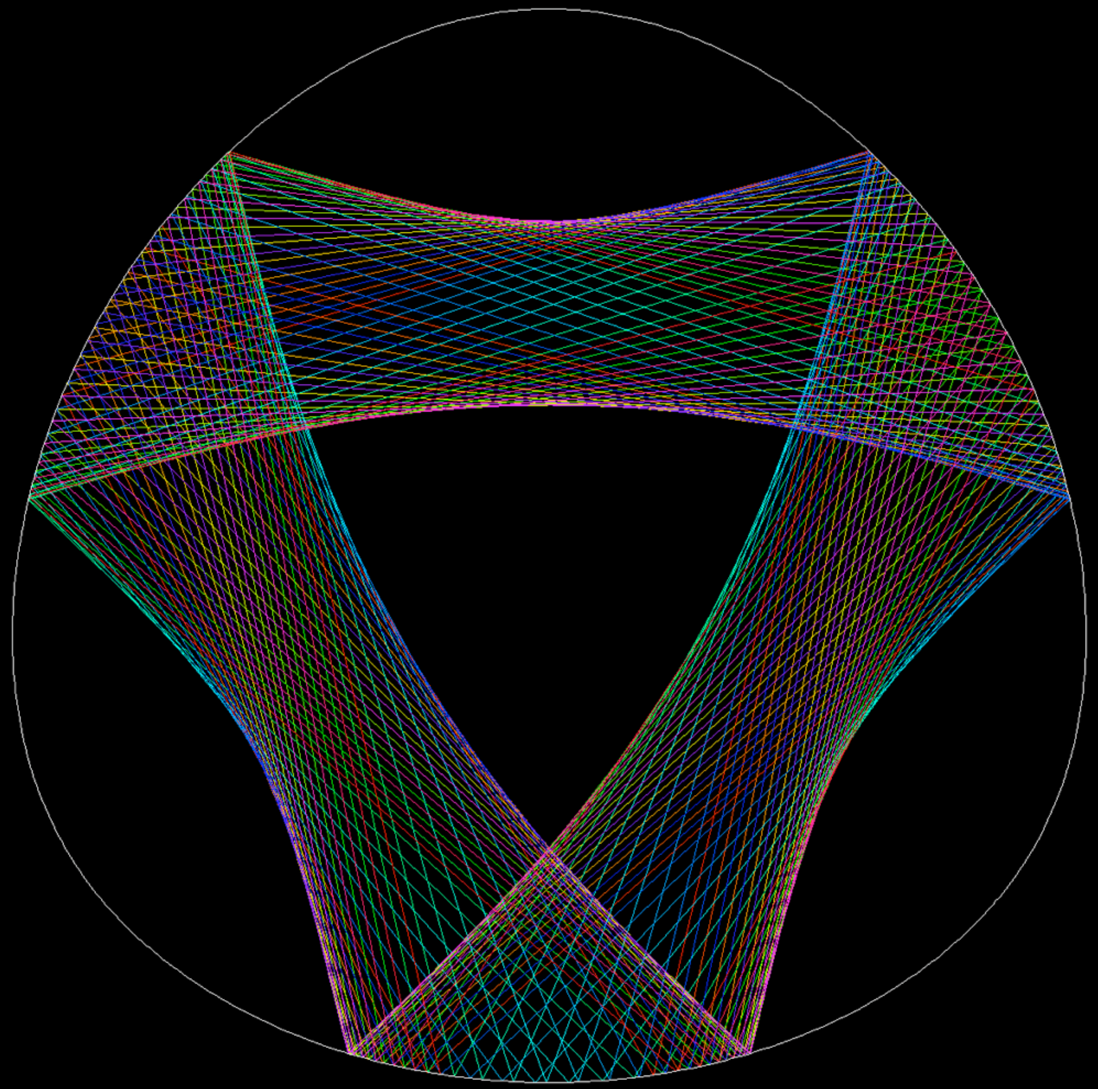
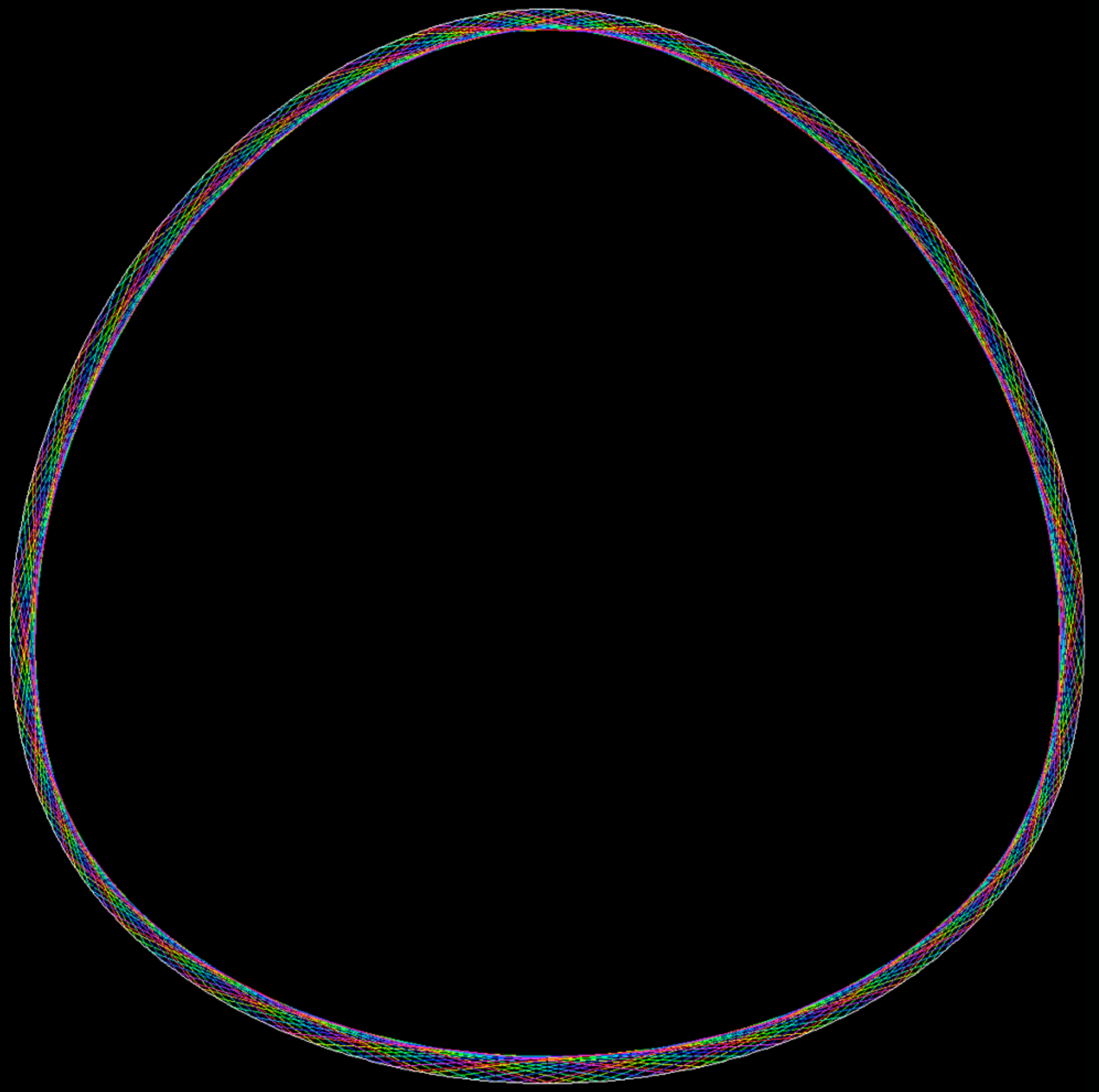
Boltzmann (1844-1906) hard sphere gas
Artin (1898-1962) in 1924, billiard in hyperbolic plane
Hadamard (1865-1963)-Hedlund-Hopf, geodesic flow
Birkhoff (1884-1944) in 1927, model for 3-body problem

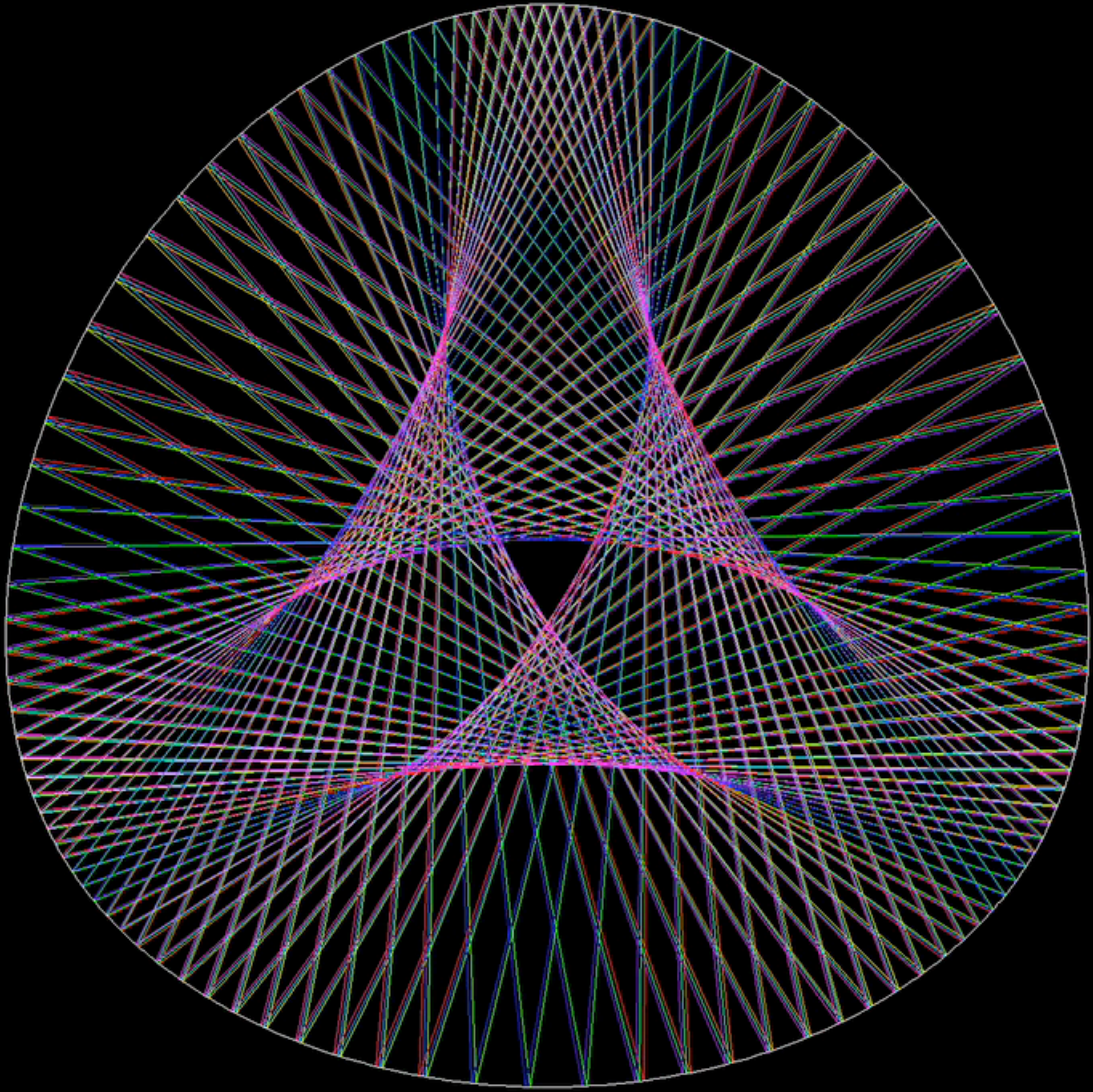






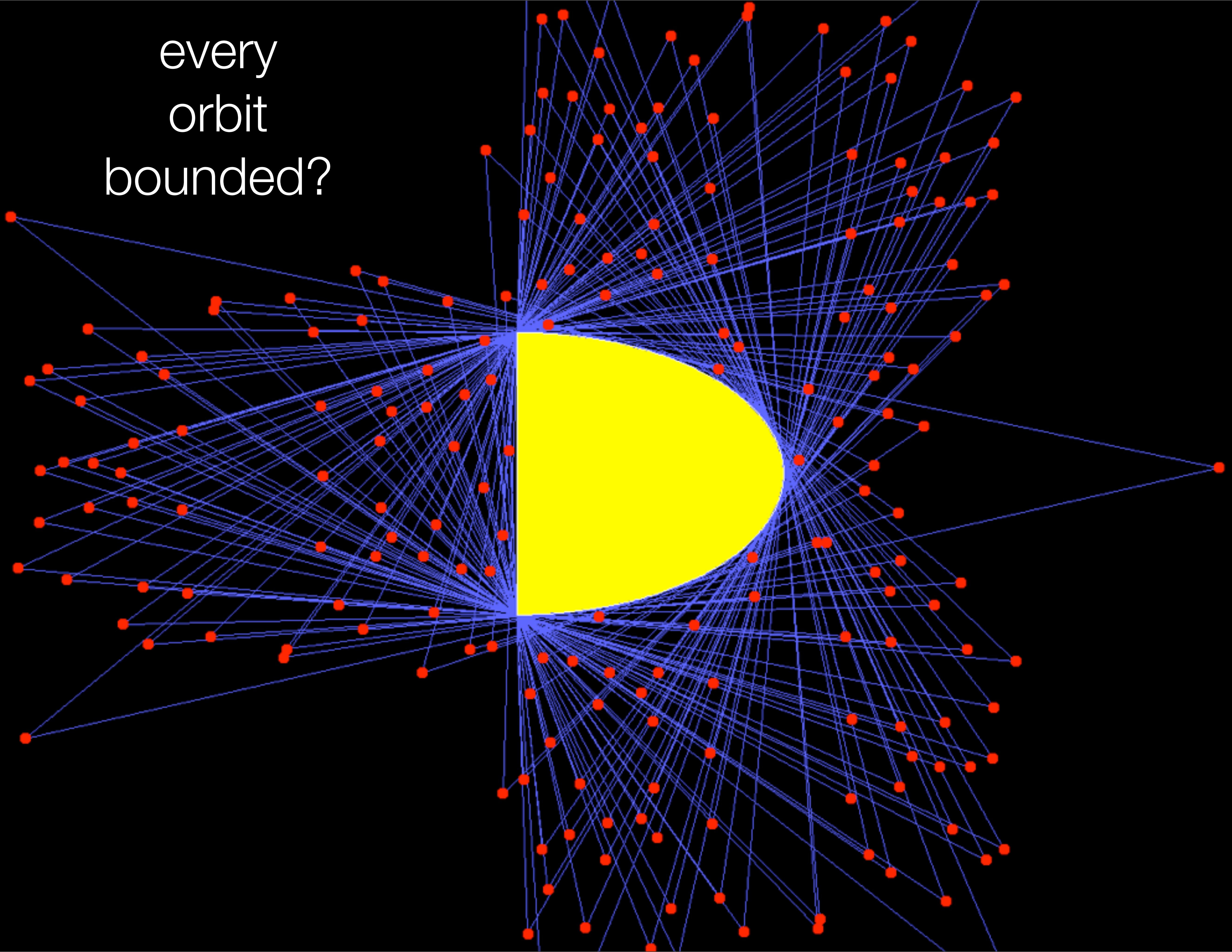


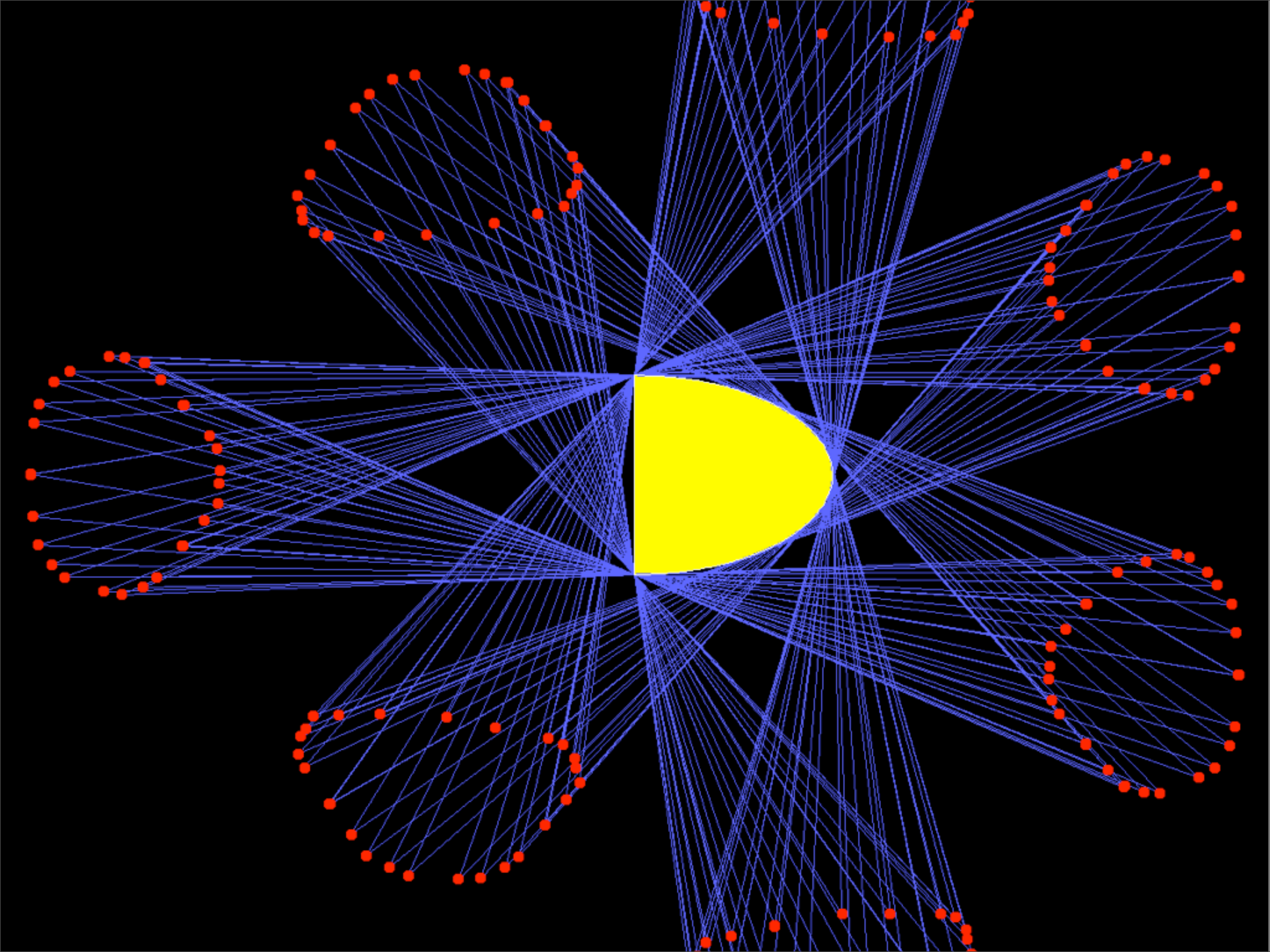






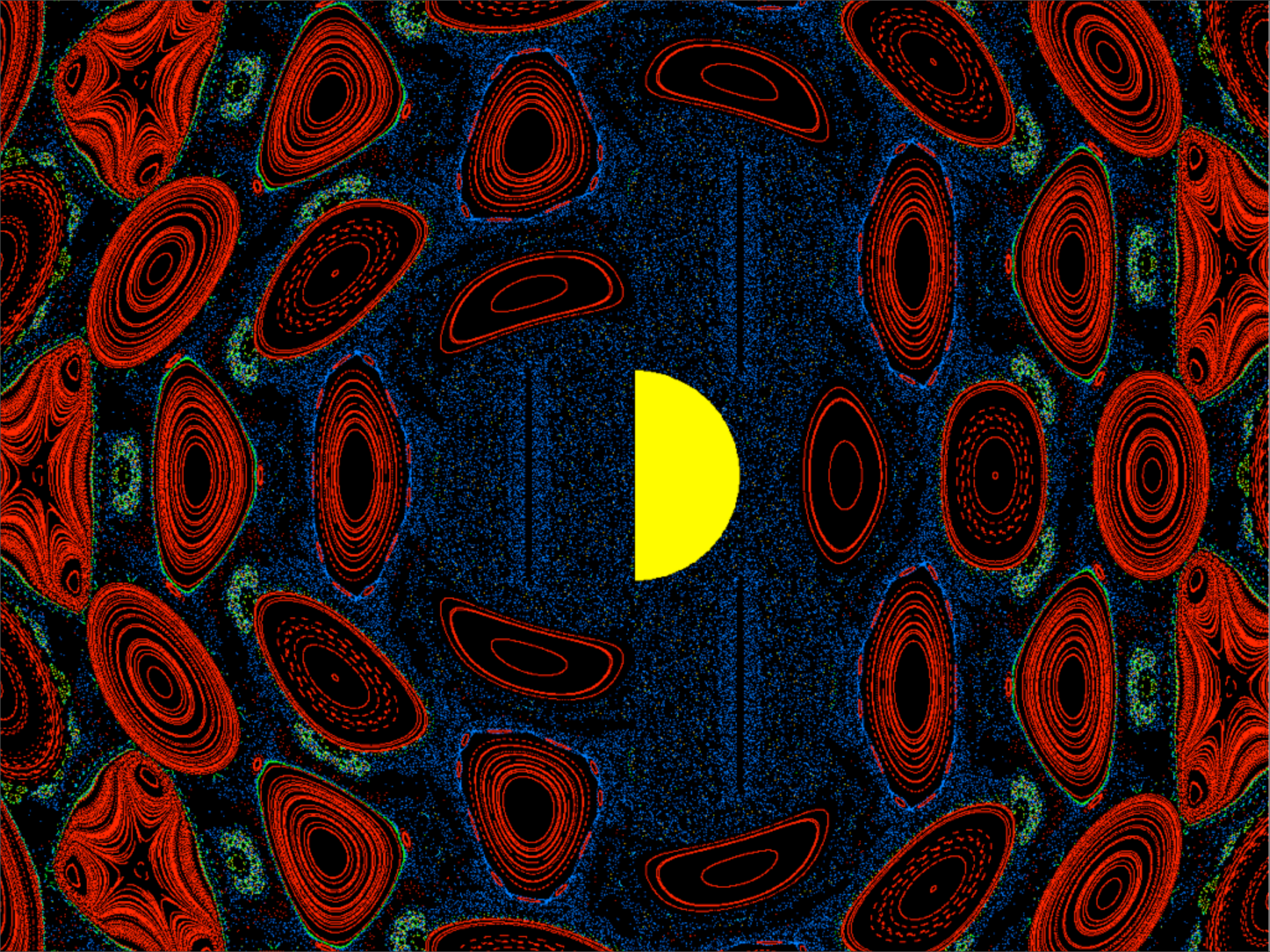


A brother of billiards is dual billiards.

every
orbit
bounded?







What is chaos?

updated 11:06 p.m. EST, Sat December 29, 2007

Transcript: Chaotic scene at zoo

STORY HIGHLIGHT
- **NEW:** Police spokes
- **NEW:** Zoo personne
- **NEW:** For several m
- Attack kills teen Carl

Next Article in U.S. »

SAN FRANCISCO, California (AP) -- Police radio transcripts from the night of a deadly tiger attack revealed a chaotic scene at the San Francisco Zoo, as zookeepers struggled to sedate the animal and medics refused to enter until they knew they would be safe.

San Francisco Zoo Director Manuel Mollinedo and Police Chief Heather Fong leave a press conference Friday.

Zoo employees also initially questioned whether early reports of the December 25 attack were coming from a mentally unstable person, according to an 18-page log of communications from police dispatchers to officers and emergency responders at the scene.

Police spokesman Sgt. Neville Gittens declined to comment beyond the transcript released late Friday. Authorities have never indicated their response was hindered by any delays, and the police chief has praised officers for their quick action and collaborative work with the zoo staff.

Zoo officials on Saturday did not immediately return messages seeking comment.

The attacks killed 17-year-old Carlos Sousa Jr., whose throat was slashed while he tried to scare away the tiger. Two of Sousa's friends suffered bite and claw injuries. Paul Dhaliwal, 19, and Kulbir Dhaliwal, 23, were released from the hospital Saturday. Watch what investigators are looking at »

The first report of an attack -- a male bleeding from the head -- came in at 5:08 p.m.

PAKISTAN IN CRISIS

Pakistan 'in grip of chaos and anarchy'

Hopes for Jan. 8 vote dim as electoral offices torched, opposition renews call for boycott

Dec 30, 2007 04:30 AM

BILL SCHILLER
ASIA BUREAU

ED WRAY/AP PHOTO

Supporters of Benazir Bhutto's Pakistan People's Party light candles during a ceremony in Lahore, yesterday. Some 10,000 people chanted anti-government slogans while holding prayers for slain Pakistani opposition leader.

ISLAMABAD – Pakistan slumped deeper into chaos and recrimination yesterday as the death toll climbed and the prospects for a decisive Jan. 8 election in this blood-soaked country appeared to grow ever more remote.

Officials said at least 46 people have now died in riots, looting and shooting following the Thursday night assassination of charismatic opposition leader Benazir Bhutto.

Early today, two suspected suicide bombers prematurely detonated their bomb near the residence of a senior leader of the ruling party in Bahawalnagar in eastern Pakistan, police said. The men were not far from the residence of Ijazul Haq, a senior leader of the Pakistan Muslim League-Q party, when their bomb exploded and killed them, district police chief Zafar Abbas Bukhari told Associated Press

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Africa

Kenya faces chaos after disputed Kibaki victory

By Barney Jopson in Nairobi
Published: December 30 2007 18:37 | Last updated: December 30 2007 18:37

Kenya was on the brink of chaos and its democratic credentials in tatters on Sunday night after Raila Odinga, the country's main opposition leader, lost a hotly contested presidential election that both his party and independent observers said was not credible.

The east African country has been regarded as one of the continent's most stable and open democracies, but its reputation crumbled over the weekend as the results of the poll were delayed, reports of irregularities multiplied, and violence flared.

The electoral commission said on Sunday that president Mwai Kibaki secured 46.7 per cent of the vote versus 44.3 per cent for Mr Odinga. Minutes later, as the president was sworn in for a second term in a hurried ceremony, riots broke out and killings were reported in Nairobi and other towns.

Earlier in the day Mr Odinga said: "Kenyans will not accept the results of a rigged election. No force will stop Kenyans attaining what they want. The river Nile is unstoppable. It must flow to the sea." The government, which accused him of inciting violence, blocked the broadcast of an opposition press conference held after results were announced.

EDITOR'S CHOICE
Kibaki re-elected as president of Kenya - Dec-30
Post-election chaos in Kenya worsens - Dec-30
Opposition leads in Kenya polls - Dec-28
Odinga aims to rise above tribal allegiances - Dec-26
Kibaki faces tough test in Kenya poll - Dec-27
Business backs Kenya's separation of powers - Dec-24

Koki Muli, head of the Institute of Education in Democracy, said: "This is the saddest day in the history of democracy in this country. It is a coup d'etat. The process does not have integrity and credibility. Do these people not care about legitimacy?"


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THE ULTIMATE OSCAR PREVIEW

Chaos (2008) - Video Release

OVERVIEW »

REVIEW

NEWS

PICTURES

TRAILERS

AWARDS WON

ON DVD

JASON STATHAM RYAN PHILLIPPE WESLEY SNIPES

CHAOS

BUY THE POSTER

Starring: Wesley Snipes, Ryan Phillippe, Jason Statham, Justine Waddell, Henry Czerny

Director: Tony Giglio

Studio: Lionsgate Home Entertainment

Rating: R

Genre: Action / Thriller

DVD Release Date: February 19, 2008

SYNOPSIS:

The film is described as a bank heist tale of a rookie and veteran cop, played by Ph
bank robber (Snipes) who knows too much about the inner workings of the police
has a personal connection to one of the cops. ~ *The Hollywood Reporter*

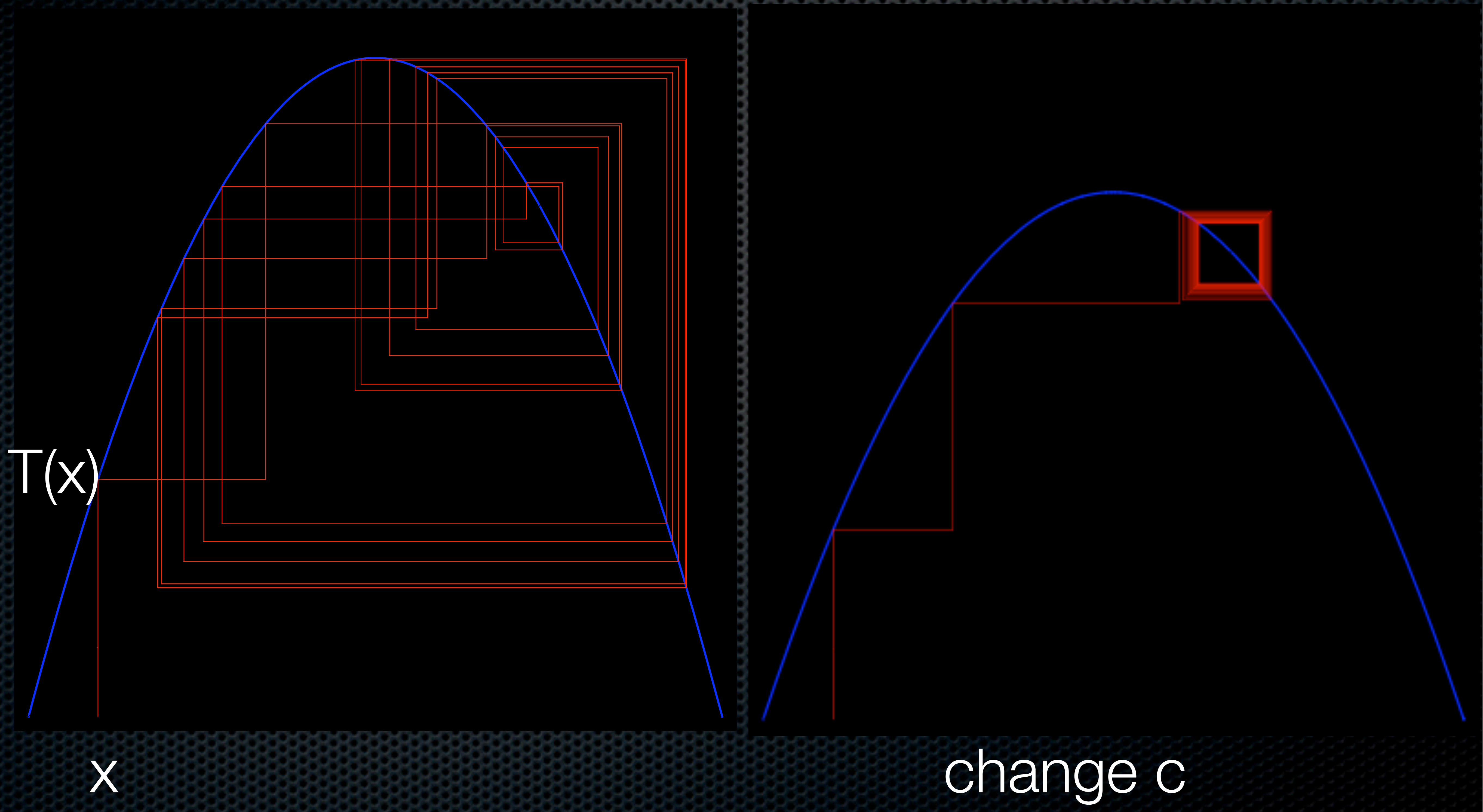
PHOTOS:

THERE ARE NO PICTURES AVAILABLE FO

TRAILERS & CLIPS:

Jurassic Park 1993

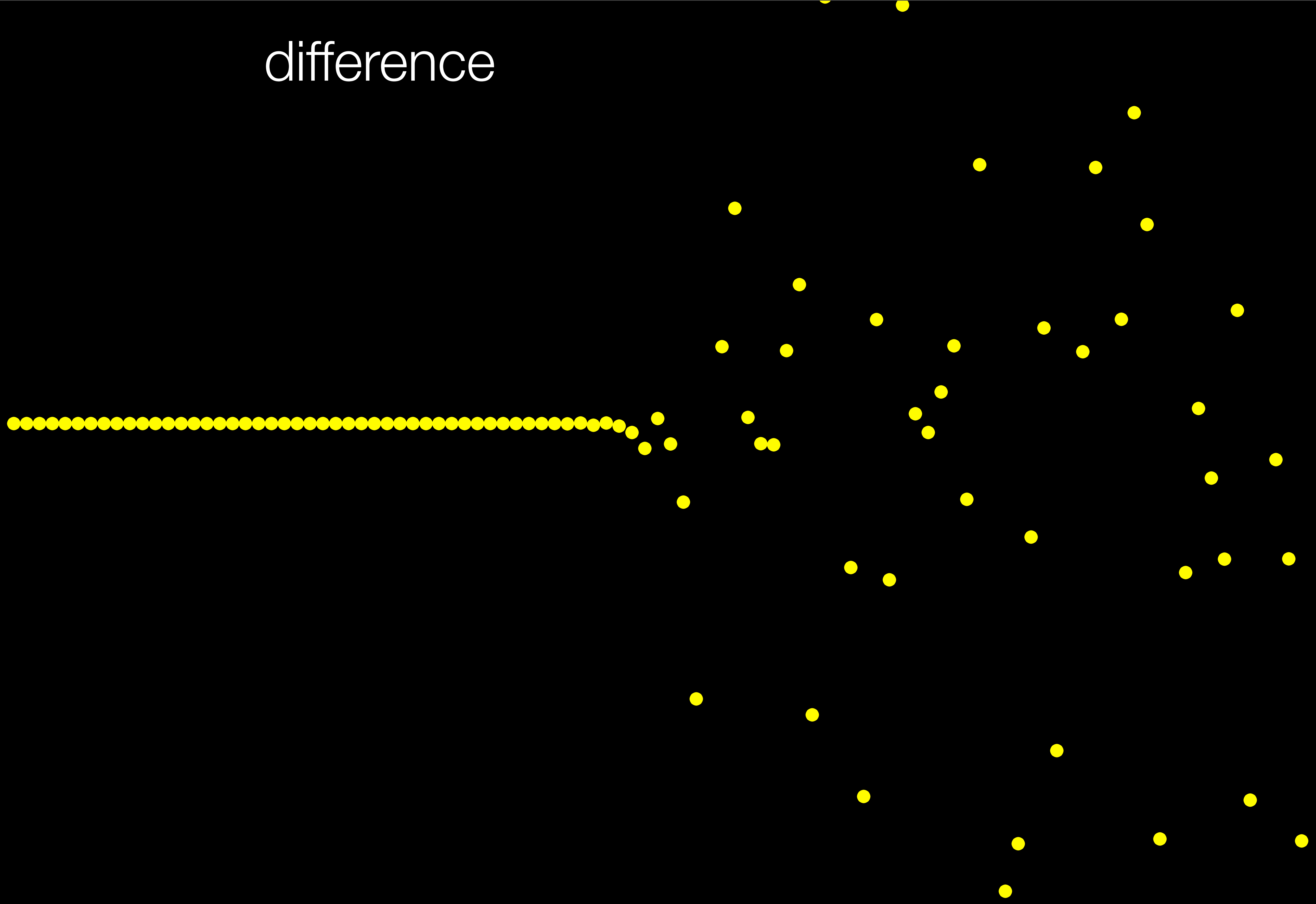
Possibly the simplest demonstration of “chaos” is an experiment with interval maps.

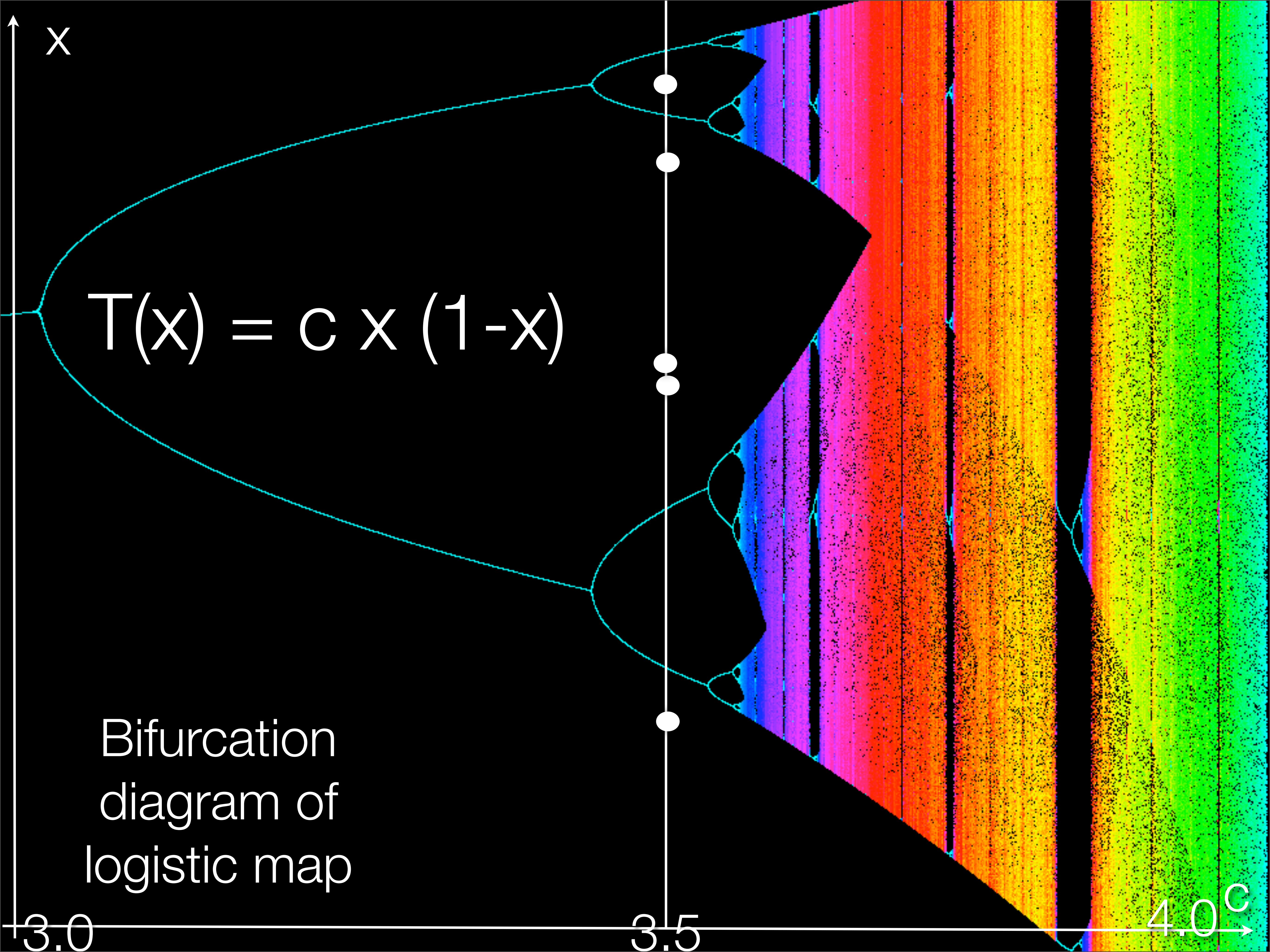


- $T(x) = 4x(1-x)$
- $S(x) = 4x - 4x^2$

we start with the same point!

difference




x
T(x) = c x (1-x)
Bifurcation diagram of logistic map
3.0
3.5
4.0
c

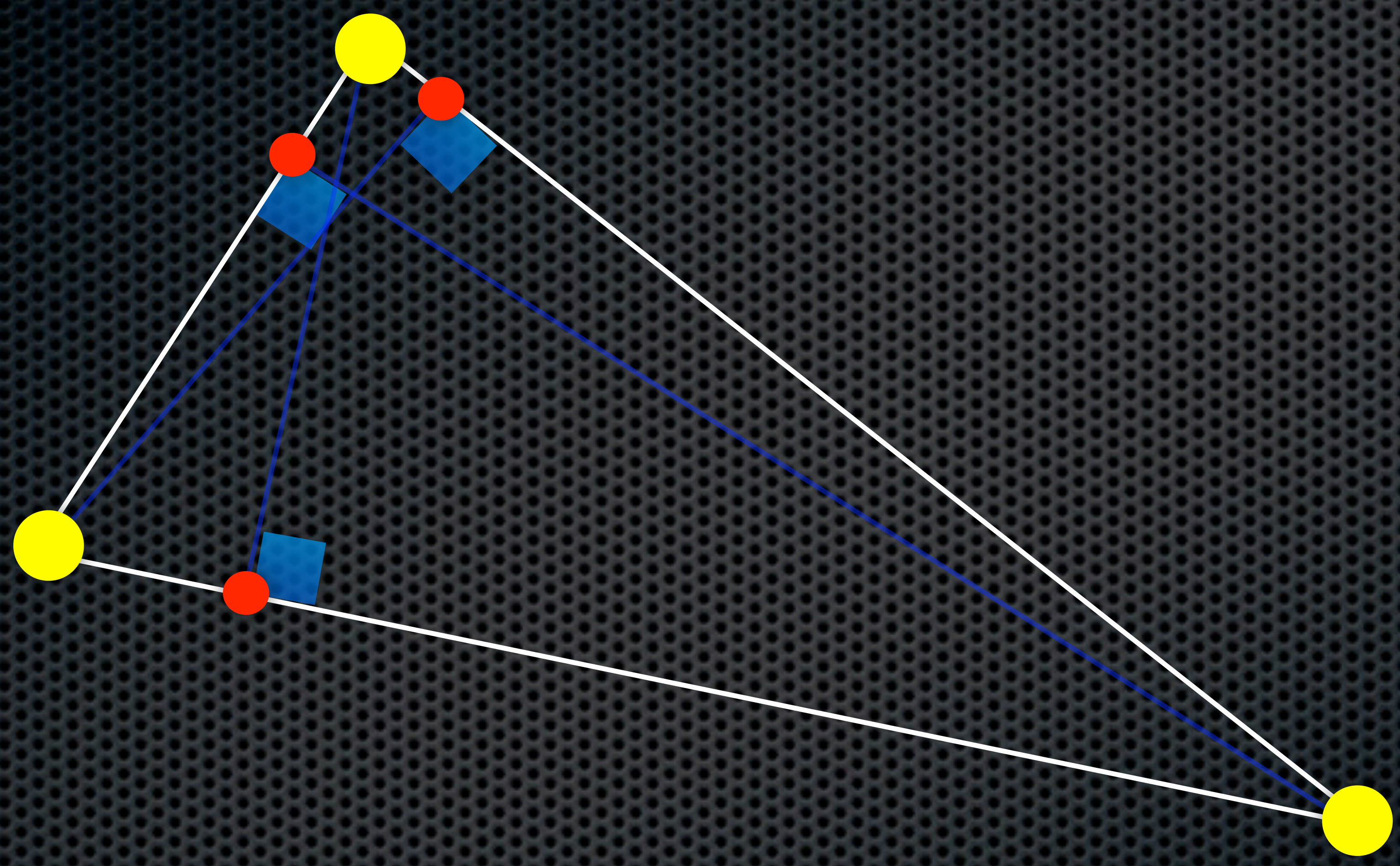
In the rest of the talk, we look at examples which lead to other fields.

- probability theory
- geometry
- astronomy
- sociology
- disaster prevention
- music composition
- poetry
- number theory
- chemistry
- mechanics
- christallograpy
- cryptology
- choreography

probability: random walks

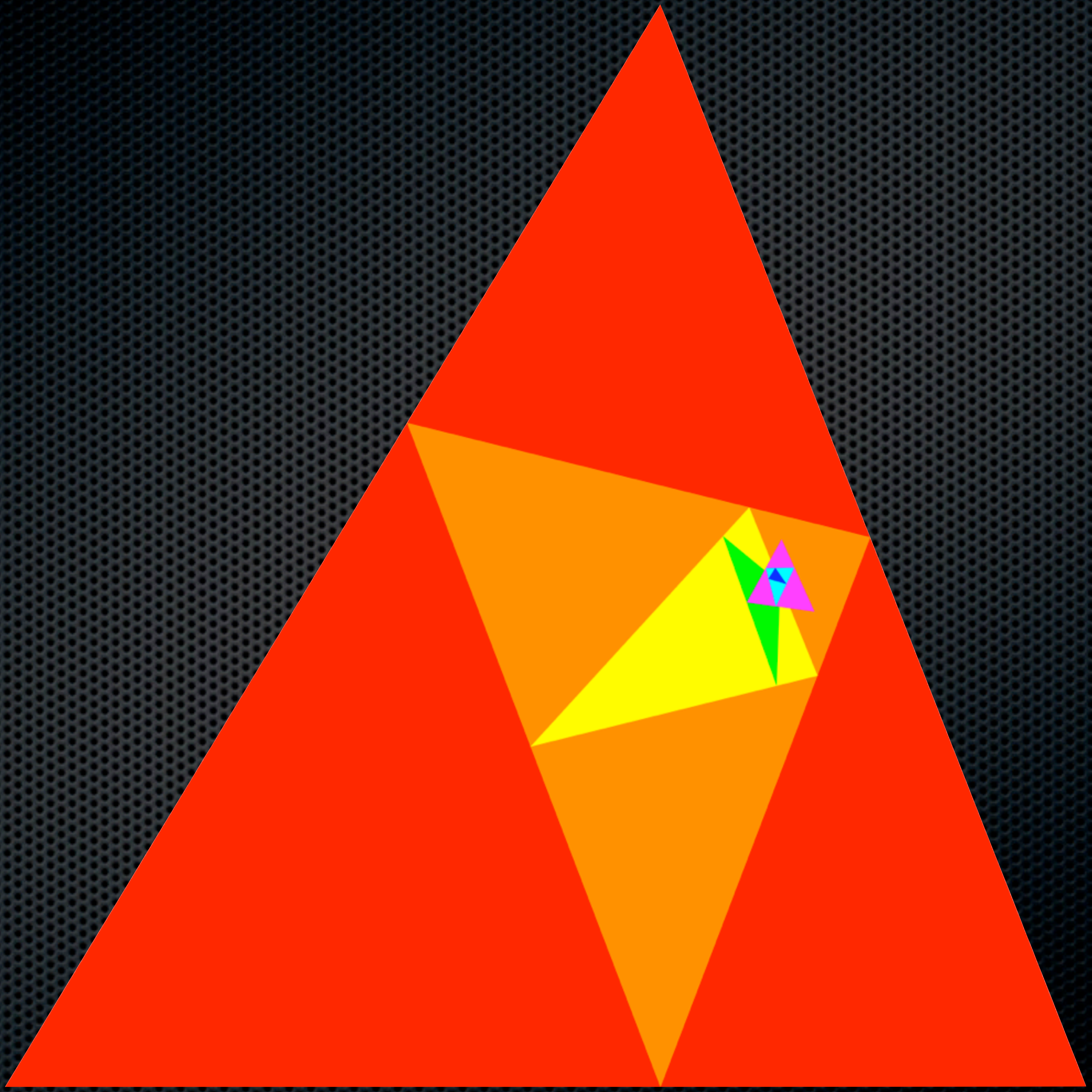
dependence
and
independence

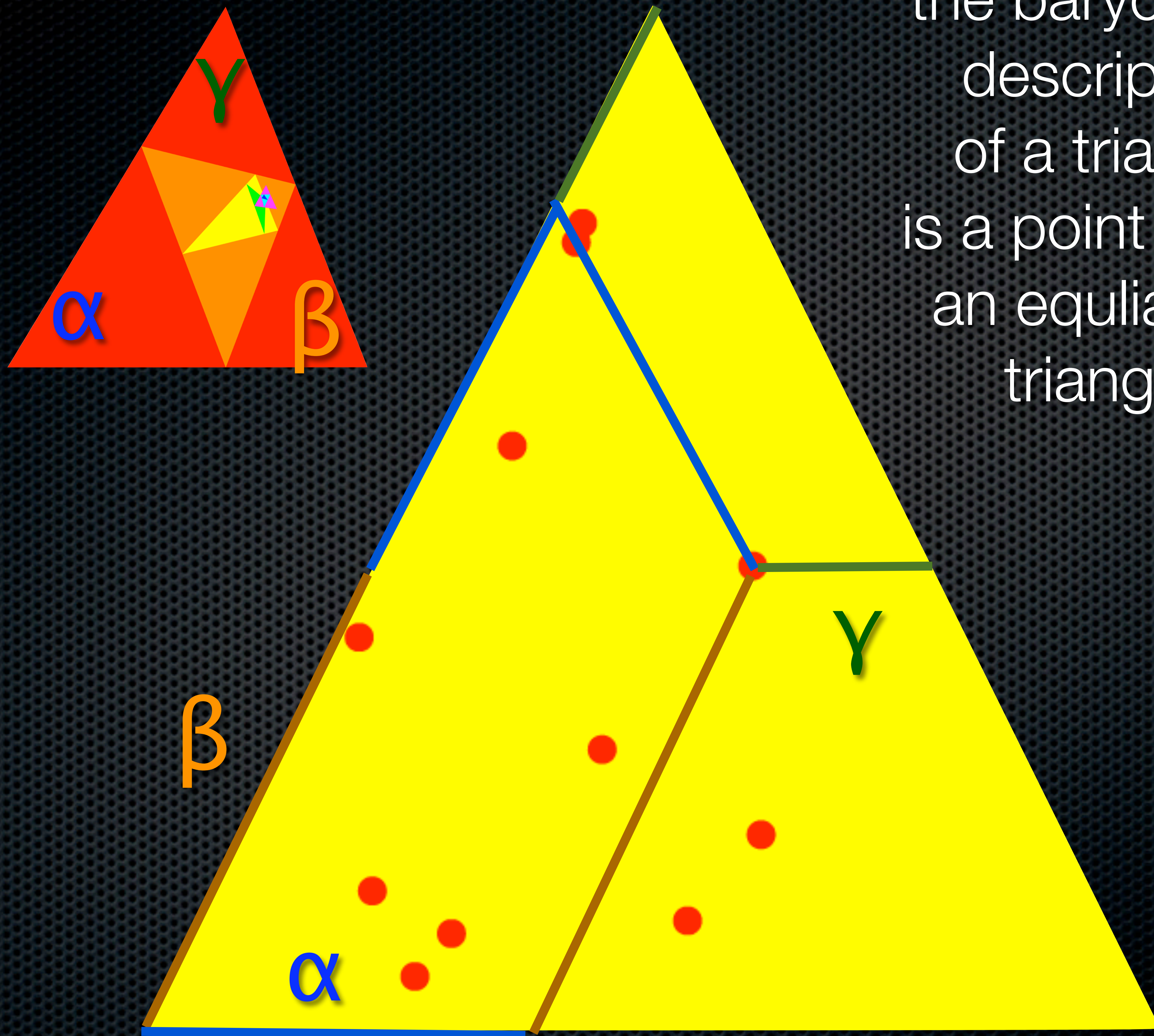
planimetry: the pedal map

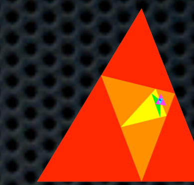


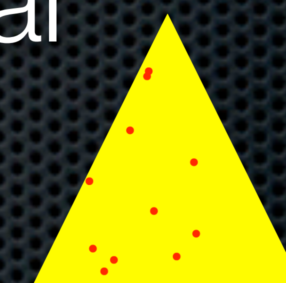
A dynamical system on
the set of triangles

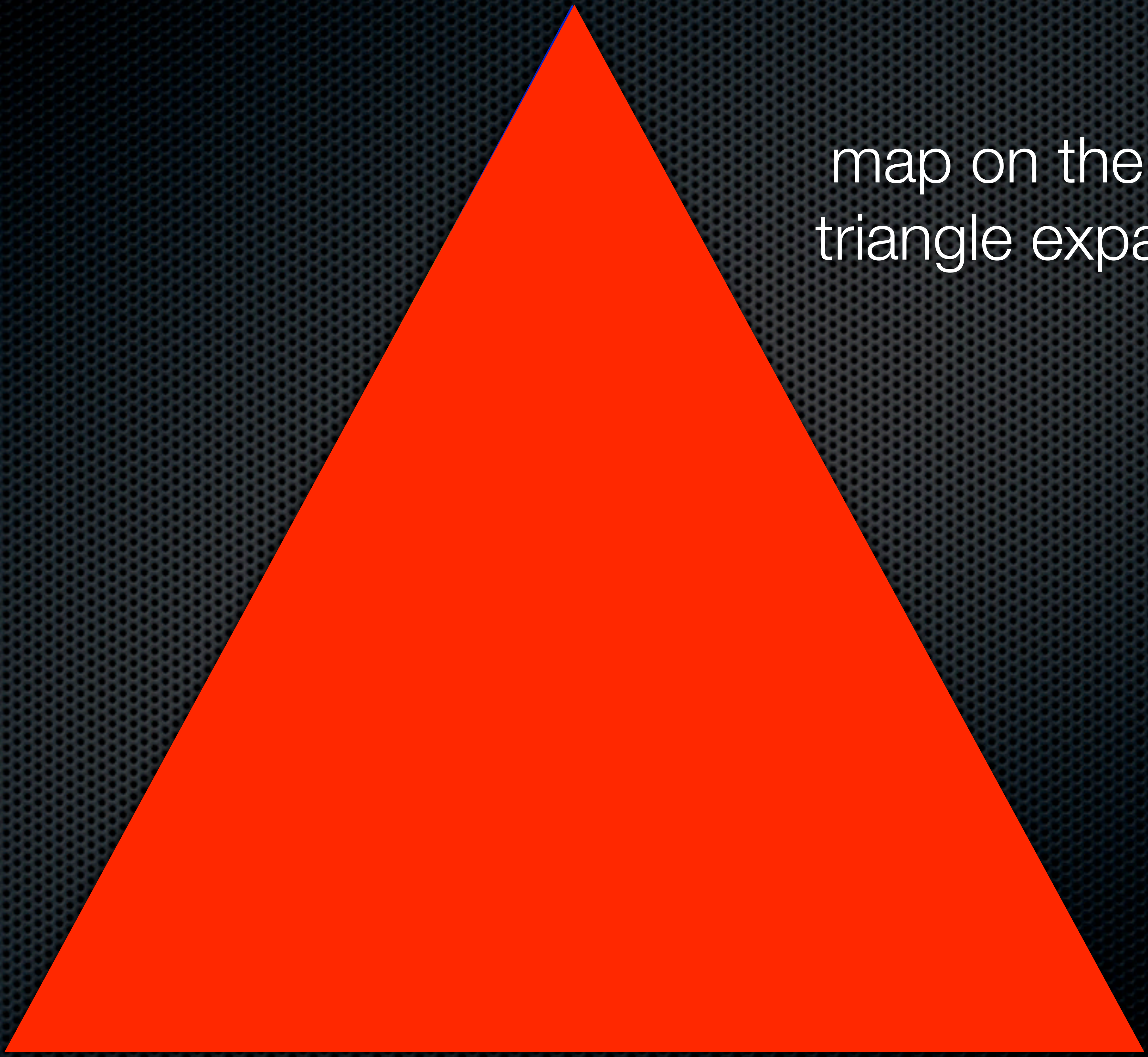
Kingston and Singe,
analyzed by P.D Lax



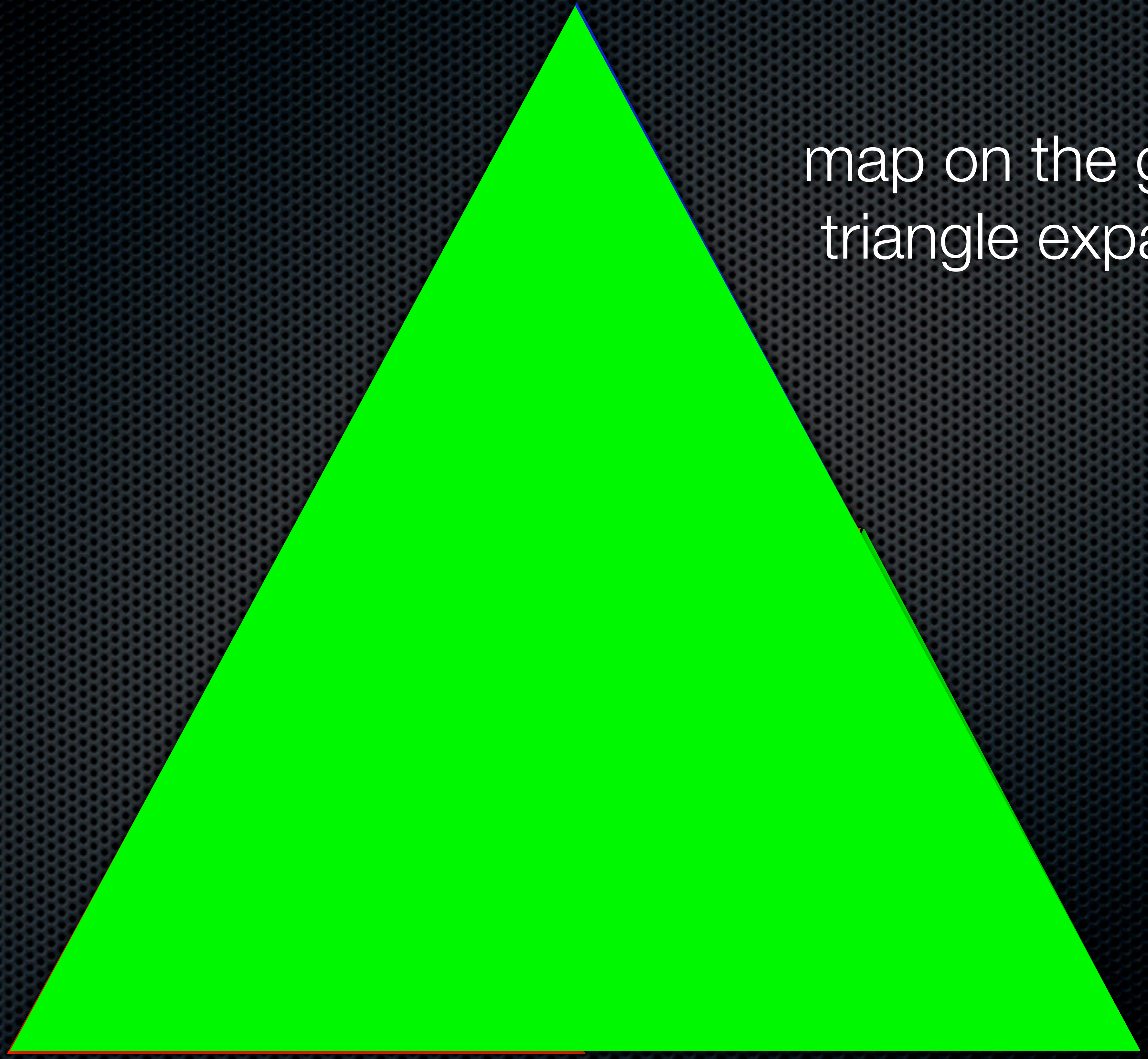


the barycentric description of a triangle  is a point inside an equilateral triangle.

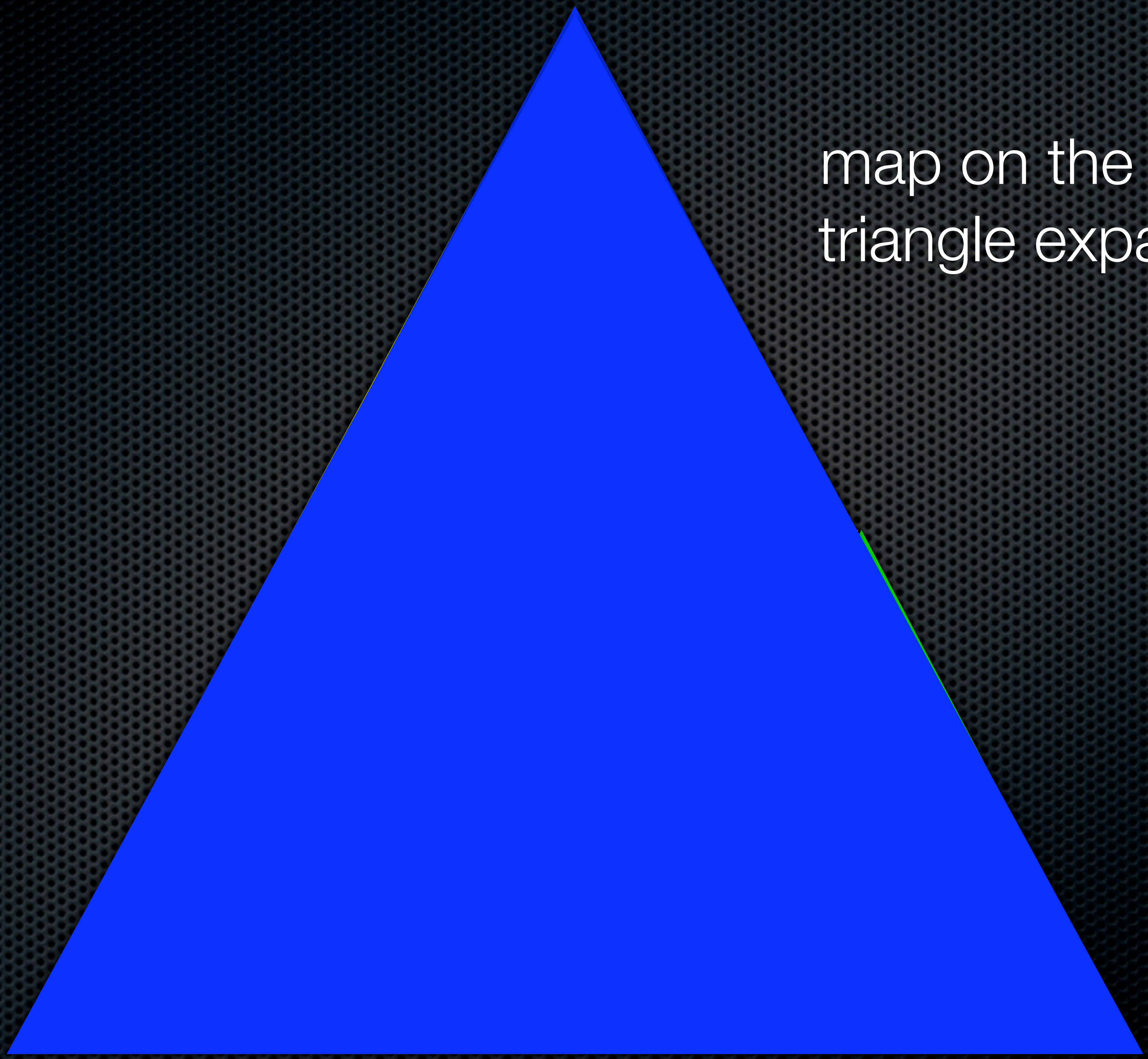




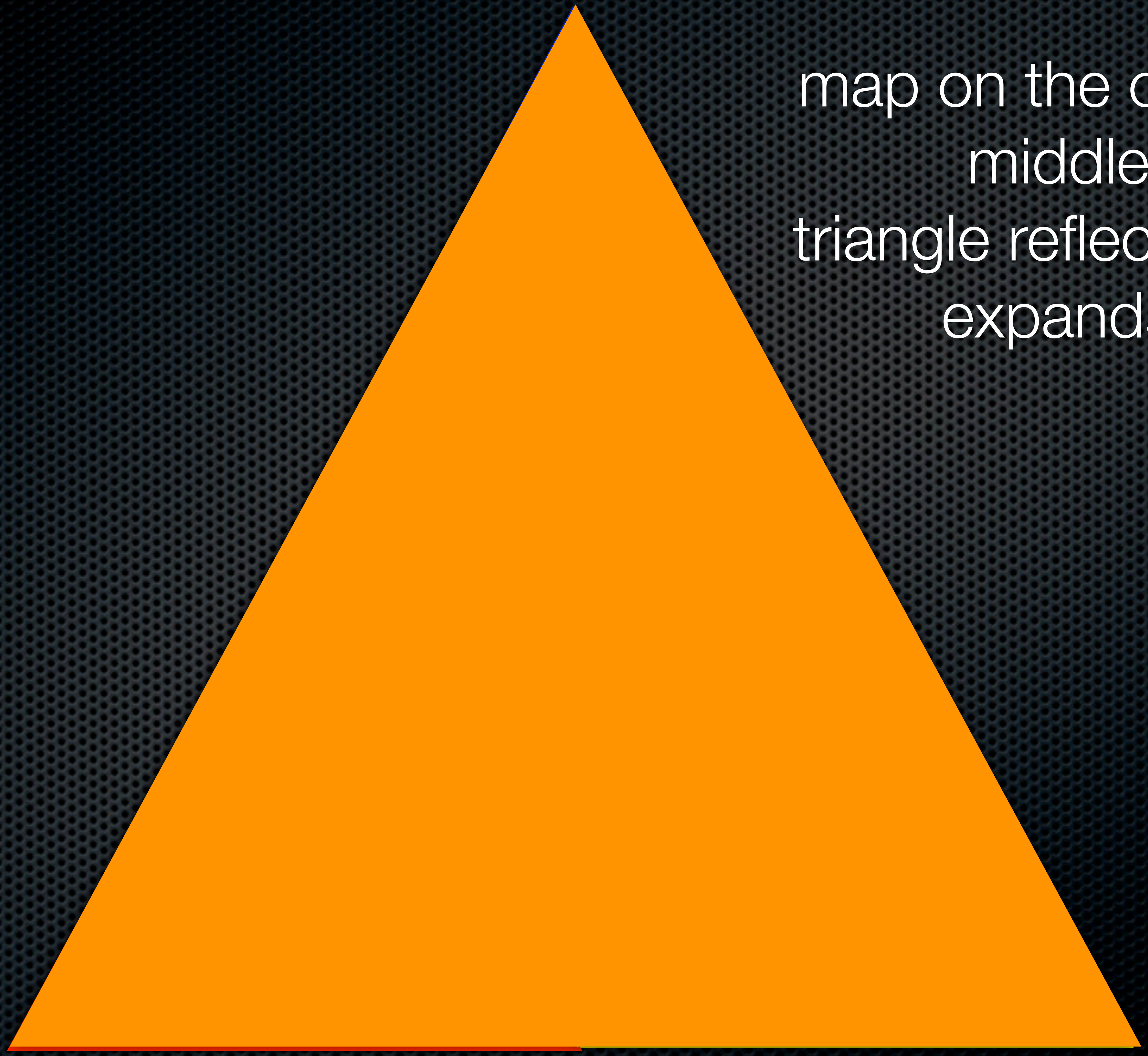
map on the red triangle expands



map on the green triangle expands

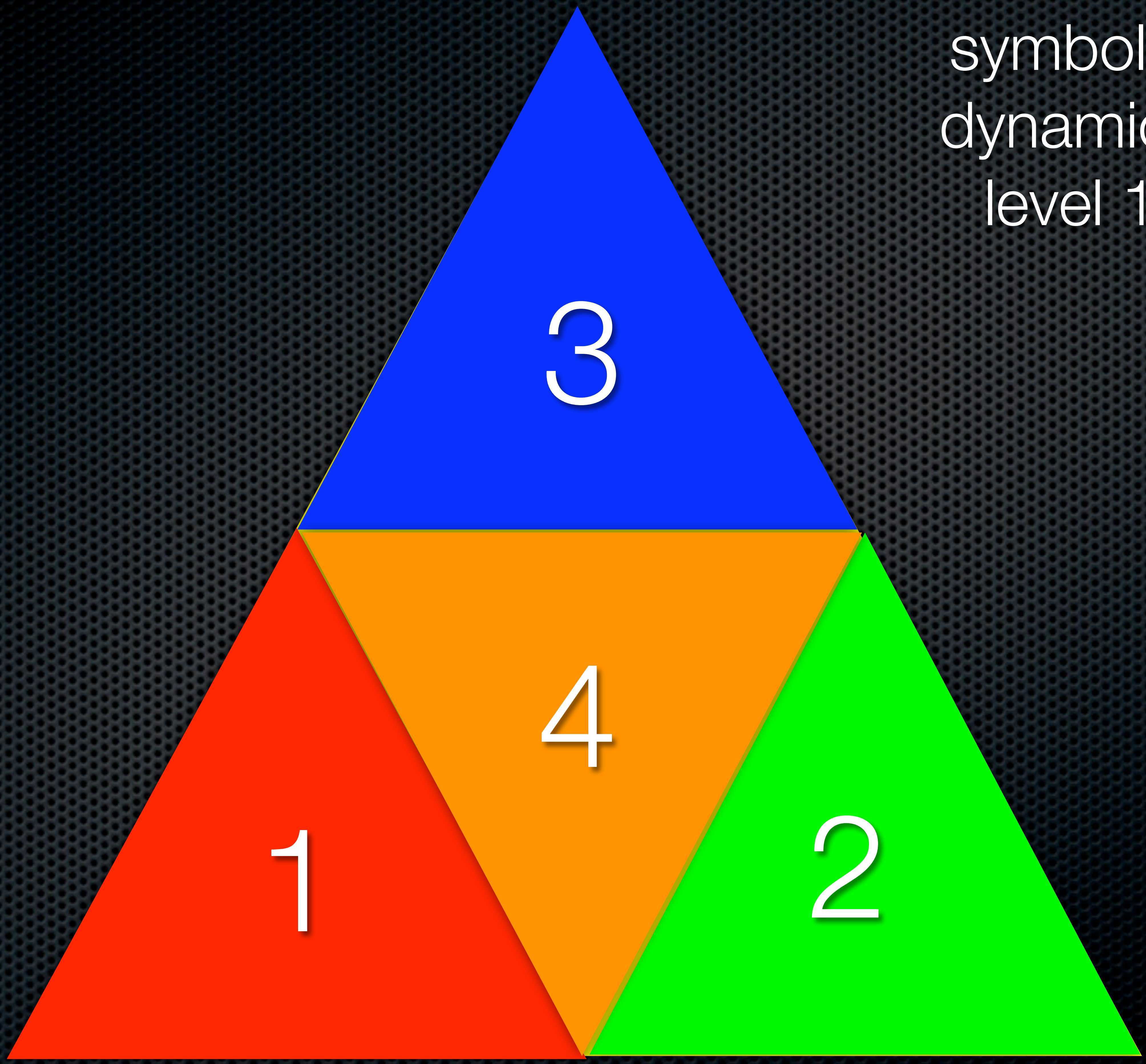


map on the blue triangle expands

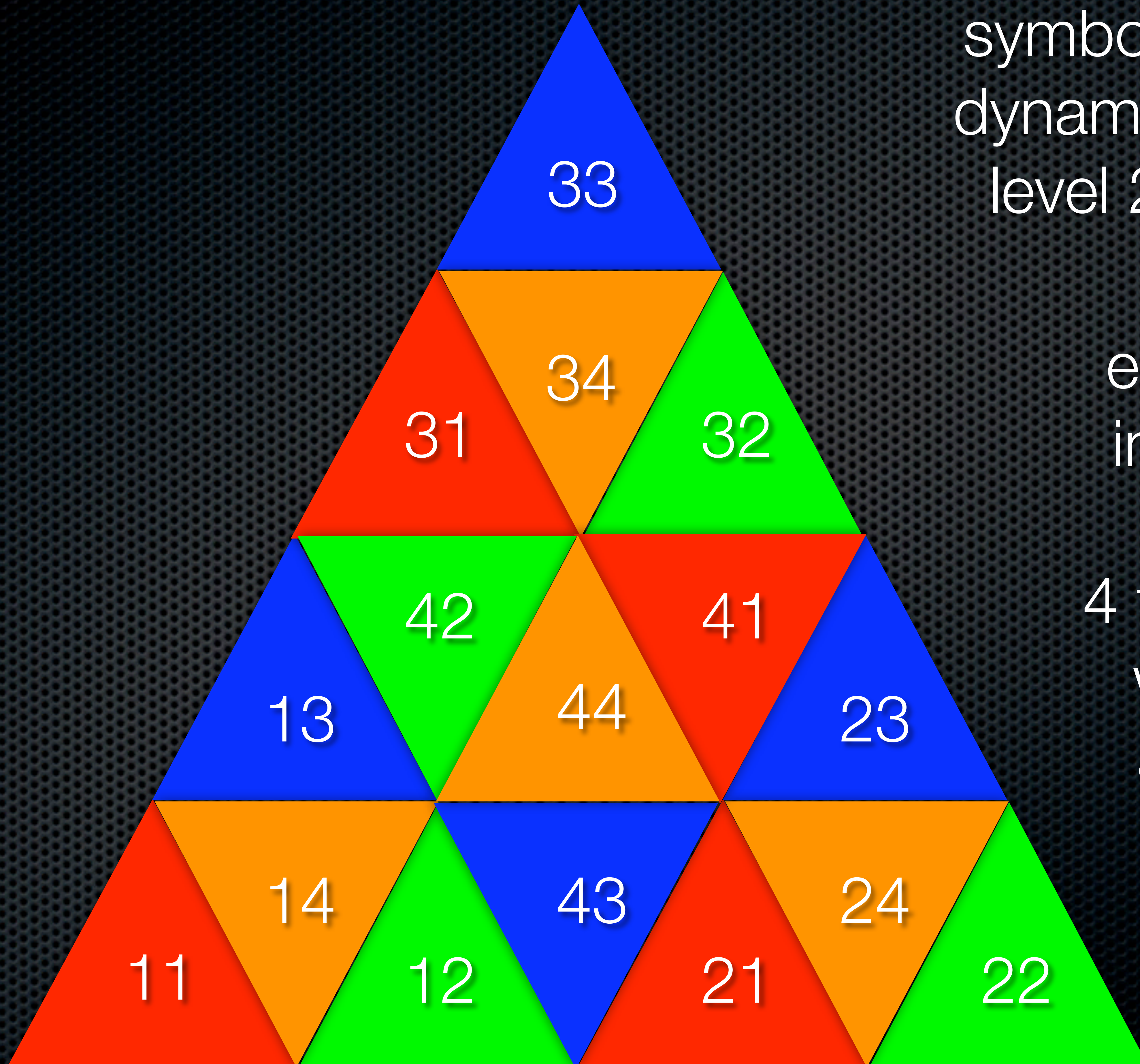


map on the orange middle
triangle reflects and expands

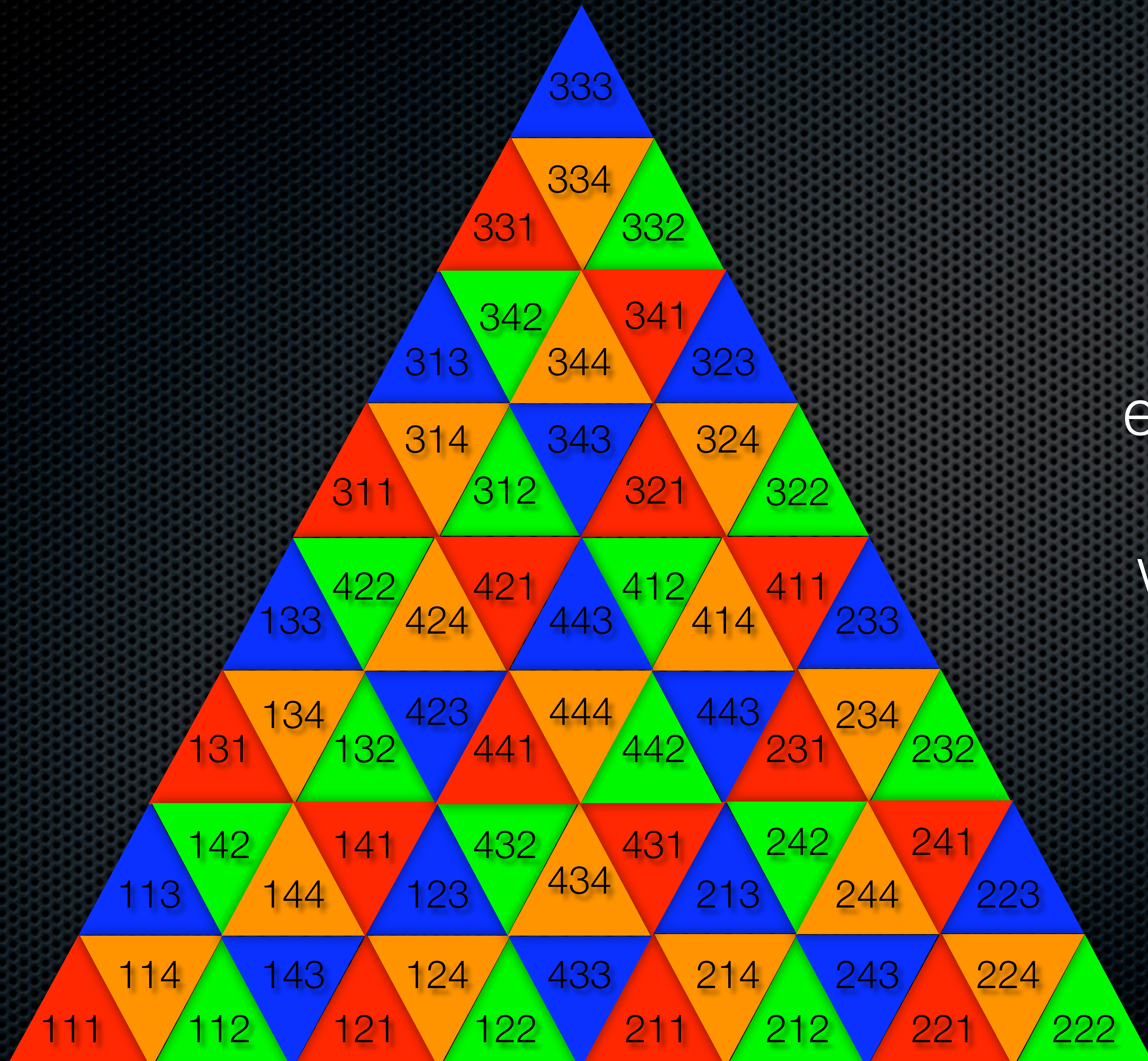
symbolic
dynamics
level 1



symbolic dynamics level 2.



encodes in which of the 4 triangles we are after 2 steps

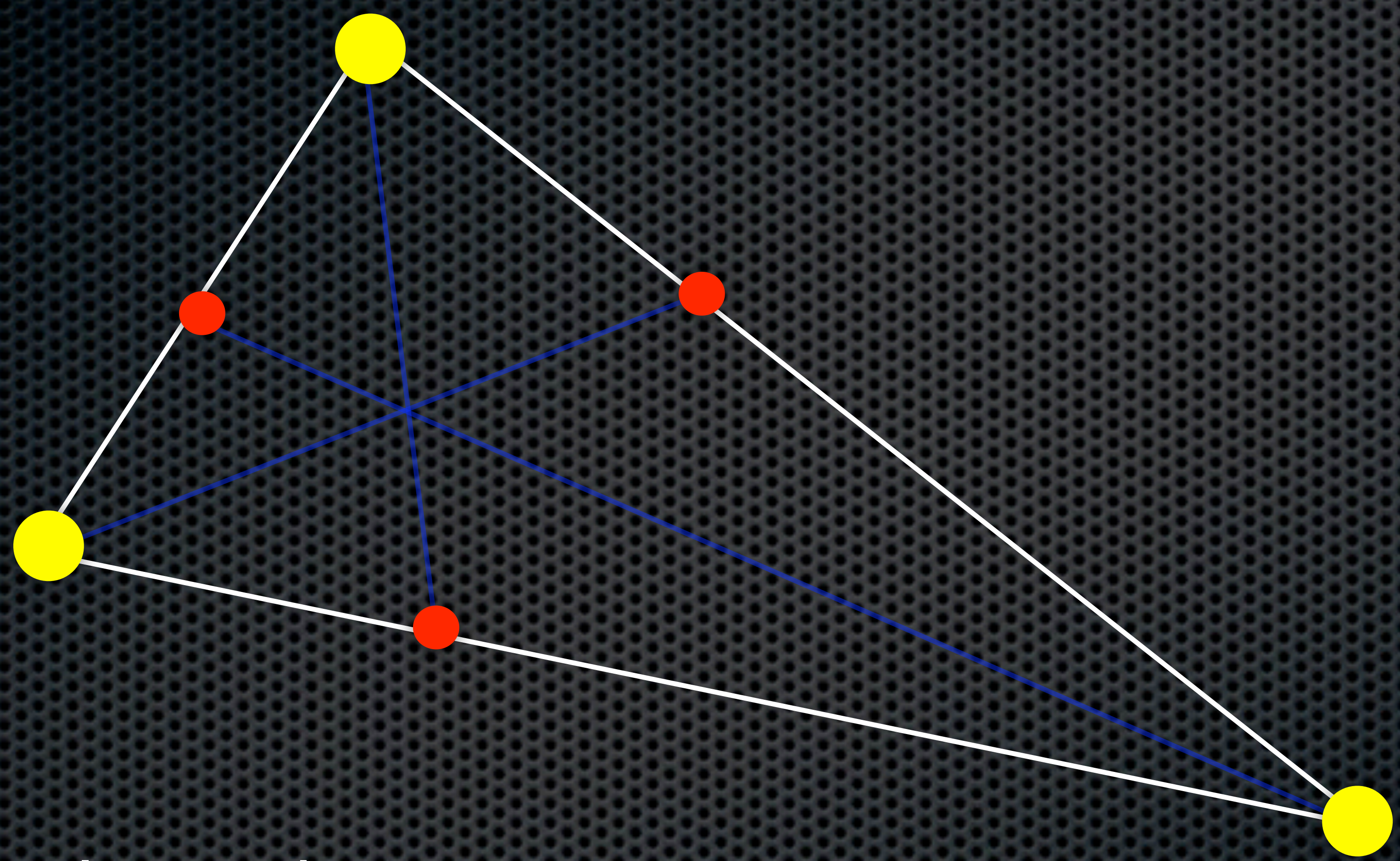


symbolic dynamics level 3.

...

every triangle is coded by which of the 4 original triangles it visits

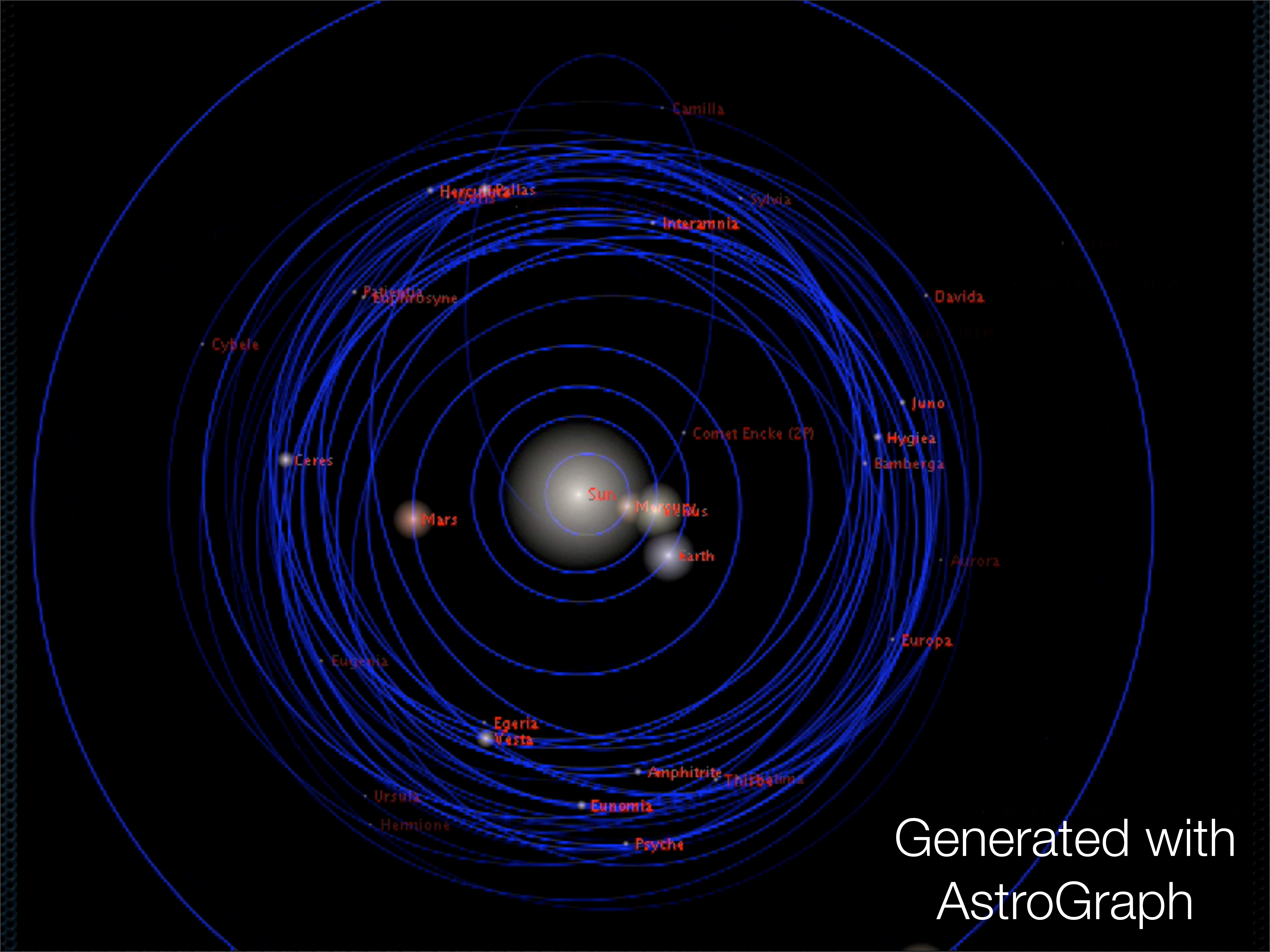
something unexplored?



Explore the map on triangles if one replaces orthogonal lines by angle bisectors.

astronomy: planetary motion

Source: NASA



Generated with AstroGraph

Astronomy: asteroid motion

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In this photo made by the Mars rover Opportunity and released in this Oct. 2006 file photo, by NASA shows a view of the "Victoria crater" looking southeast from "Duck Bay." A newly discovered asteroid has a 1 in 75 chance of slamming into the Red Planet on Jan. 30, 2008, scientists said Thursday, Dec. 20, 2007. If the asteroid does smash into Mars, it'll likely aim near the equator close to where the rover Opportunity has been exploring the Martian plains since 2004. (AP Photo/NASA, JPL, CORNELL)

Asteroid May Hit Mars in Next Month

By ALICIA CHANG – **1 day ago**

LOS ANGELES (AP) — Mars could be in for an asteroid hit. A newly discovered hunk of space rock has a 1 in 75 chance of slamming into the Red Planet on Jan. 30, scientists said Thursday.

"These odds are extremely unusual. We frequently work with really long odds when we track ... threatening asteroids," said Steve Chesley, an astronomer with the Near Earth Object Program at NASA's Jet Propulsion Laboratory.

The asteroid, known as 2007 WD5, was discovered in late November and is similar in size to an object that hit remote central Siberia in 1908, unleashing energy equivalent to a 15-megaton nuclear bomb and wiping out 60 million trees.

Scientists tracking the asteroid, currently halfway between Earth and Mars, initially put the odds of impact at 1 in 350 but increased the chances this week. Scientists expect the odds to diminish again early next month after getting new observations of the asteroid's orbit, Chesley said.

"We know that it's going to fly by Mars and most likely going to miss, but there's a possibility of an impact," he said.

If the asteroid does smash into Mars, it will probably hit near the equator close to where the rover Opportunity has been exploring the Martian plains since 2004. The robot is not in danger because it lies outside the impact zone. Speeding at 8 miles a second, a collision would carve a hole the size of the famed Meteor Crater in Arizona.

In 1994, fragments of the comet Shoemaker-Levy 9 smacked into Jupiter, creating a series of overlapping fireballs in space. Astronomers have yet to witness an asteroid impact with another planet.

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AFP - Dec 21, 2007

Full coverage »

The Catalina Sky Survey opens the 60-inch Mount

"We estimate such impacts occur on Mars every

Friday Februrary 1, 2008:

2007 TU24 (610 m diameter) zooms by in a distance of 550000 km, comes back in 2027

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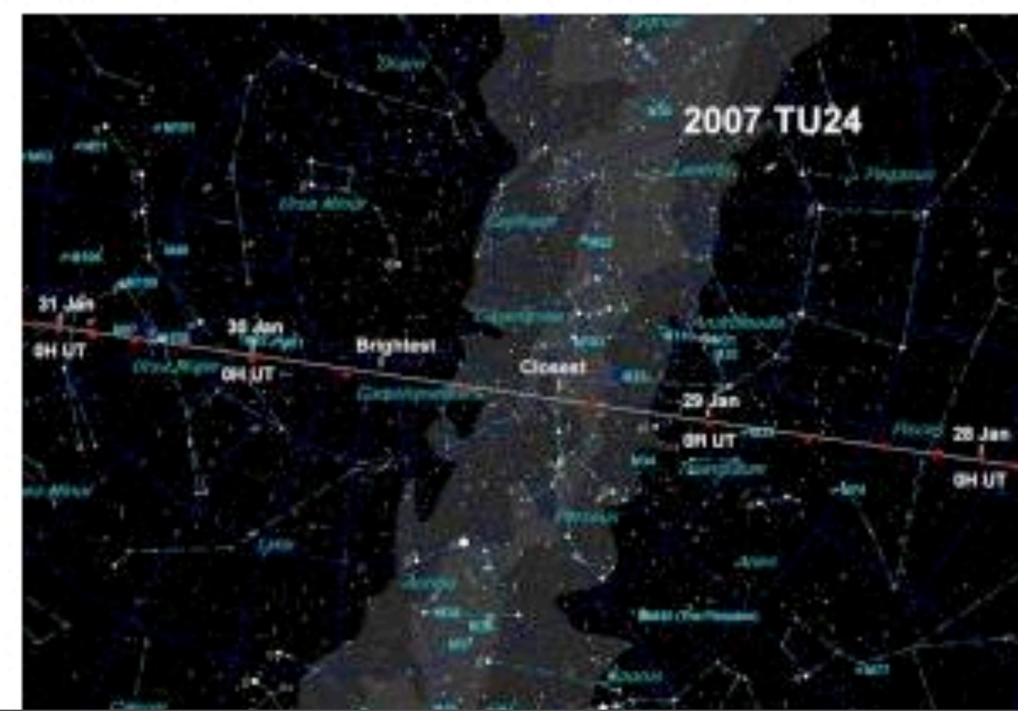
Science News

Asteroid To Make Rare Close Flyby Of Earth January 29

ScienceDaily (Jan. 24, 2008) — Scientists are monitoring the orbit of asteroid 2007 TU24. The asteroid, believed to be between 150 meters (500 feet) and 610 meters (2,000 feet) in size, is expected to fly past Earth on Jan. 29, with its closest distance being about 537,500 kilometers (334,000 miles) at 12:33 a.m. Pacific time (3:33 a.m. Eastern time). It should be observable that night by amateur astronomers with modest-sized telescopes.



Asteroid 2007 TU24 was

January 30, 2008:

2007 WD 5 (610 m diameter) zooms by Mars.

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Asteroid: MARS ATTACK

by Barry Artiste | December 21, 2007 at 09:40 am | 6551 views | 3 comments

It will be interesting to view this as a televised asteroid attack if scientists are proven correct in their assertion that an asteroid impact on Mars would be on a scale of 15 megaton nuclear bomb.

Asteroid could collide with Mars in late January: astromomers

This image provided by NASA's Hubble Space Telescope shows a close-up of the red planet Mars when it was closest to the Hubble Space Telescope - just 88 million kilometers away from Earth on Dec. 18, at 11:45 p.m. Universal Time (6:45 p.m. EST). (THE

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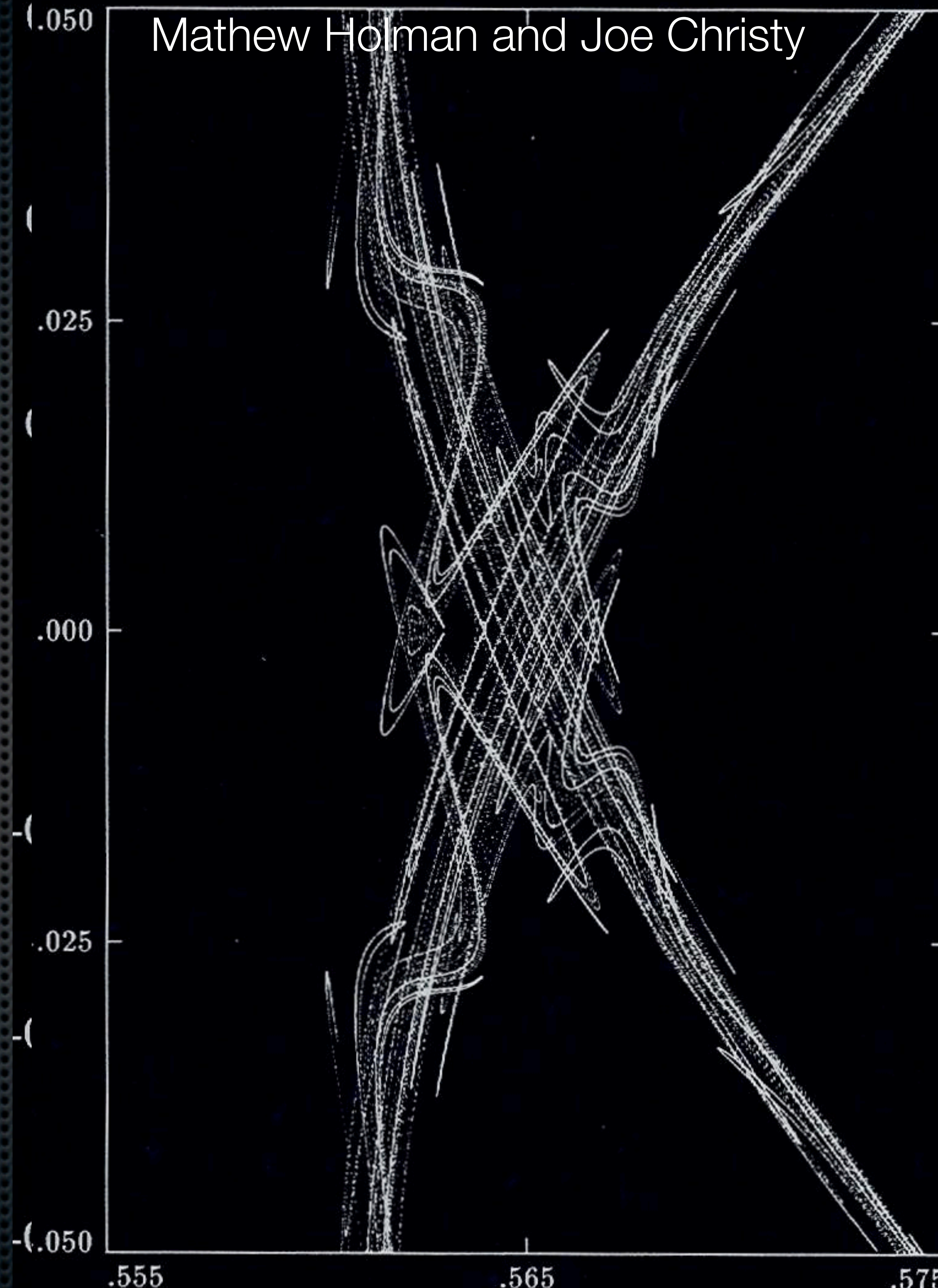
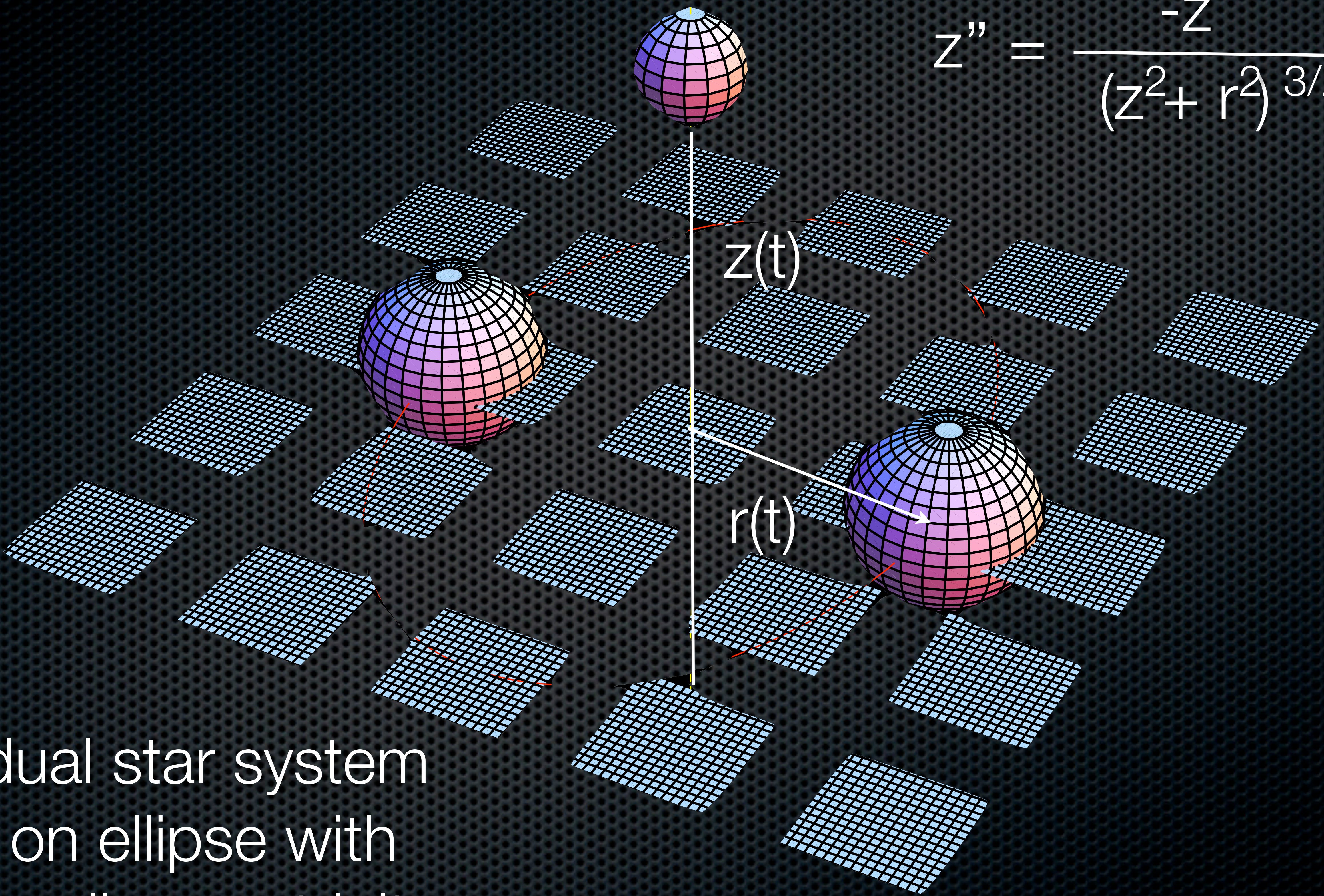
Henry Poincare

image credit right: AIP New Methods of Celestial Mechanics, Intro by D. Goroff



$$z'' = \frac{-z}{(z^2 + r^2)^{3/2}}$$



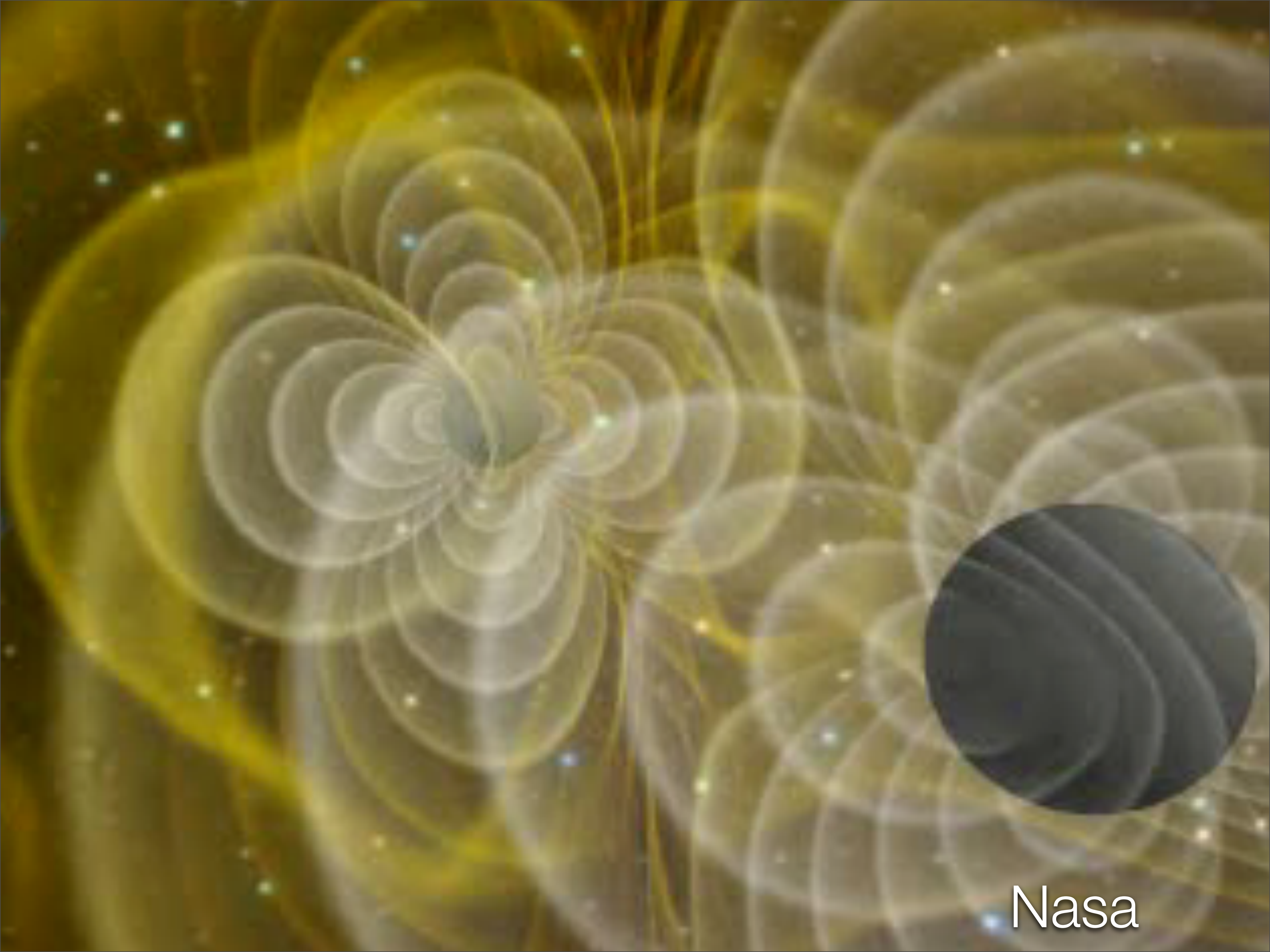
dual star system
on ellipse with
small excentricity

Random Calendar

Sitnikov theorem: for every sufficiently small excentricity, there exists m, such that for any choice of numbers like d_1, d_2, d_3, ... >m, there exists an initial condition such that the first planet year is d_1, the second d_2 etc...

Cosmology: blackhole collision

partial differential
equations, much
much more complicated


Nasa

...much too complicated. Lets look at something simpler

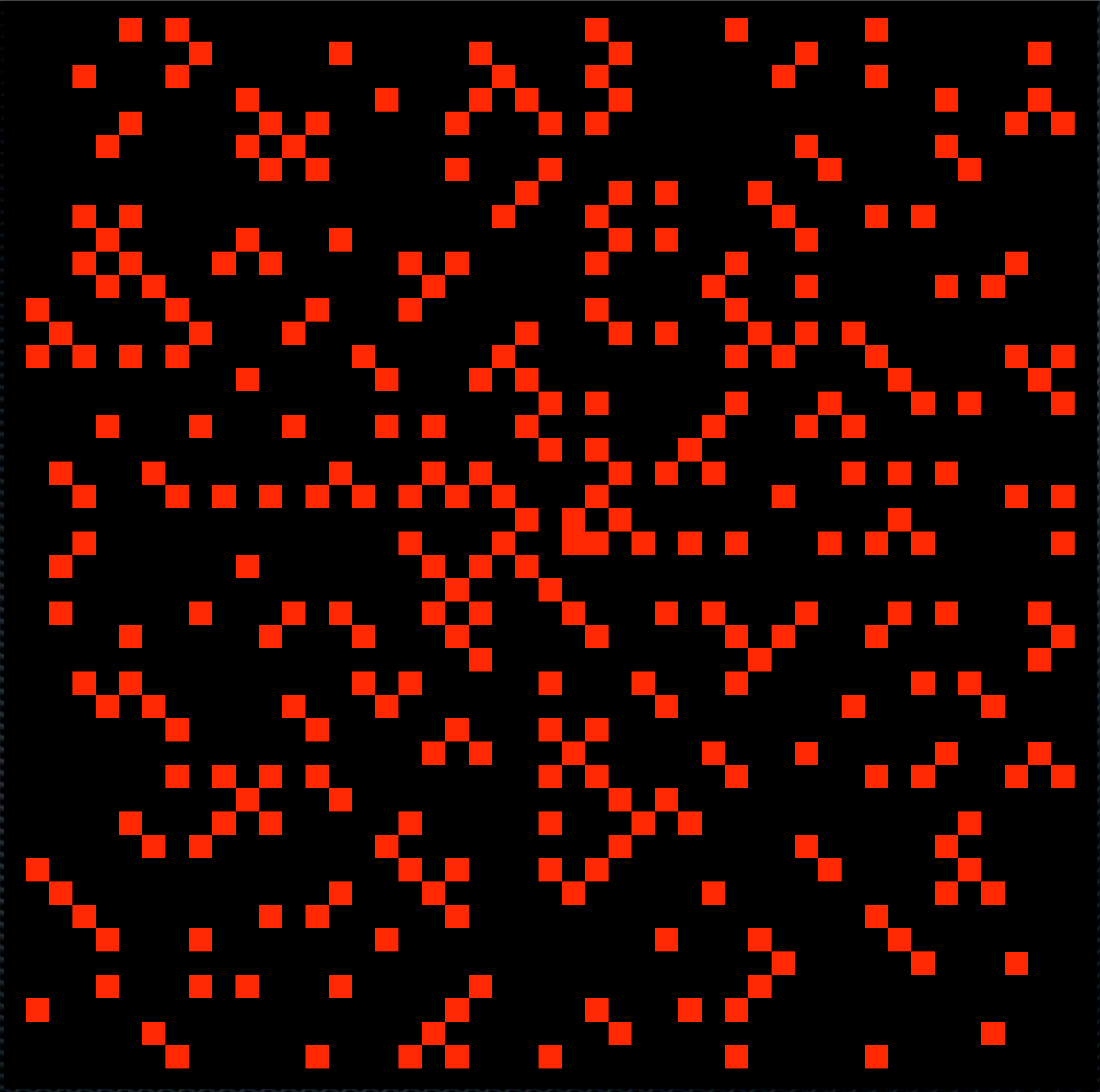
number theory: primes

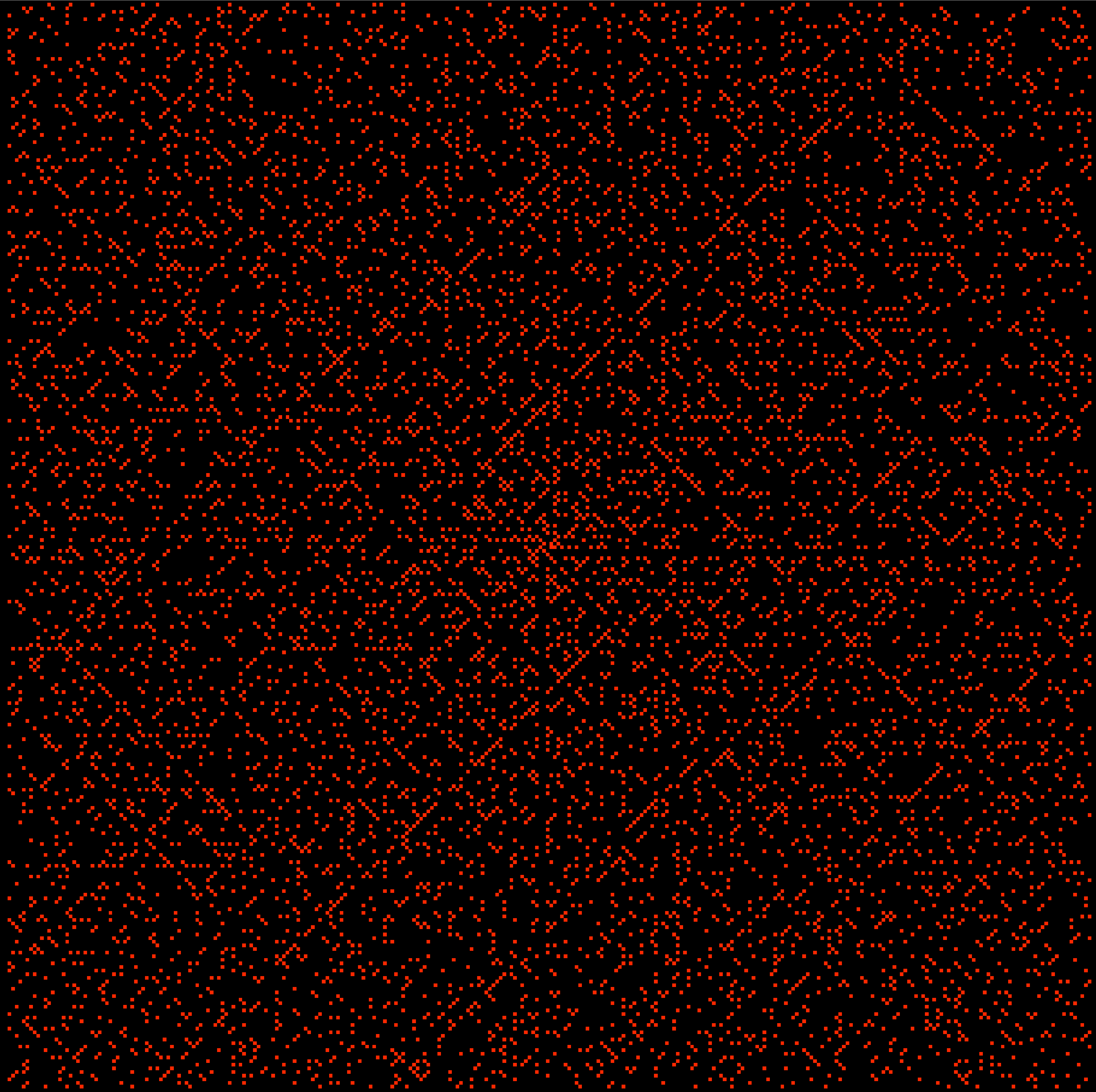
are there some
regularities in the
distribution of primes?

65	64	63	62	61	60	59	58	57	90
66	37	36	35	34	33	32	31	56	89
67	38	17	16	15	14	13	30	55	88
68	39	18	5	4	3	12	29	54	87
69	40	19	6	1	2	11	28	53	86
70	41	20	7	8	9	10	27	52	85
71	42	21	22	23	24	25	26	51	84
72	43	44	45	46	47	48	49	50	83
73	74	75	76	77	78	79	80	81	82

Ulam Spiral

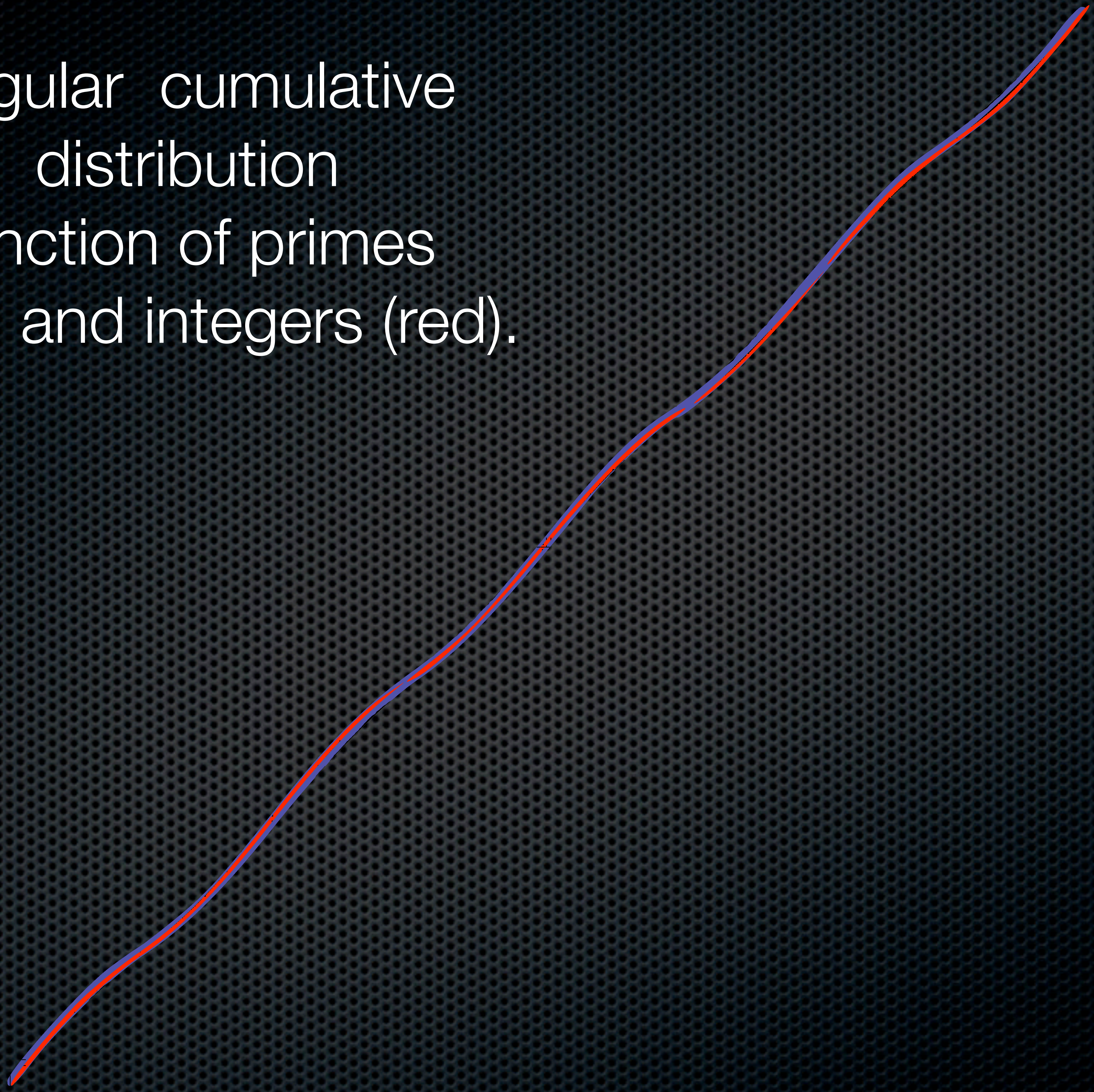
197	196	195	194	**193**	192	**191**	190	189	188	187	186	185	184	183	240
198	145	144	143	142	141	140	**139**	138	**137**	136	135	134	133	182	**239**
199	146	**101**	100	99	98	**97**	96	95	94	93	92	91	132	**181**	238
200	147	102	65	64	63	62	**61**	60	**59**	58	57	90	**131**	180	237
201	148	**103**	66	**37**	36	35	34	33	32	**31**	56	**89**	130	**179**	236
202	**149**	104	**67**	38	**17**	16	15	14	**13**	30	55	88	129	178	235
203	150	105	68	39	18	**5**	4	**3**	12	**29**	54	87	128	177	234
204	**151**	106	69	40	**19**	6	1	**2**	**11**	28	**53**	86	**127**	176	**233**
205	152	**107**	70	**41**	20	**7**	8	9	10	27	52	85	126	175	232
206	153	108	**71**	42	21	22	**23**	24	25	26	51	84	125	174	231
207	154	**109**	72	**43**	44	45	46	**47**	48	49	50	**83**	124	**173**	230
208	155	110	**73**	74	75	76	77	78	**79**	80	81	82	123	172	**229**
209	156	111	112	**113**	114	115	116	117	118	119	120	121	122	171	228
210	**157**	158	159	160	161	162	**163**	164	165	166	**167**	168	169	170	**227**
211	212	213	214	215	216	217	218	219	220	221	222	**223**	224	225	226





The code conspiracy

angular cumulative distribution function of primes (blue) and integers (red).

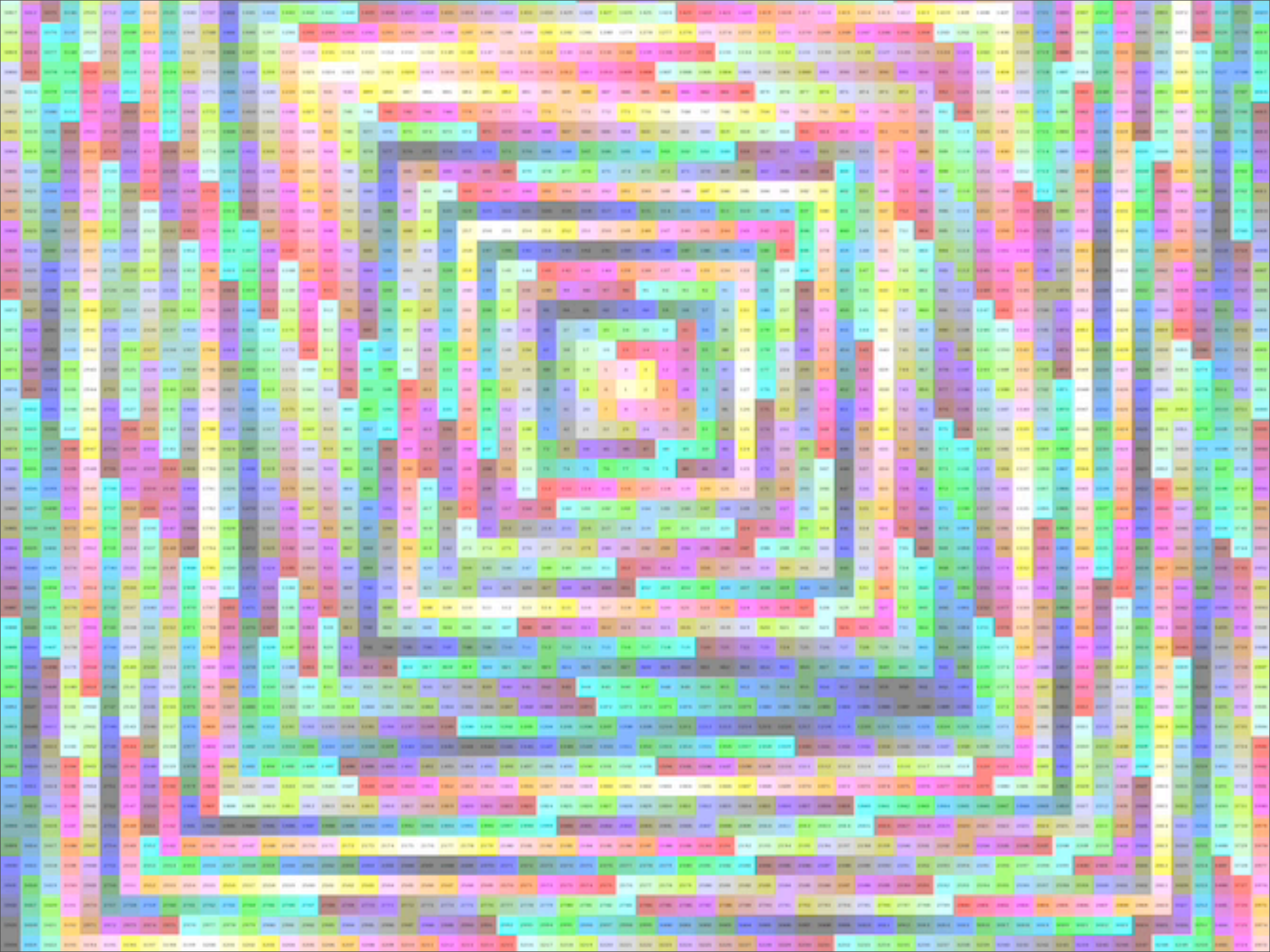


Number theory: Dickson system

T(n) = sum of proper factors of n

T(6) = 1+2+3 = 6 fixed point of T

A dynamical system on
the set of natural numbers



Number theory: Collatz system

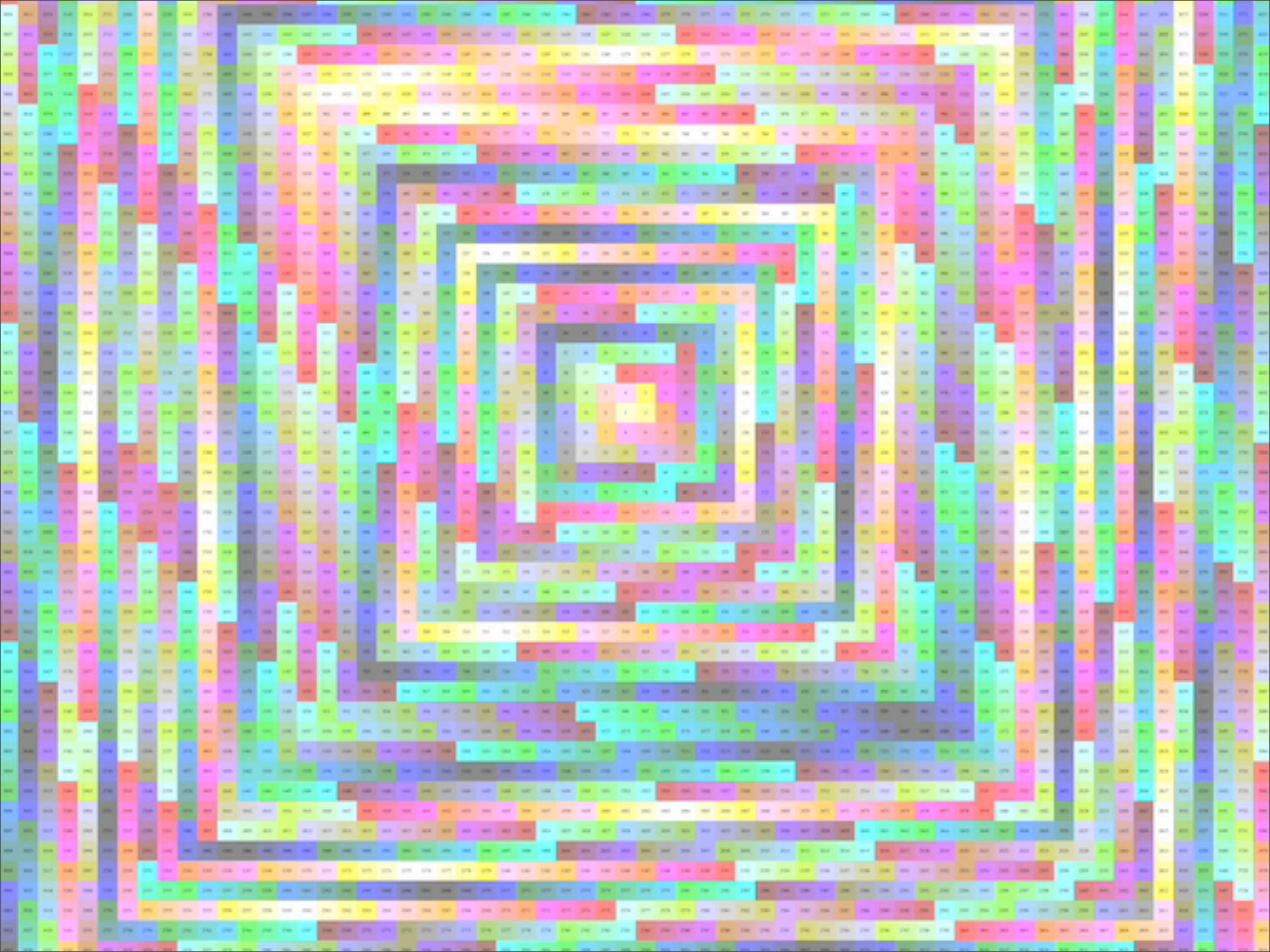
A dynamical system on
the set of natural numbers

Collatz system

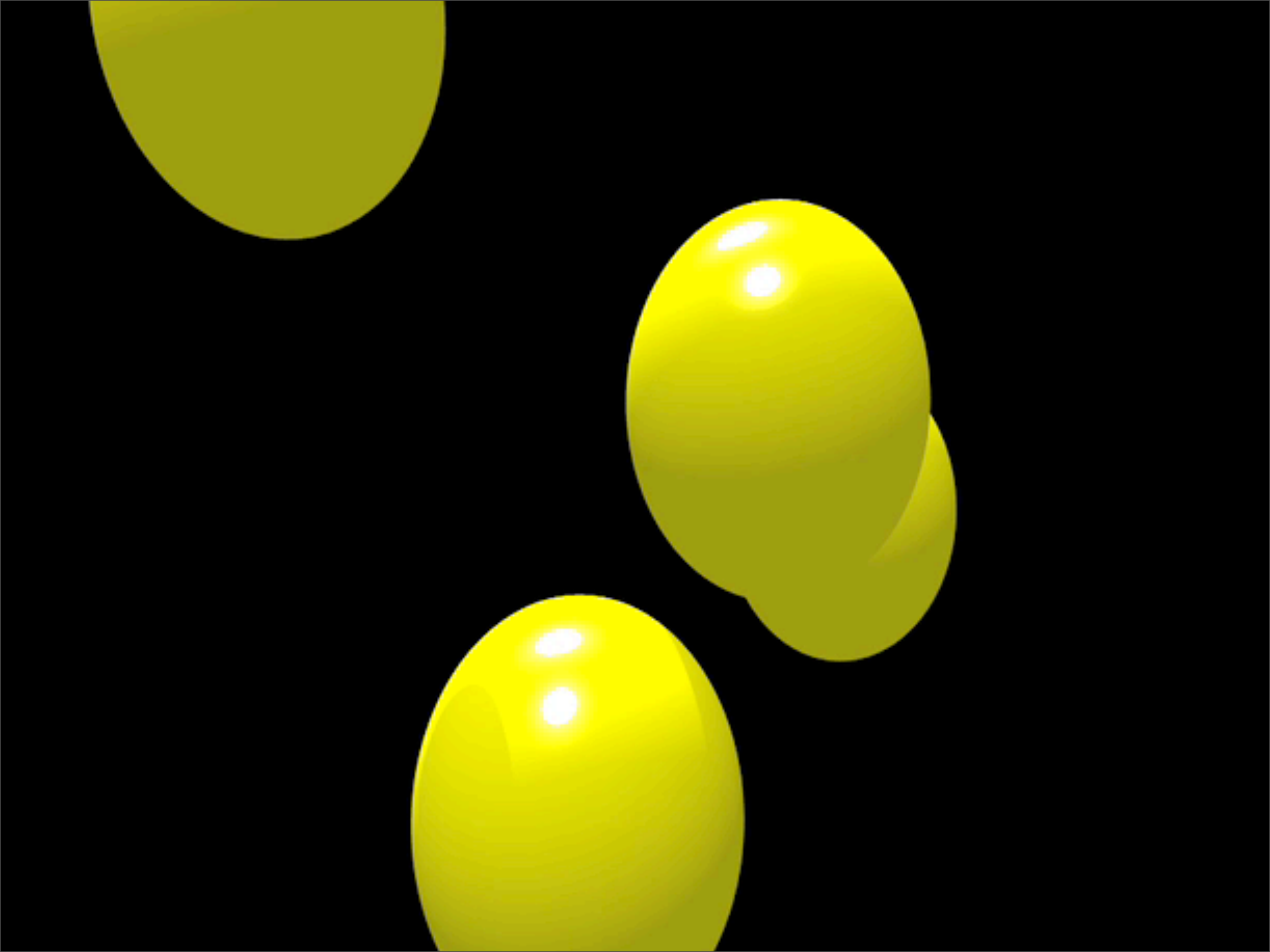
$$T(n) = \begin{cases} n/2 & n \text{ even} \\ 3n+1 & n \text{ odd} \end{cases}$$

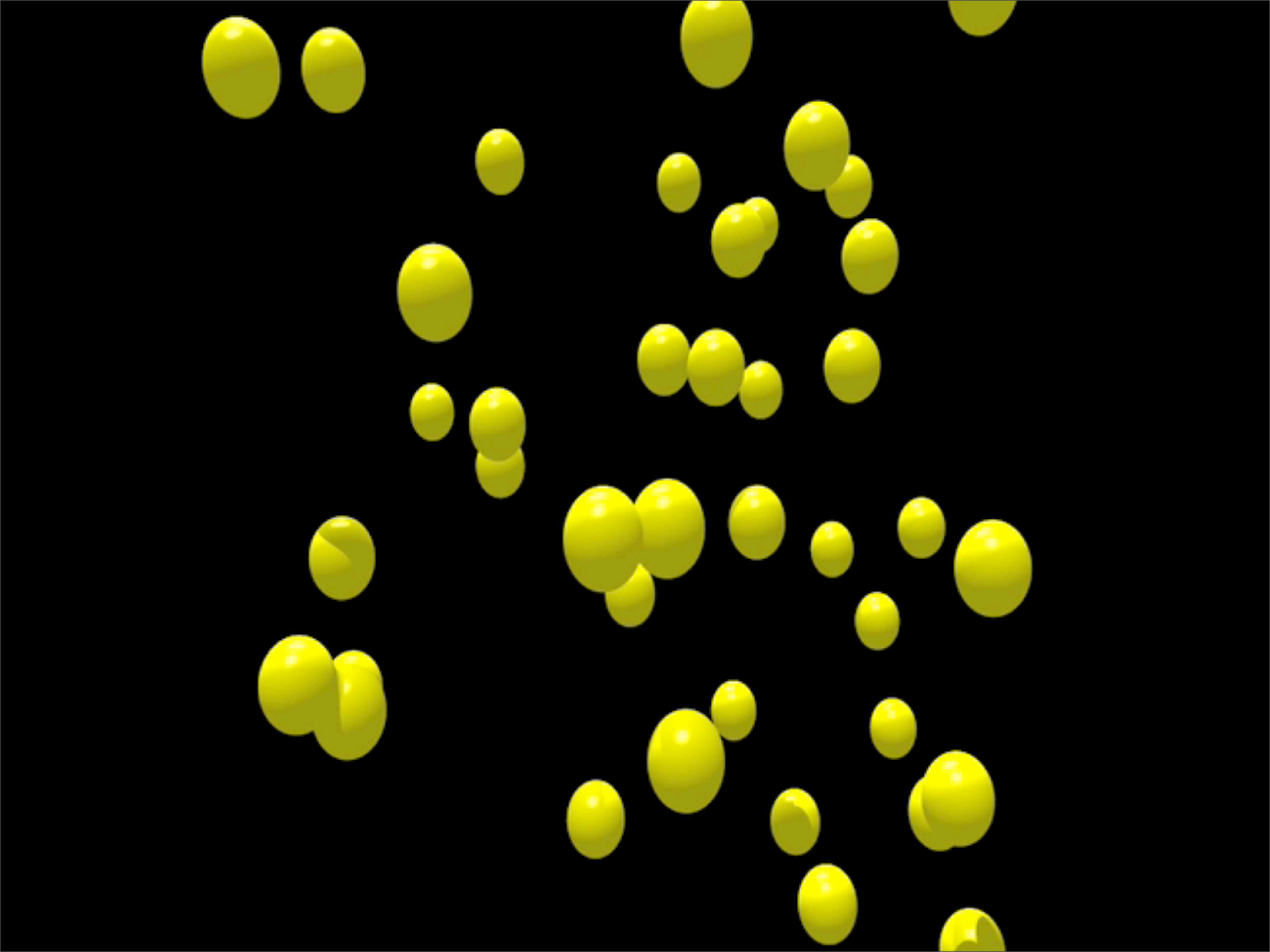
Example:

6 - 3 - 10 - 5 - 16 - 8 - 4- 2 - 1

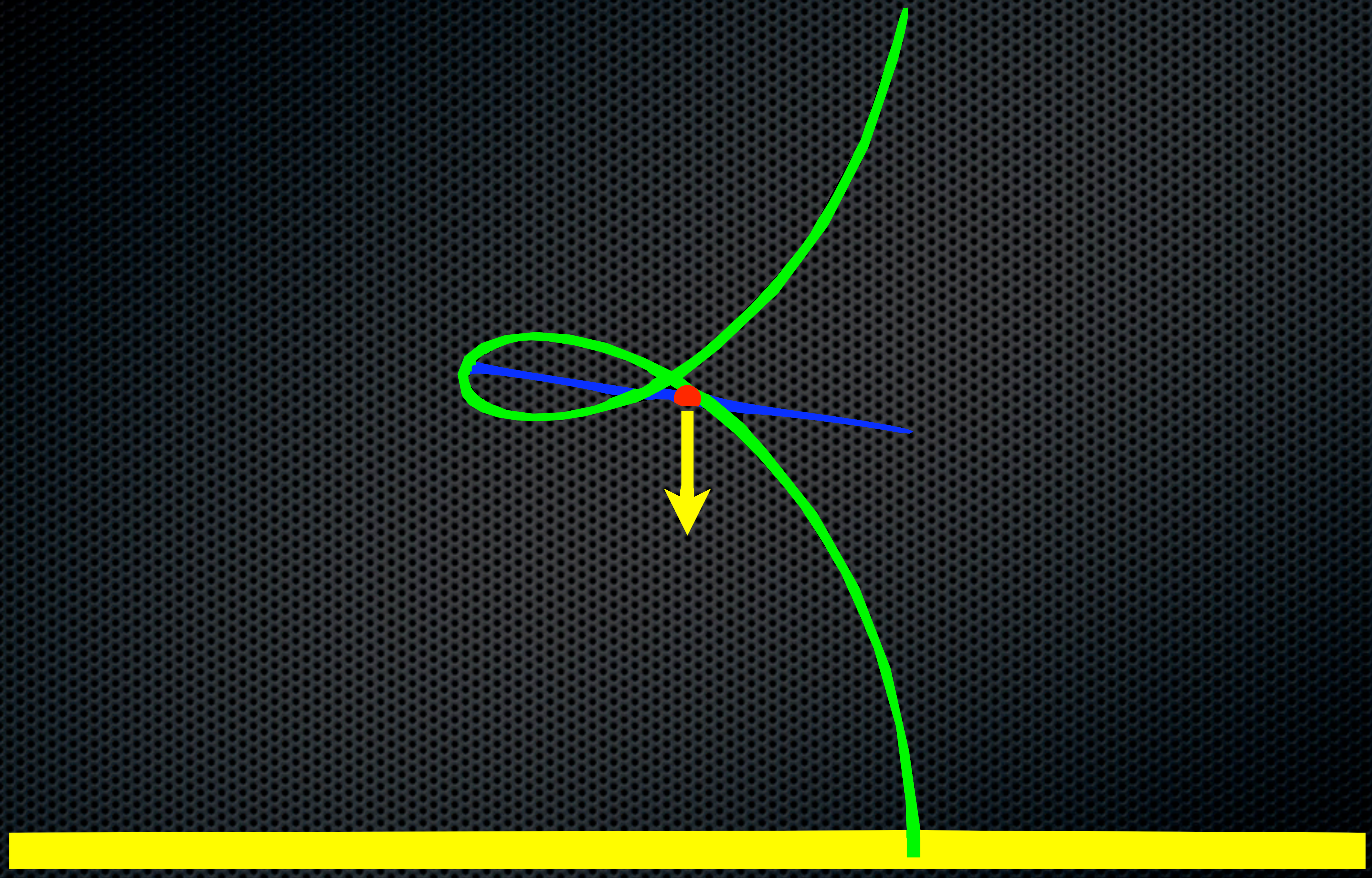


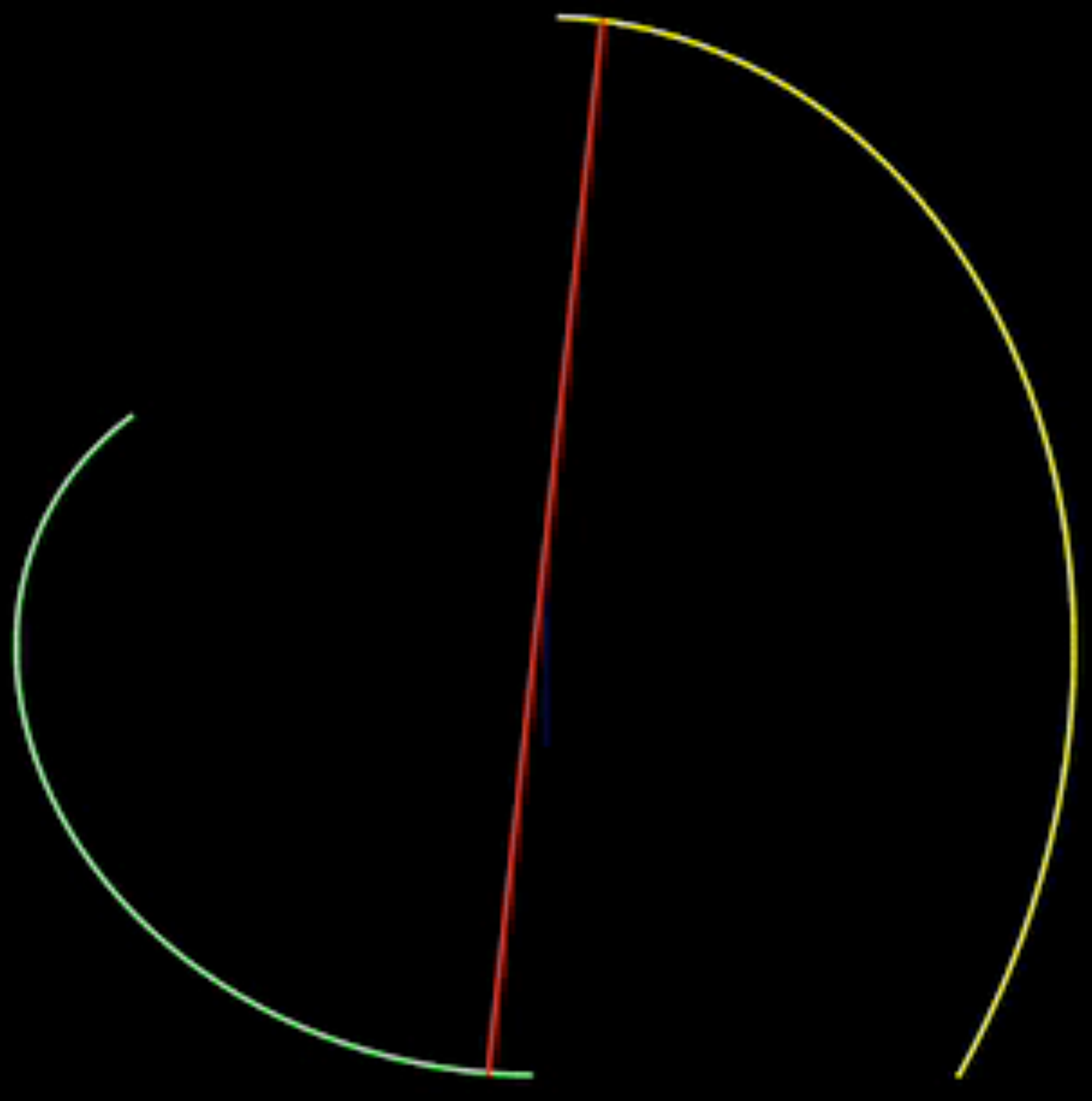
Mechanics: from particle motion

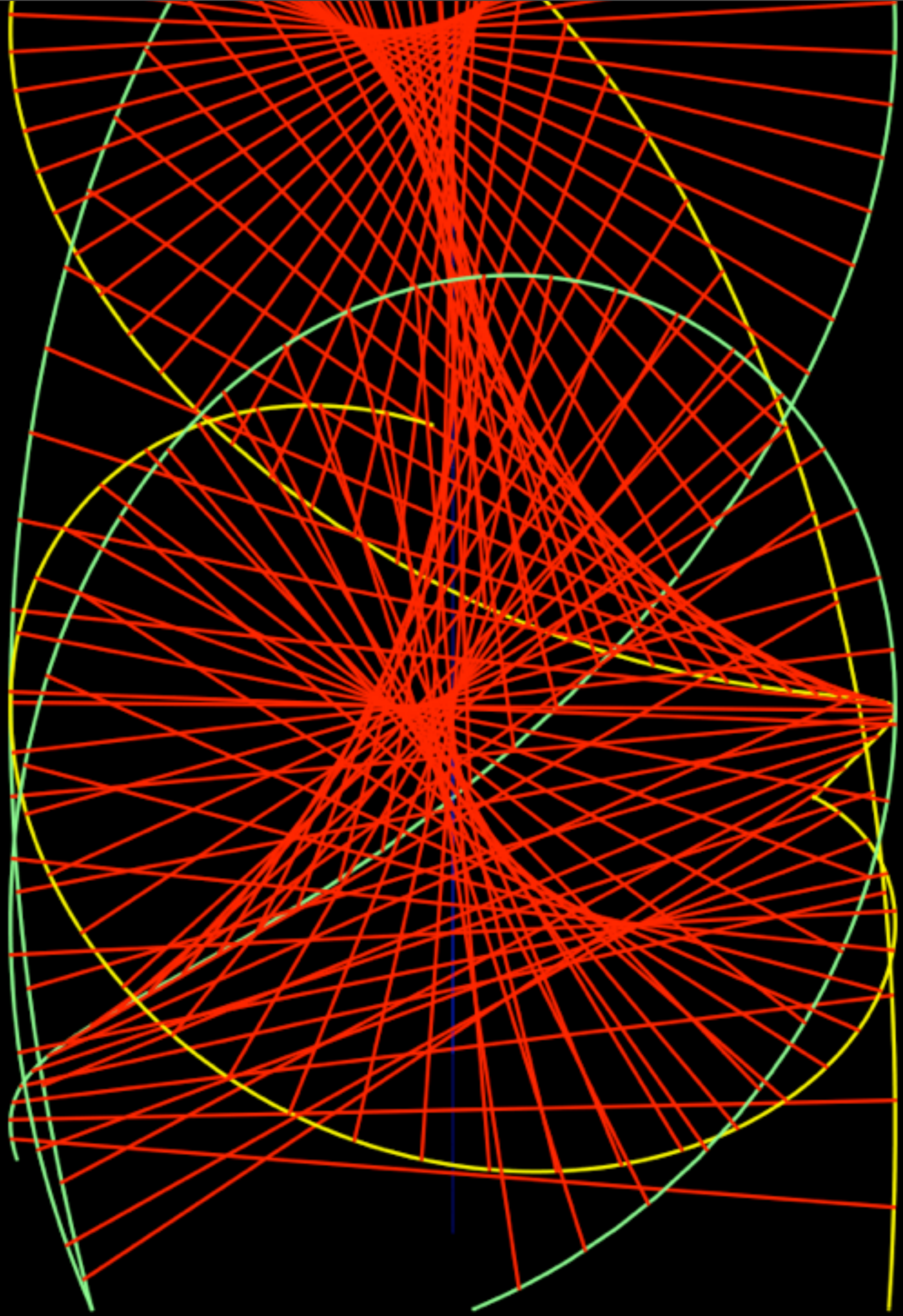


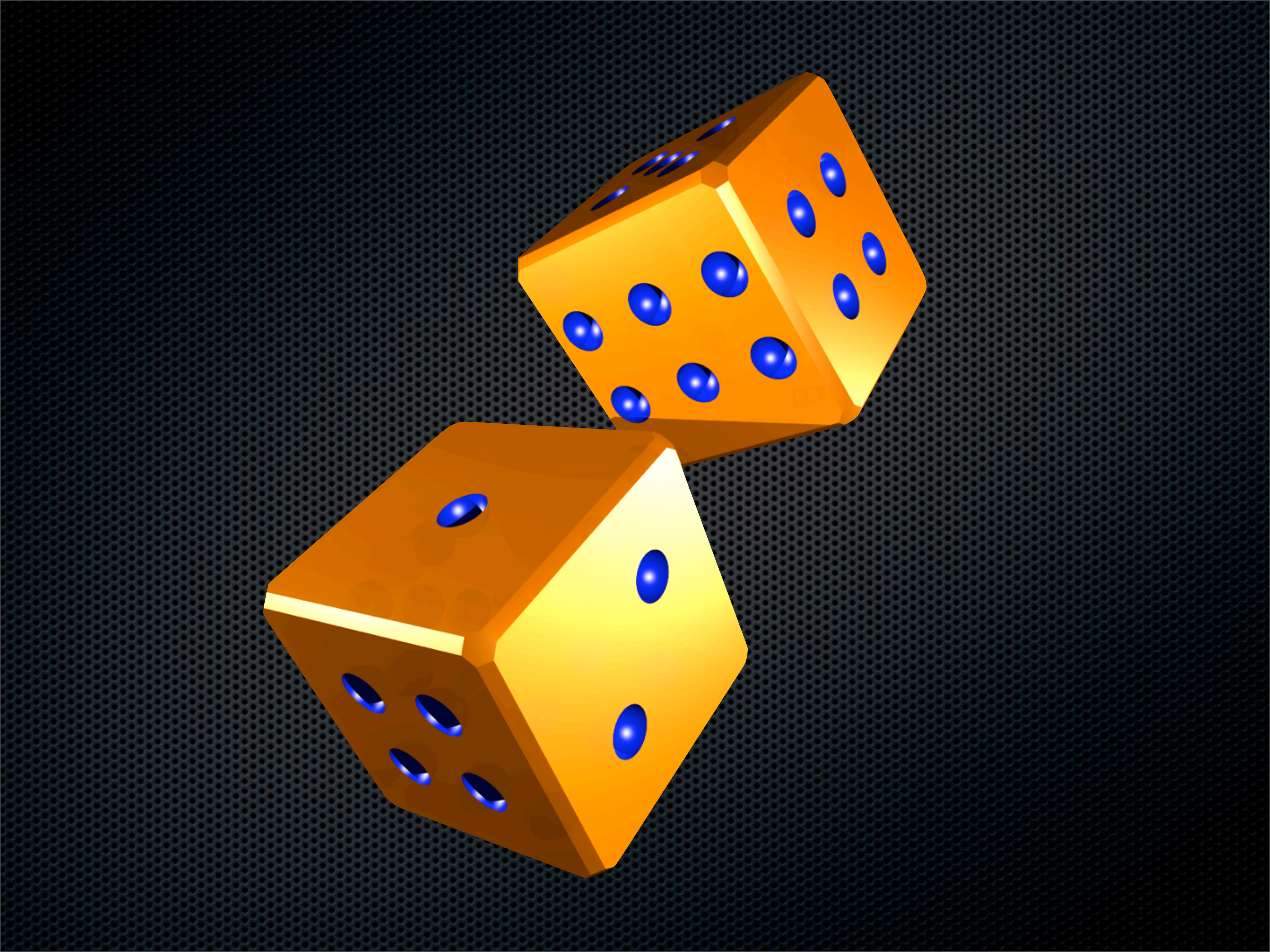


To falling coins



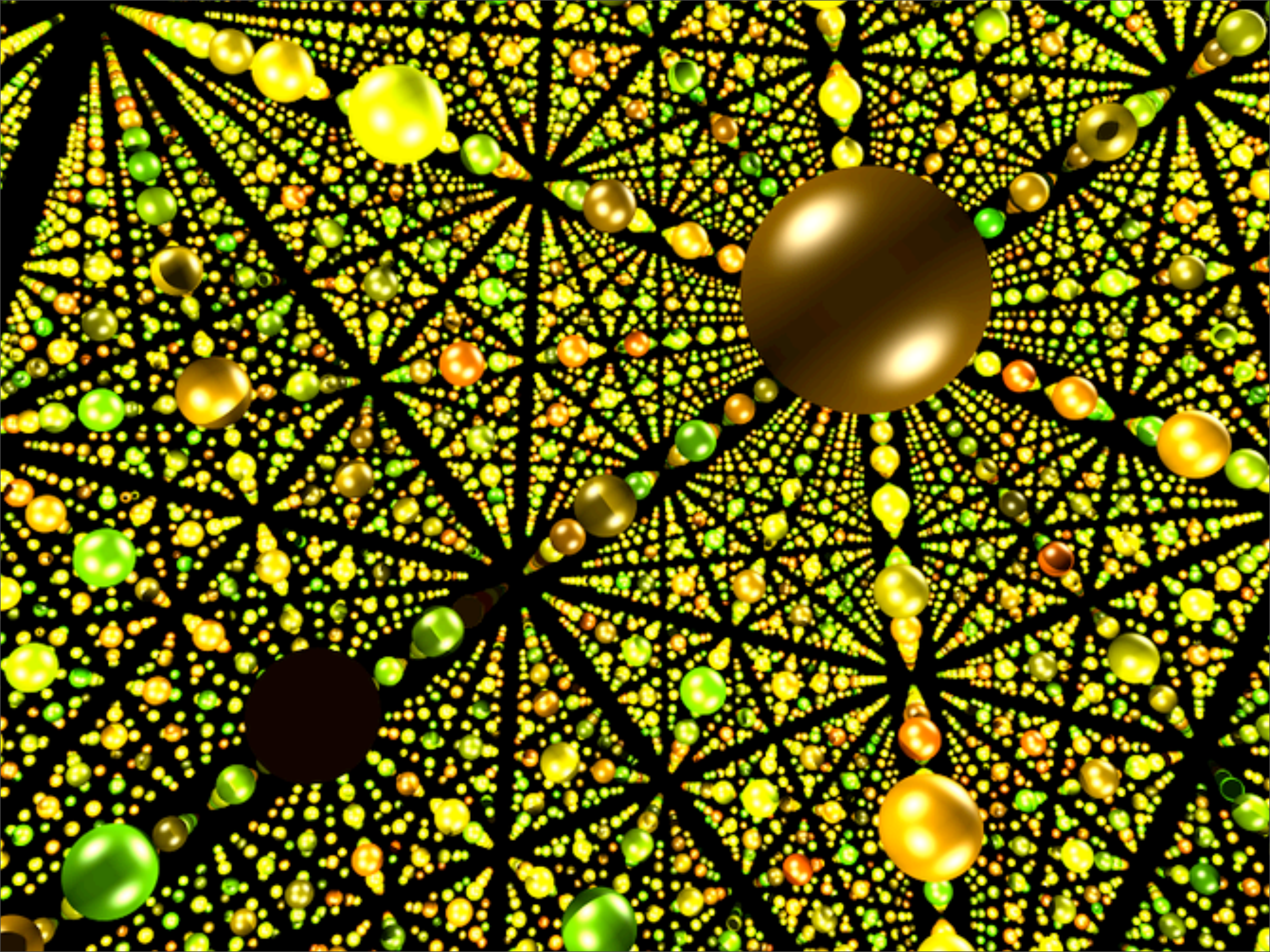


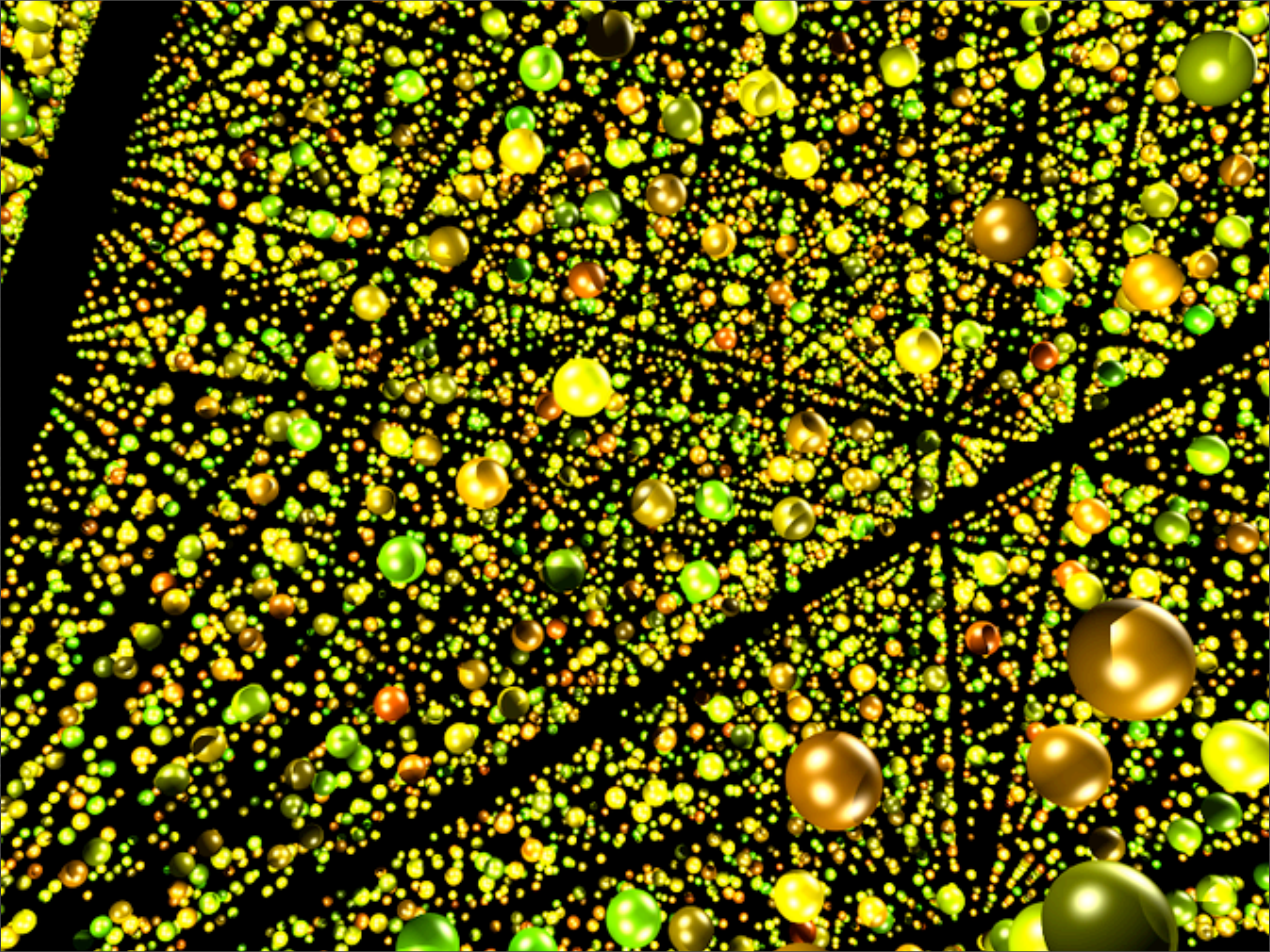


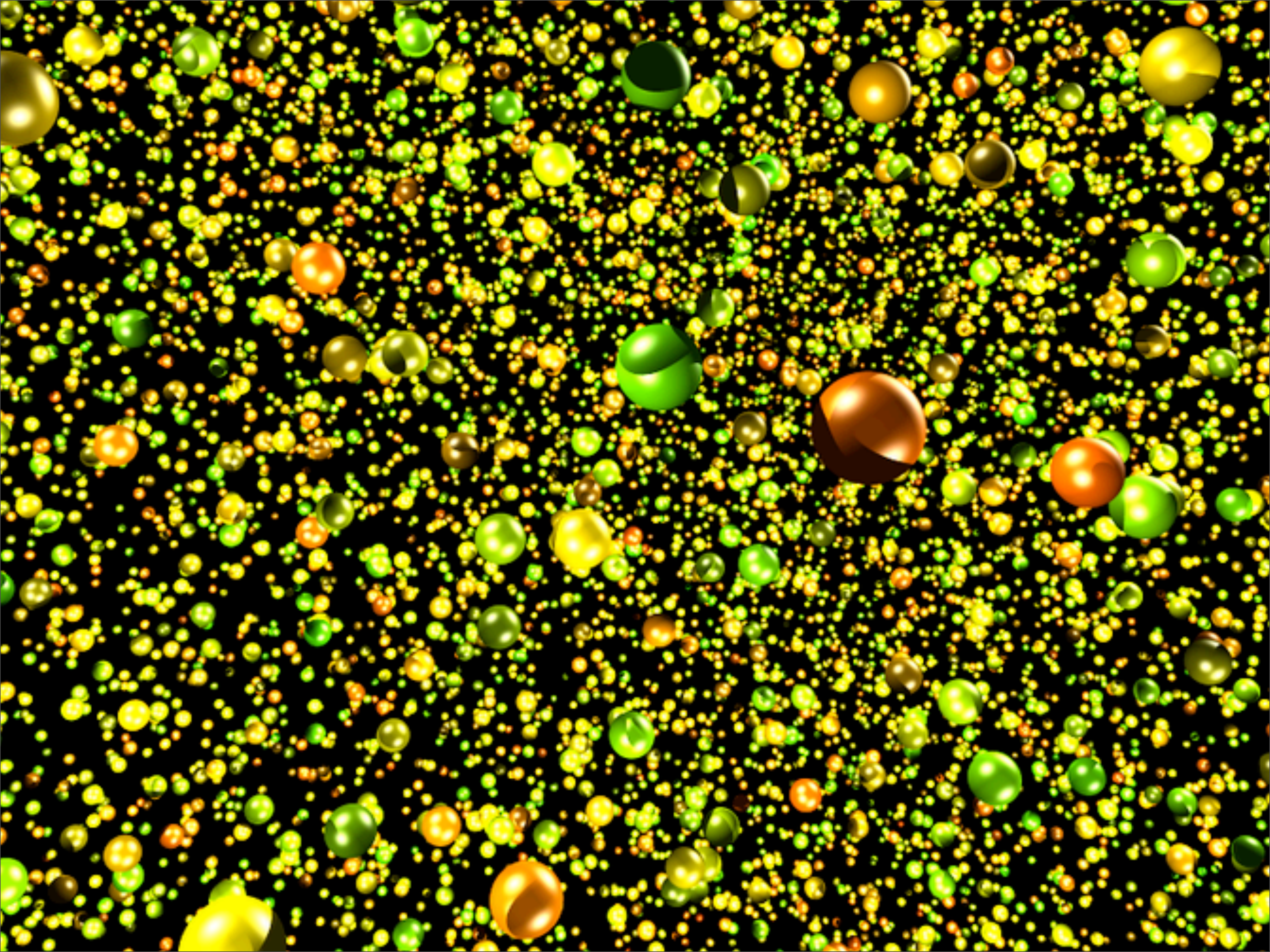


solid state physics: crystals

Order and
disorder







Sociology: spatial dynamics

Does it pay off to cooperate? The May-Novak CA

8	8	8	8	8
8	7	7	7	8
8	7	8b	7	8
8	7	7	7	8
8	8	8	8	8

every person changes to becomeas the guy with the best payoff in its neighborhood.

bad

good

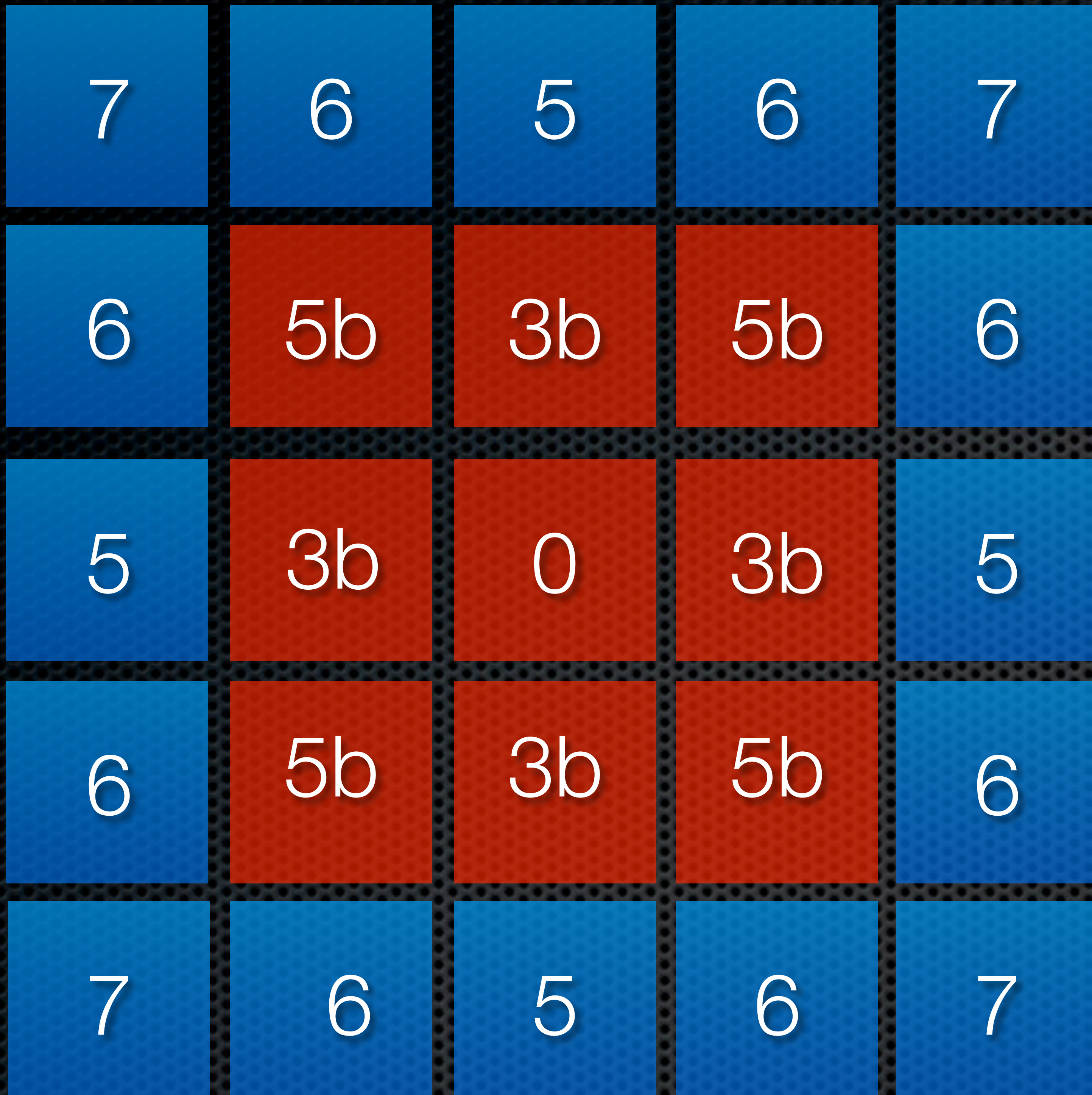
x: number of bad neighbors

y: number of good neighbors

$$\begin{vmatrix} 1 & 0 \\ b & 0 \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} p \\ q \end{vmatrix}$$

p: payoff for good guy

q: payoff for bad guy



bad

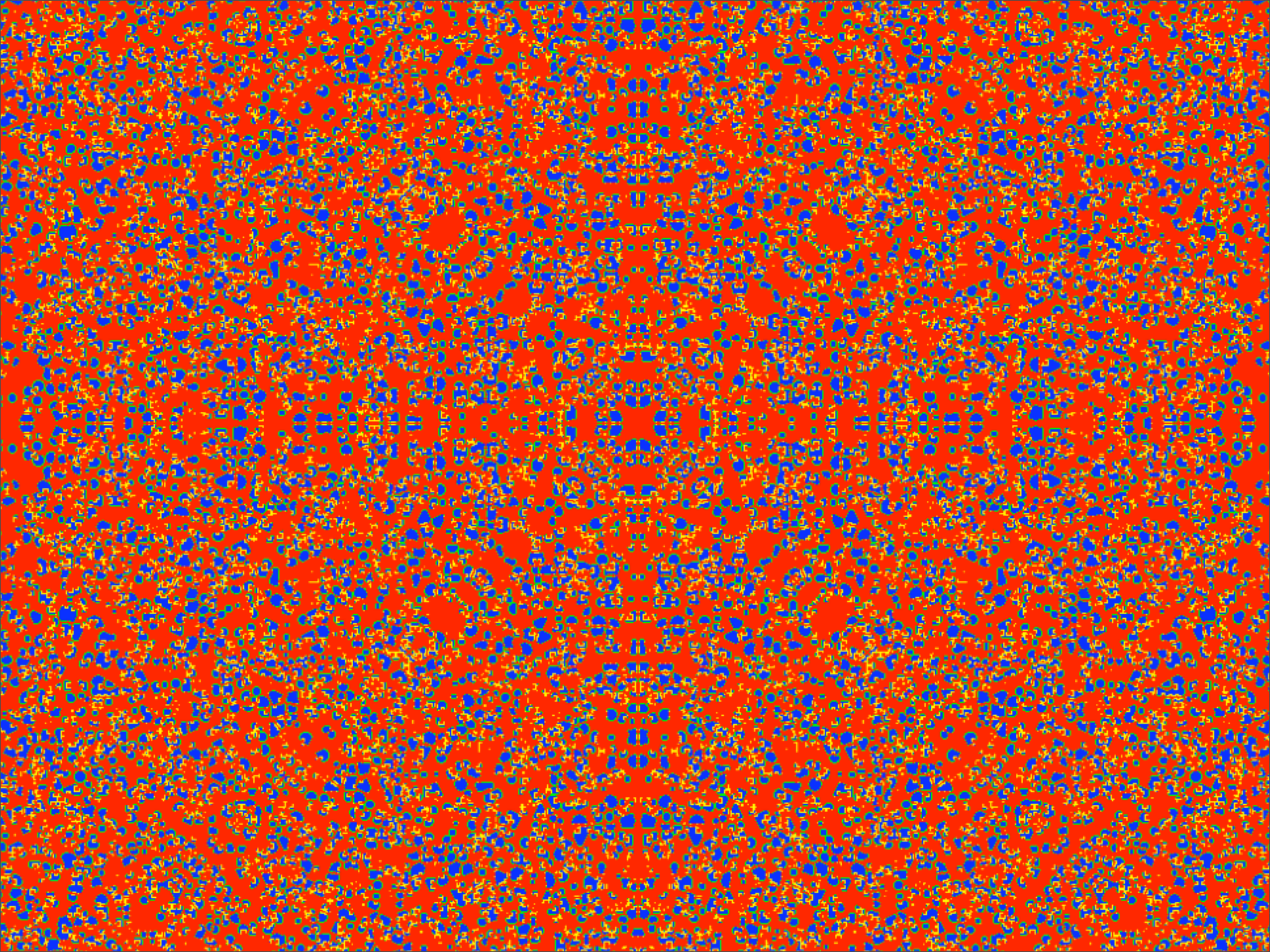
good

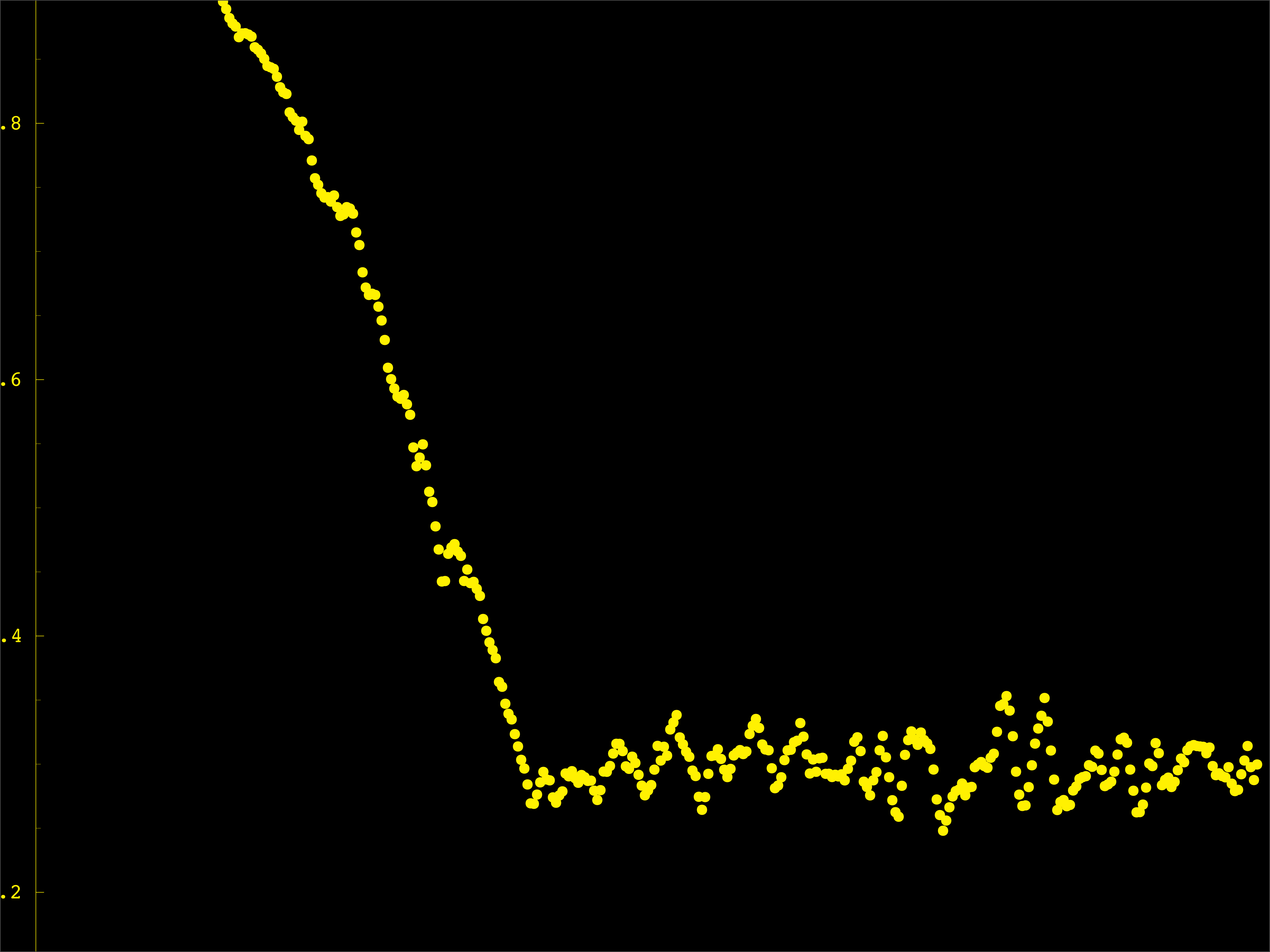
x: number of bad neighbors

y: number of good neighbors

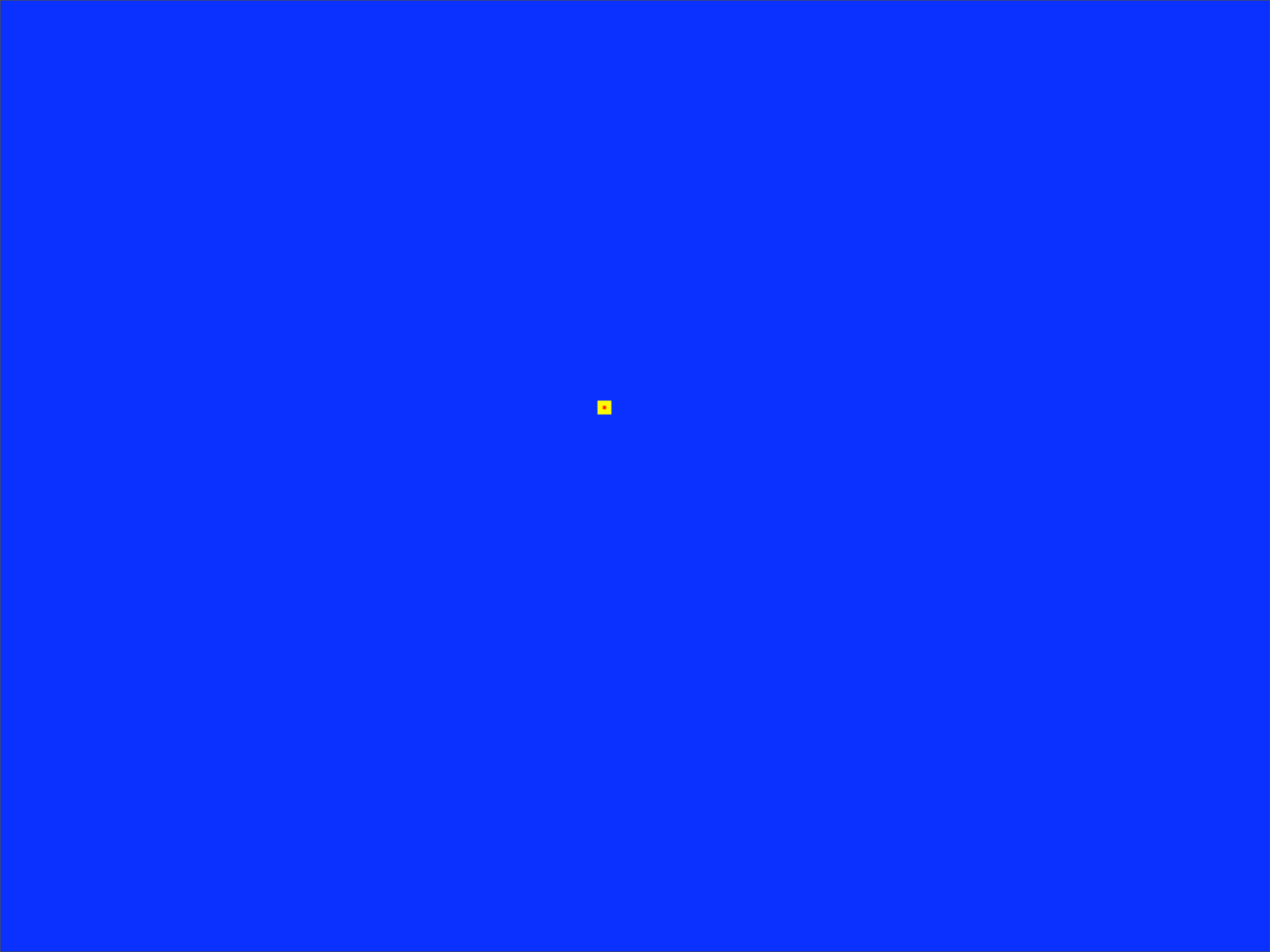
$$\begin{vmatrix} 1 & 0 \\ b & 0 \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} p \\ q \end{vmatrix}$$

for b>1, it has payed off to become bad in the neighborhood of the bad guy. We have a gang. Badness spreads

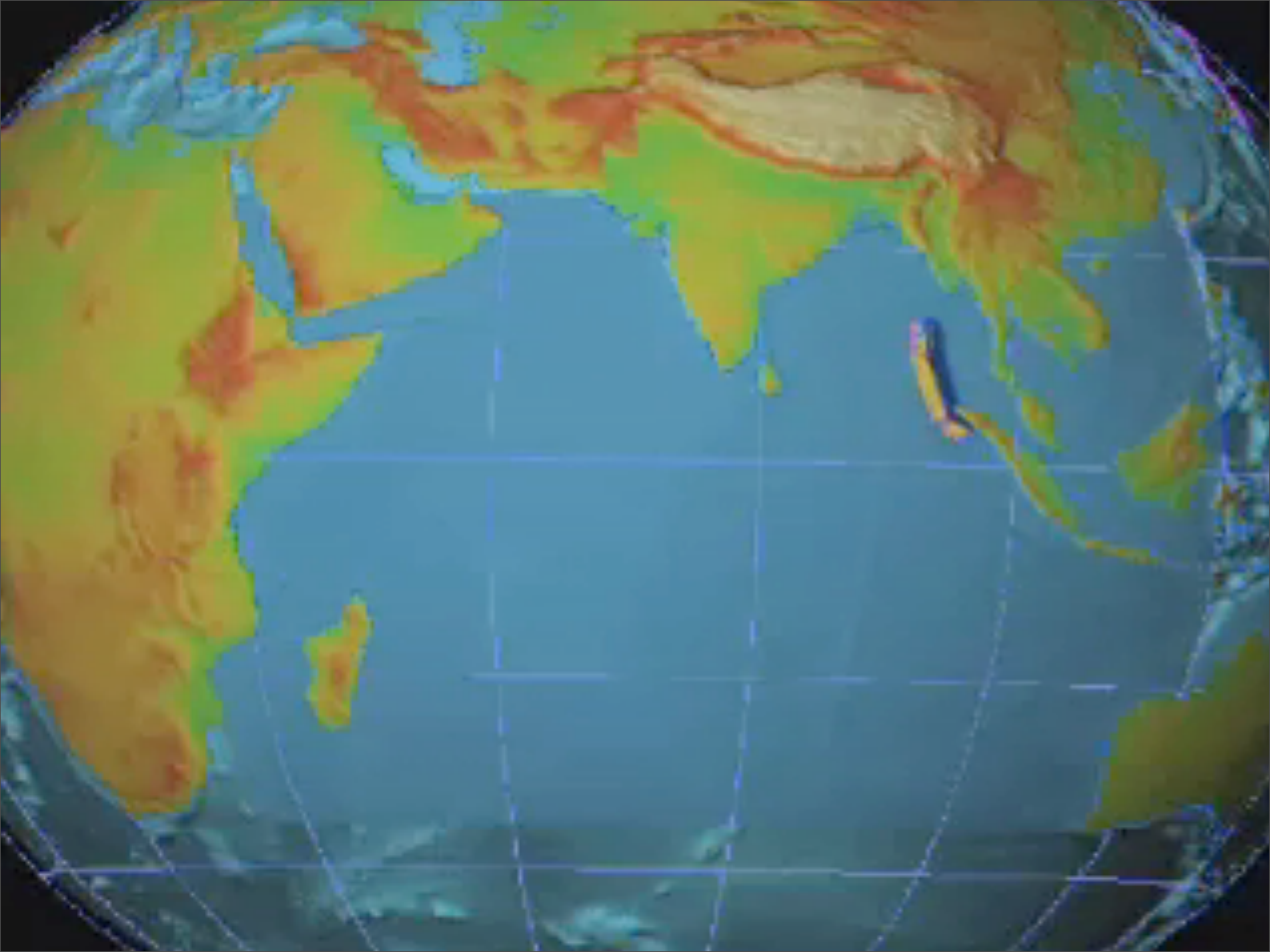



.8
.6
.4
.2

$$\begin{vmatrix} 1 & 1.1 \\ 1.65 & 0 \end{vmatrix}$$



Optics: wave front motion



Music: structure

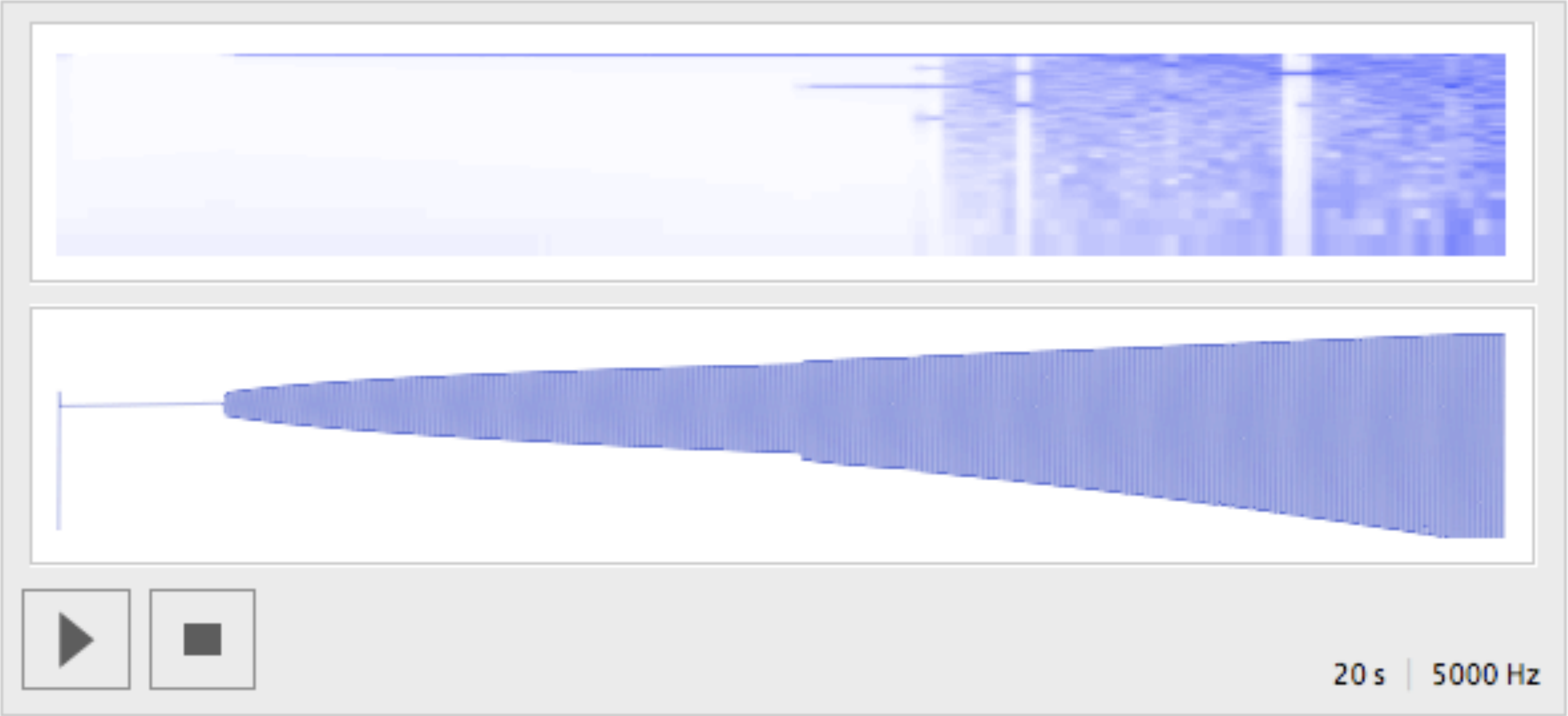
Dynamical systems to
generate or vary
harmonies.

Bifurcation diagram of the logistic map

In[31]:=

```
n=100000; f = Compile[{x,t},Evaluate[(2.9+1.1*t/n) x (1 - x)]];
ListPlay[FoldList[f, Random[], Range[n]], SampleRate -> 5000]
```

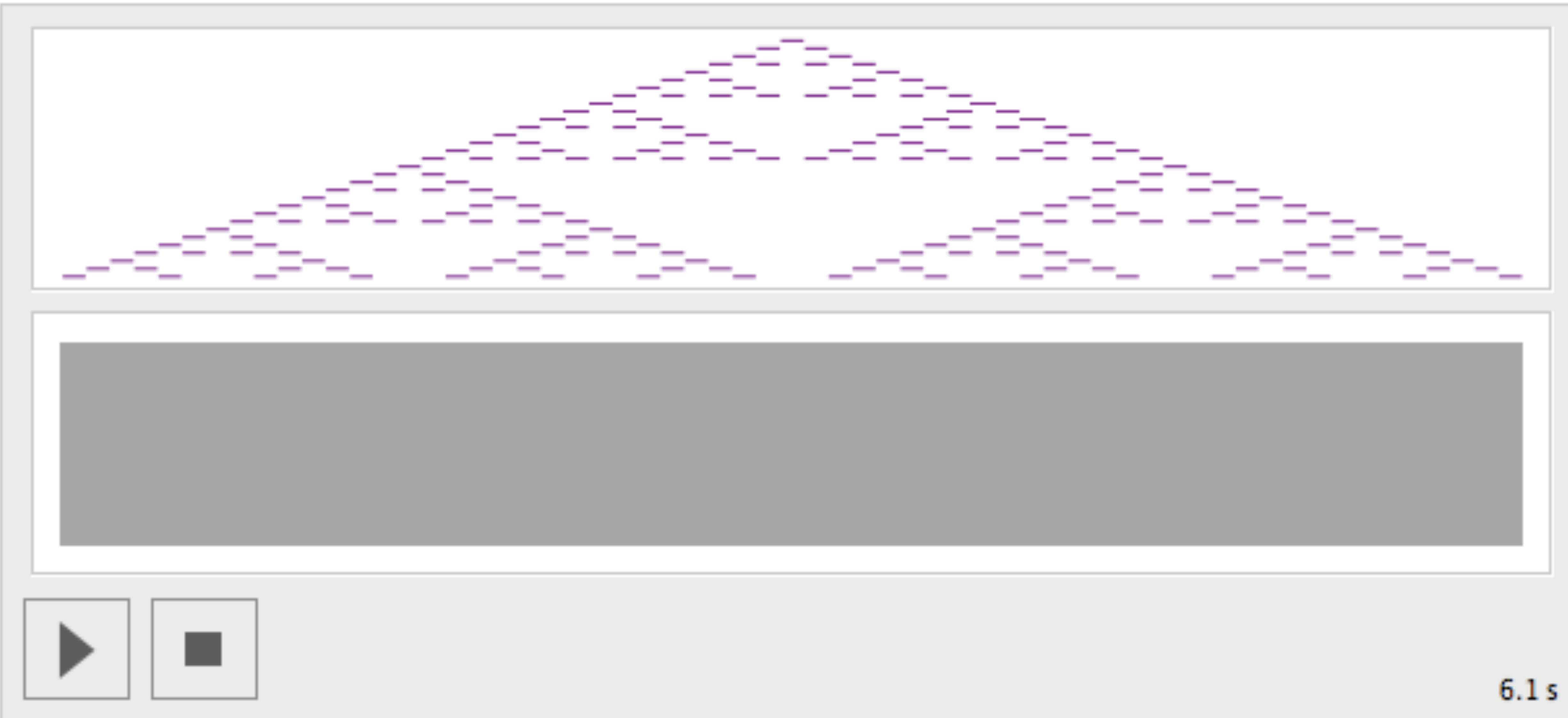
Out[32]=



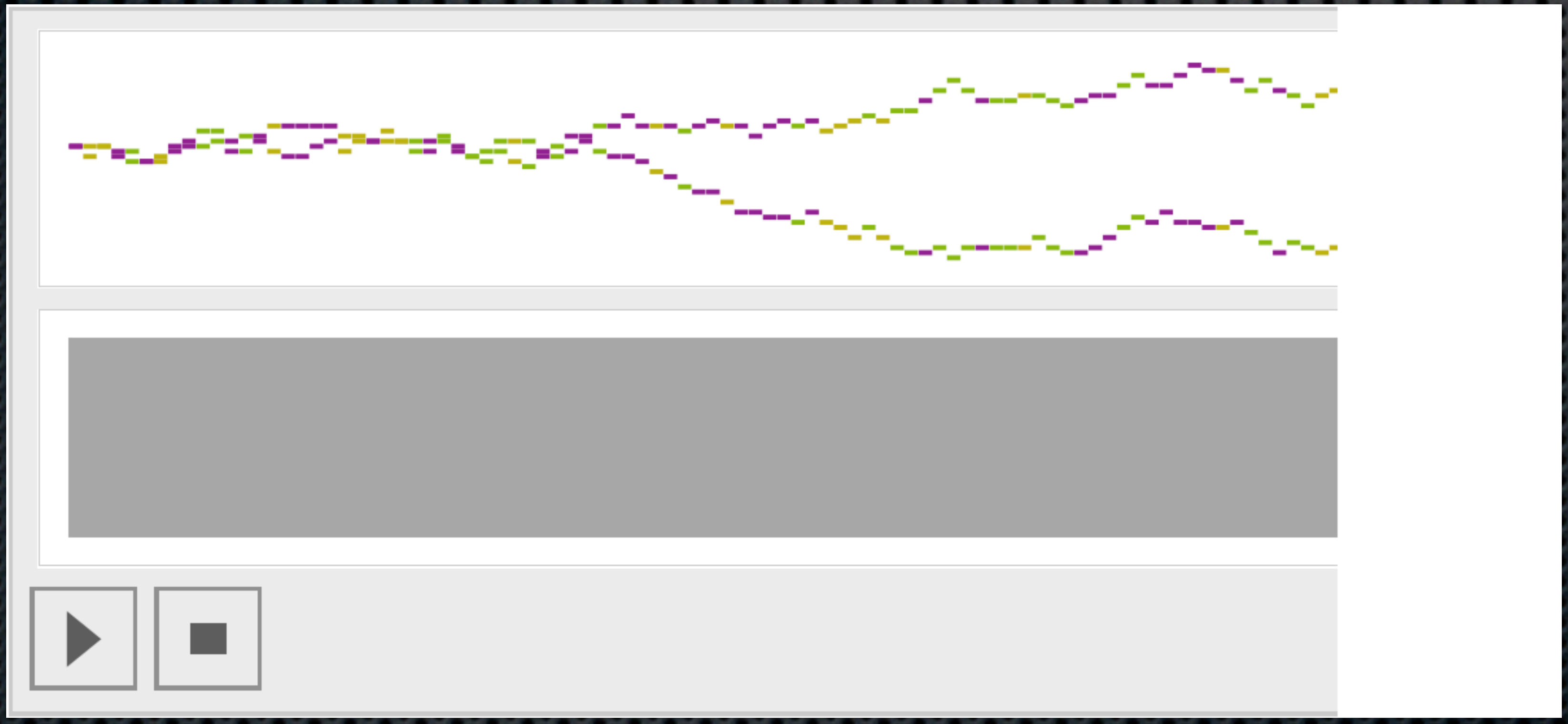
CellularAutomaton 90

In[35]:=
```
Sound[SoundNote[DeleteCases[3 Range[31] Reverse[#], 0] - 48, .1] & /@
  Transpose[CellularAutomaton[90, {{1}, 0}, 30]]]
```

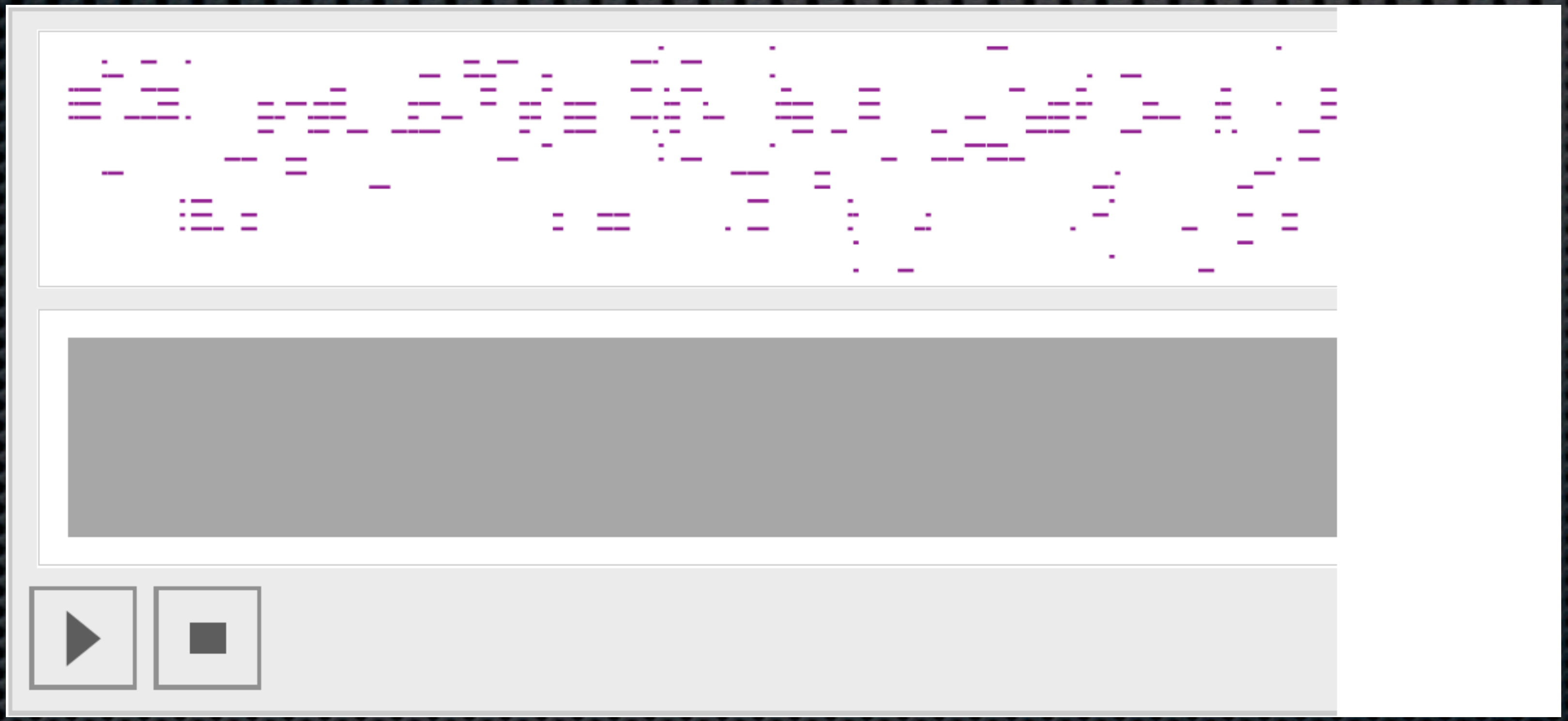
Out[35]=



random melodies



random accords



Phillipp Fischer

fractal music 2

inspired by Martin Gaertner and Diana Debby (Tufts)

Improvisation principle:

there are certain transitions which work, others which do not work. Just take a random choice of a good transition.

```
A = {(*c  c# d  d# e  f  f# g  g# a  a# h  C  C# D  D# E  F  F# G  G# A  A# H  *)
(*c*) {1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0},
(*c#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*d*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*d#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*e*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*f*) {1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0},
(*f#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*g*) {1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
(*g#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*a*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*a#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*h*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*C*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*C#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*D*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*D#*){1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
(*E*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*F*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*F#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*G*) {1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0},
(*G#*){0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
(*A*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
(*A#*){1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0},
(*H*) {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}};
AT = Transpose[A]; T[x_] := RandomChoice[Position[AT[[x]], 1]][[1]];
markovchain[m_] := NestList[T, 1, m];
cminor = {1, 4, 8, 13}; cmajor = {1, 5, 8, 13}; e = {1, 1, 1, 1}; dim = 4;
cisminor = cminor + e; cismajor = cmajor + e;
dminor   = cminor + 2 e; dmajor = cmajor + 2 e;
disminor = cminor + 3 e; dismajor = cmajor + 3 e;
eminor   = cminor + 4 e; emajor = cmajor + 4 e;
fminor   = cminor + 5 e; fmajor = cmajor + 5 e;
fisminor = cminor + 6 e; fismajor = cmajor + 6 e;
gminor   = cminor + 7 e; gmajor = cmajor + 7 e;
gisminor = cminor + 8 e; gismajor = cmajor + 8 e;
aminor   = cminor + 9 e; amajor = cmajor + 9 e;
aisminor = cminor + 10 e; aismajor = cmajor + 10 e;
hminor   = cminor + 11 e; hmajor = cmajor + 11 e;
minors   = {cminor, cisminor, dminor, disminor, eminor, fminor, fisminor, gminor, gisminor, aminor, aisminor, hminor};
majors   = {cmajor, cismajor, dmajor, dismajor, emajor, fmajor, fismajor, gmajor, gismajor, amajor, aismajor, hmajor};
accordlist = Flatten[{minors, majors}];
accords = Partition[accordlist, dim]; songlength = 30;
sn[{s_, t_}] := SoundNote[s, t]; chain = markovchain[songlength];
aa1 =  Table[sn[{accords[[chain[[k]]]], 1}], {k, songlength-1}];
uu2 =  Flatten[Table[{RandomSample[accords[[chain[[k]]]]], RandomSample[accords[[chain[[k]]]]]},{k,songlength}]];
aa2 =  Table[sn[{uu2[[l]],1/(2 dim)}],{l,Length[uu2]}]
uu3 =  Flatten[Table[RandomSample[accords[[chain[[k]]]]],{k,songlength}]];
aa3 =  Table[sn[{uu3[[l]],1/dim}],{l,Length[uu3]}]

markovsong1 := Sound[Prepend[aa1, "Piano"]];  ss1 = markovsong1;
markovsong2 := Sound[Prepend[aa2, "Harp"]]; ss2 = markovsong2;
markovsong3 := Sound[Prepend[aa3, "Cello"]]; ss3 = markovsong3;
Export["niceaccords1.midi", ss1, "MIDI"]
Export["niceaccords2.midi", ss2, "MIDI"]
Export["niceaccords3.midi", ss3, "MIDI"]
~
```

Transition matrix

This is a Markov chain. Lets try that.

Mathematica program which generated this

Musical variations from a chaotic mapping

Diana S. Dabby
Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 27 March 1995; accepted for publication 15 February 1996)

A chaotic mapping provides a technique for generating musical variations of an original work. This technique, based on the sensitivity of chaotic trajectories to initial conditions, produces changes in the pitch sequence of a piece. A sequence of musical pitches $\{p_i\}$, i.e., any piece ranging from Bach (or earlier) to contemporary music, is paired with the x-components $\{x_i\}$ of a Lorenz chaotic trajectory. Each p_i is marked on the x axis at the point designated by its x_i. In this way, the x axis becomes a pitch axis configured according to the notes of the original composition. Then, a second chaotic trajectory, whose initial condition differs from the first, is launched. Its x-components trigger pitches on the pitch axis (via the mapping) that vary in sequence from the original work, thus creating a variation. There are virtually an unlimited number of variations possible, many appealing to expert and nonexpert alike. © *1996 American Institute of Physics.* [S1054-1500(96)00502-7]

now at Tufts

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Art intersects science: Dabby creates music out of chaos

March 18, 1998

Fifteen years ago, Diana Dabby was enjoying a flourishing career as a pianist and composer, performing in New York's Weill (Carnegie) Recital Hall and abroad. Then one day she found a series of articles on computer music -- all written by mathematicians, computer scientists or electrical engineers -- which led her back to college and into the field of engineering. In 1987 she became a graduate student in MIT's Department of Electrical Engineering and Computer Science (EECS) and in 1995 received the PhD for her thesis, "Musical Variations from a Chaotic Mapping."

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requiring some intrinsic

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analyses given in Secs.
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the same spirit as com-
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netimes an inversion is
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der no obligation to use
the variation technique.
d, or rejected. The artist

Poetry:

Random words
Words with correlations
Real poems

When is a poem good?

Two poems. Can you find the author?

O mistress mine, where are you roaming?
O stay and hear! your true-love's coming
That can sing both high and low;
Trip no further, pretty sweeting,
Journey's end in lovers' meeting--
Every wise man's son doth know.
What is love? This not hereafter;
Present mirth hath present laughter;
What's to come is still unsure:
In delay there lies no plenty,--
Then come kiss me, Sweet and twenty,
Youth's a stuff will not endure.

the mellow rose gave us the graceful misery
a happy flower gave you a burning desire
you desired a sad girl
she cried kindly
the rose danced
the delicate flower felt me
a mellow girl fondled us passionately
a sweet despair died from the spiritless girl
a spiritless kiss surrendered passionately
the nice girl danced burning

ADAM

```
(*Mathematica by O. Knill, adapted from Java program by Nandy Millan  *)

Adjectives={placid,mellow,nice,sad,happy,noisy,cold,warm,beautiful,delicate,
sweet,soft,graceful,pretty,captivating,desiring,languishing,fragile,exquisite,
graceful,deafening,spiritless,heartbroken,stormy,carnal,frenetic,burning};
Adverbs={soon,late,desperately,passionately,slowly,softly,carefully,tenderly,gently,quickly,kindly};
Articles={the,a}; Ditverbs={gave,told}; Opronouns={me,you,him,her,us,them};Spronouns={I,you,he,she,we};
Itverbs={cried,fell,died,came,anguished,surrendered,shouted,whispered,danced,dreamed};
Nouns={heart,girl,mouth,love,lover,flower,sun,breeze,rose,kiss,misery,despair,
passion,desire,warmth,fire,flame,fantasy};
Preps={from,for,with,in} Pronouns={I,you,he,she,it,we,they,me,him,her,us,them};
Tverbs={loved,kissed,touched,felt,desired,cuddled,fondled,enjoyed}
Verbs={loved,escaped,sang,cried,hoped,fell,died,came,slept,shouted,kissed,touched,felt,
whispered,danced,desired,dreamed}

Adjective:=RandomChoice[Adjectives];Adverb:=RandomChoice[Adverbs];Ditverb:=RandomChoice[Ditverbs];
Itverb:=RandomChoice[Itverbs];Noun:=RandomChoice[Nouns];Opronoun:=RandomChoice[Opronouns];
Prep:=RandomChoice[Preps];Pronoun:=RandomChoice[Pronouns];Spronoun:=RandomChoice[Spronouns];
OPronoun:=RandomChoice[OPronouns];Tverb:=RandomChoice[Tverbs];Verb:=RandomChoice[Verbs];
Article=RandomChoice[Articles]; Subject:=RandomChoice[{{Article,Noun},{Spronoun},{Article,Adjective,Noun}}];
Predicatelist:={{Adverb},{Prep,Article,Adjective,Noun},{Prep,Opronoun},{Article,Adjective,Noun},
{Opronoun},{Article,Adjective,Noun,Adverb},{Opronoun,Adverb},{Opronoun,Article,Adjective,Noun}};
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Predicate:=RandomChoice[Table[{Verblist[[i]],Predicatelist[[i]]},{i,Length[Verblist]}]];
Object:=RandomChoice[{{},{Adverb},{Subject},{Opronoun},{Prepsubject}}];
Verbobject:={Verb,Object}; Prepsubject:={Prep,Subject}; Subjectpredicate:={Subject,Predicate};
Continuation:=RandomChoice[{{},{"and",Subjectpredicate}}];
Sentence:=Flatten[RandomChoice[{{Subject,Verbobject,Continuation}}]]; Poem:=Do[Print[Sentence],{10}];
Poem
```
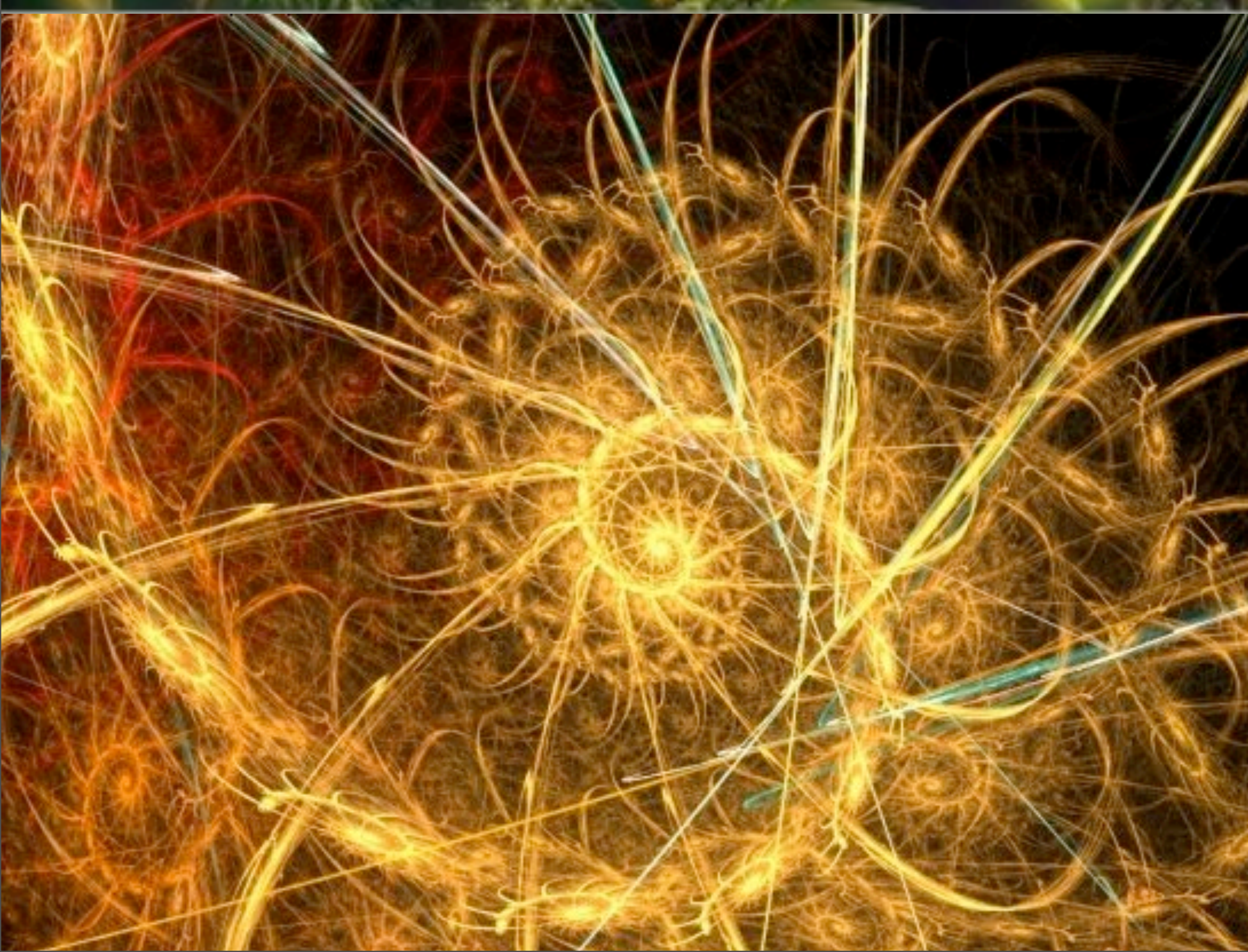
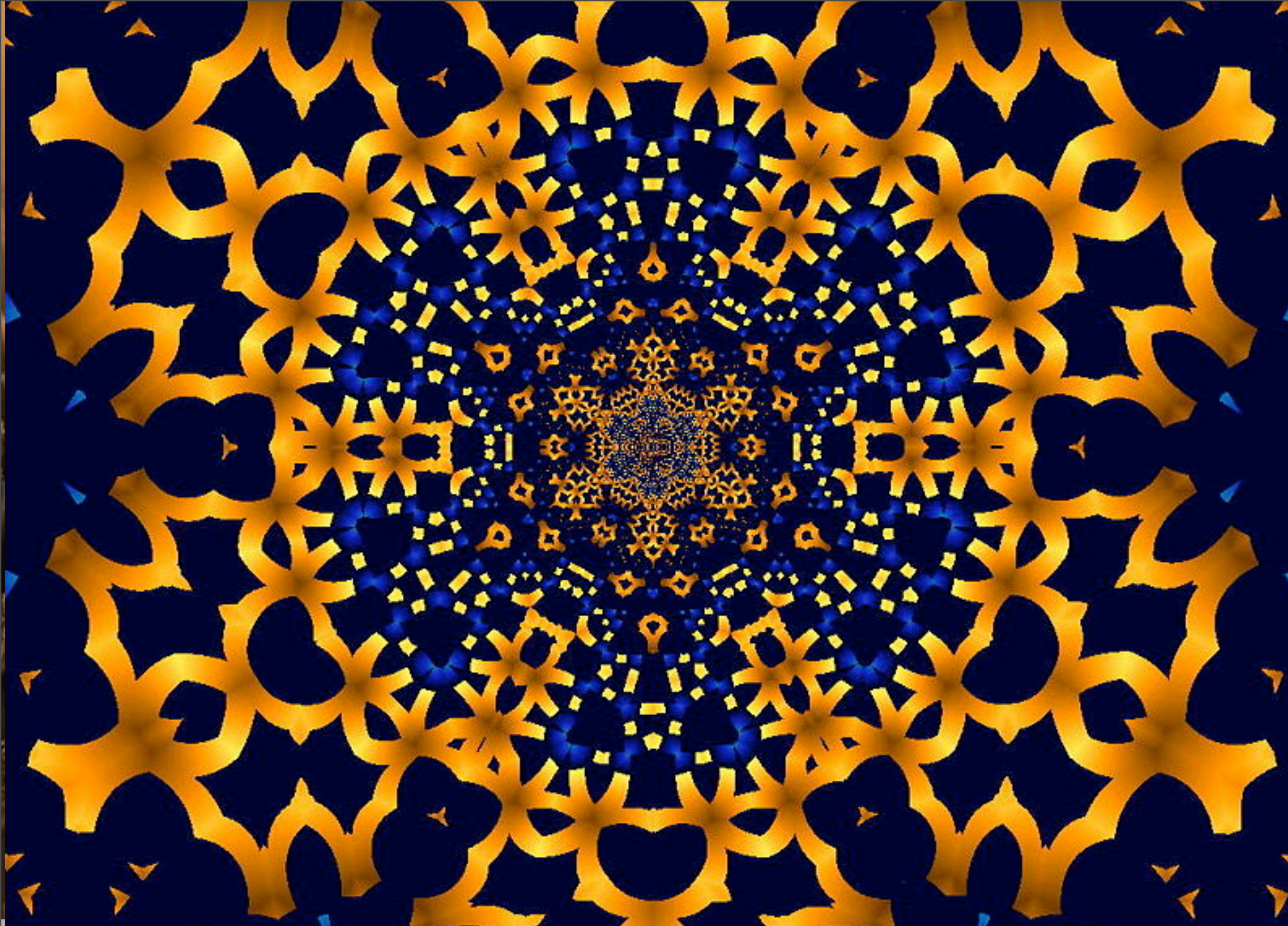

Fractal Art

generating
“paintings”






Market:


UDA
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'Father of Fractals' takes on the stock market

Seeing a snowflake pattern in the NASDAQ

Katharine Stoel Gammon, News Office Correspondent
November 16, 2006

Benoit Mandelbrot is world-famous for making mathematical sense of irregular shapes--clouds that are not round, mountains that are not cones, coastlines that are not smooth, and now, stock markets that are not as simple as previously thought.

Mandelbrot, known as the "father of fractal geometry," spoke Nov. 8 to a crowd of more than 200 people in Room 10-250. His talk was the first in a series sponsored by the MIT Molecular Frontiers Club. Molecular Frontiers is a new international alliance of scientists, including several at MIT, intended to inspire young people to get involved in science. The title of Mandelbrot's lecture was "The Mandelbrot Set and Fractals in Finance."

Mandelbrot coined the term "fractals" in 1975 to describe shapes that appear similar at all levels of magnification and are also called "infinitely complex." Examples of fractal-like structures in nature include snowflakes, rivers, broccoli flowers and systems of blood vessels.

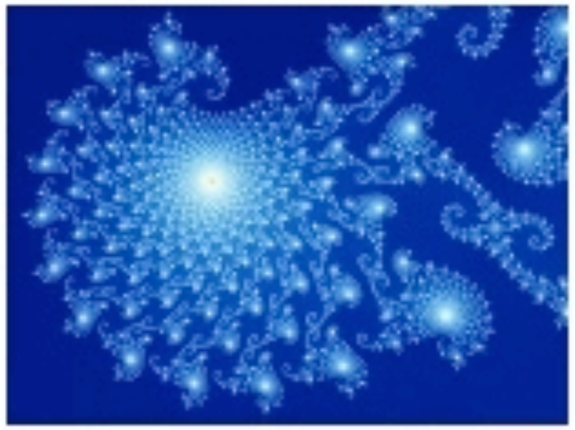


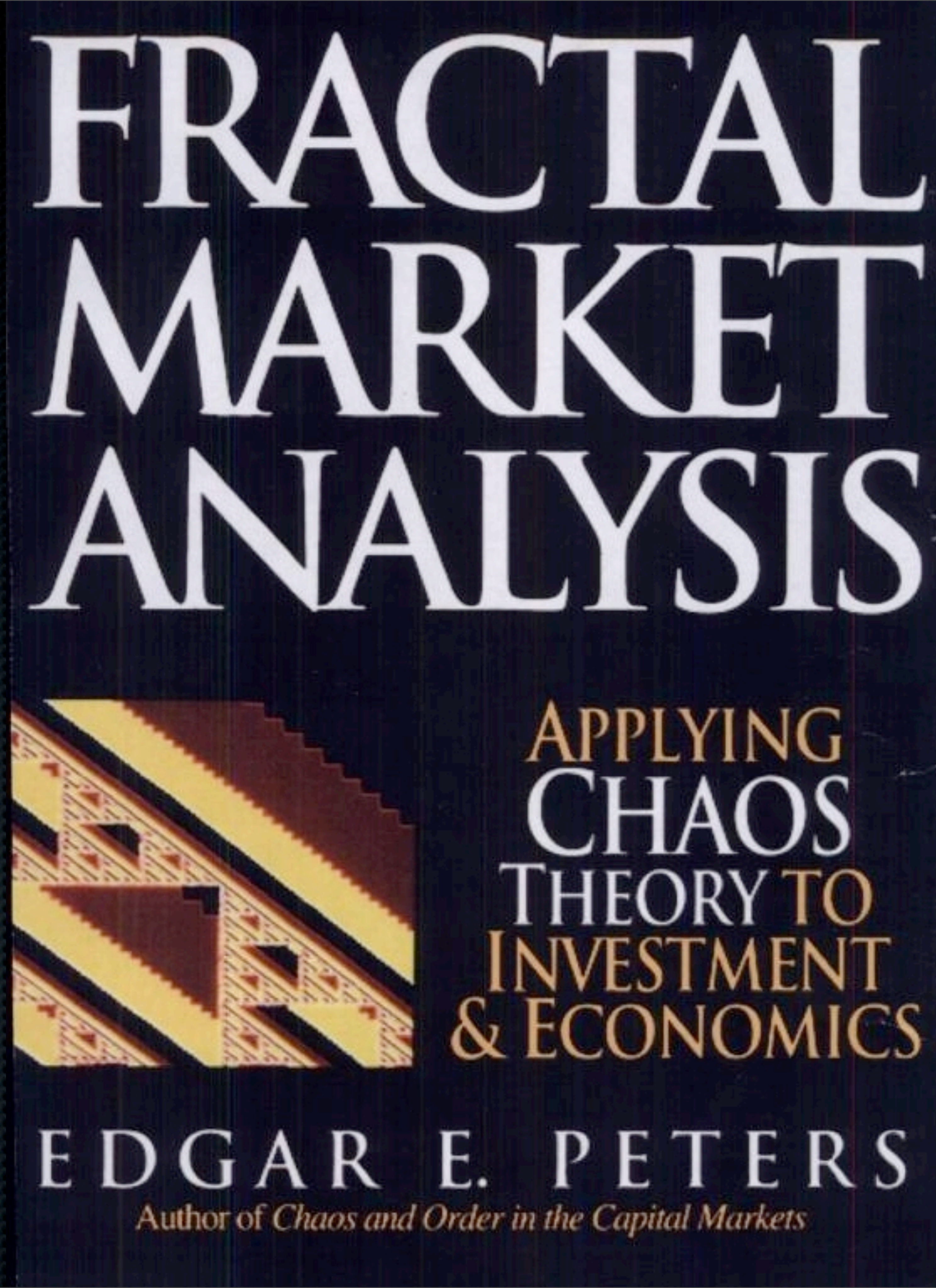
Image / Wolfgang Beyer

Partial view of the Mandelbrot set fractal.
Enlarge image

Photo courtesy / Yale Mathematics Department

Benoit B. Mandelbrot

TOOLS



FMH: fractal market hypothesis.

Instead of Brownian motion (independence) have correlations. Fractal dimension as a measure of volatility (standard deviation of security prize change.


BASED ON AN ORIGINAL IDEA BY
Brian Price and Mike Betar

Choreography

generating
motion from dynamical
systems

Using Chaos to Generate Variations on Movement Sequences

Elizabeth Bradley and Joshua Stuart[1]

Department of Computer Science
University of Colorado
Boulder, Colorado, USA 80309-0430
[lizb,stuartj]@cs.colorado.edu

Chaos, **8**:800-807 (1998)

Abstract

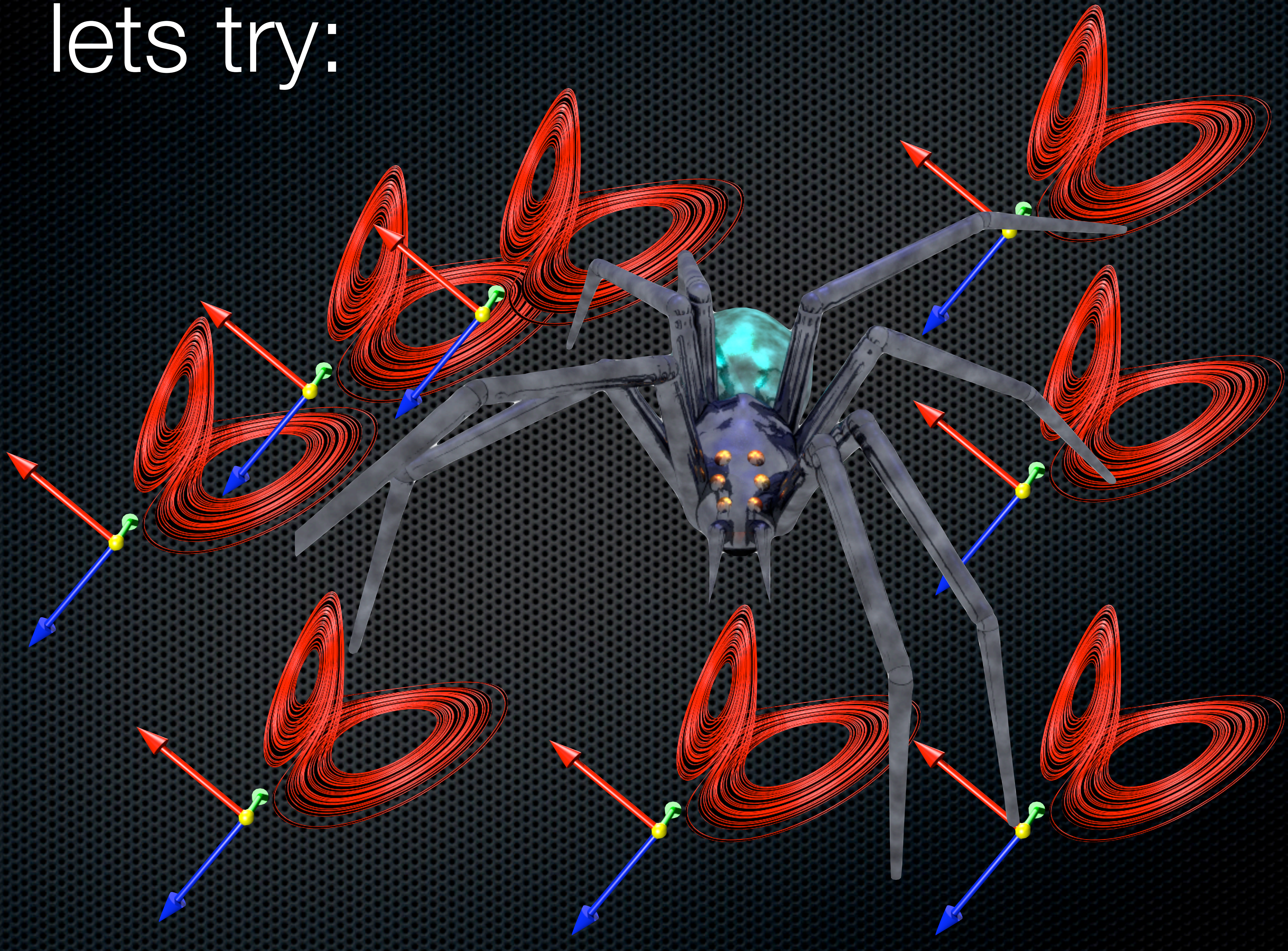
Josh Stewart, UC Santa Cruz

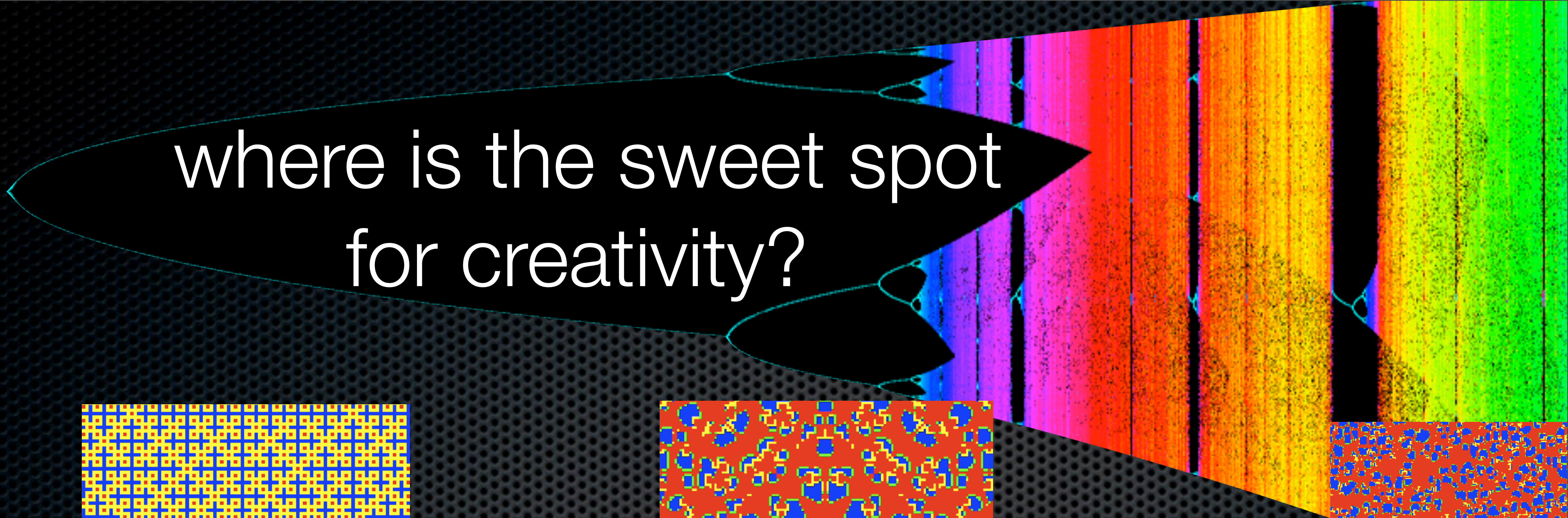
Using Lorentz attractor

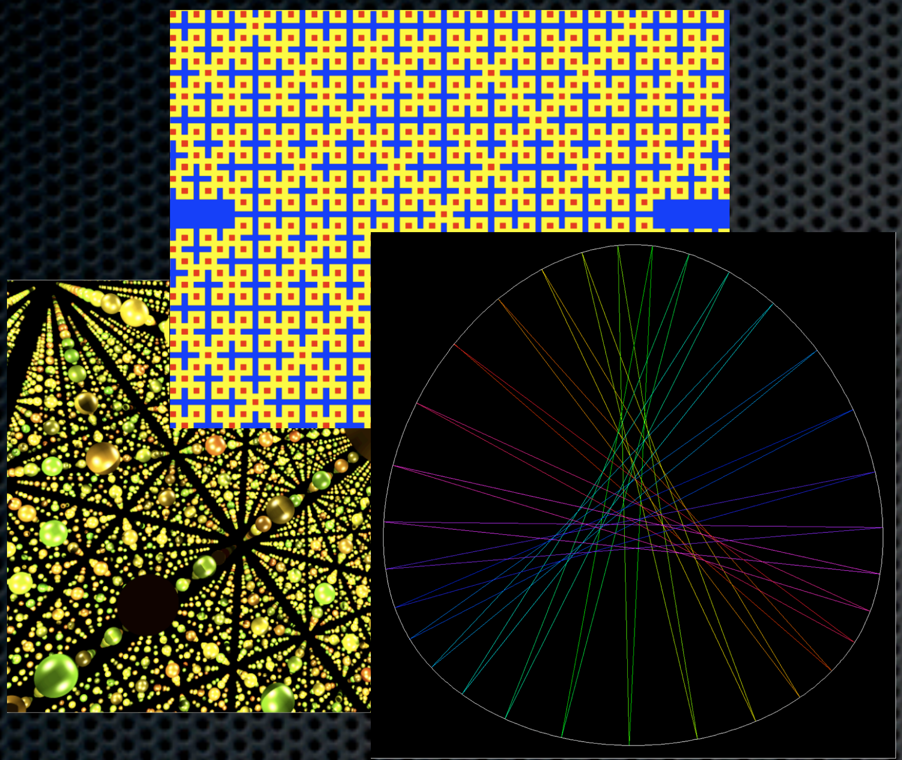
Using Roessler attractor

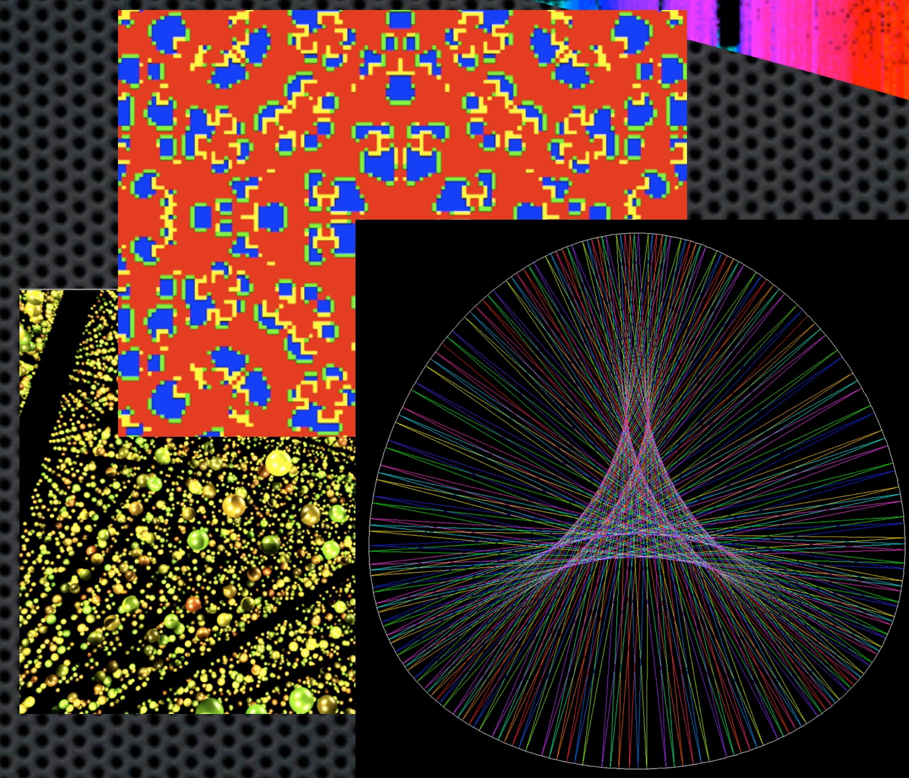
What algorithm was used to generate these moves?

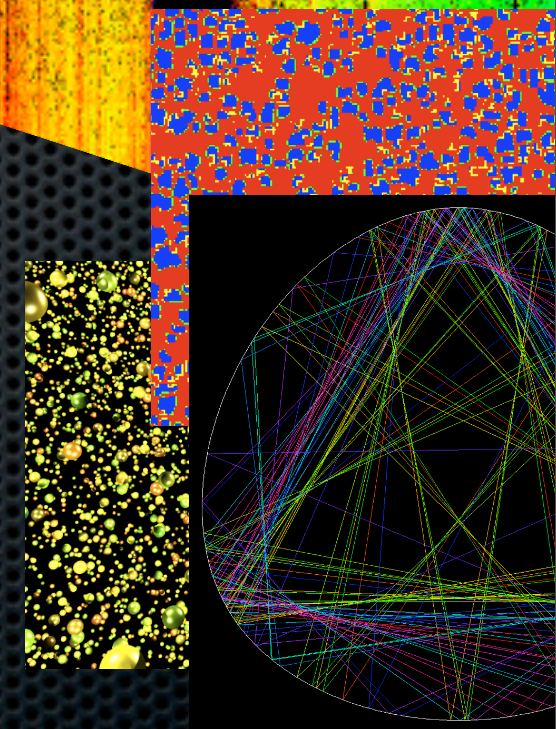
lets try:











periodic
integrable
boring

correlations
structure and
surprise

random
erratic
boring

Credits

- CAS: mathematica
- raytracing: povray
- spider uses povray code of Rafael Ghiglia, 2001
- mandelbrot: xaos fractal zoomer with additional scripts
- choreography sequence: bradley and Stuart (Colorado)
- trading sequence: Movie: “Trading Places”
- chaos sequence: Movie: “Jurassic Park”
- movie scene from “the bank”
- movie scene from “Grease”
- primes on Ulam spiral: movie “Code conspiracy”
- cellular automaton accord: wolfram research
- Blackhole merger: NASA
- Poem by William Shaespear
- Poem generator A.D.A.M. by Nandy Millar
- fractal paintings: Kelly Dietrich
- 3 body problem picture by Mathew Holman and Joe Christy in forword by D. Goroff to Poincare’s “Nouvelle Methods”.