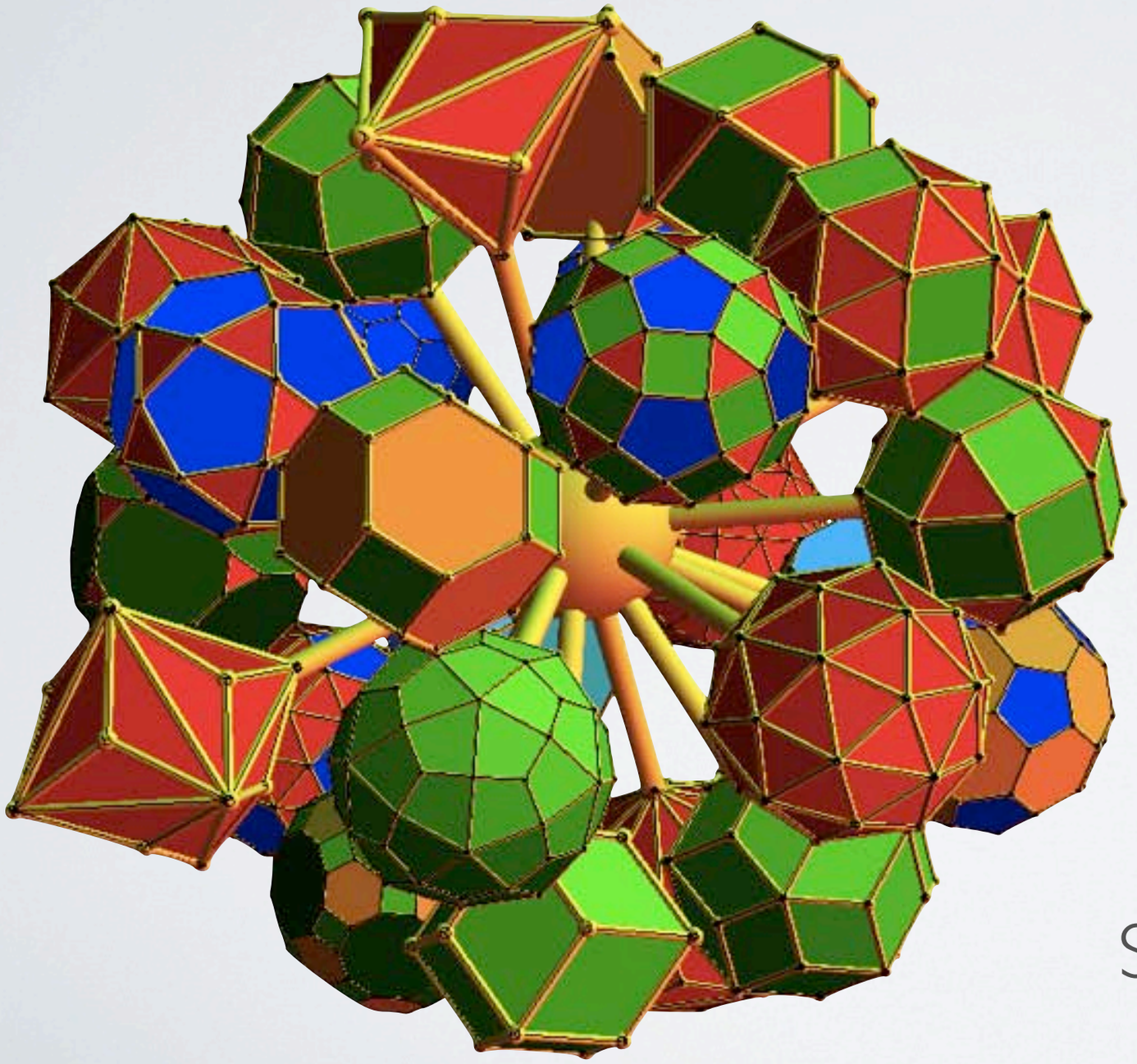
FROM ARCHIMEDES TO 3D PRINTING



Oliver Knill
Harvard University

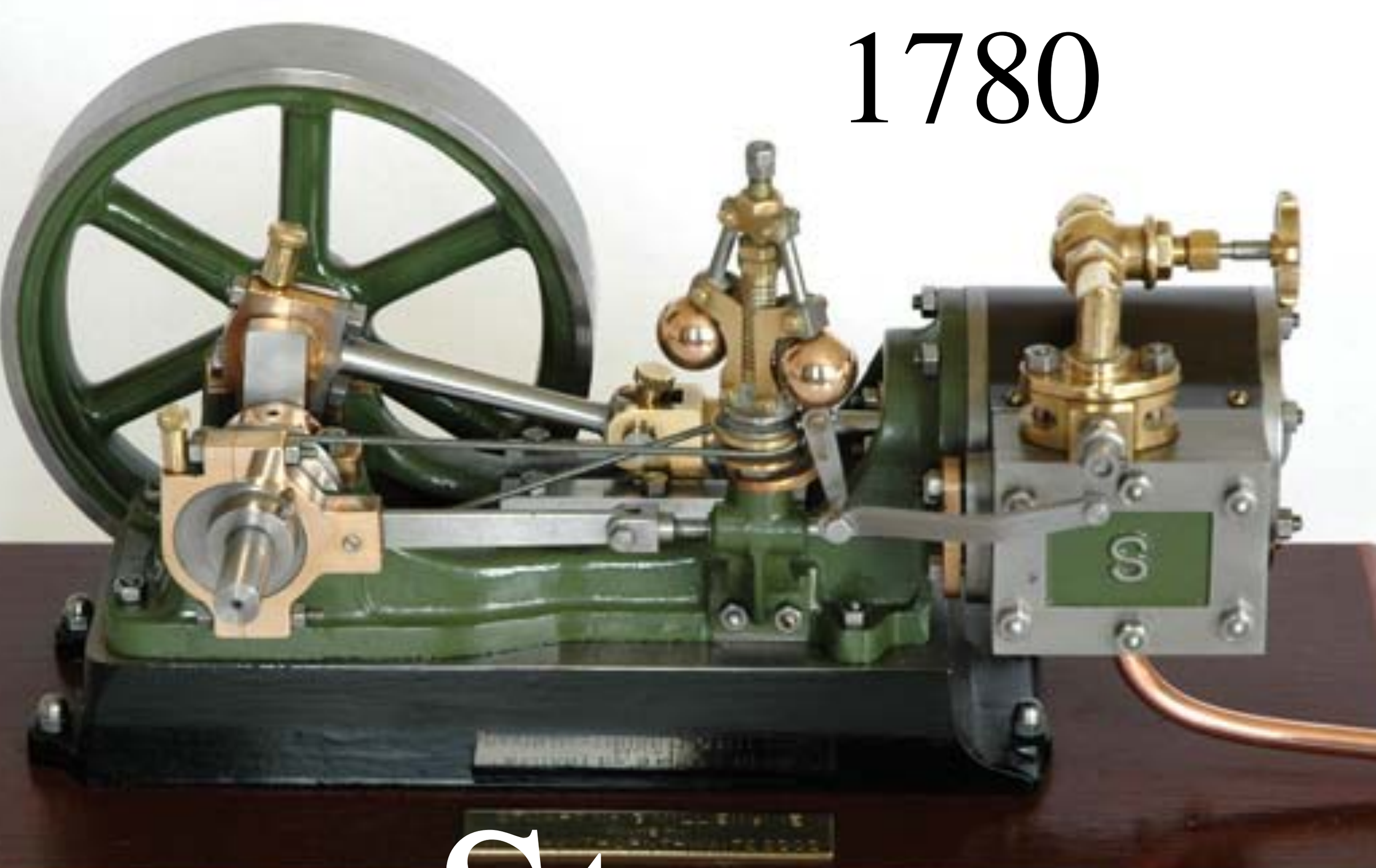
Science Initiative Competition,
Boston, 2013

REVOLUTIONS

INDUSTRIAL

Steam Engine, Textile	1780
Automotive, Chemistry	1850
PC, Space 3D Print	1970

1780



Steam

1880
Steel

1980

Stars

Nasa: Horsehead

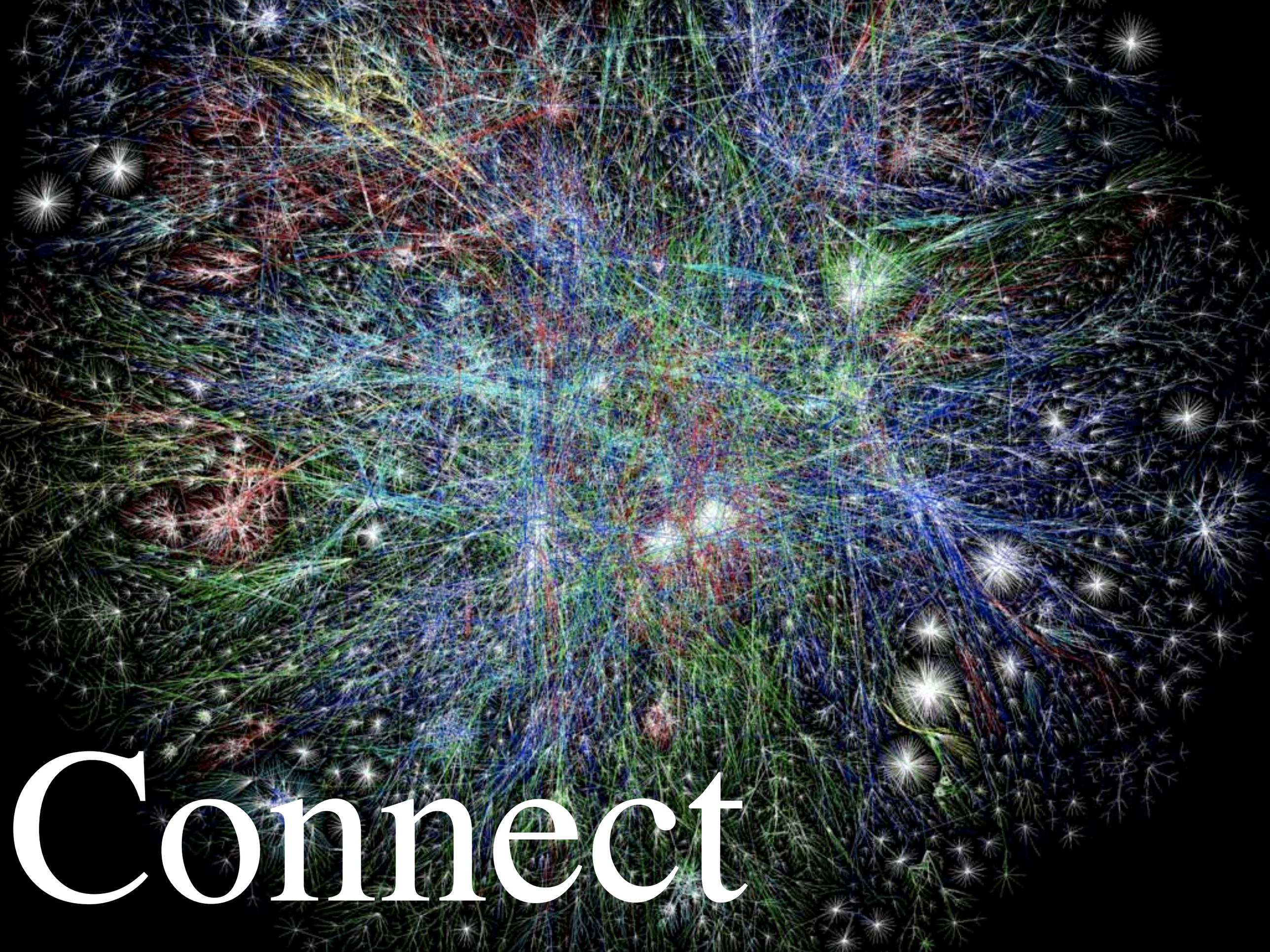
INFORMATION

Press	1439
Telegraph	1880
Web	1970

Print

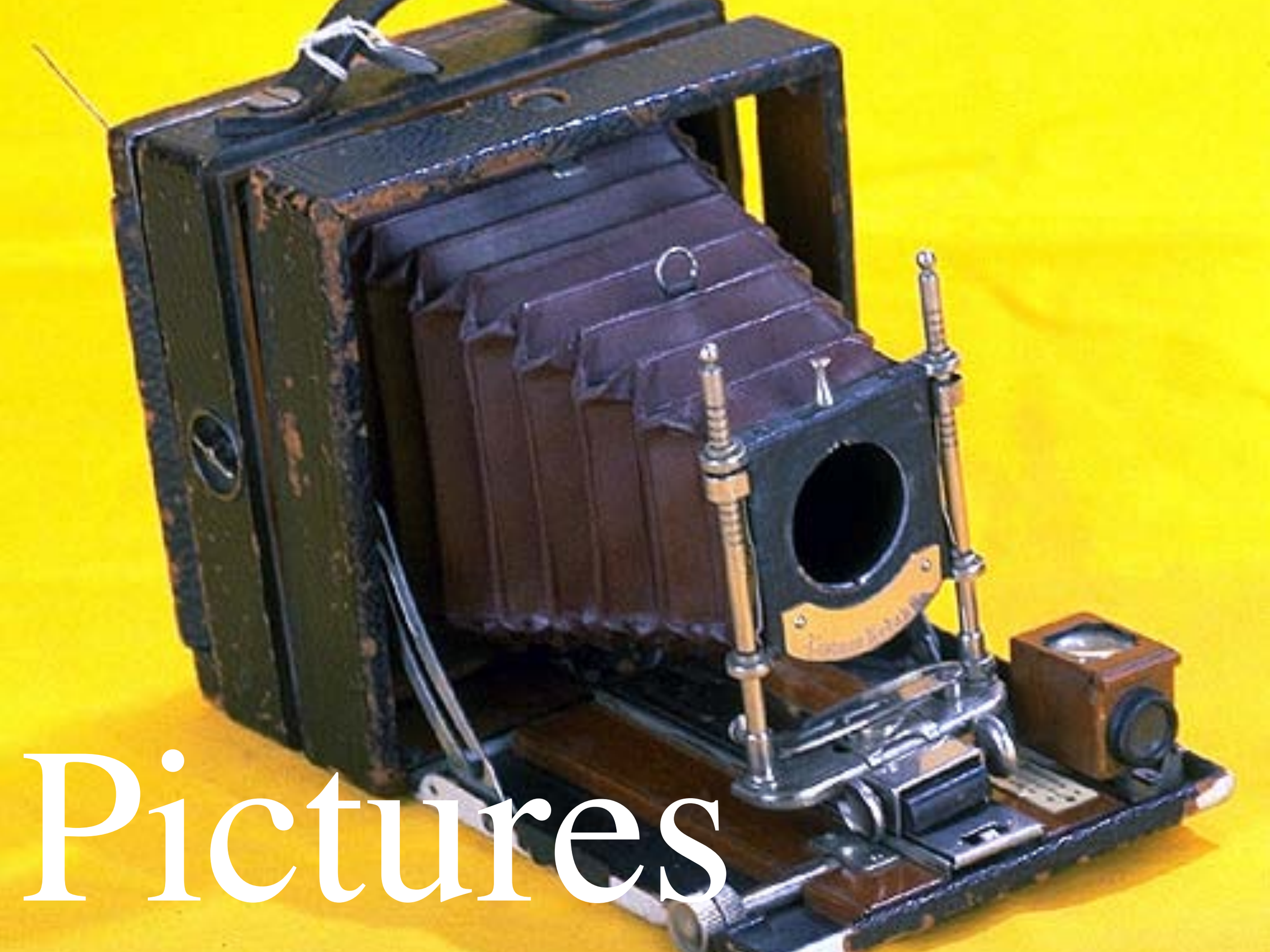
The quick brown fox jumps over the lazy
dog and feels as if he were in the seventh
heaven of Typography together with Her-
mann Zapf, the most famous artist of the
Bacher

Talk


Connect

MEDIA

Photo	170 years ago
Film	130 years ago
3D Print	30 years ago



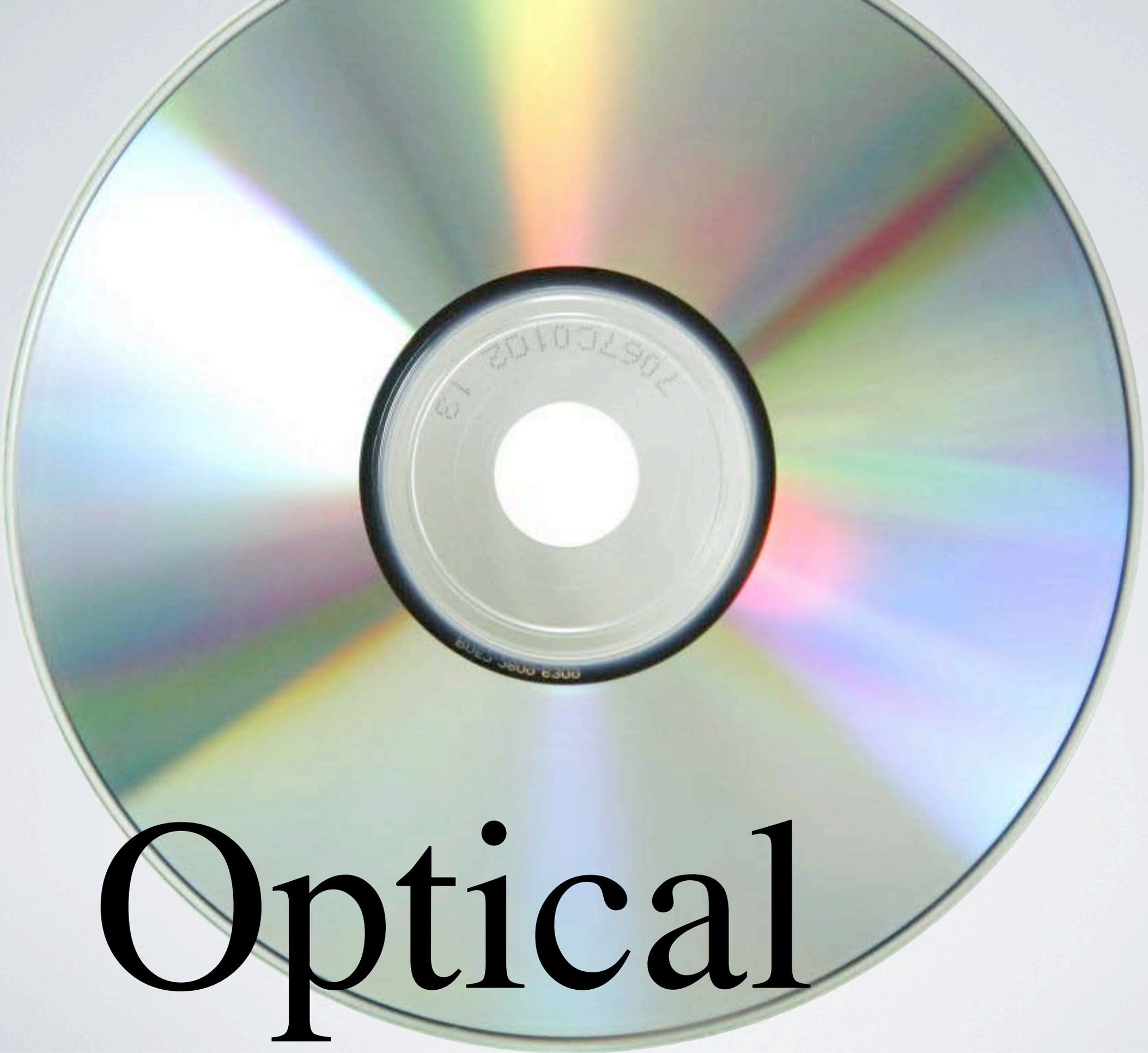
Pictures

Movies

Objects

STORAGE

CD, DVD	1958
Hard drives	1956
NAND flash	1980



Optical

Magnetic

Electric

PERCEPTION

Microscope	420 years ago
Telescope	400 years ago
3D Scan	50 years ago

Micro

Macro

Meso

EDUCATION

New Math	1970
Math Wars	1990
MOOC	2010

2 = A NUMBER
1 = A NUMBER

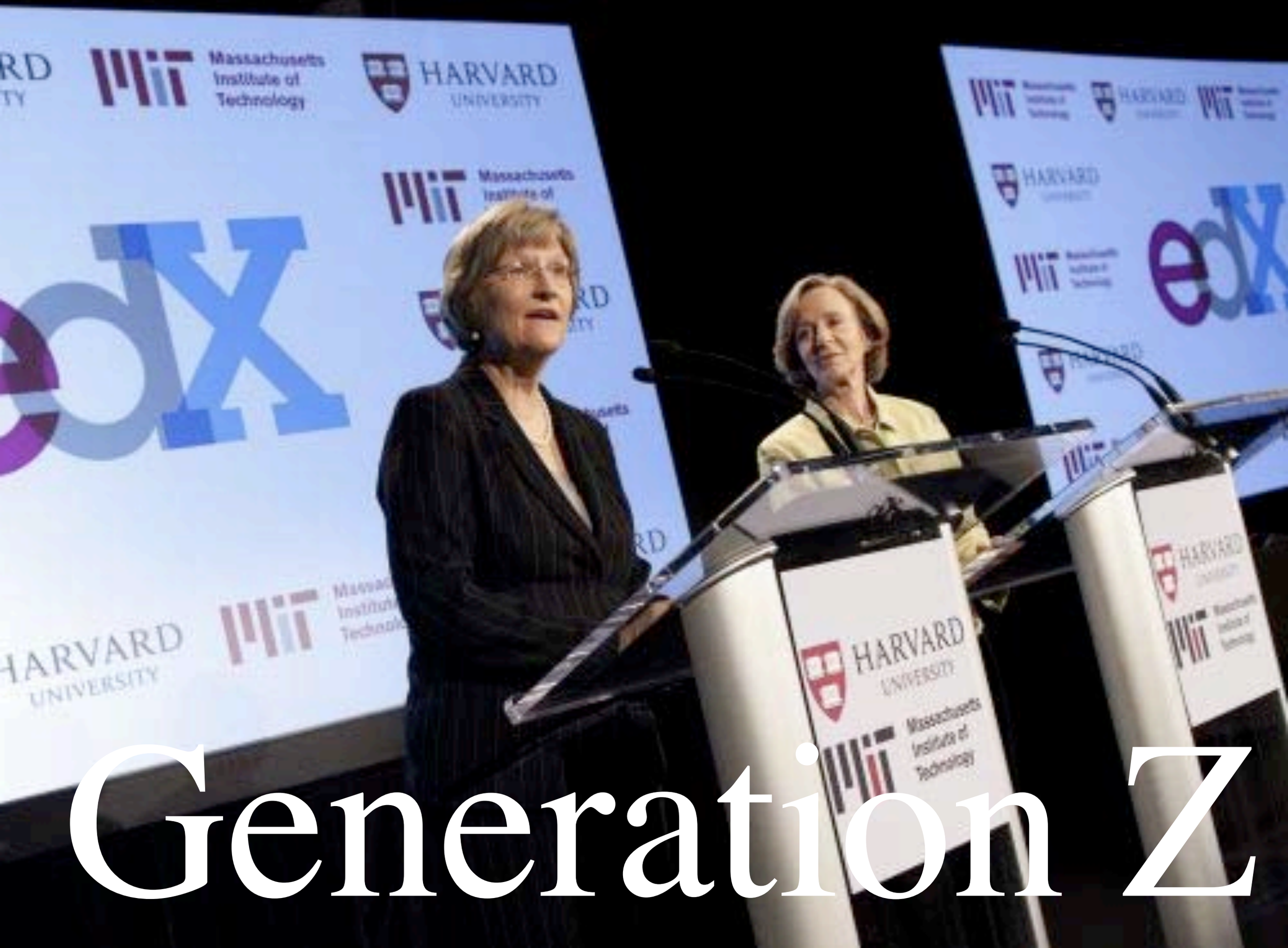
2=1

Generation X

MATH

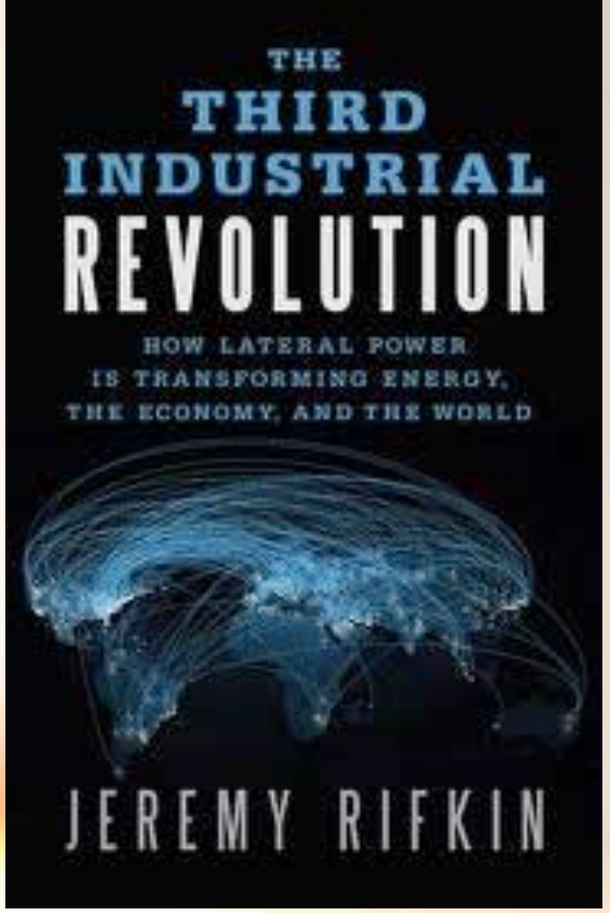
STAR WARS

Generation Y



Generation Z

JEREMY RIFKIN



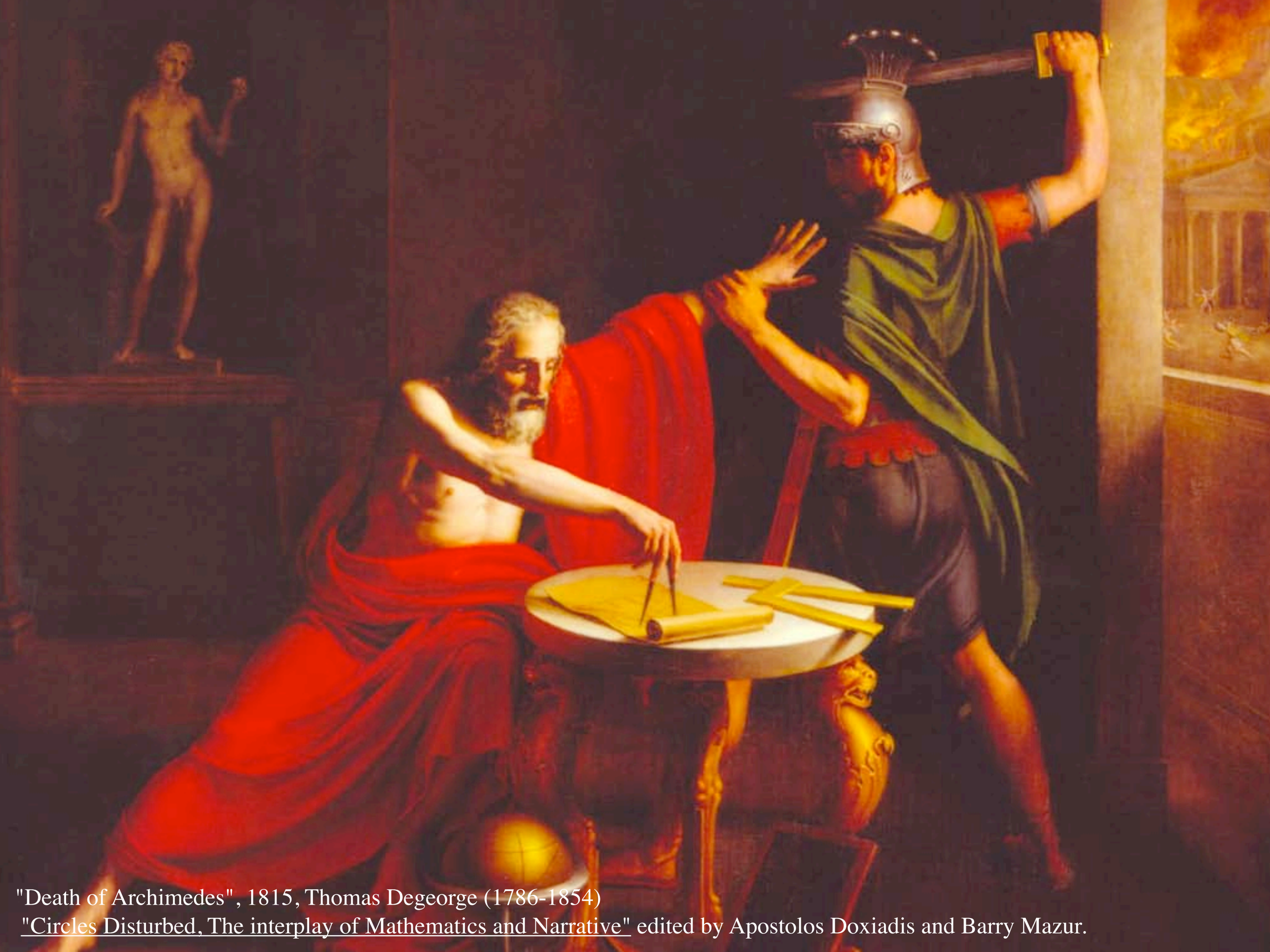
"manufacturing becomes digital, personal, and affordable"

ARCHIMEDES

$u = x^2$
$du = 2x\,dx$
$\frac{du}{2x}$
$84 \int e^{-u} \frac{1}{2}\,du$
$= \frac{84(-e^{-u})}{2} = \frac{84}{2}(-e^{-x^2}$
$-\boxed{42}$
$- \cos(t) + 1$
$- \sin(t) + t$
$+ 2\pi$
Elizabeth Slavkovsky

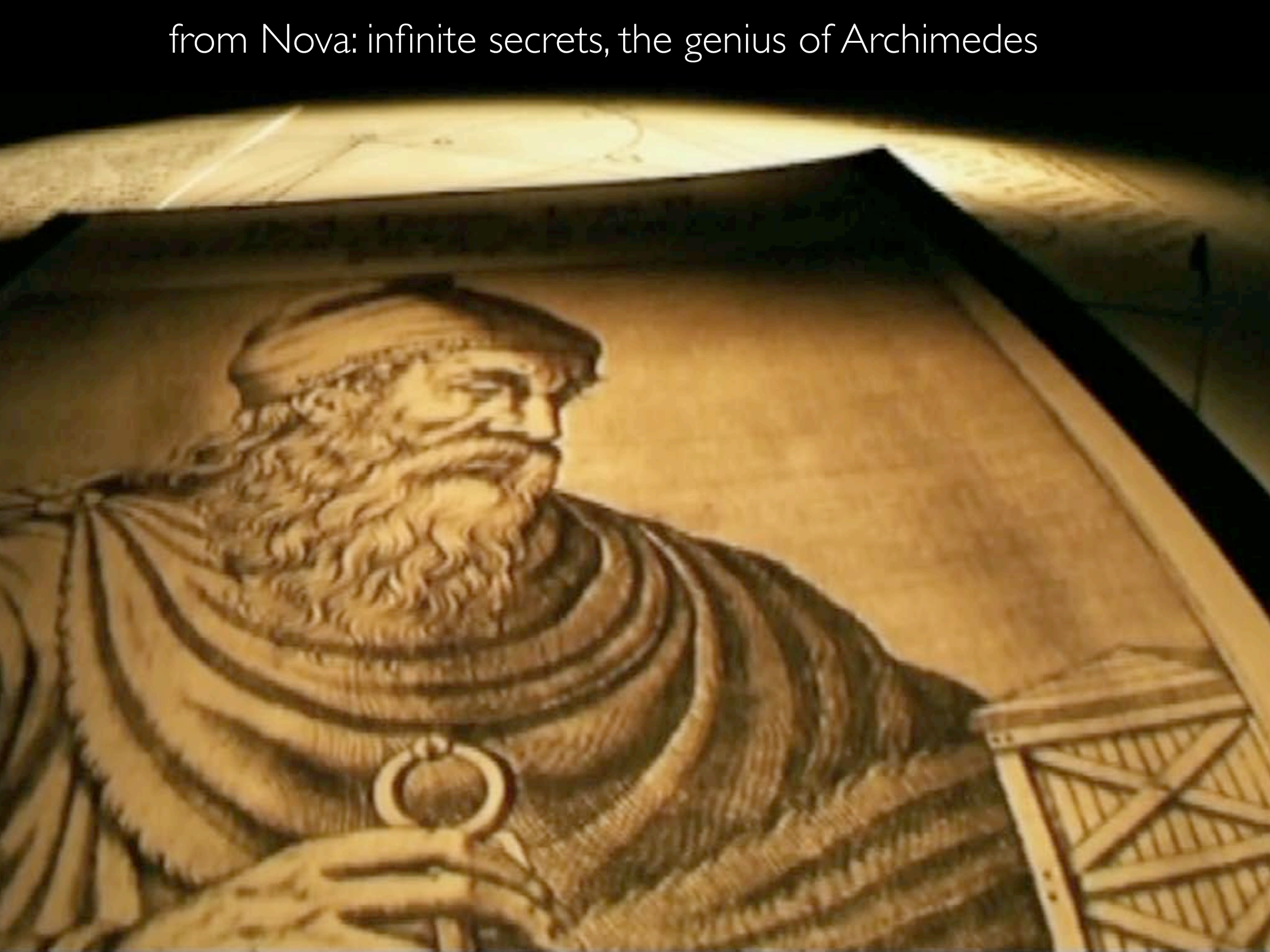
Domenico Fetti, Archimedes, 1620

Jose de Ribera, Portrait of Archimedes, 1630



"Death of Archimedes", 1815, Thomas Degeorge (1786-1854)
"Circles Disturbed, The interplay of Mathematics and Narrative" edited by Apostolos Doxiadis and Barry Mazur.

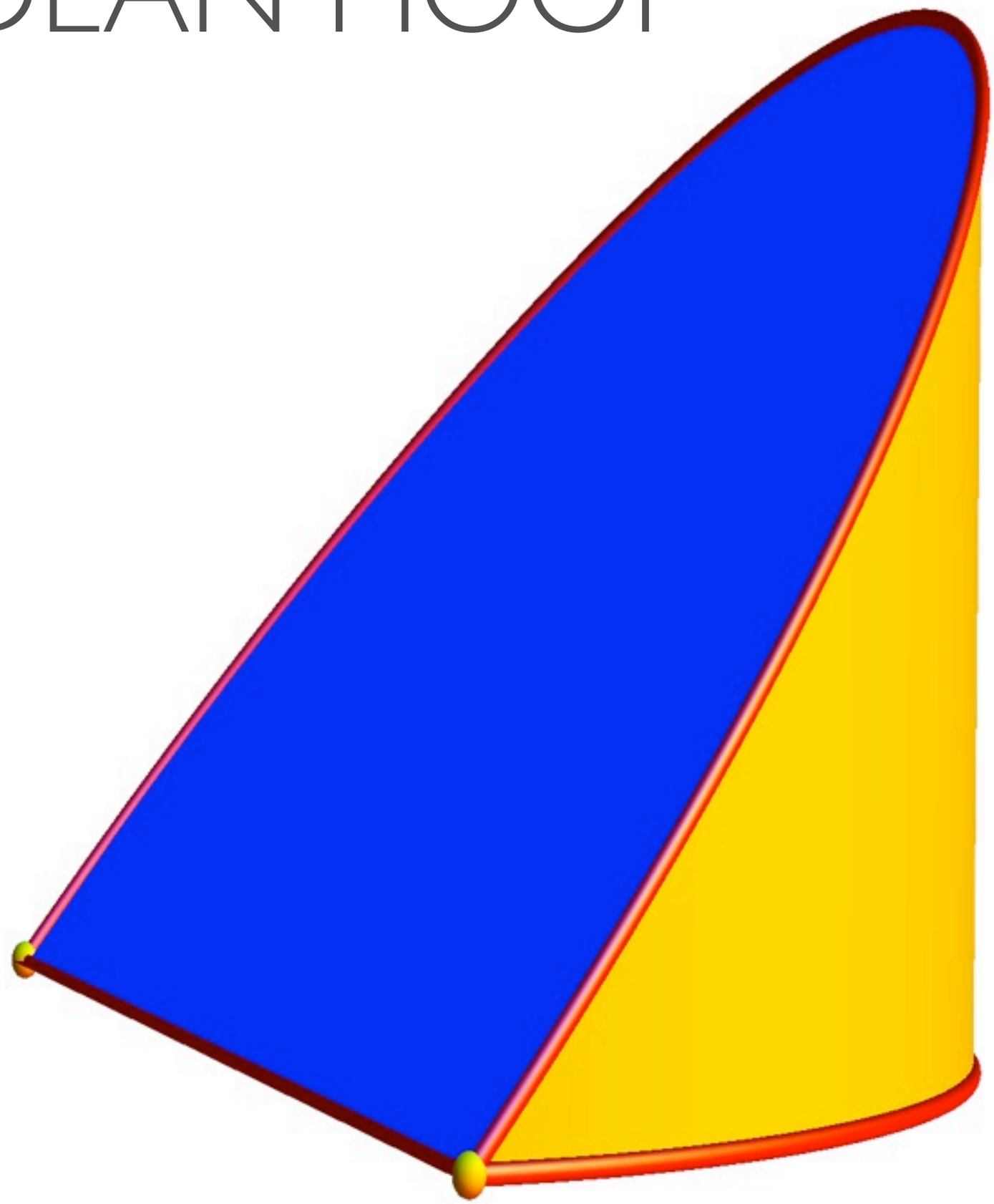
from Nova: infinite secrets, the genius of Archimedes

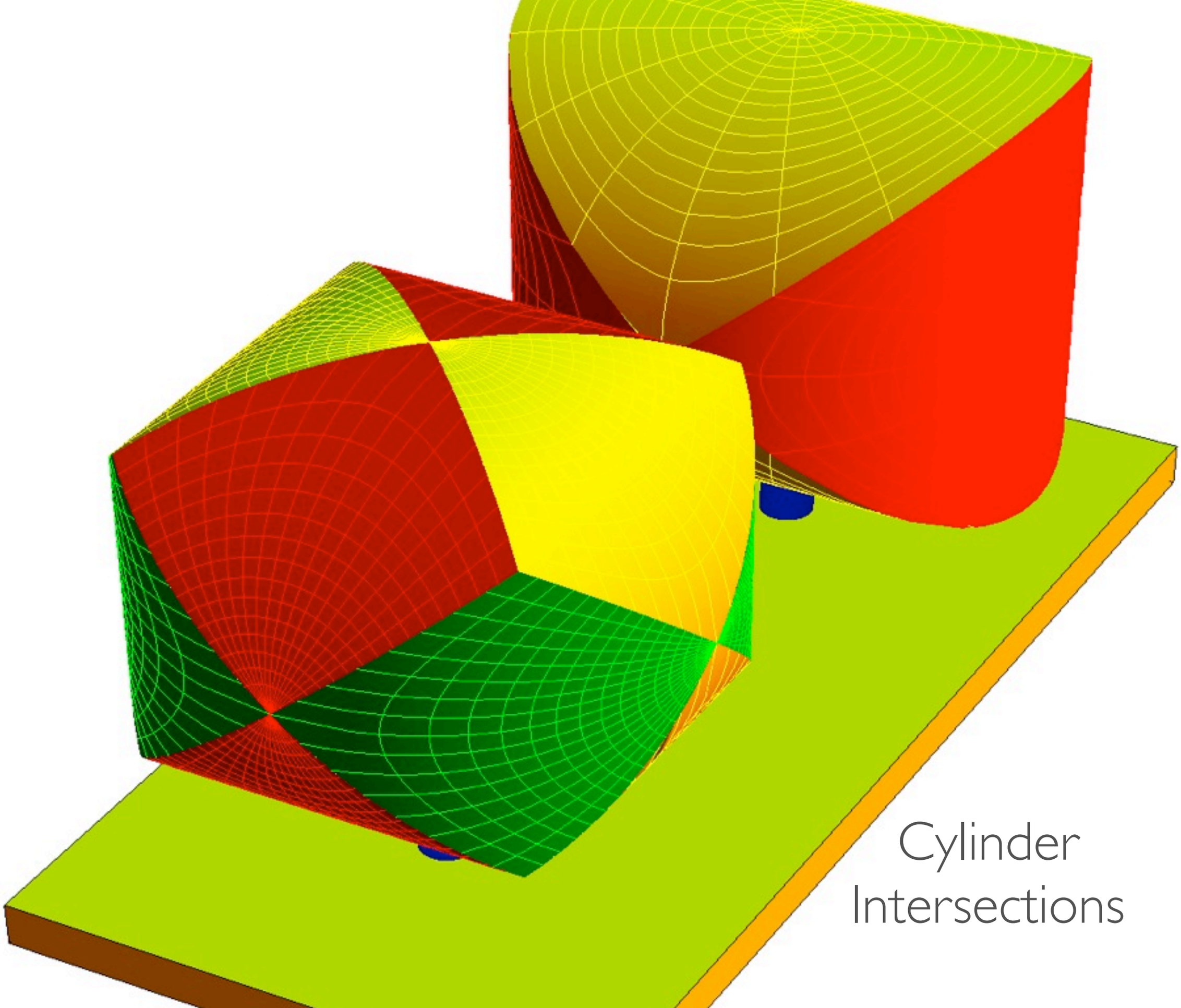


ILLUSTRATING ARCHIMEDEDS

ARCHIMEDEAN HOOF

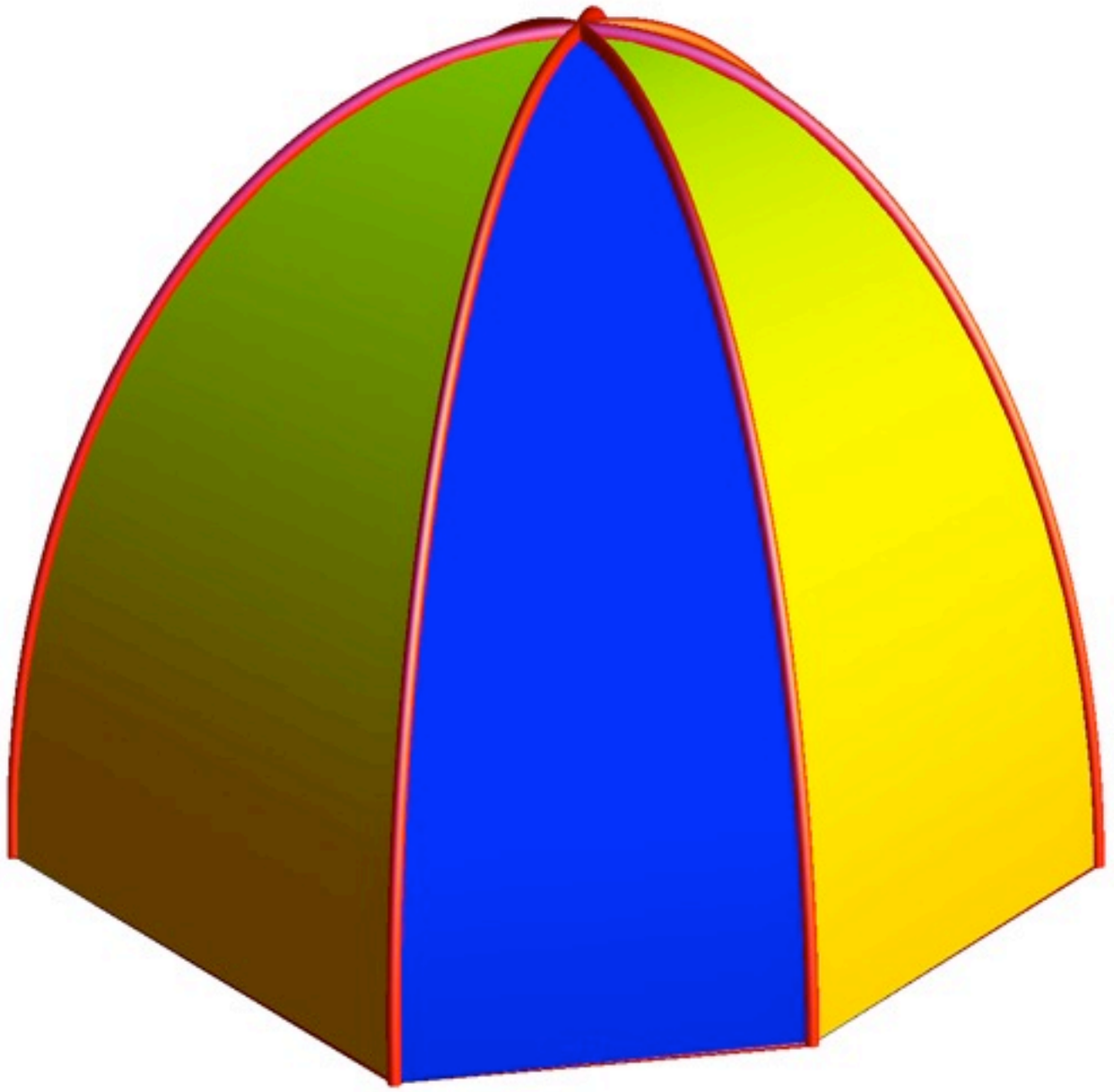
"Birth place of calculus"



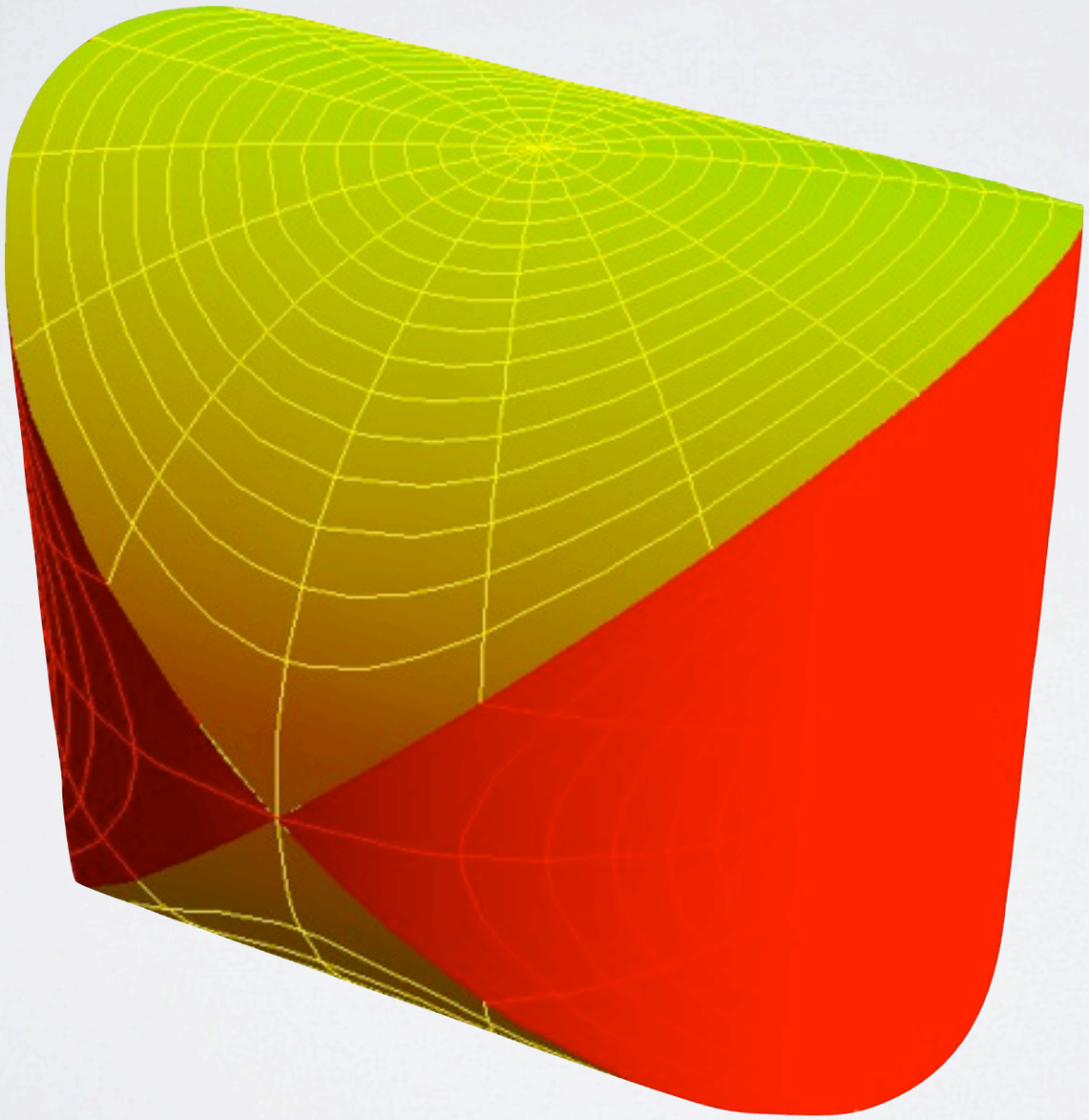


Cylinder Intersections

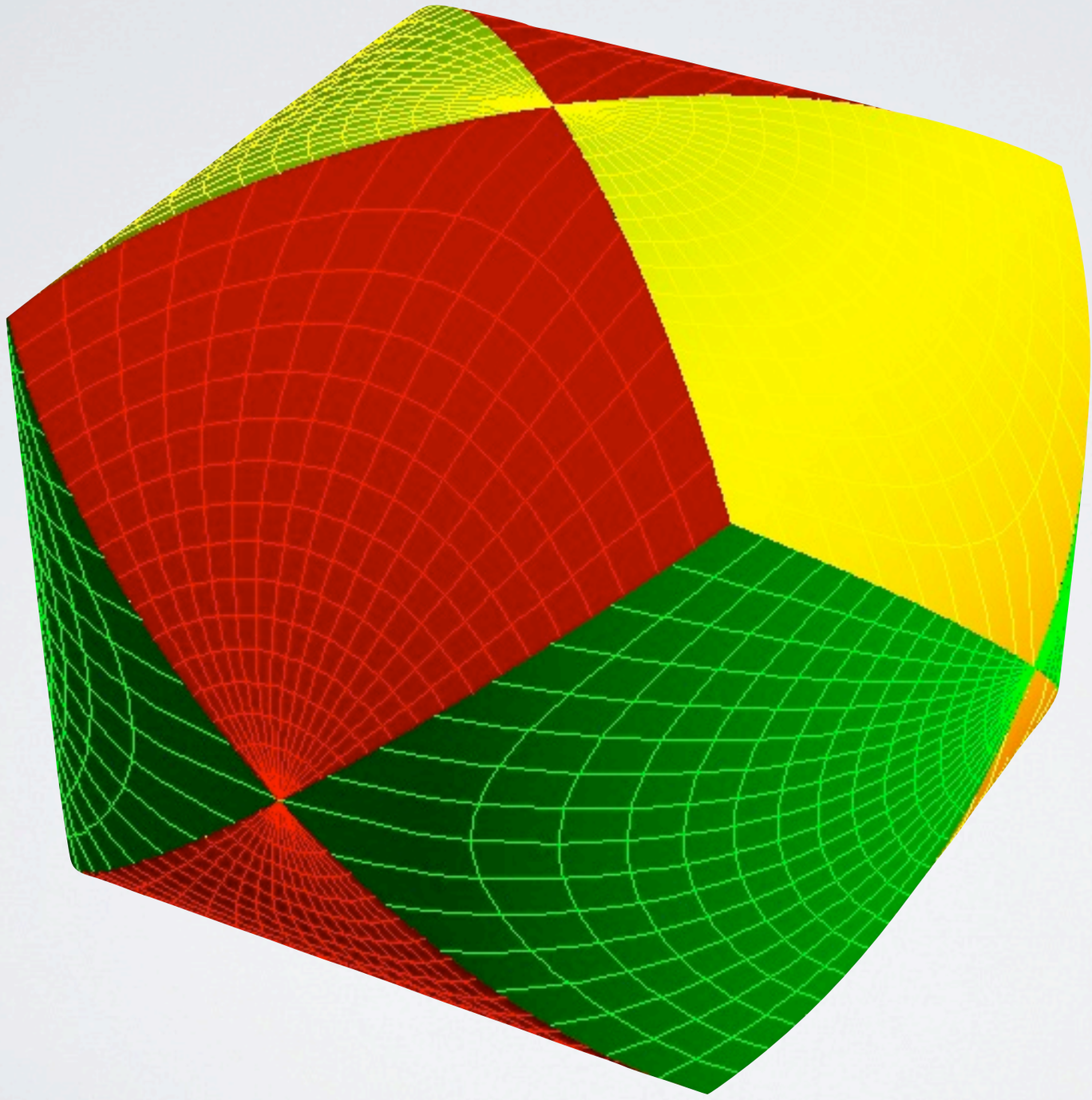
ARCHIMEDEAN DOMES



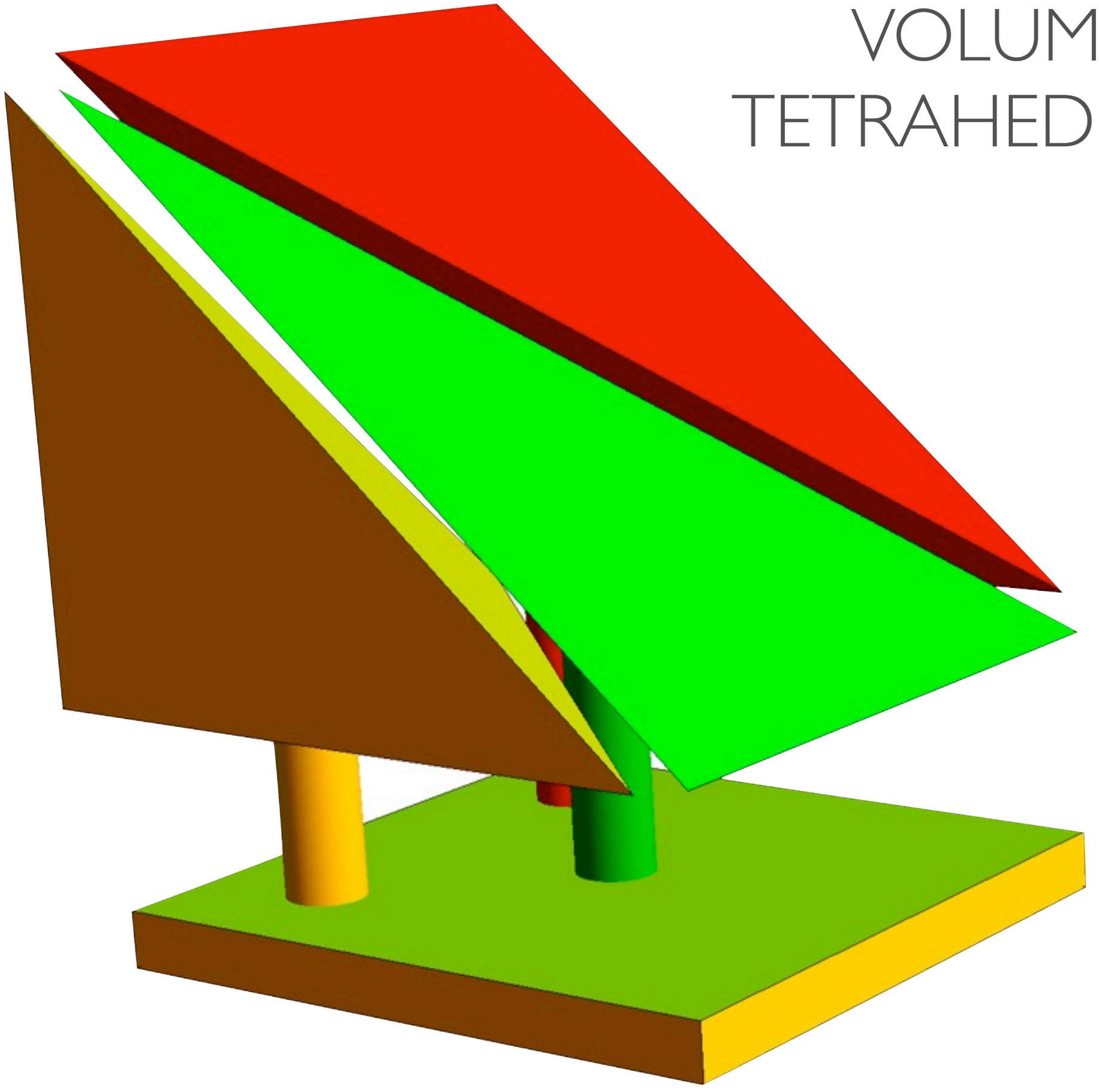
2 CYLINDERS

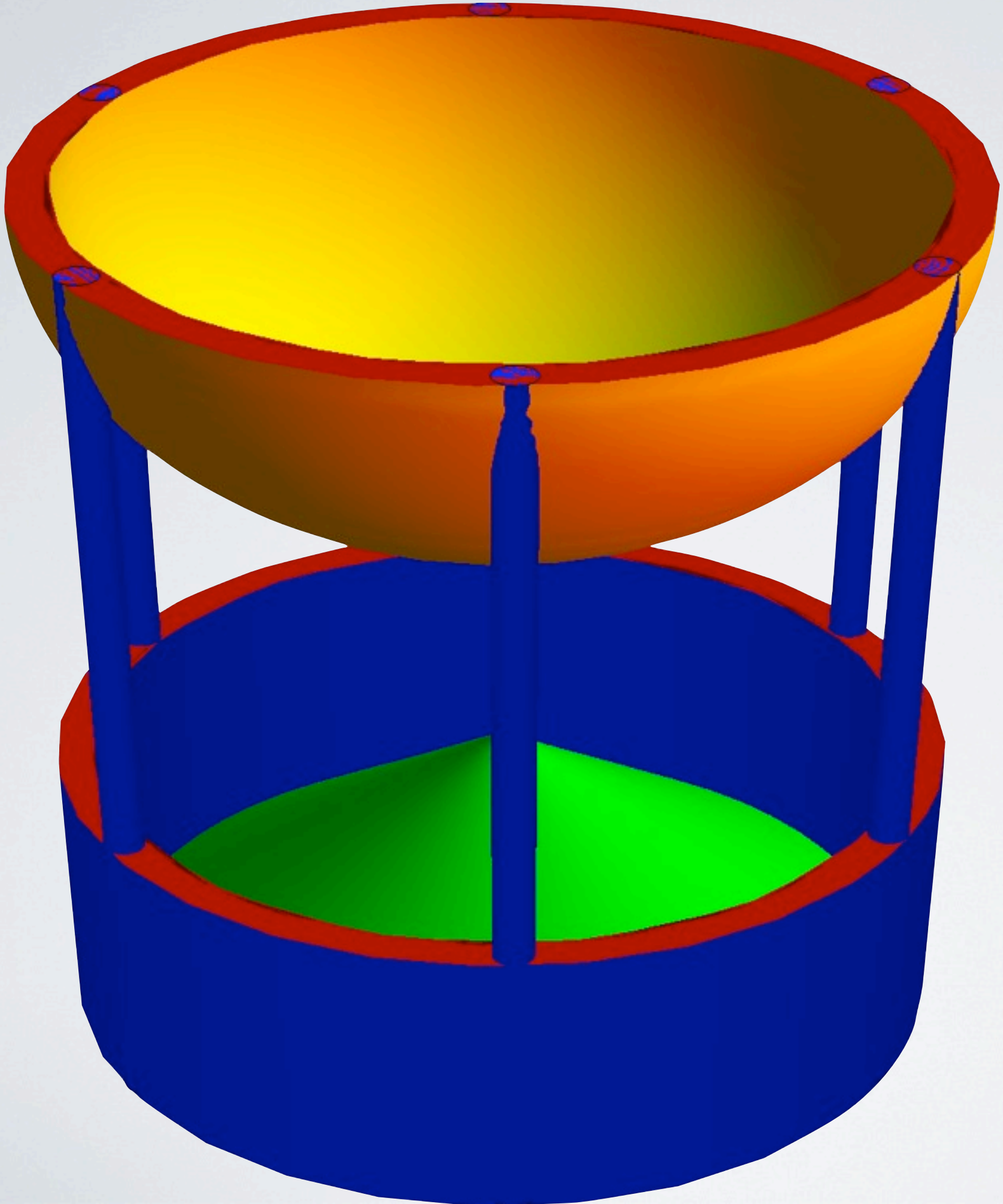


3 CYLINDERS

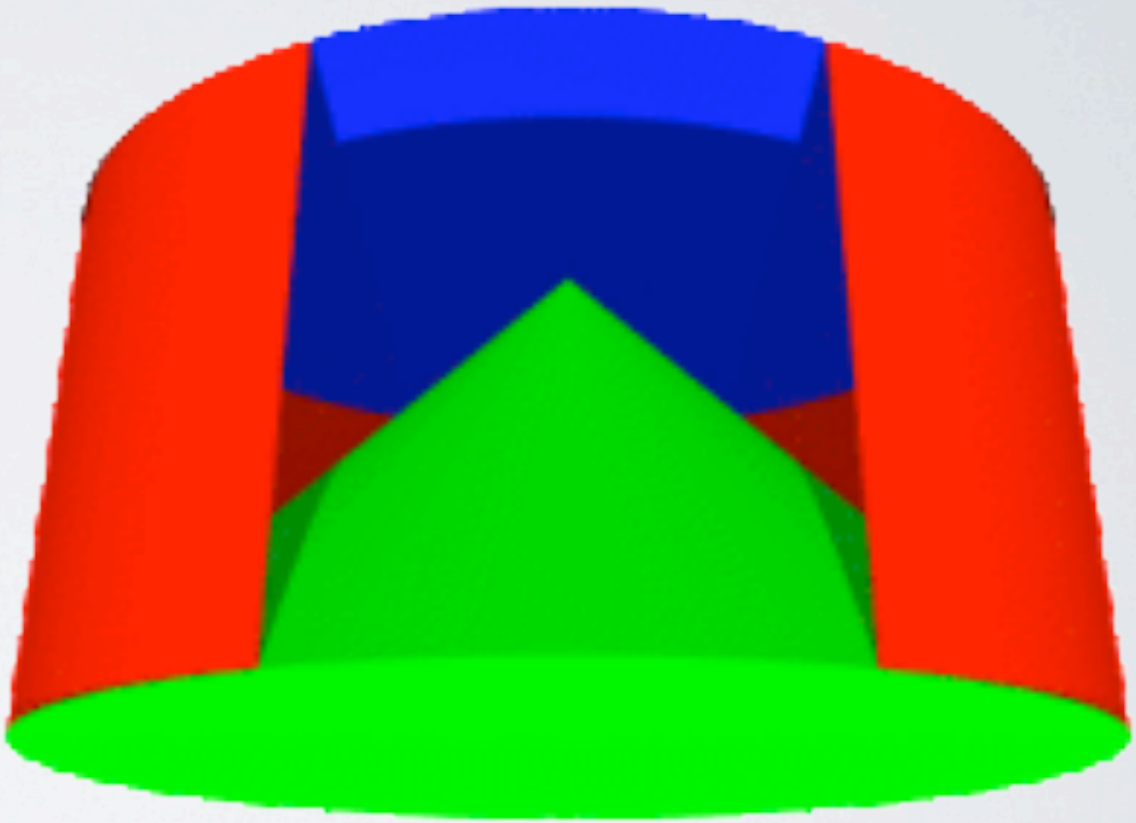


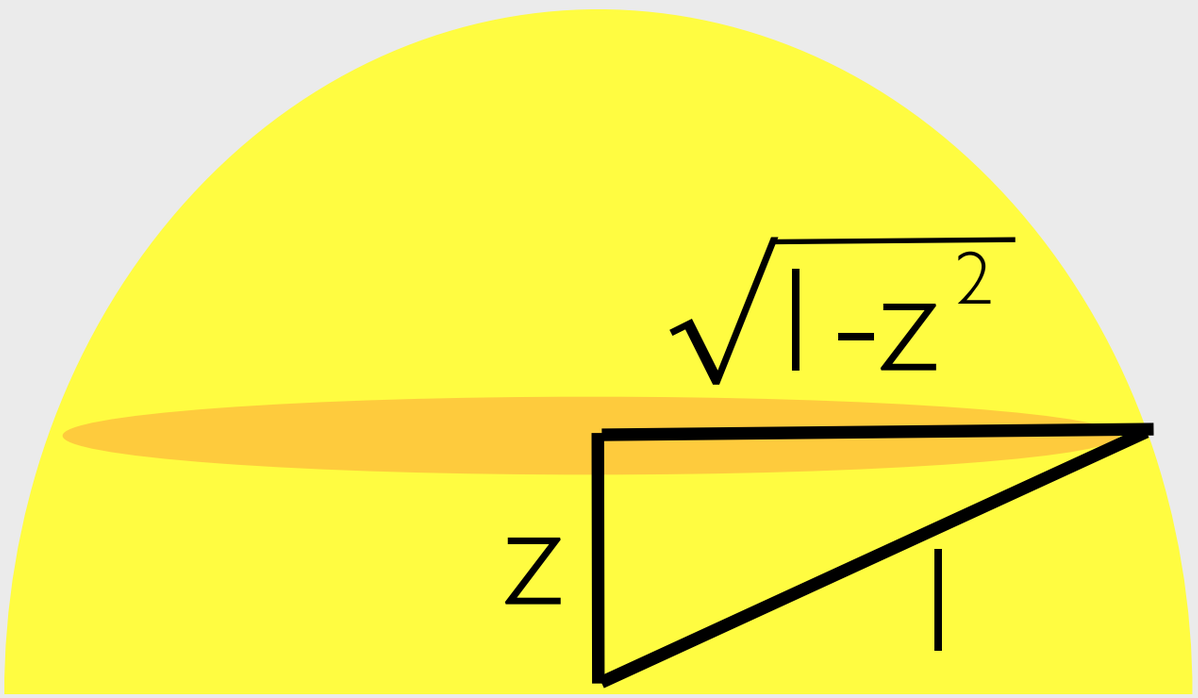
VOLUME TETRAHEDRON

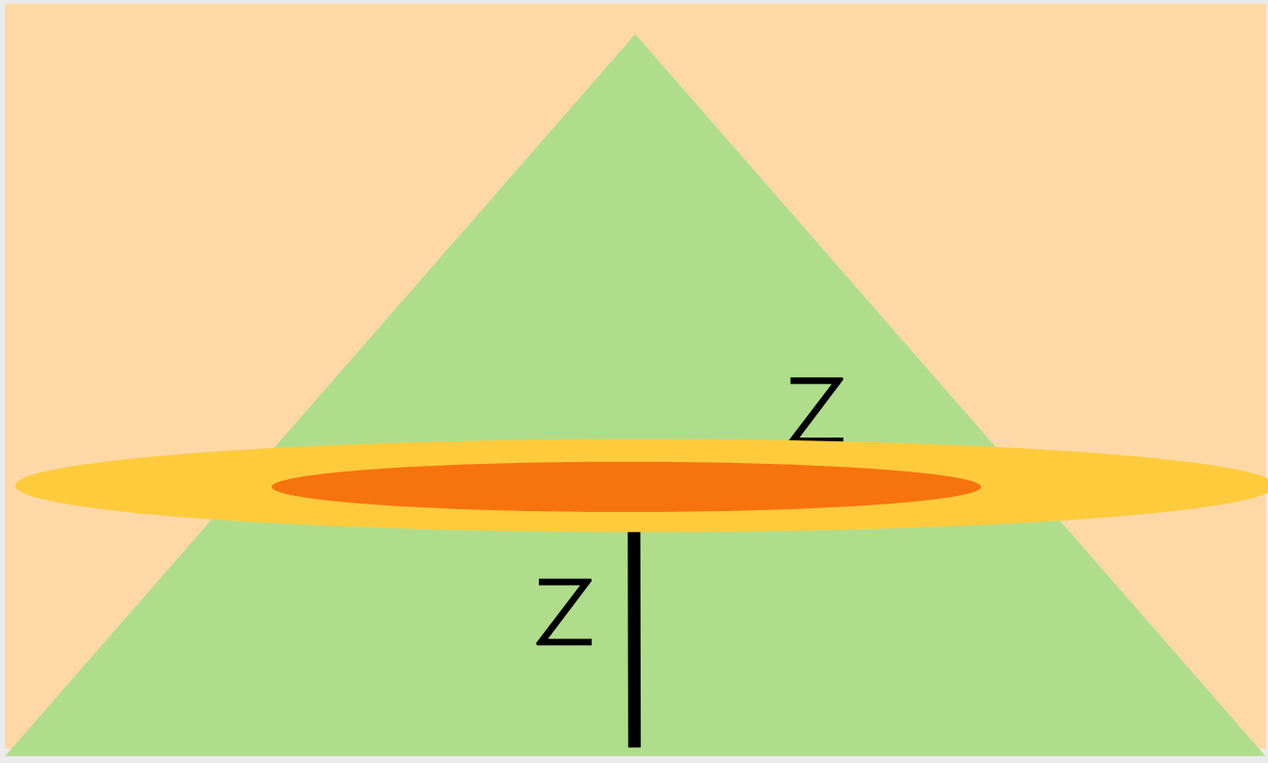




SPHERE PROOF





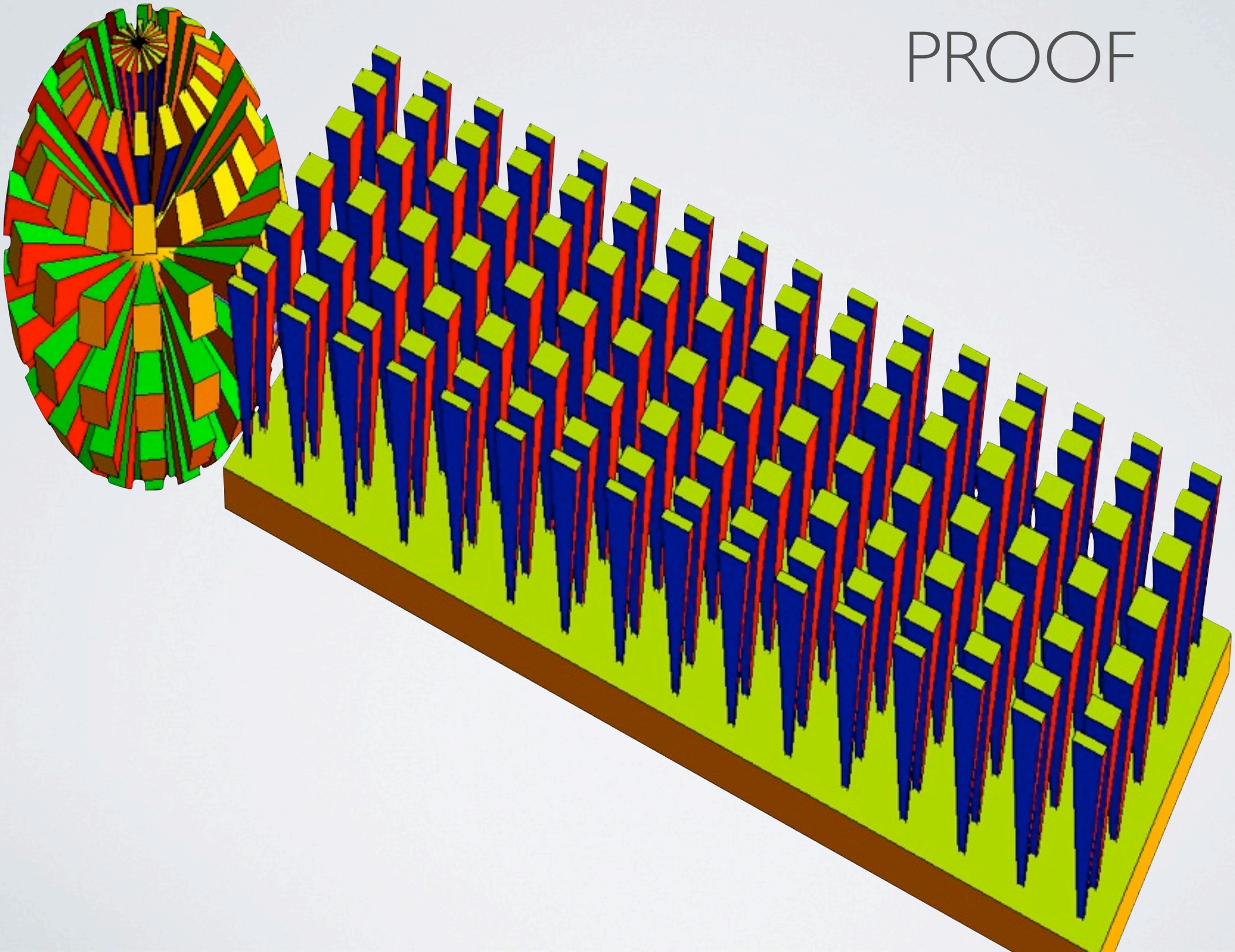


$$A(z) = \pi(1-z^2)$$

$$A(z) = \pi - \pi z^2$$

Image U.S. Geological Survey

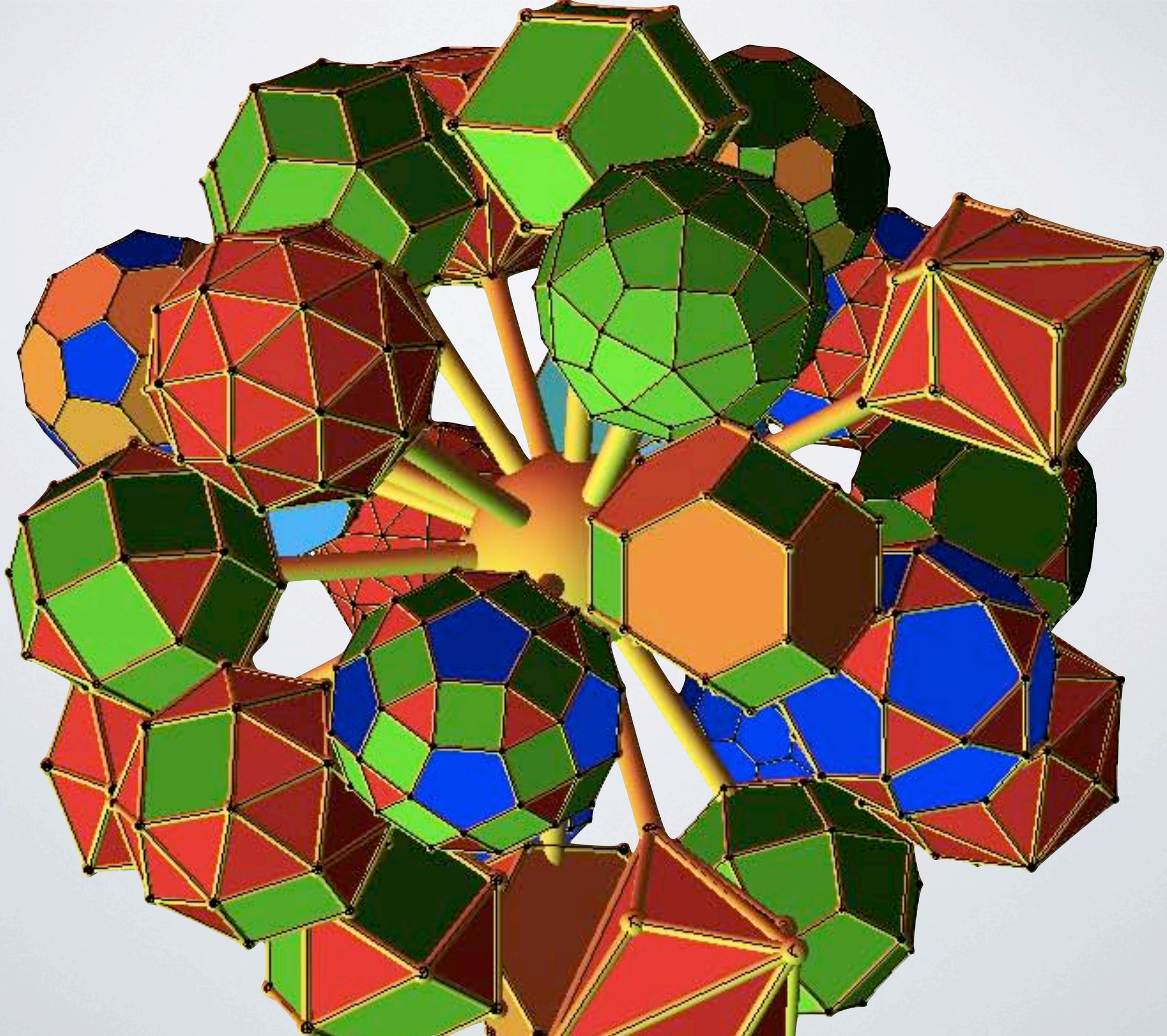
ARCHIMEDEAN TYPE PROOF

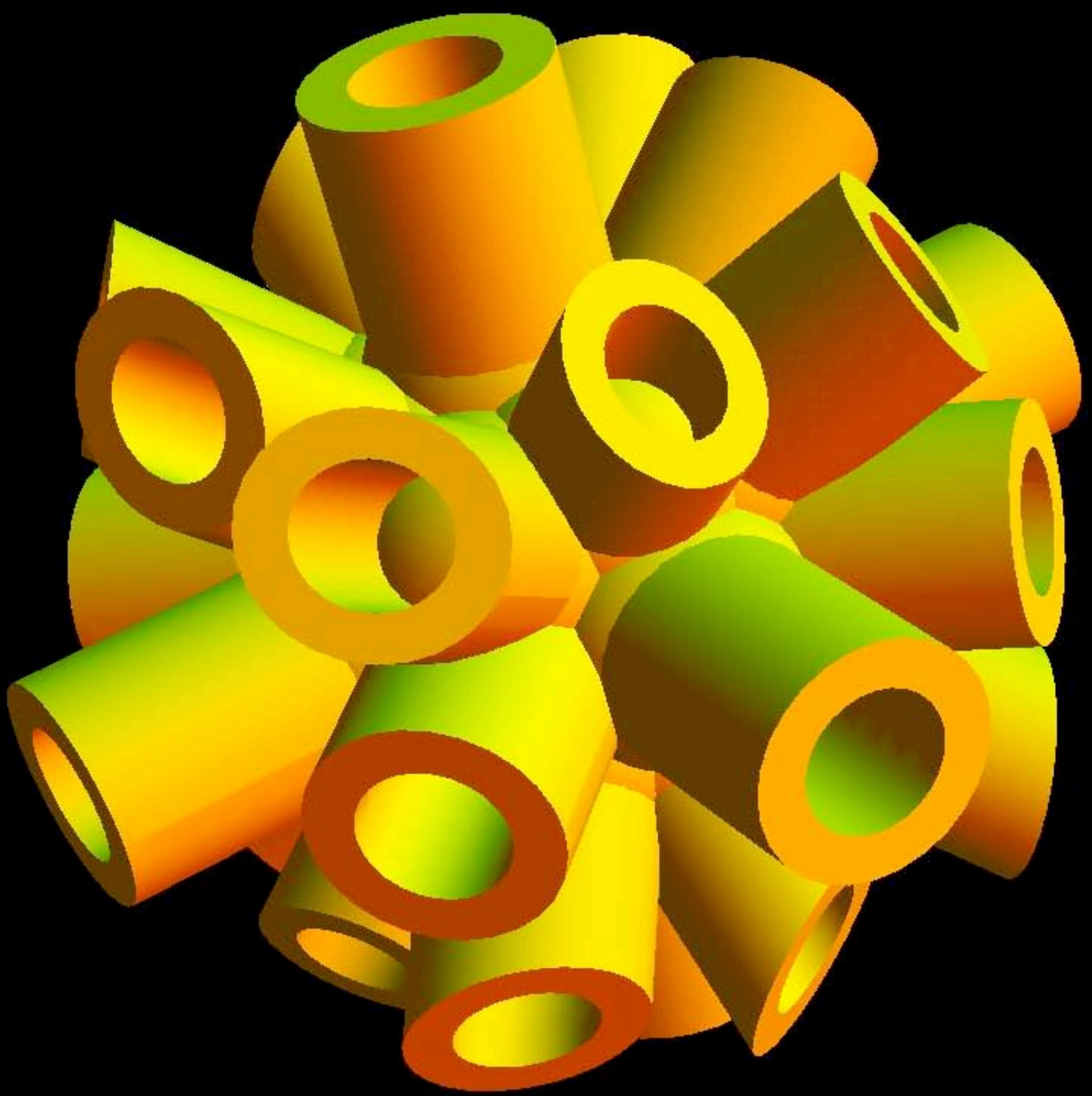


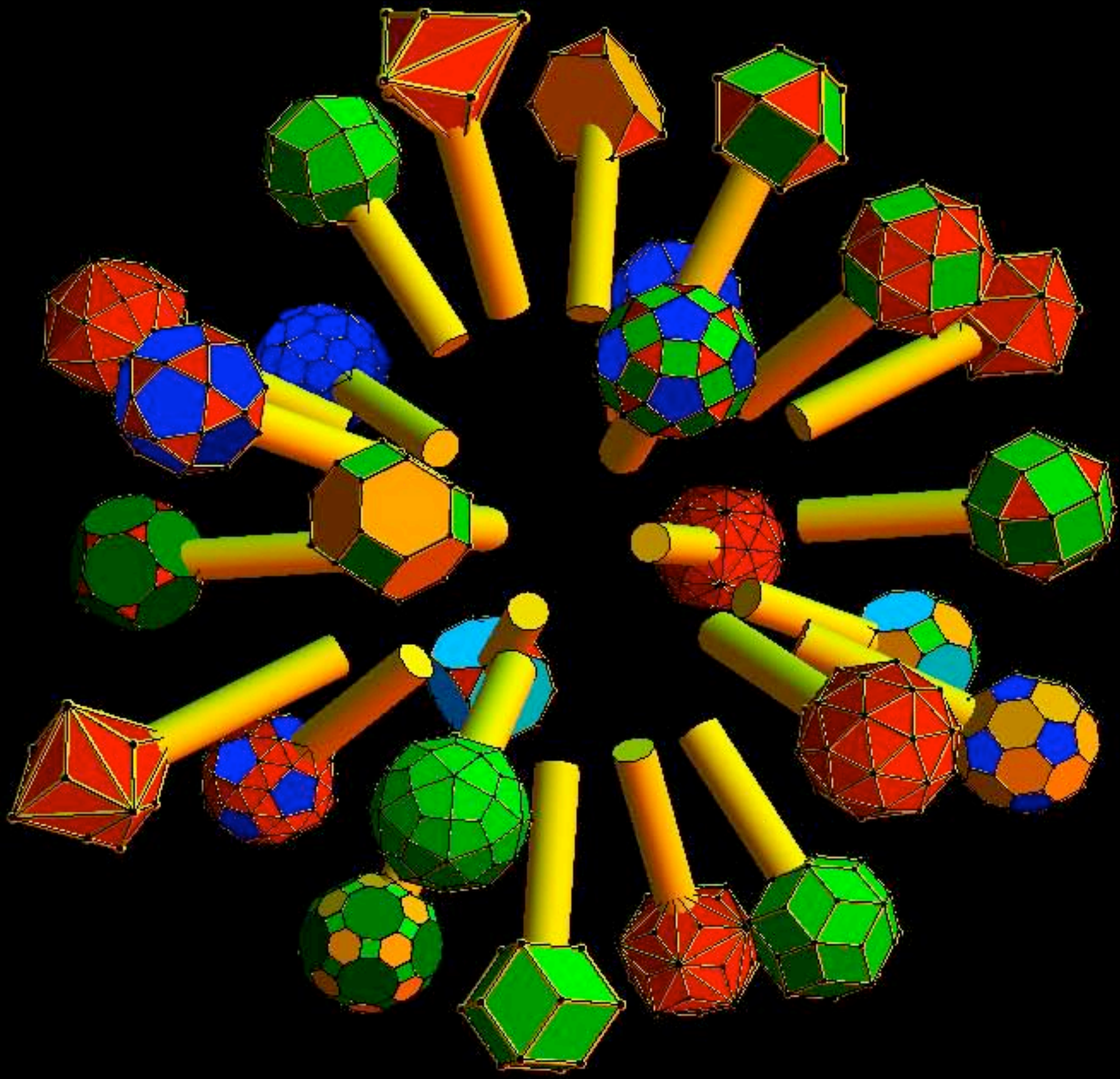
ARCHIMEDES ENGINEERING

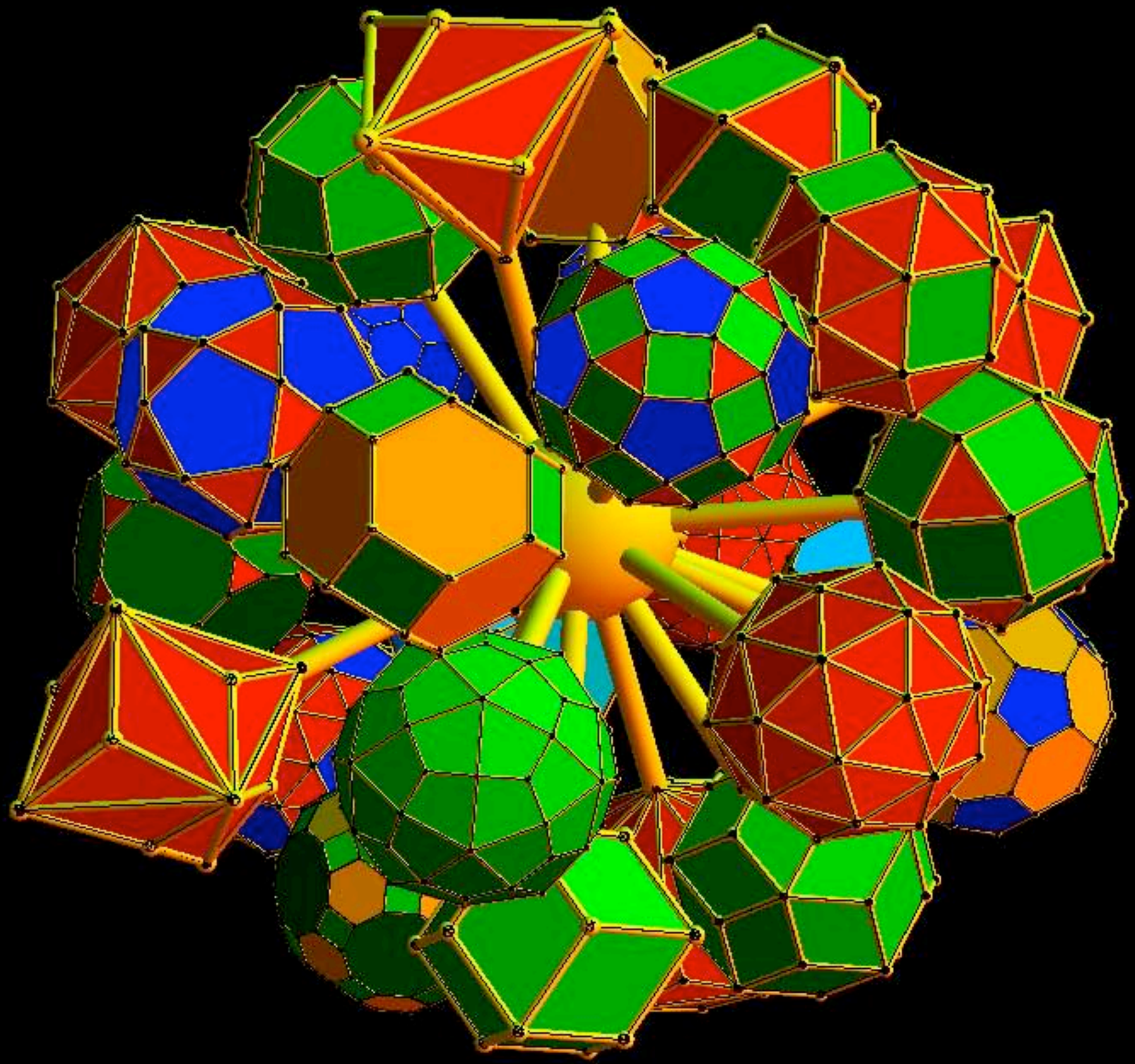
Nova: Ancient Computer

ARCHIMEDEAN SOLIDS



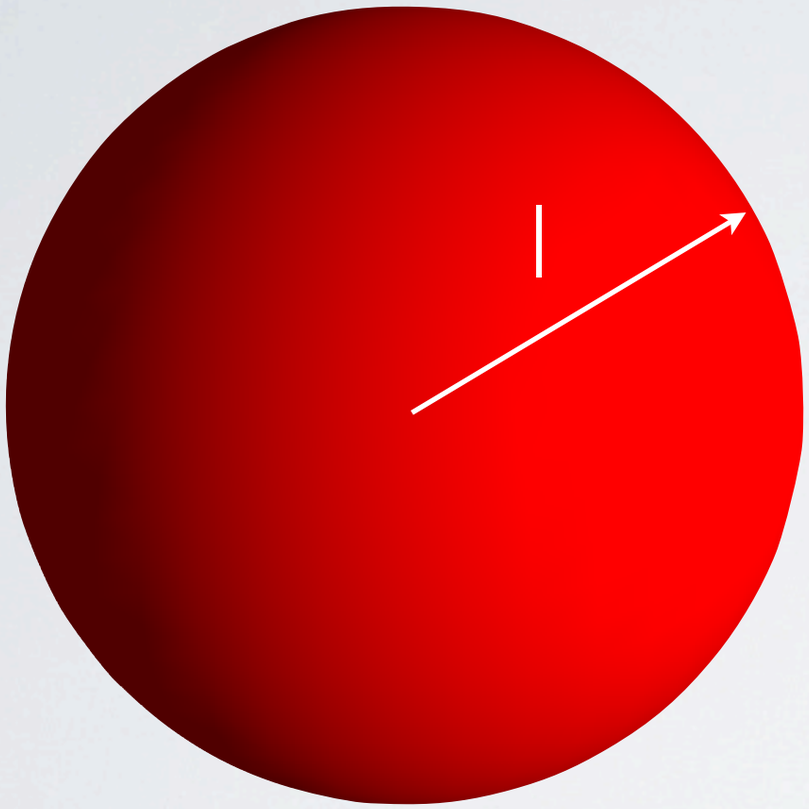






ARCHIMEDES SOLVED AN OPEN PROBLEM AT THAT TIME, THE COMPUTATION OF THE SPHERE VOLUME, 2300 YEARS LATER, CAN WE COMPUTE THE VOLUME OF THE MANDELBULB?

SPHERE

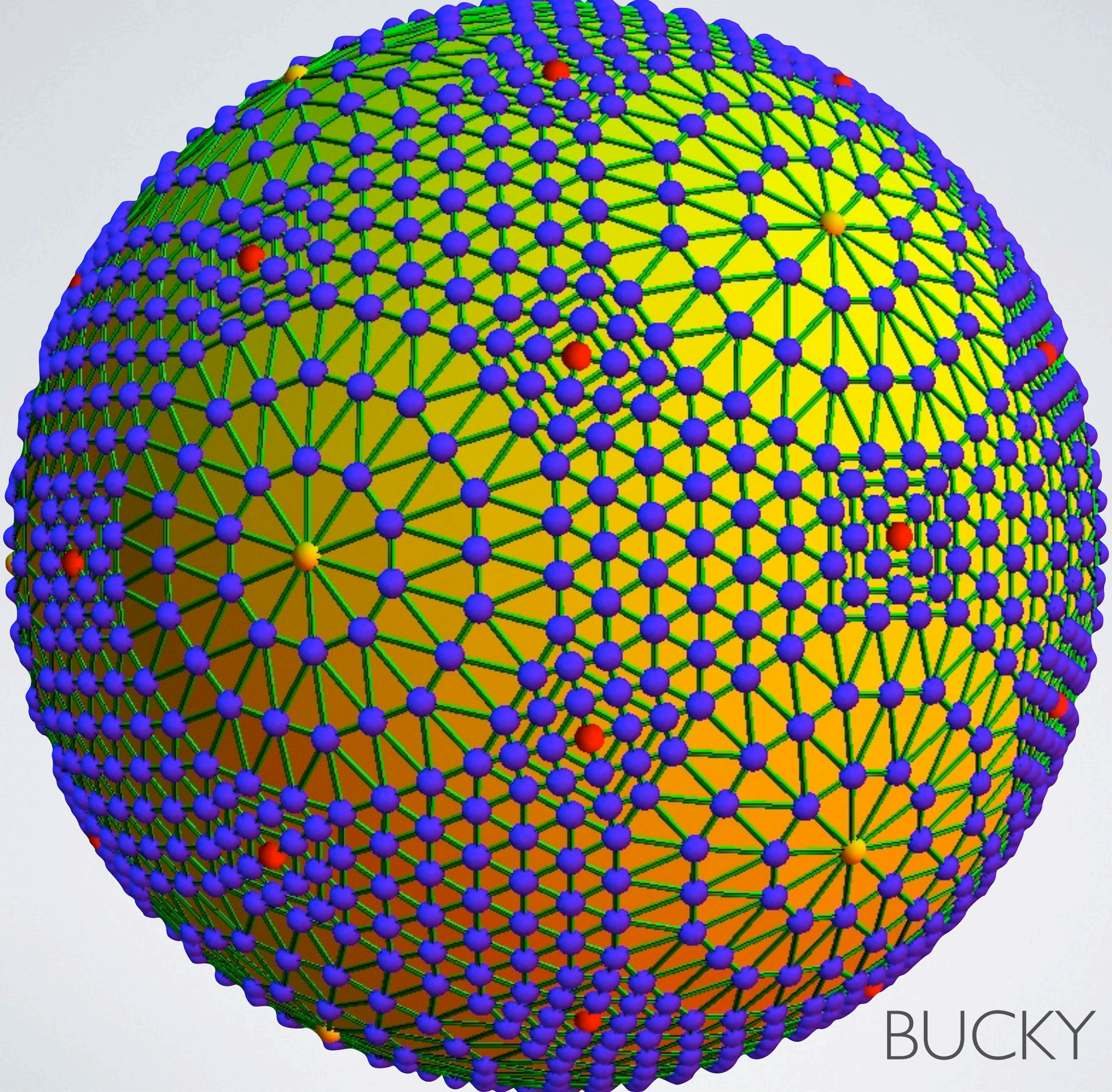


$4\pi/3$

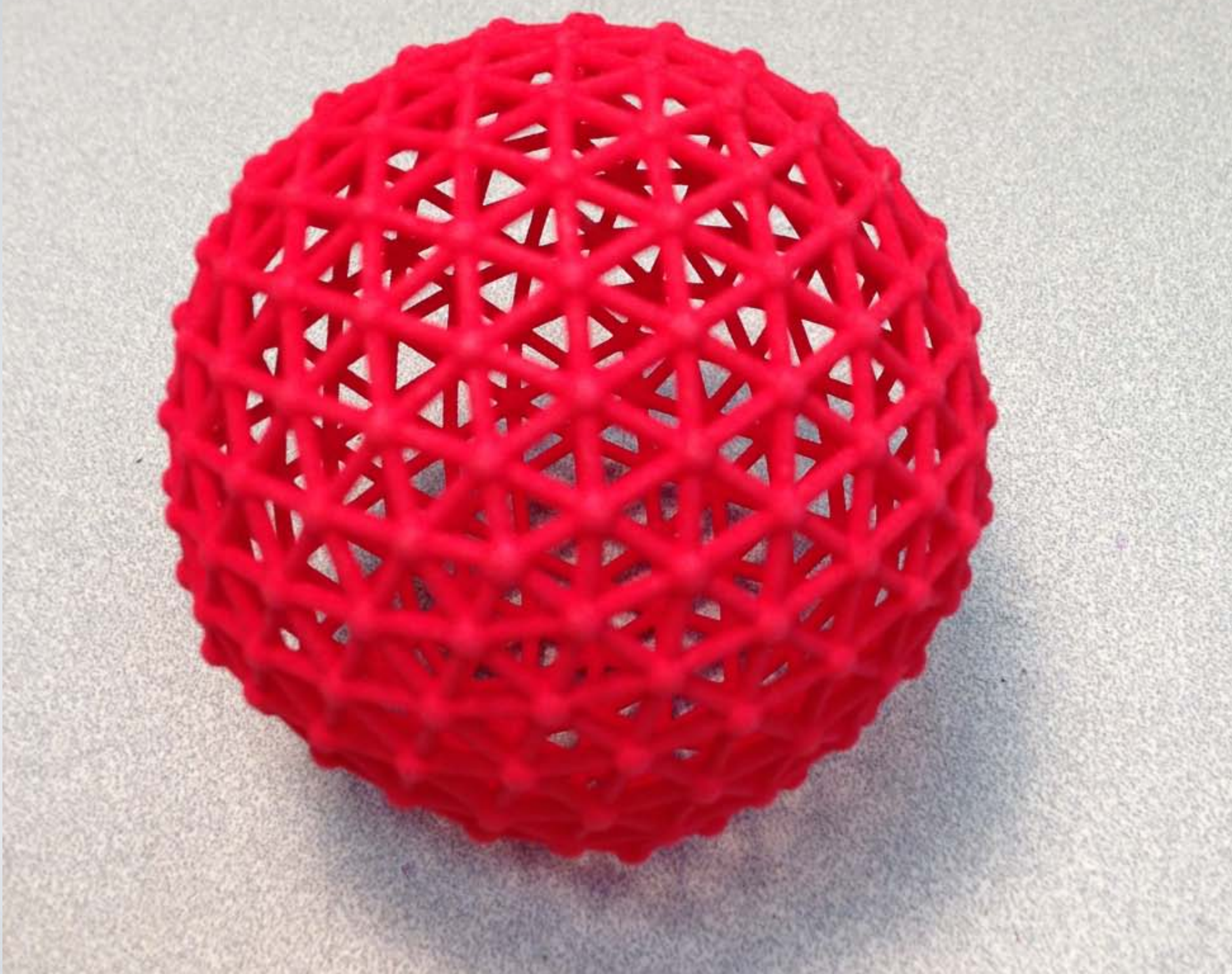
MANDELBULB

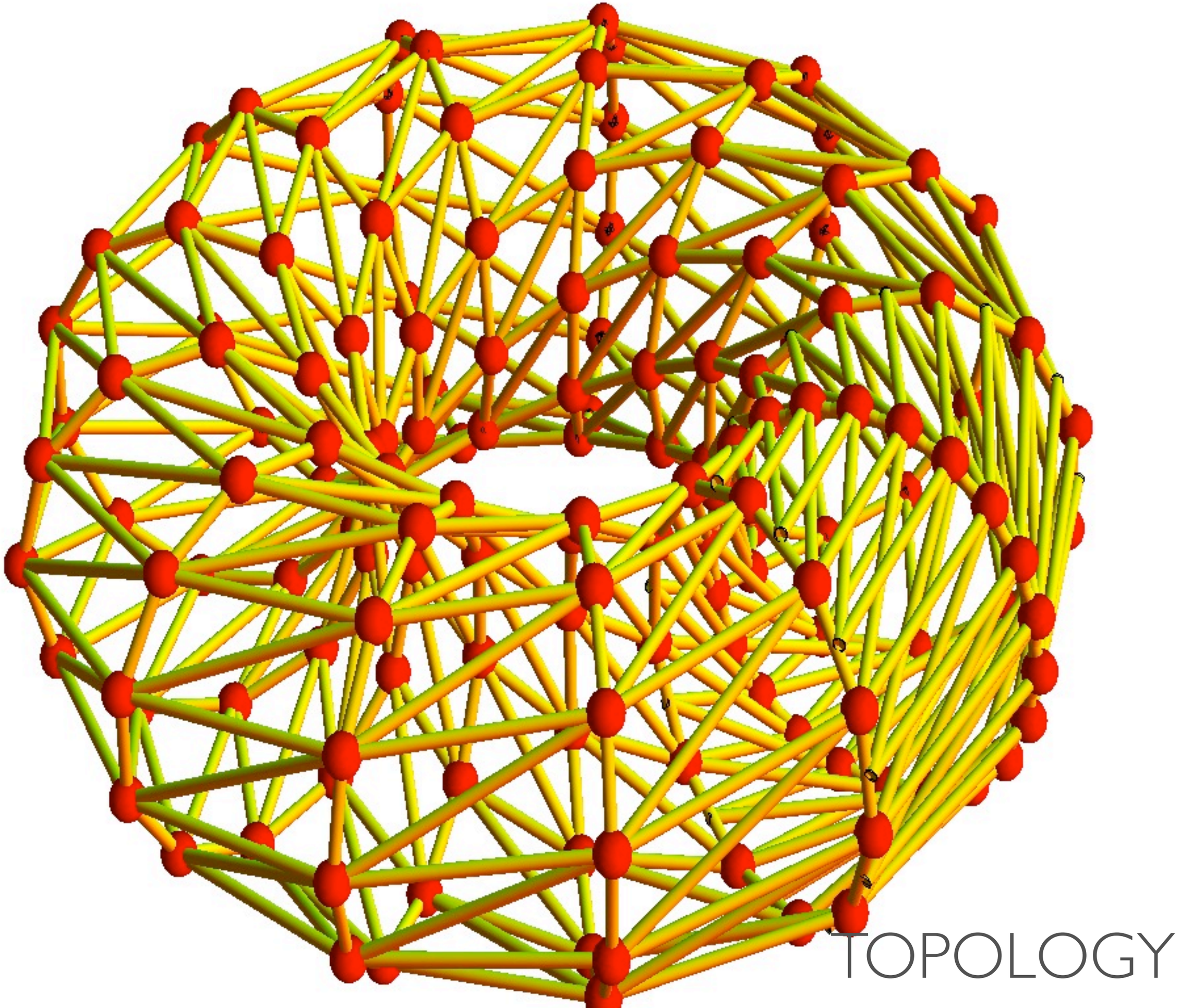
???

MORE ILLUSTRATIONS



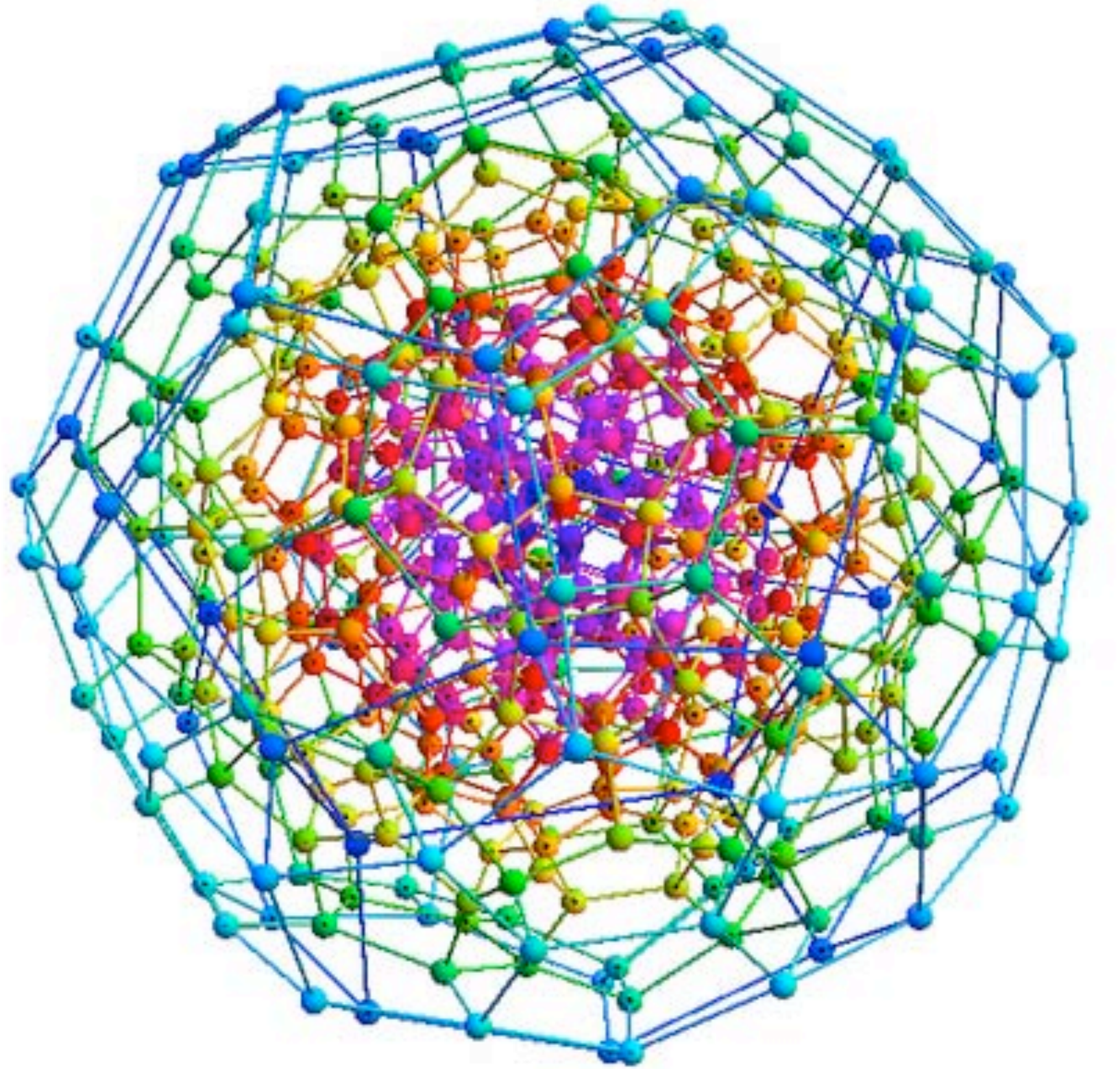
BUCKY BALLS



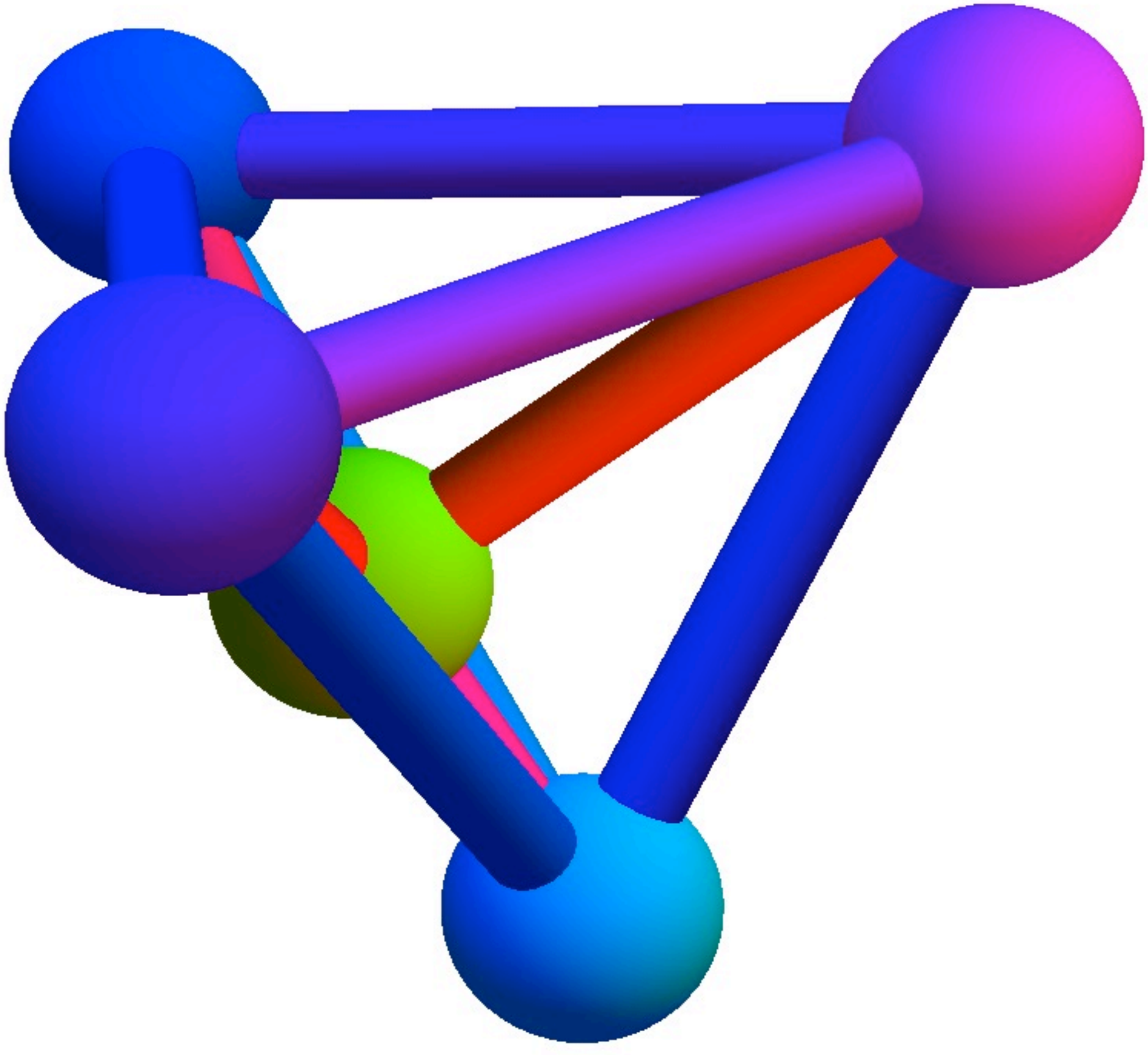


TOPOLOGY

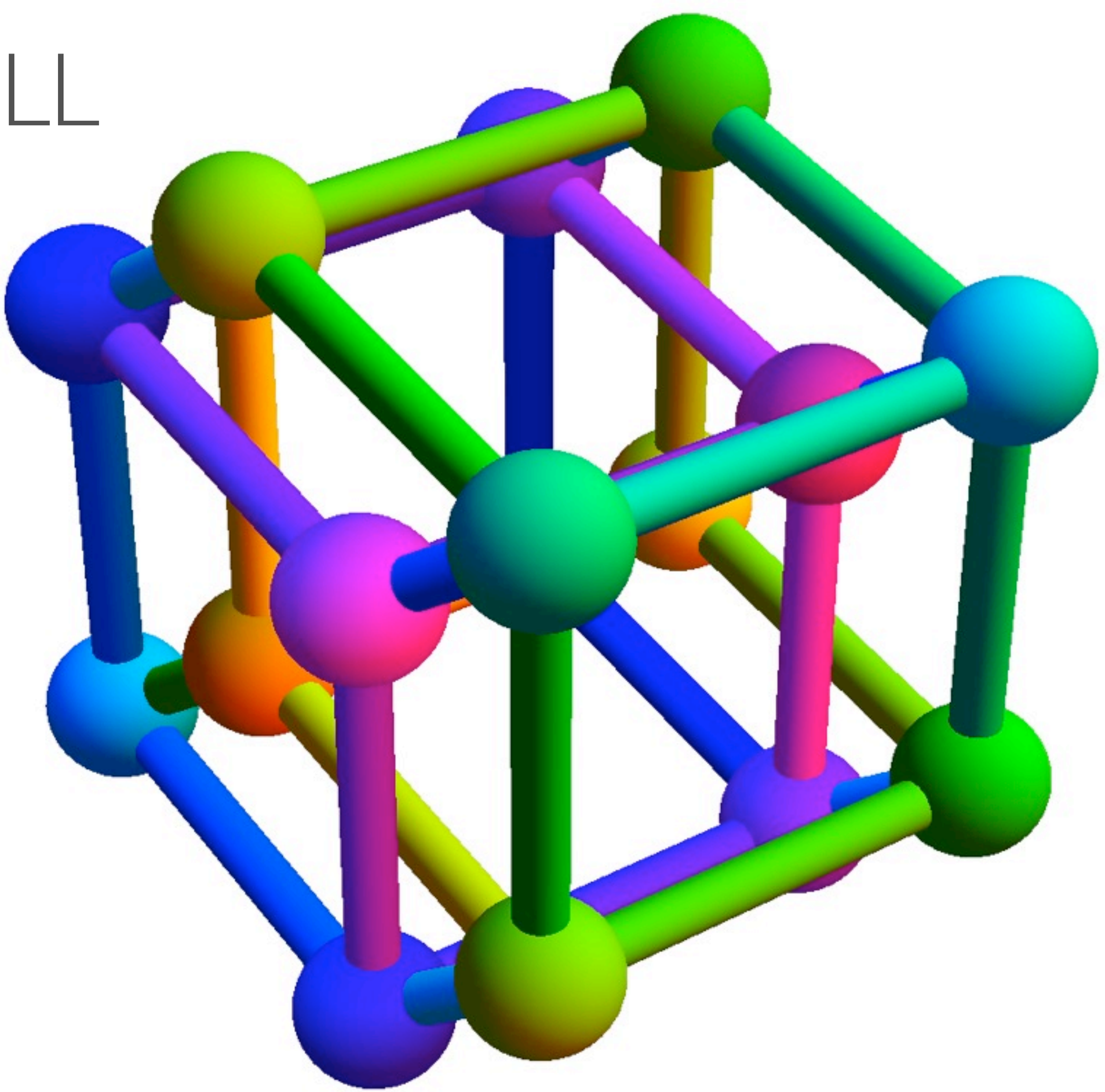
HIGHER DIMENSIONS



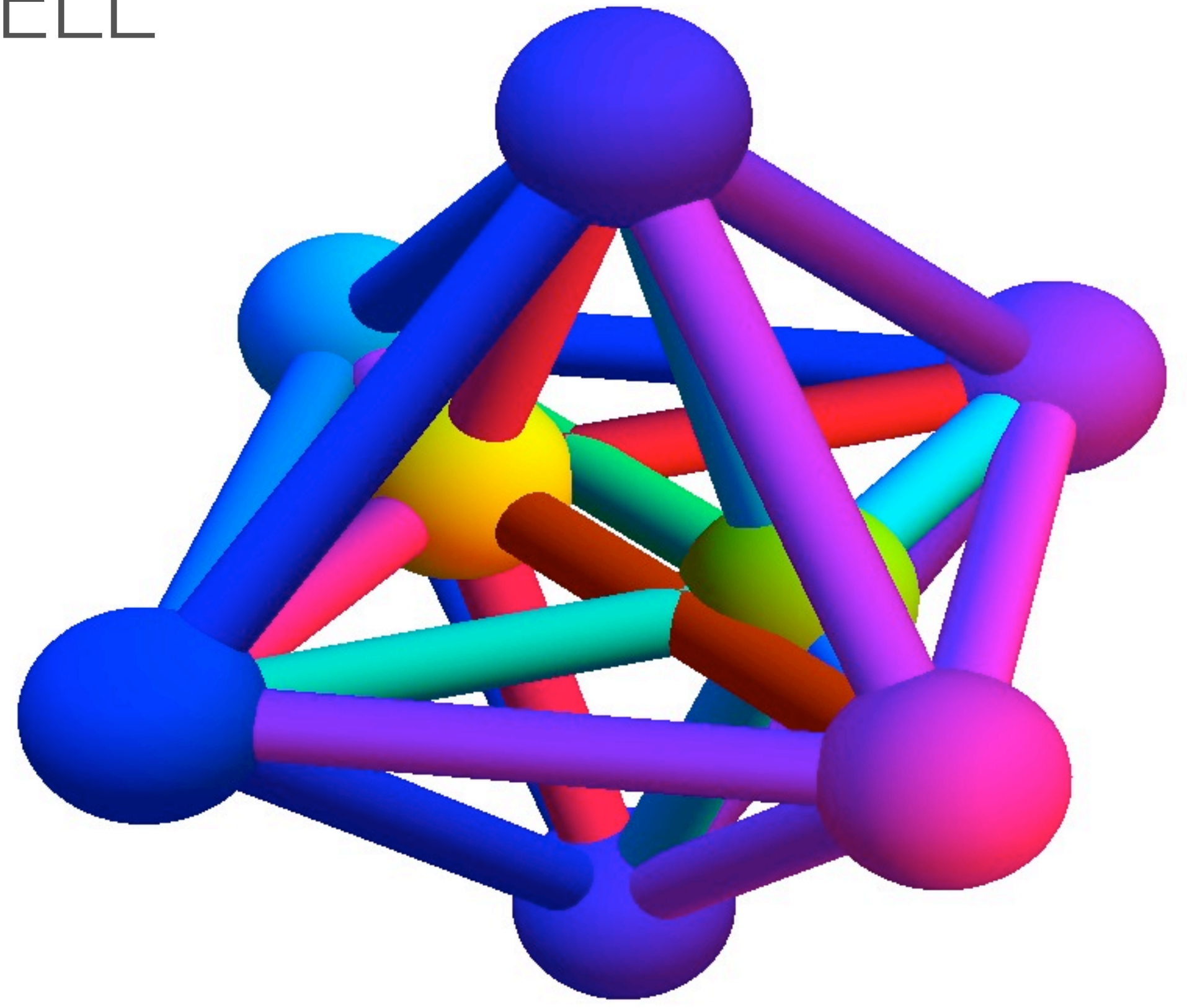
5 CELL



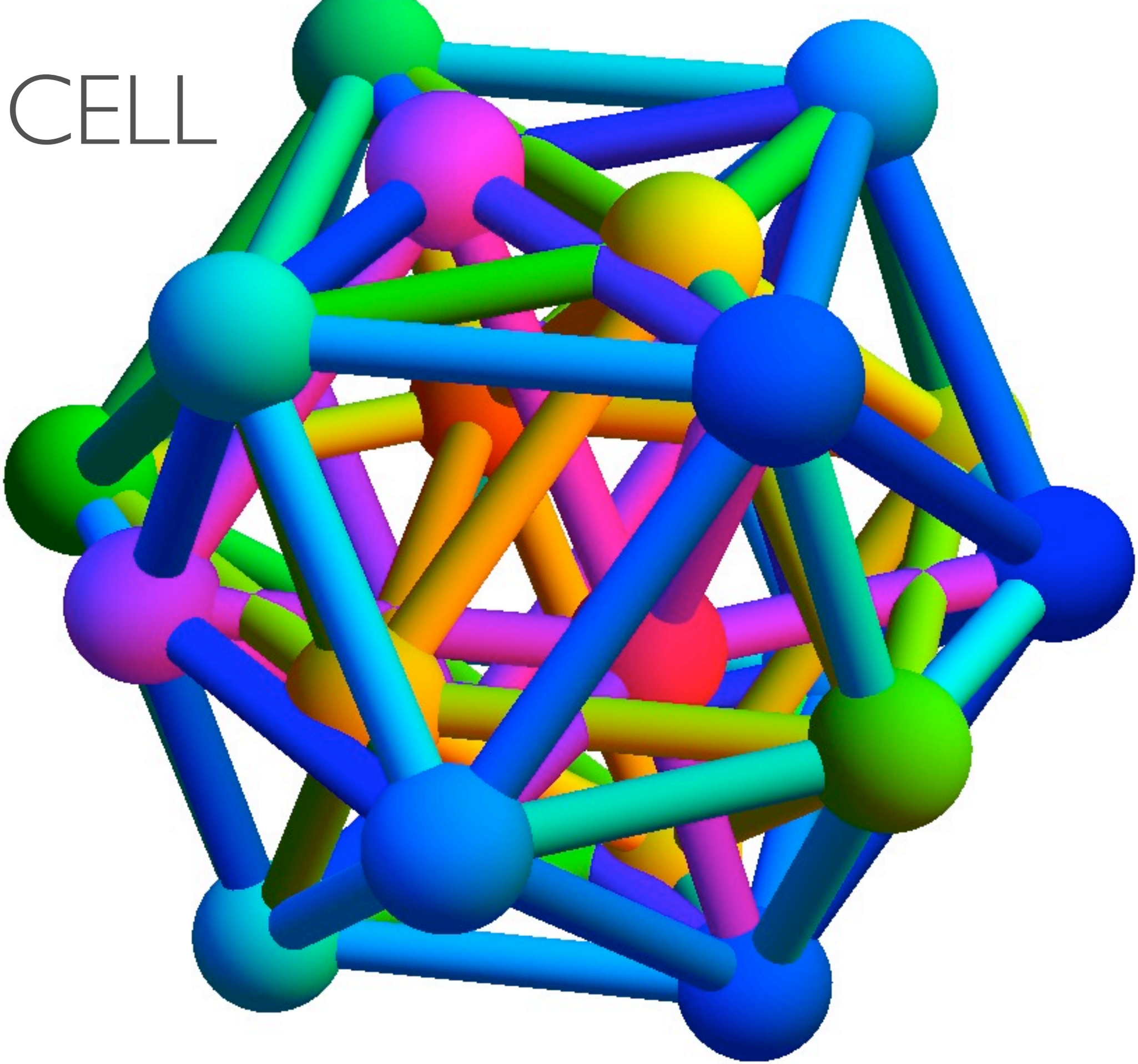
8 CELL



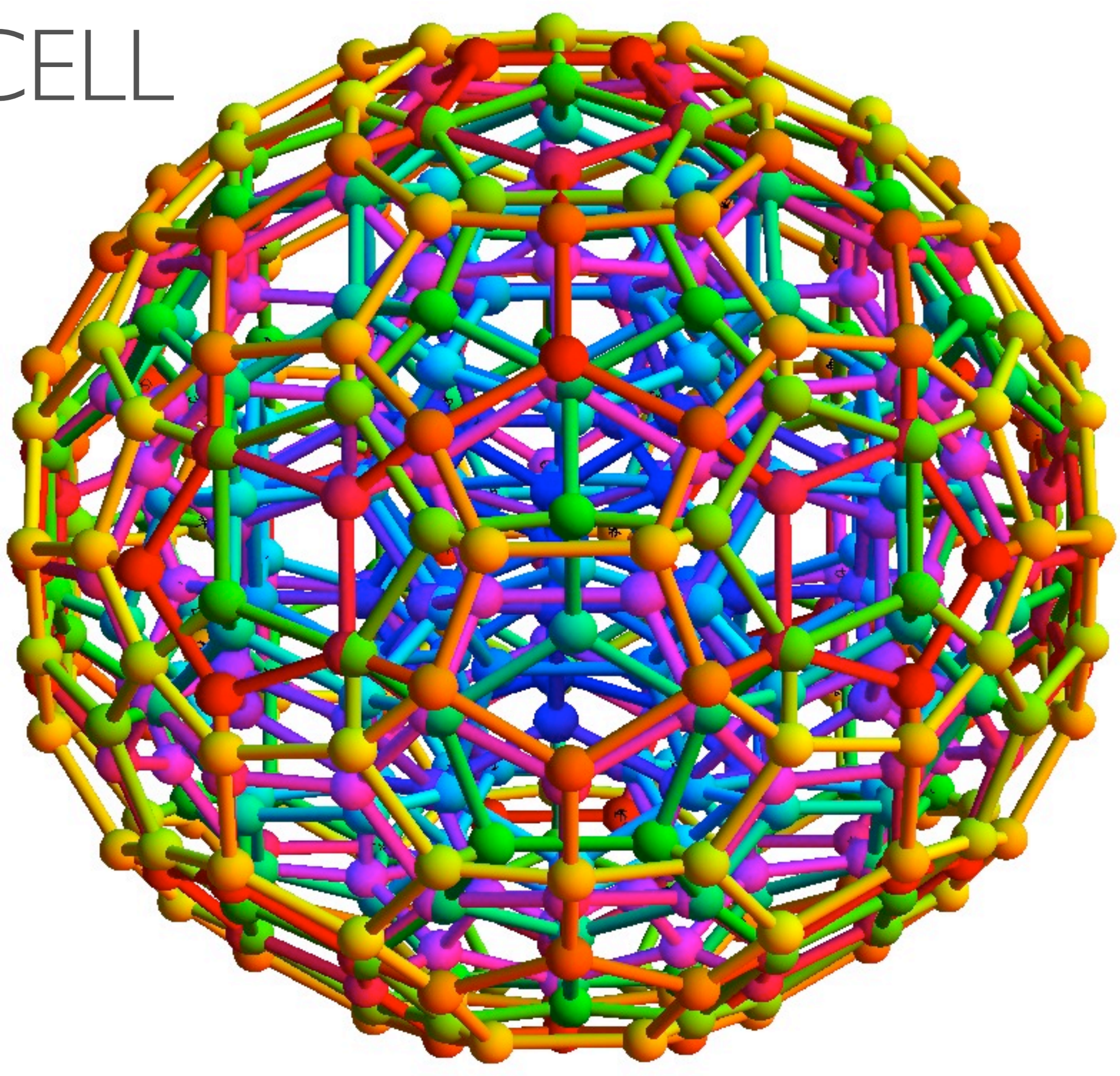
16 CELL



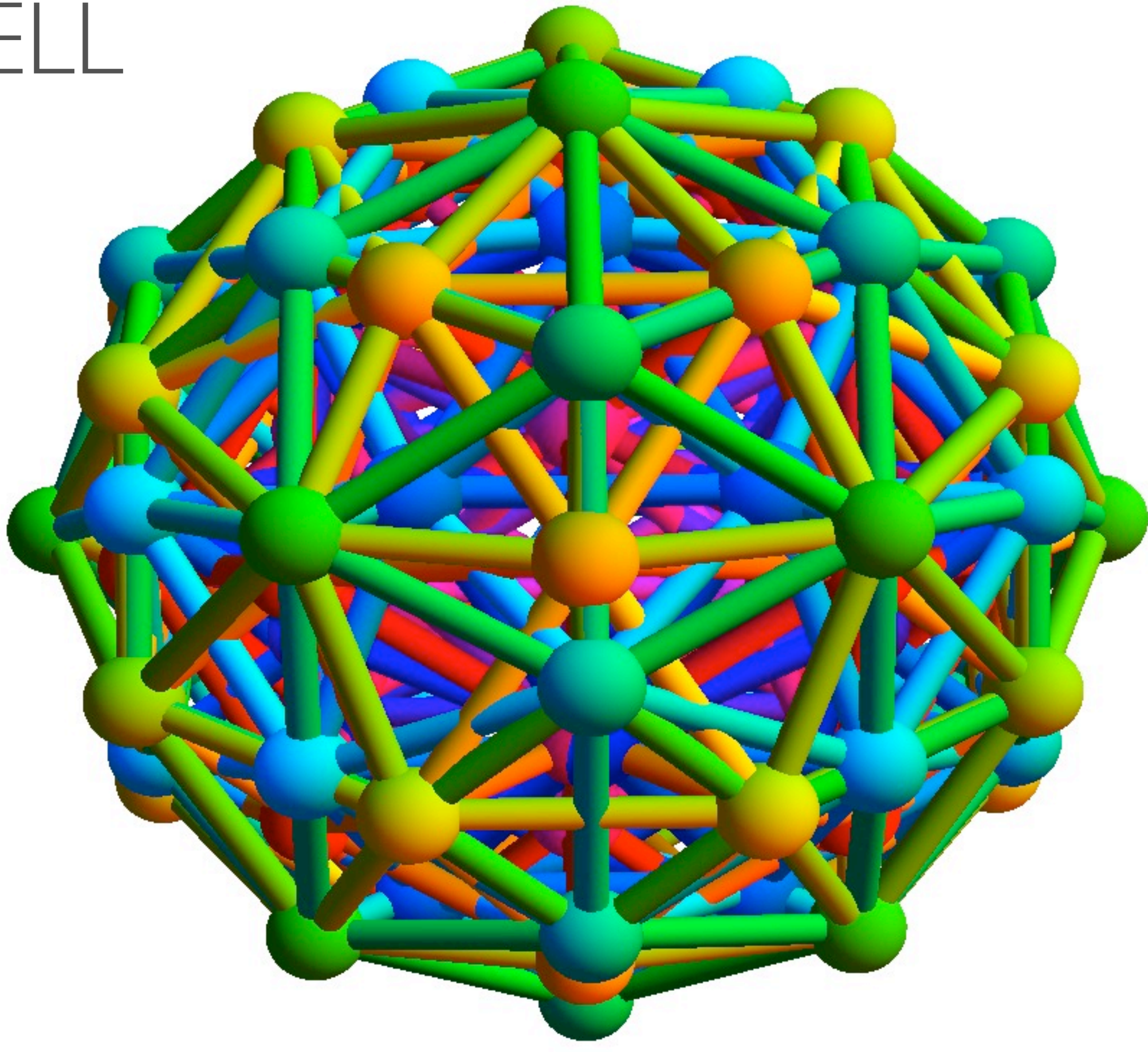
24 CELL

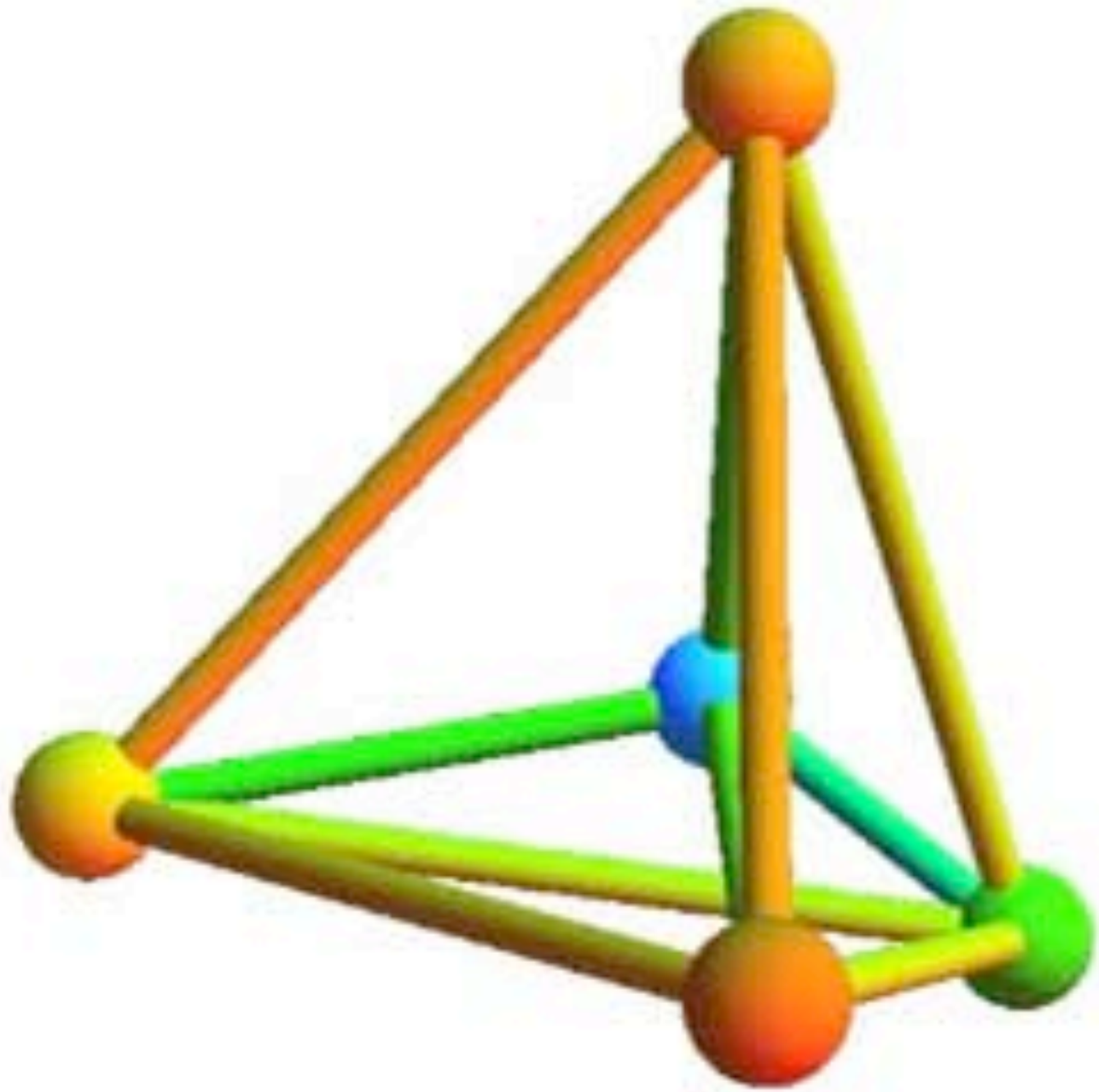


120 CELL

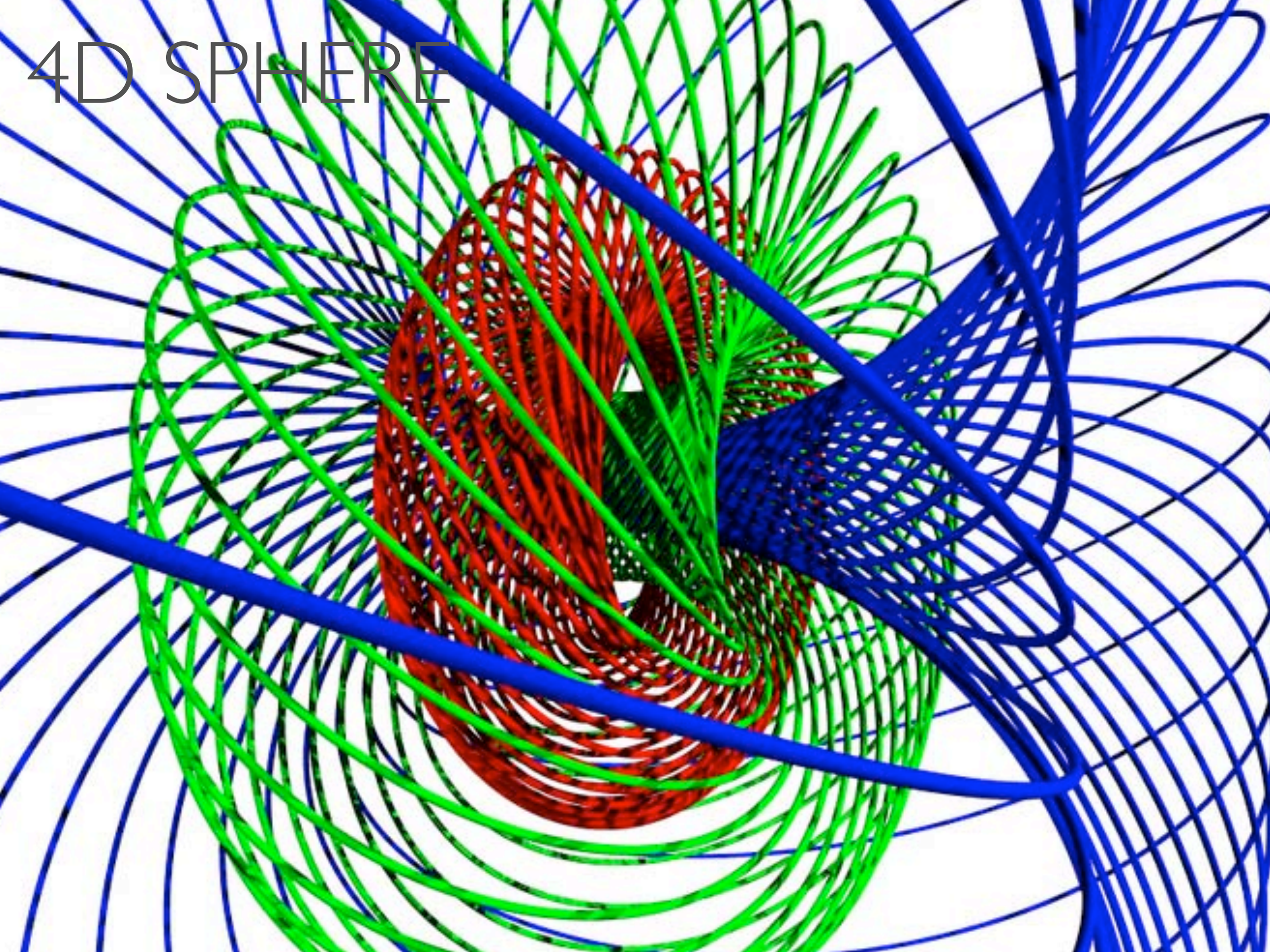


600 CELL



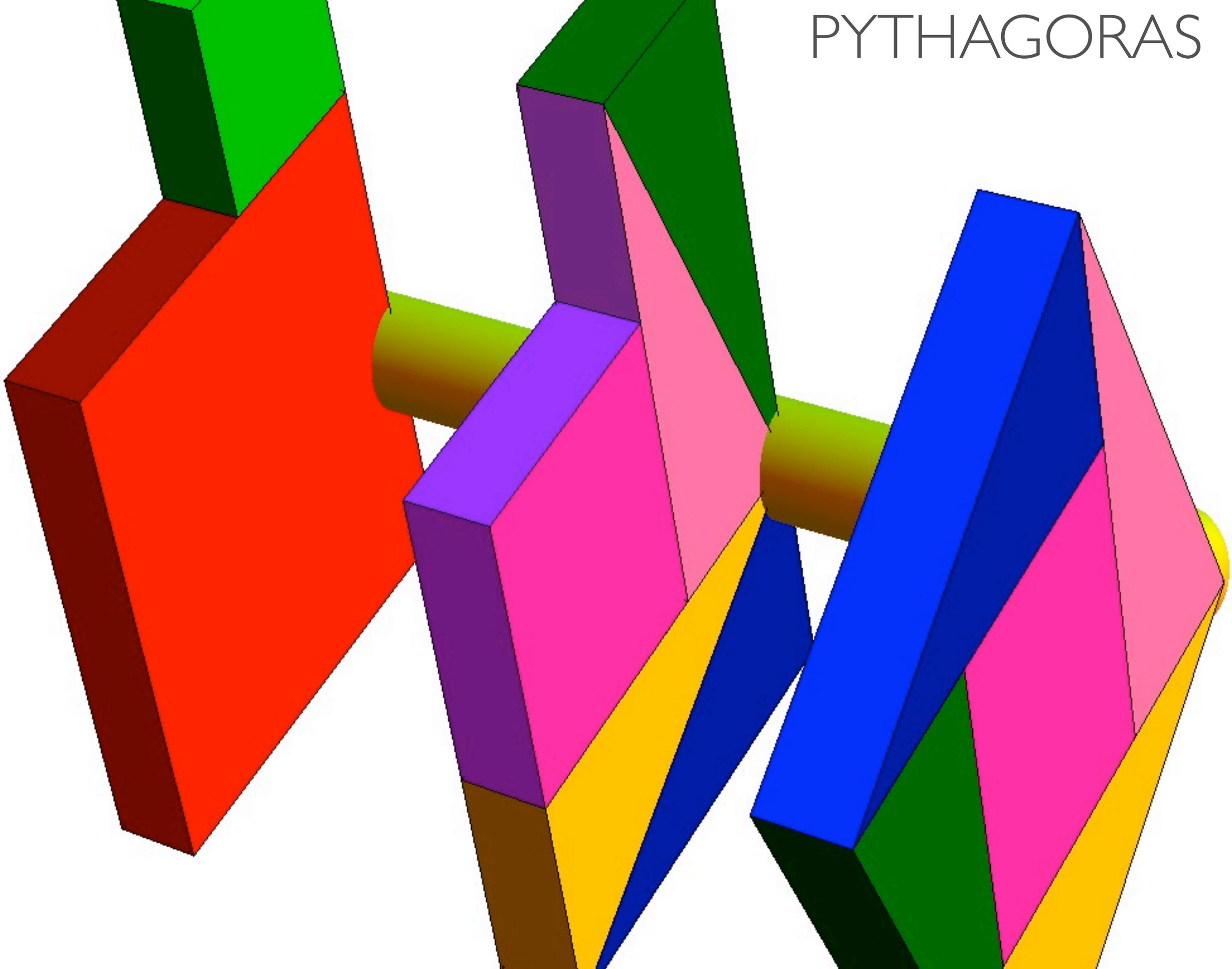


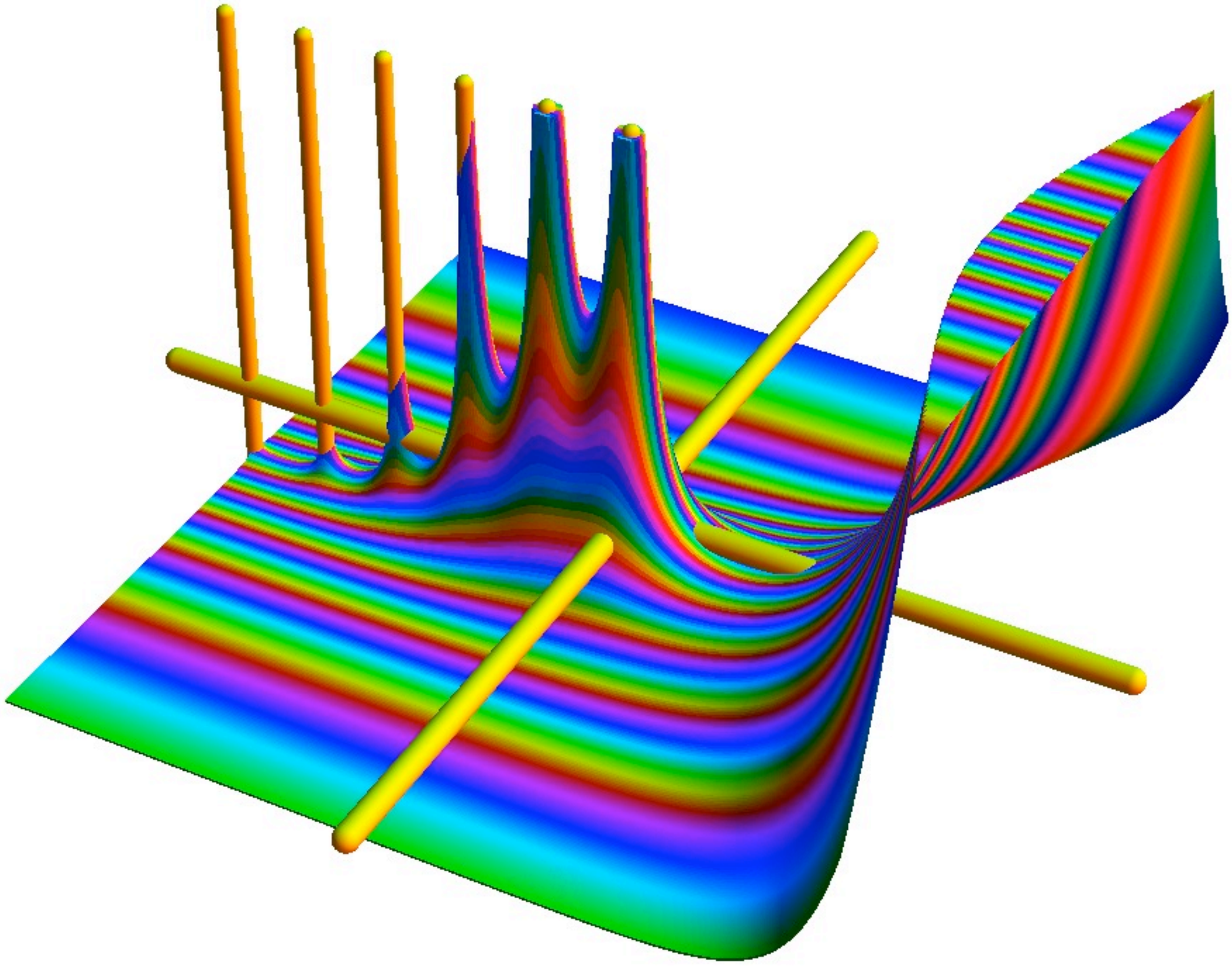
4D SPHERE

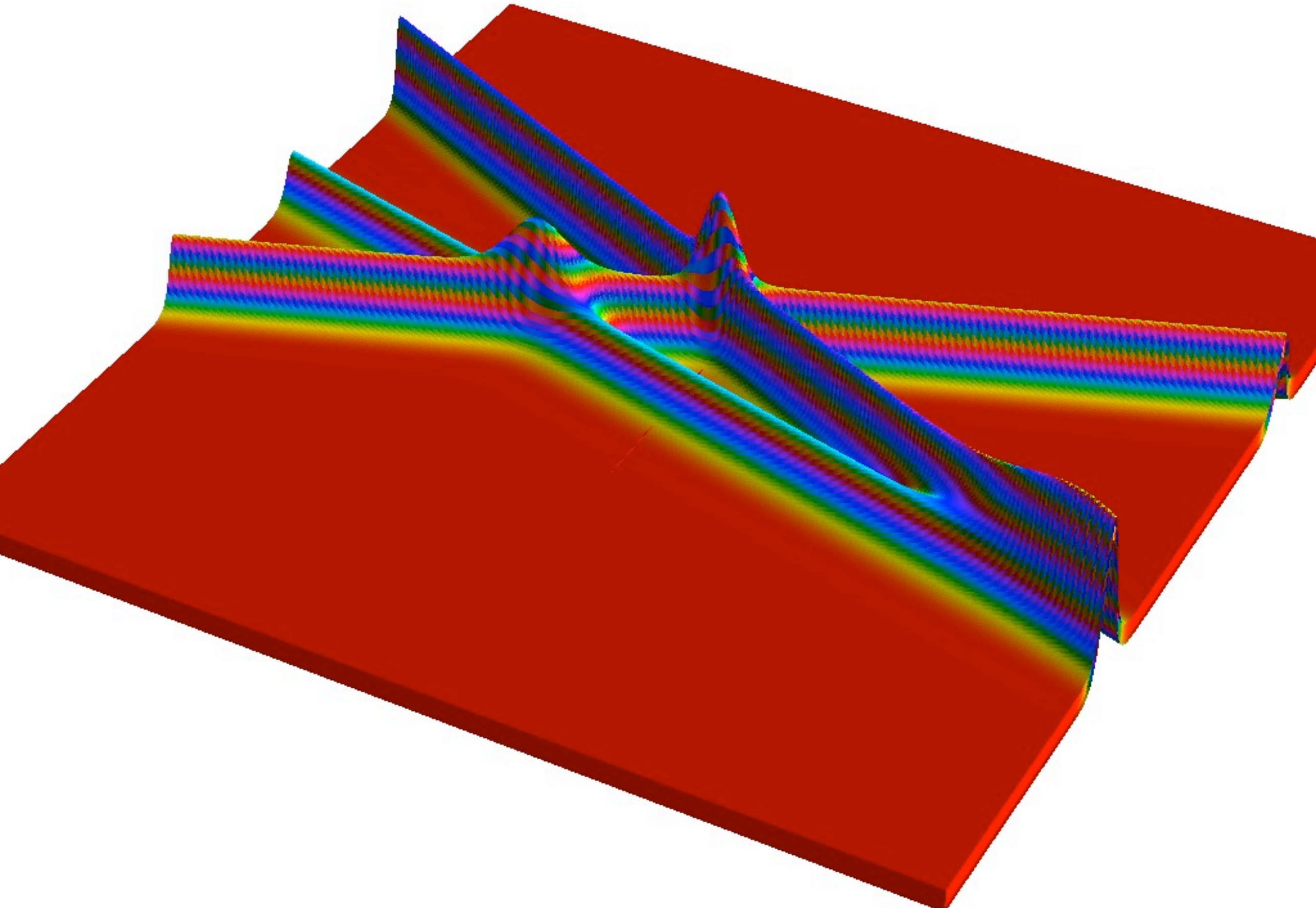


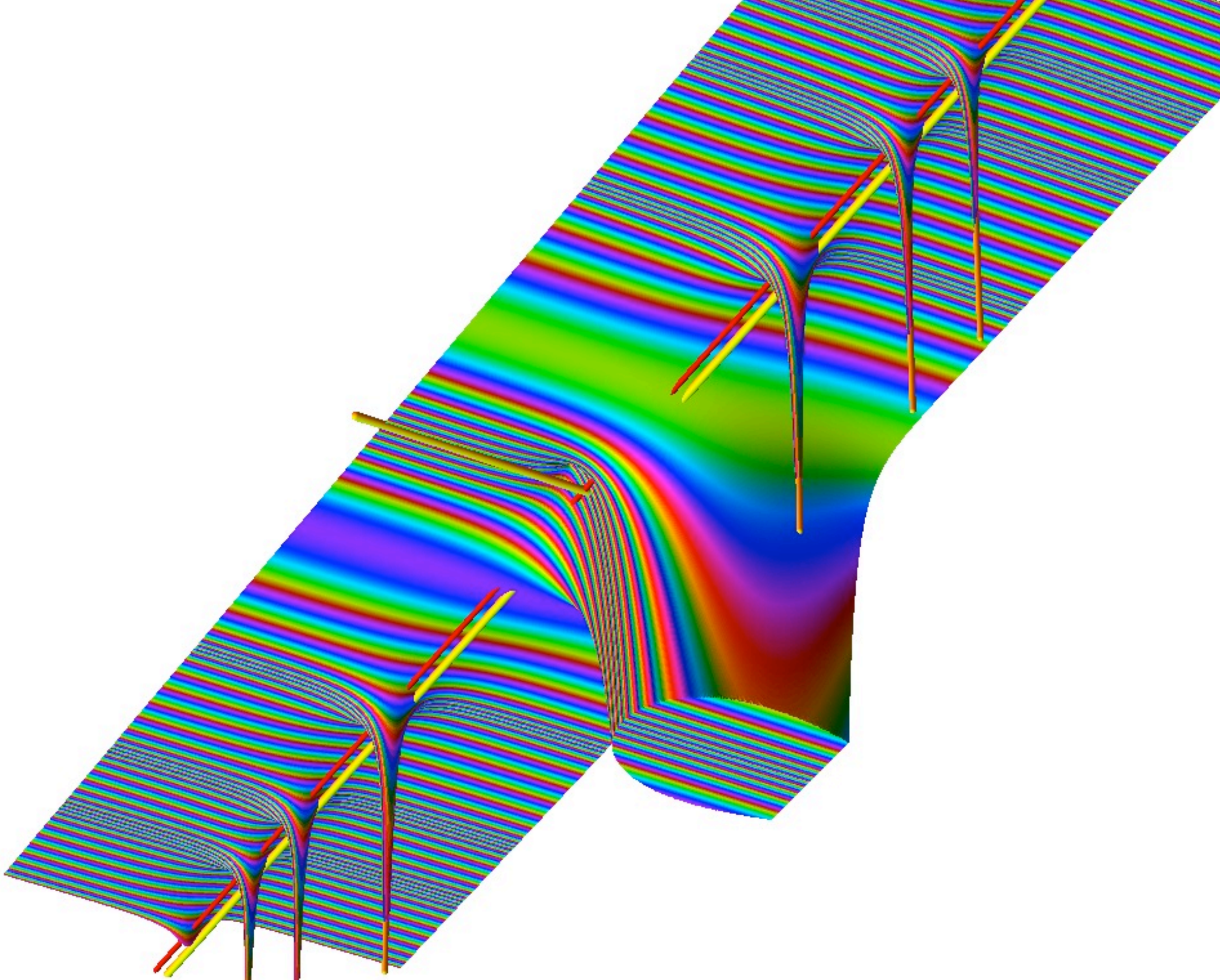
PRINTING MATHEMATICS

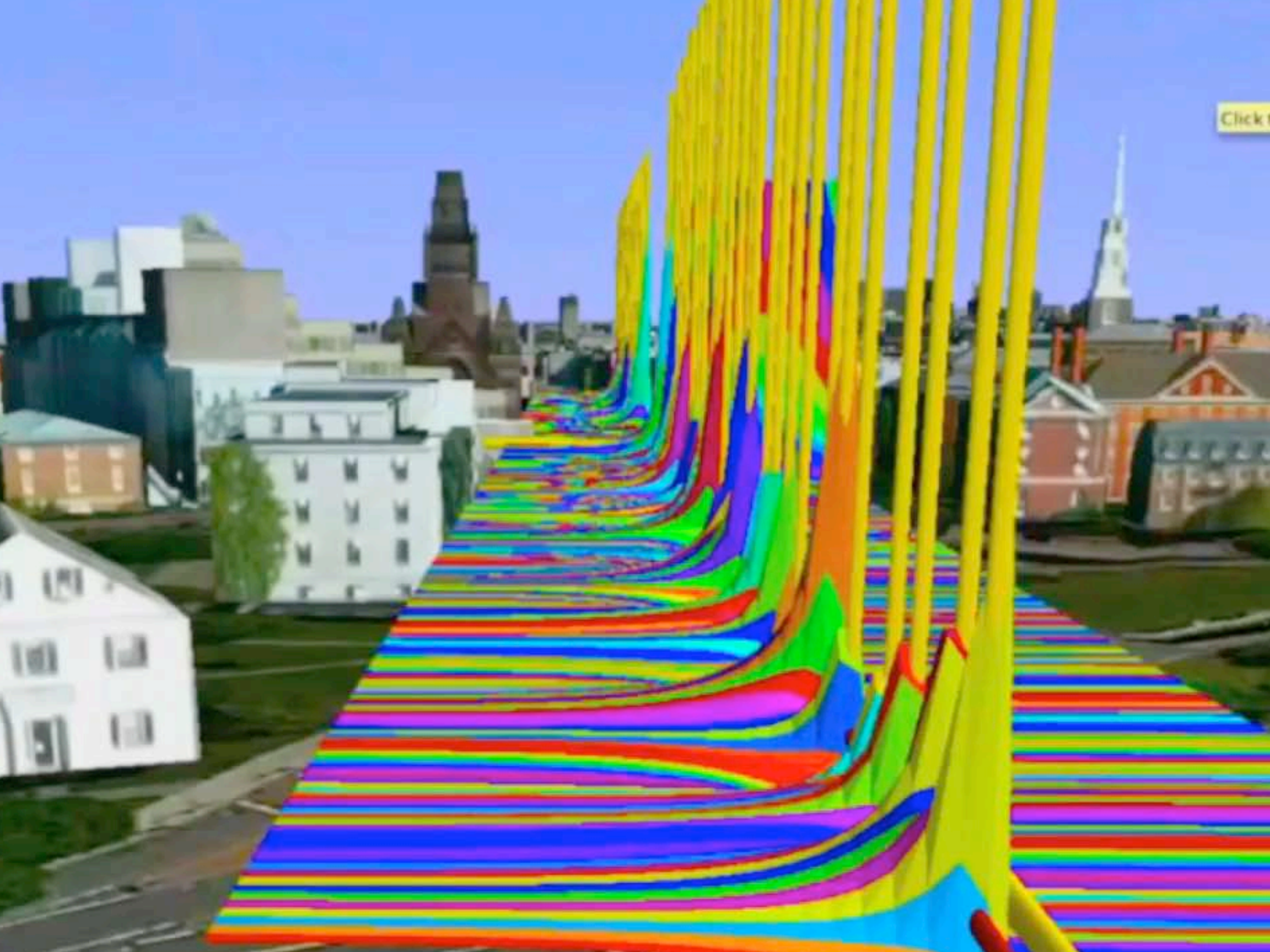
PYTHAGORAS

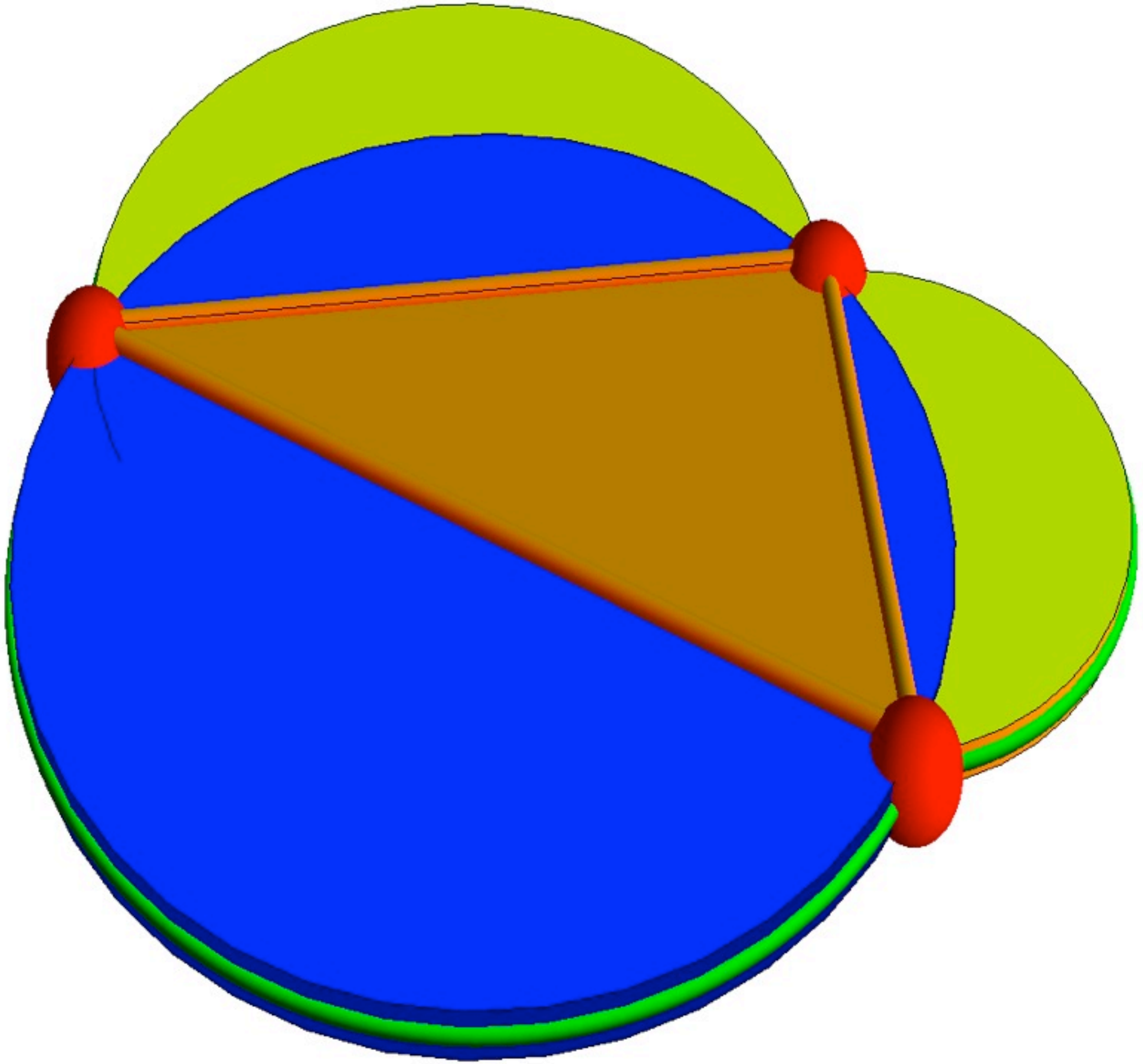


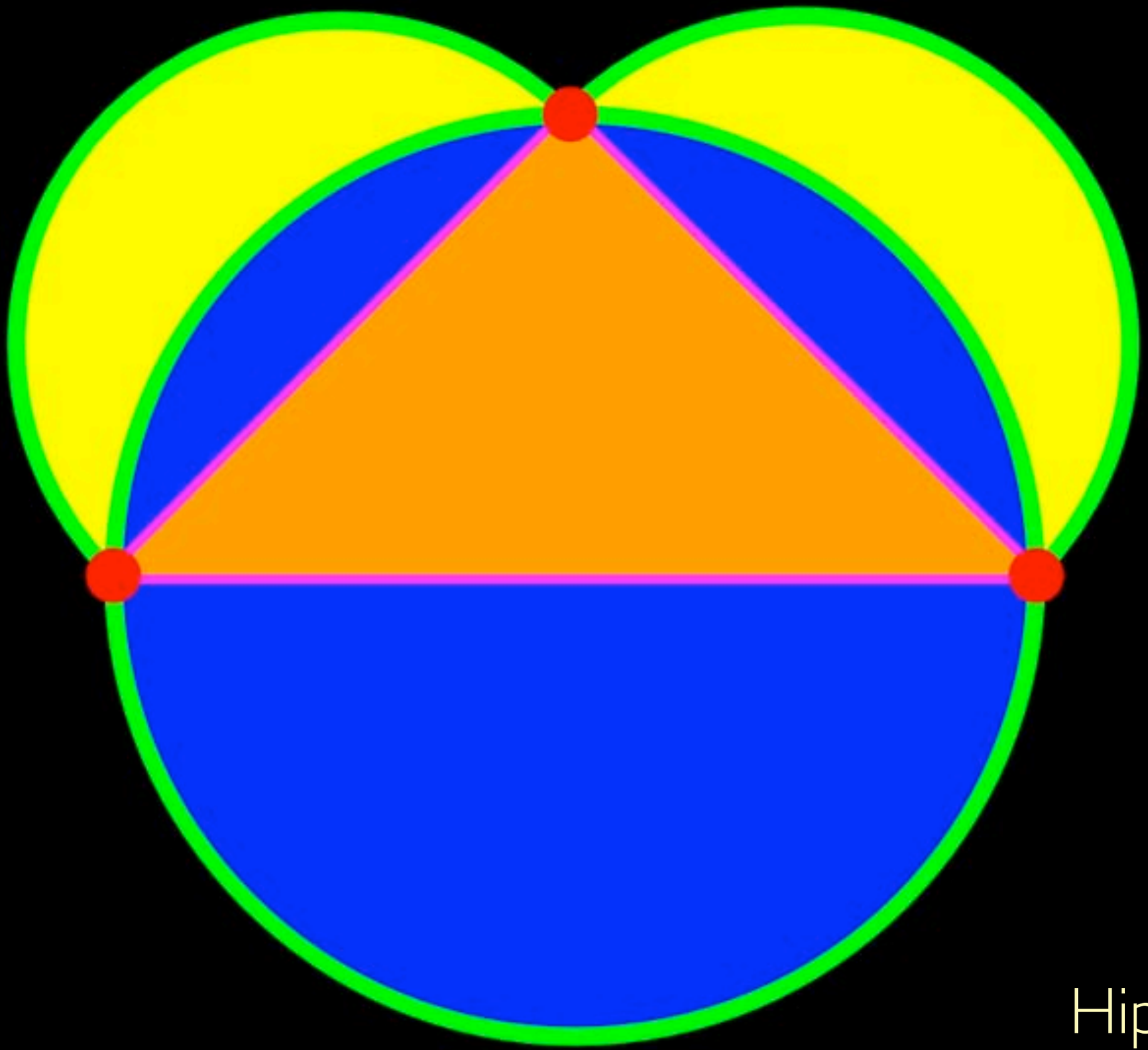




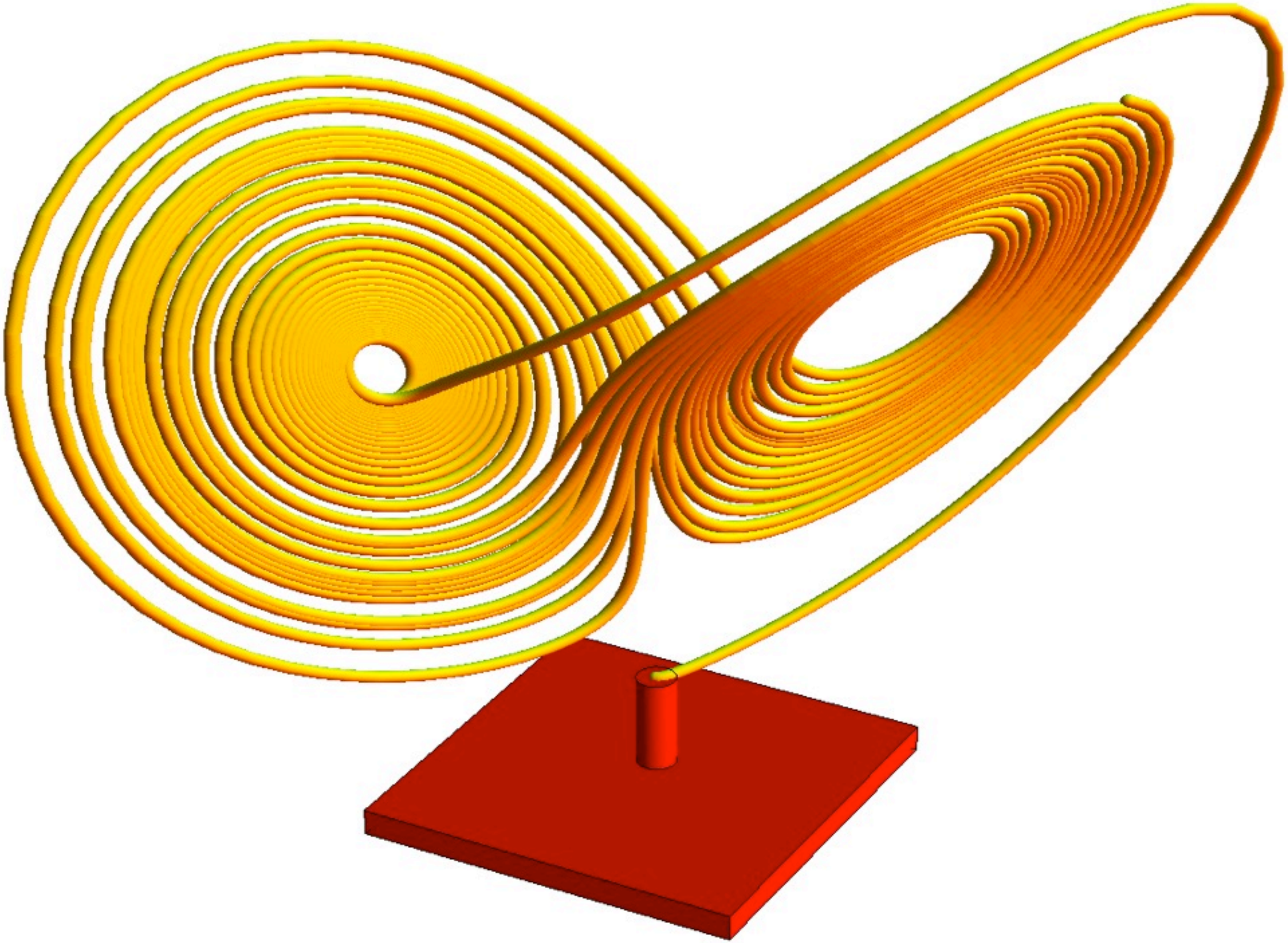


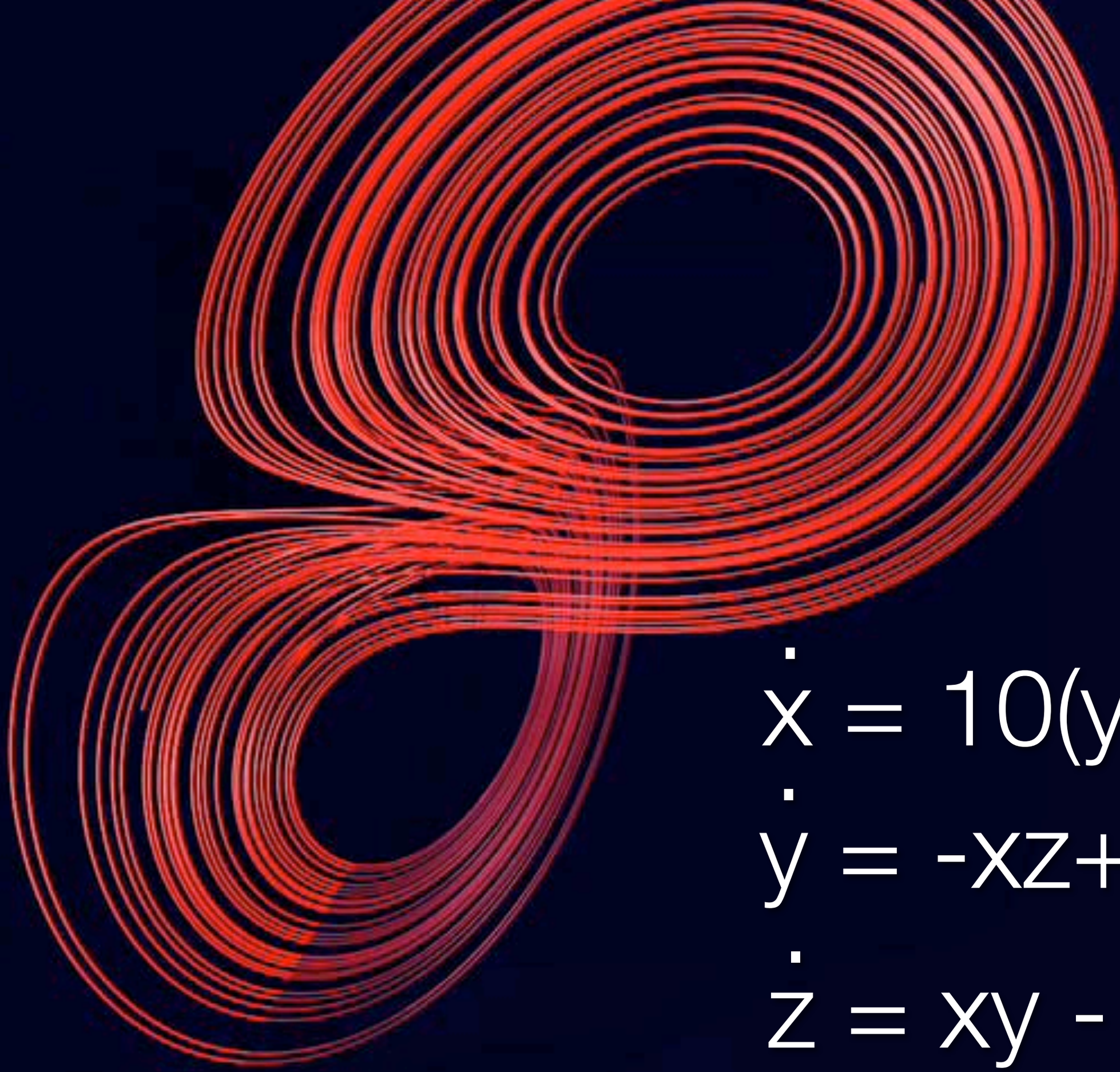






Hipocrates

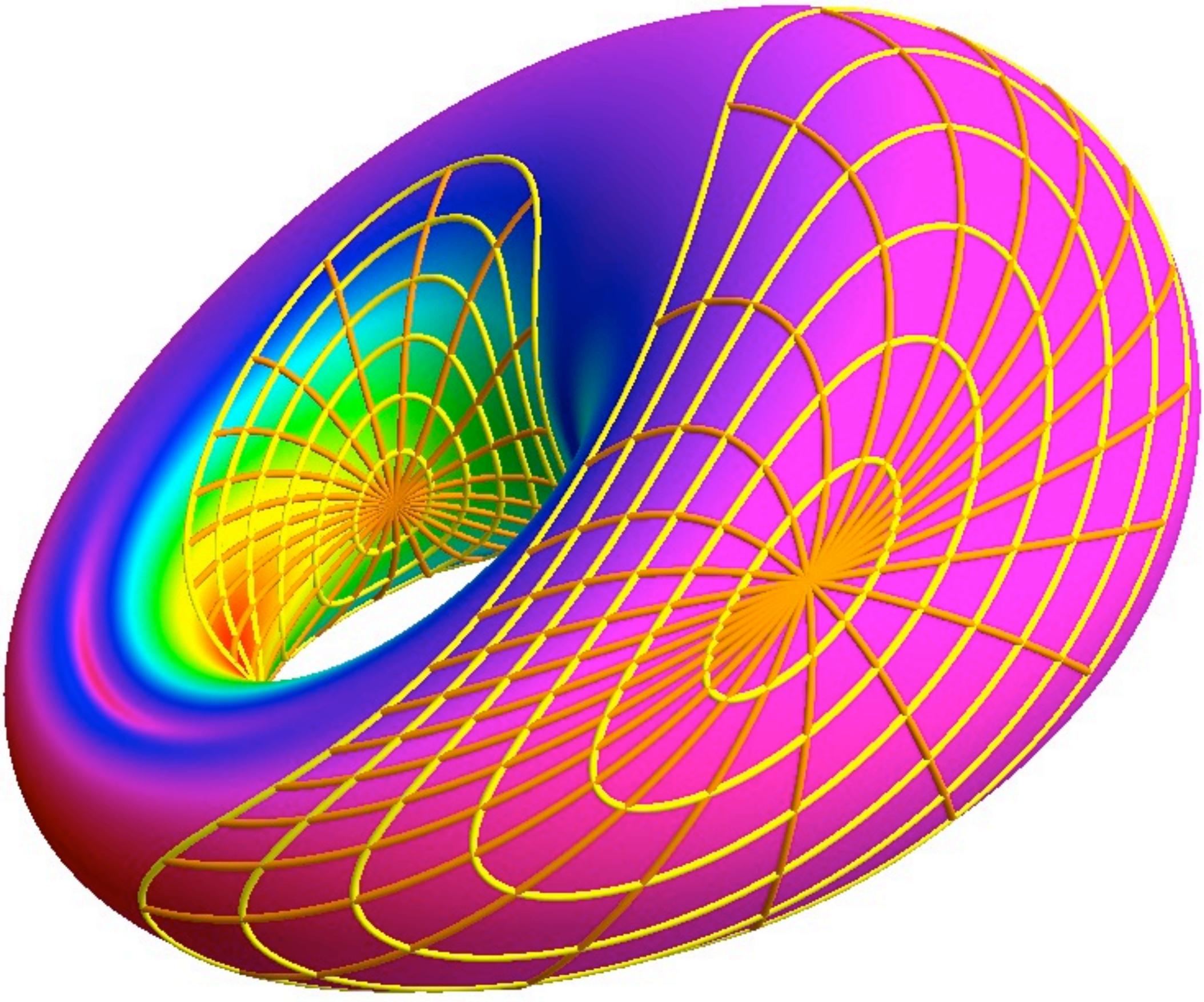


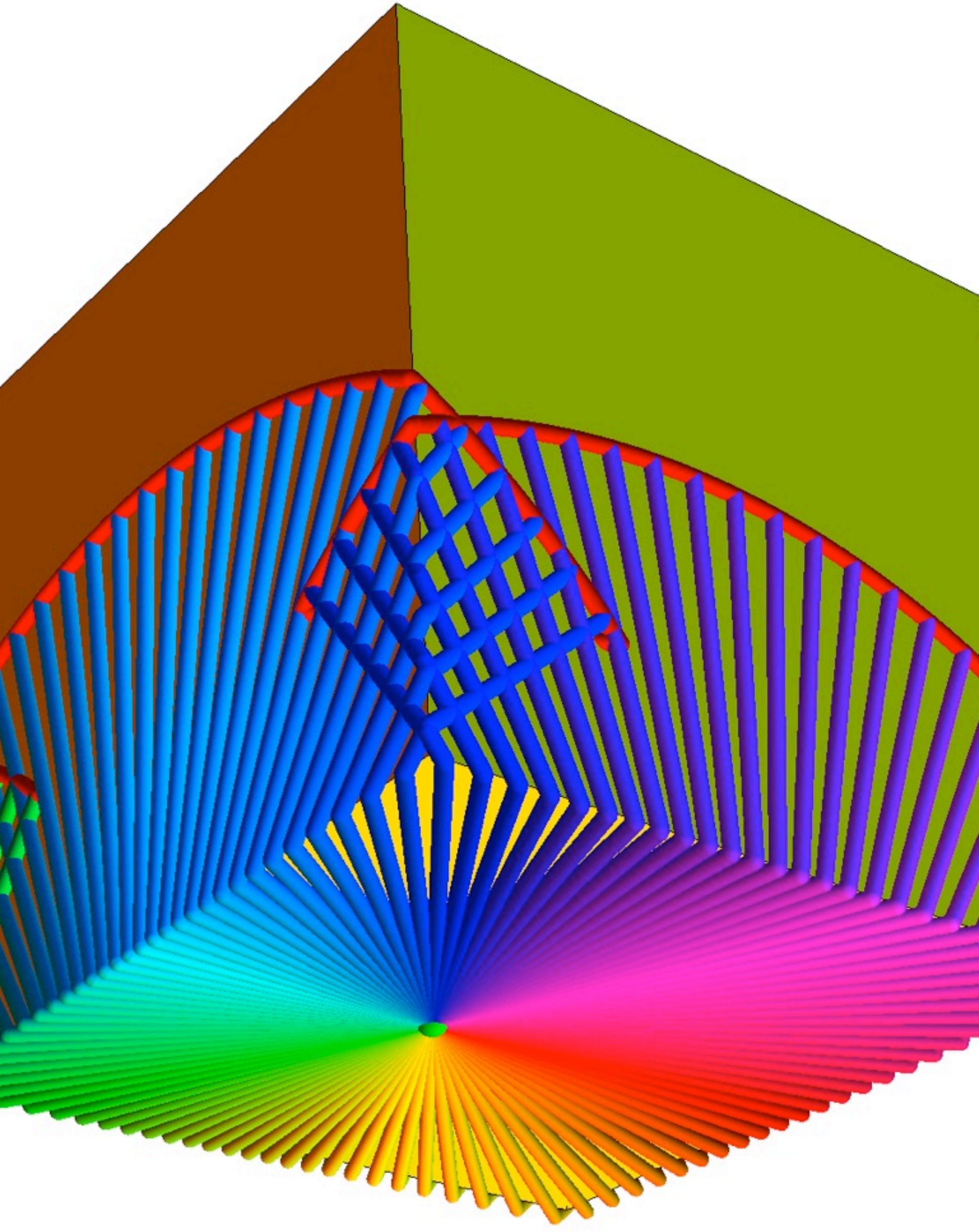


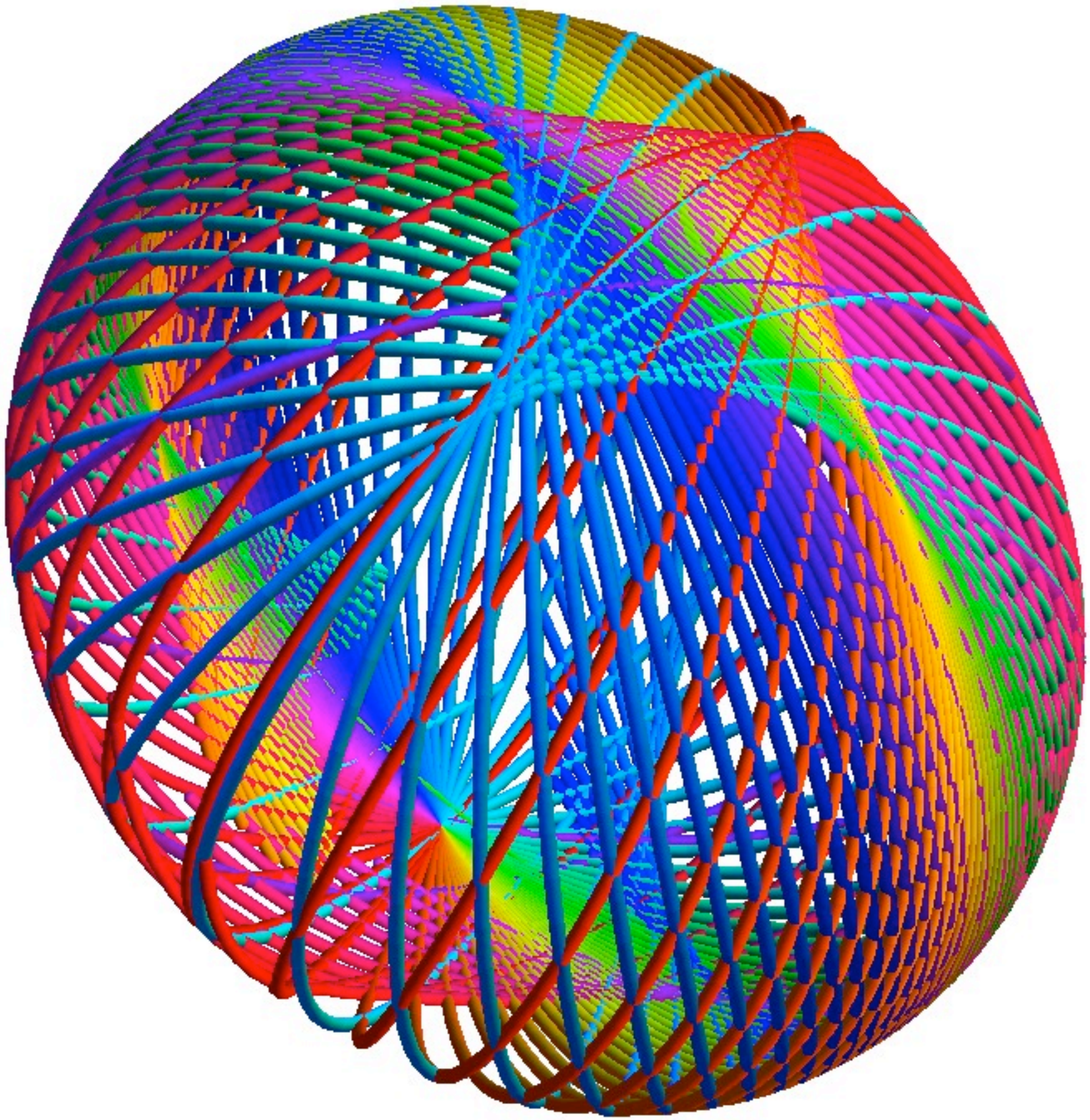
$$\dot{x} = 10(y-x)$$

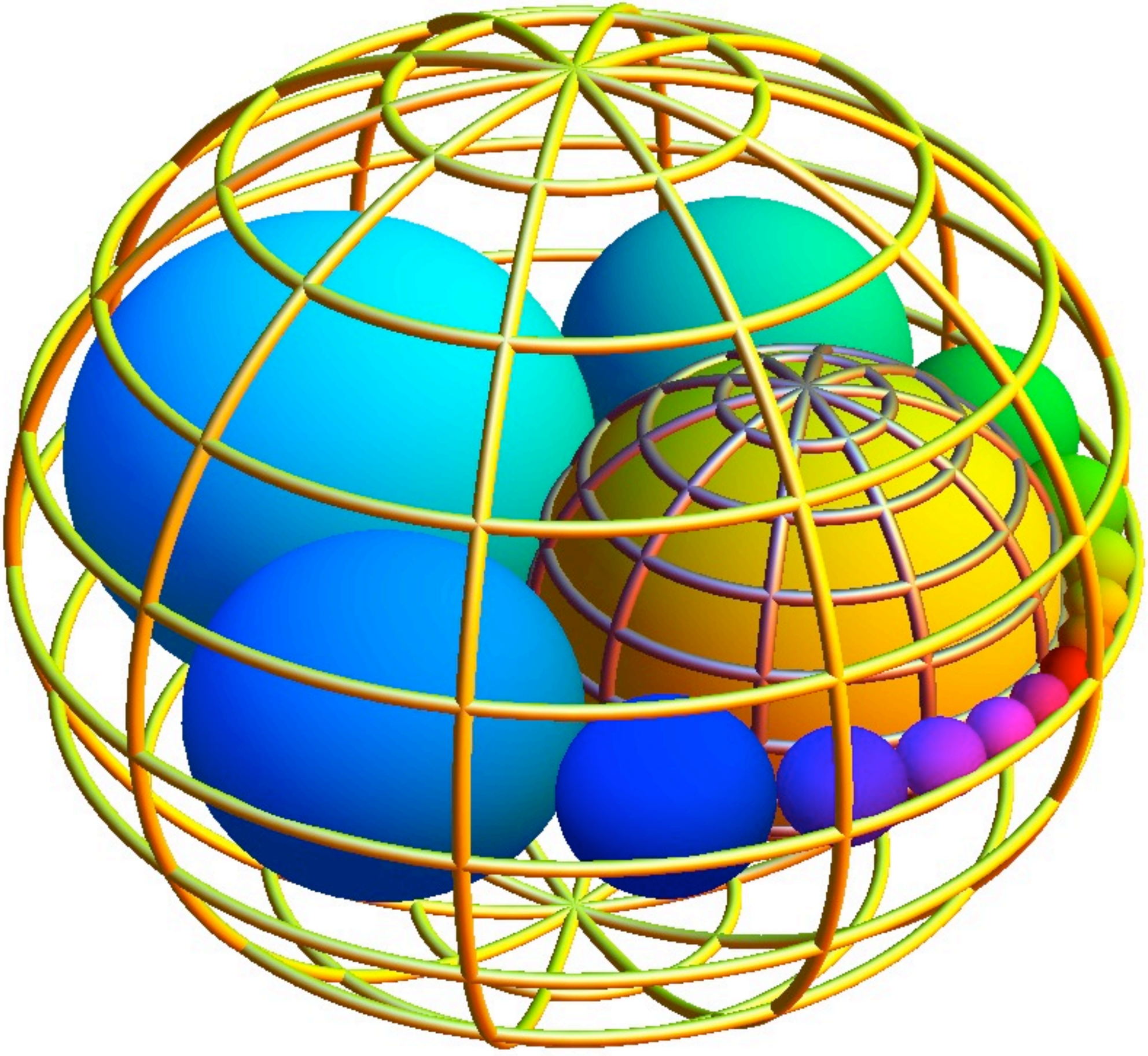
$$\dot{y} = -xz+28x-y$$

$$\dot{z} = xy - z\frac{8}{3}$$

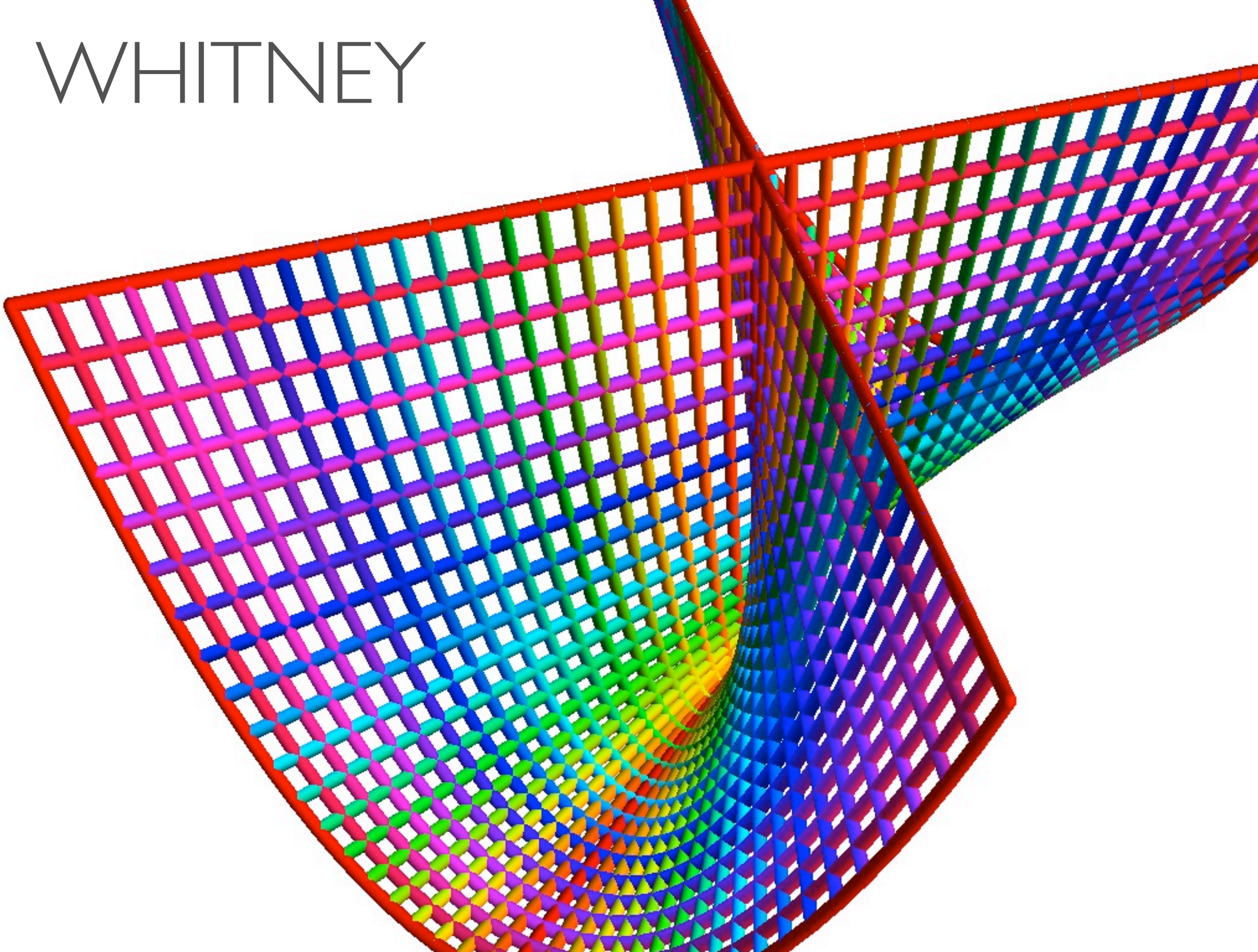


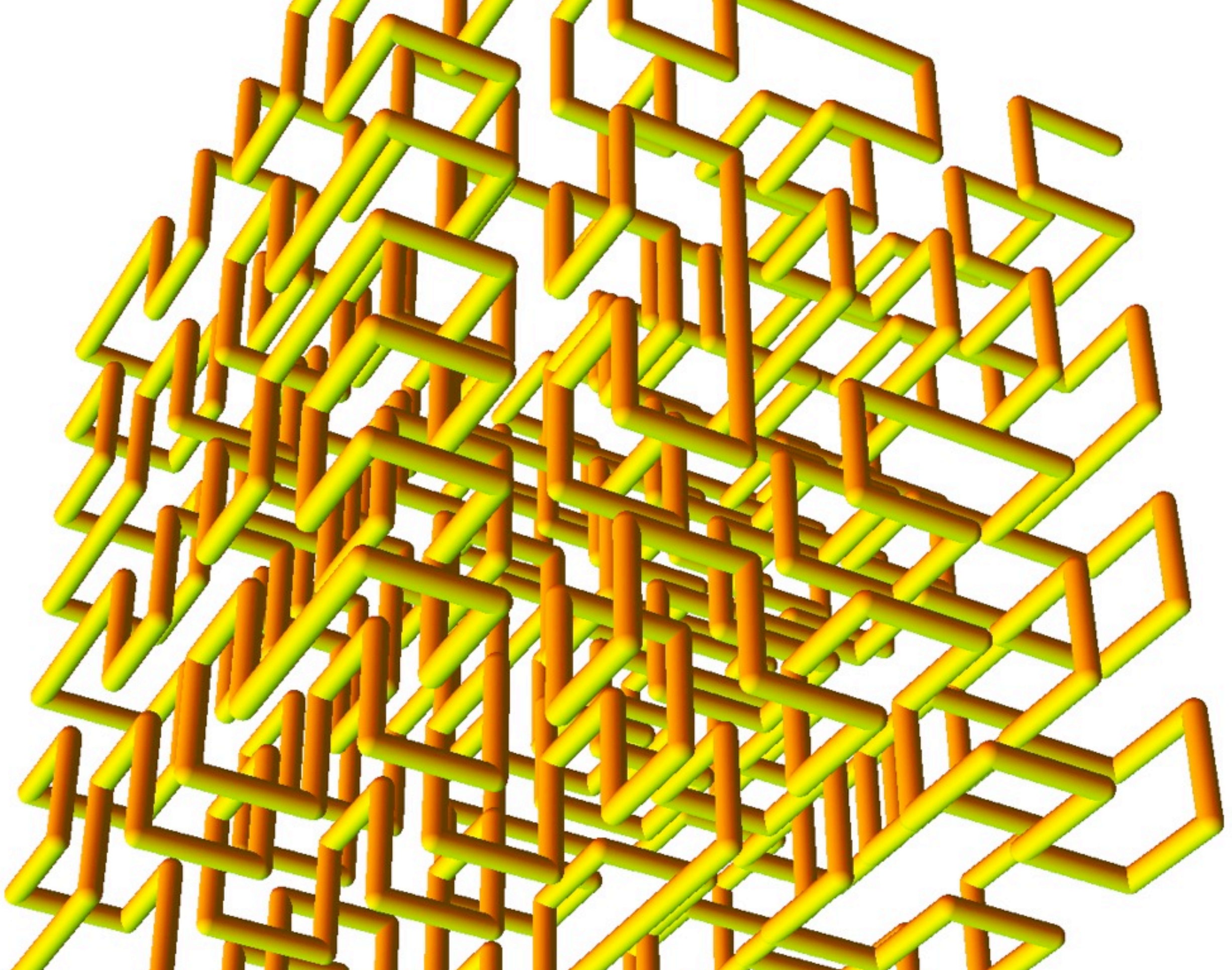


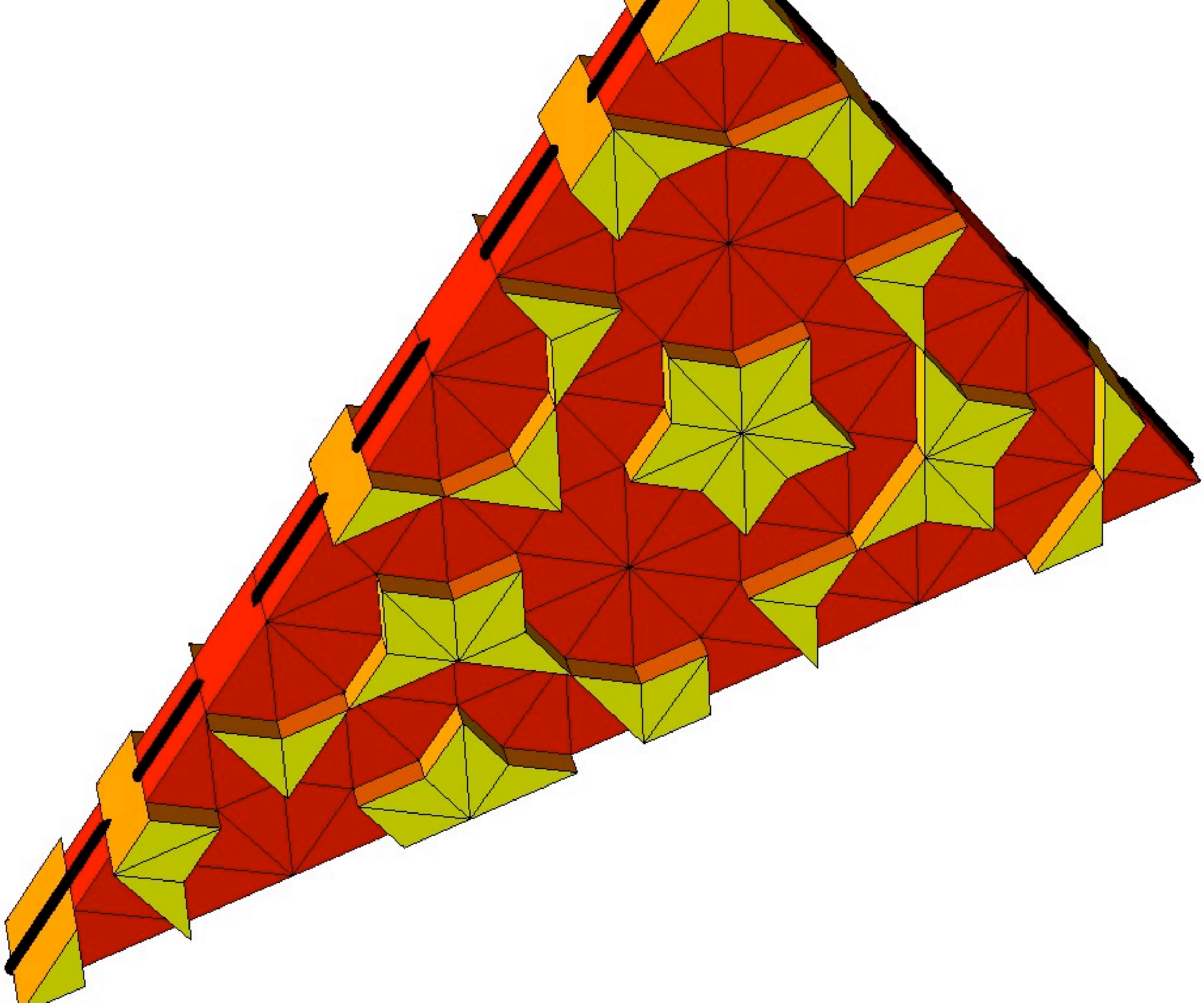


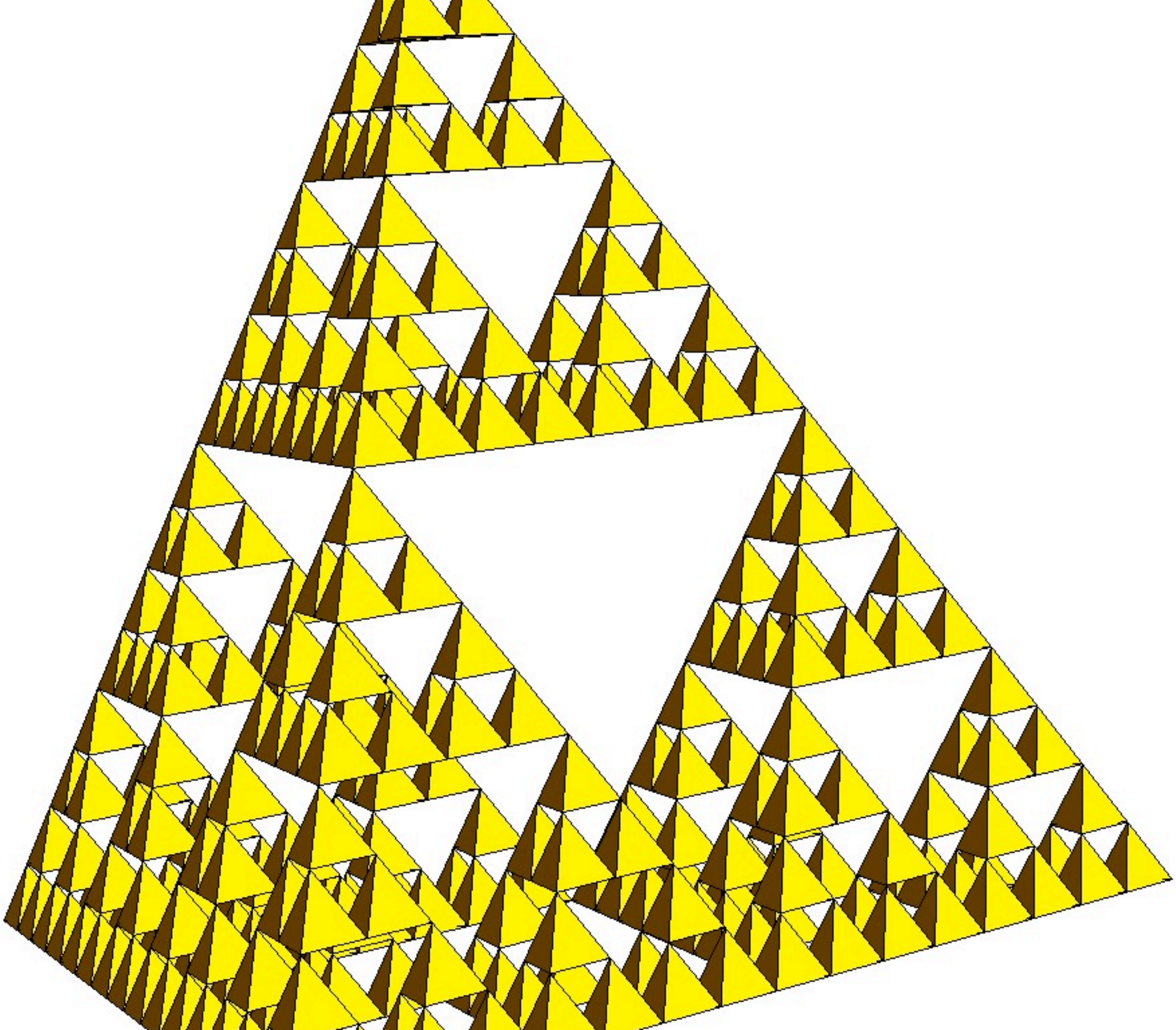


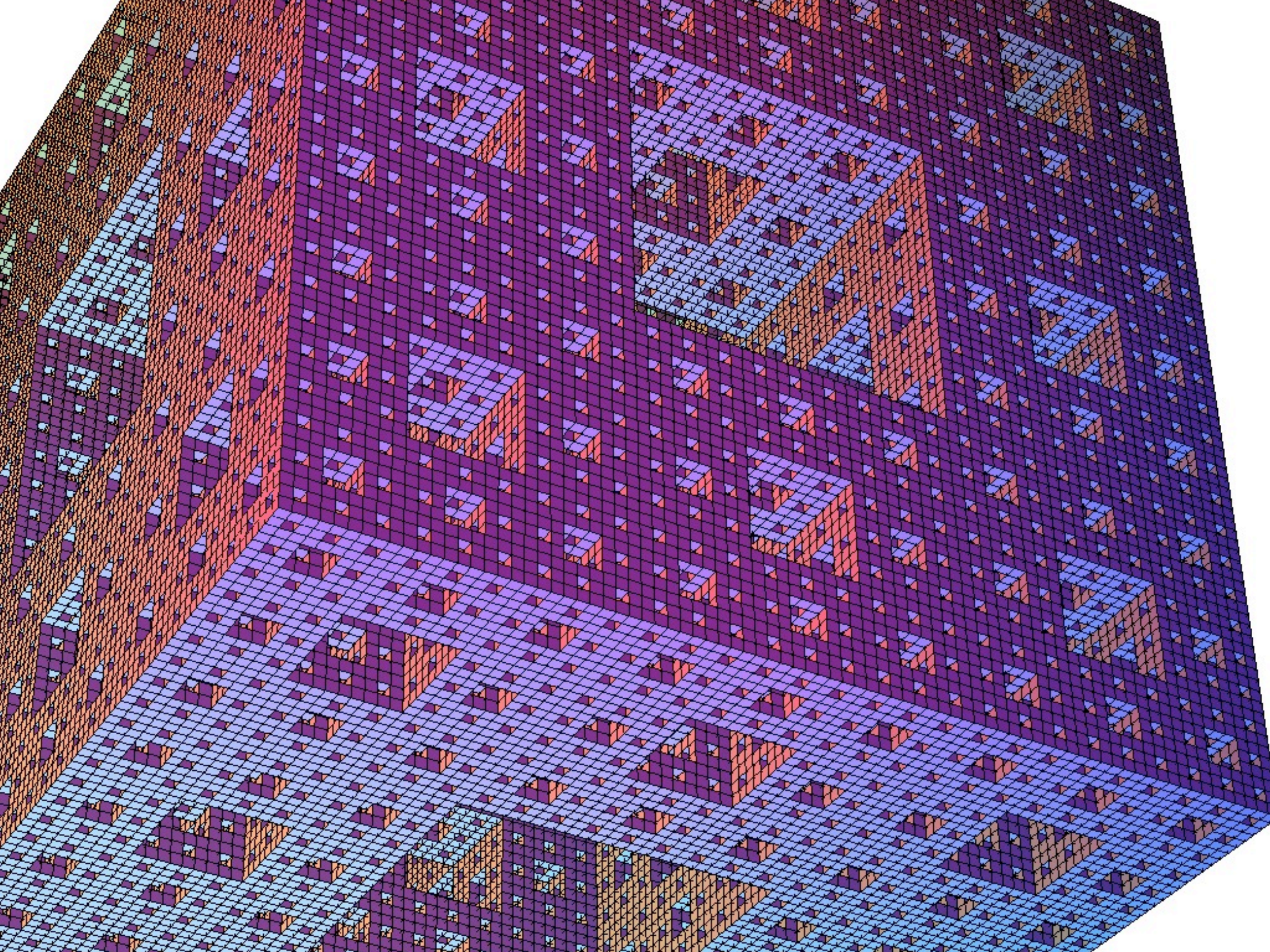
WHITNEY

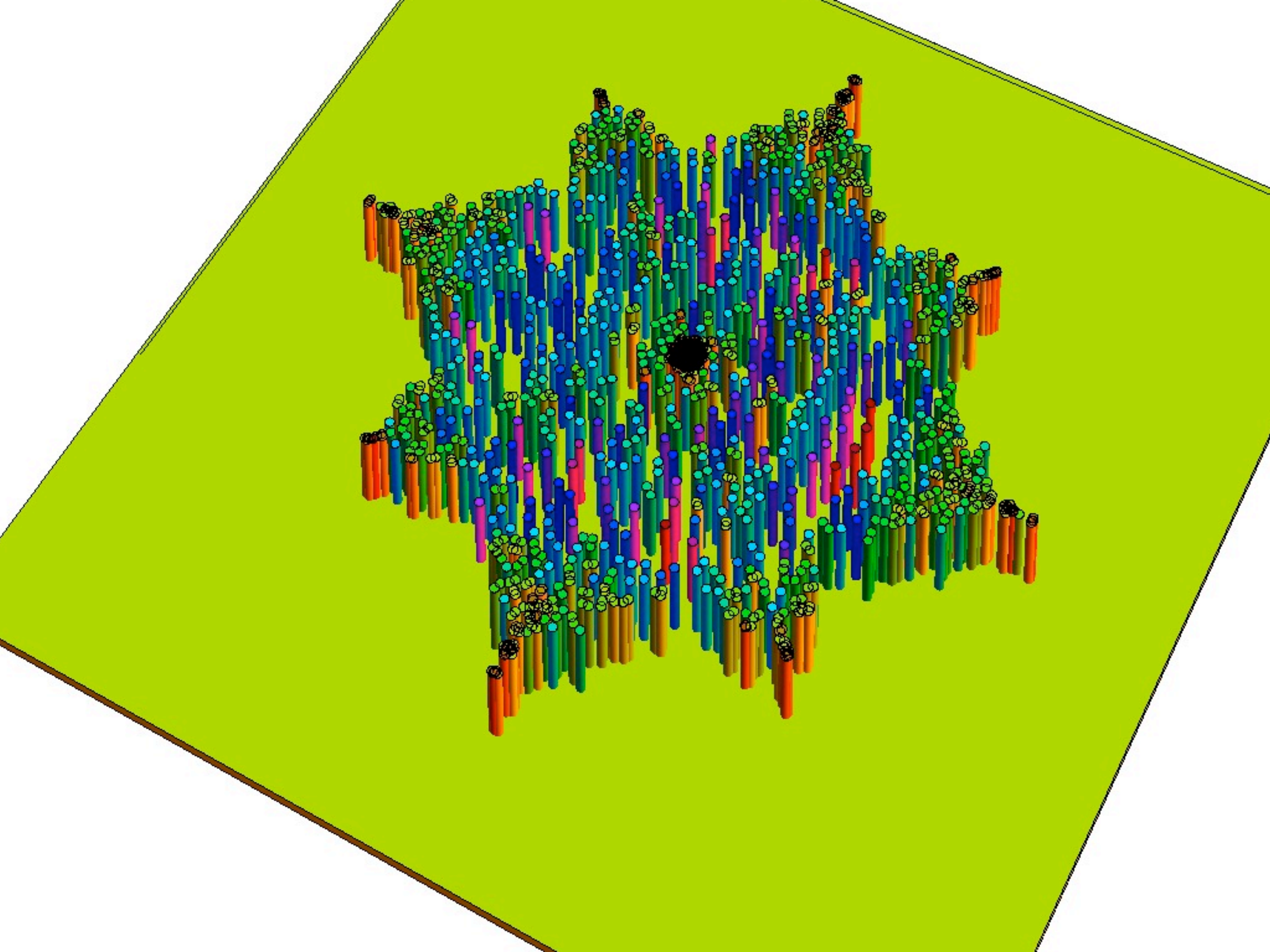


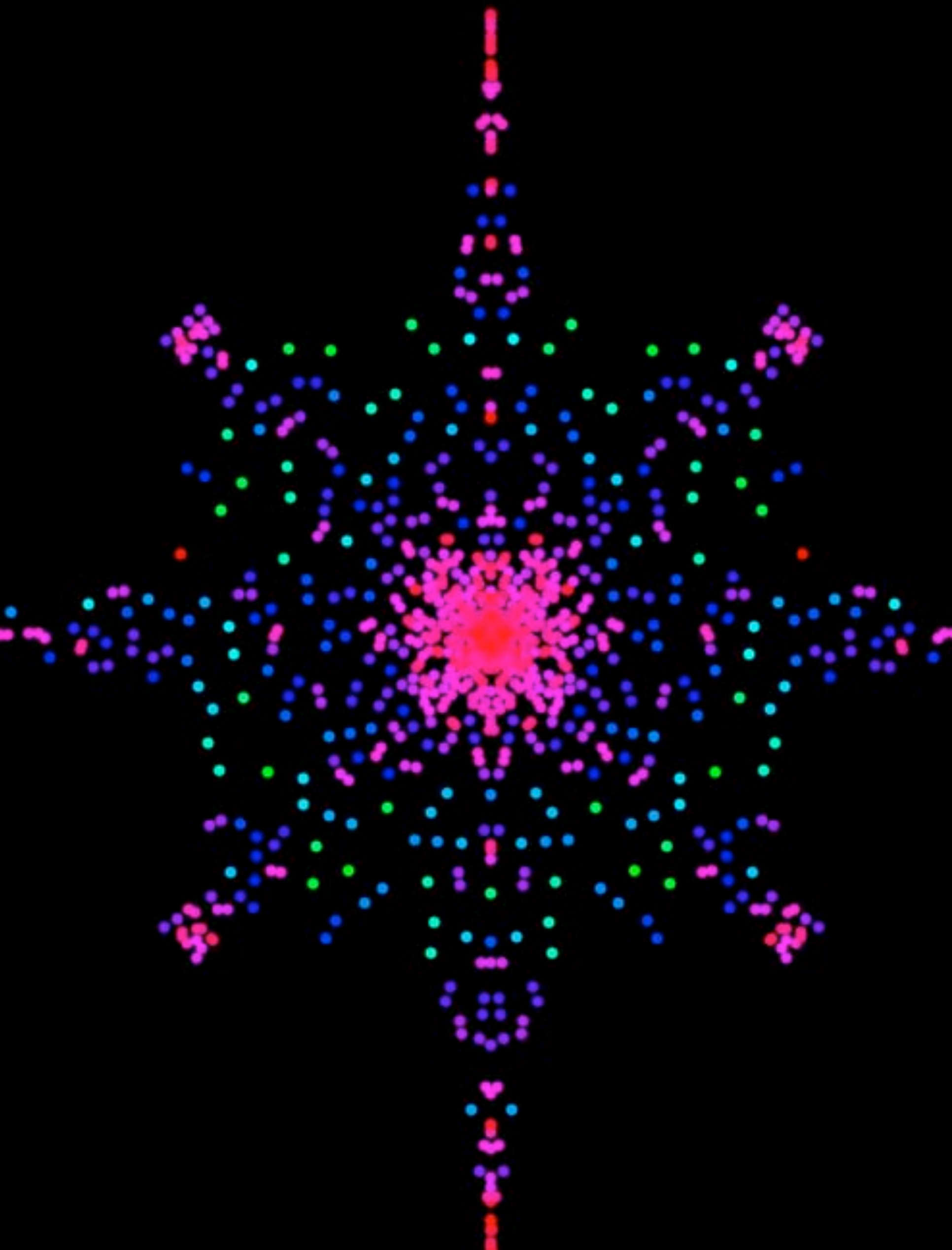


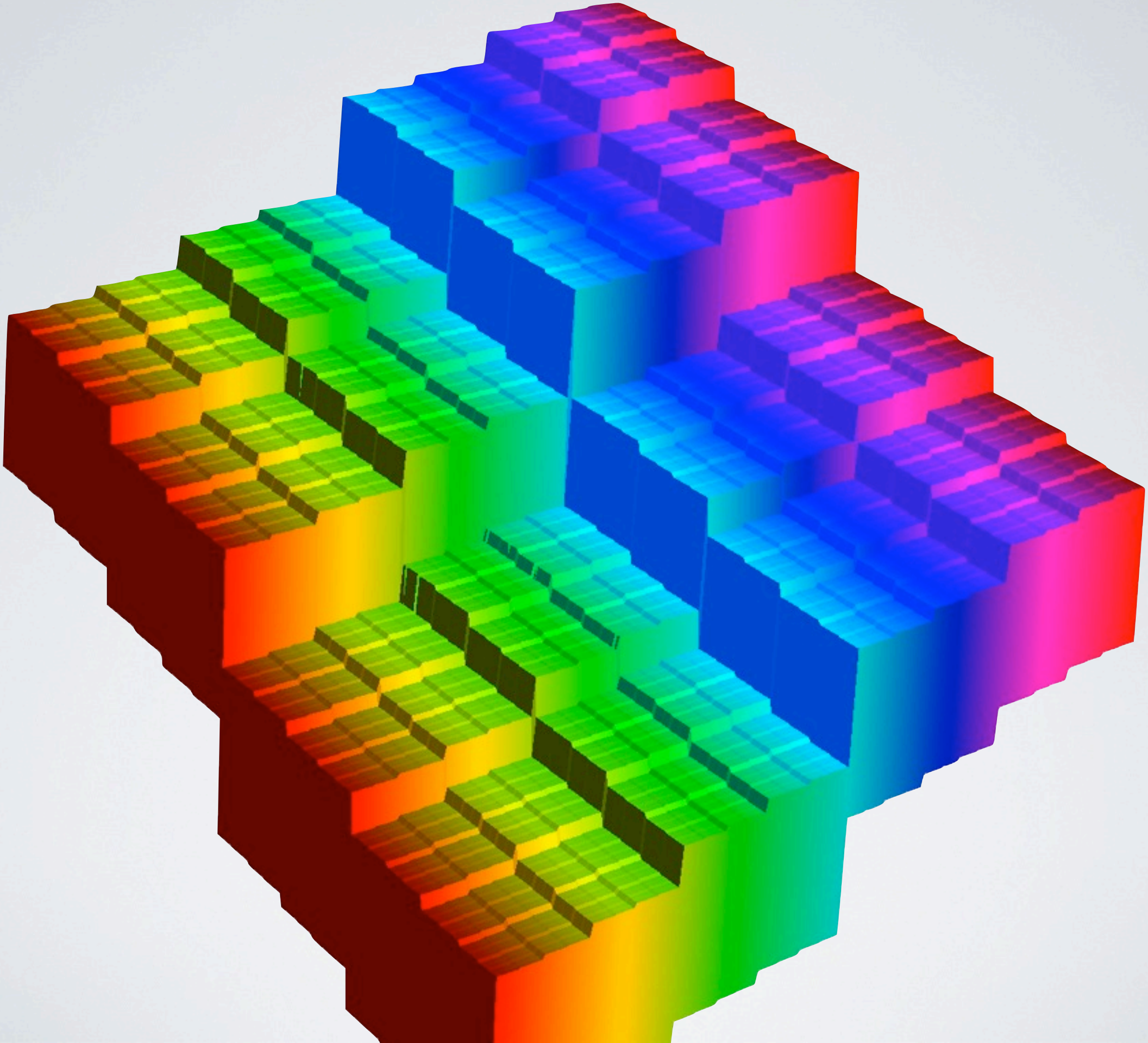


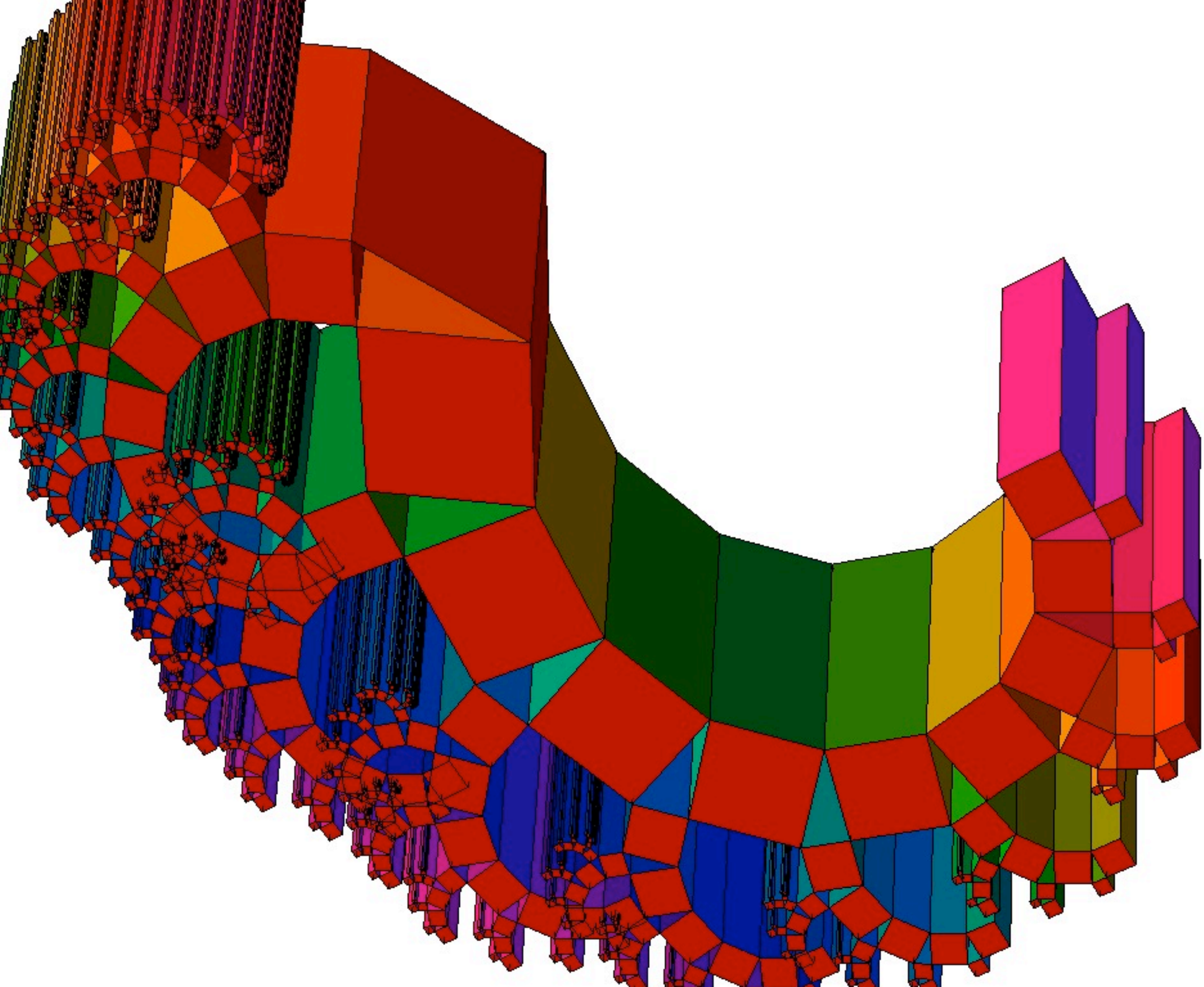


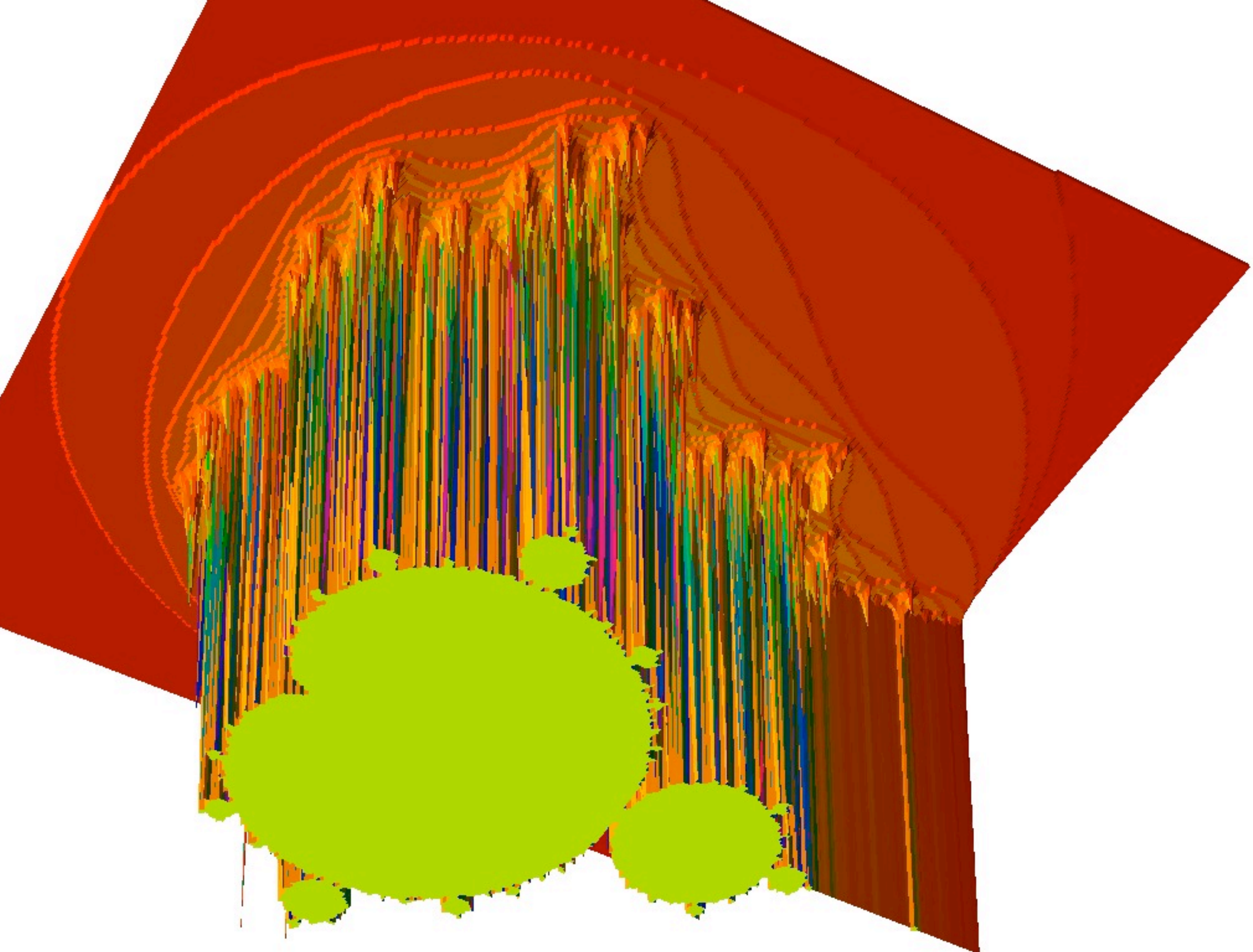


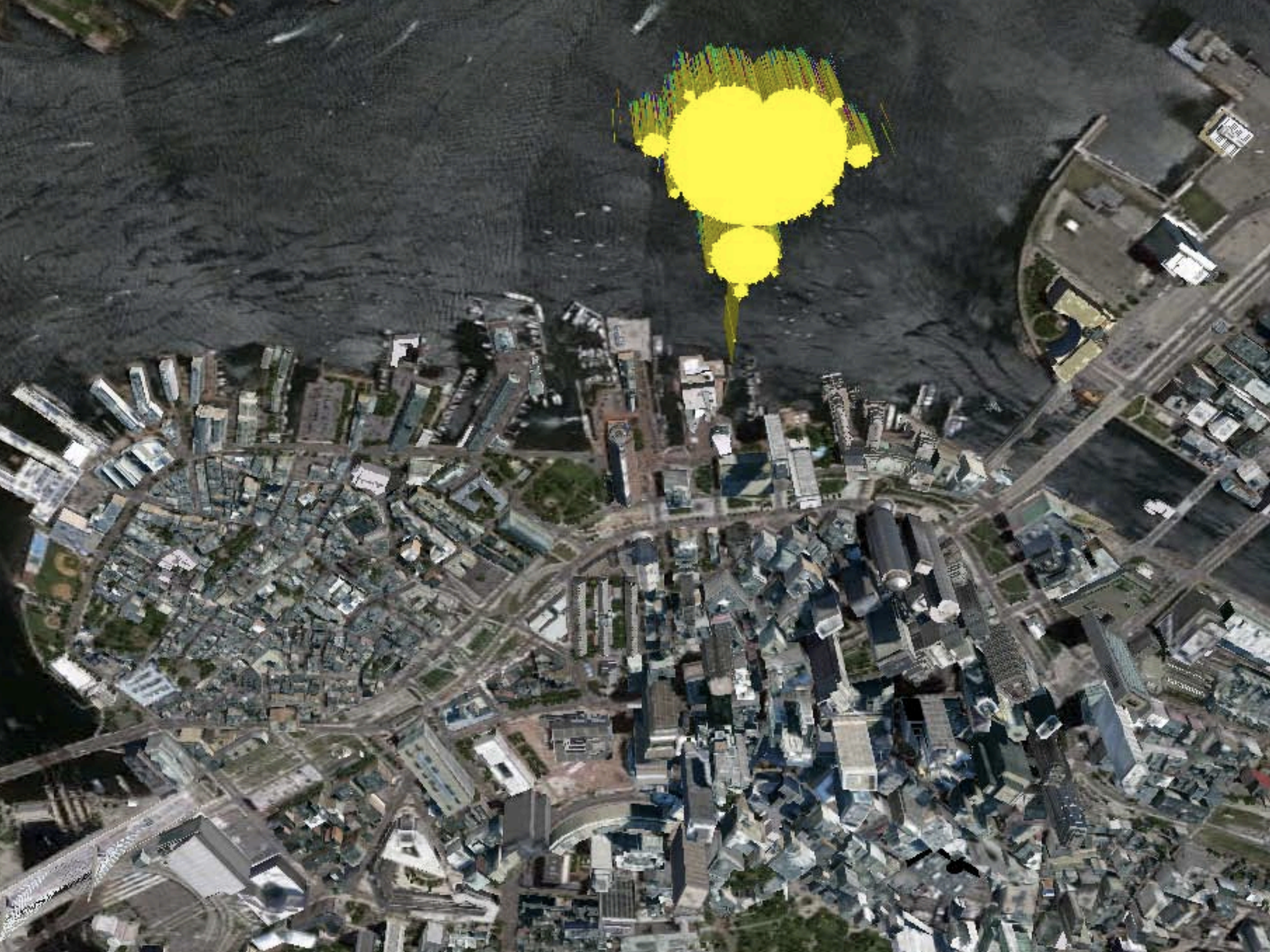








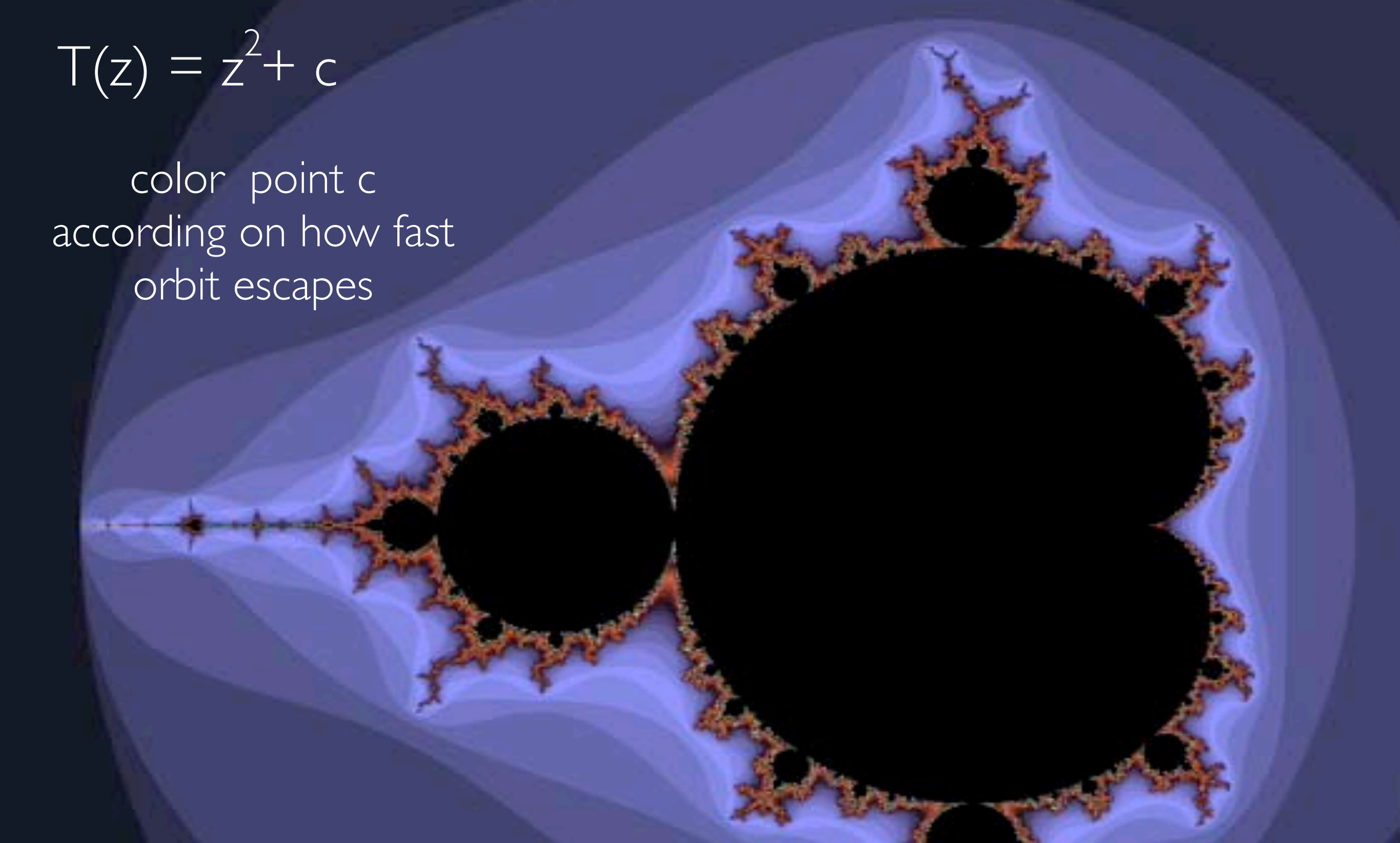




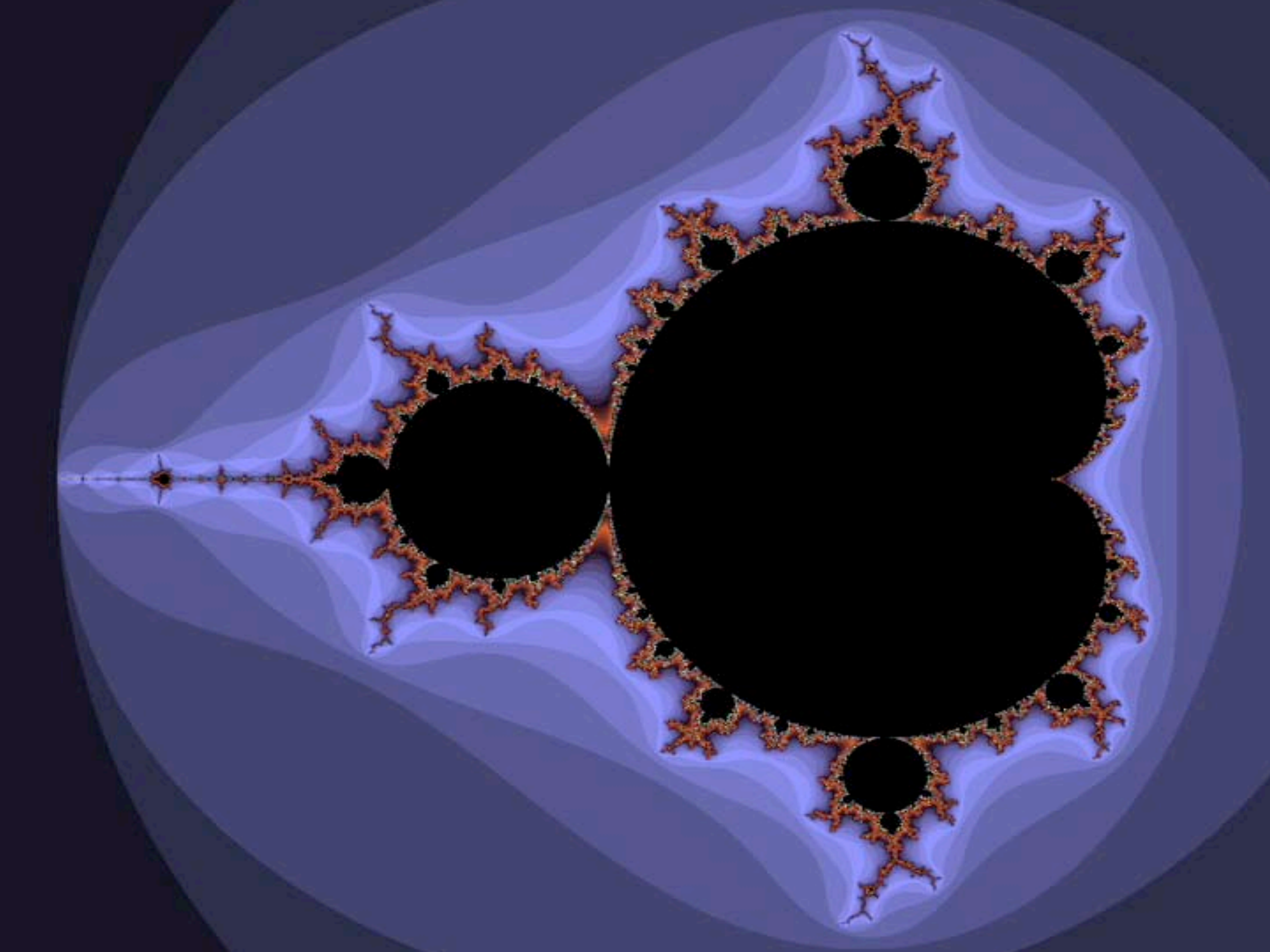
W
30
33
250
240
230
220
-60
-65
-70
-75
-60
-65
FLAPS 100%

$T(z) = z^2 + c$

color point c
according on how fast
orbit escapes



Mandelbrot parameter set. 1978
(Brooks Matelski, Hubbard
Mandelbrot, 1980)



THE MANDELBULB

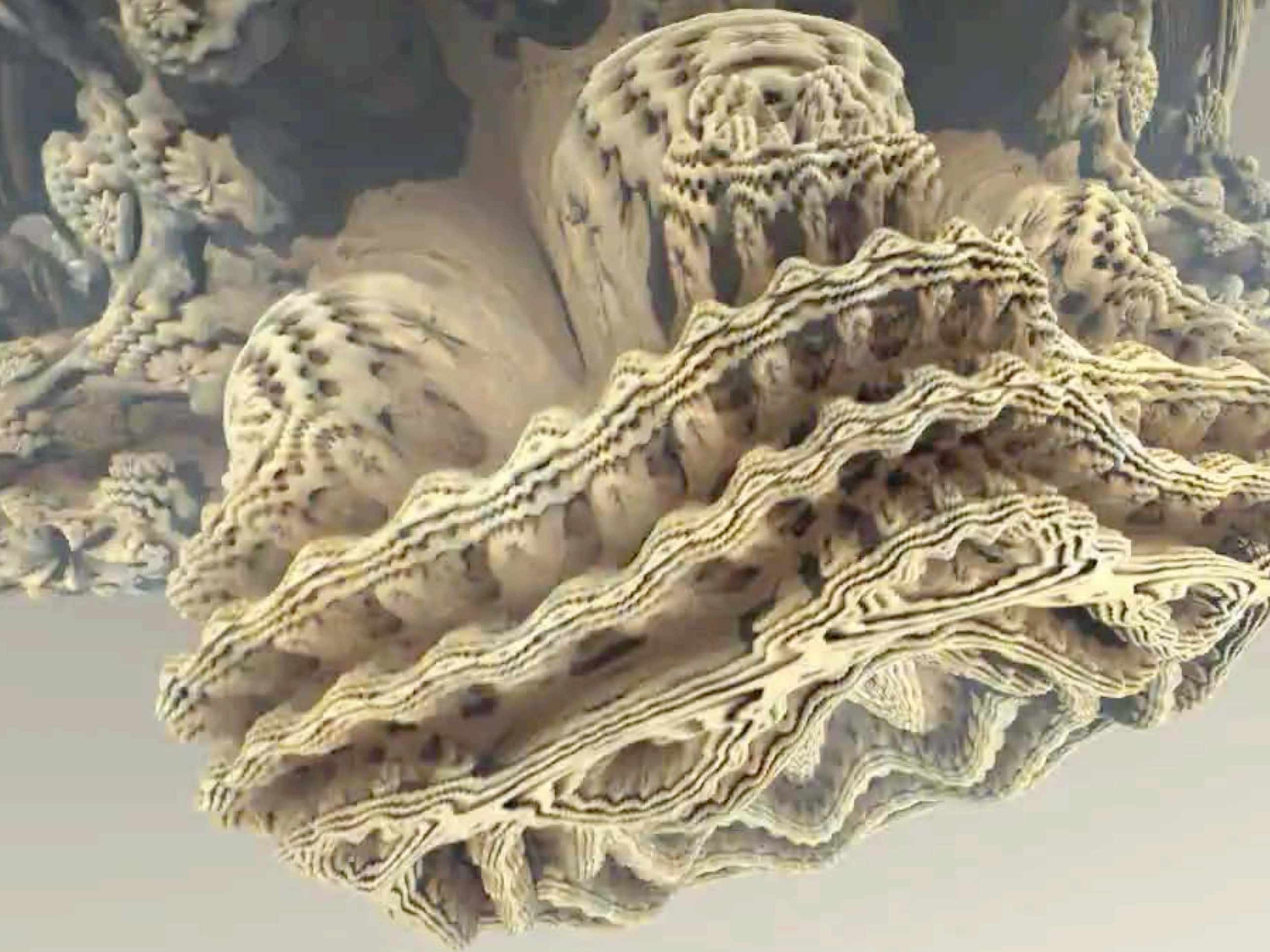
A youtube and internet star

DEFINITION

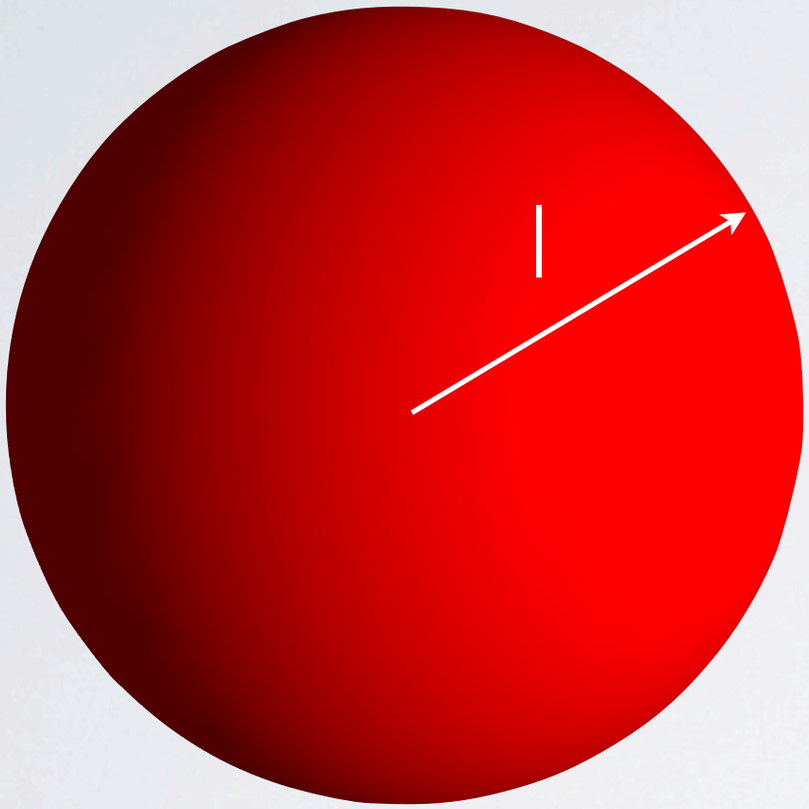
iterate in 3D spherical coordinates and look which c lead to bounded orbits

$$z^8 \longrightarrow z + c$$

Daniel White 2007, inspired by Rudy Rucker



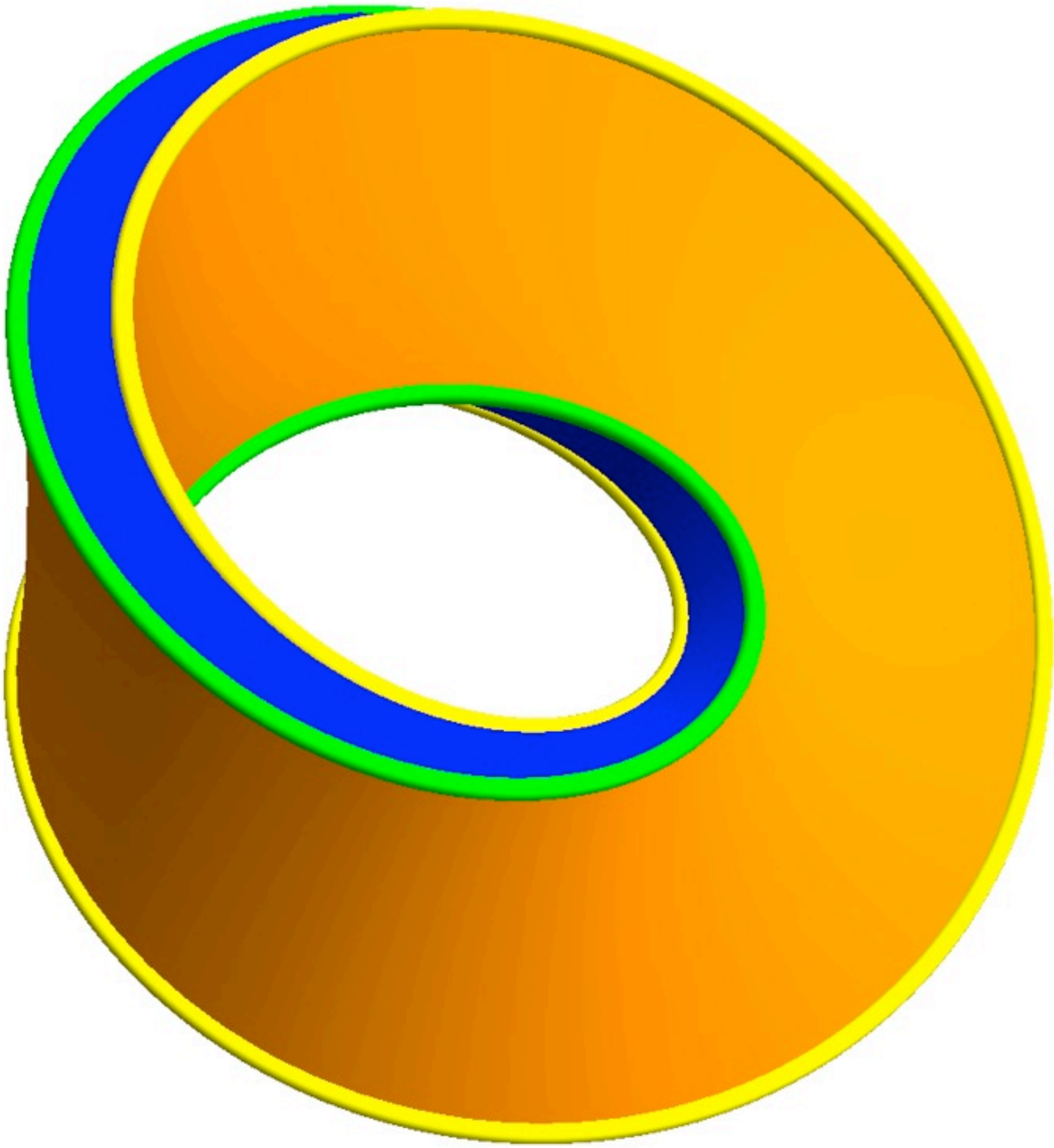
SPHERE

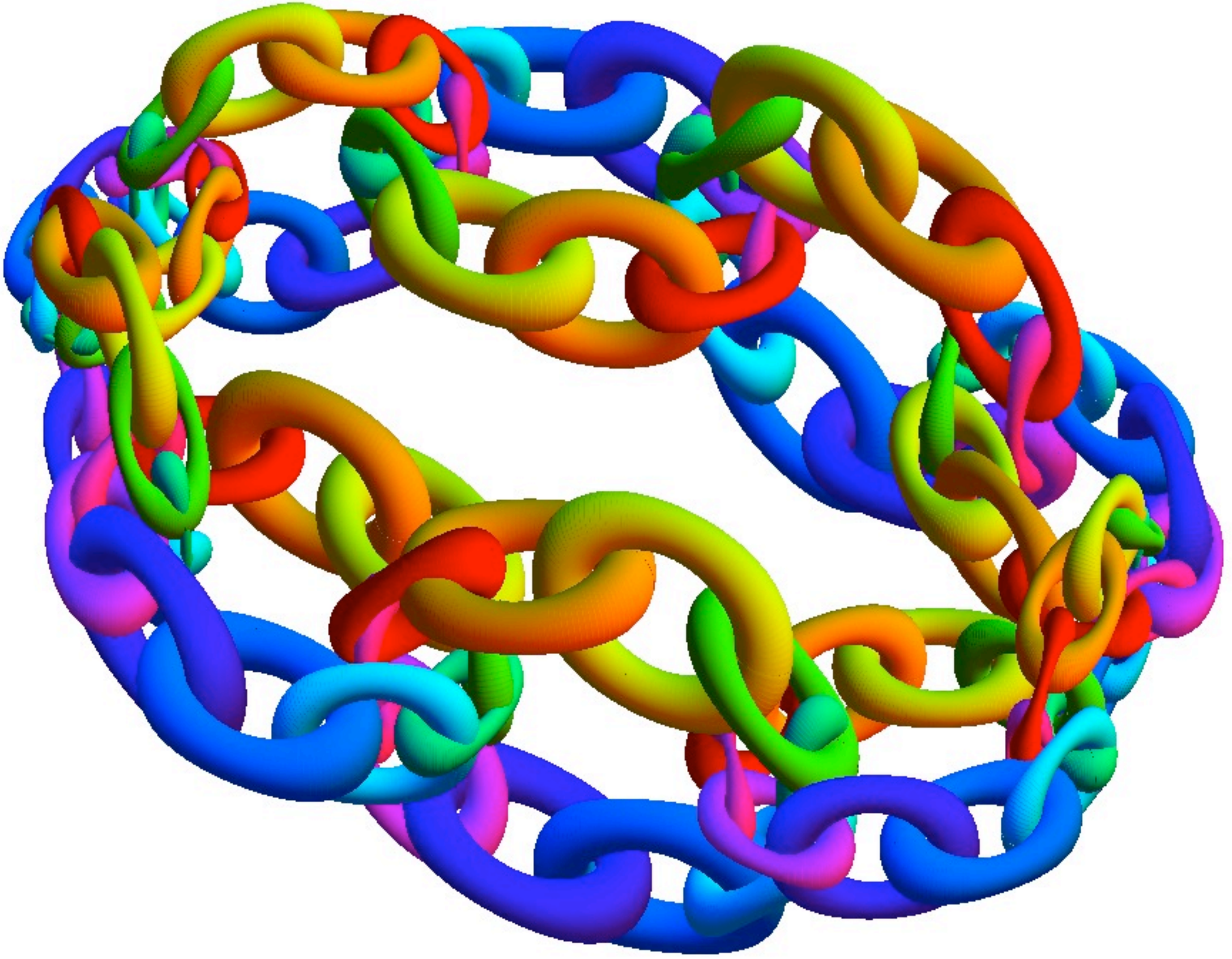


$4\pi/3$

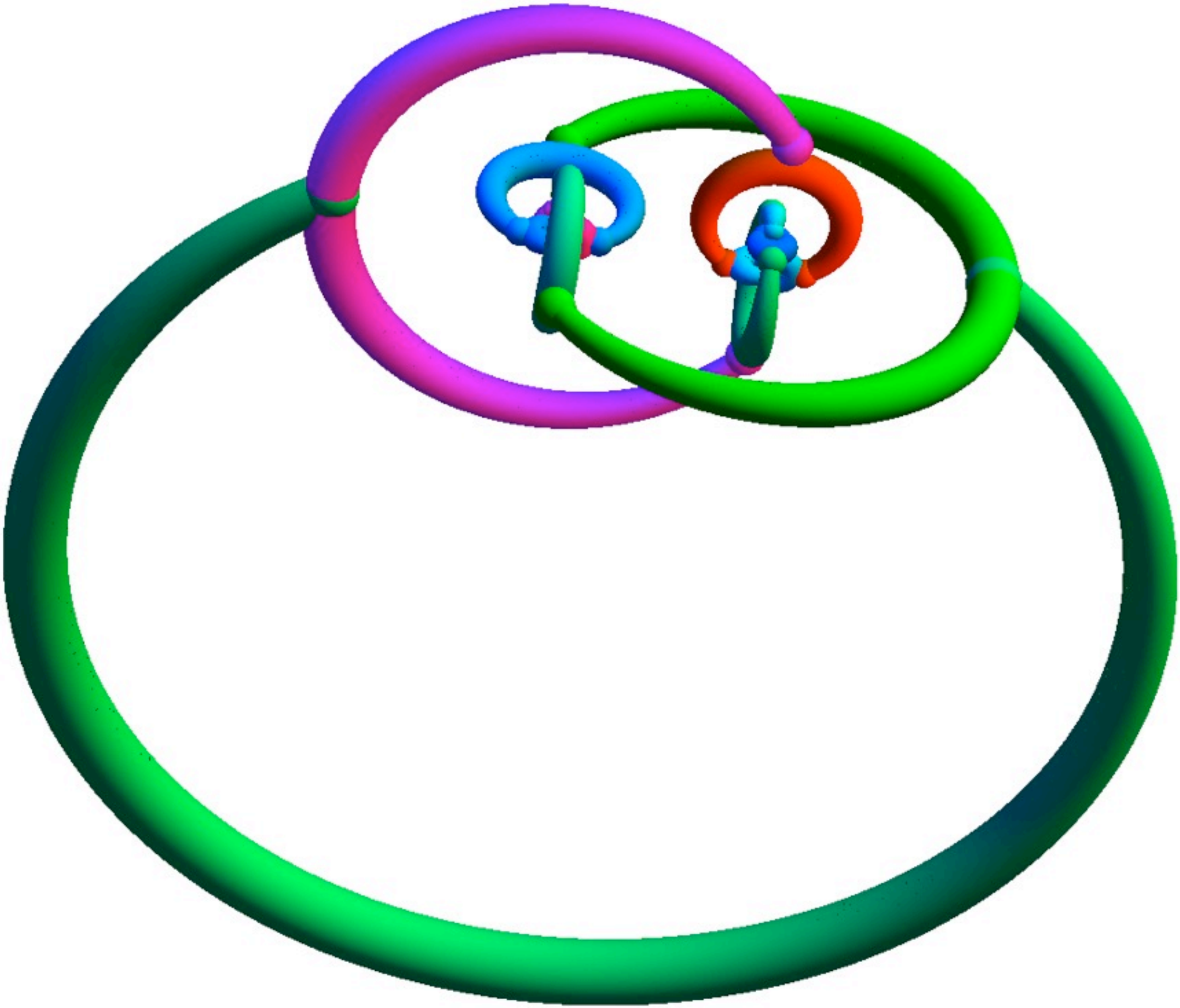
MANDELBULB

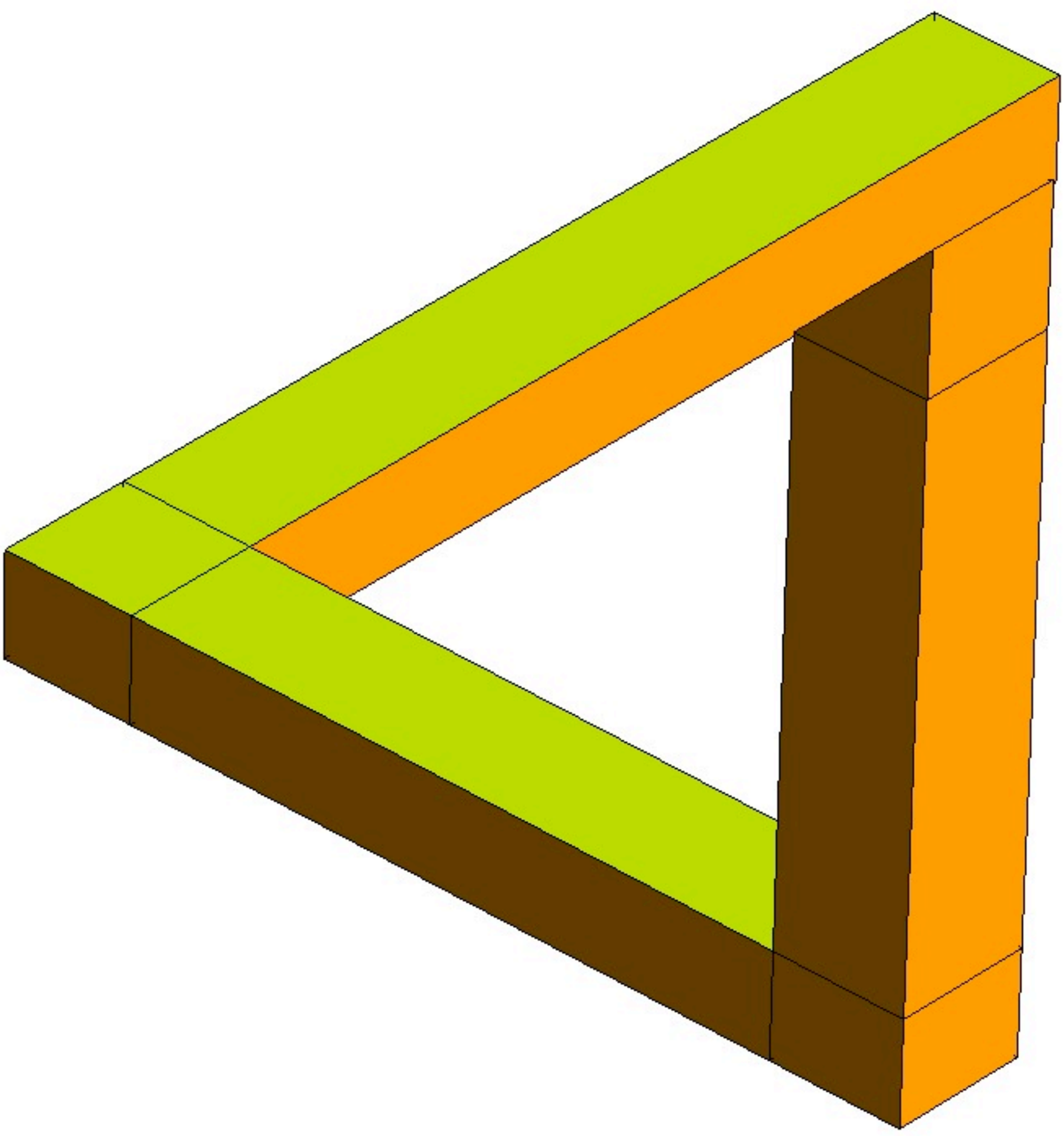
???

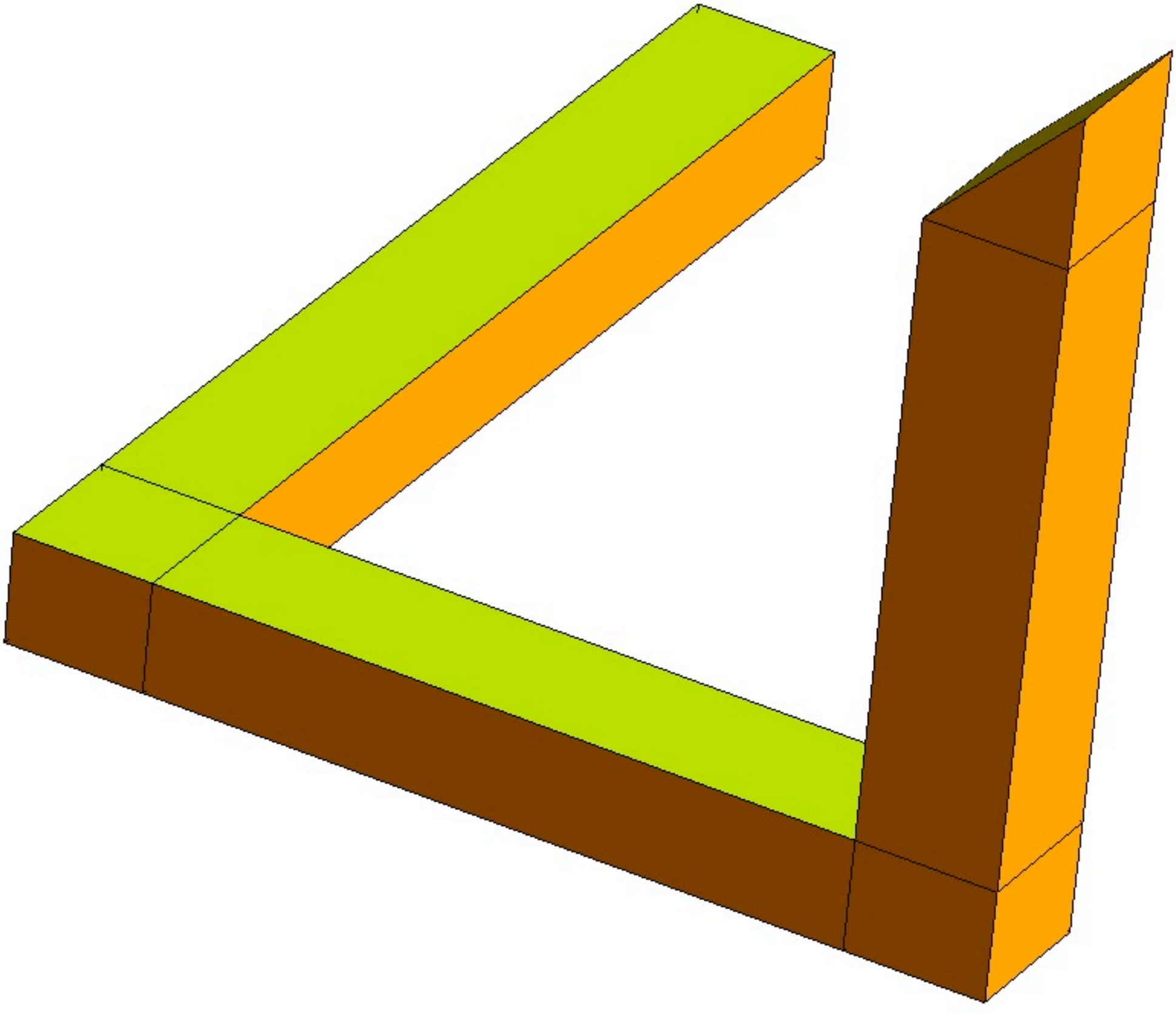




KNOTS



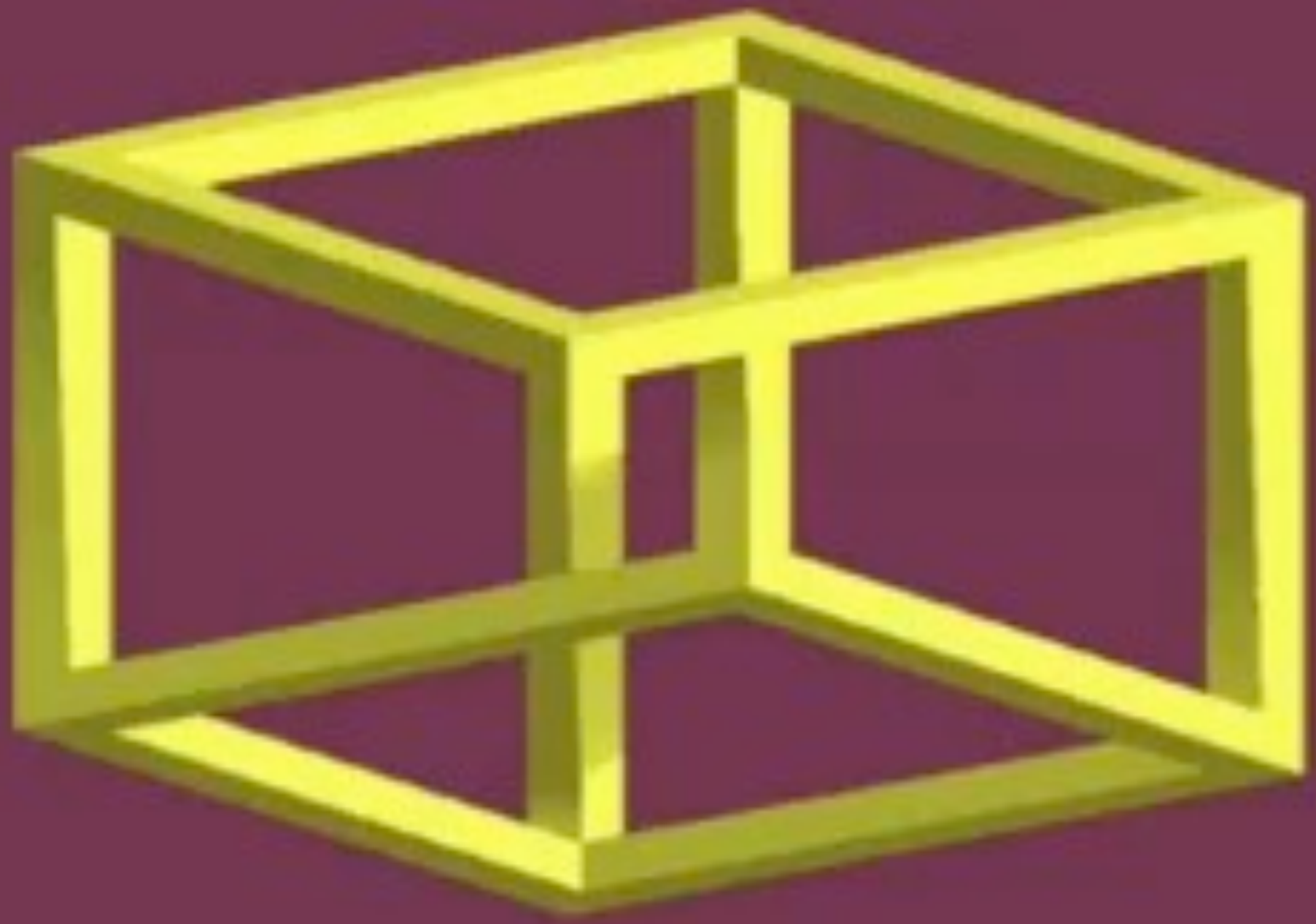


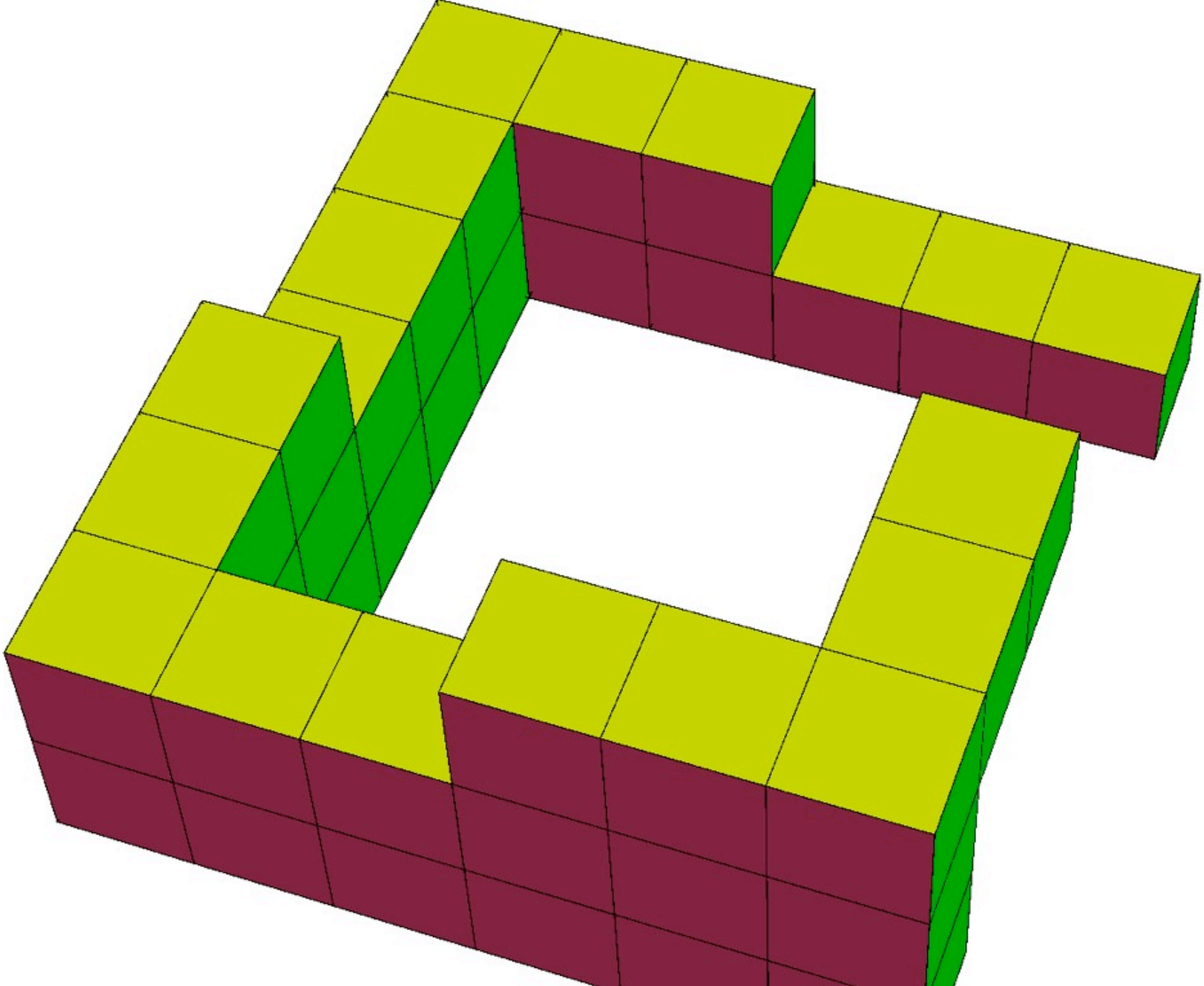


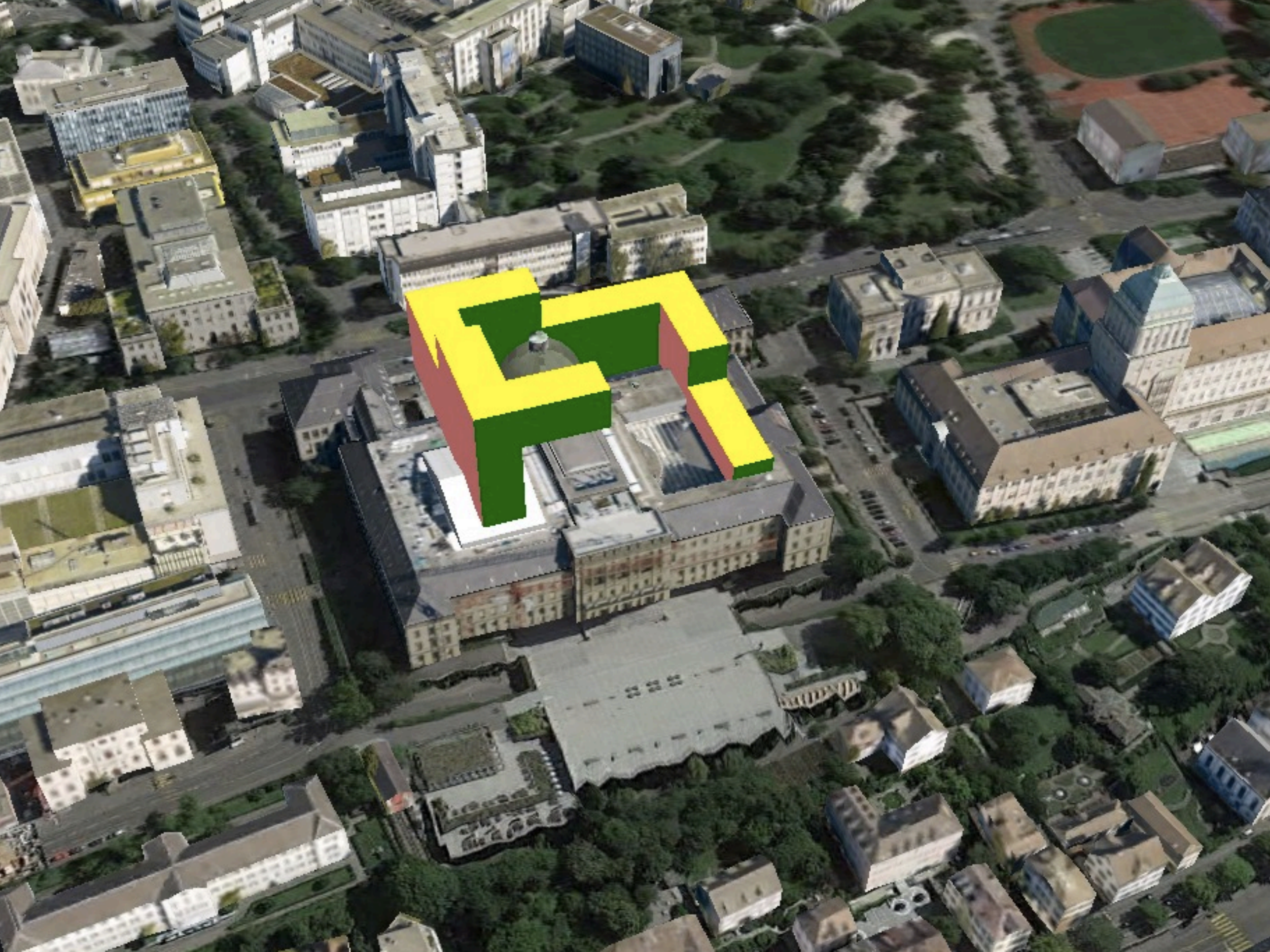
Gershon Elber,
Technion

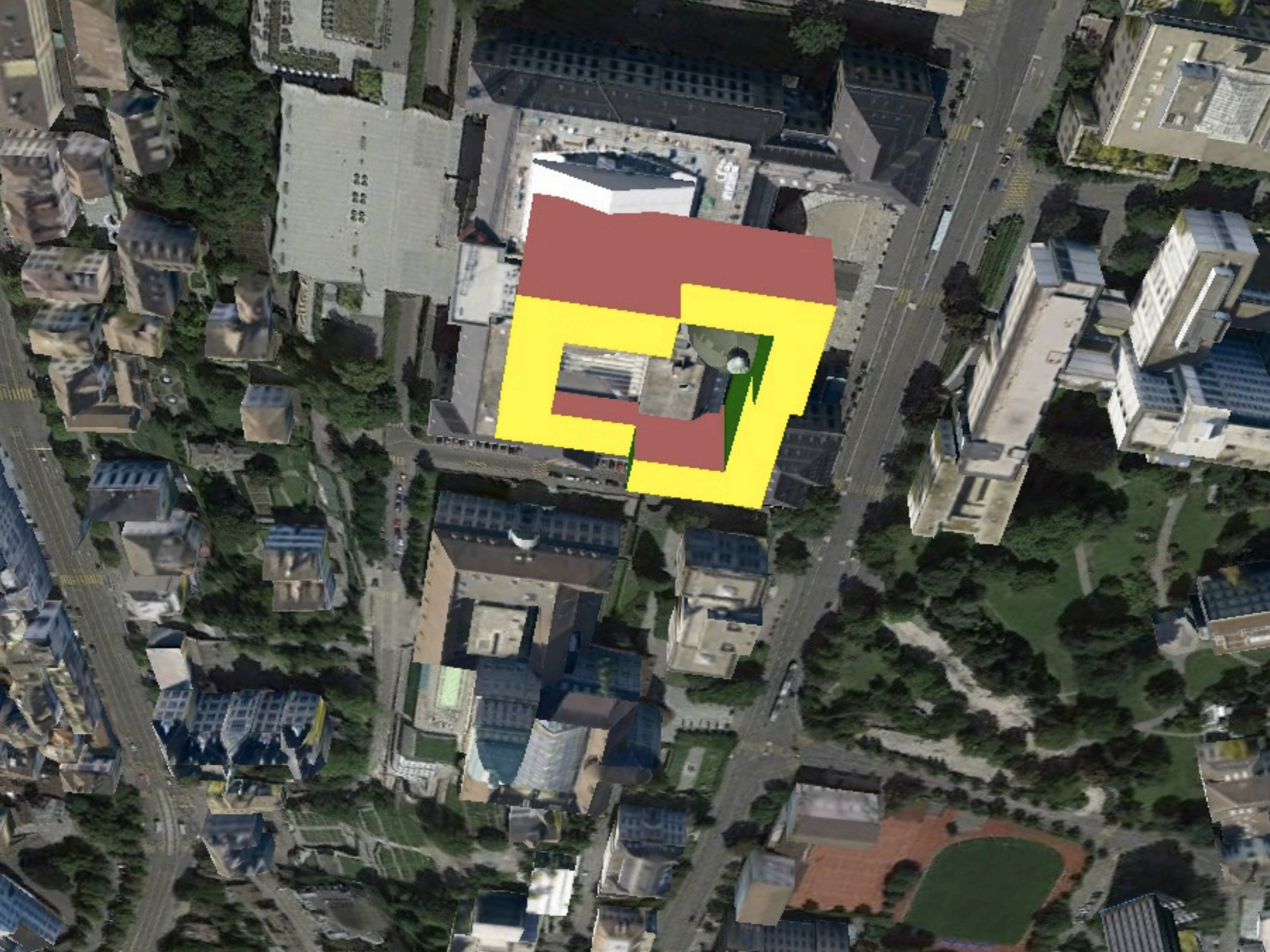
Can you imagine a solid which looks like this?

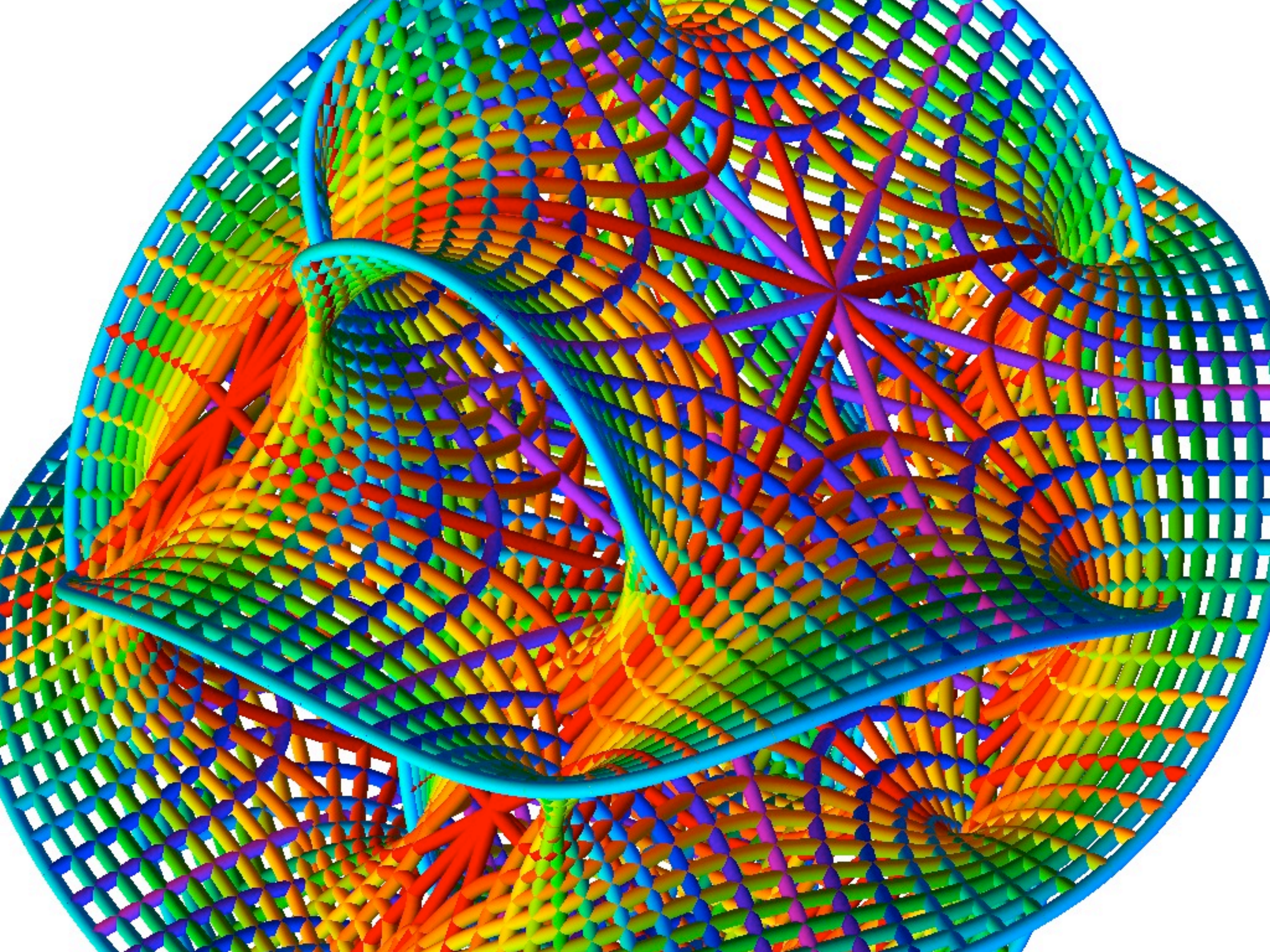
Gershon Elber,
Technion

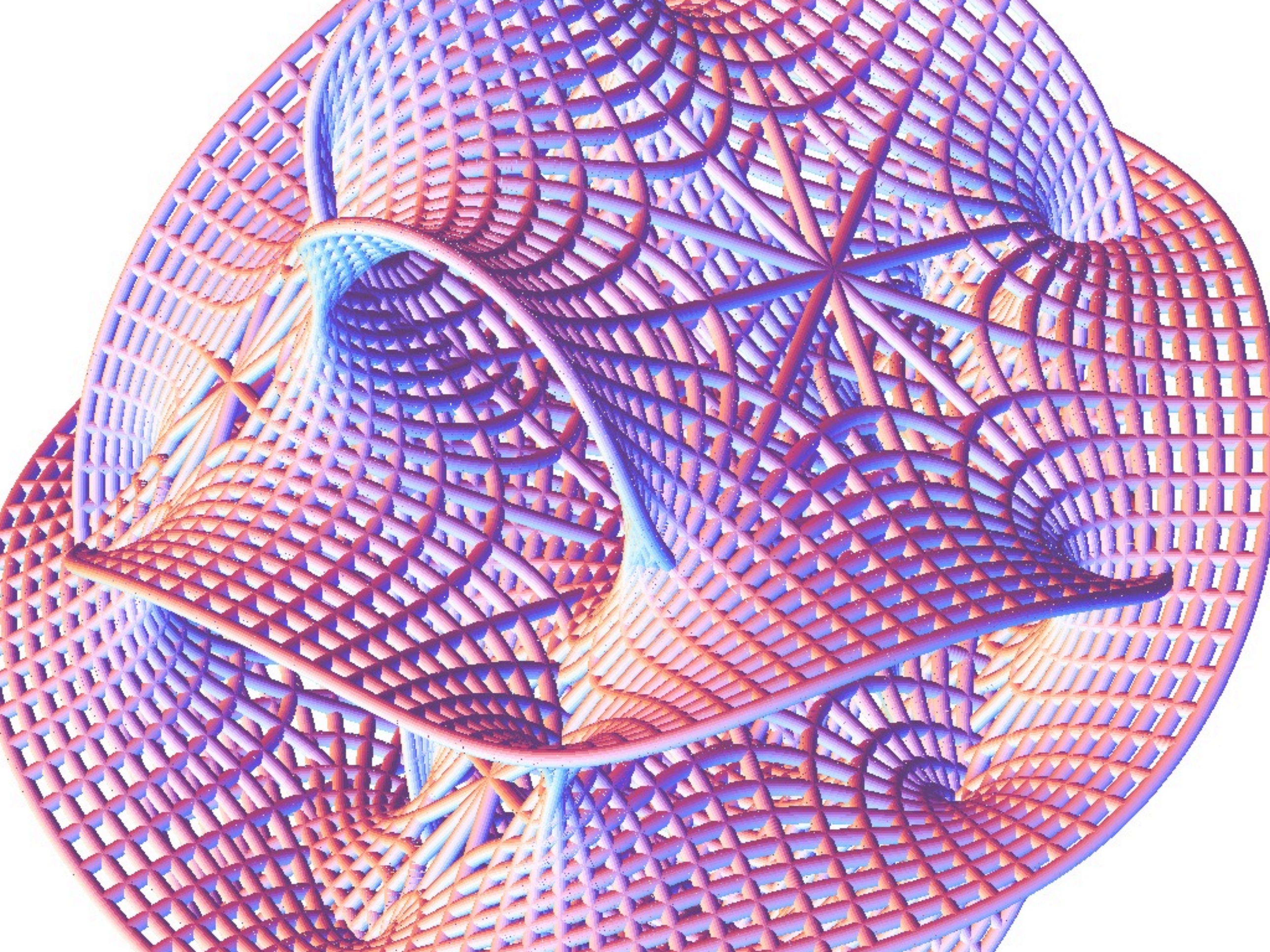


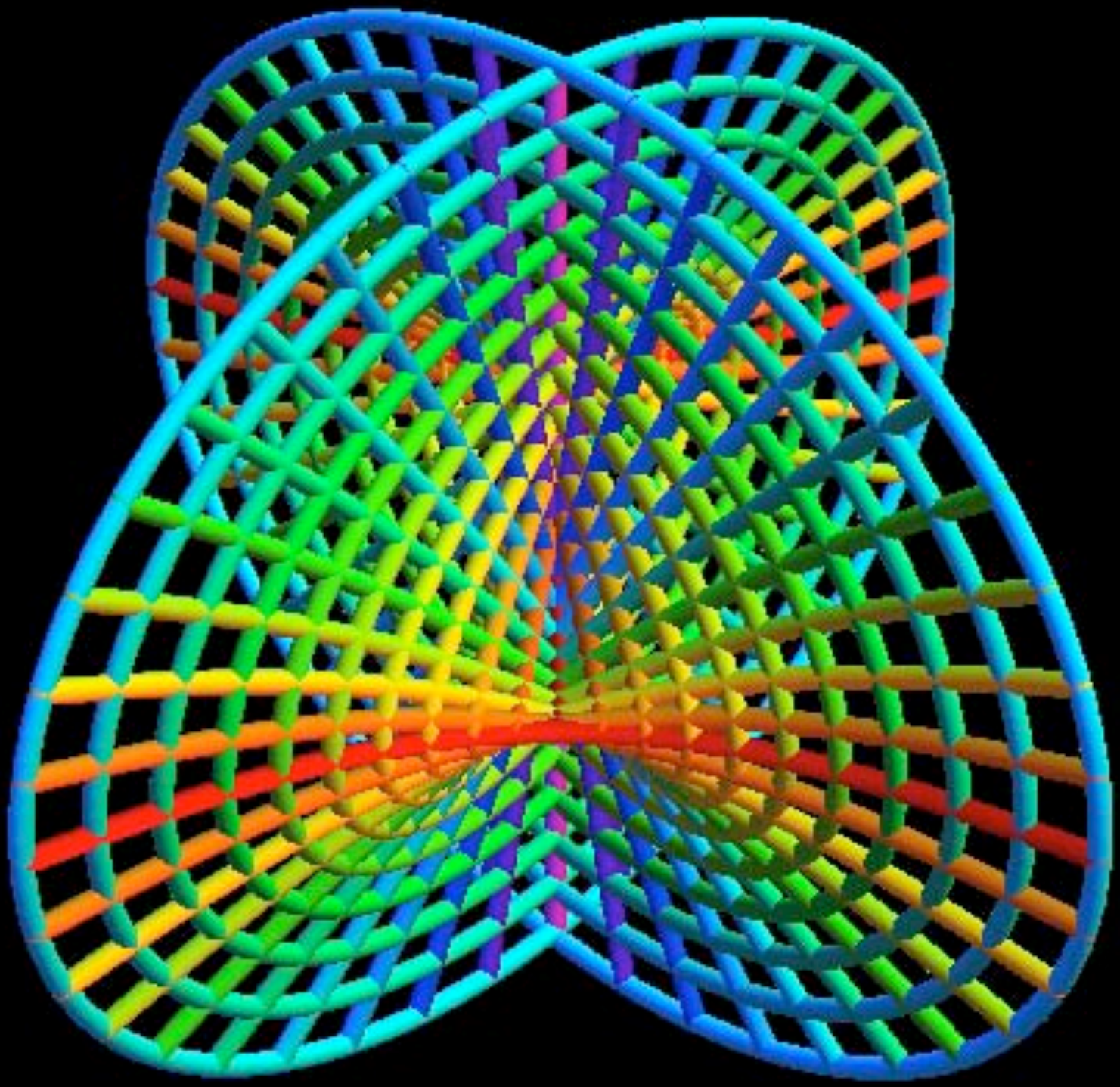


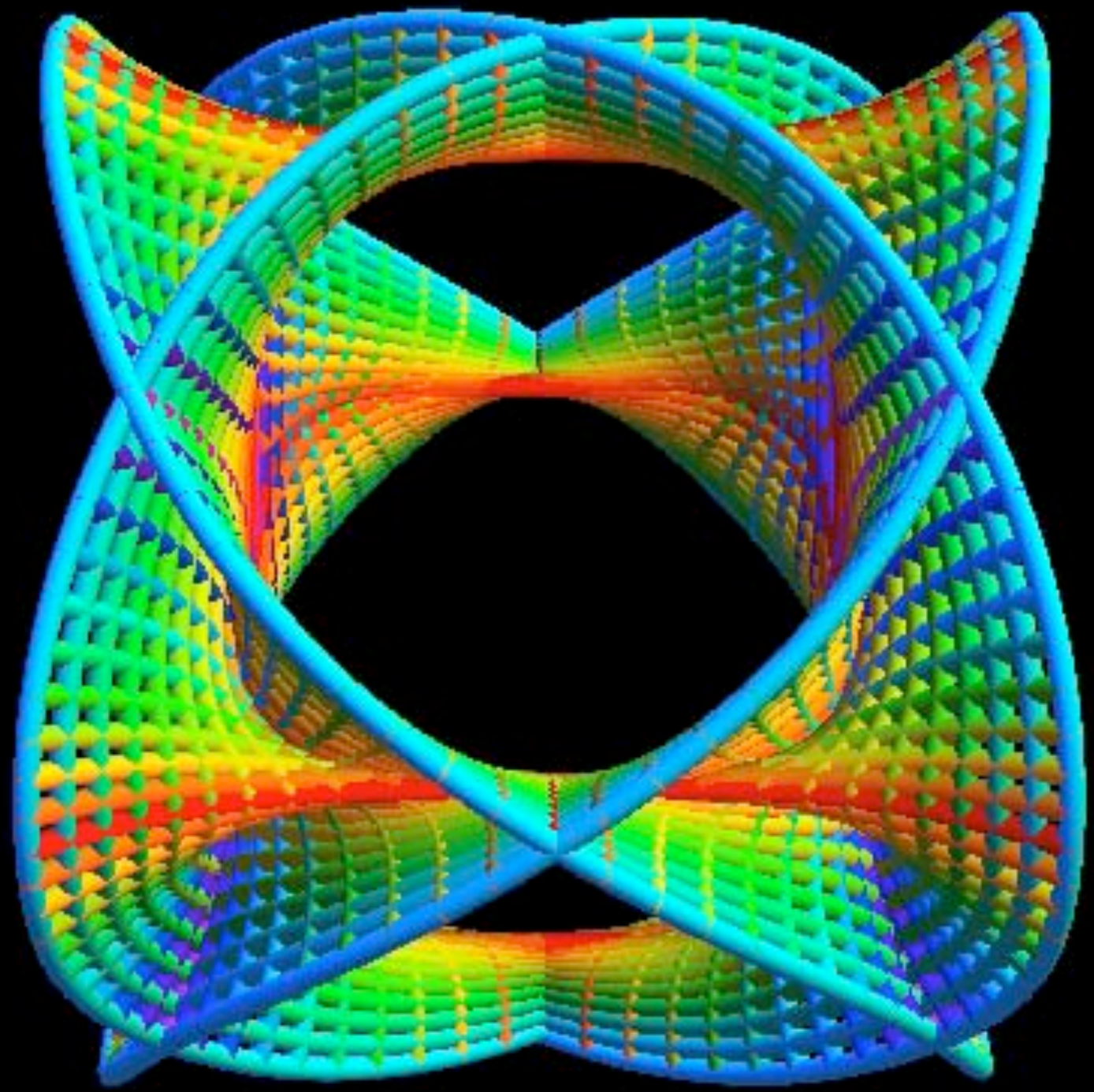


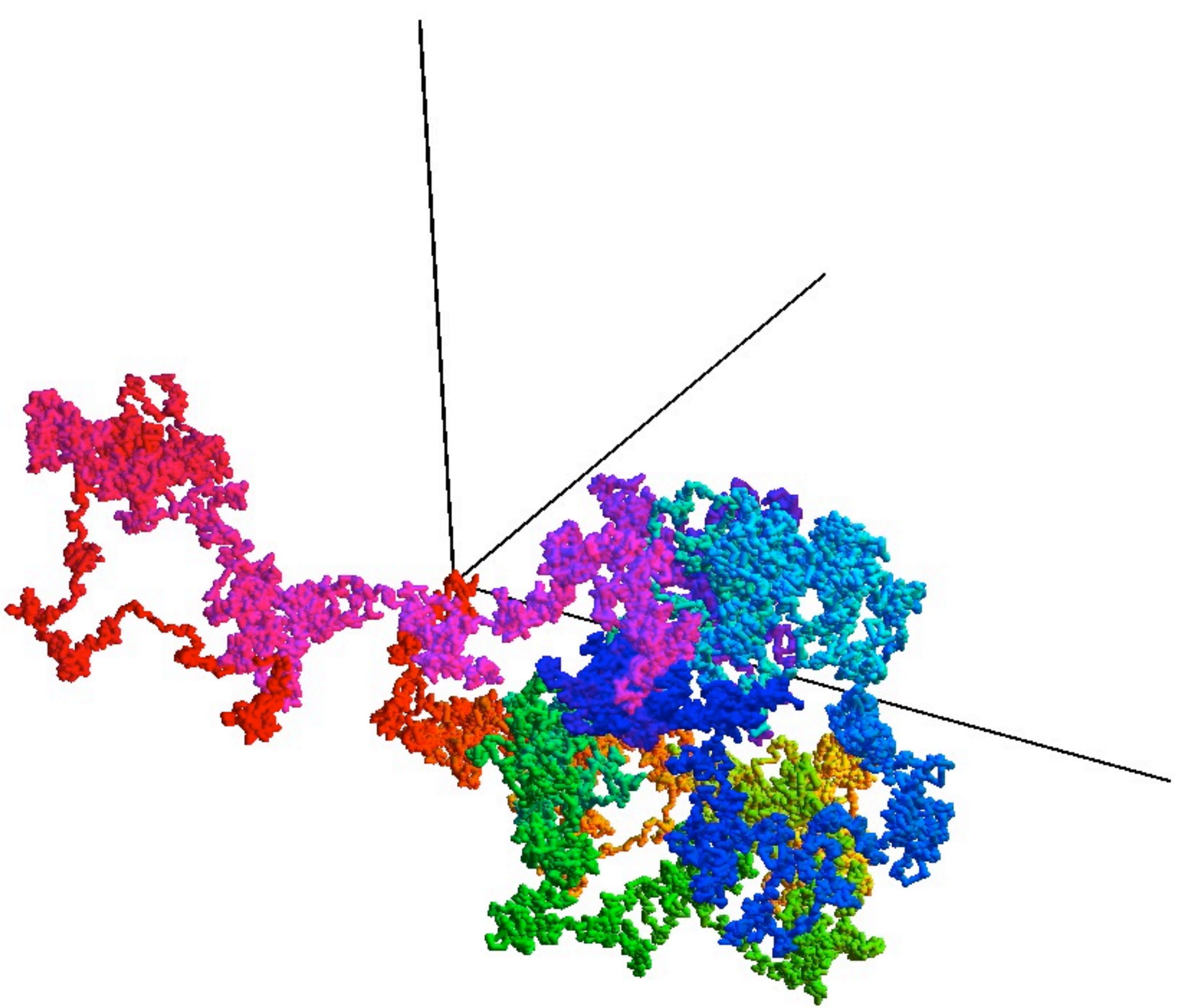


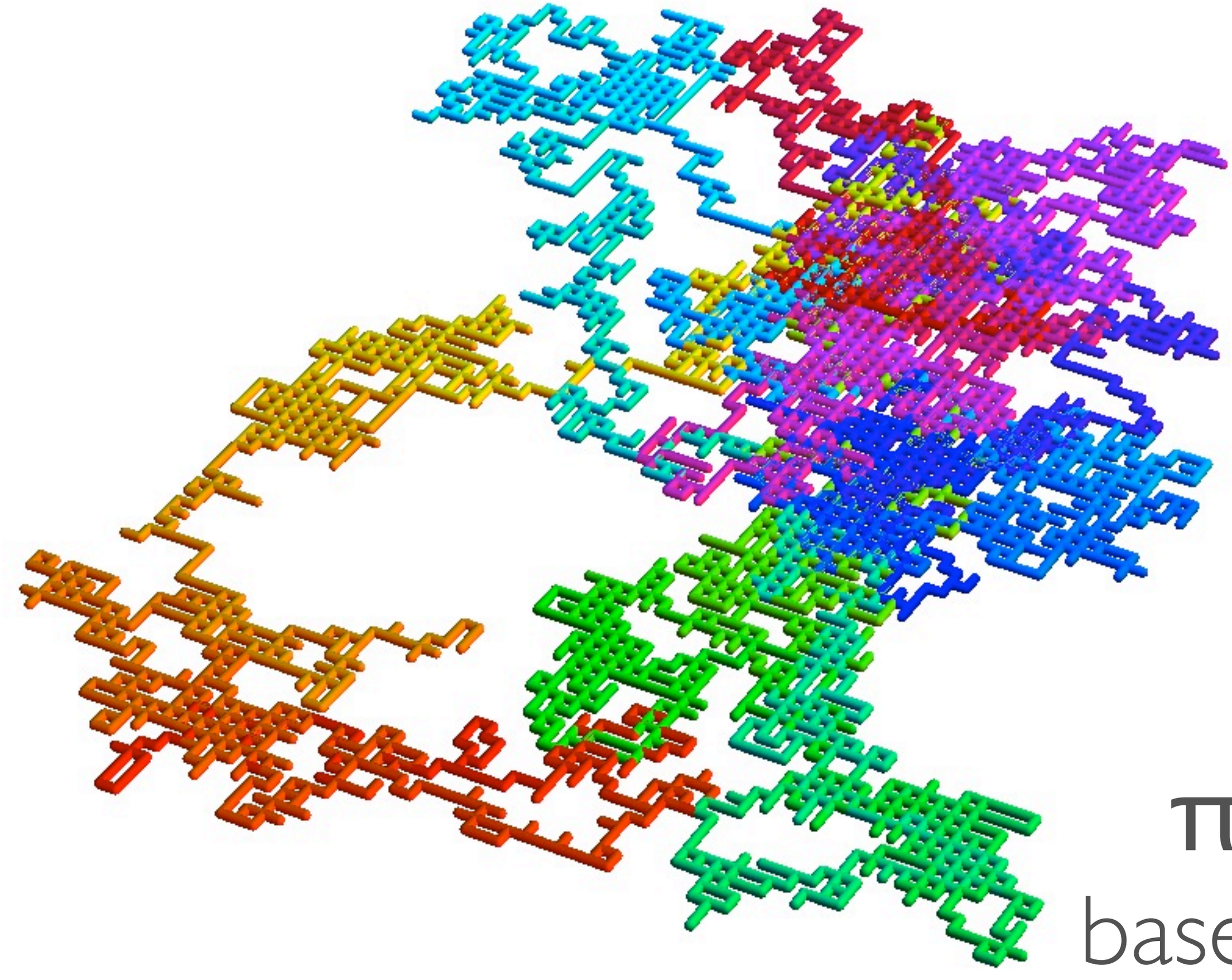




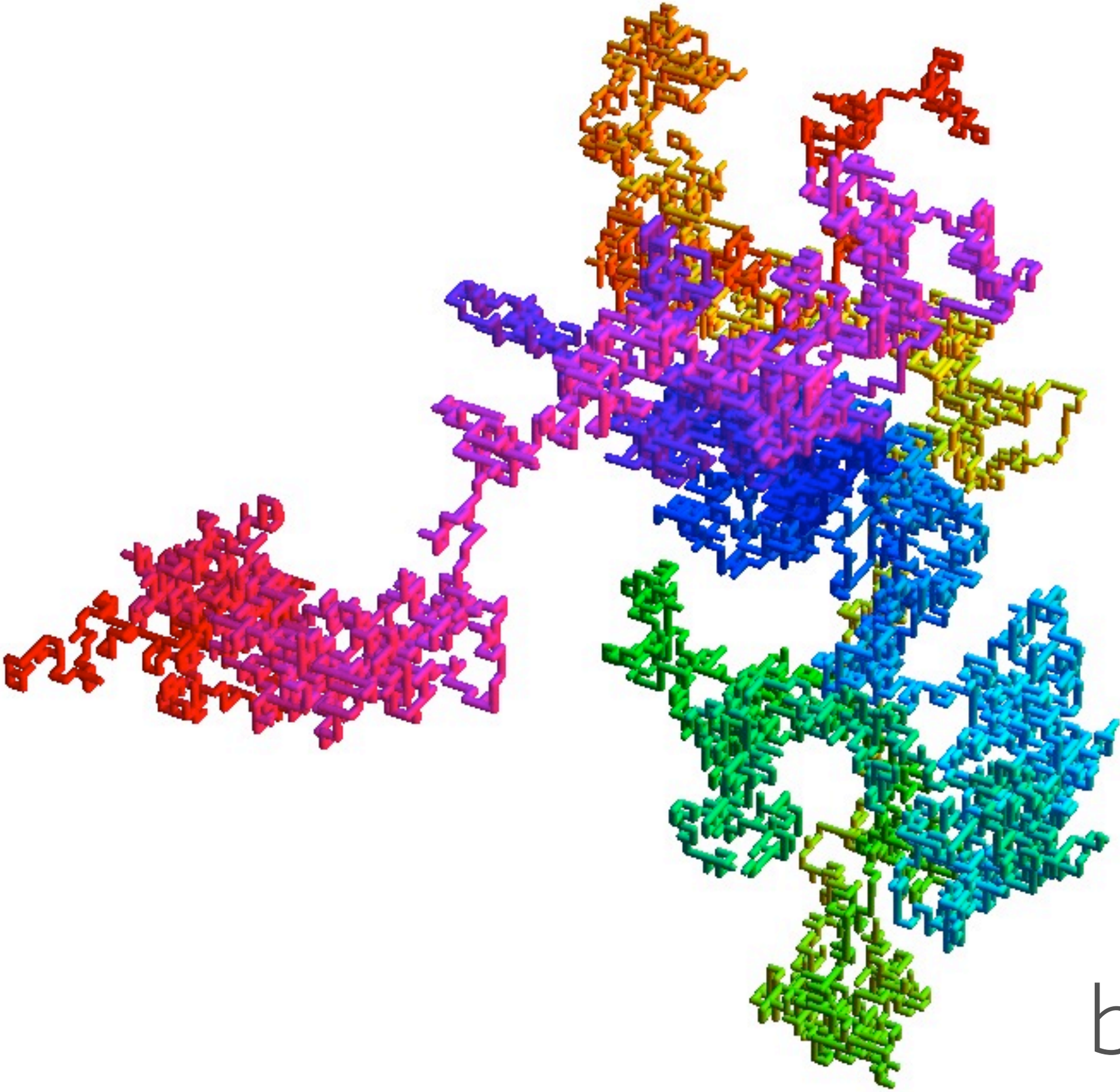








π
base 4



π

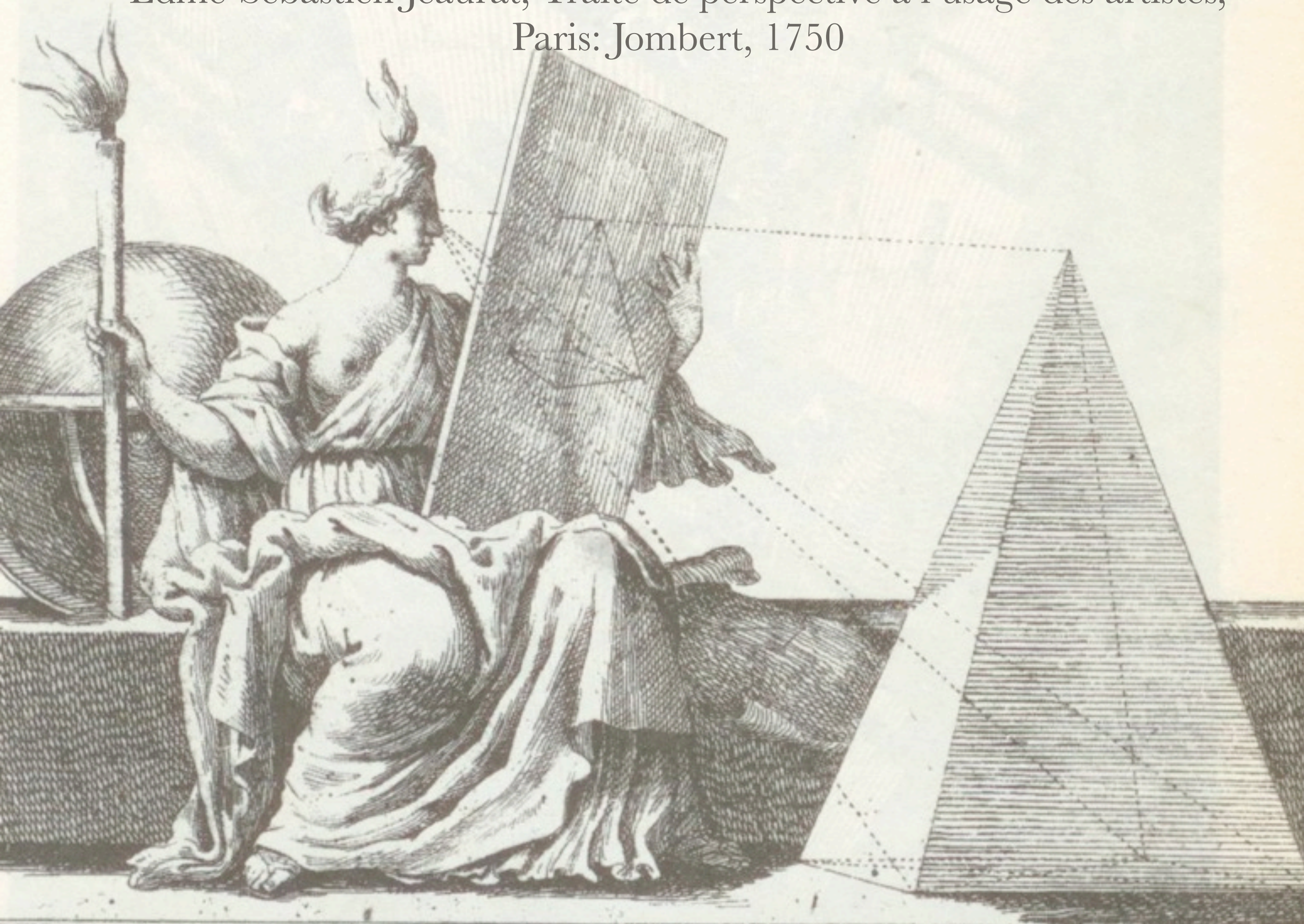
base 6

3D SCANNING

Art

Albrecht Duerer,, 1471-1528

Edme-Sebastien Jeaurat, Traite de perspective a l'usage des artistes,
Paris: Jombert, 1750



From Images to 3D-Models

Institute of Computer Science and Applied Mathematics

Multimedia Information Processing

C | A | U Christian-Albrechts-Universität zu Kiel

M | I | P

Wassar Street
36
45
44
20R
48
43

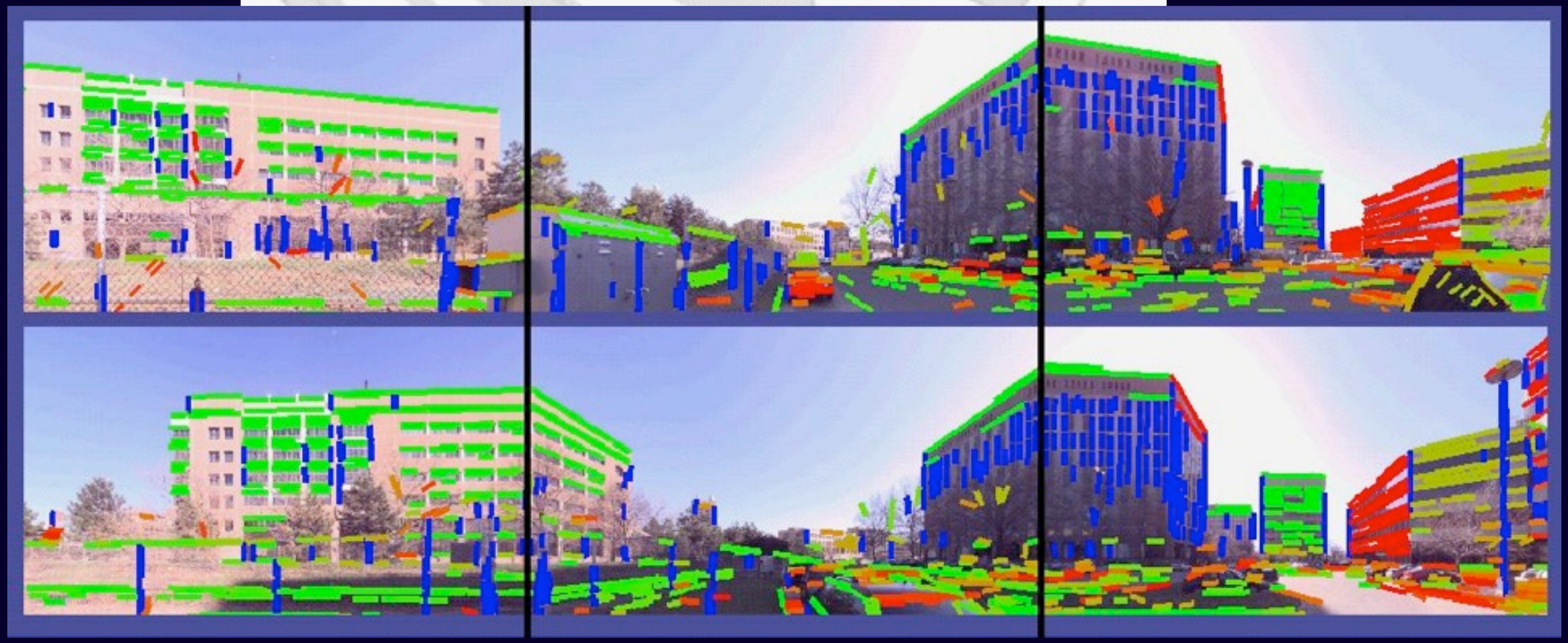


Figure 10: On the left is a photograph of Michelangelo's David. On the right is a rendering made from our model. Constructed at a resolution of 1.0 mm, the model is watertight and contains 4 million polygons. Its surface reflectance was computed from digitized color images as described in section 4.2. The photograph was taken under uncalibrated conditions, so its viewpoint and illumination differ slightly from those of the rendering. The raw data for this part of the statue was acquired at a resolution of 0.29 mm and contained 480 million polygons and 2,000 color images. Using one processor of an SGI Onyx2, it took 12 hours to align merge, and map color onto this model.

The statue	
height without pedestal	517 cm
surface area	19 m^2
volume	2.2 m^3
weight	5,800 kg
Our raw dataset	
number of polygons	2 billion
number of color images	7,000
losslessly compressed size	32 GB

Other statistics	
total size of scanning team	22 people
staffing in the museum	3 people (on average)
time spent scanning	360 hours over 30 days
man-hours scanning	1,080
man-hours post-processing	1,500 (so far)

Stanford, Wash U. Cyberware Inc.

FACADE, a program developed
at the University of
California at Berkeley, can
be used to reconstruct
architectural scenes directly
from photographs.

Facade

In aerial photography, the problem arises of matching partial shots of an area which is too large to fit in a single photo. Assuming the area to be flat, it is possible to use perspectives to make overlapping parts coincide perfectly: the correspondence between matching points in two photographs is a homography, being the composition of the perspectives of the two photos (figure 4.7.3.1), and thus can be composed with another homography to give the identity. By proposition 4.5.10, it is enough to match four points in the two images to obtain a perfect correspondence. See [BUR, 36–51].

Figure 4.7.3

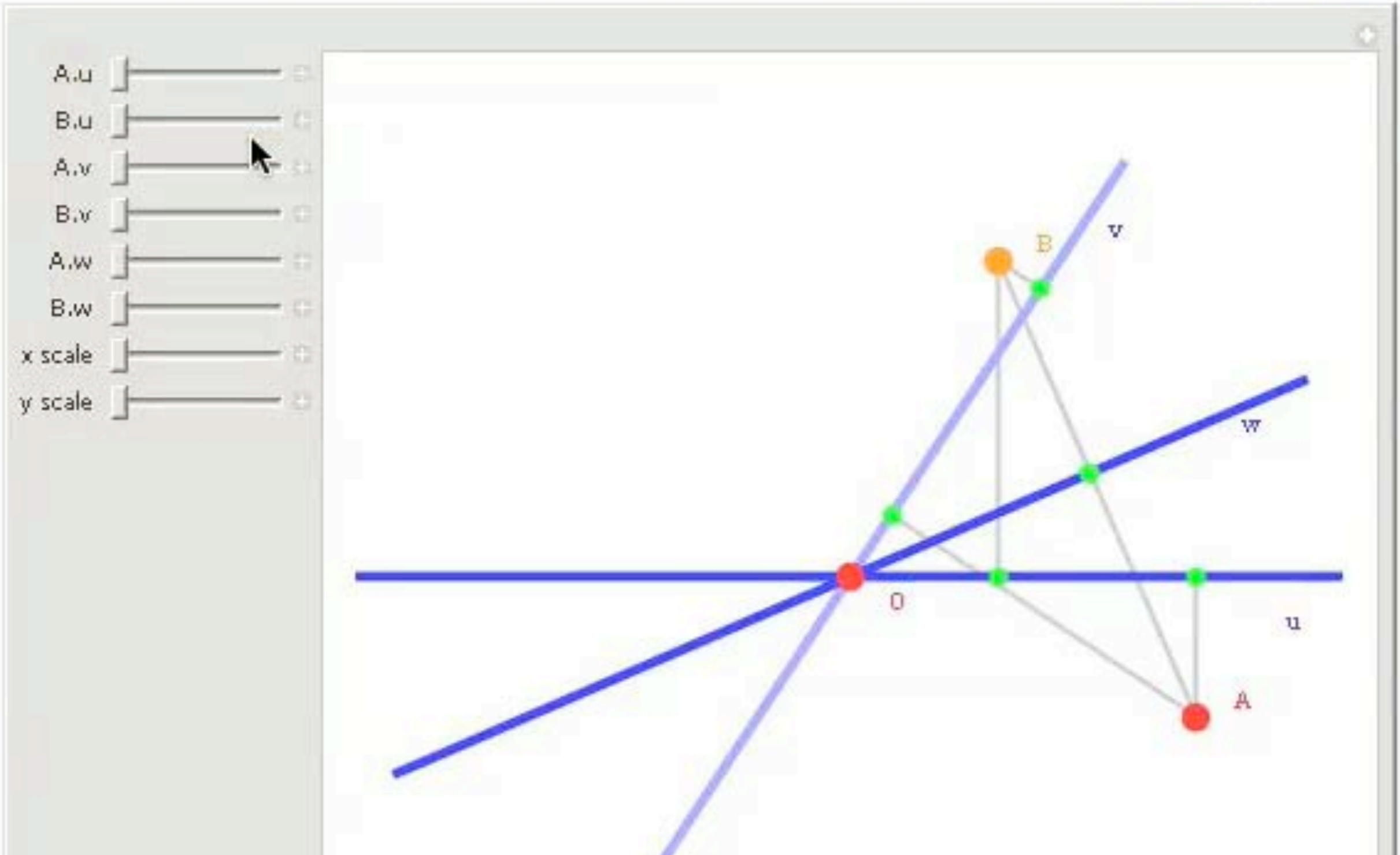
Photogrammetry

Berger, Geometry I

Ullman's Theorem in Two Dimensions

DOWNLOAD LIVE VERSION >>

watch web preview >>



GO!SCAN

JAMES WALKER
BORN IN BURLINGTON,
AUGUST 16. 1796.
DIED AT CAMBRIDGE DECEMBER 23. 1874.
PASTOR OF THE HARVARD CHURCH IN CHARLESTOWN
1818 – 1839
OVERSEER OF HARVARD COLLEGE 1825–1836
FELLOW 1834–1853 PROFESSOR 1839–1853
PRESIDENT 1853–1860 OVERSEER 1864–1874

KINECT

XBOX

DEMO