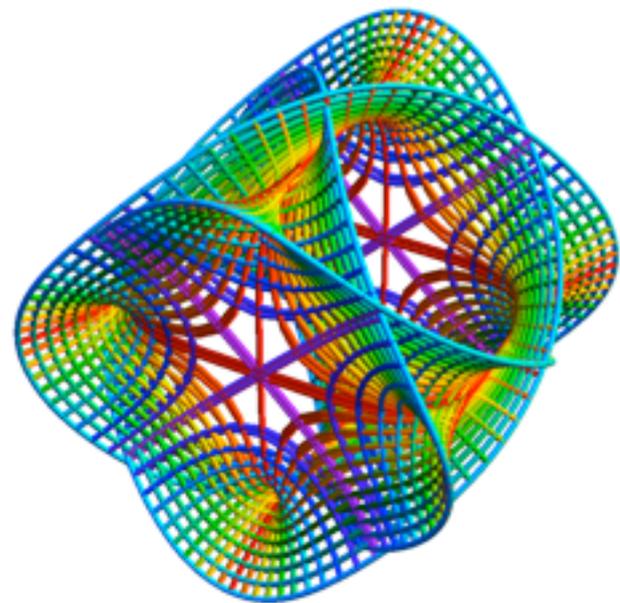
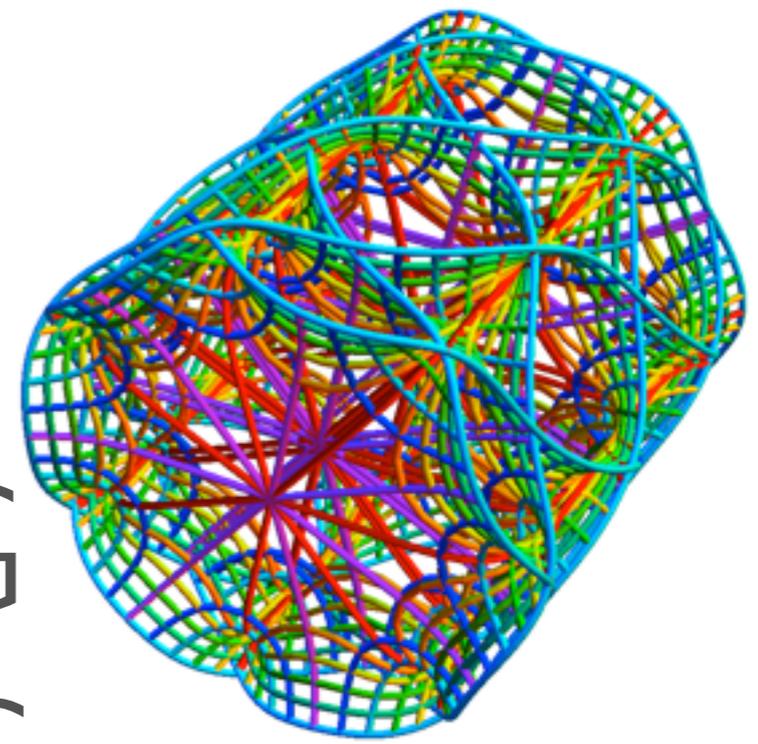


ILLUSTRATING
MATHEMATICS
USING
3D PRINTERS



Oliver Knill

Harvard University

Elizabeth Slavkovsky

May 6, Trieste, 2013



*"We live in the best of
all possible worlds."*



Gottfried Leibniz in *Théodicée*

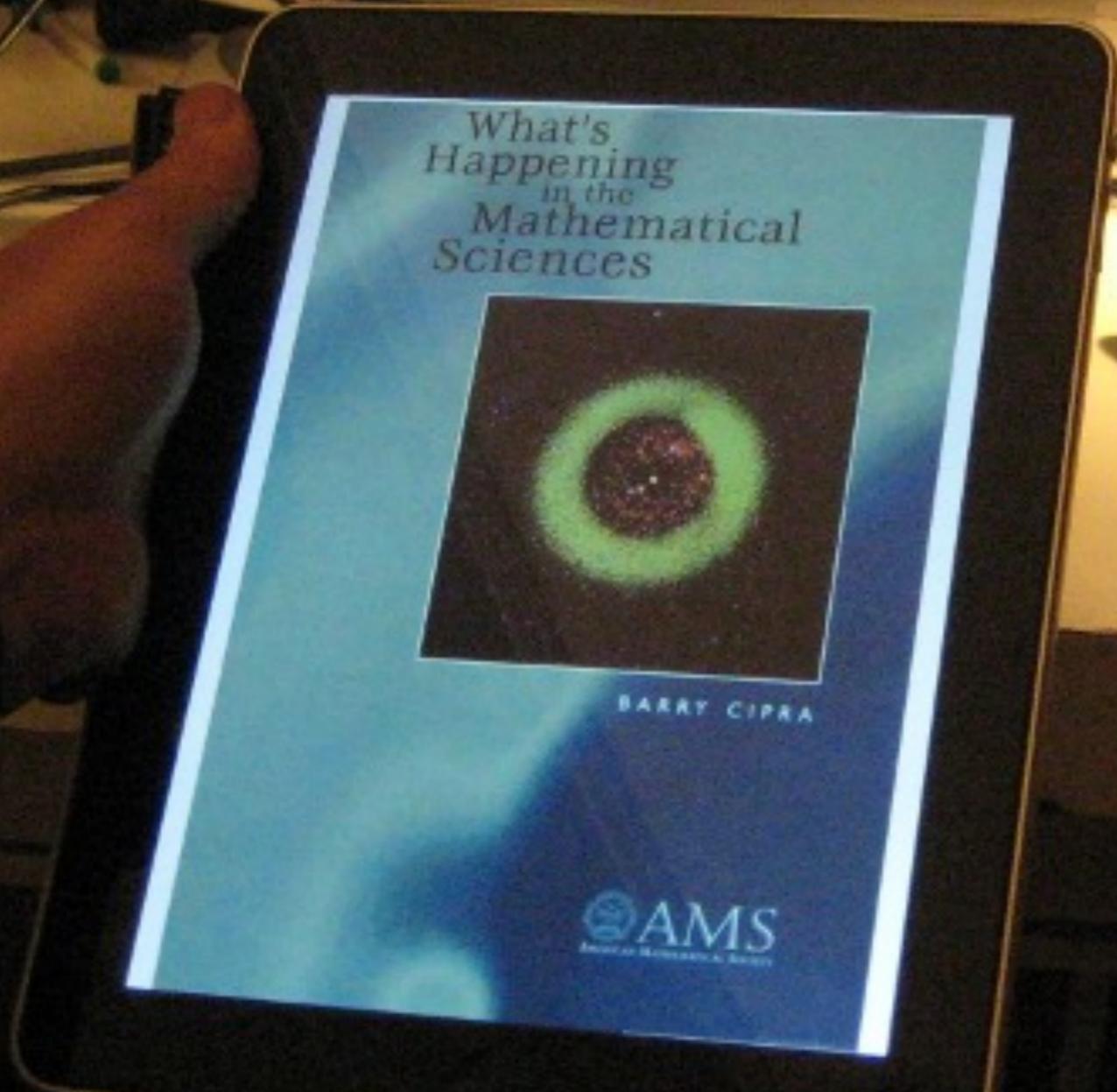
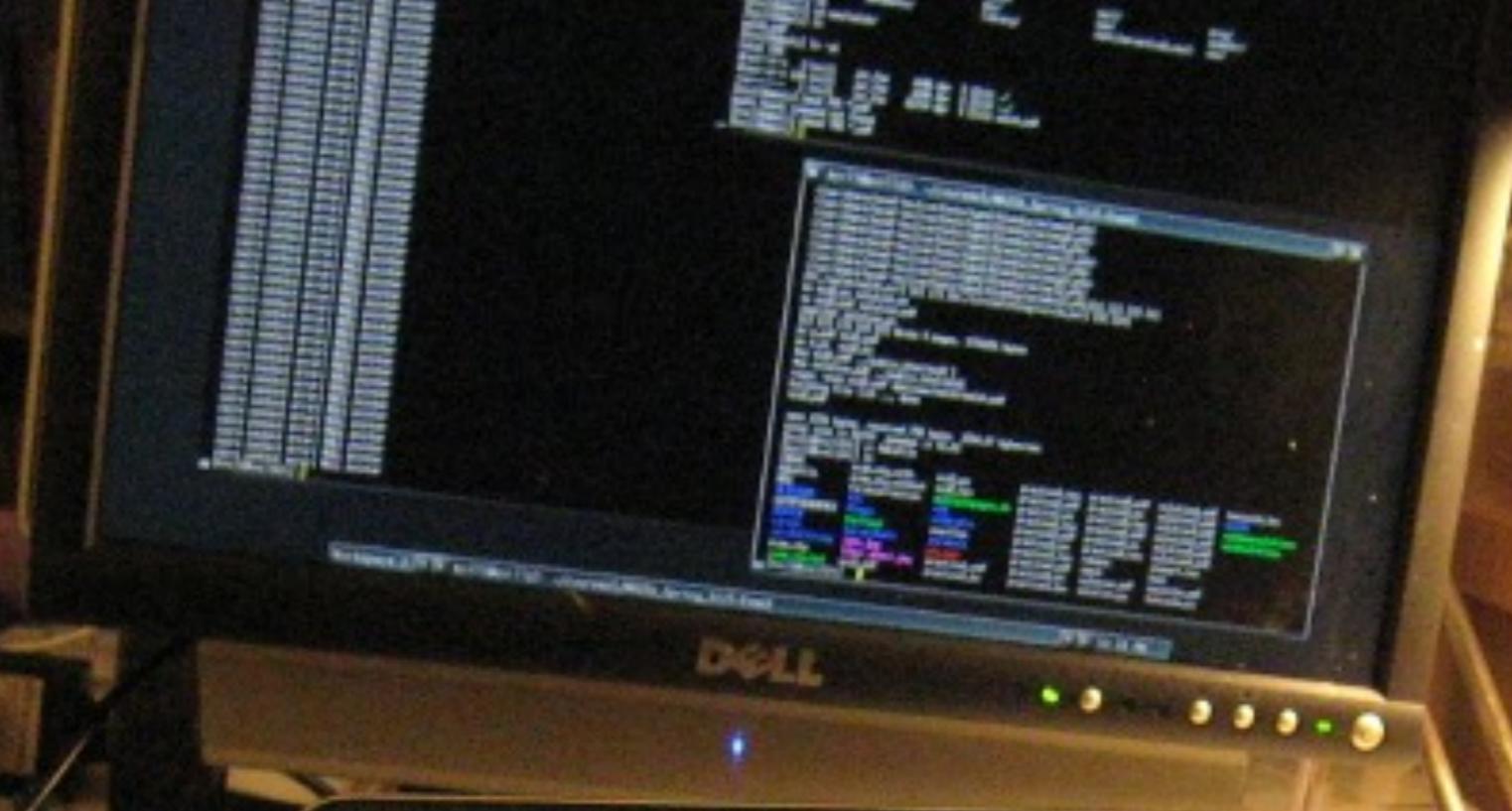
We seem to
live also in the
best of all
possible times!

Why? We
experience
several exciting
revolutions at
once.



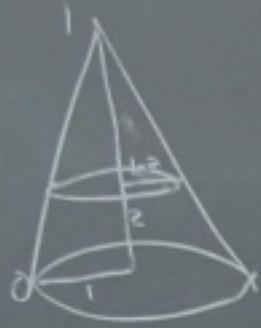
2010

Information
Revolution

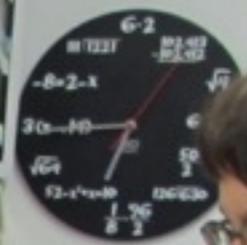




Liz has
just won a
prize for her
work!



$$\int_0^1 (1-z)^2 dz = \left[-\frac{(1-z)^3}{3} \right]_0^1 = \frac{1}{3}$$

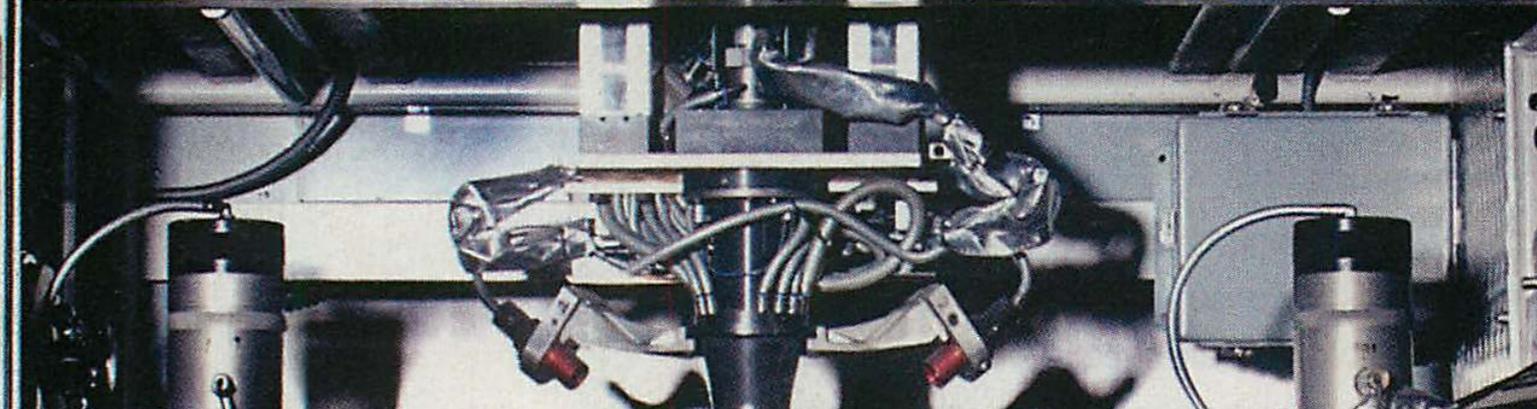
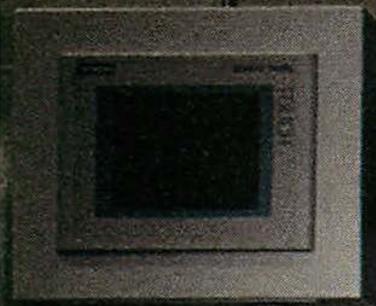


EMRE
PROGRAM

3D printing in the
news. Examples
from last couple of
days:

Scientific american

To Print the Impossible





DE TITLOW

CE-PRESIDENT OF PRODUCT MANAGEMENT
CORPORATION

National Geographic

Revolutions Everywhere



Harvard Bee





**Controlled Flight of a
Biologically-inspired,
Insect-scale Robot**

And of course:

LOW-COST 3D PRINTING

FOR SCIENCE, EDUCATION

 SUSTAINABLE DEVELOPMENT



The Abdus Salam
International Centre
for Theoretical Physics



Low-cost 3D Printing for Science, Education & Sustainable Development

Low-cost, three-dimensional (3D) desktop printing, although still in its infancy, is rapidly maturing, with seemingly unlimited potential. The hope is that this cutting-edge 3D technology will open new dimensions to science and education, and will make a marked impact in developing countries.

This book gives a reasonable, first overview of current research on 3D printing. It aims to inspire curiosity and understanding in young scholars and new generations of scientists to motivate them to start building up their own 3D printing experiences and to explore the huge potential this technology provides –with the final goal of putting learning literally in their hands.

Thanks, Enrique,
Carlo and Marco!

2013

Editors: E. Canessa ♦ C. Fonda ♦ M. Zennaro

Graphic design by C. Fonda.
Cover photo courtesy of G. Fior.
Published by the ICTP, © 2013.

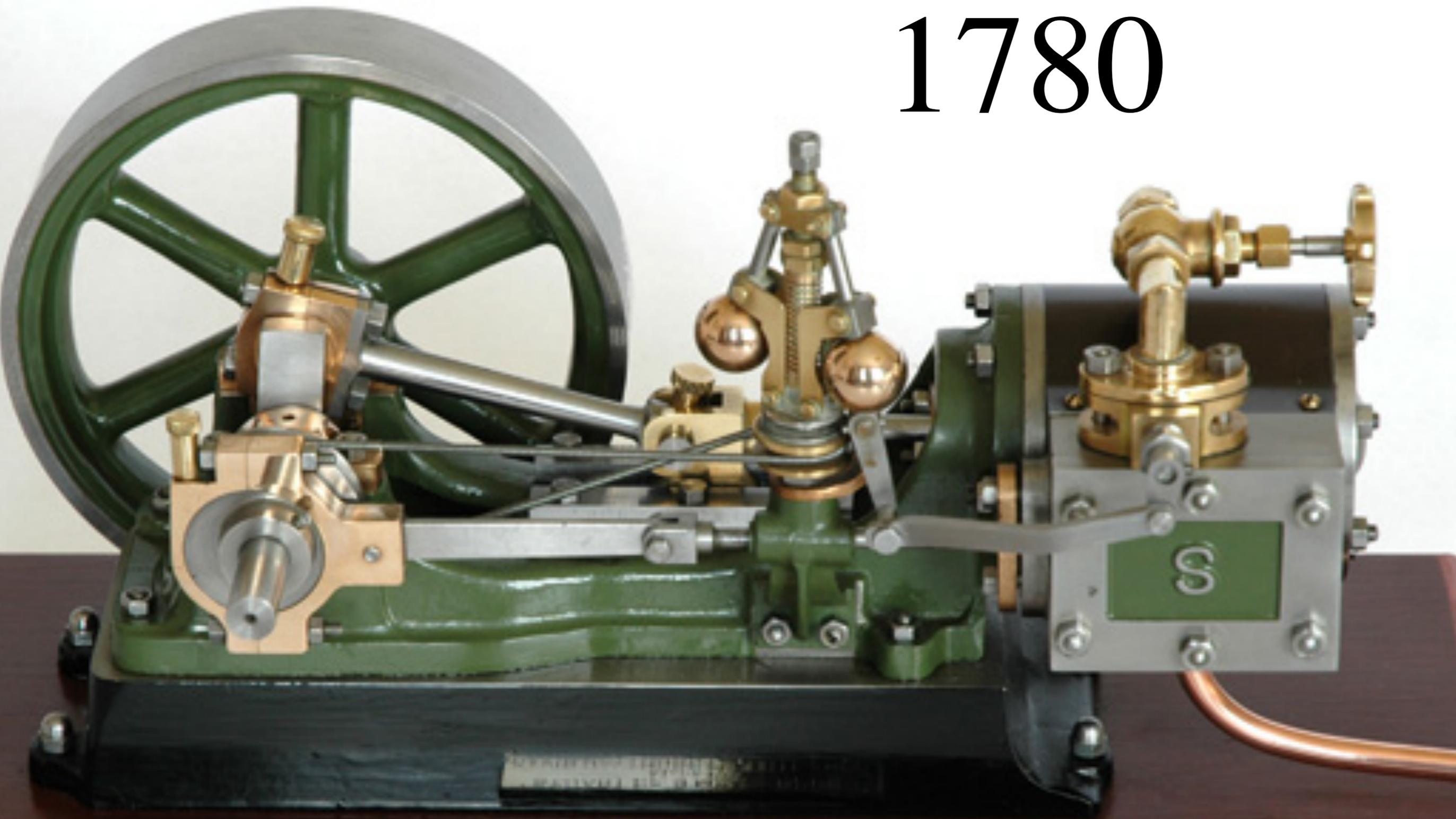


ISBN 92-95003-48-9



Industrial Revolutions

1780



Steam



1880

Steel



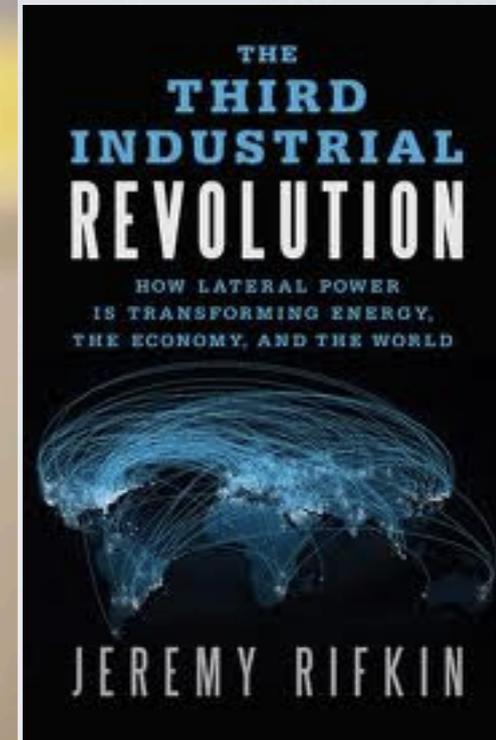
1980

Stars

Nasa: Horsehead

INDUSTRIAL REVOLUTIONS

Steam Engine, Textile	1780
Automotive, Chemistry	1850
Rapid Prototyping Personal Computing	1970



JEREMY RIFKIN

"Third Industrial Revolution"

"manufacturing becomes digital, personal, and affordable"

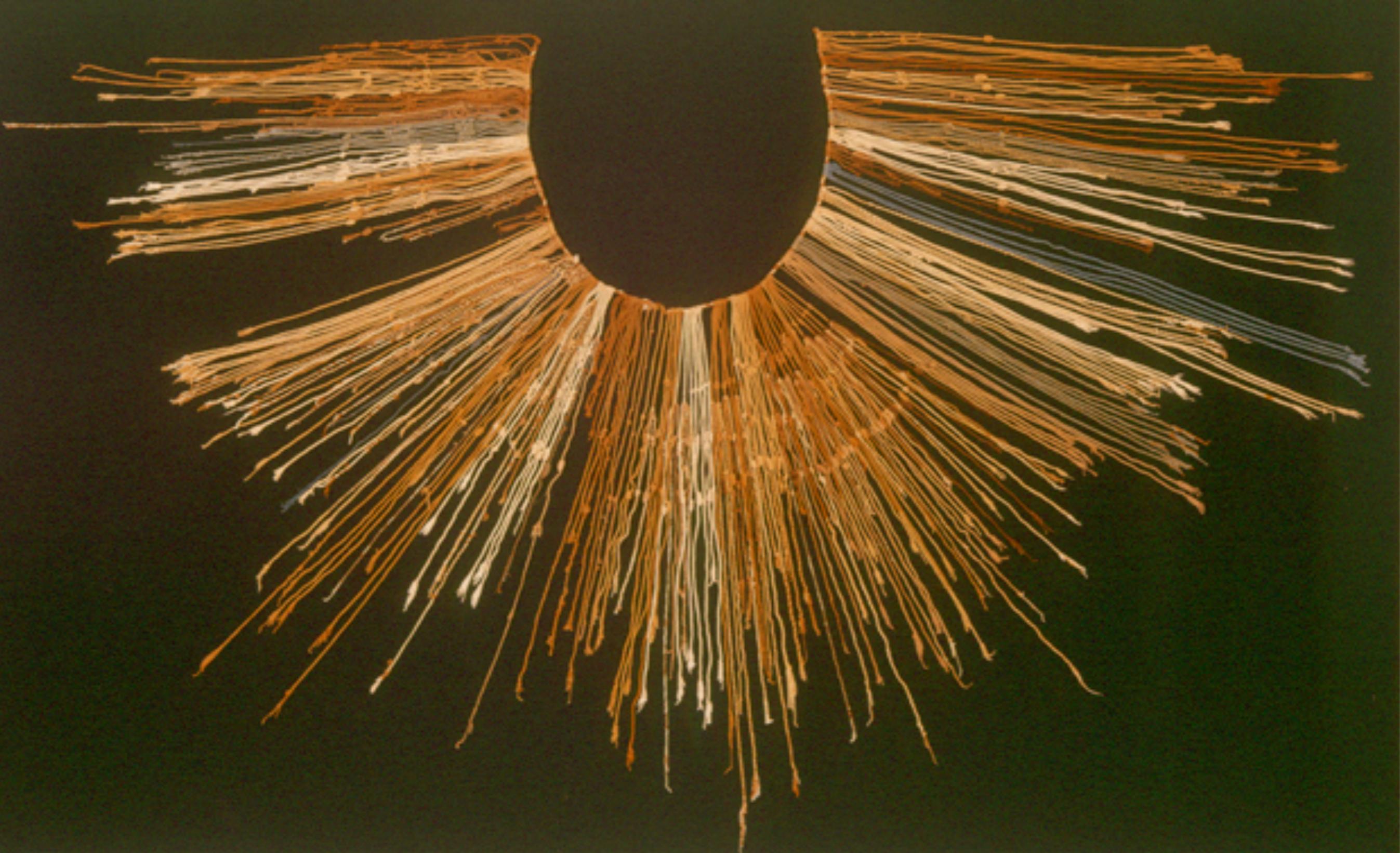
INFORMATION REVOLUTIONS

Gutenberg Press	1439
Telegraph	1880
Personal Computer Cell Phone,Internet	1970

THIS IS A STEP OF
MANY:



ISHANGO, 20000 YEARS AGO
AFRICA



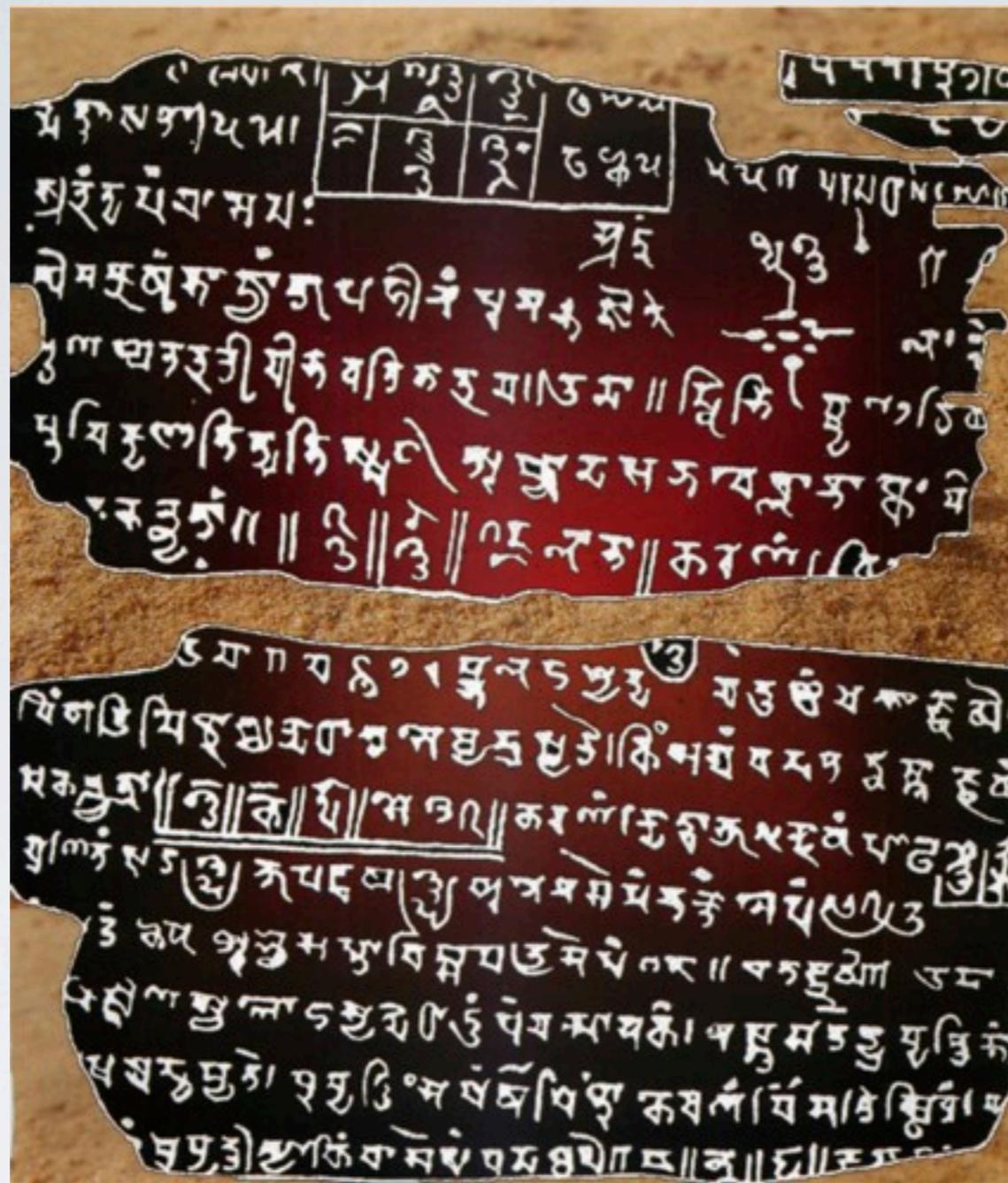
QUIPU, 5000 YEARS AGO
ANDES



PLIMPTON 322, 4000 YEARS AGO
BABYLONIA



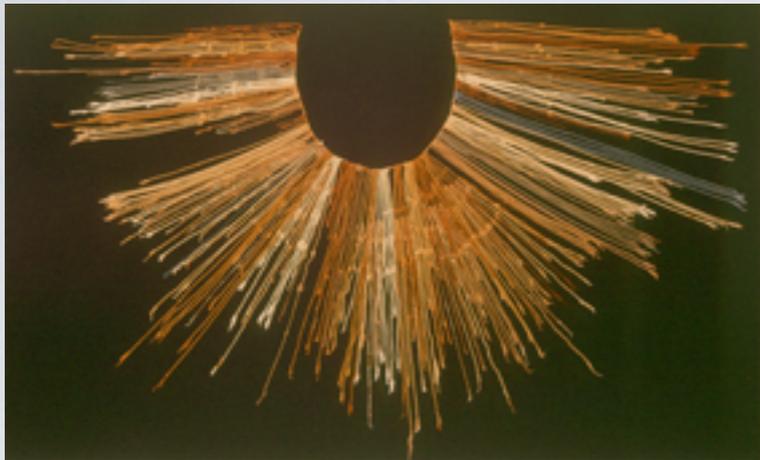
RYND, 4000 YEARS, EGYPT



BAKSHALY, 2500, INDIA



MAYAN, 2200, CENTRAL
AMERICA



Quipu

3000 BC



Rynd

1600 BC



Maya

150 AD



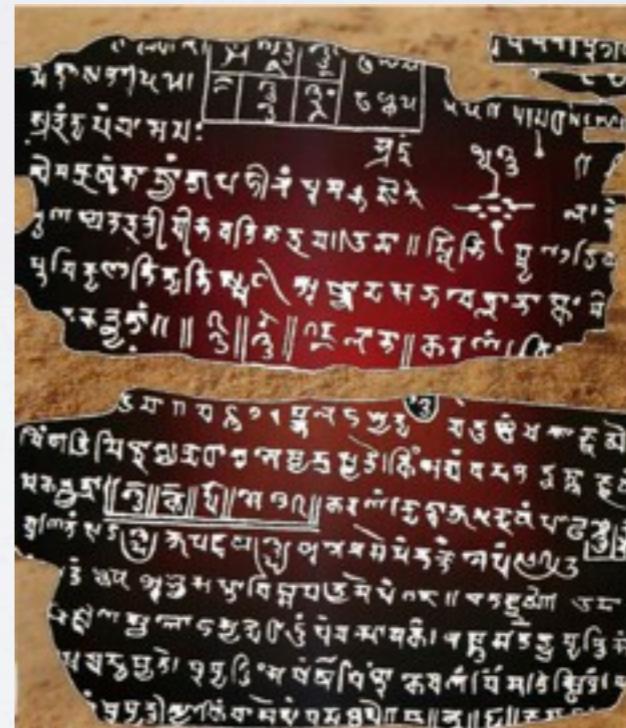
Ishango

80000 BC



Plympton

1800 BC



Bakshali

400 AD

COMMUNICATION REVOLUTIONS



Figures

6'000 years ago

Books

560 years ago

Photo

170 years ago

Film

130 years ago

3D Print

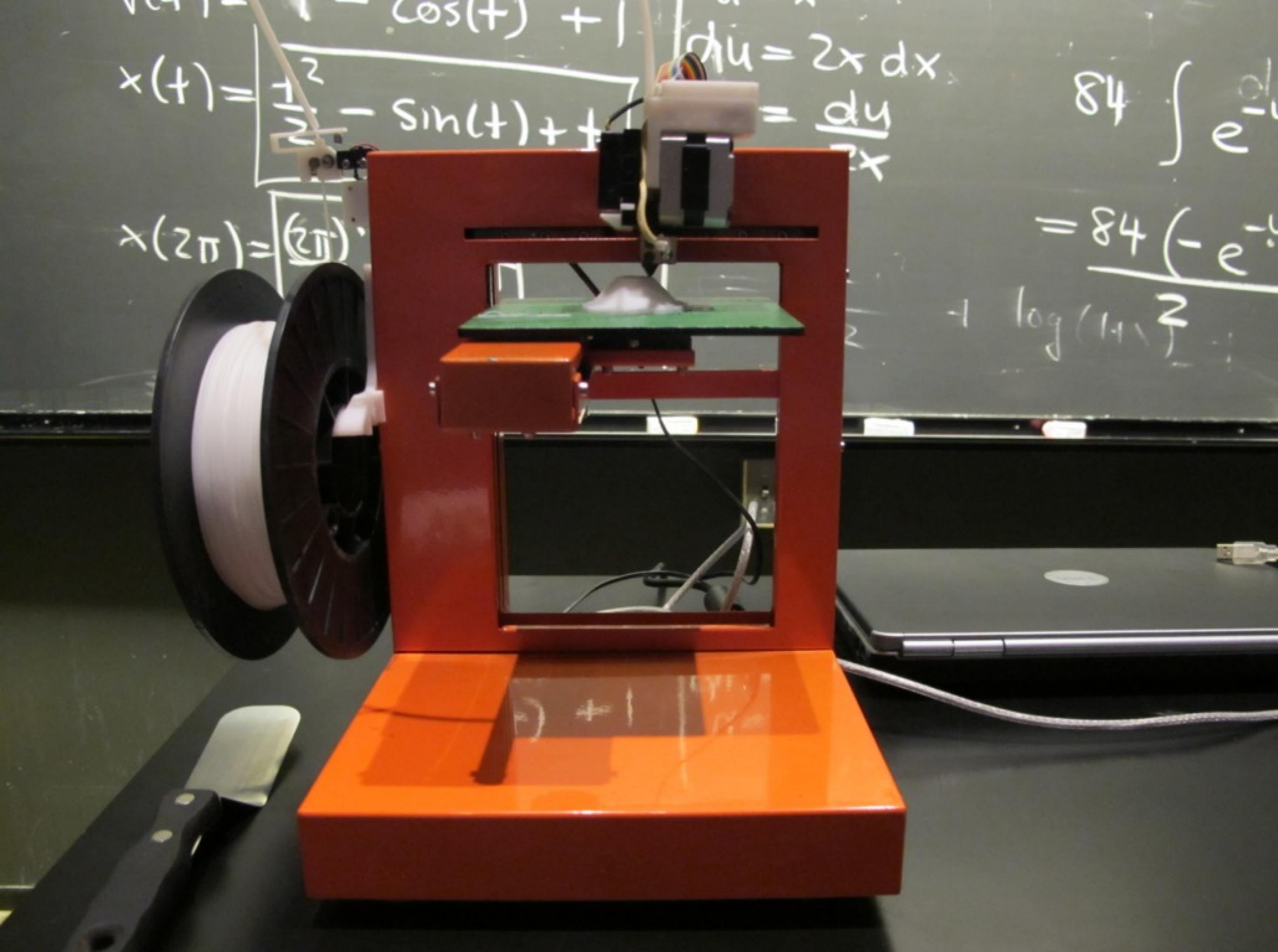
30 years ago



PRINTING, 600 YEARS



LASER PRINTER, 50 YEARS



$$x(t) = \frac{t^2}{2} - \sin(t) + t$$

$$x(2\pi) = \sqrt{(2\pi)}$$

$$\cos(t) + 1 \quad du = 2x dx \quad \frac{dy}{dx}$$

$$84 \int e^{-x}$$

$$= 84 \frac{-e^{-x}}{-1} + \log(1+x^2)$$



3D SCANNER



PERCEPTION REVOLUTIONS



Eye Glass

730 years ago

Microscope

420 years ago

Telescope

400 years ago

Xrays

110 years ago

MRI

60 years ago

3D Scan

25 years ago

CLASSROOM REVOLUTIONS

Abacus	1500 years ago
Blackboard	1000 years ago
Computer Algebra	50 years ago
Calculator	40 years ago
Powerpoint	30 years ago
3D printed Models	15 years ago

Some Timelines

word wide web → HTML5

→ java → flash → flash

Computer Games →

Educational TV VHS

DVD

YouTube

since 30ies: slide projectors

3Dprint

since 40ies: overhead projectors

since 60ies: PRS

power point

keynote

Computer algebra systems

Maxima

macyma

Mathematica, Maple, Matlab

Reduce

Cayley

Magma

Pocket calculators

Programmable calculators

Graphing calculators

Calculators with CAS

Ipad Alpha



70ies

80ies

90ies

now

Schaltinterface für TI-59, TI-58, TI-57

Es sind schon viele gute Bauanleitungen für ein Schaltinterface in CHIP publiziert worden. Warum denn ein anderes bauen? Ich suchte eine Schaltung, die es möglich macht, mehrere Kanäle zu steuern. Dafür eignet sich die Anzeige des Rechners als Schnittstelle sehr gut.

Eine 7-Segment-Anzeige ist einfach aufgebaut. Wenn zum Beispiel Leitung 1 und 8 angesteuert werden, so leuchtet das oberste Segment. Umgekehrt: Wenn das oberste Segment leuchtet, kann bei 1 und 8 eine Spannung abgegriffen werden.

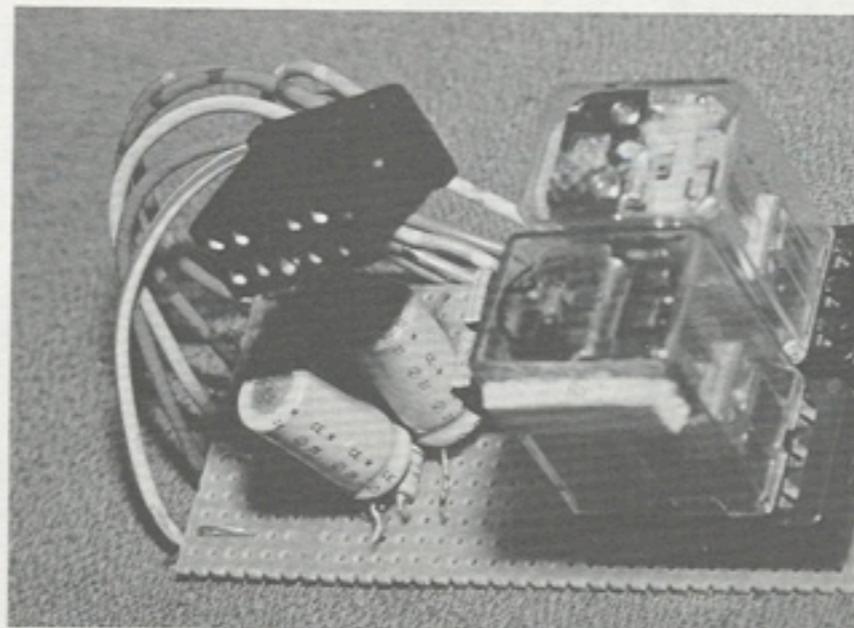
Diese Spannung wird nun verstärkt und schaltet ein Relais. In der Anzeige können die Zahlen 0 bis 9 stehen. Betrachten wir das obere und mittlere waagerechte Segment. Es reichen die Ziffern 1, 4, 7, 8, um zwei Kanäle voll zu steuern. Bei den Modellen TI-58, TI-59 sind zehn Ziffern da. Es könnten also 20 Kanäle gesteuert werden. Das gibt aber Probleme:

- Die Kosten für 20 Kanäle sowie der Arbeitsaufwand wären hoch.
- Es müssen 22 Leitungen vom Rechnerinnern nach draußen geführt werden.
- Im Extremfall müssen 20 Relais durchgeschaltet werden.

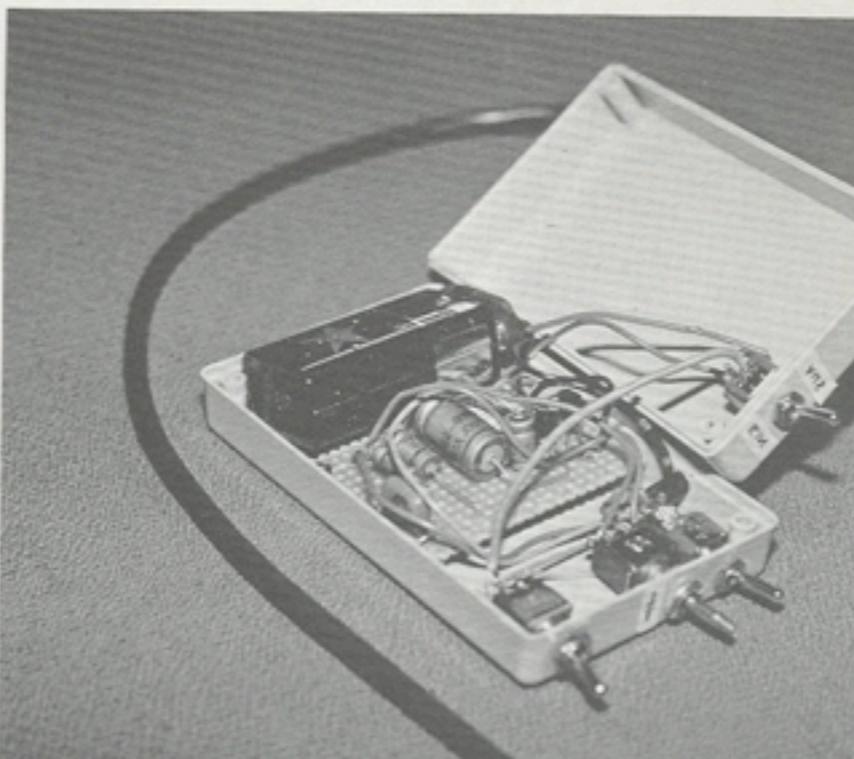
Ich habe mich deshalb entschlossen, zuerst nur zwei Kanäle auszubauen.

Die Schaltung

Der Schaltplan ist sehr einfach und kann auch von Computerfans aufgebaut werden, die über keine große Erfahrung verfügen.



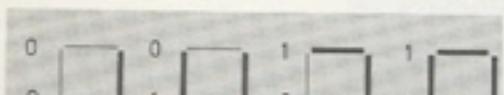
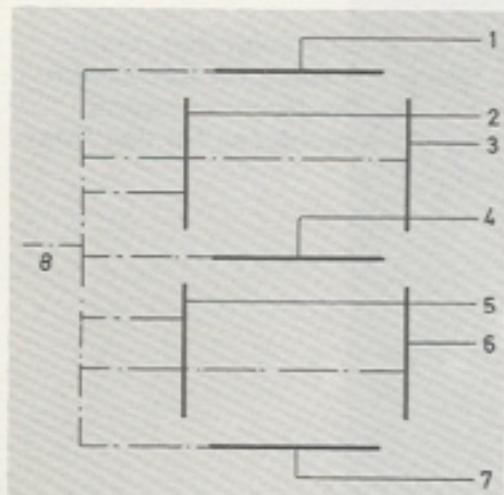
Versuchsaufbau des Schaltinterface mit zwei Relais



den, die über keine große Erfahrung verfügen.



1982



2300 YEARS

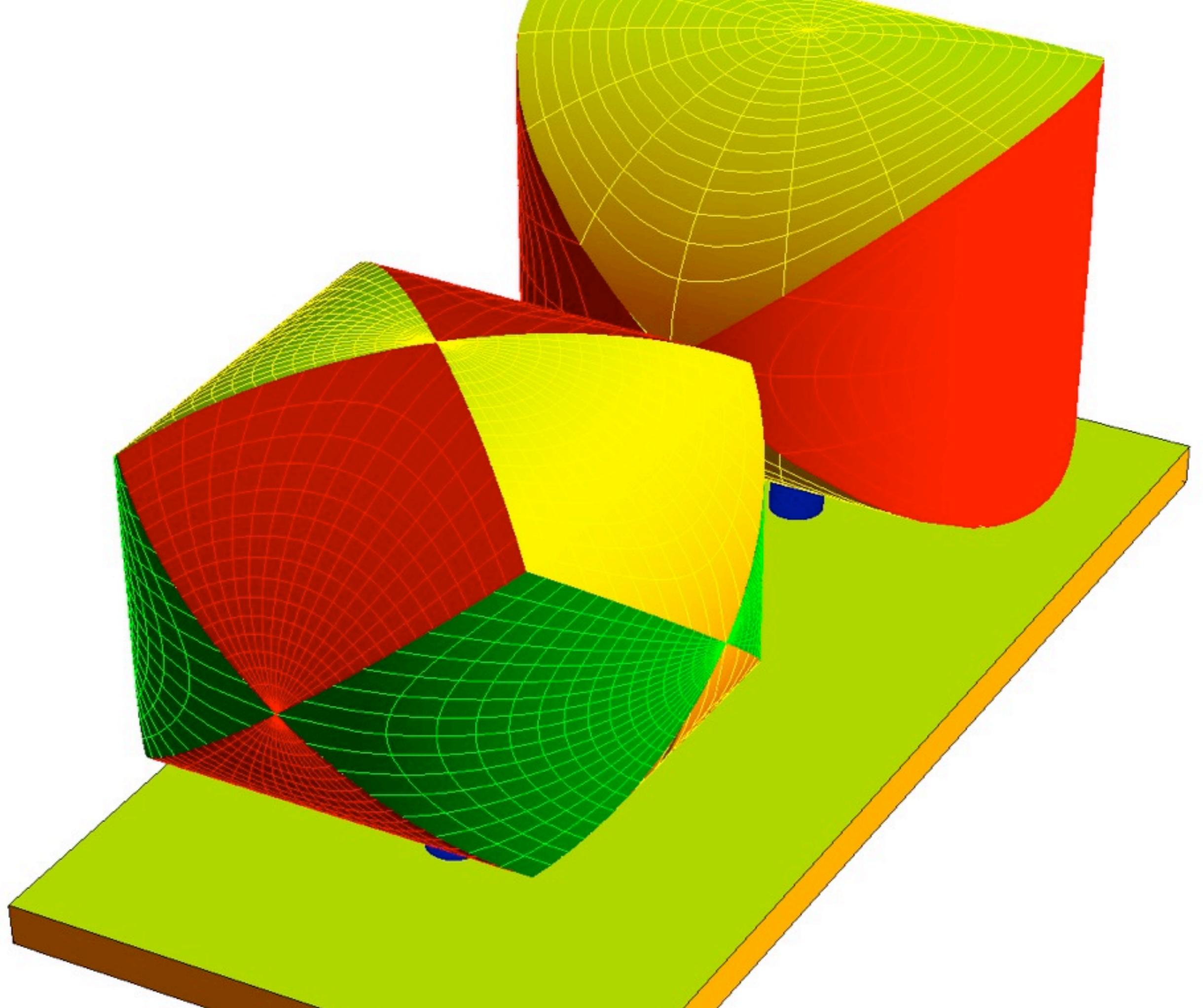
Archimedes

287-212 BC

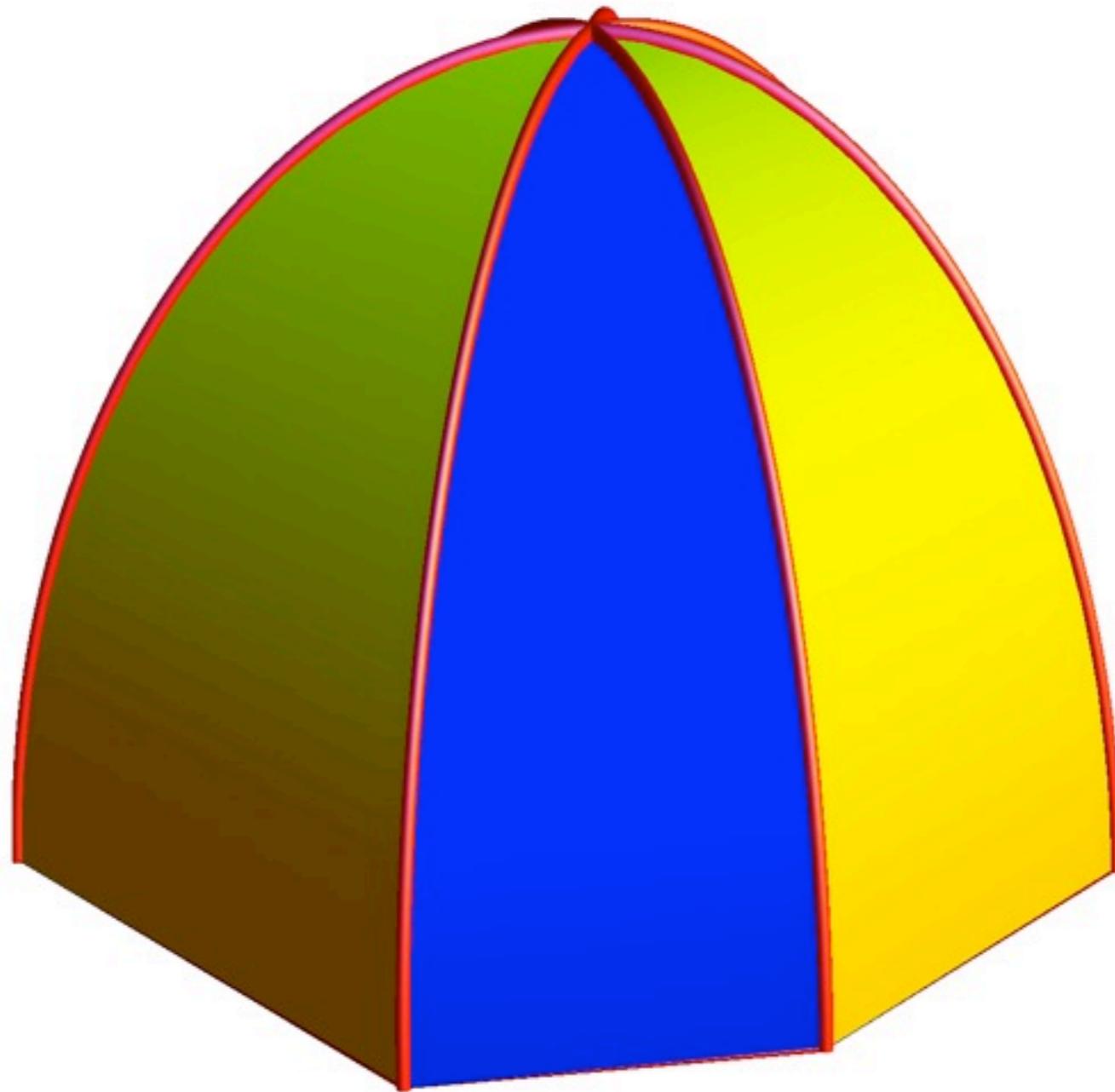
Domenico Fetti, Archimedes, 1620



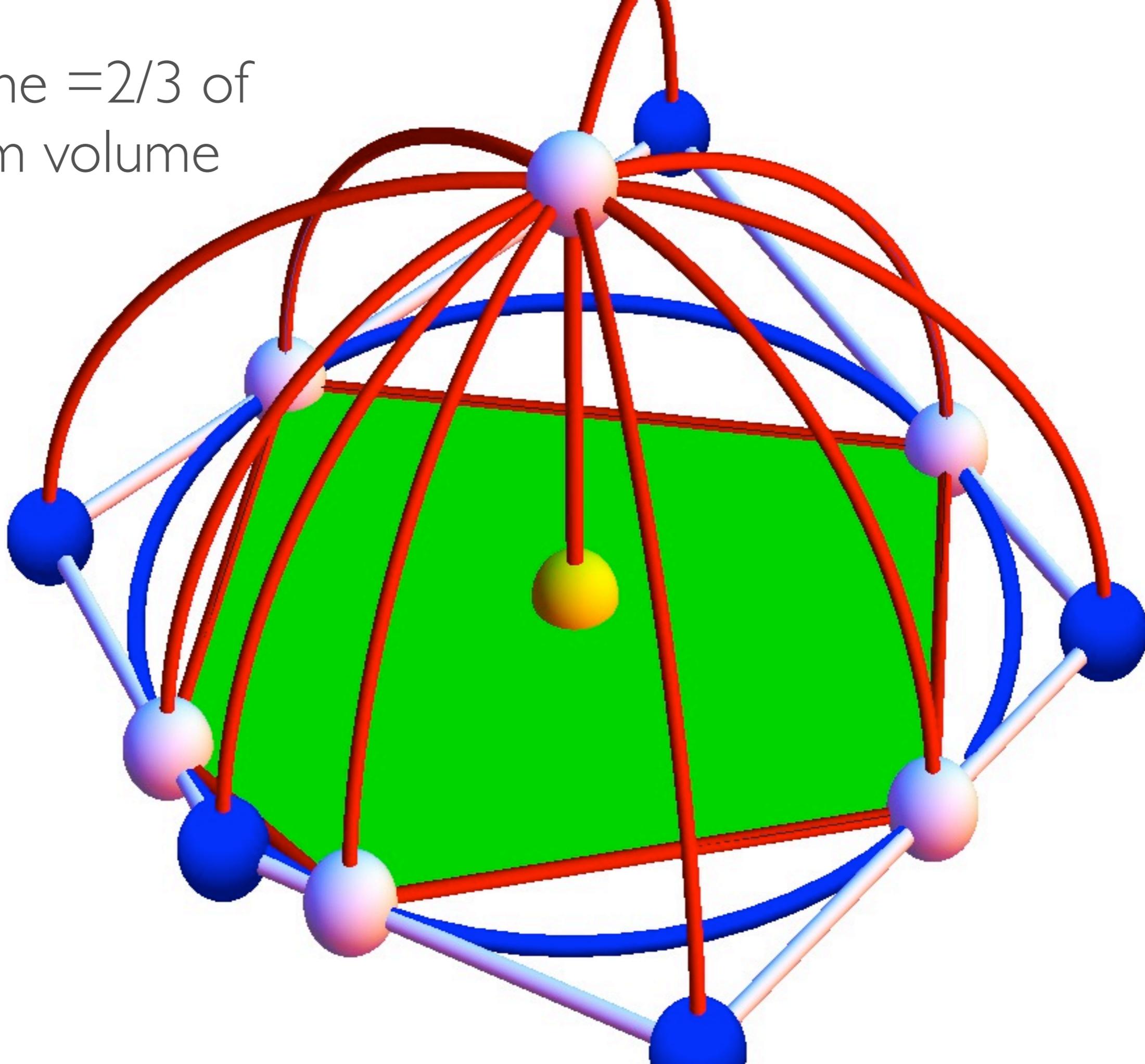
287-212 BC



ARCHIMEDEAN DOME

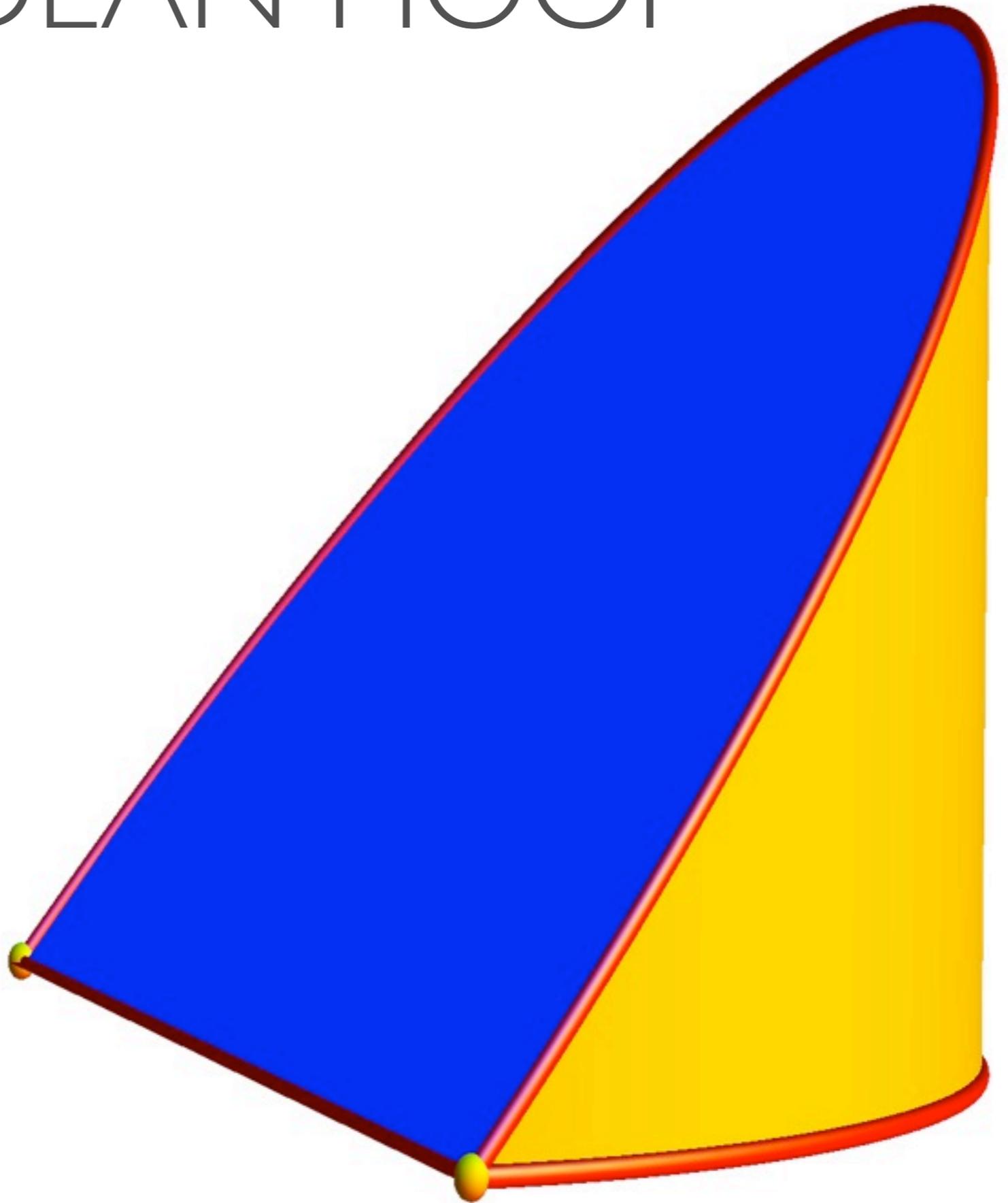


Volume = $\frac{2}{3}$ of
prism volume



ARCHIMEDEAN HOOF

"Birth place of
calculus"



CAVALIERI PRINCIPLE

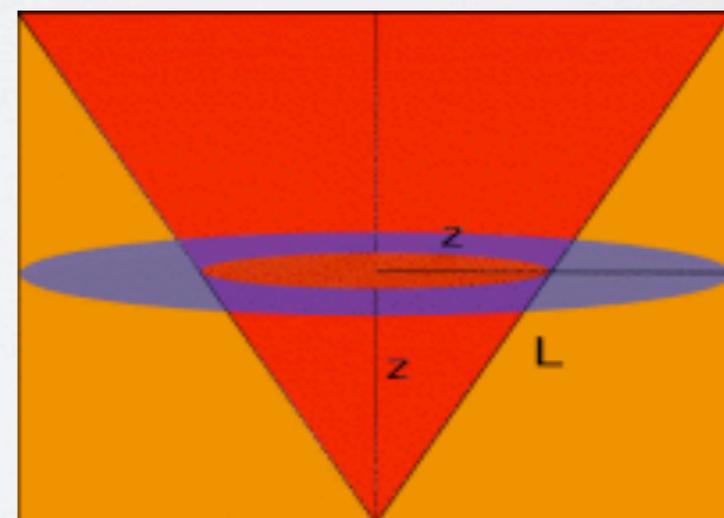
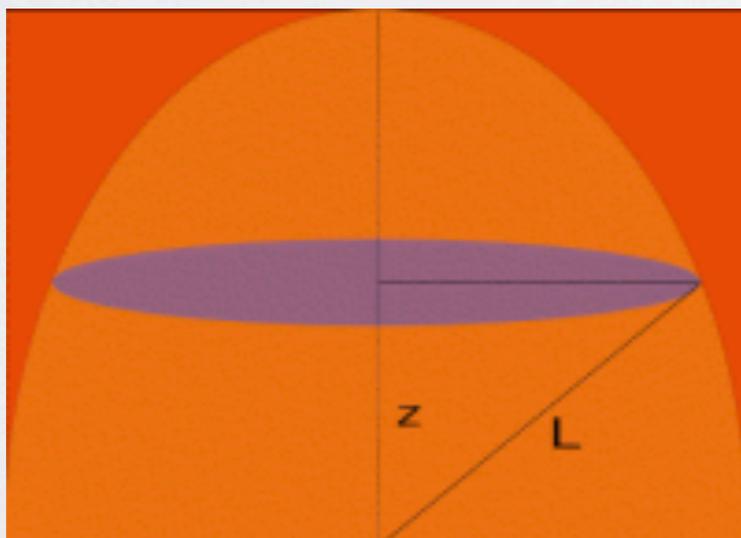
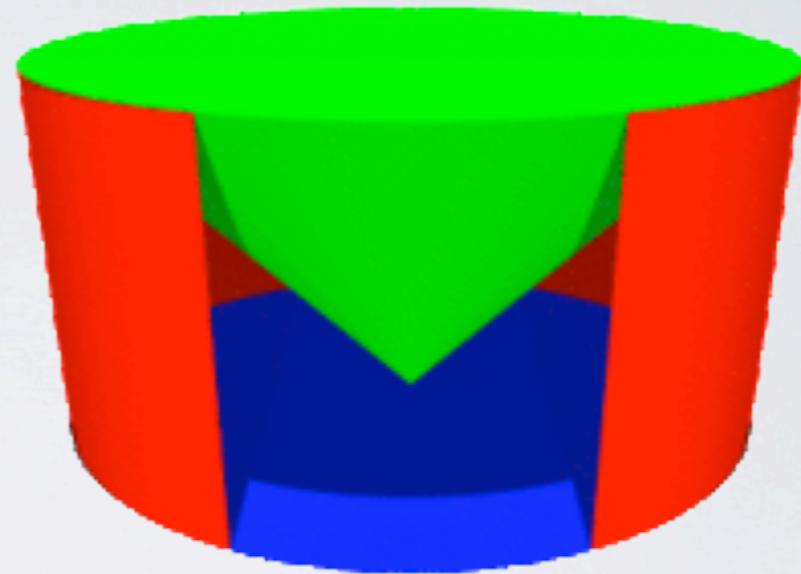
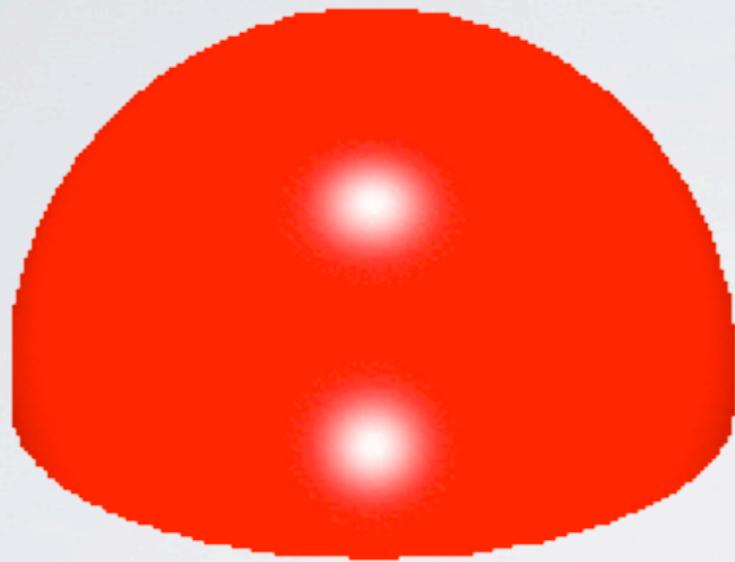


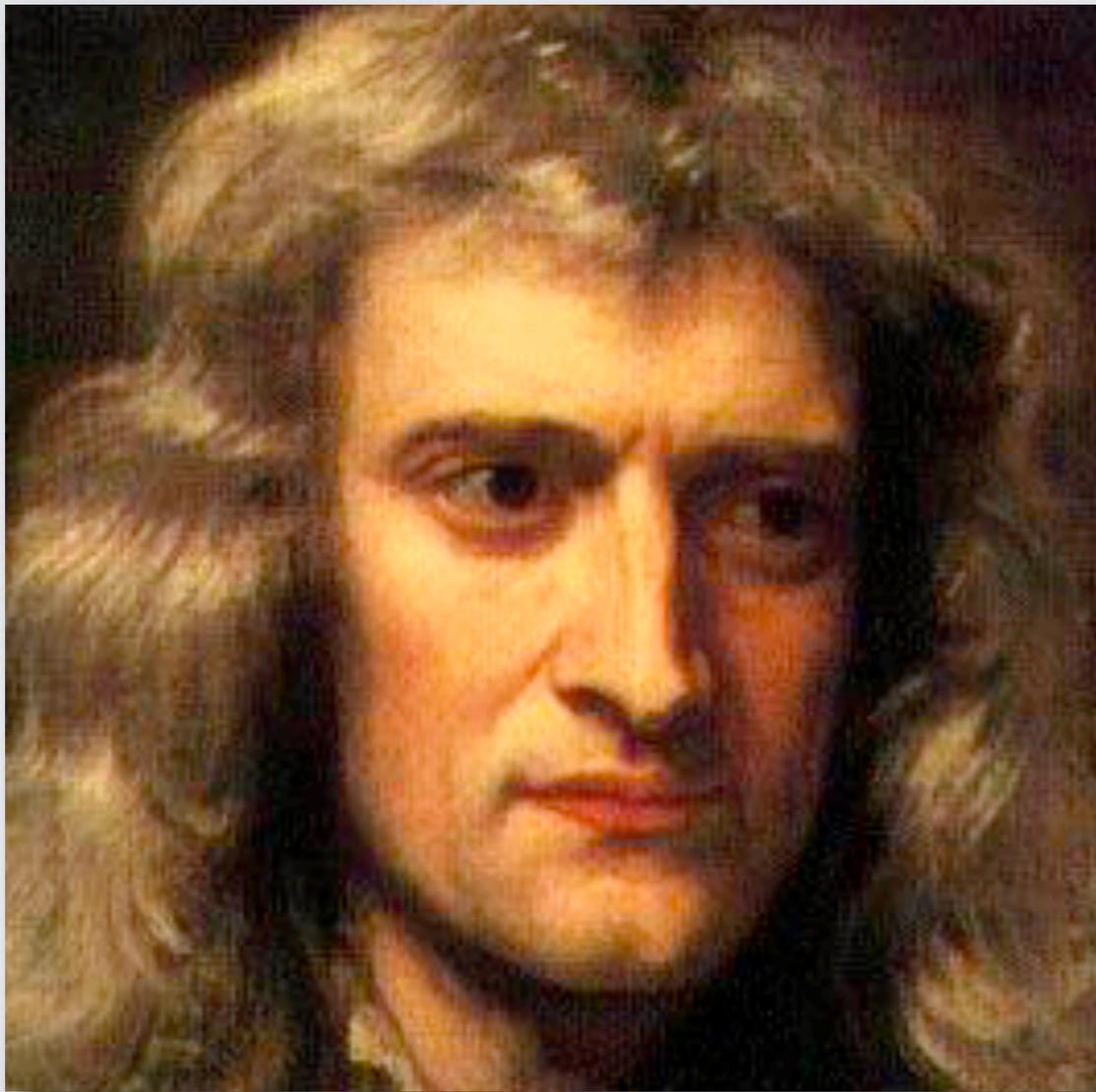




Image U.S. Geological Survey

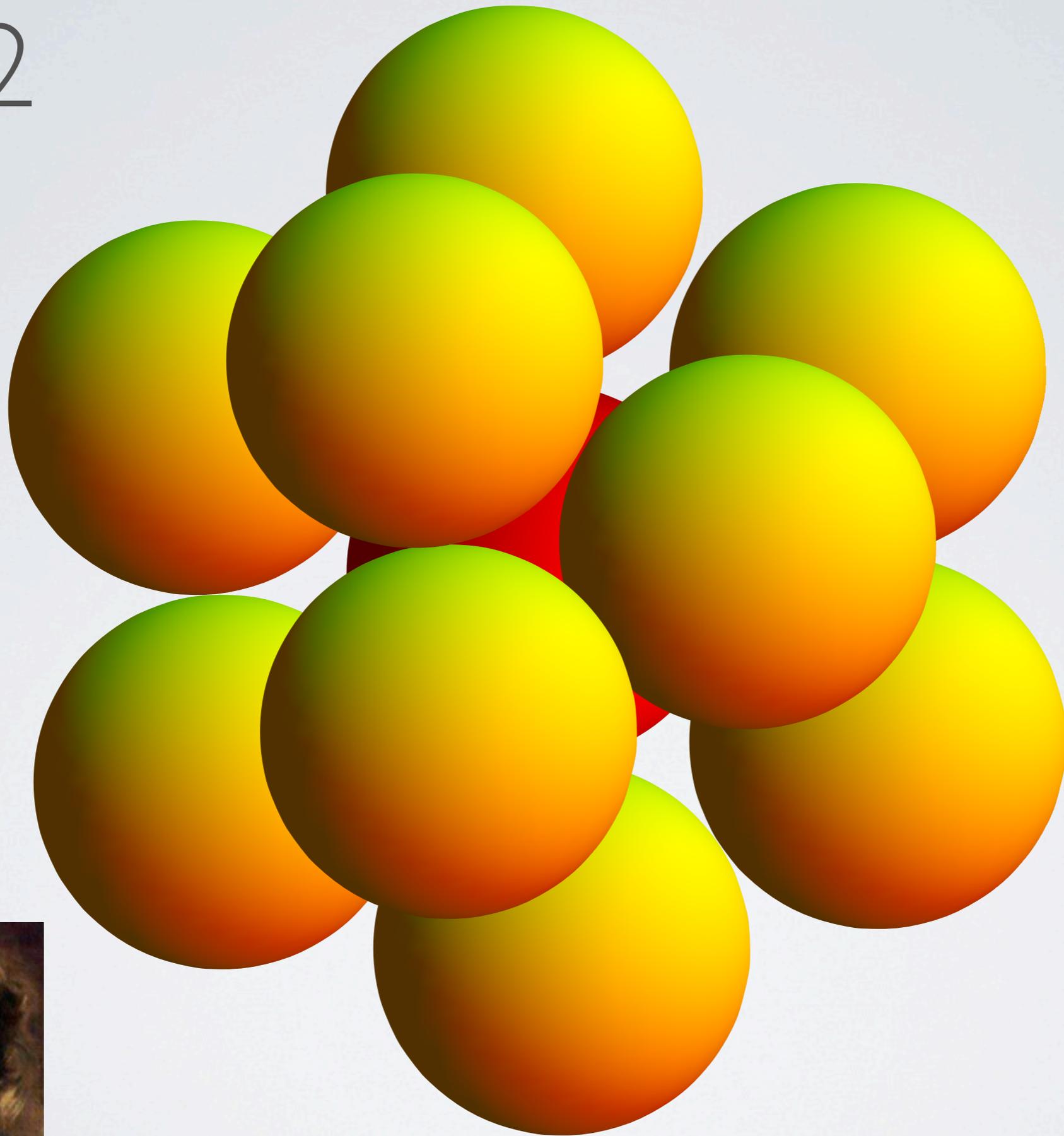
MORE ABOUT PROOFS
in *Mathematics*

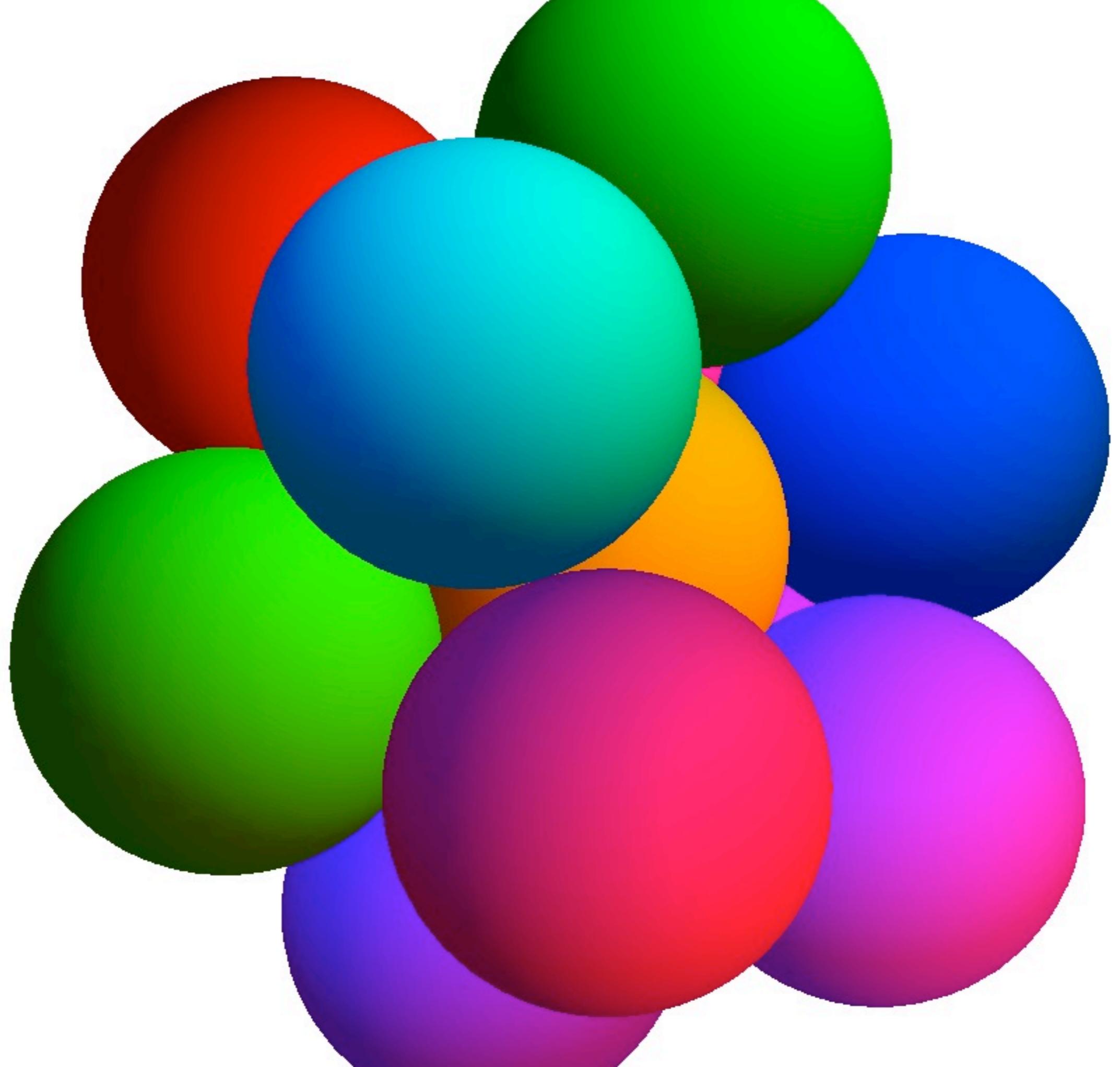
NEWTON - GREGORY



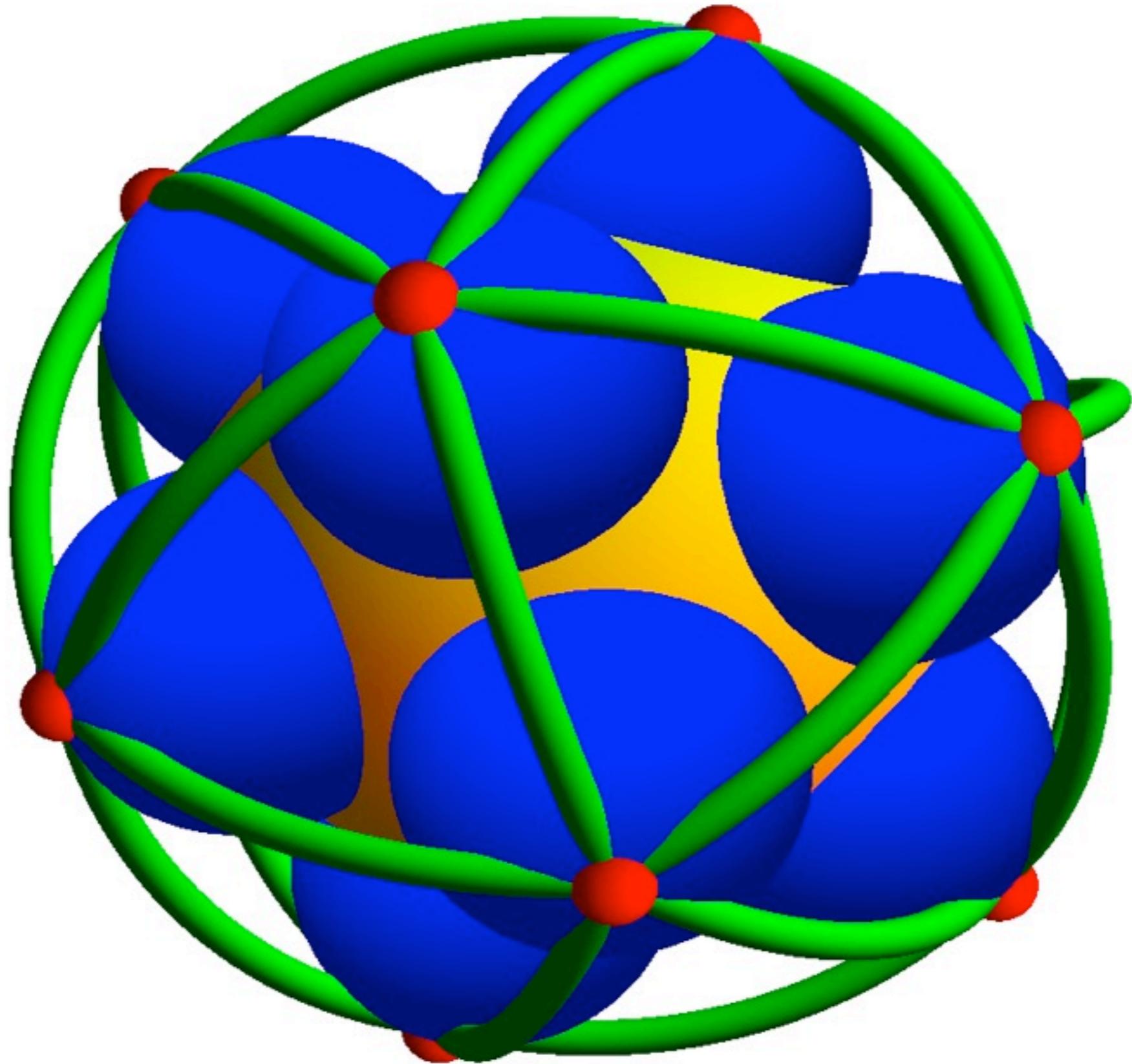
12

13

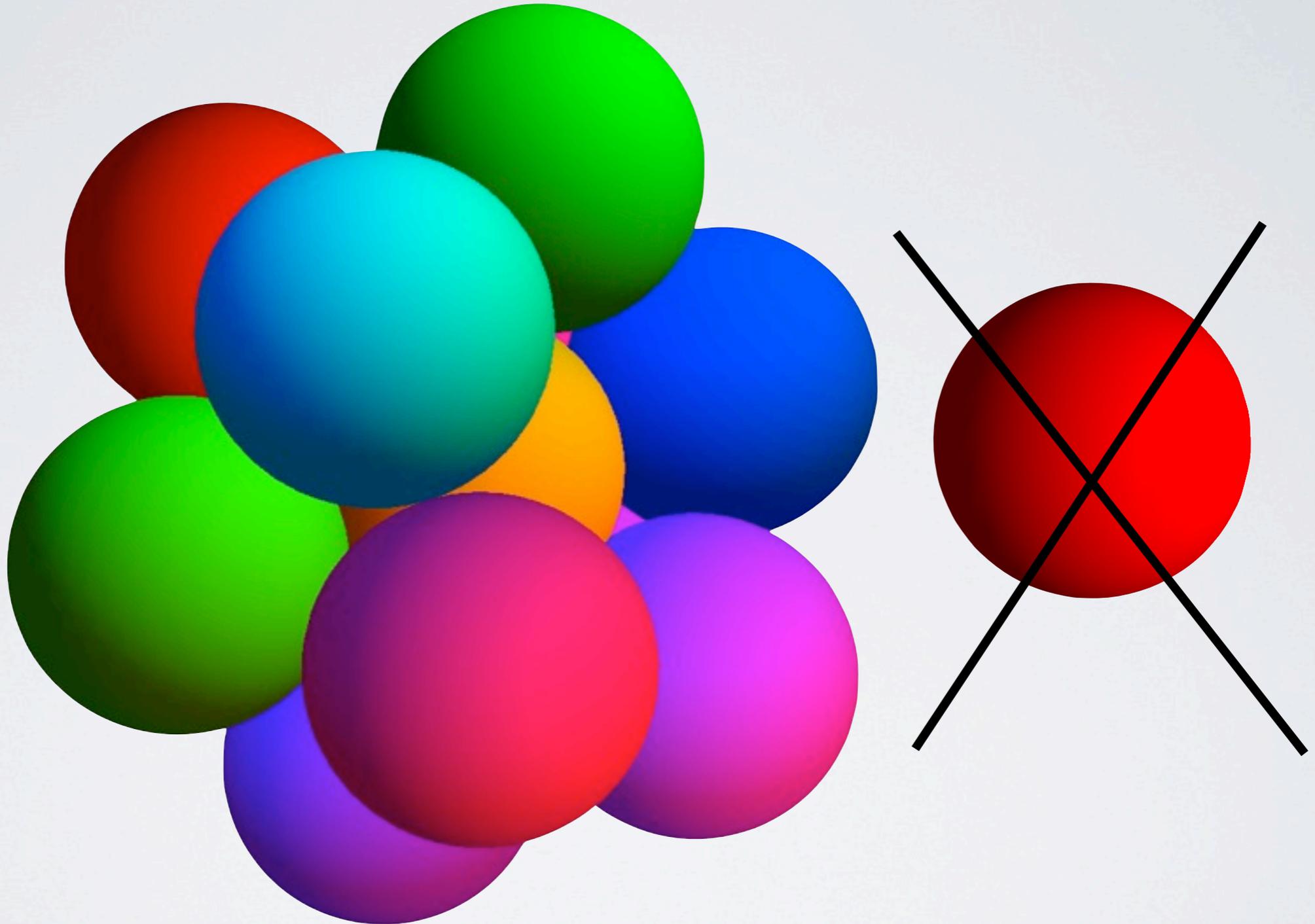




ICOSAHEDRAL ARRANGEMENT



NEWTON WON



ONLY IN 1953!

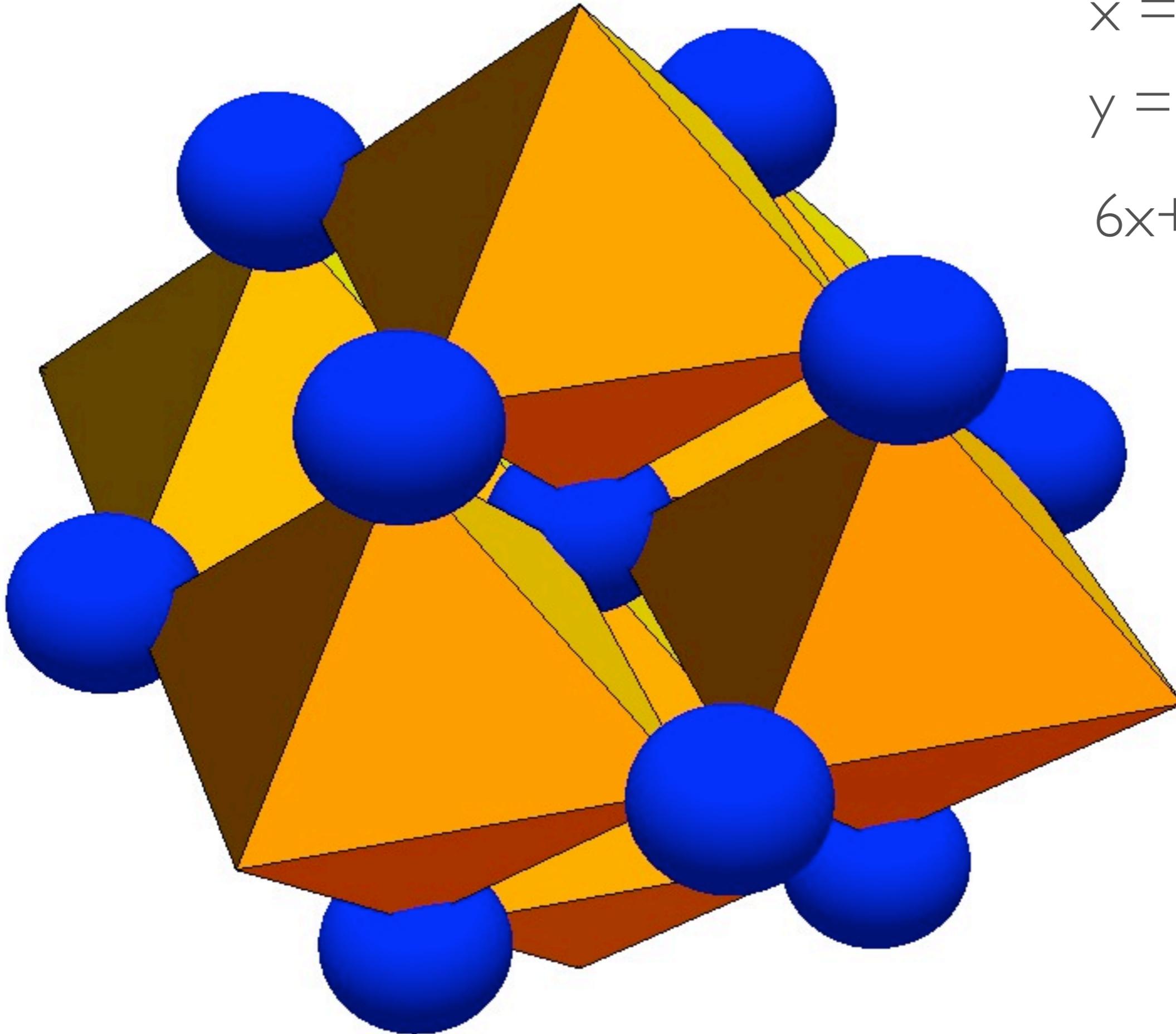
OCTAHEDRON AND TETRAHEDRON

$$x = \text{Vol}(\text{O})$$

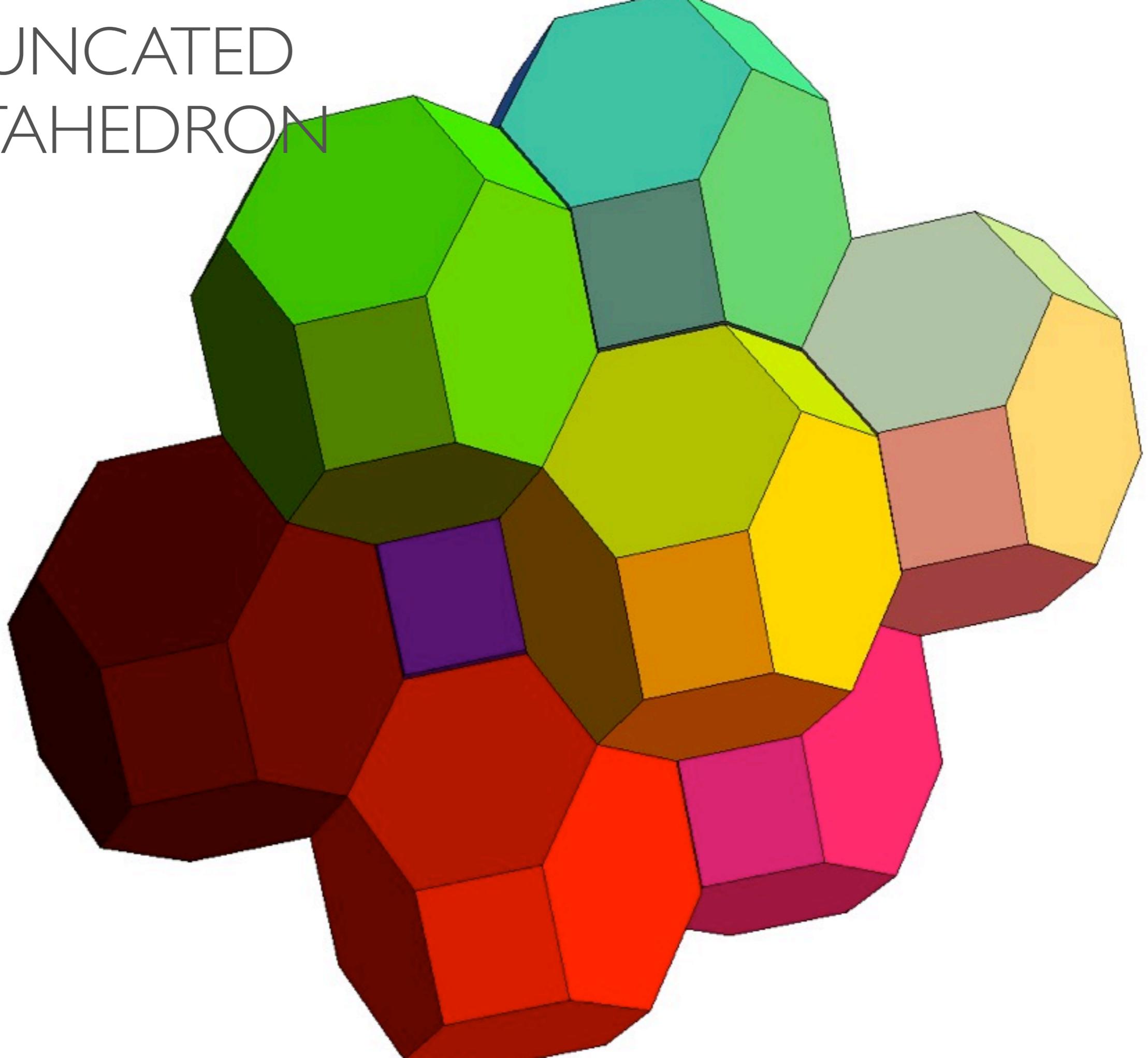
$$y = \text{Vol}(\text{T})$$

$$6x + 8y = 8x$$

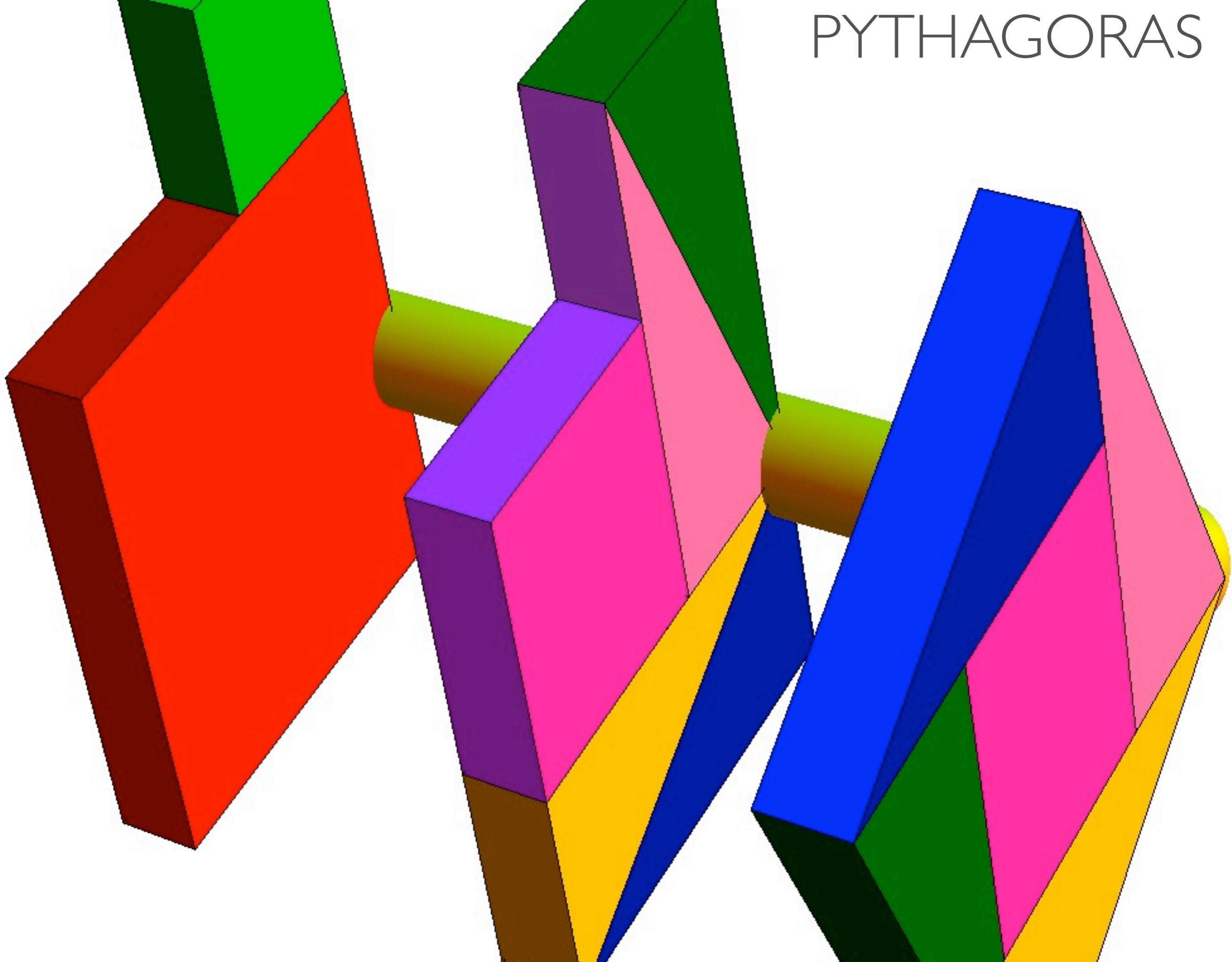
$$y = x/4$$

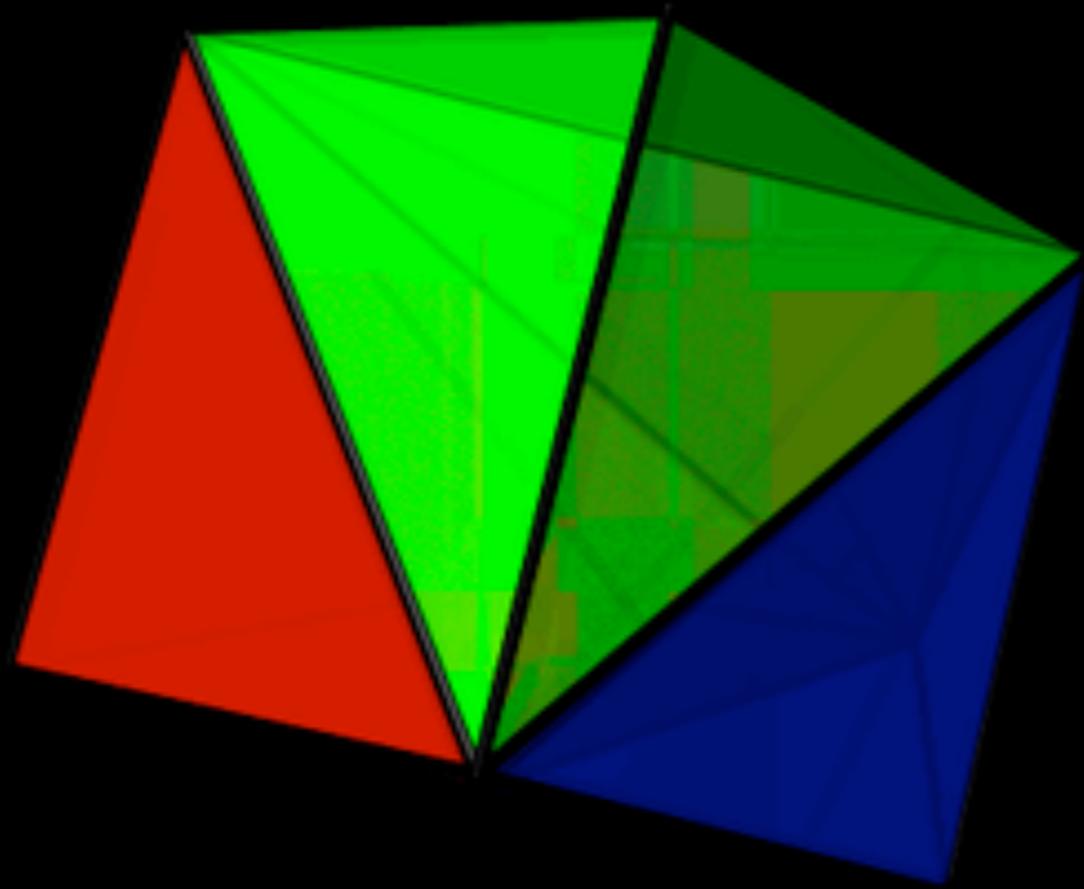


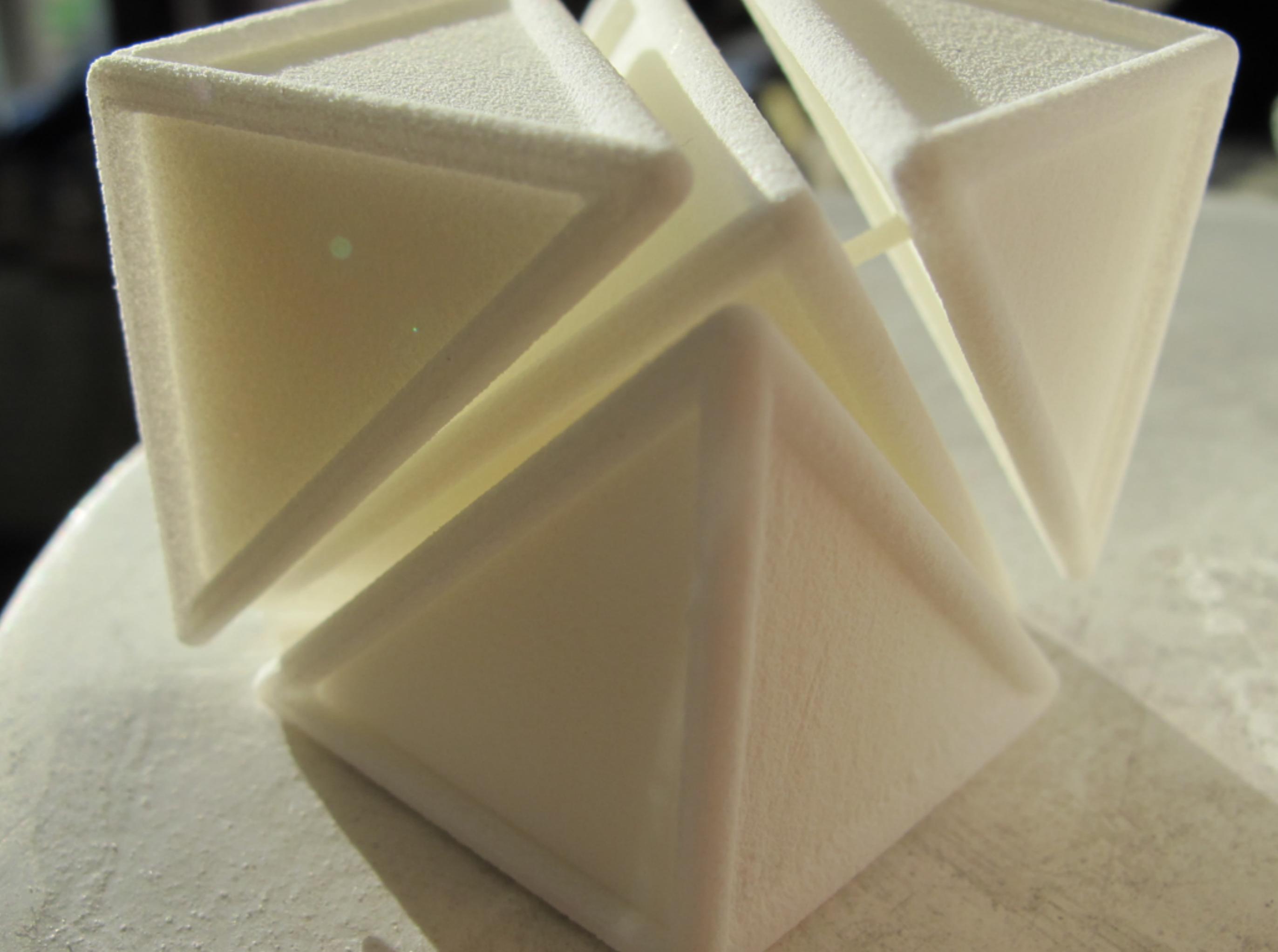
TRUNCATED OCTAHEDRON



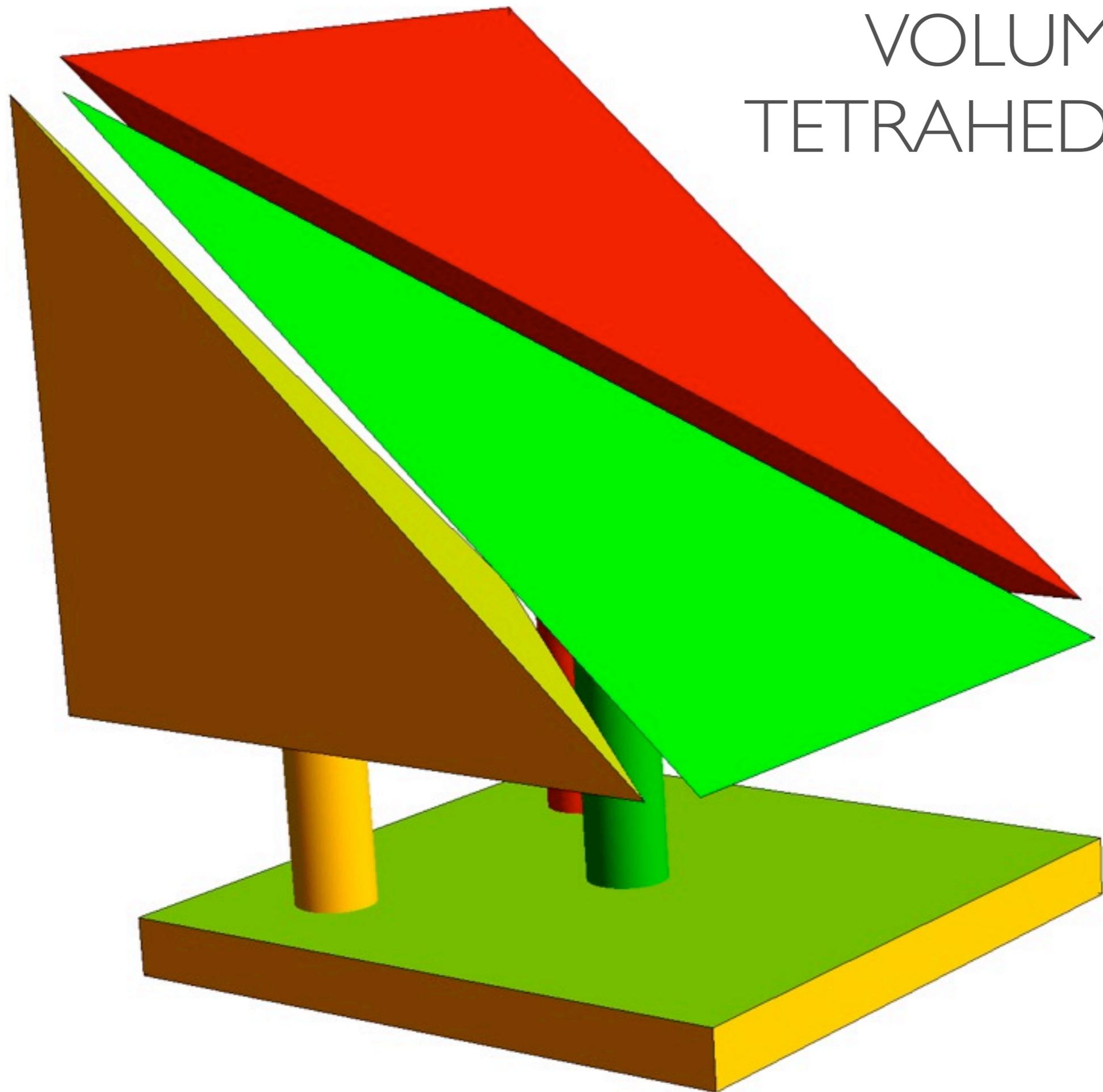
PYTHAGORAS







VOLUME TETRAHEDRON



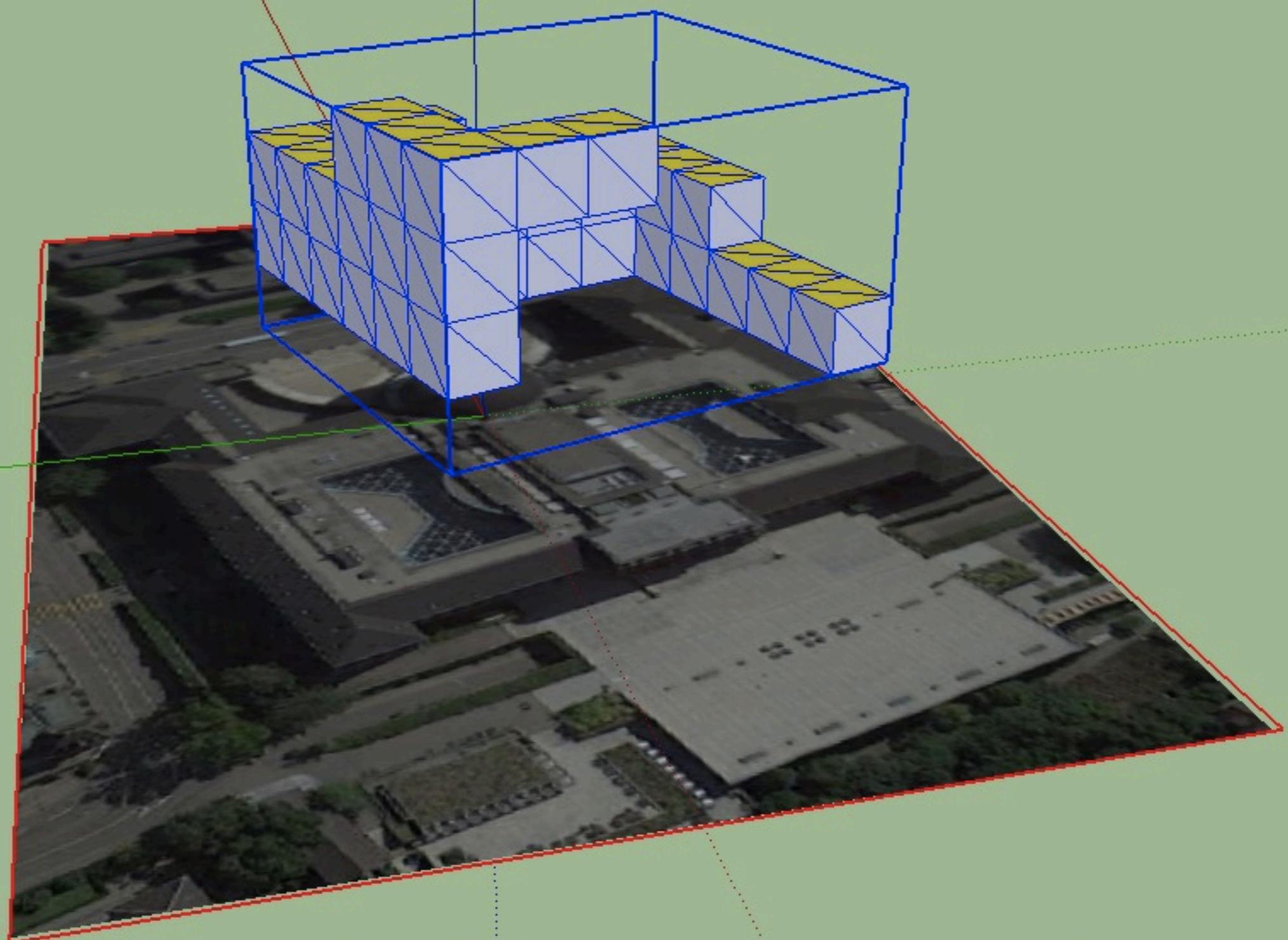
TOOLS FOR
PRODUCING
MATH MODELS

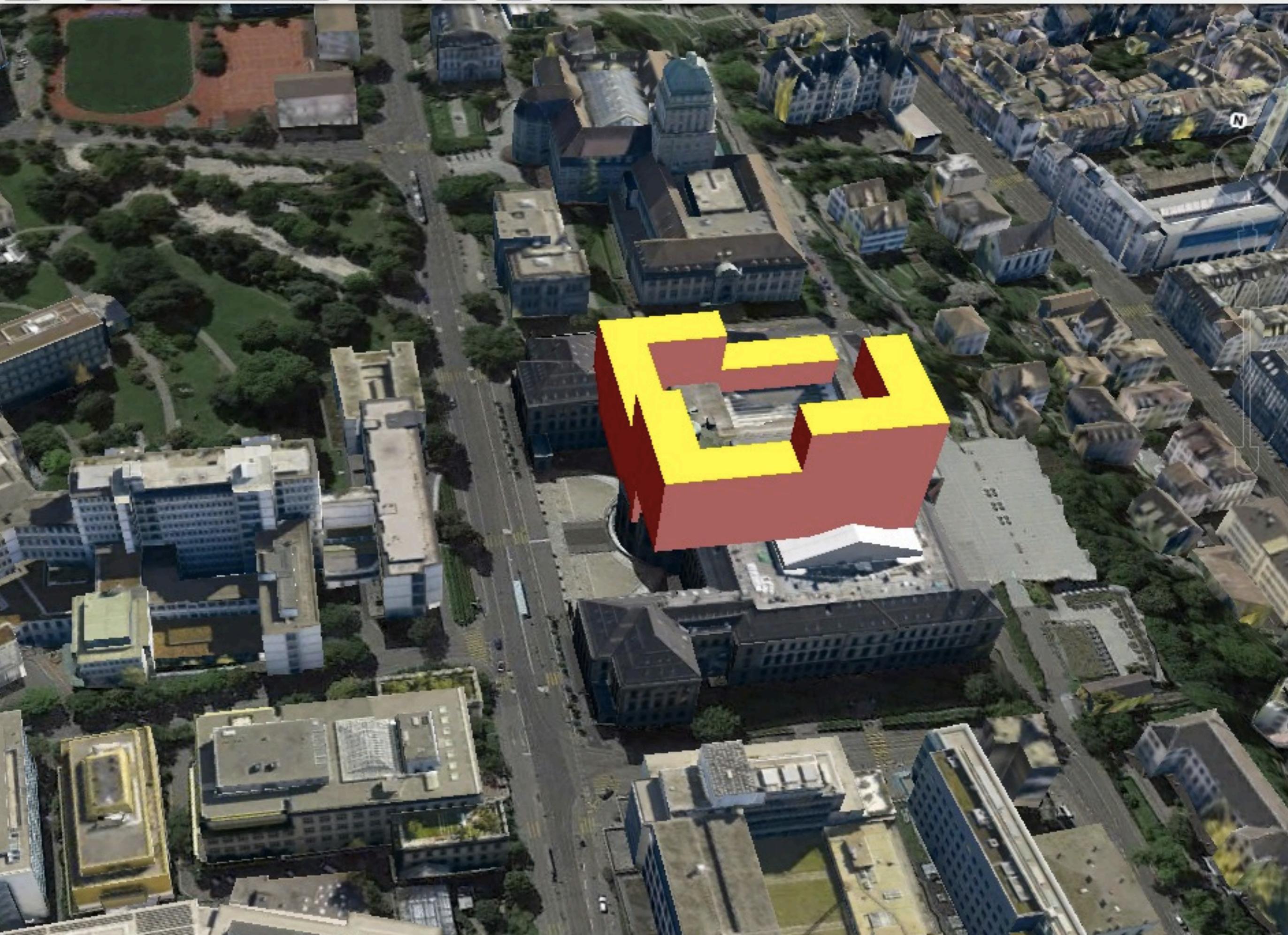
TOOLS

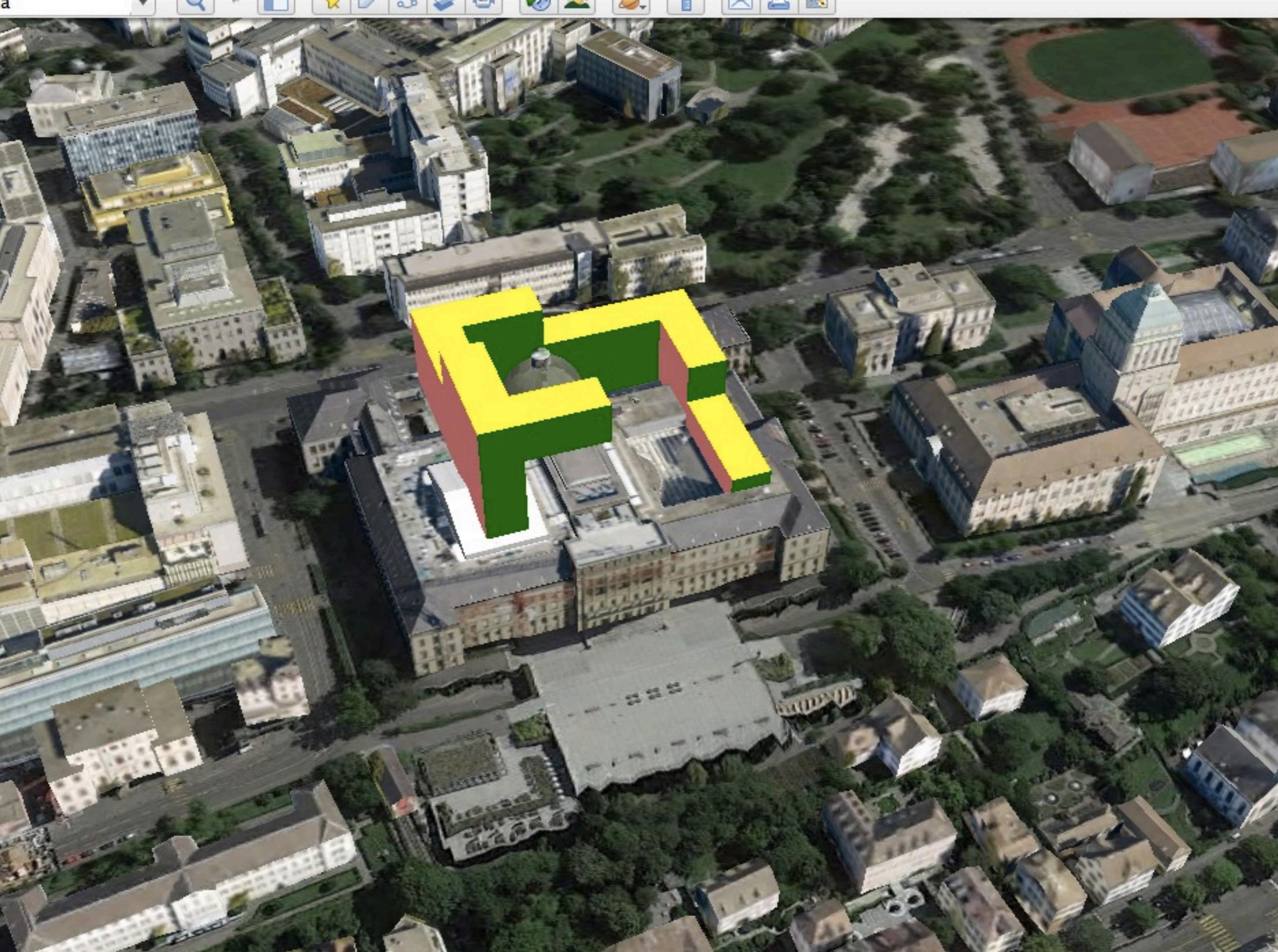
- Modelers: like Blender, Trimble Sketchup
- CAD: AutoCAD
- Generate STL directly: python, C, etc.
- Computer Algebra systems: Mathematica
- Intelligent CAD: OpenCAD

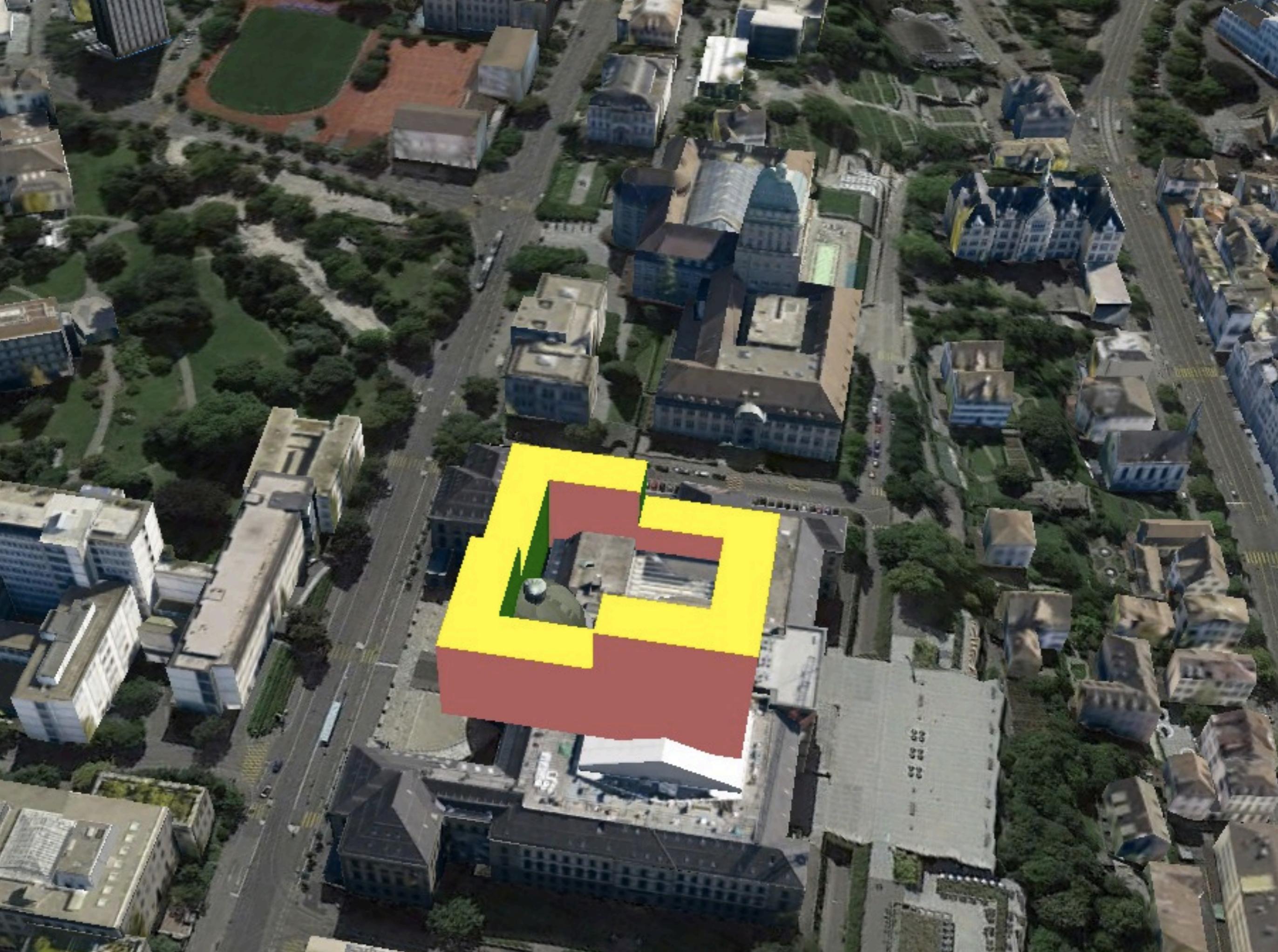


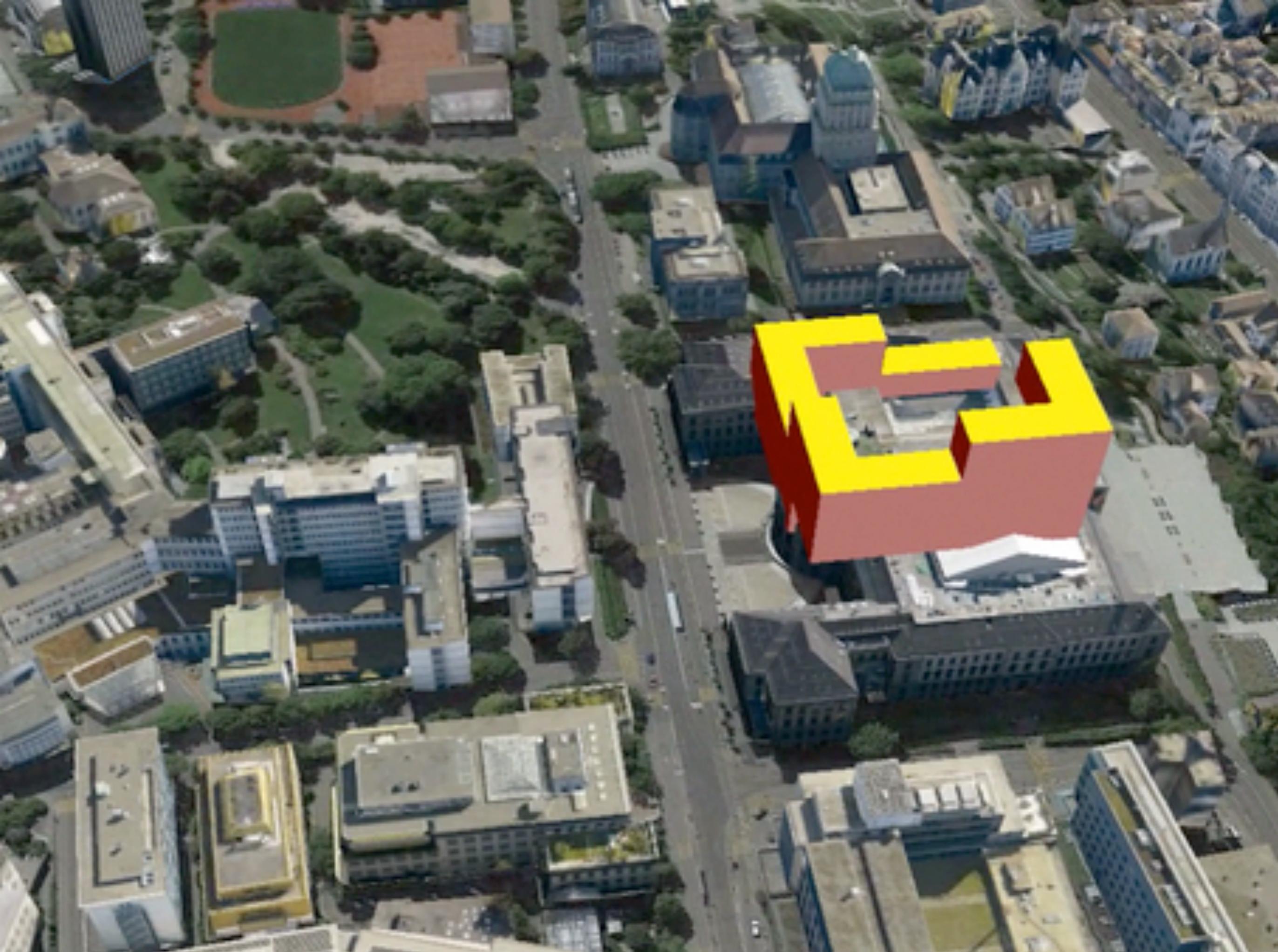
SKETCHUP





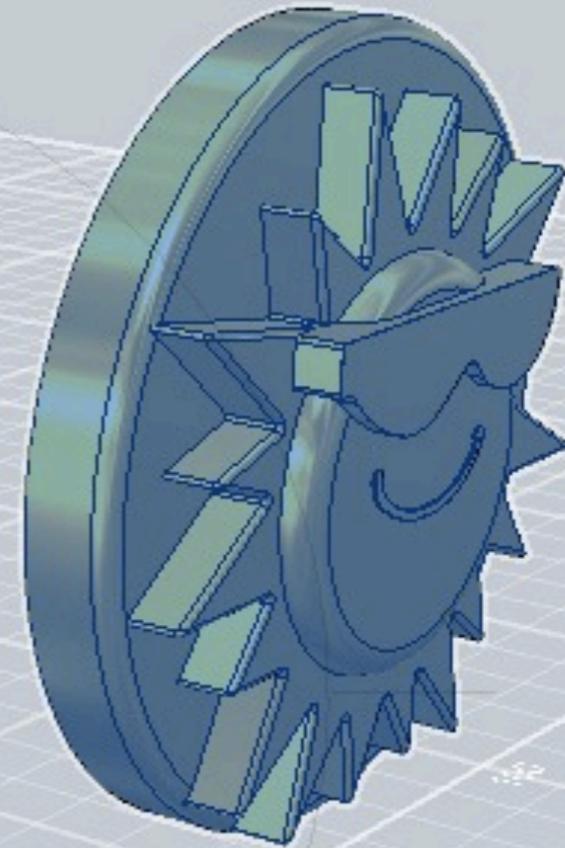




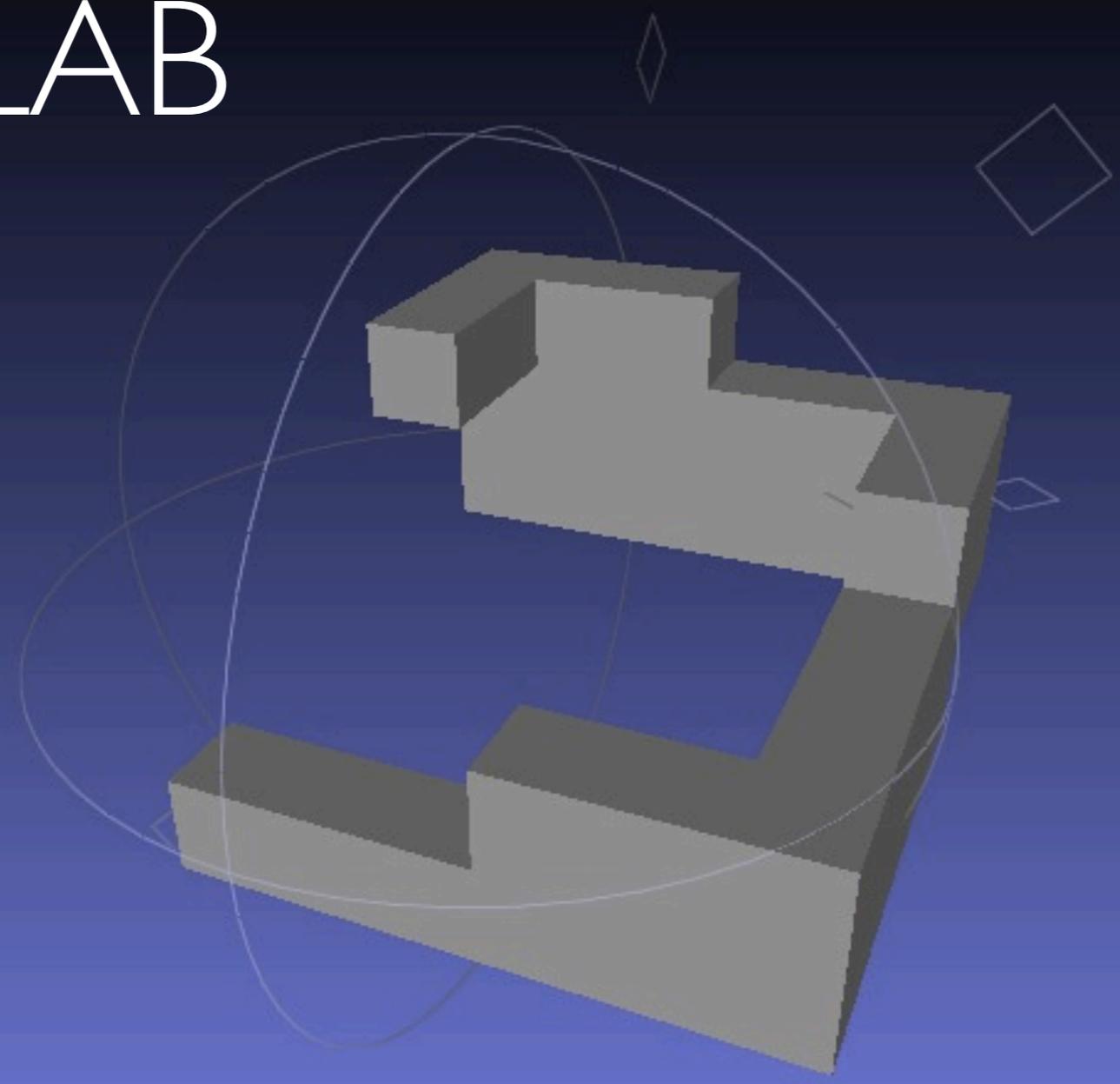




123 DESIGN



MESHLAB

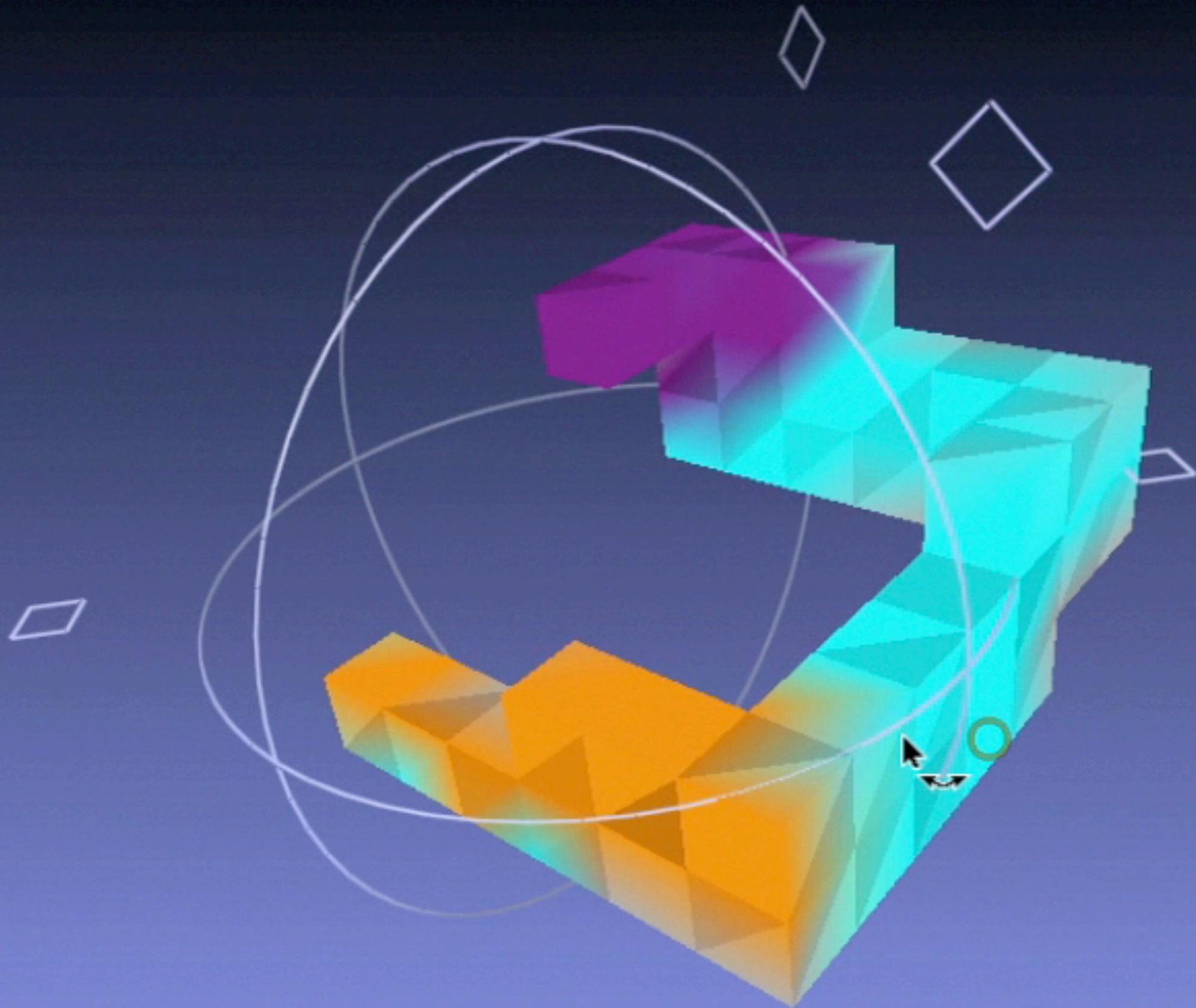


Current Mesh: stairs.stl
Vertices: 122 (122)
Faces: 444 (444)

50

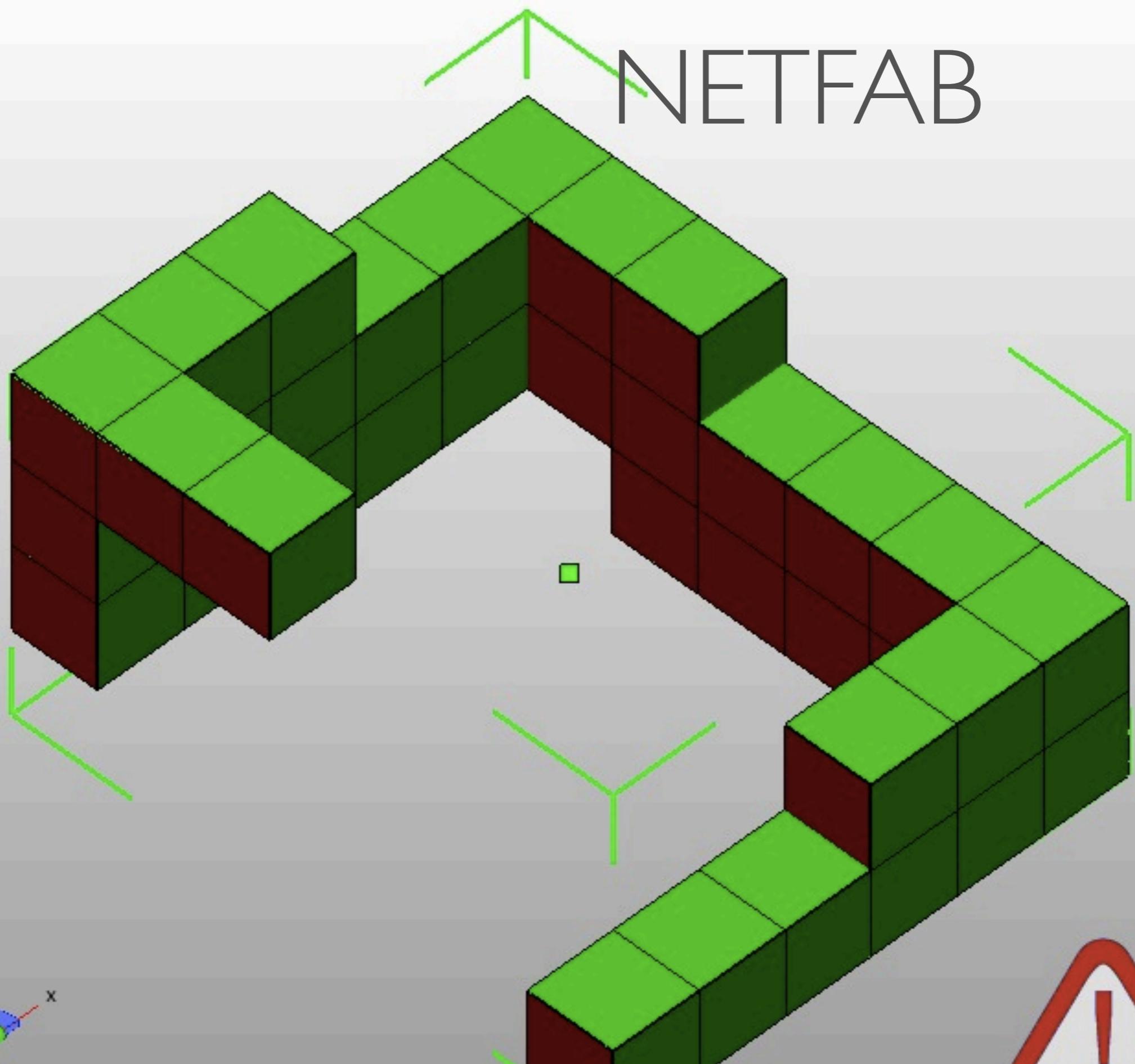
20

hidden poly



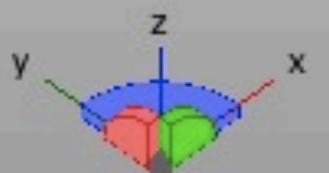


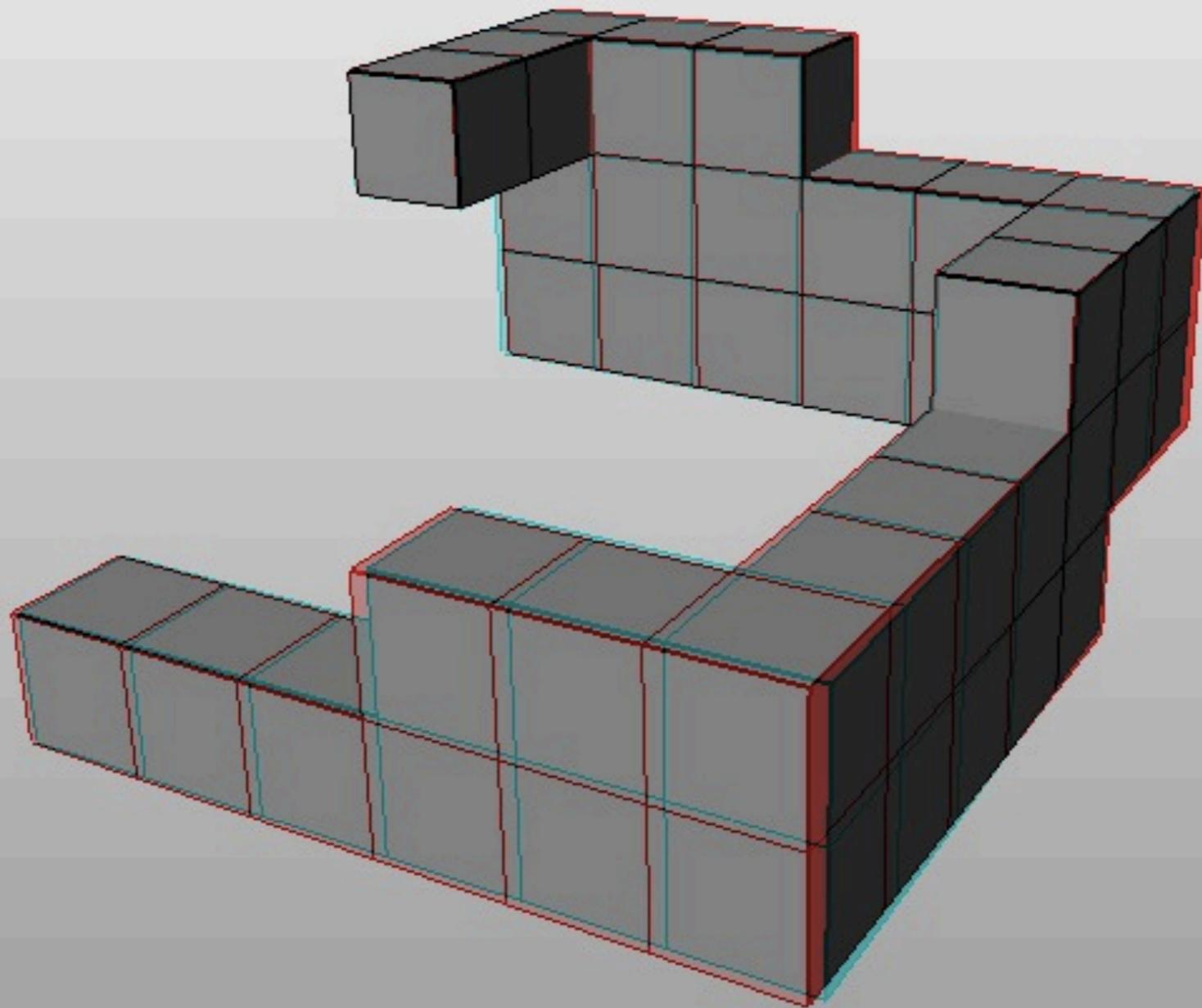
NETFAB



Ins

123D
Design





Context area

Parts
Slices

Settings

Depth:

Color scheme

Navigation controls: a central red dot surrounded by four directional arrows (up, down, left, right).

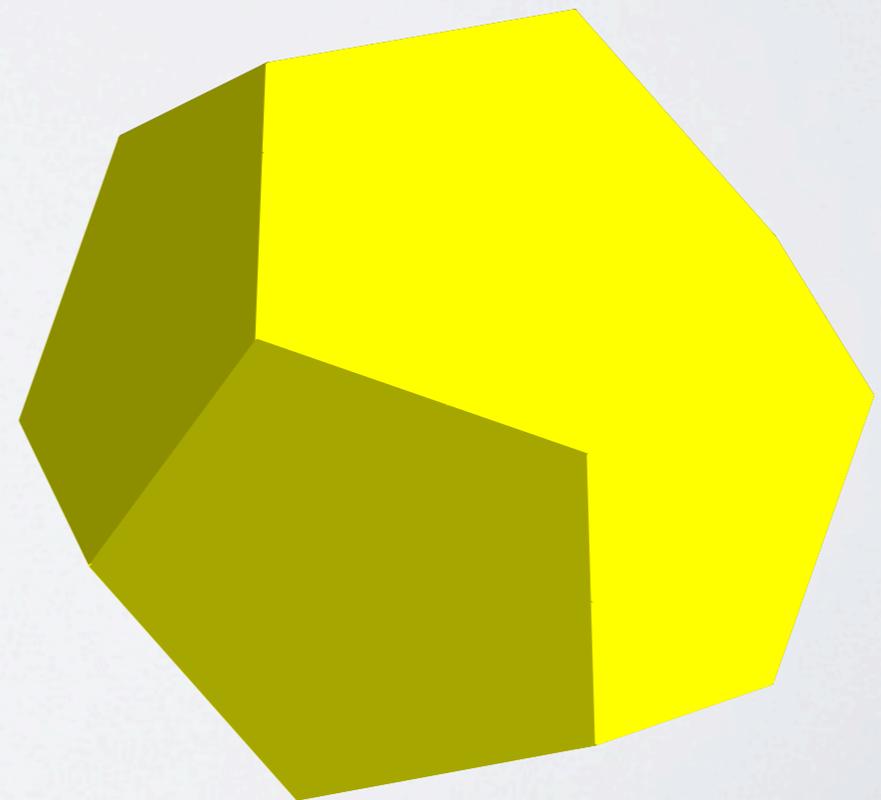
Name

Input field for naming the object.

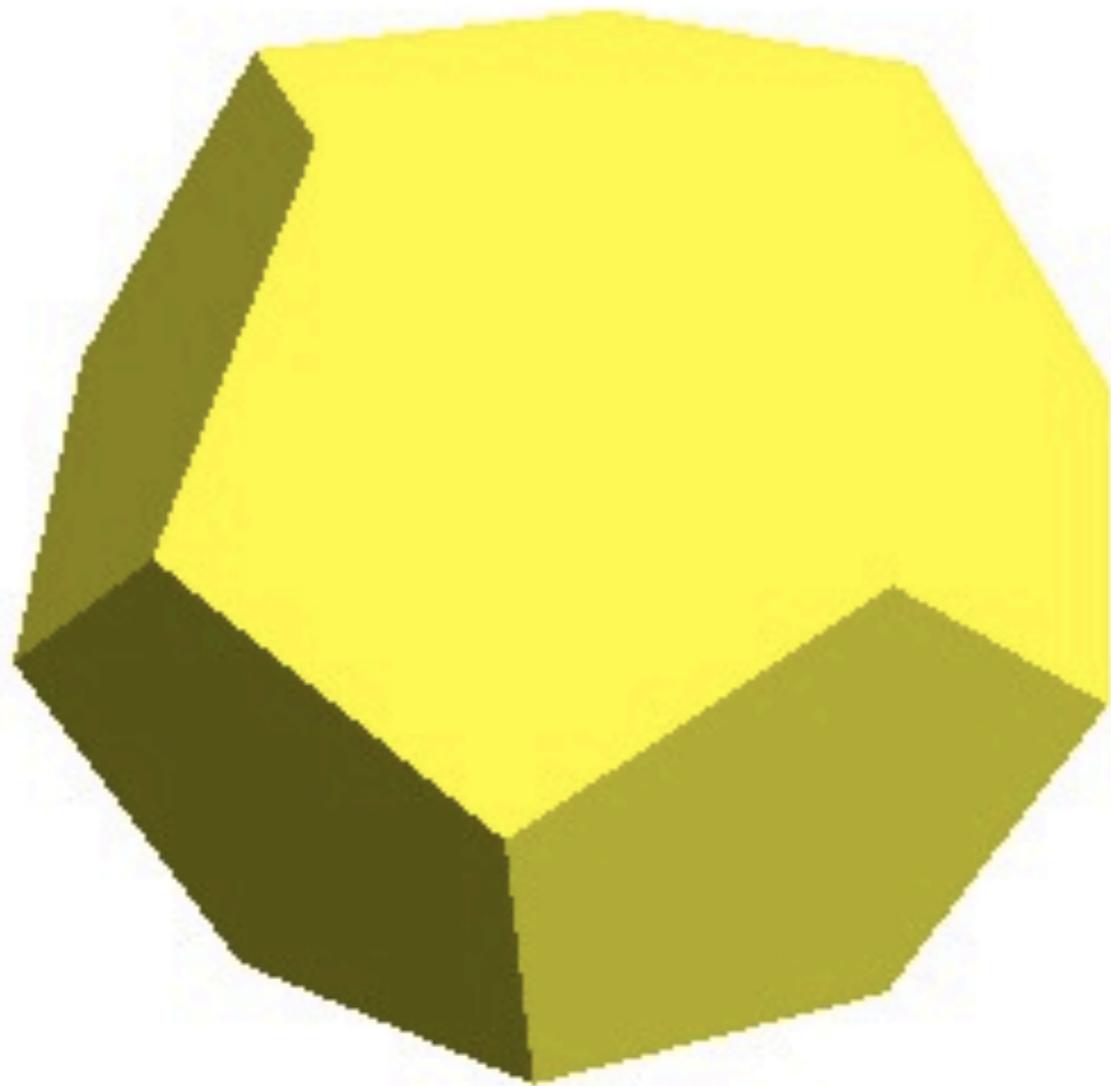
Loop list

MATHEMATICA

```
S = Graphics3D[{Yellow, PolyhedronData["Dodecahedron"][[1]]};  
Export["dodecahedron.3ds", S, "3DS"];  
Import["dodecahedron.3ds"]
```



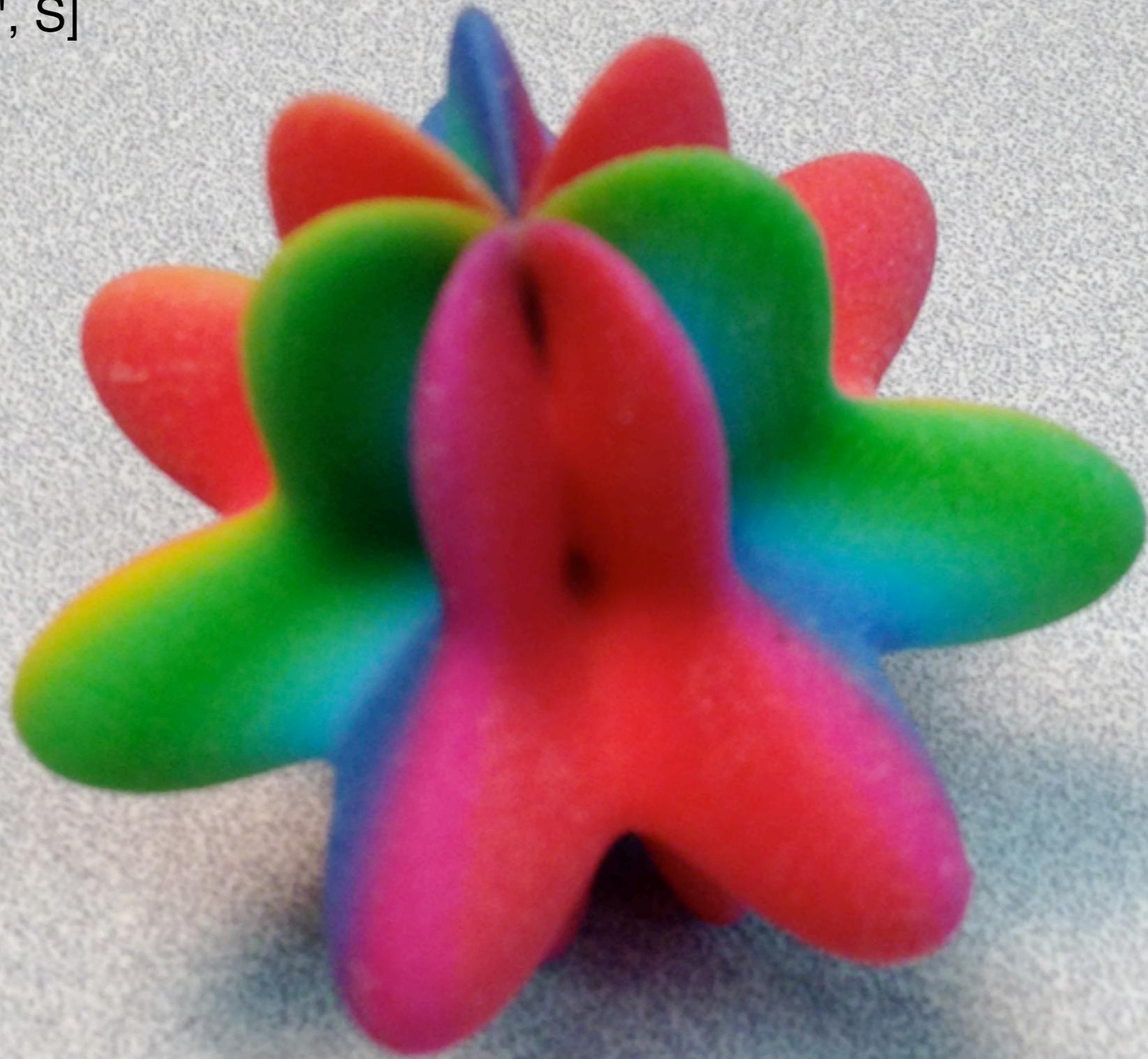
```
S = Graphics3D[{Yellow, PolyhedronData["Dodecahedron"][[1]]}];  
Export["dodecahedron.3ds", S, "3DS"];  
Import["dodecahedron.3ds"]
```



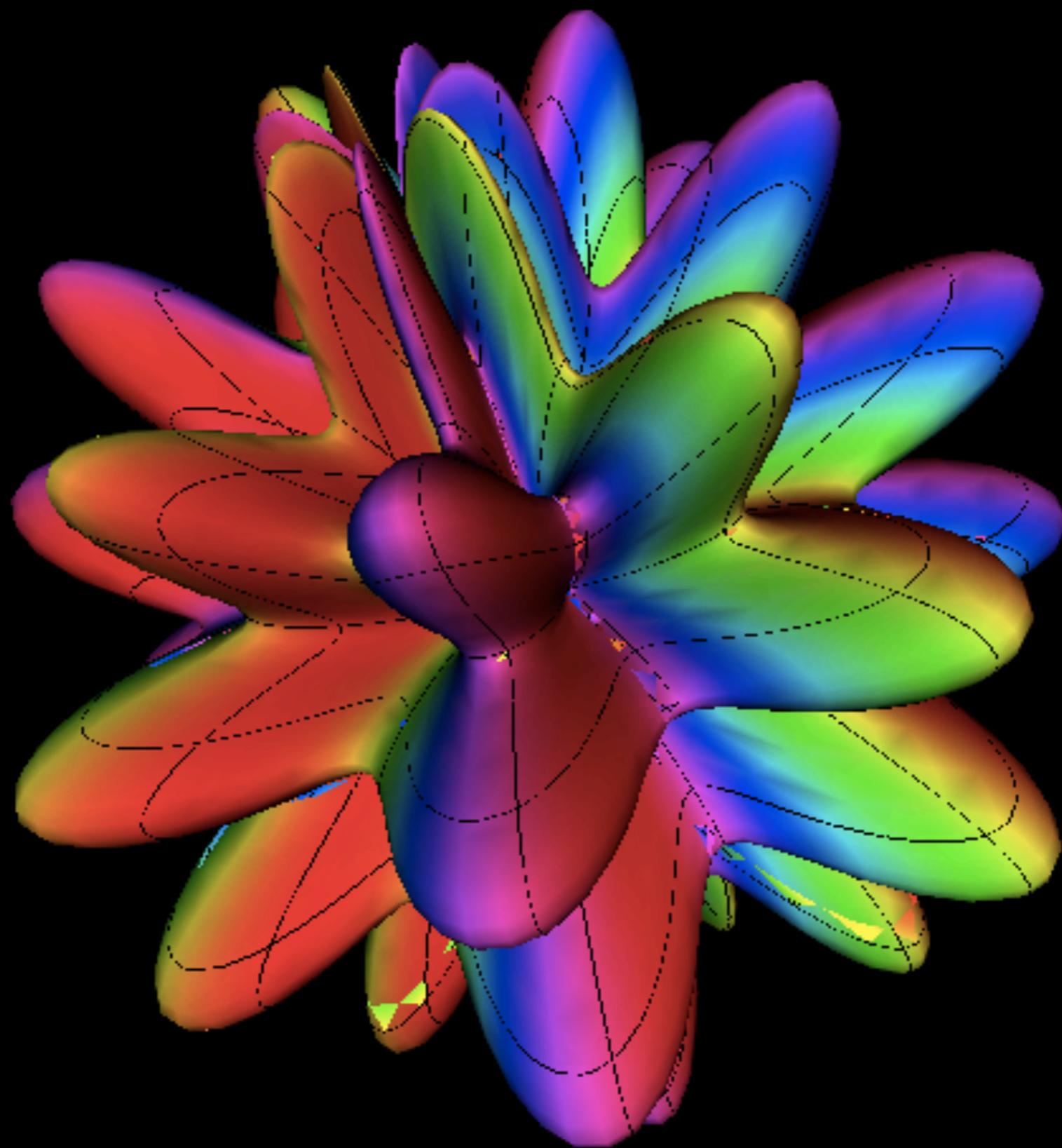
```
S = Graphics3D[{Yellow, PolyhedronData["Dodecahedron"][[1]]};  
Export["dodecahedron.3ds", S, "3DS"]; Import["dodecahedron.3ds"]
```



```
S = ParametricPlot3D[  
  5 (3 + Sin[11 s] - Sin[7 t]) {Cos[t] Sin[s], Sin[t] Sin[s], Cos[s]}, {t, 0, 2 Pi}, {s, 0, Pi},  
  ColorFunction -> Function[{x, y, z, u, v}, Hue[(Sin[11 u])^2]]];  
Export["sphere.x3d", S]
```



Free WRL



AND JUST
LEARNED
FROM
THE BOOK

COMBINE TOOLS

3D Modeling with OpenSCAD - Part 1

Sebastian Büttrich

pITLab, IT University of Copenhagen, Denmark

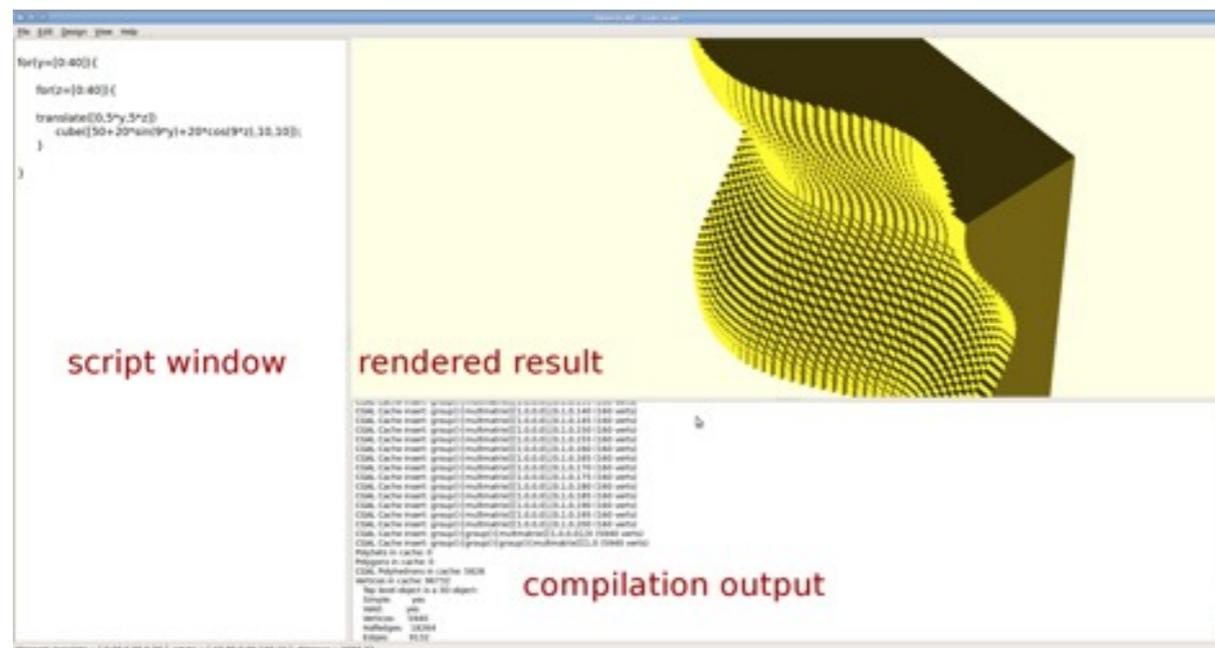
sebastian@itu.dk

On the way from idea to finished 3D print, there are a number of different steps to perform. Starting with the design of a CAD file or the capture of an existing object, followed by the conversion to an STL file, possibly some post-processing/repair work, and finally to the conversion to a printer-executable gcode file.

Your first steps in 3D printing might be based on 3D designs found on the internet, but when you are getting serious, you will want to design your own, or improve existing designs, rather than just replicating the work of others. We will focus on the design step here –*i.e.*, the production of 3D models and export of STL files.

There are many software tools available, and the following two URLs are good starting points for learning about them:

- http://www.reprap.org/wiki/Useful_Software_Packages
- https://en.wikipedia.org/wiki/Comparison_of_3D_computer_graphics_software



Designing in OpenSCAD

An example motivated by the book

3D Modeling with OpenSCAD - Part 2

Marius Kintel

OpenSCAD developer, Austria

maris@kintel.net

Some words from the author

OpenSCAD grew out of the RepRap community, more exactly out of the 3D printing activities at the Metalab (<http://metalab.at>), a hackerspace in Vienna, Austria.

The idea of OpenSCAD was born because we lacked a free software design tool for rapidly and iteratively creating mechanical parts. The existing tools at the time were too time consuming to use and changing details often required full remodeling. Commercial CAD tools which solve these problems do exist. However, apart from them being prohibitively expensive, they weren't Open Source and we felt that the world needed a better Open Source design tool. The basic idea of OpenSCAD was to allow people to describe their 3D models beginning with basic building blocks, and iteratively build from there. Additionally, we wanted it to be possible to parametrically describe shapes and positions in order to facilitate customizations and adaptations without having to go through time consuming and boring remodeling tasks.

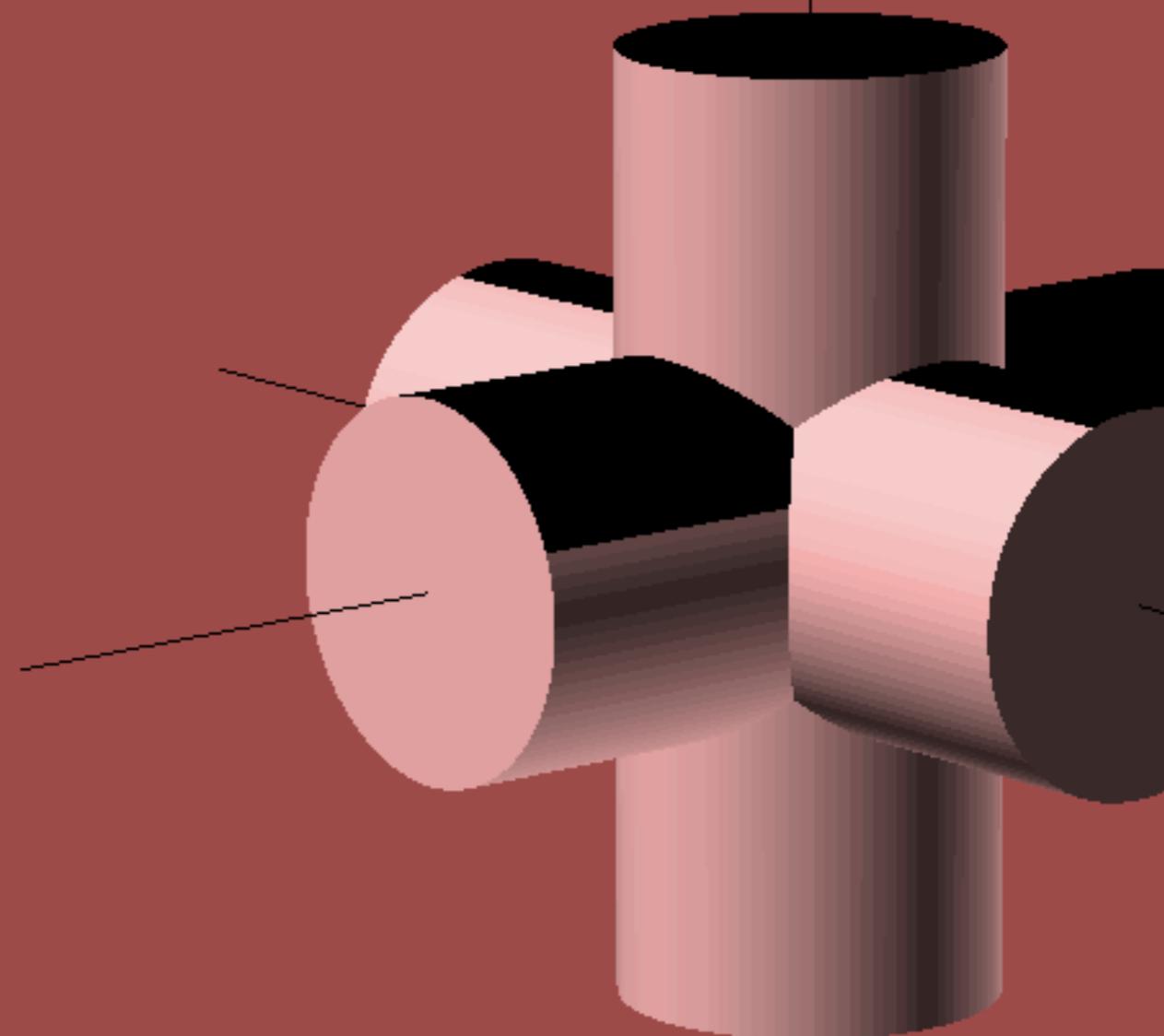
Early on, we realized that OpenSCAD would have severe limitations in terms of creating geometric shapes, so we decided to enable users to model more complex building blocks in their software of choice. OpenSCAD can then import these files for further modeling while you at any time can go back and change the basic geometry without having to redo the work already done in OpenSCAD. Keeping source code as the user interface also has an important emergent property in that people are enticed to share their designs, as well as their design intentions. This also makes it possible to change, reuse, or in other ways build on the existing ideas and designs of other people.

Parametric designs

One of the primary strengths of OpenSCAD is that it supports parametric designs. Parametric in this context means that you can create logical building blocks, which take certain parameters and in return create a 3D component satisfying those parameters. Examples of parameters can be *Object sizes*, *Nut and bolt holes*, *Object descriptors* (e.g. *number of teeth in a gear*) or *Design elements* (text to emboss onto a design).

```
module cyl(a){  
  rotate(90,a)  
  cylinder(r=10,h=50,  
  center=true,  
  $fn=100);  
}
```

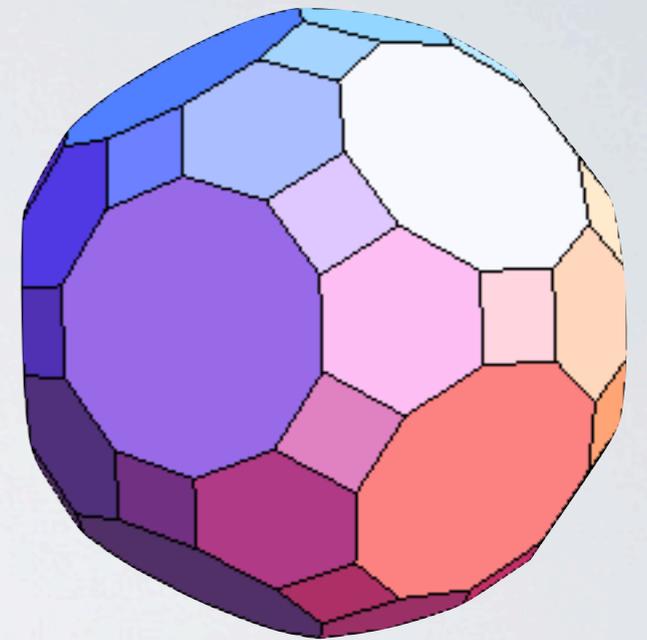
```
intersection() {  
  cyl([0,0,0]);  
  cyl([1,0,0]);  
  cyl([0,1,0]);  
}
```



```

A=PolyhedronData["GreatRhombicosidodecahedron","Faces"];
points=10*A[[1]]; triangles = A[[2,1]]-1;
WriteString["example.scad","polyhedron("];
WriteString["example.scad","points = "];
Write["example.scad",N[points]];
WriteString["example.scad",","];
WriteString["example.scad","triangles = "];
Write["example.scad",triangles];
WriteString["example.scad",");"]
Run["cat example.scad |tr \"{\\" \"[\"|tr \"}\" \"\"}\" >tmp; mv tmp
example.scad"];

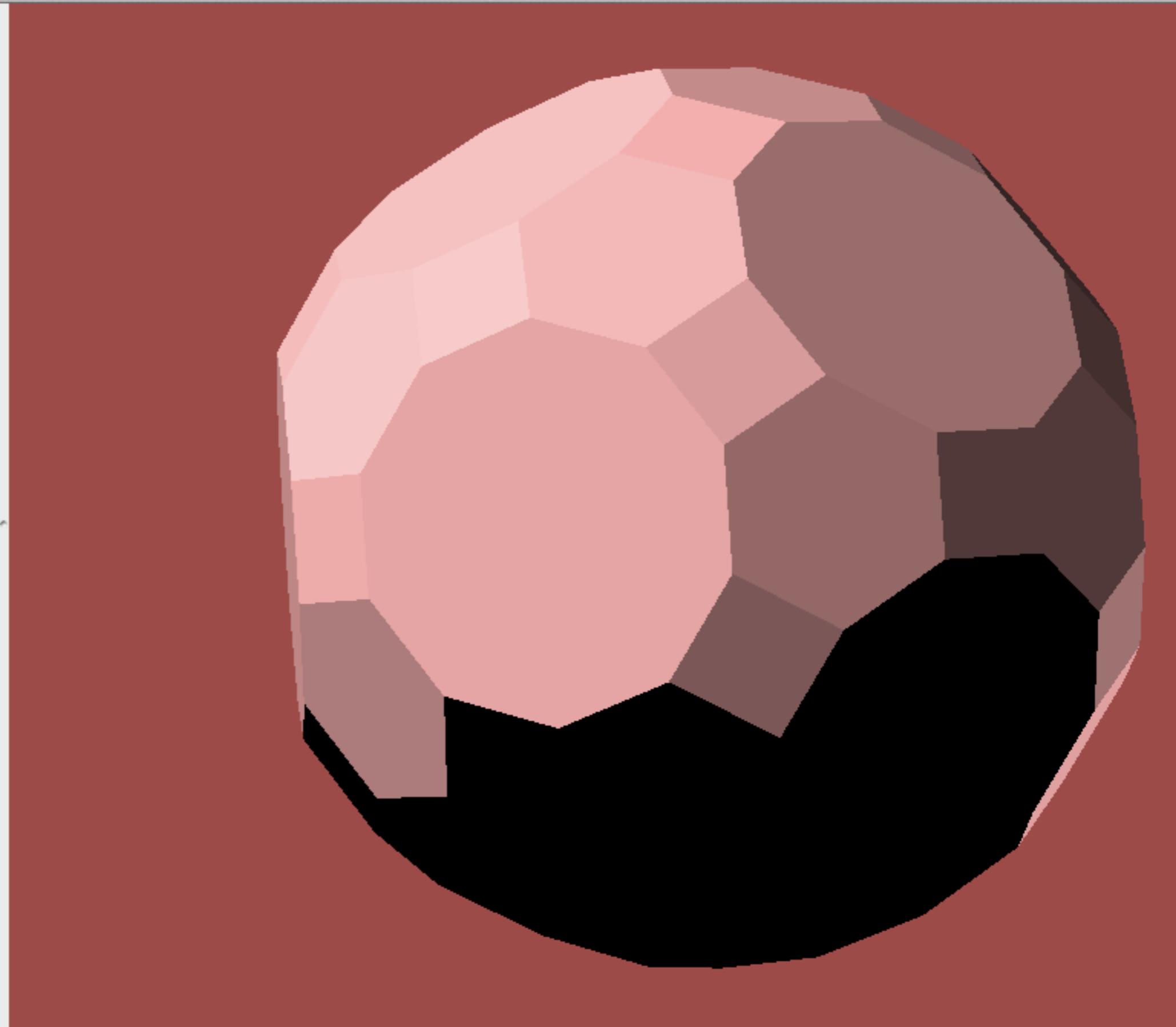
```



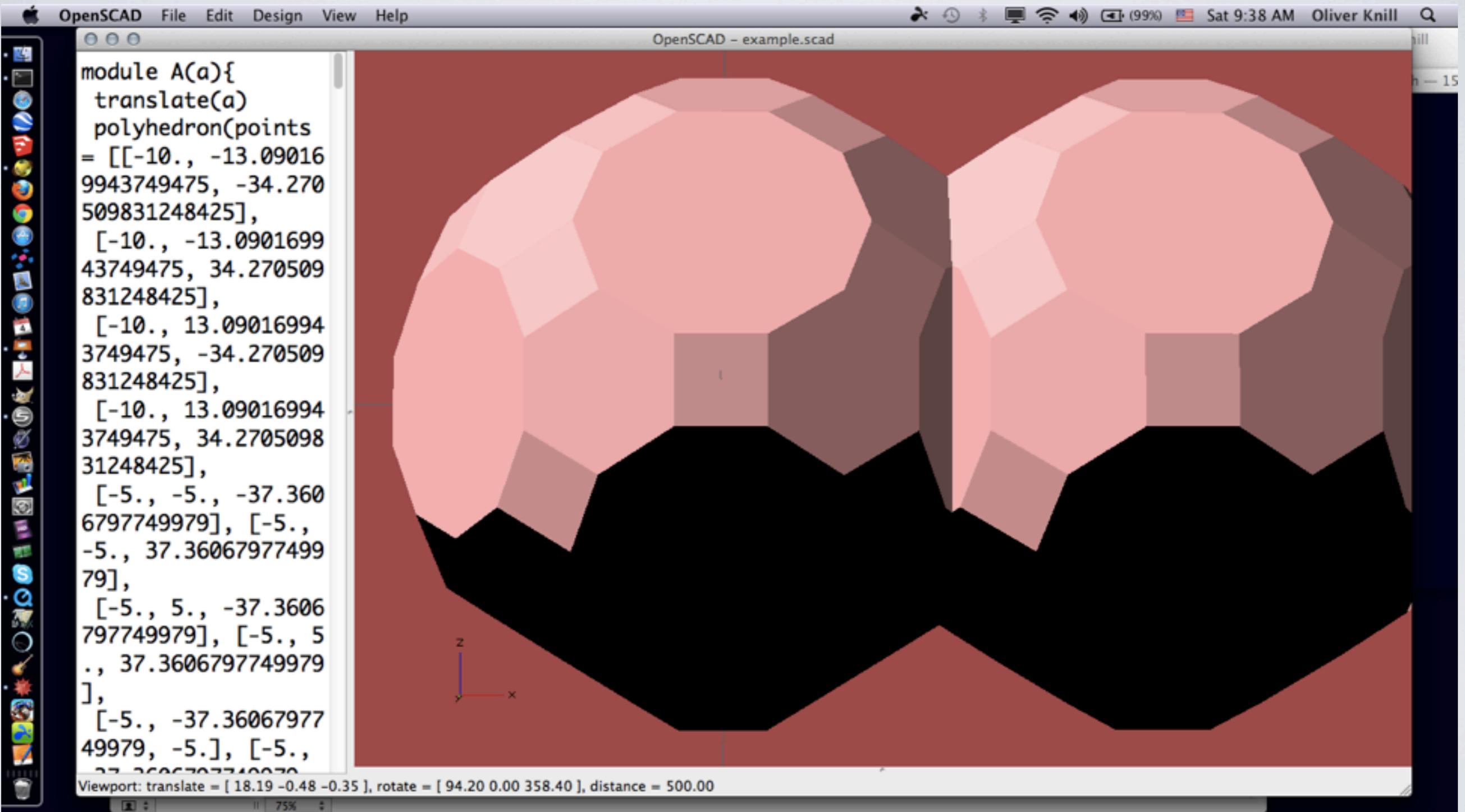
```

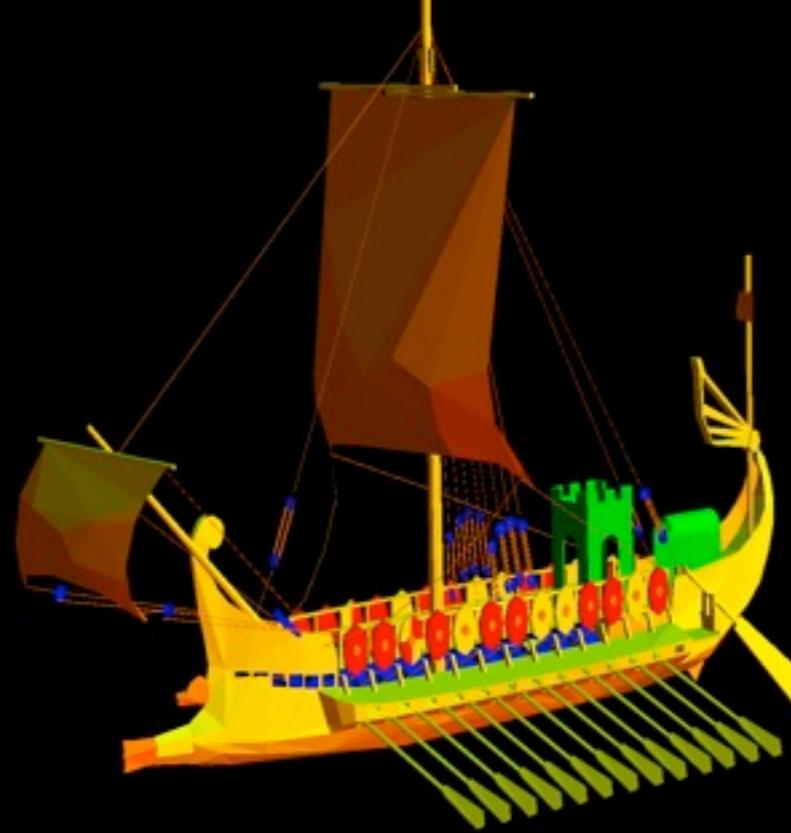
ts = [[-10., -13.090169943749475, -34.27050
169943749475, 34.270509831248425],
69943749475, -34.270509831248425],
69943749475, 34.270509831248425],
.3606797749979], [-5., -5., 37.360679774997
3606797749979], [-5., 5., 37.3606797749979]
797749979, -5.], [-5., -37.3606797749979, 5
3988749895, -31.18033988749895],
3988749895, 31.18033988749895], [-5., 37.36
.],
97749979, 5.], [-5., 21.18033988749895, -31
],
988749895, 31.18033988749895], [5., -5., -3
],
606797749979], [5., 5., -37.3606797749979],
06797749979], [5., -37.3606797749979, -5.],
97749979, 5.], [5., -21.18033988749895, -31
],
988749895, 31.18033988749895], [5., 37.3606
,
7749979, 5.], [5., 21.18033988749895, -31.1
88749895, 31.18033988749895], [10., -13.090
248425], [10., -13.090169943749475, 34.2705
9943749475, -34.270509831248425],
9943749475, 34.270509831248425],
248425, -10., -13.090169943749475],
248425, -10., 13.090169943749475],
248425, 10., -13.090169943749475],
248425, 10., 13.090169943749475],
248425, -18.090169943749473, -16.1803398874
248425, -18.090169943749473, 16.18033988749
248425, 18.090169943749473, -16.18033988749
248425, 18.090169943749473, 16.180339887498
749473, -16.18033988749895, -29.27050983124
749473, -16.18033988749895, 29.270509831248
749473, 16.18033988749895, -29.270509831248
749473, 16.18033988749895, 29.2705098312484
749475, -34.270509831248425, -10.],
nslate = [ 0.00 0.00 0.00 ], rotate = [ 55.00 0.00 25.00 ], distance = 500.00

```

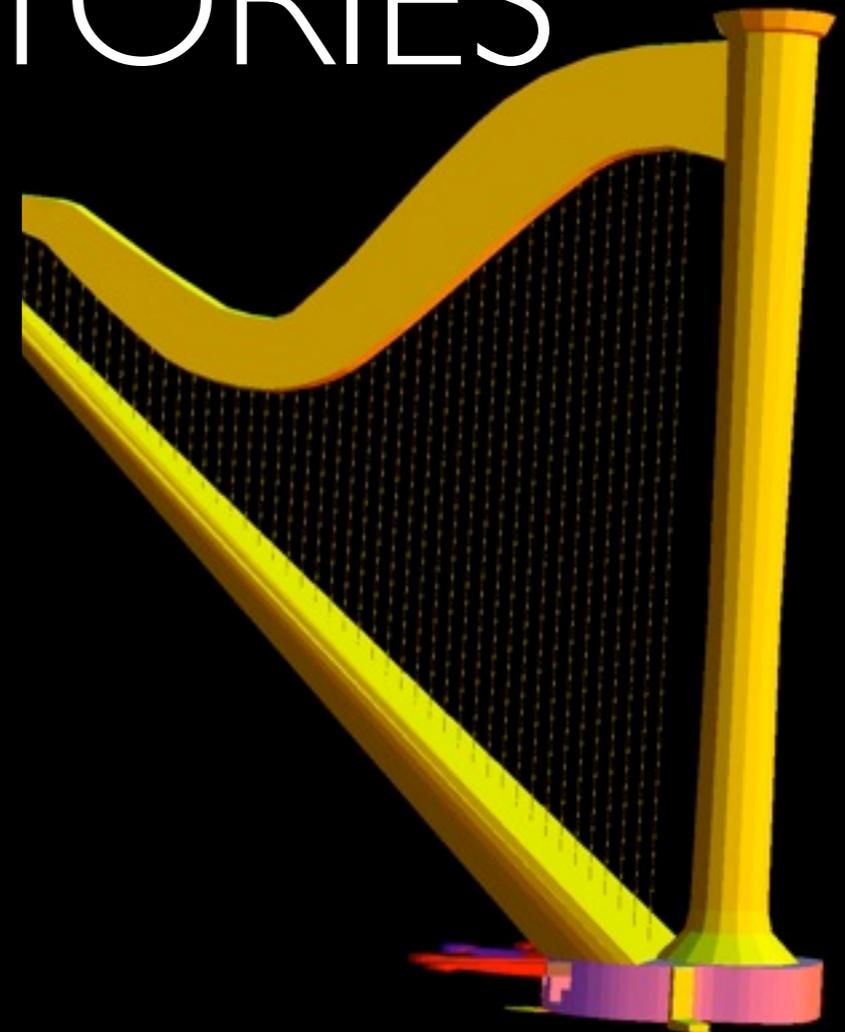
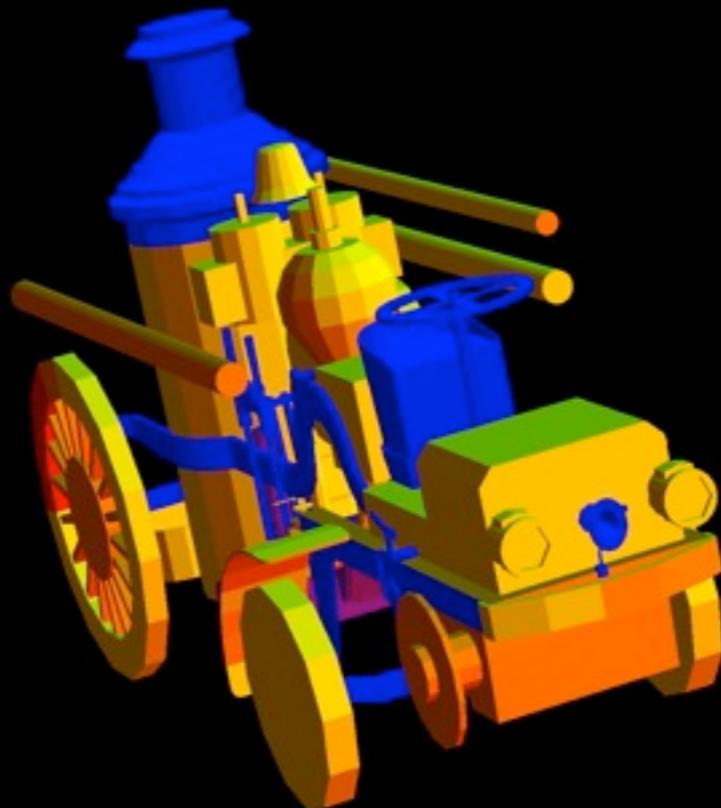
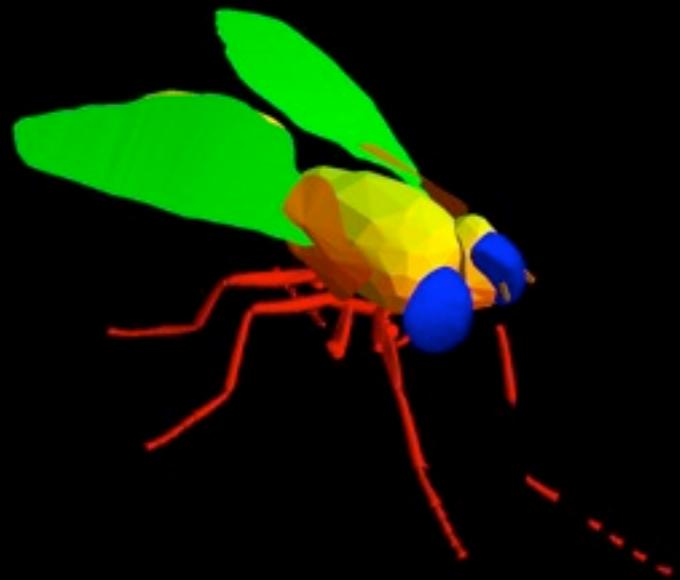


BOOLEAN OPERATIONS

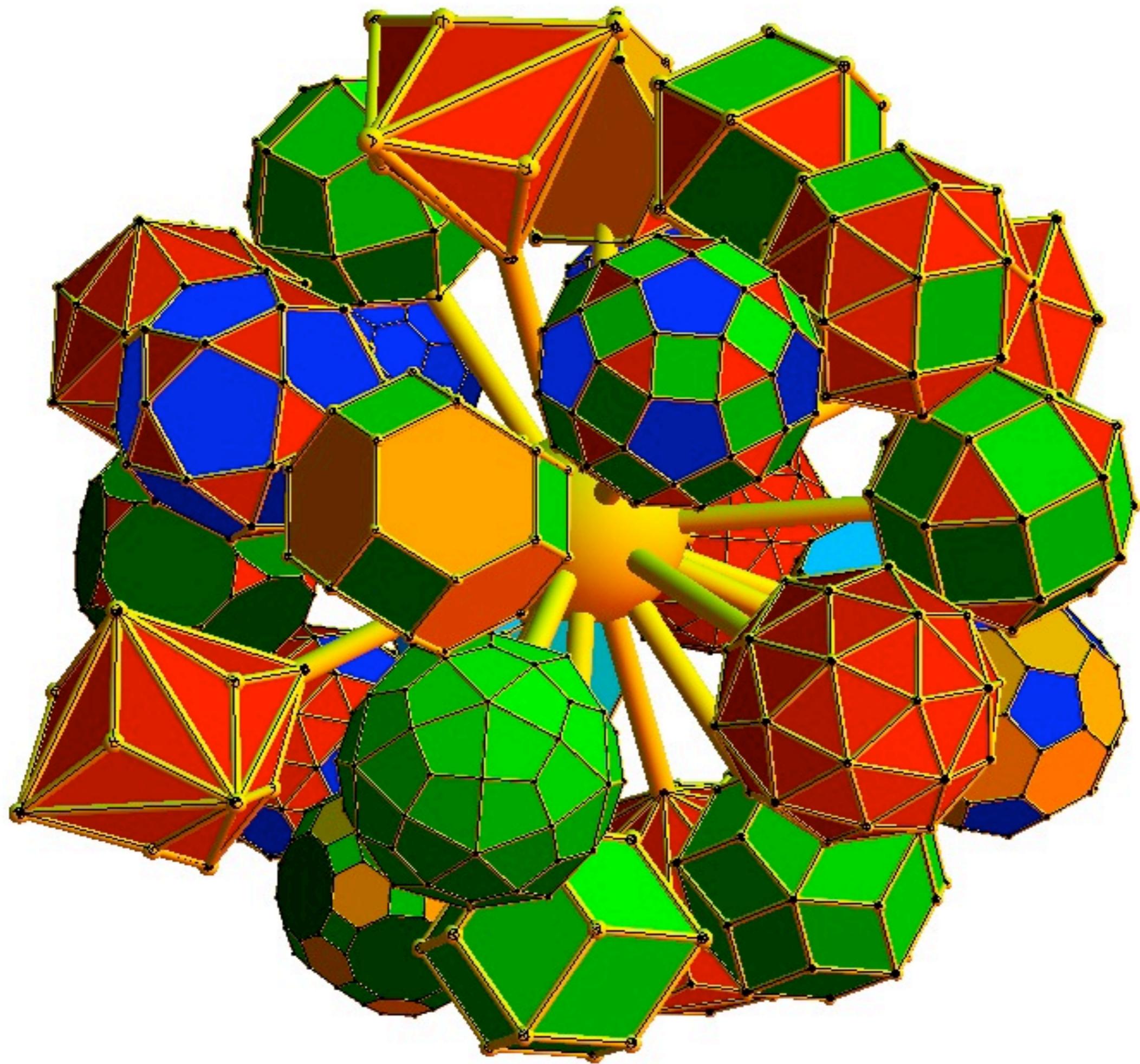


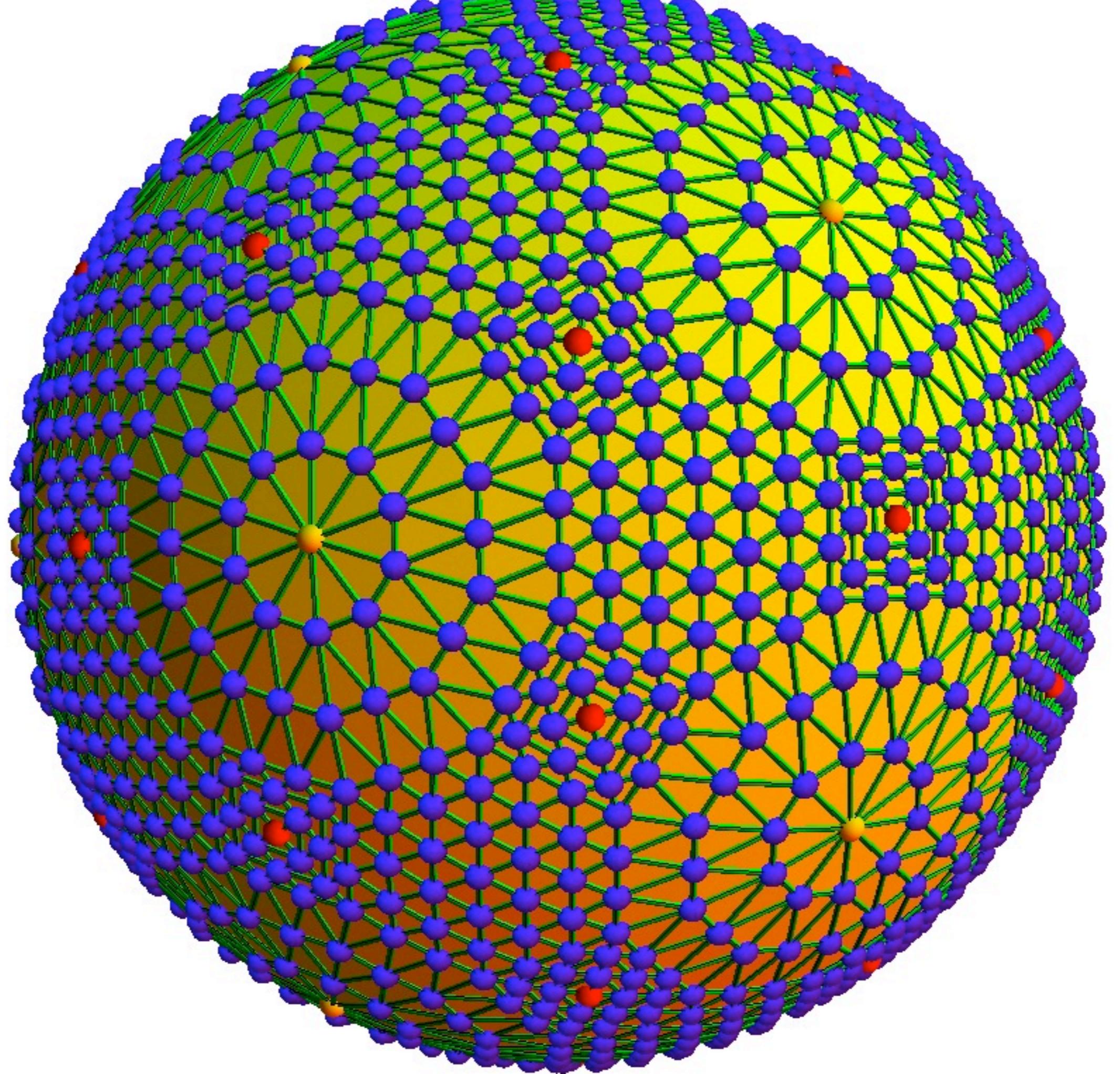


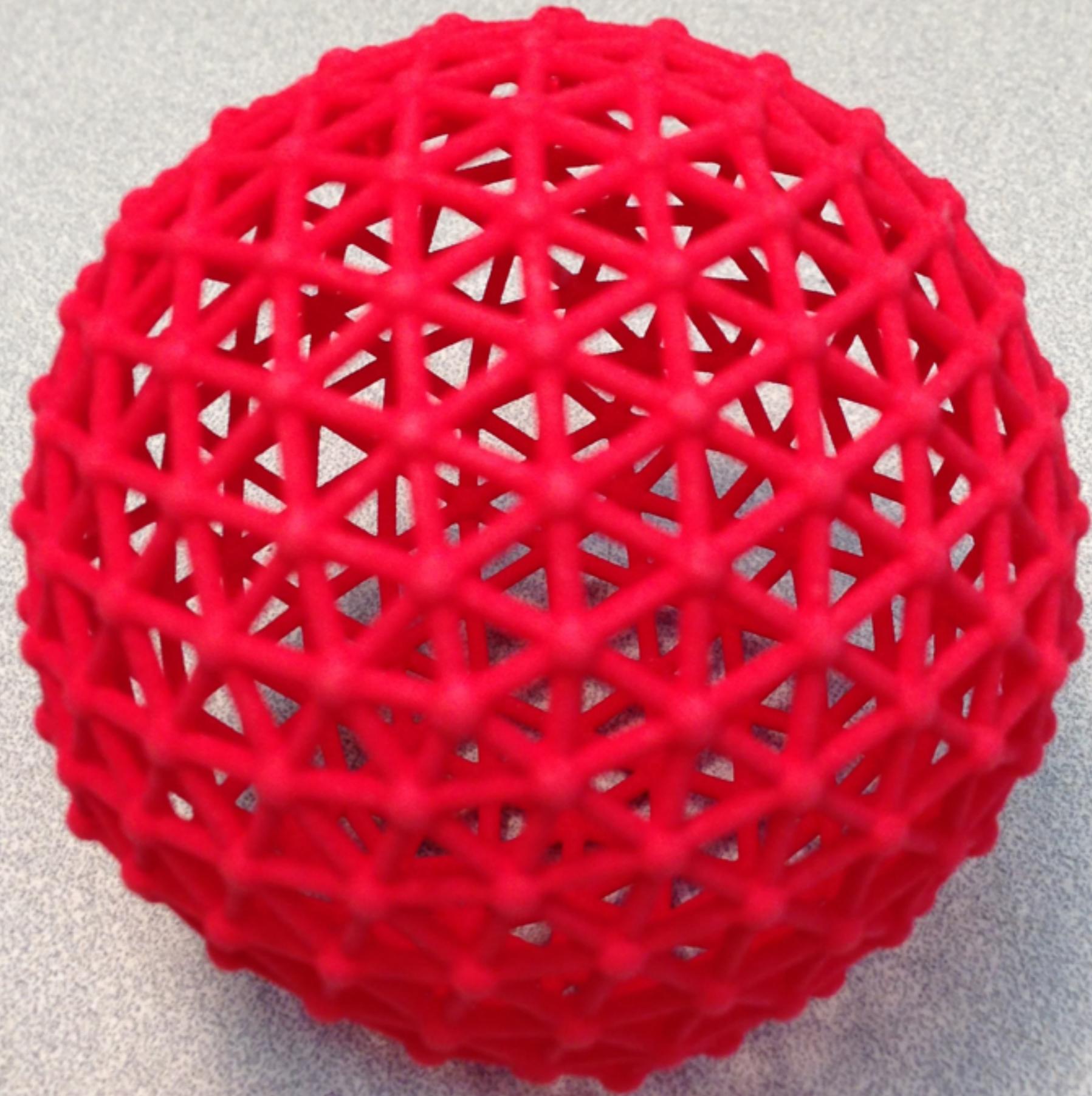
EXISTING 3D REPOSITORIES

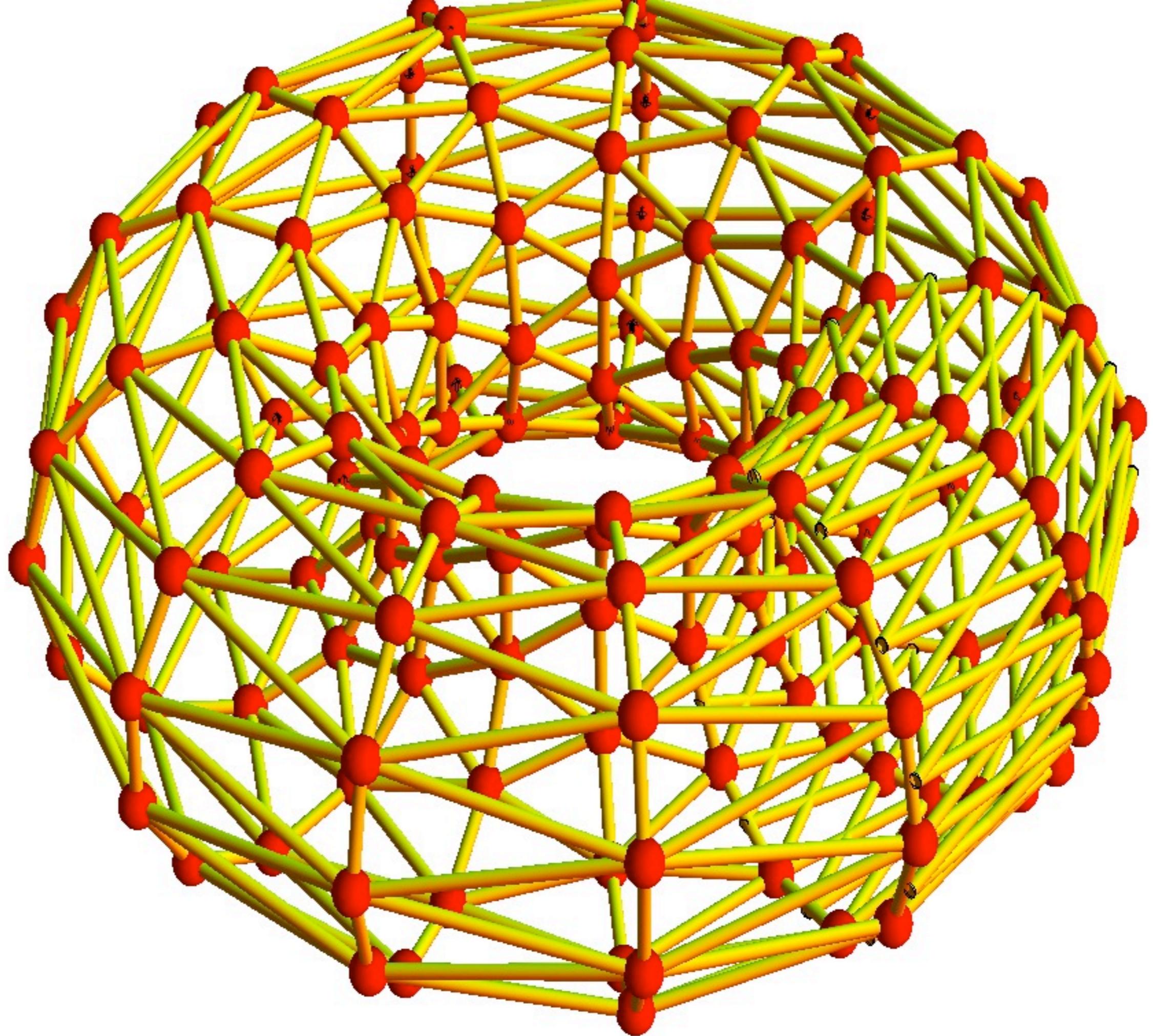


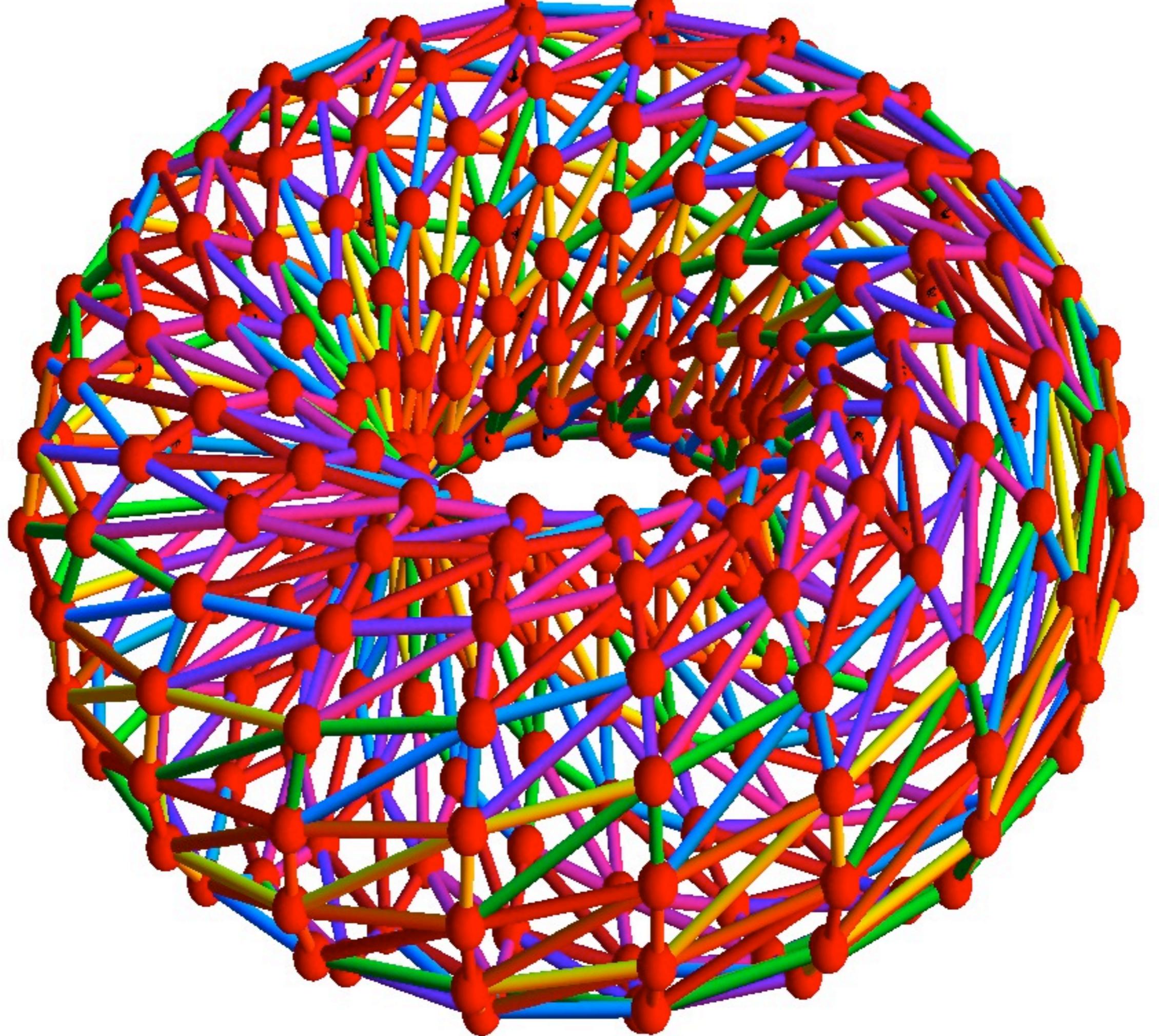
ILLUSTRATING MATHEMATICS

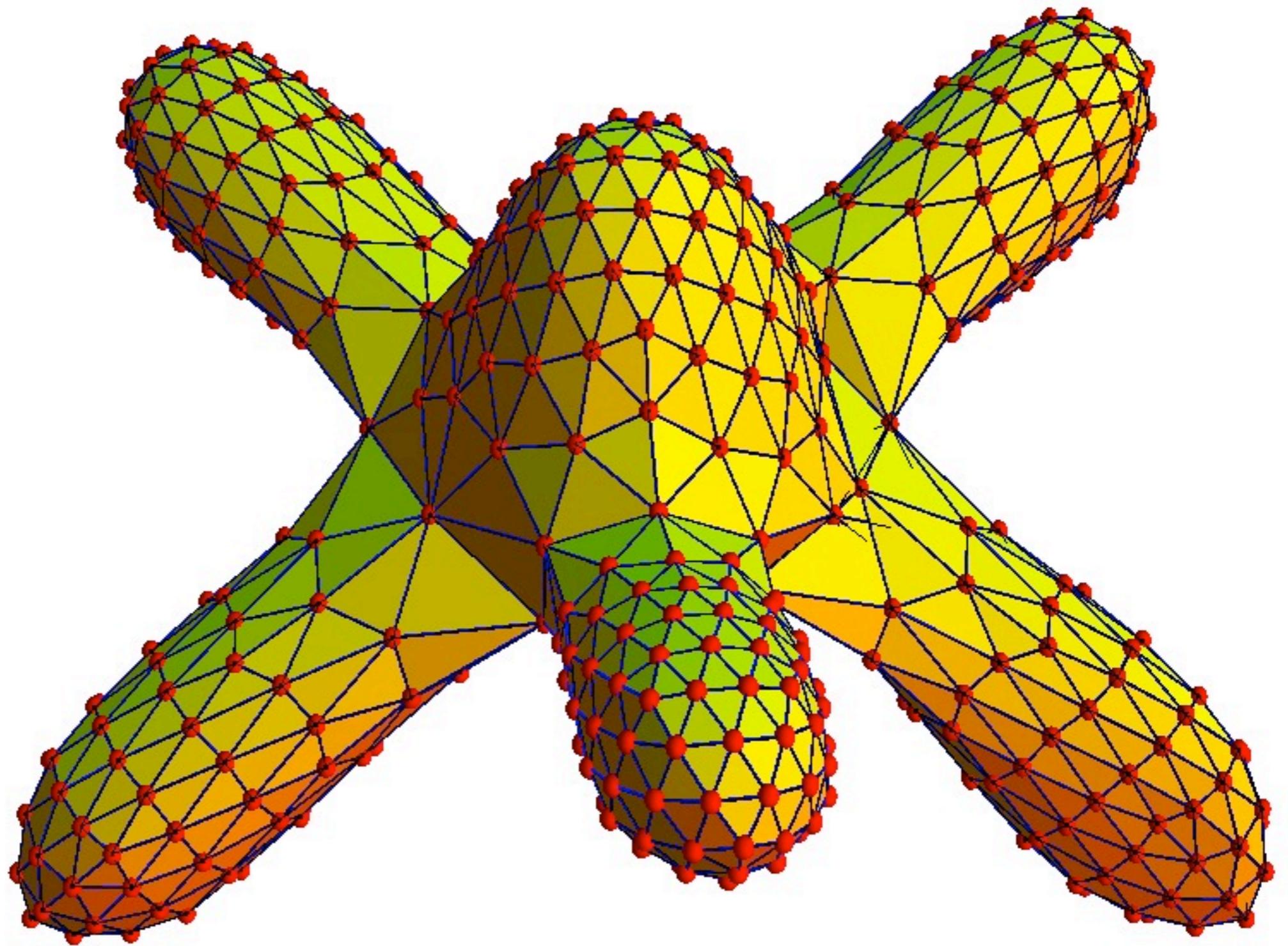




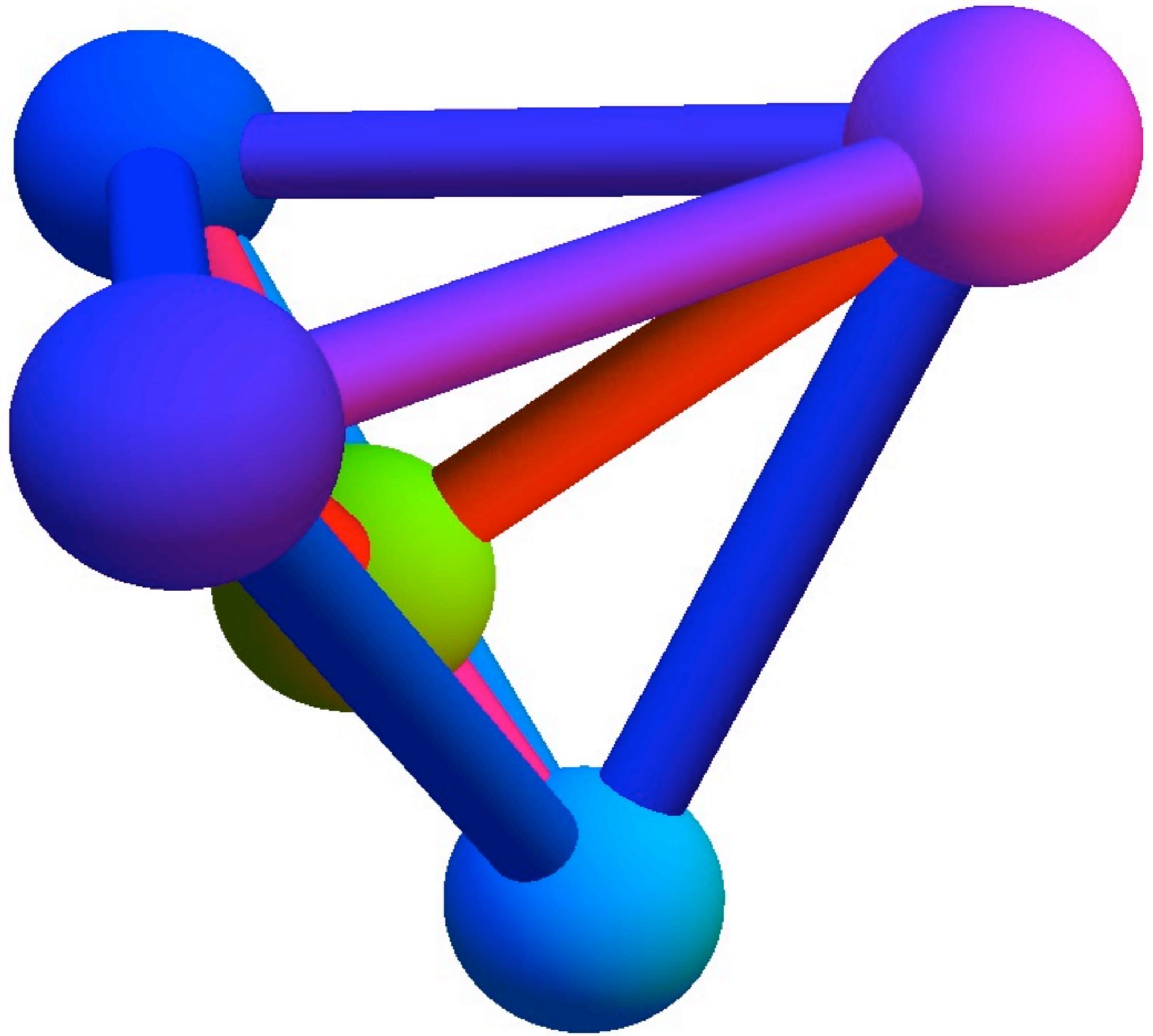




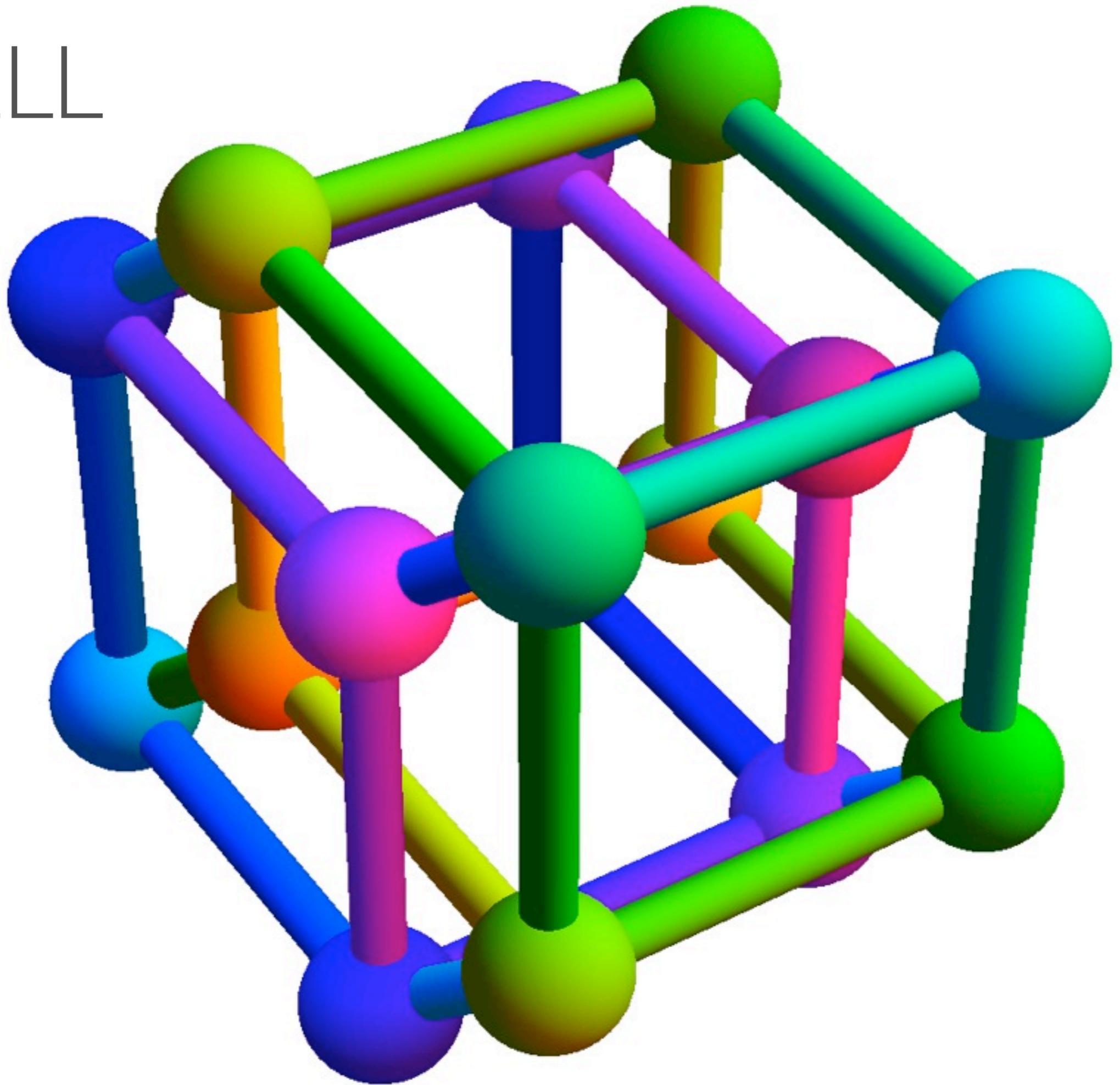




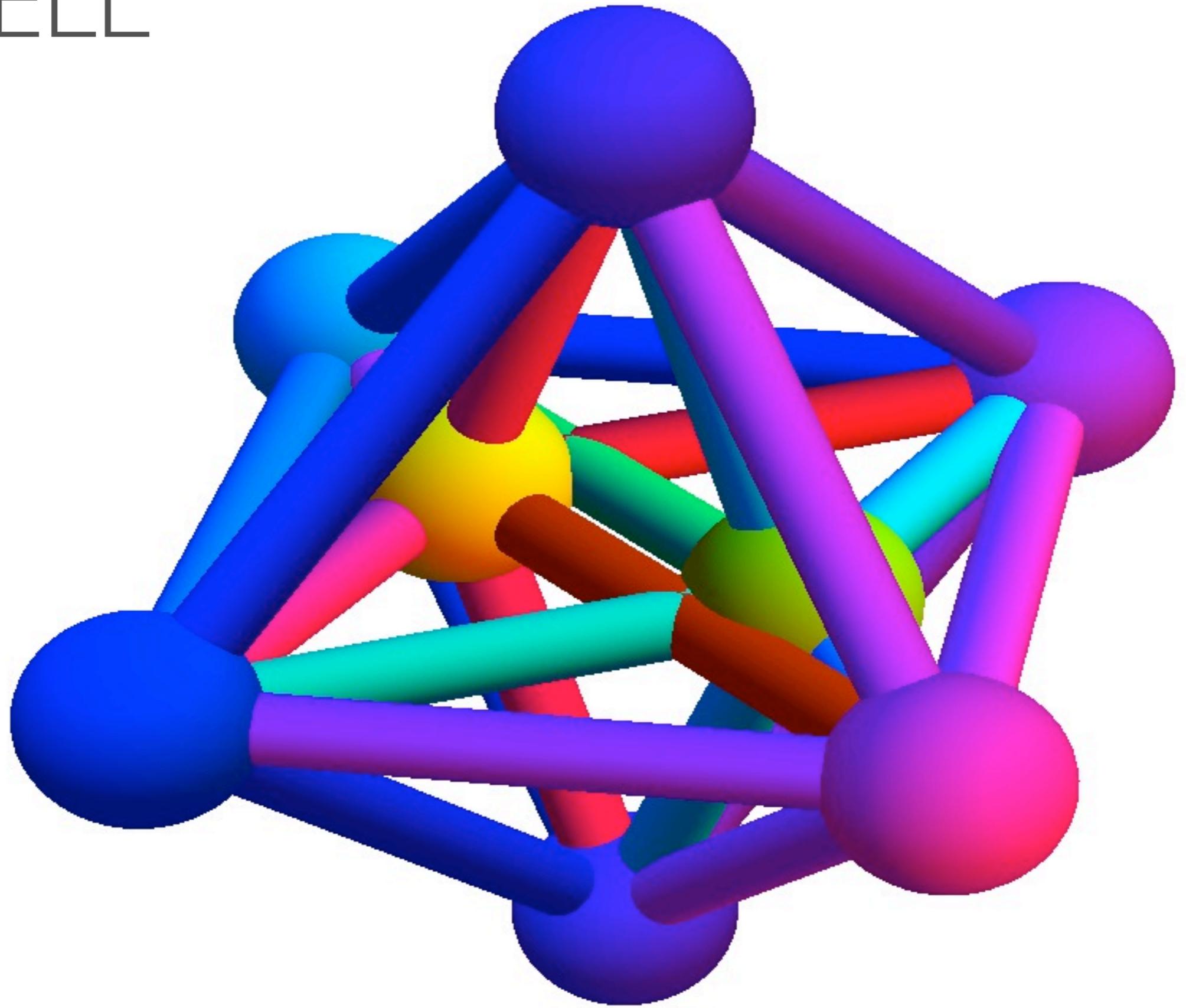
5 CELL



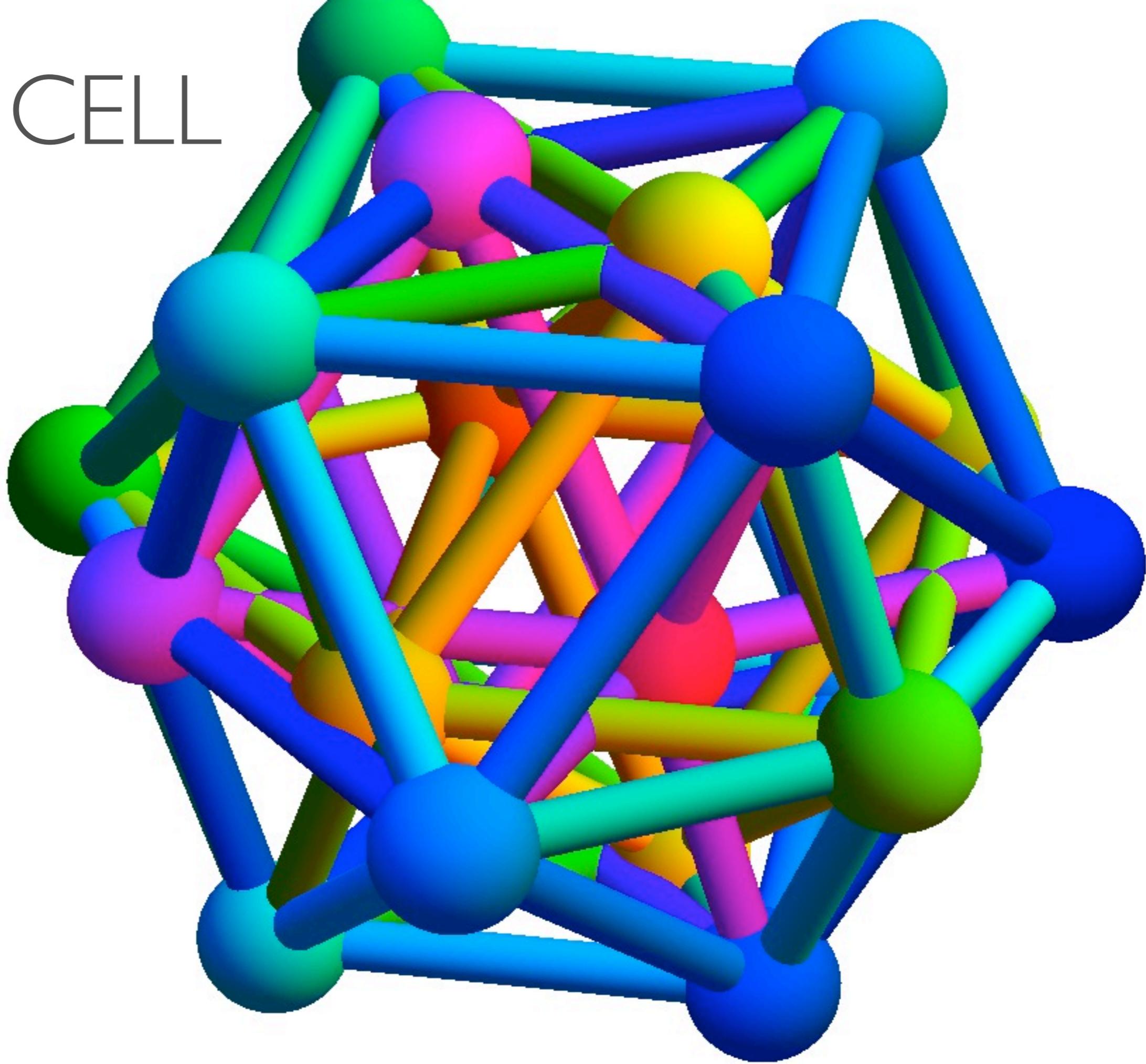
8 CELL



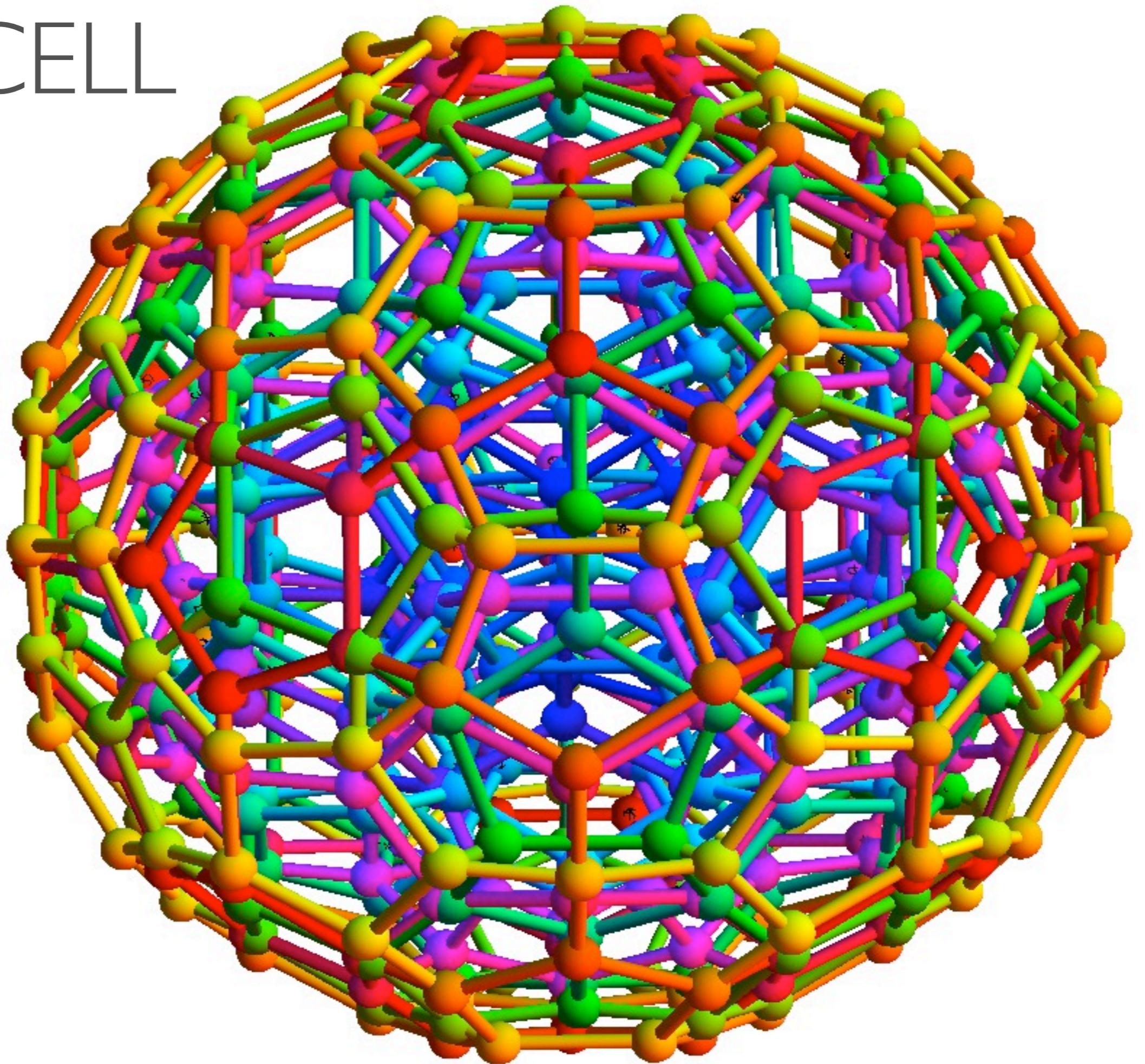
16 CELL



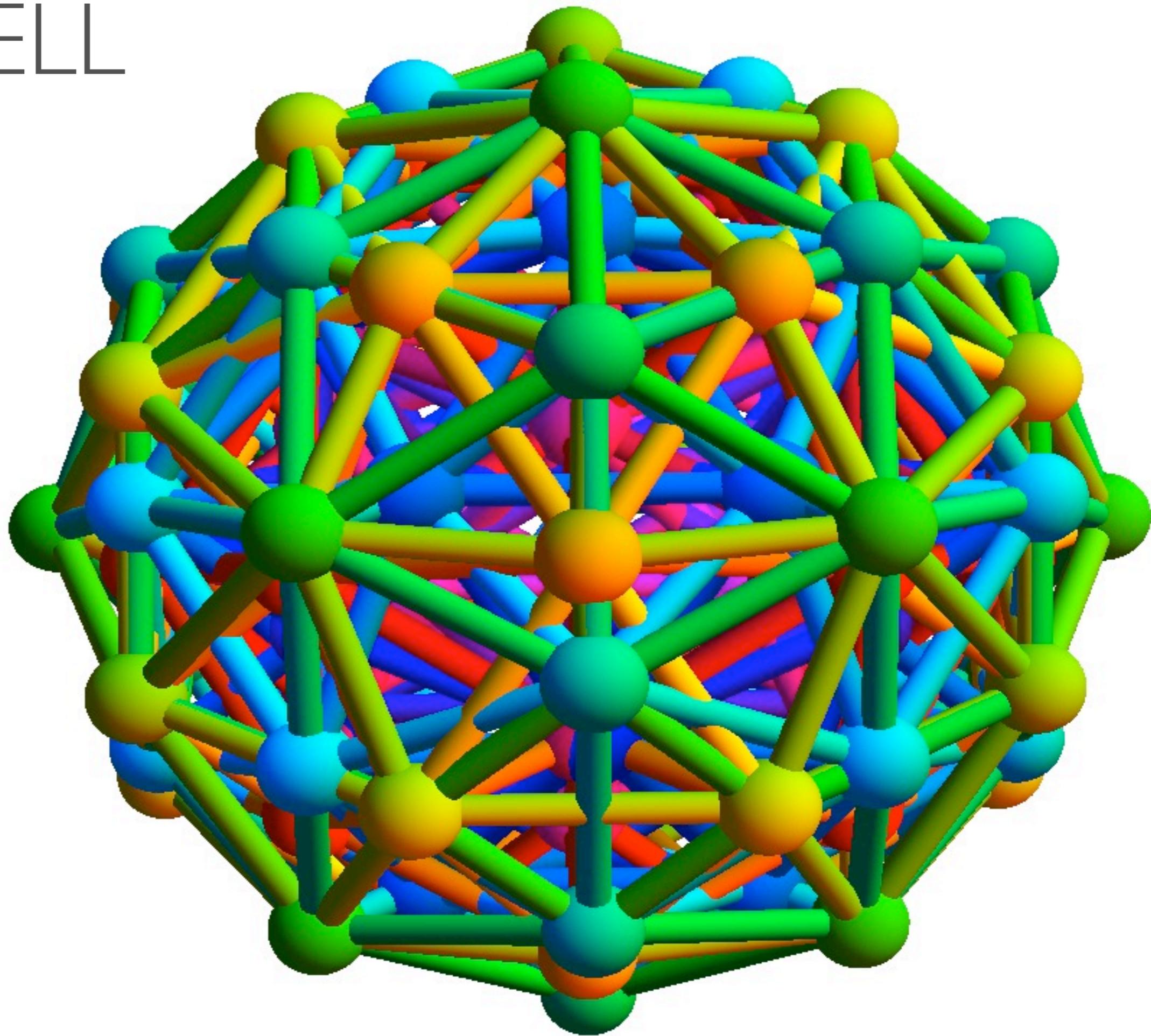
24 CELL

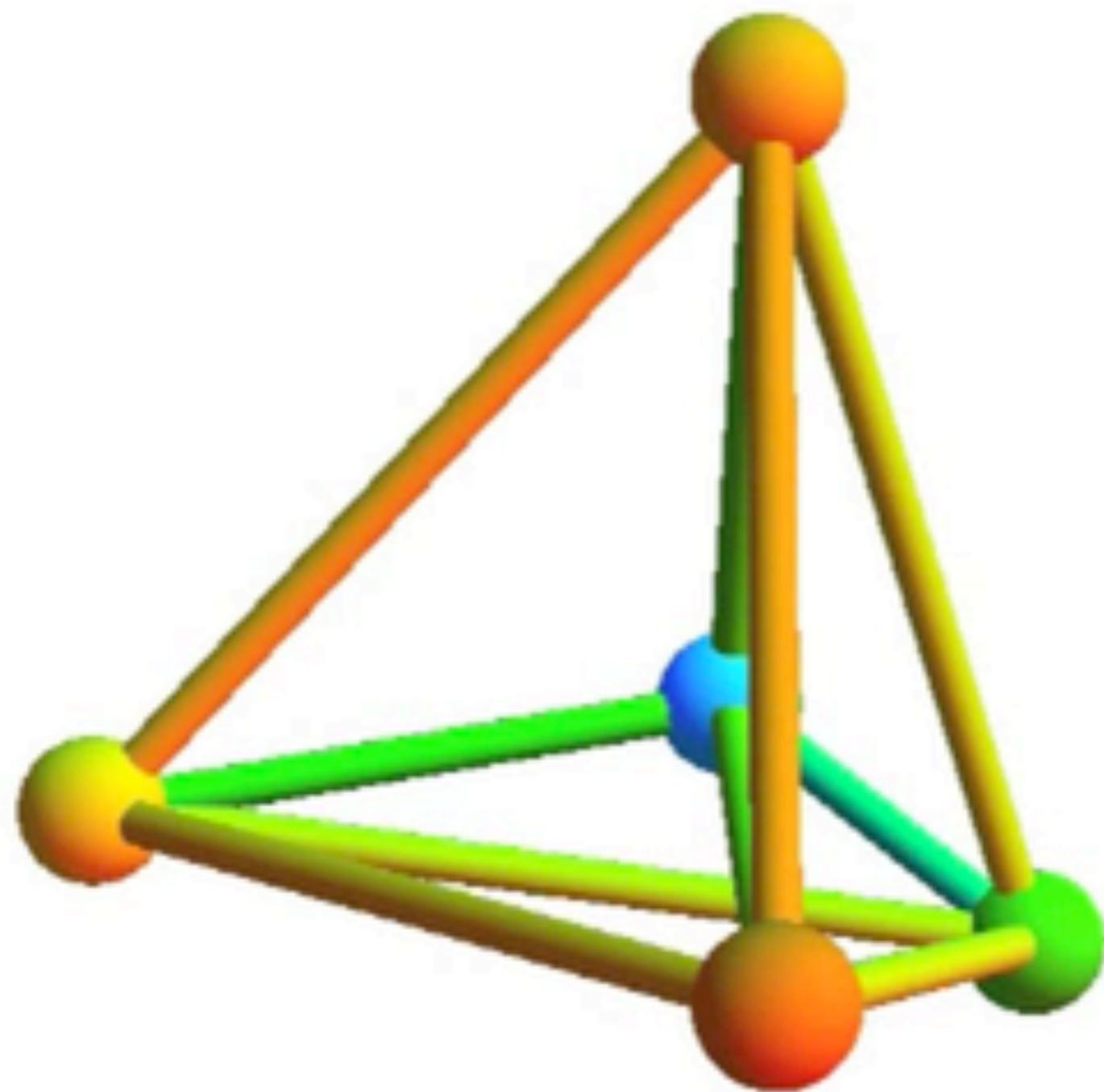


I 20 CELL



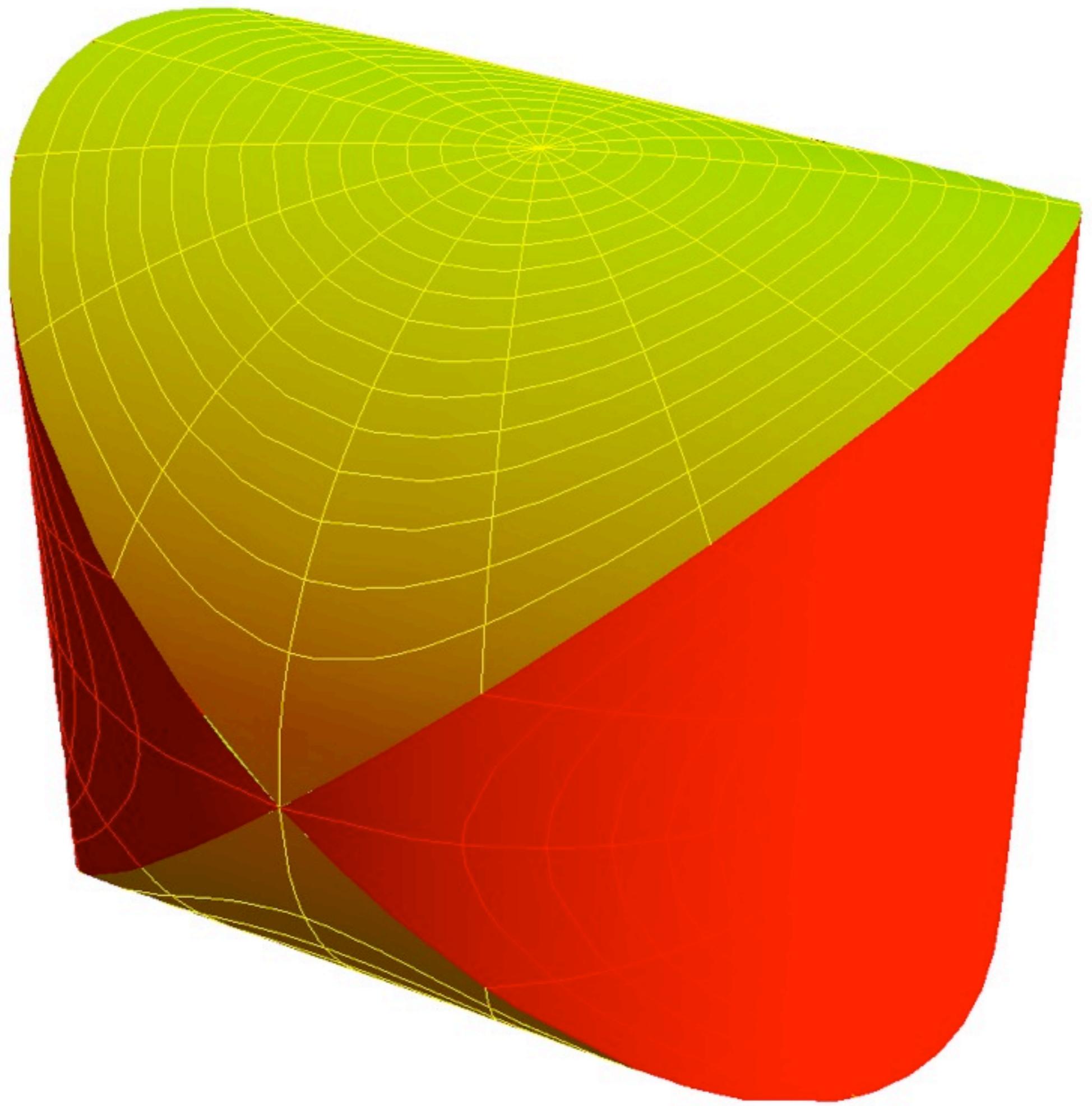
600 CELL

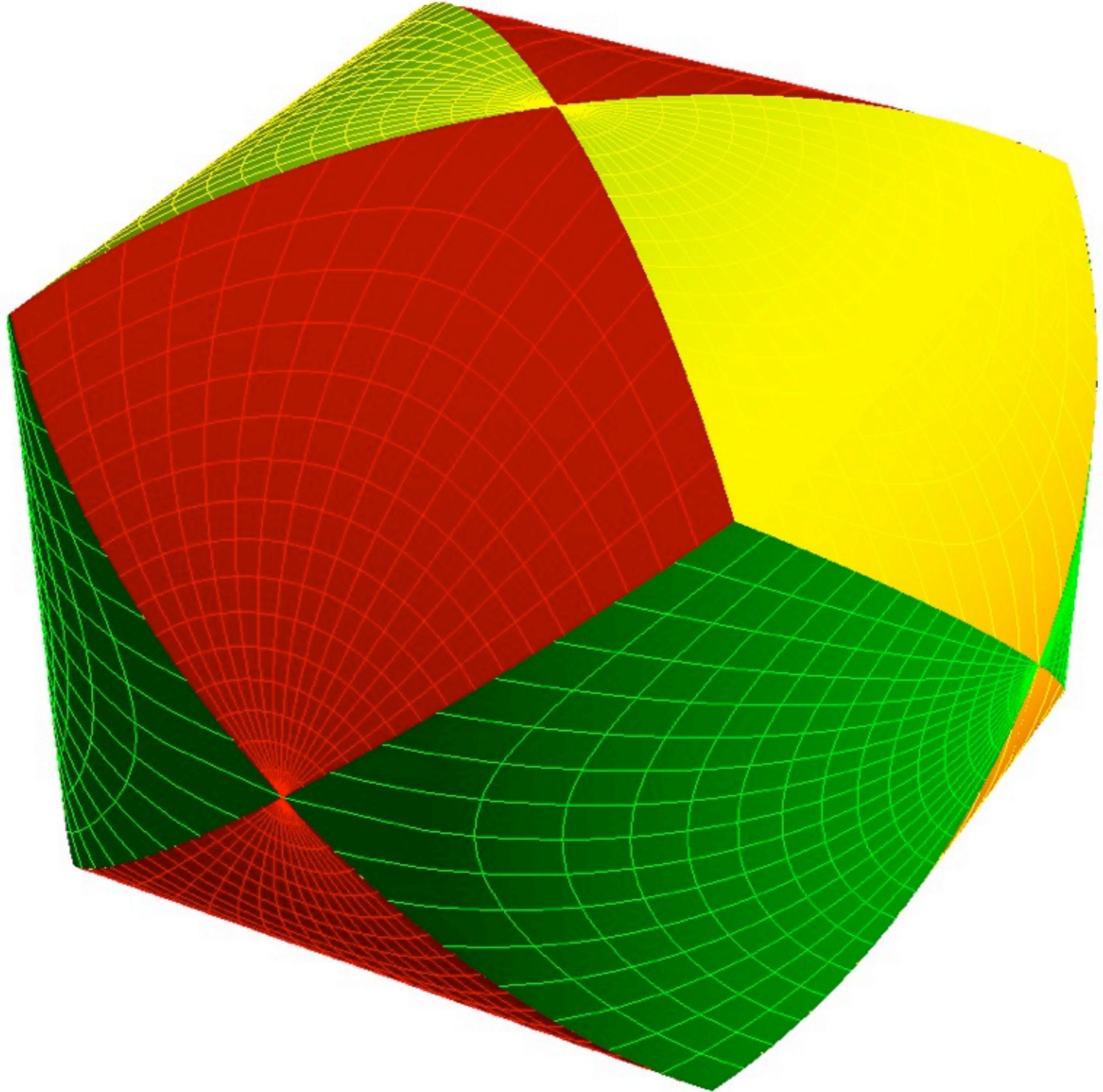




2D Infinitely Polygons
3D Five Platonic solids
4D Six Platonic solids
5D Three







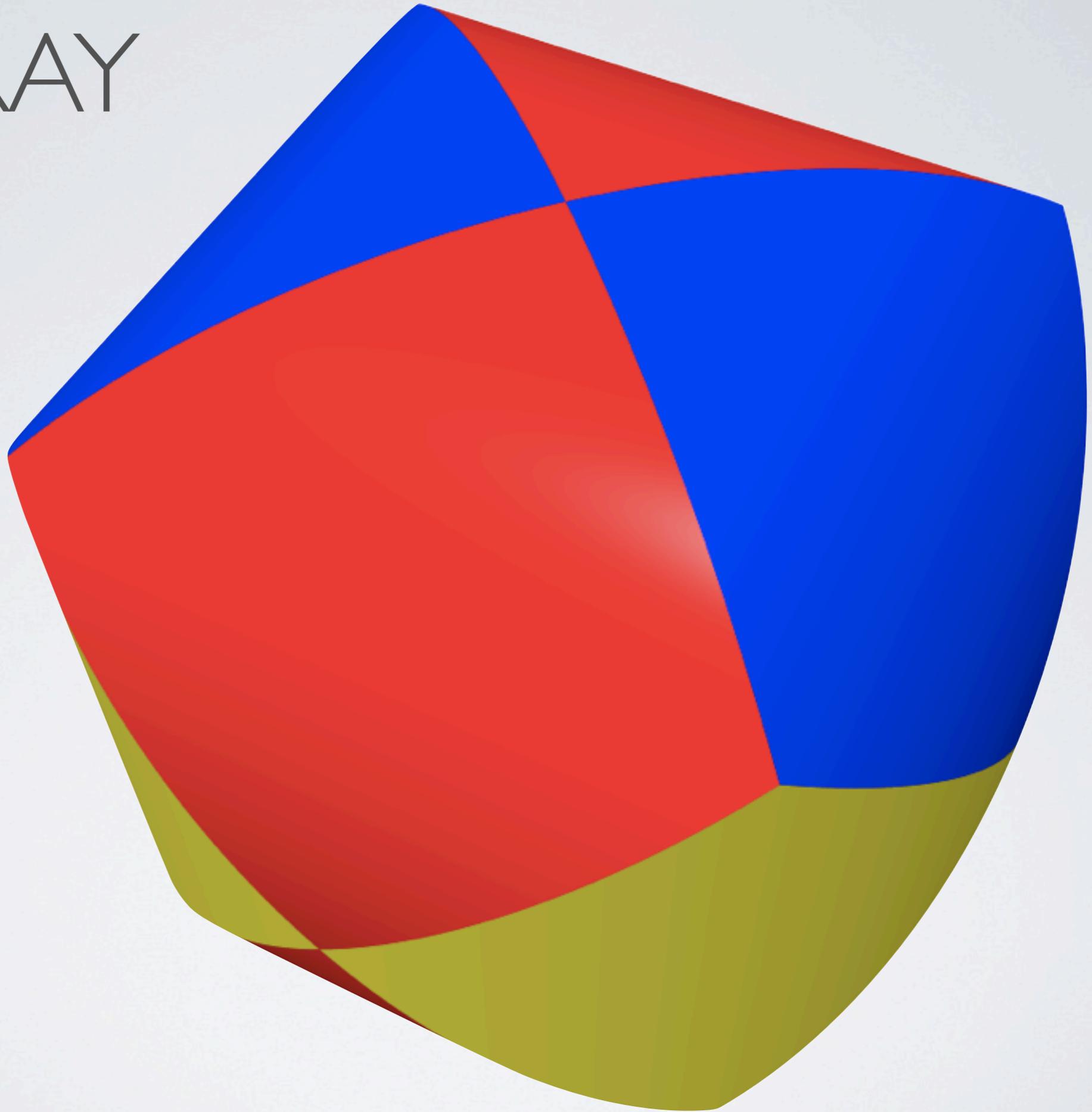
POVRAY

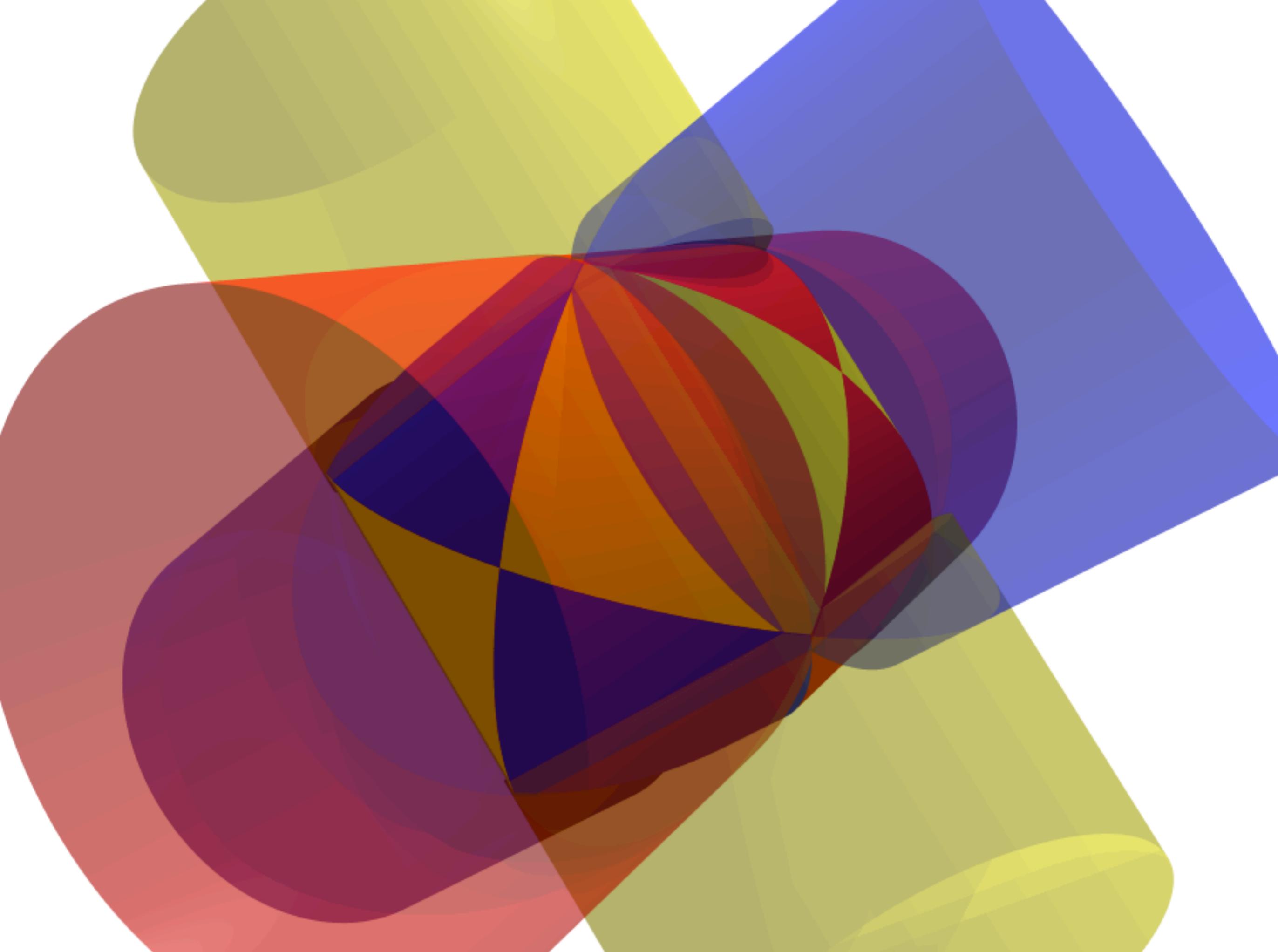
```
camera{up y right x location <1,3,-2> look_at <0,0,0>}  
light_source { <0,300,-100> color rgb <1,1,1> }  
background { rgb<1,1,1> }
```

```
#macro r(c) pigment{rgb c} finish {phong 1 ambient 0.5} #end
```

```
intersection {  
  cylinder{<-1,0,0>,<1,0,0>, 1 texture {r(<1,0,0>)}}  
  cylinder{< 0,-1,0>,<0,1,0>, 1 texture {r(<1,1,0>)}}  
  cylinder{<0,0,-1>,<0,0,1>, 1 texture {r(<0,0,1>)}}  
}
```

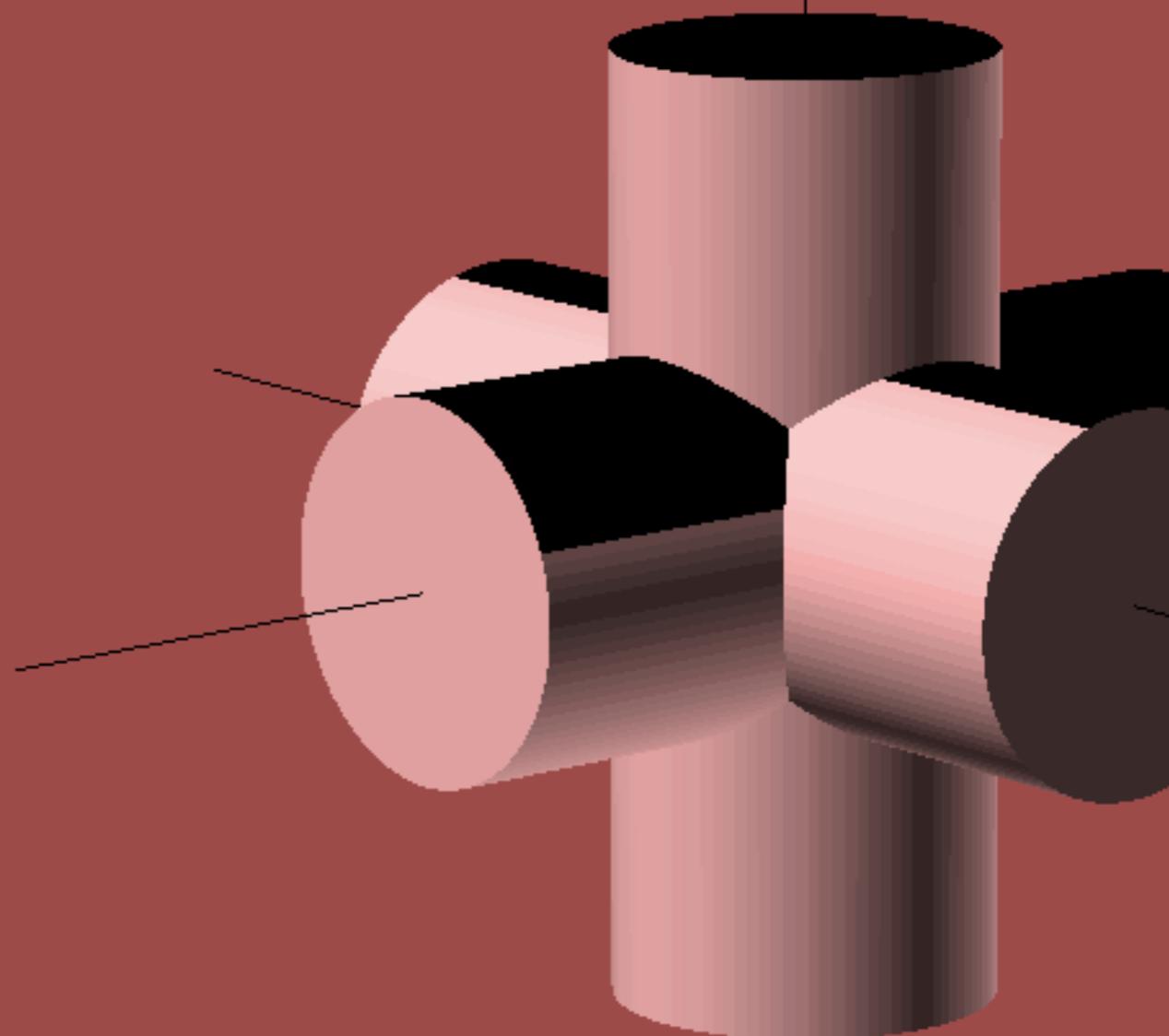
POVRAY



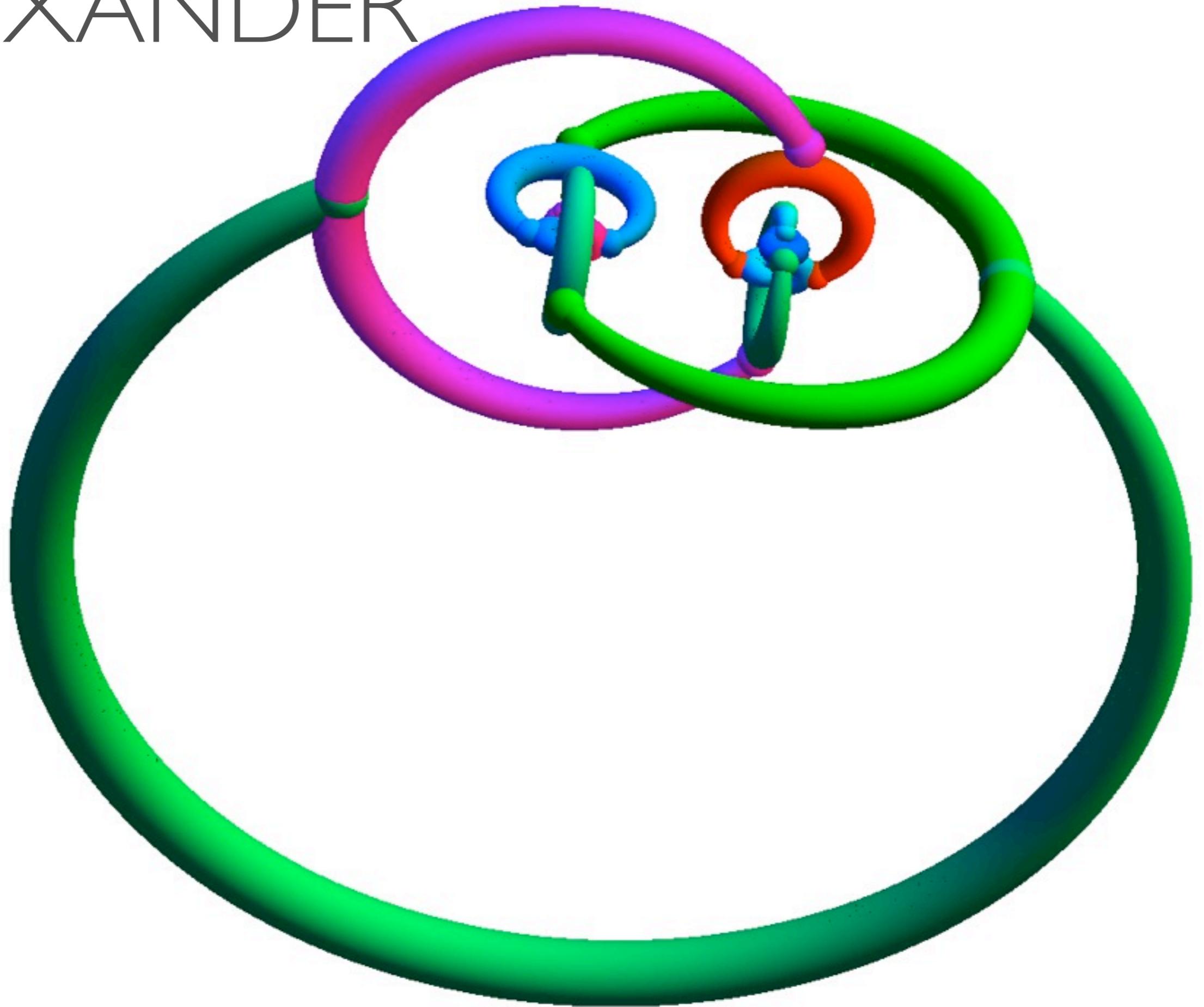


```
module cyl(a){
  rotate(90,a)
  cylinder(r=10,h=50,
  center=true,
  $fn=100);
}

intersection() {
  cyl([0,0,0]);
  cyl([1,0,0]);
  cyl([0,1,0]);
}
```



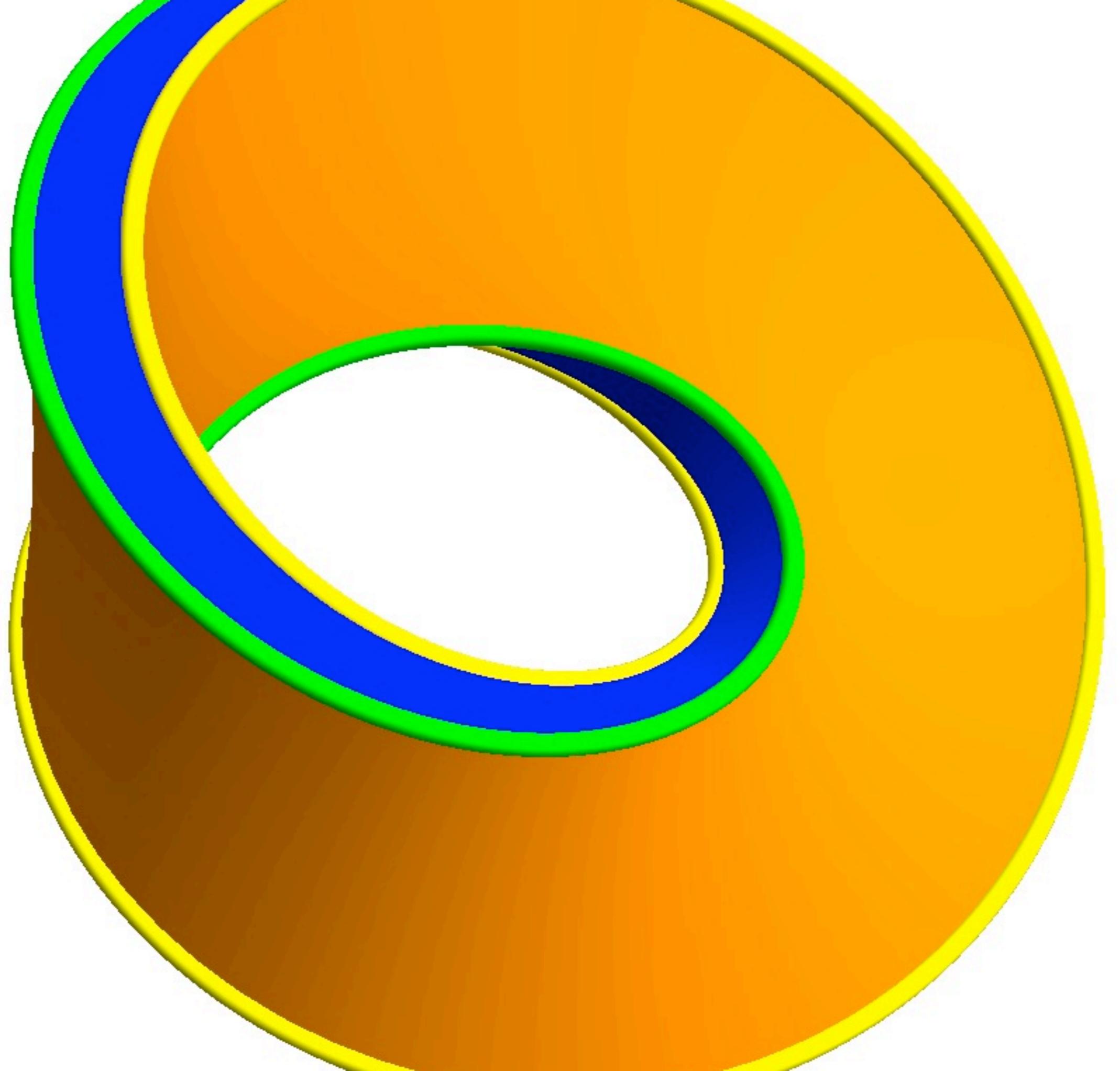
ALEXANDER

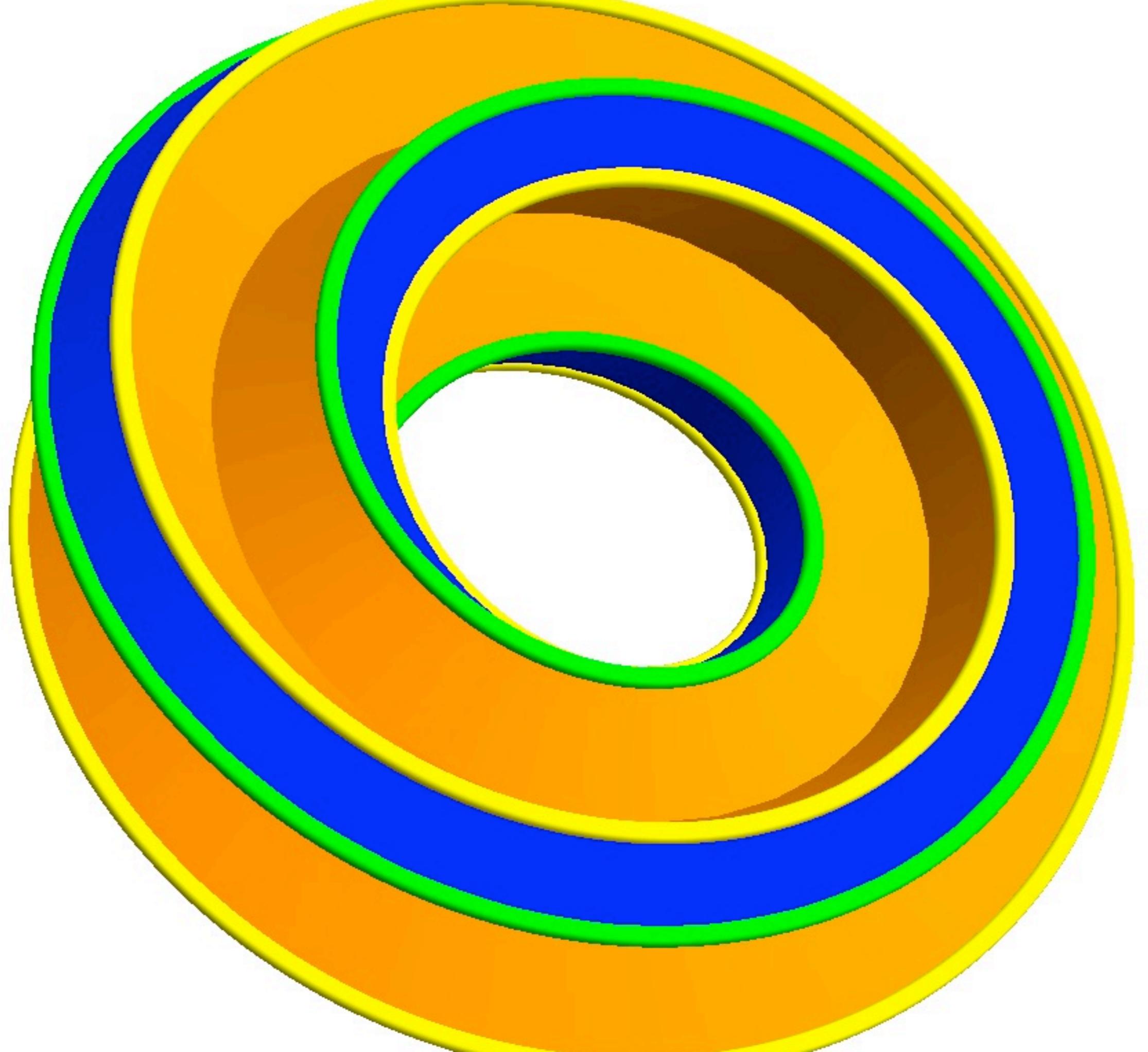


KNOTS

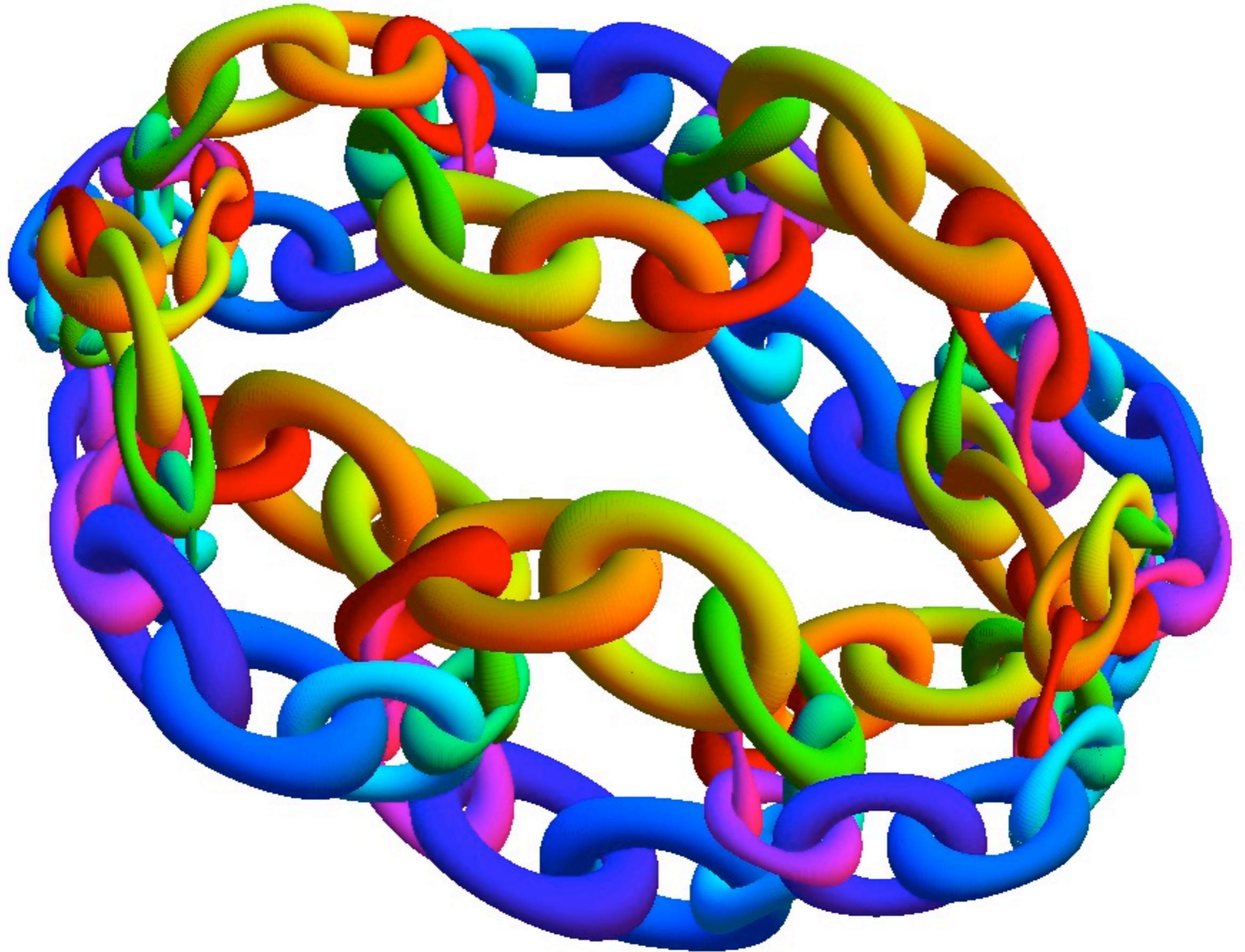


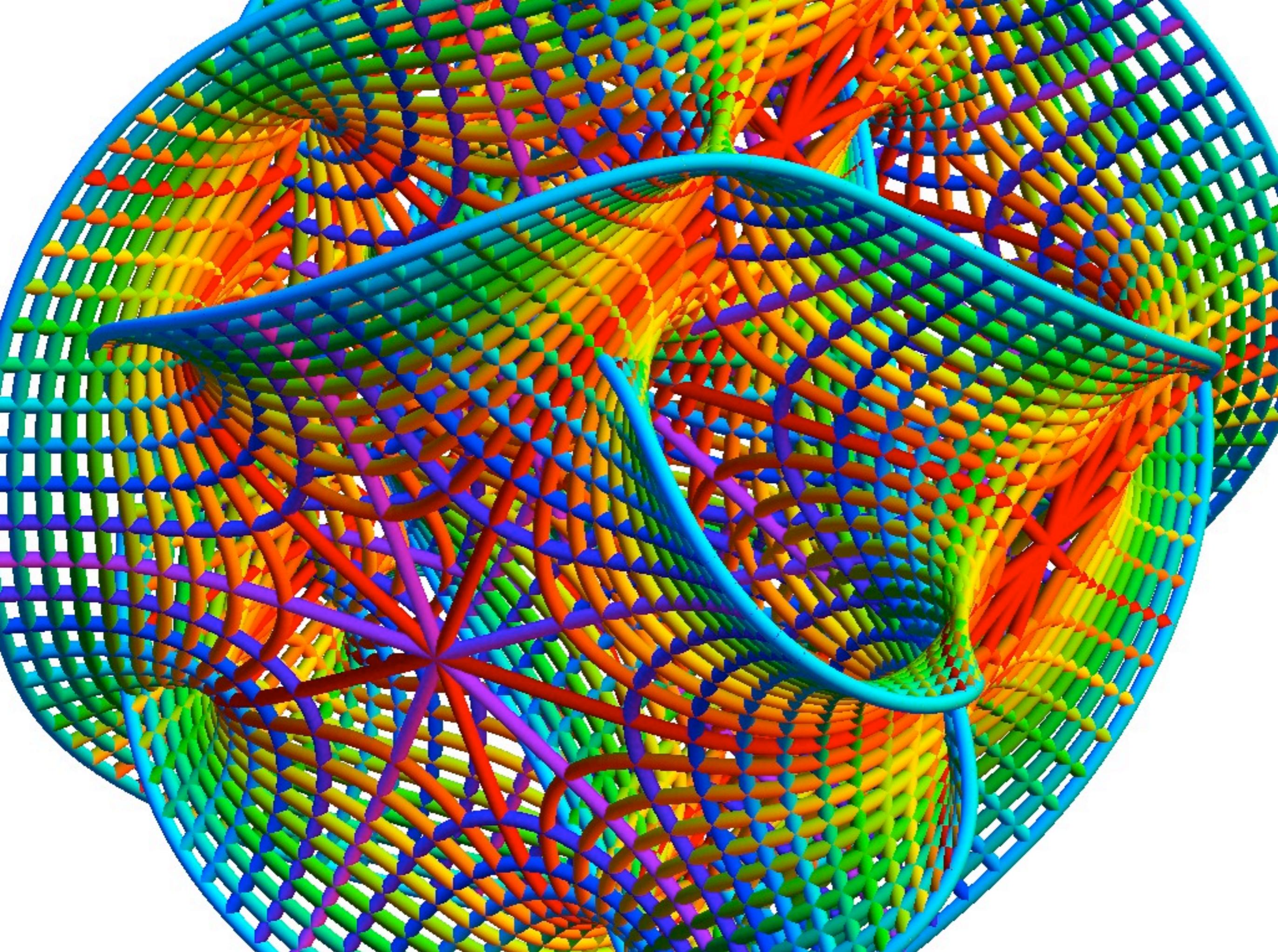


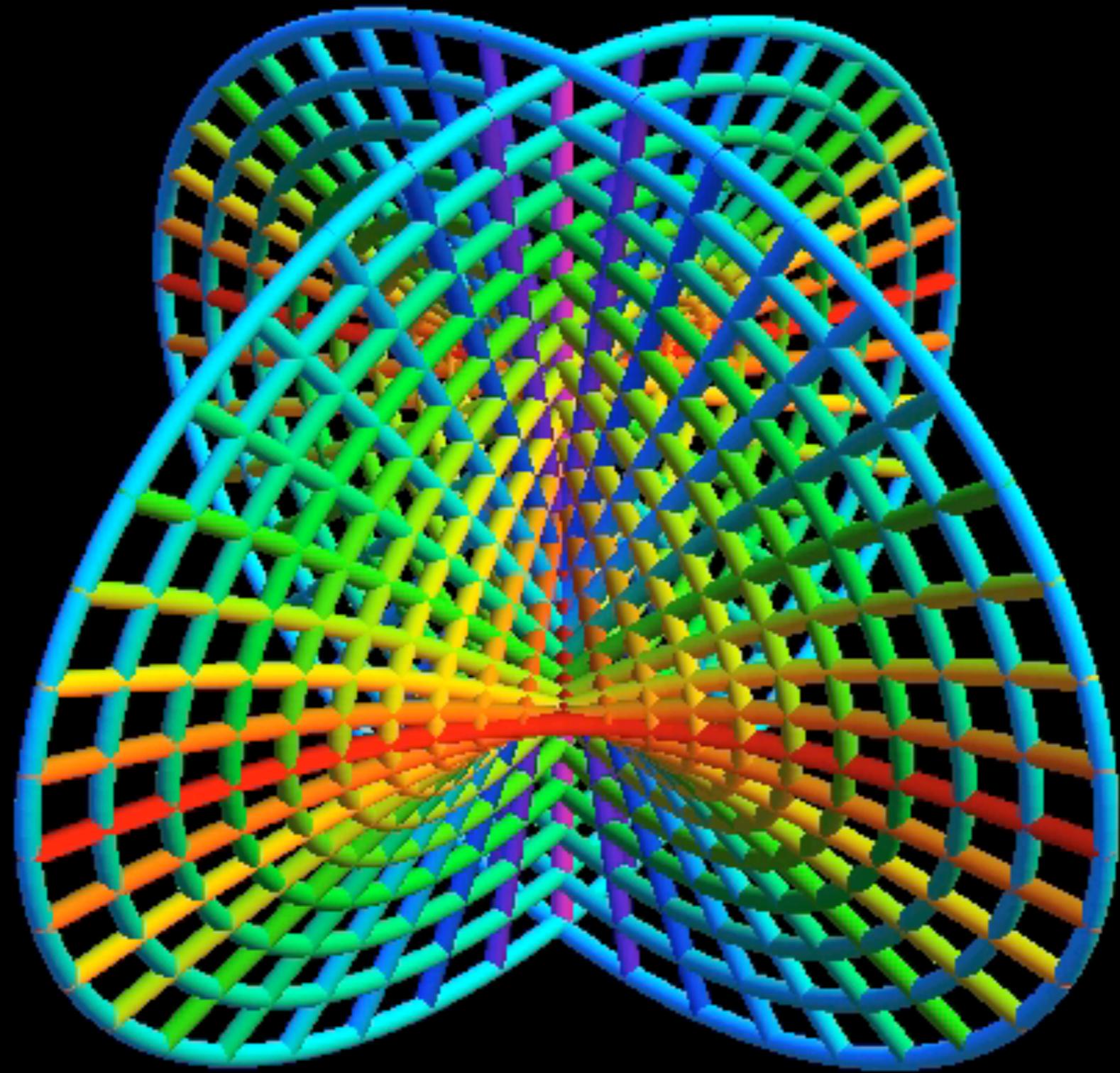


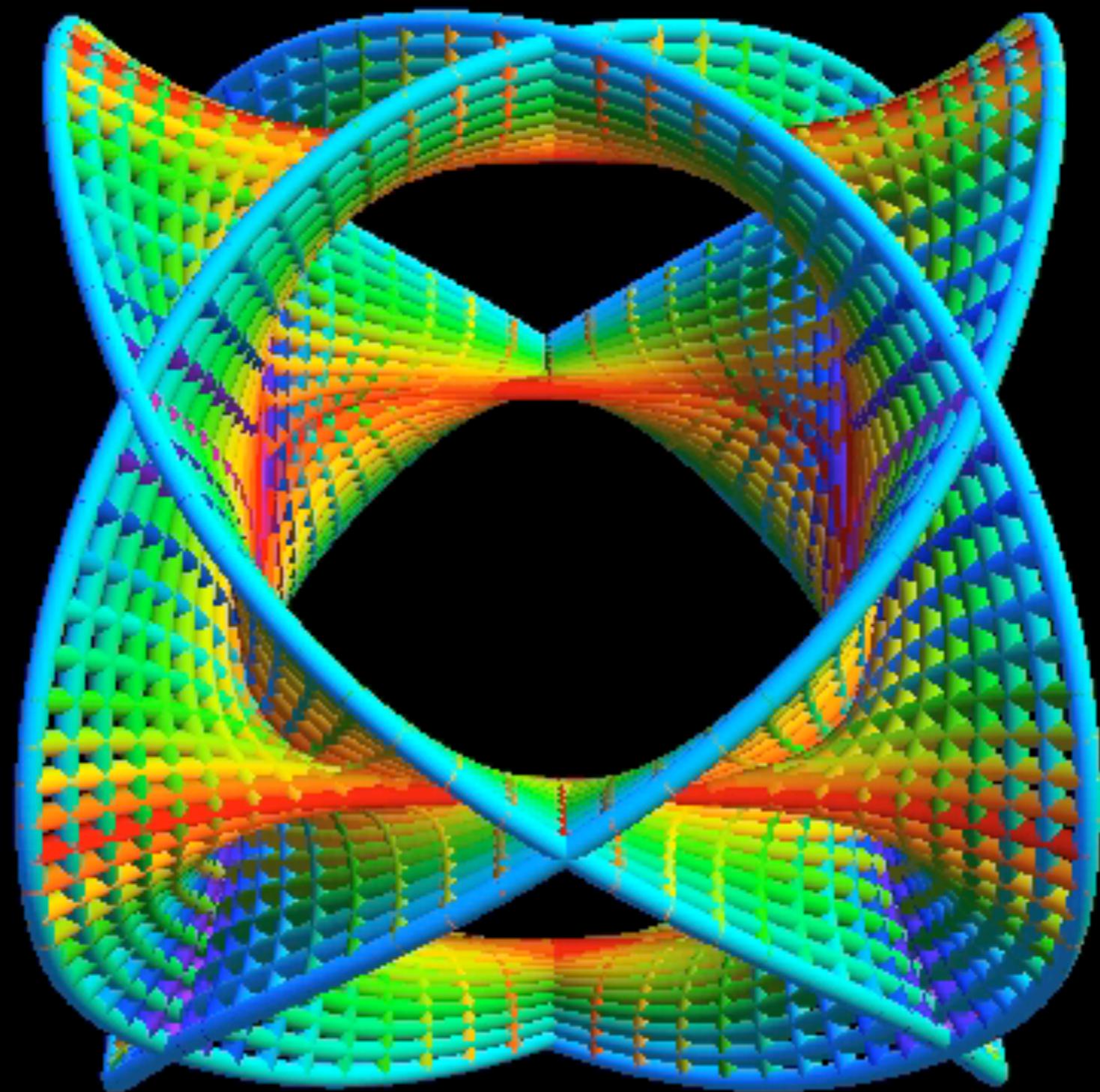


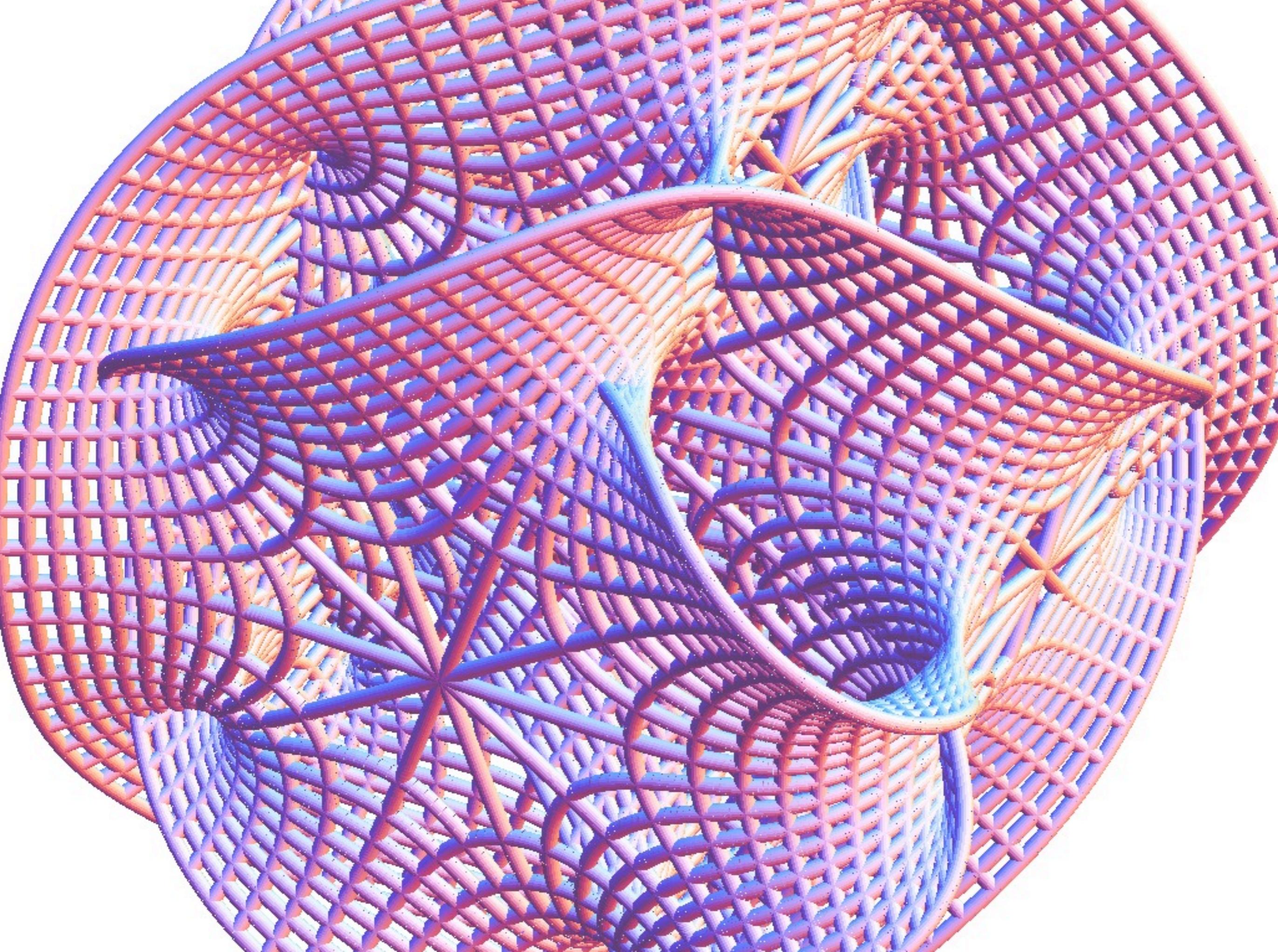
ANTOINE

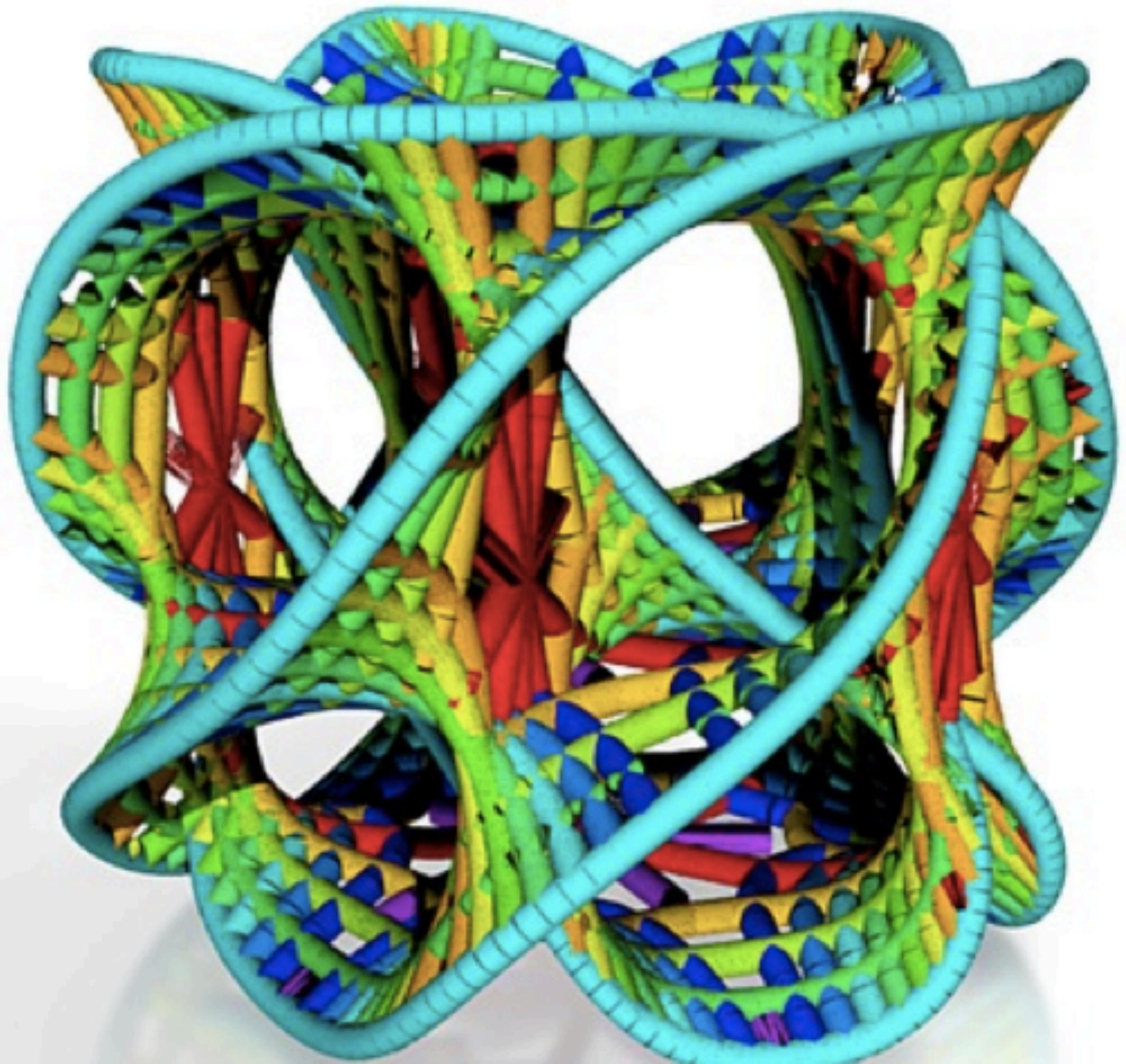














Calabiyau5

Edit Model

SHAPEWAYS

White Strong & Flexible \$35.22

\$35.22

Add to Cart



Calabiyau5

Edit Model

White Strong & Flexible \$35.22

\$35.22

Add to Cart



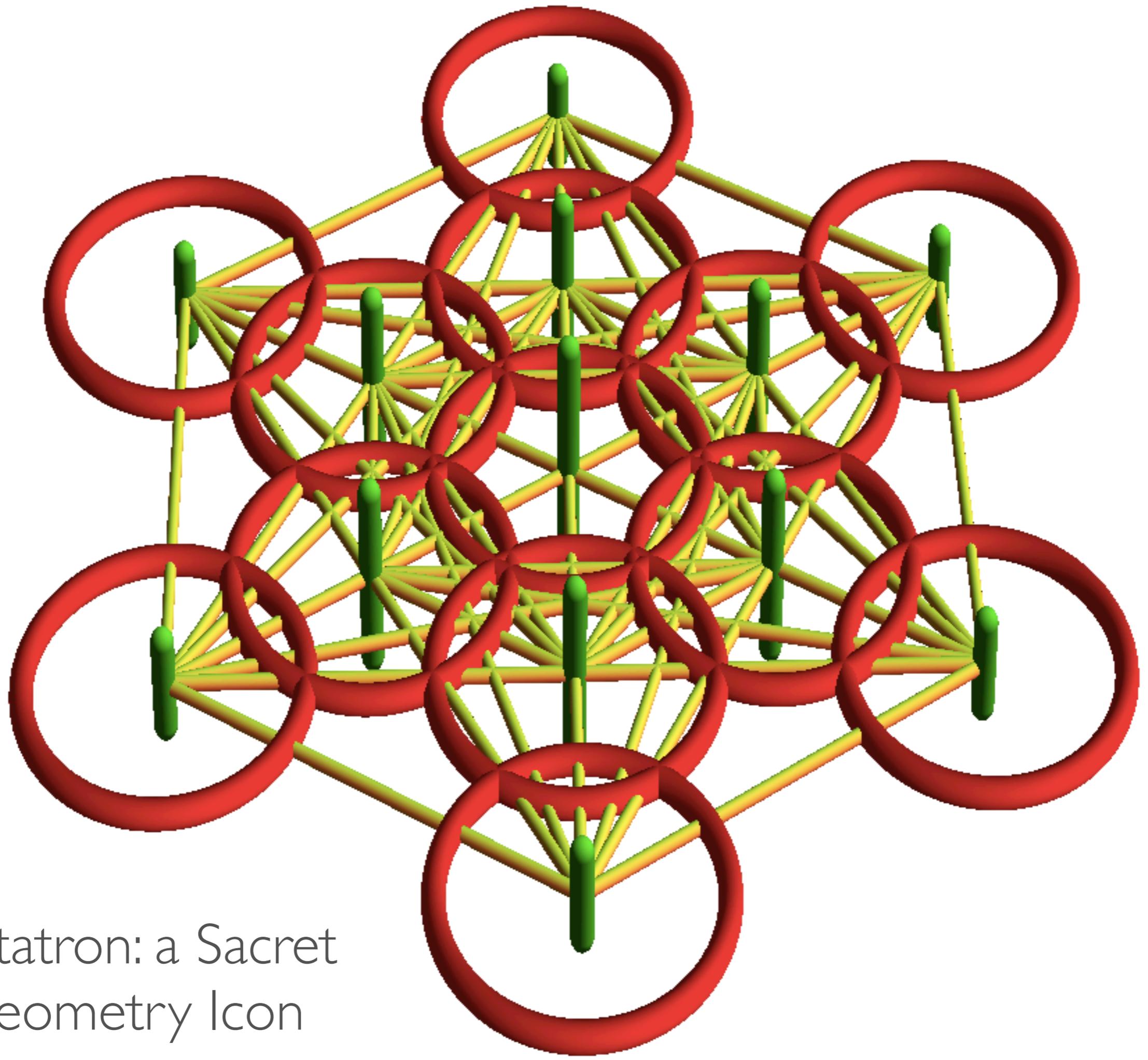
Calabiyau5a

Edit Model

White Strong & Flexible \$148.42

\$148.42

Add to Cart



Metatron: a Sacret
Geometry Icon

drone

search

try: dog figure cube gear gear samsung ace tower lens fujifilm cookie cutter » more »

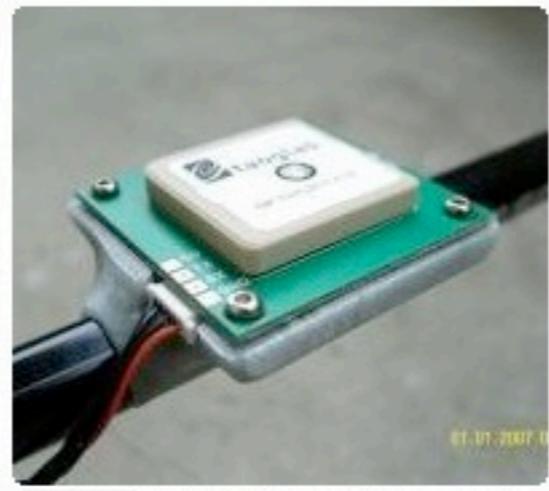
Your search for drone - 36 printable 3D models



Tags T-1 Aerial aka Hunter Killer Drone



Tags Joe's FPV QuadCopter variant



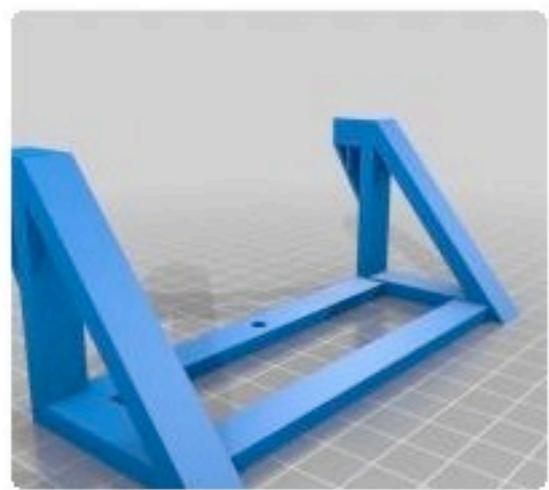
Tags 3DR Ublox GPS mount for 450 Traditional Heli



Tags Hexacopter



Tags 3DR Radio



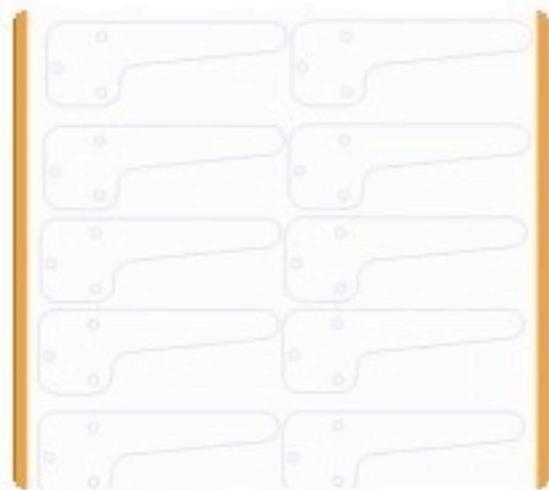
Tags ar.drone wall hanger



Tags Modular Snap-Fit Airship



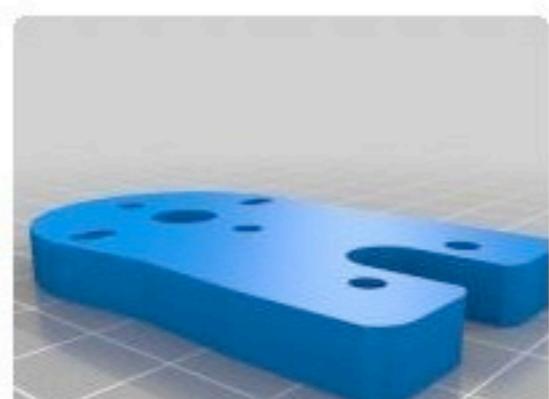
Tags Optical positioning reference



Tags Legs



Tags Arctic Tern





3D print your designs

Upload your 3D design and instantly see the price for your models. It's that easy.

- No login necessary
- Choose from a large selection of materials and colors
- Scale your model to the ideal size
- Order as many copies as you want

Sell your designs

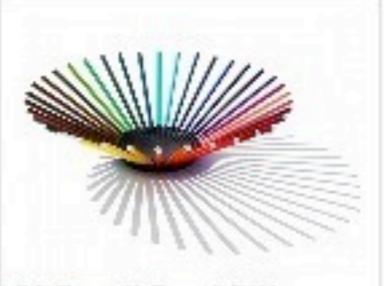
Want to show off your design talent and make some money at the same time? Offer your designs for sale in our gallery and once a month, we will pay you a fee for every one of your items sold.

Find the perfect gift

No matter the occasion, no matter the budget, our talented community of designers makes sure there is something for everyone in our ever growing gallery of 3D Printed objects. Browse through the collection and find the perfect gifts for the ones you love.



Chassis Wien T1
Guido Mandor... 29.77 €



36 Pencil Bowl DIY...
Michiel Corn... 35.00 €



Brass Bracelet Con...
Maaïke van d... 150.32 €



Diamonds are forev...
Eragatory 120.00 €



MEN NR.02 by DAMN ...
DAMN 103.30 €



Decision Spintop
Michael Muel... 46.21 €



Merry Bird ornamen...
Michiel Corn... 28.22 €



Klein Bottle - Mat...
Dizingof 30.00 €



Basalt 114 Double ...
Kevin Wei 72.00 €



Asia Dionysos
monomer 79.99 €



Dragonbite grip (s...
Vangelis and... 42.83 €

Browse the gallery for more designs

Upload a 3D file

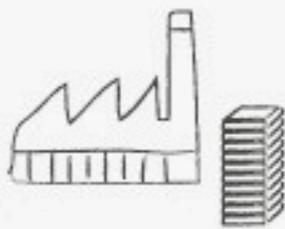
Pottery let's create! free your CREATIVITY



Innovative 3D printing service for creative people

3D Print easily

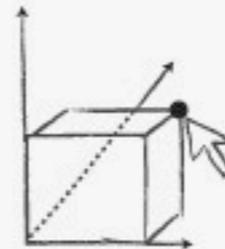
Upload your 3D file and have it printed by Sculpteo.



Upload 3D file

Design at your fingertips

We develop online 3D tools and workshops to give you the best 3D printing results. Try them !



Create objects

Sell your creations

Get inspired

Get in touch

Keep in touch with Sculpteo with our monthly newsletter.

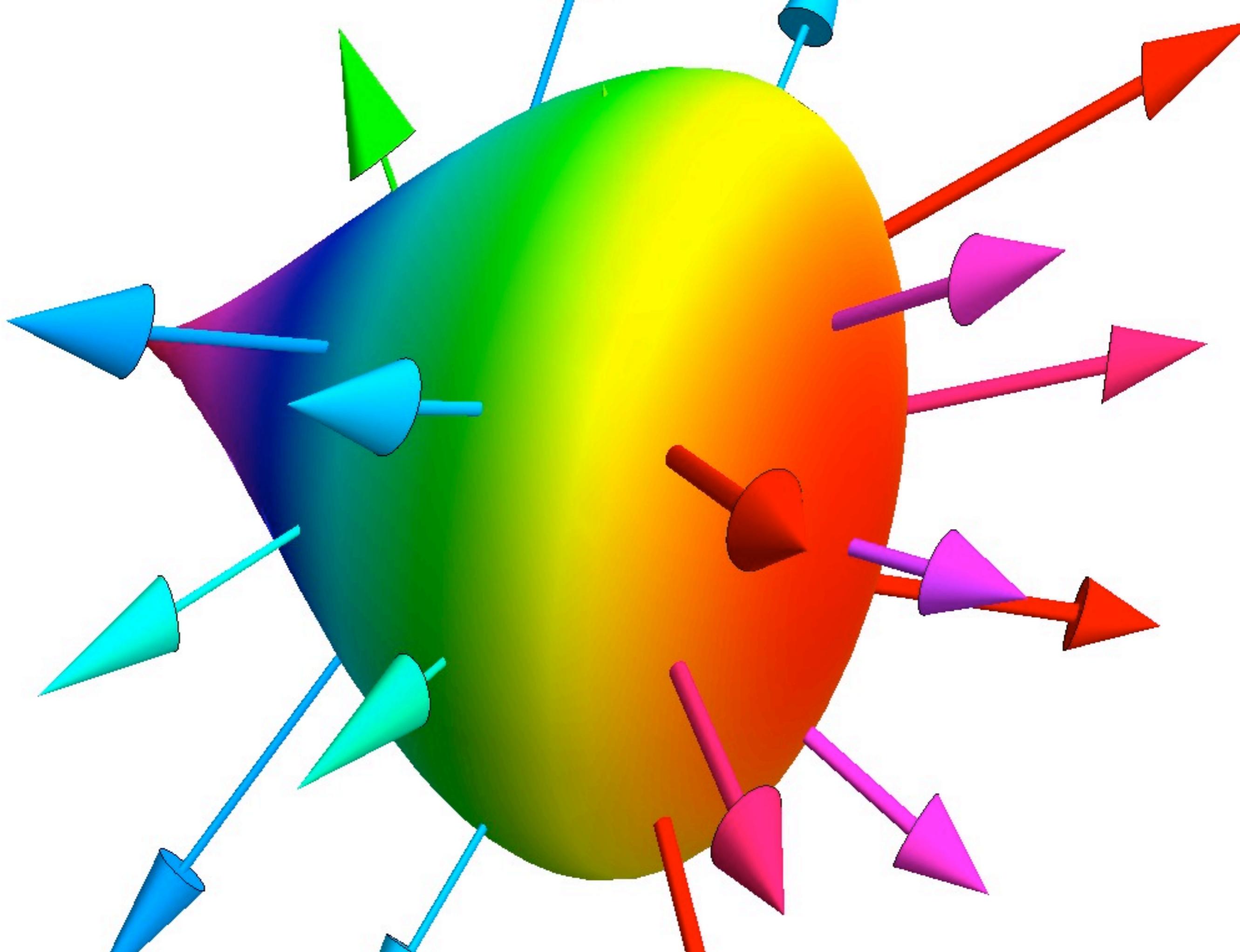
Enter your email

PARTICULE 14

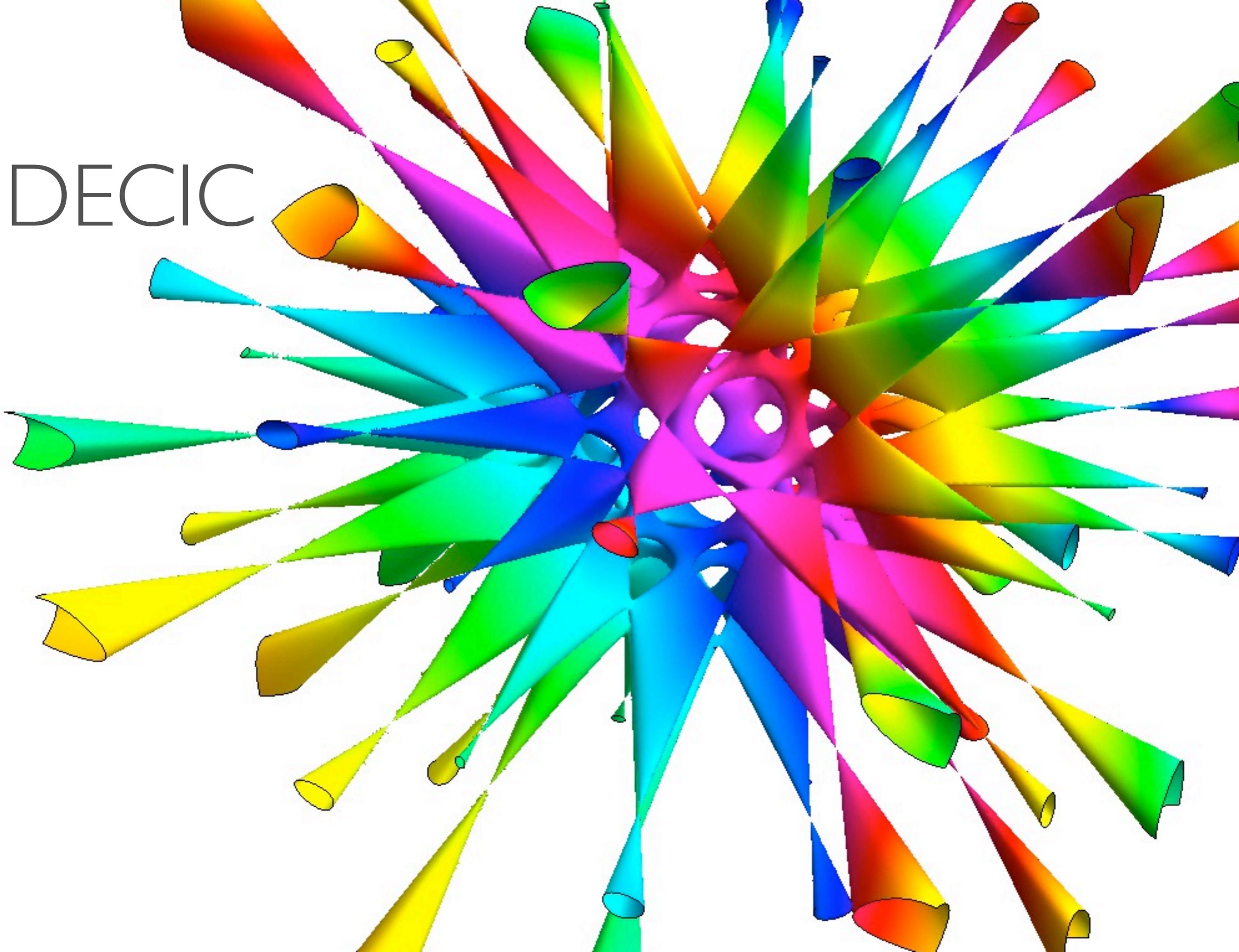
PARTICULE 14 PROJECT FOR INTERNATIONAL DESIGN WEEK OF MILAN [Learn more](#)

Follow us

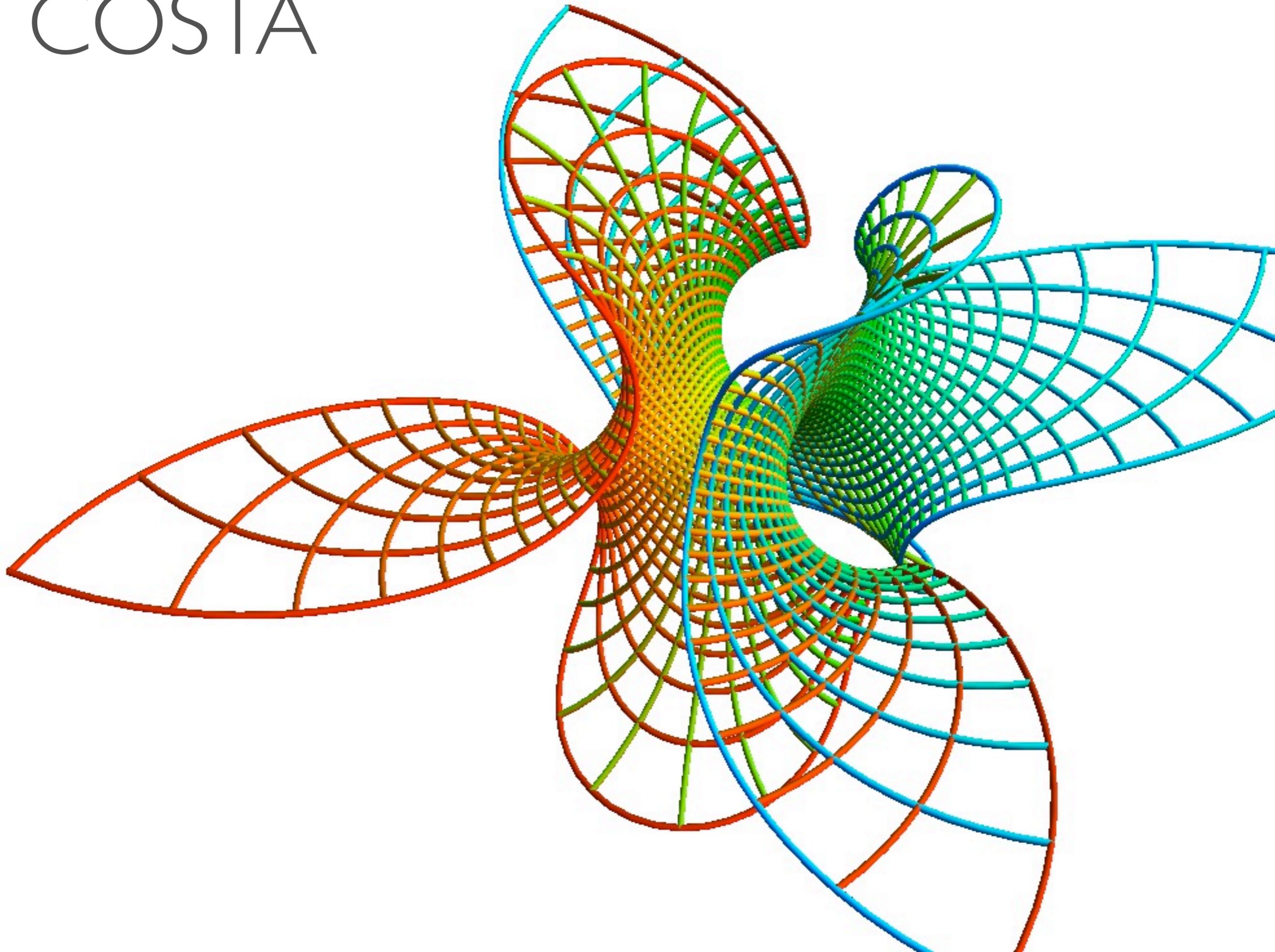




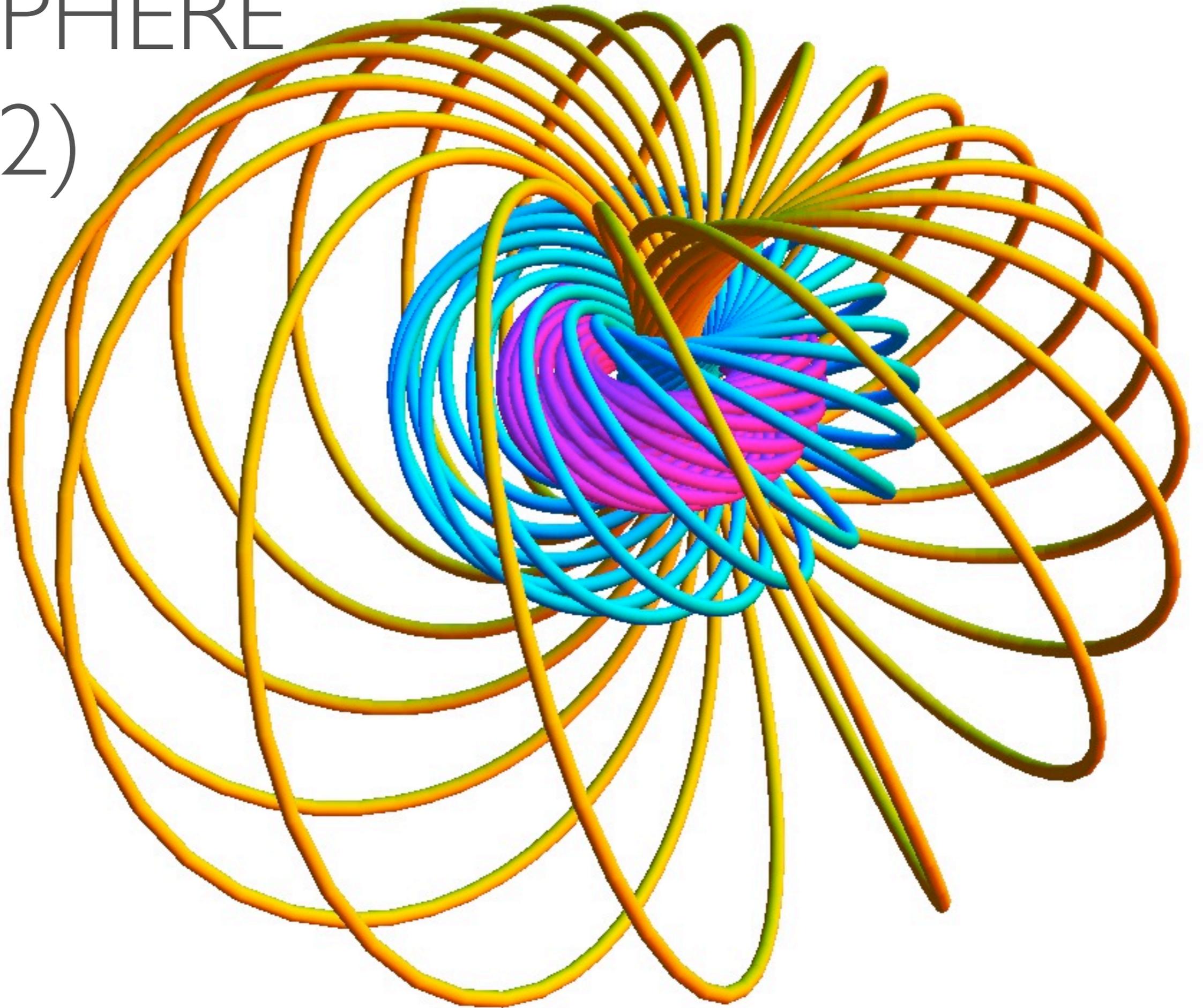
DECIC

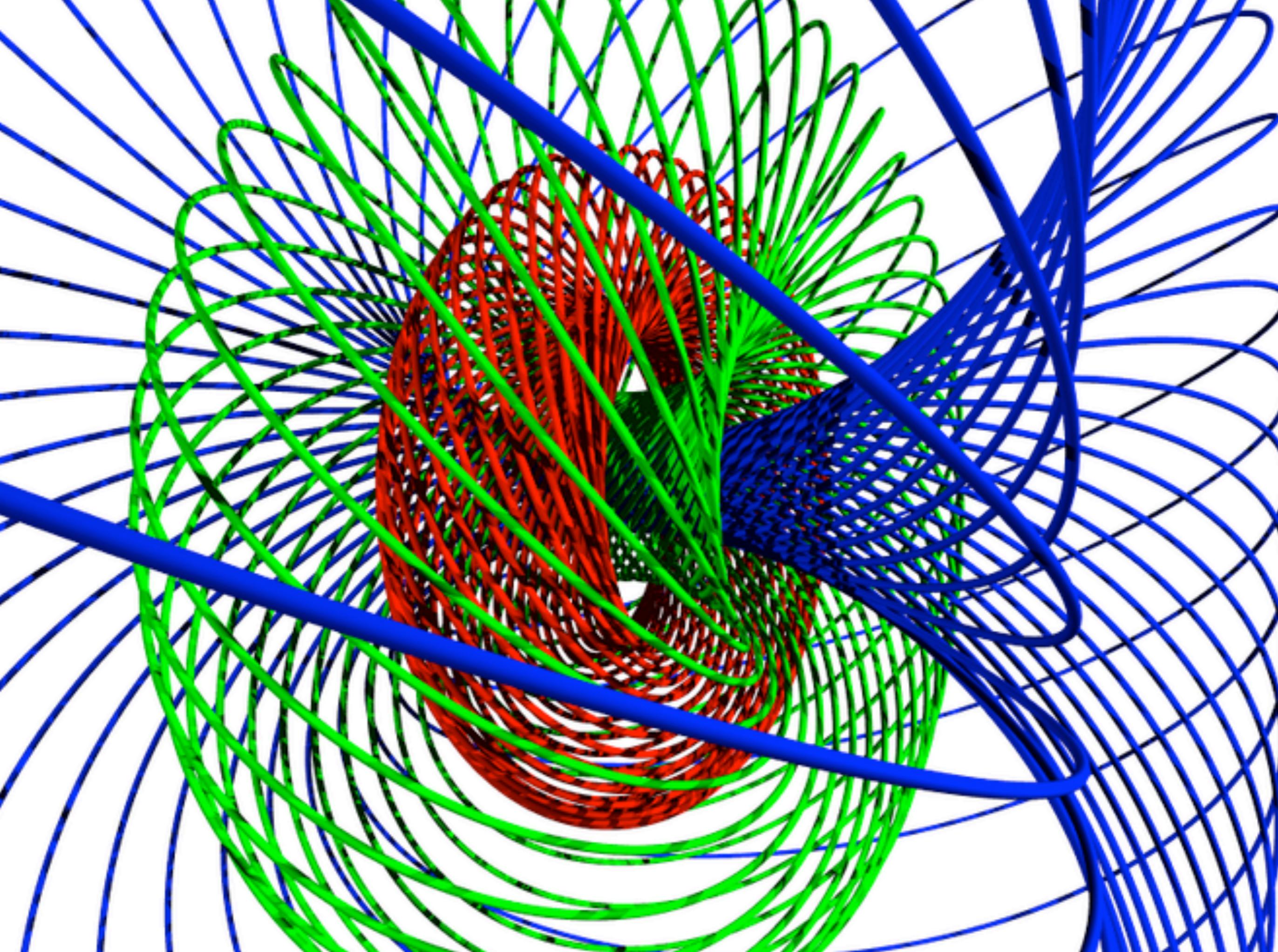


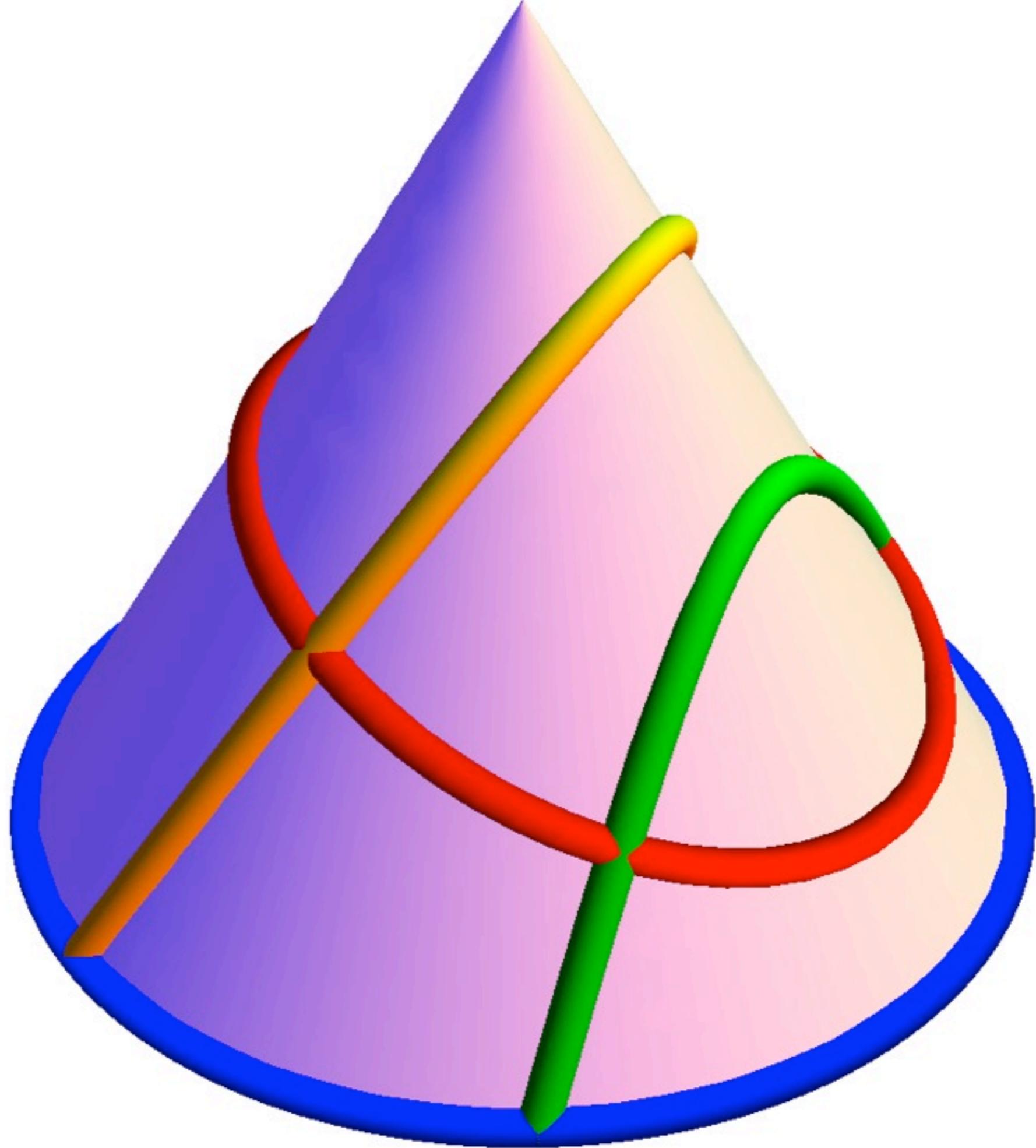
COSTA



3D SPHERE
= $SU(2)$







RACHEL WEISZ

AGORA

A FILM BY ALEJANDRO AMENÁBAR

ALEXANDRIA, EGYPT. 391 A.D.
THE WORLD CHANGED FOREVER



COMING SOON

WWW.AGORATHEMOVIE.COM



FESTIVAL DE CANNES
OFFICIAL SELECTION
OUT OF COMPETITION



ALEJANDRIA, EGIPTO 391 D.C.
EL MUNDO CAMBIÓ PARA SIEMPRE

RACHEL WEISZ

AGORA

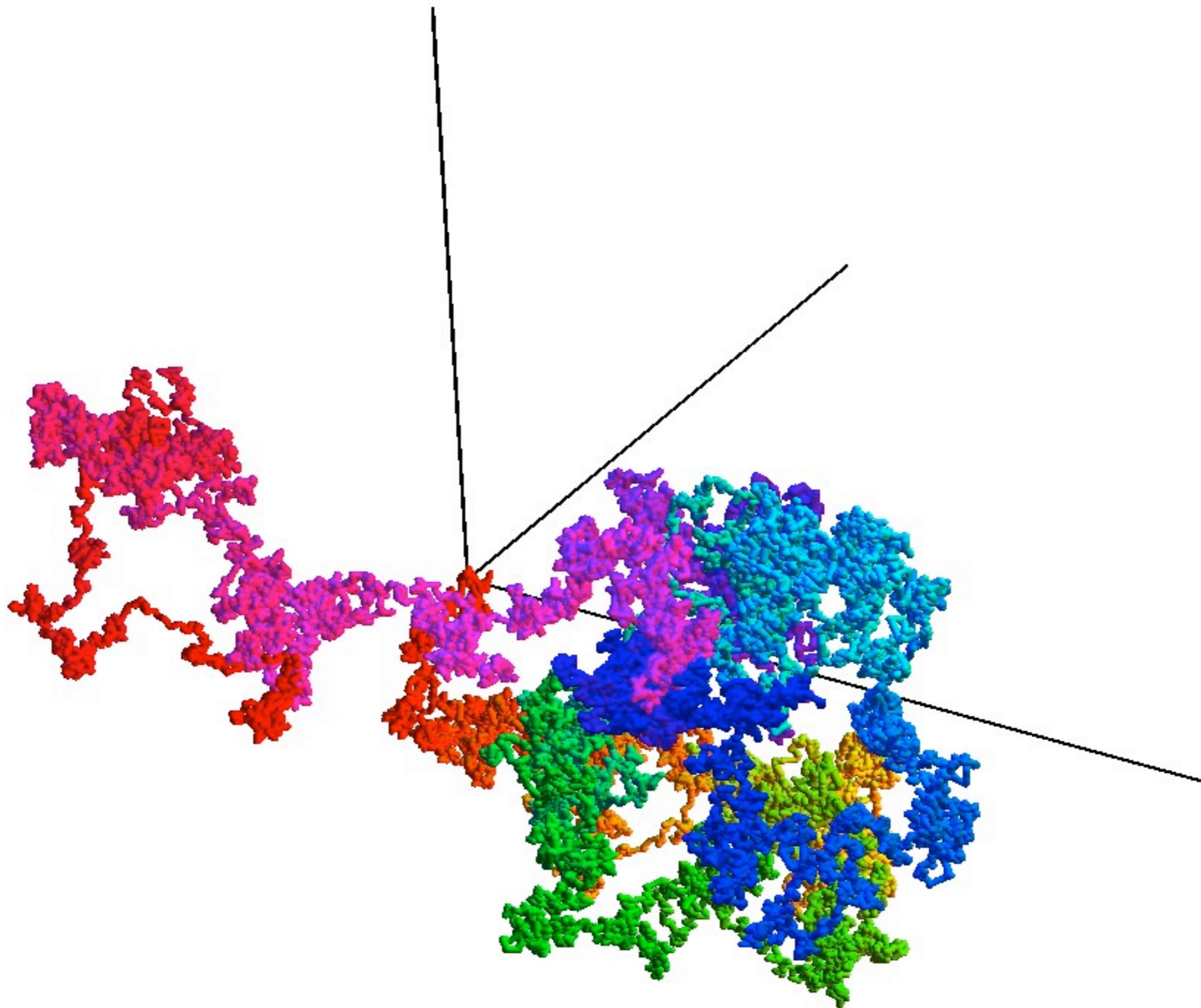
UNA PELÍCULA DE ALEJANDRO AMENÁBAR

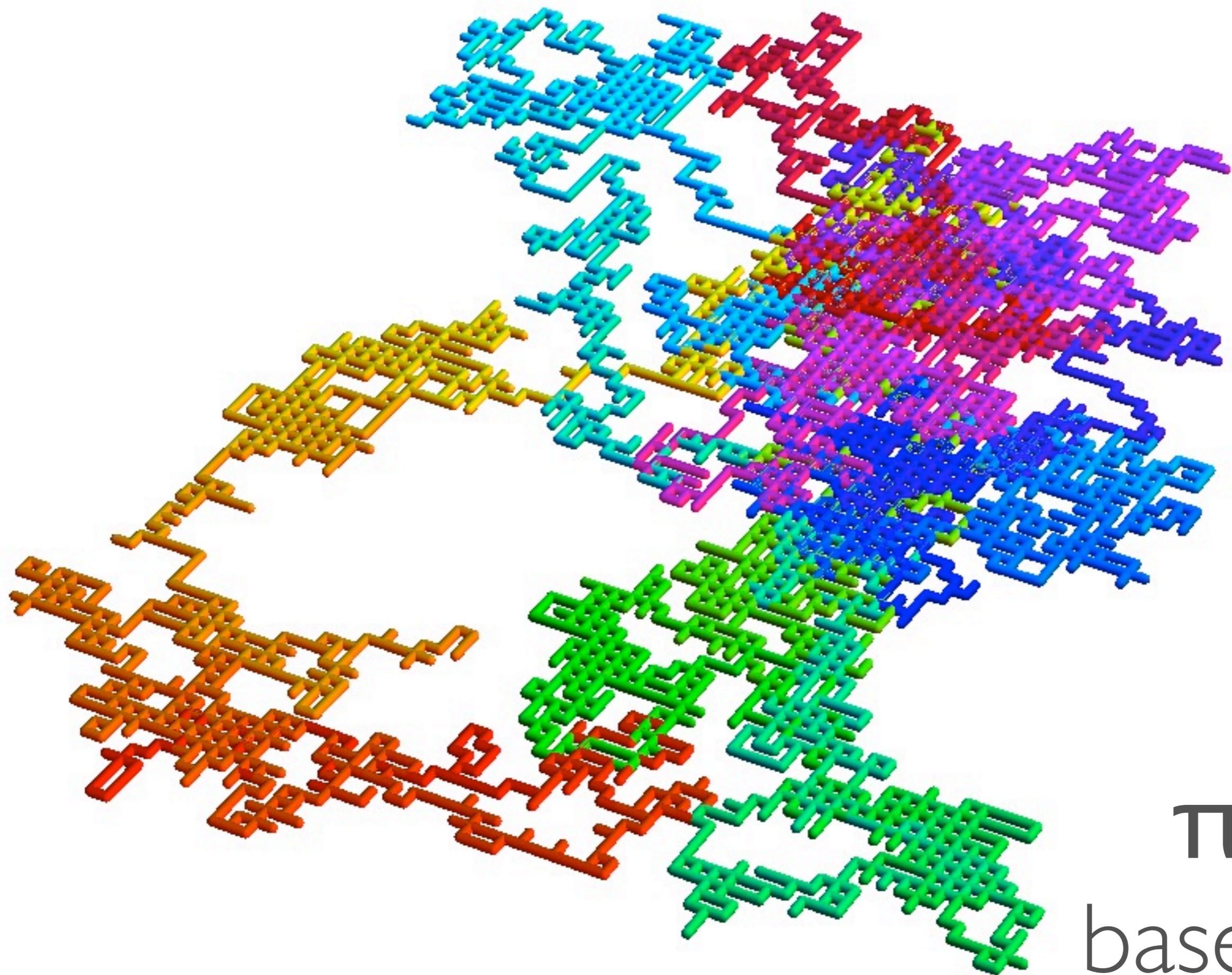
NO PRODUCCIONES HINEMPTERO - TELECINCO CINEMA / CANAL+ ESPAÑA / ICAI
RACHEL WEISZ 'AGORA' MAX MINGHELLA OSCAR ISAAC ASHRAF BARRHOM RUPERT EVANS HONAYOUN ERSHADI MICHAEL LONSDALE SAMMY SAMIR
GUIÓN Y DIRECCIÓN DE ALEJANDRO AMENÁBAR
MONTAJE DAVID GONZÁLEZ
MÚSICA JOSÉ LUIS ESCOBAR
DISEÑO DE SONIDO SANTIAGO JIMÉNEZ DE SANTIAGO
EDICIÓN DE VIDEO FERNANDO BOVIANA
ALFARDO ANDRÉS
ALEJANDRO AMENÁBAR
DISTRIBUCIÓN EN ESPAÑA ICAI
DISTRIBUCIÓN EN AMÉRICA LATINA Y CARIBE ICAI
DISTRIBUCIÓN EN FRANCIA ICAI
DISTRIBUCIÓN EN ITALIA ICAI
DISTRIBUCIÓN EN PORTUGAL ICAI
DISTRIBUCIÓN EN ARGENTINA ICAI
DISTRIBUCIÓN EN CHILE ICAI
DISTRIBUCIÓN EN COLOMBIA ICAI
DISTRIBUCIÓN EN PERÚ ICAI
DISTRIBUCIÓN EN VENEZUELA ICAI
DISTRIBUCIÓN EN ESPAÑA ICAI



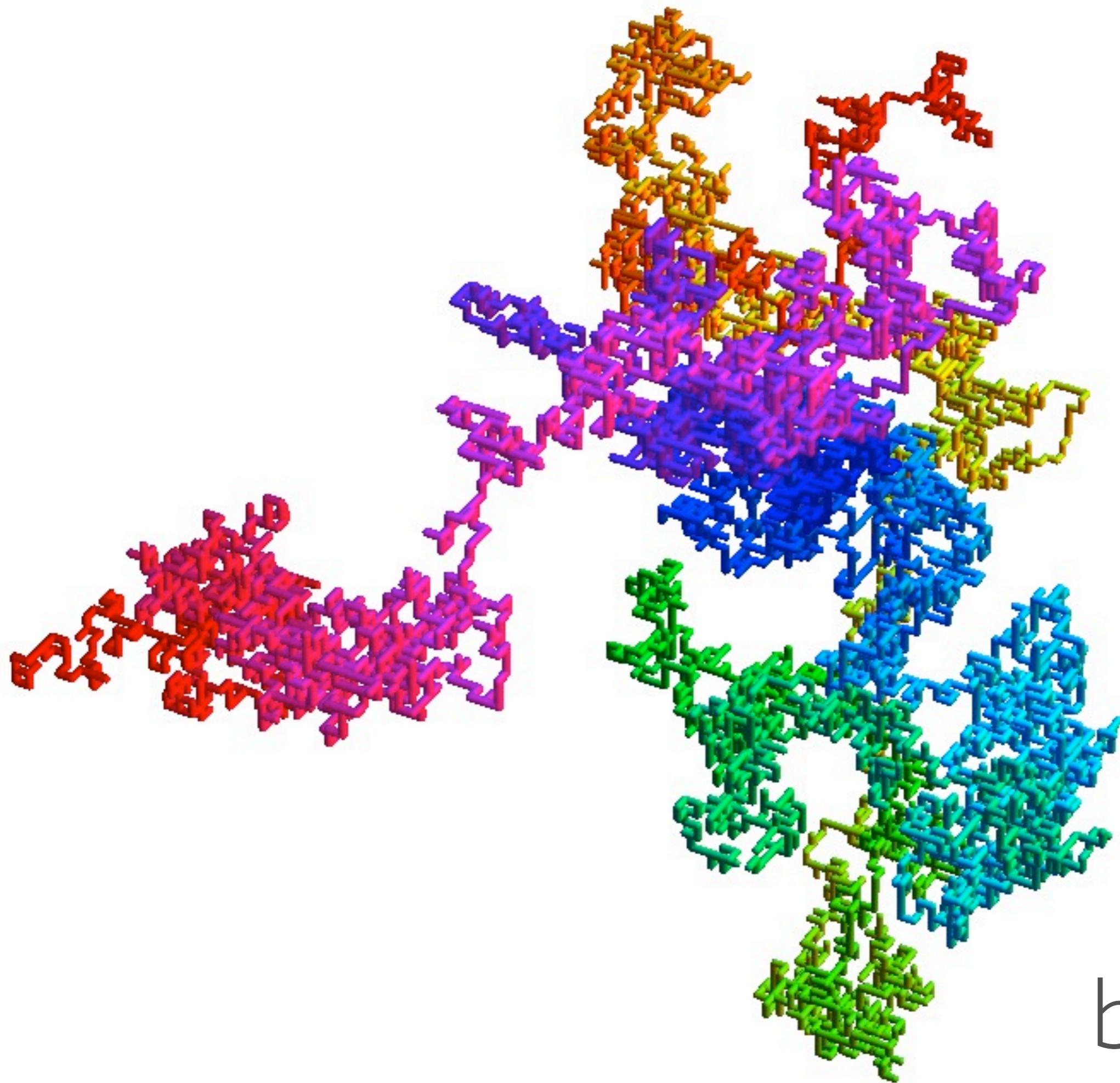


Agora

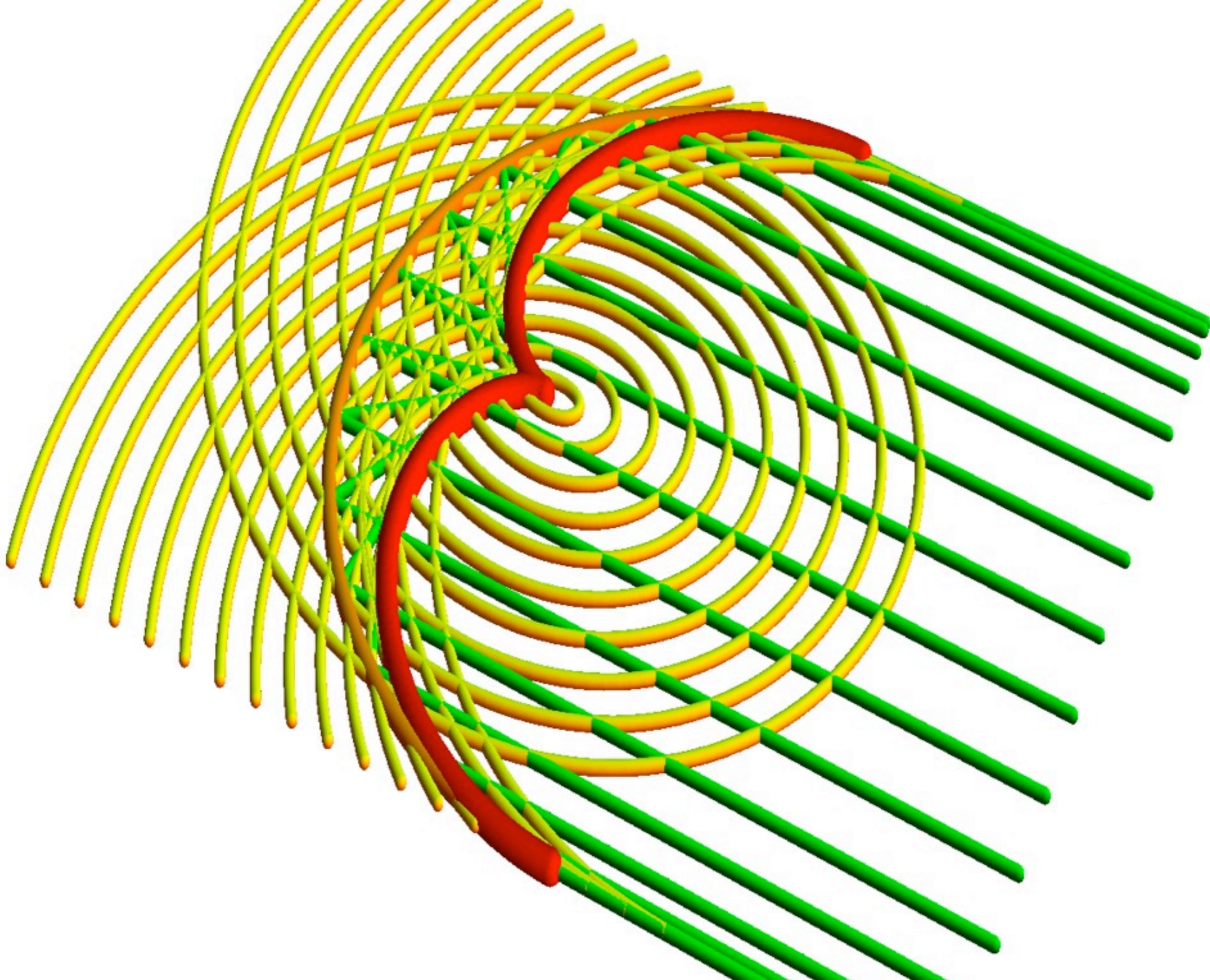


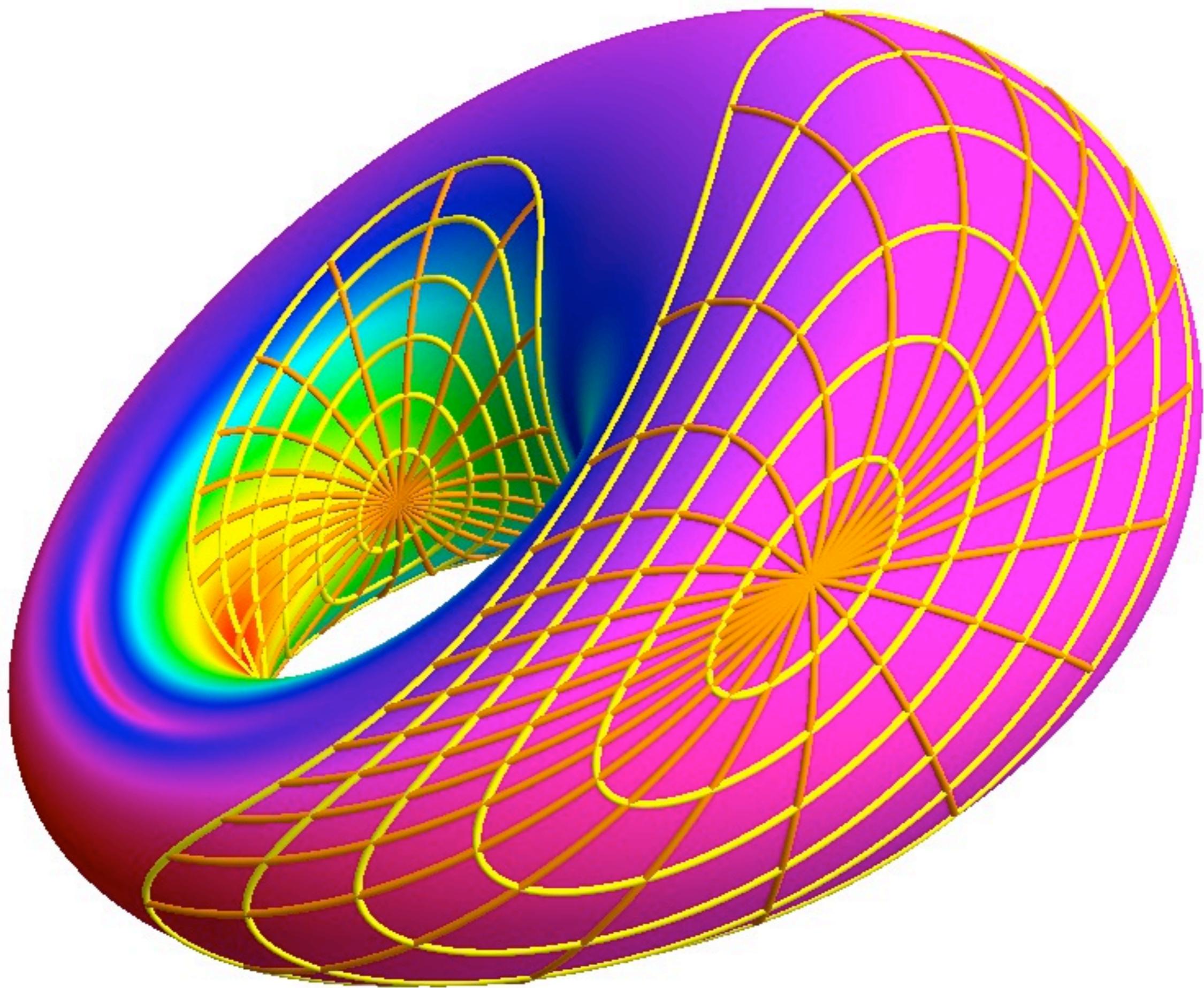


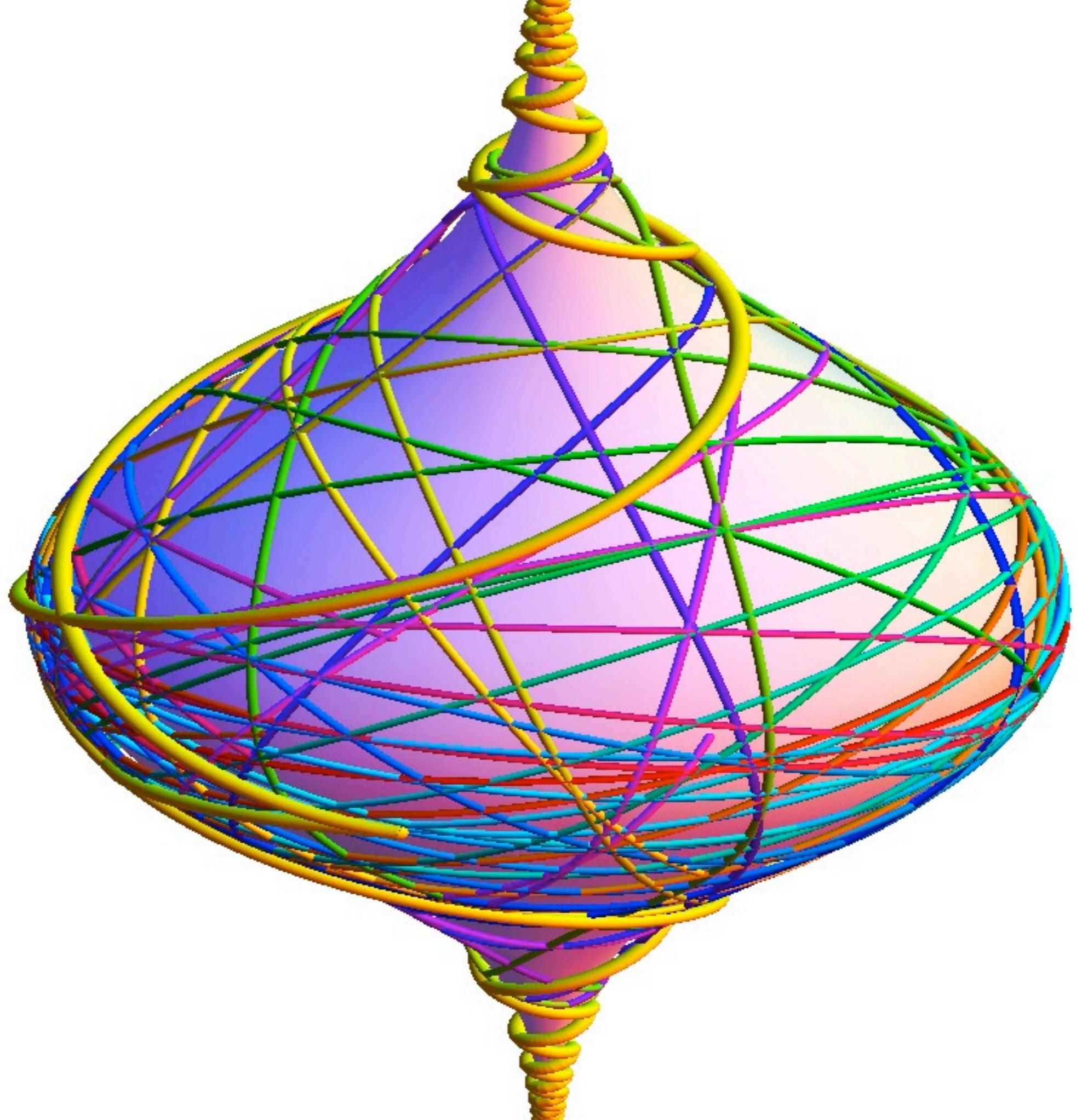
π
base 4

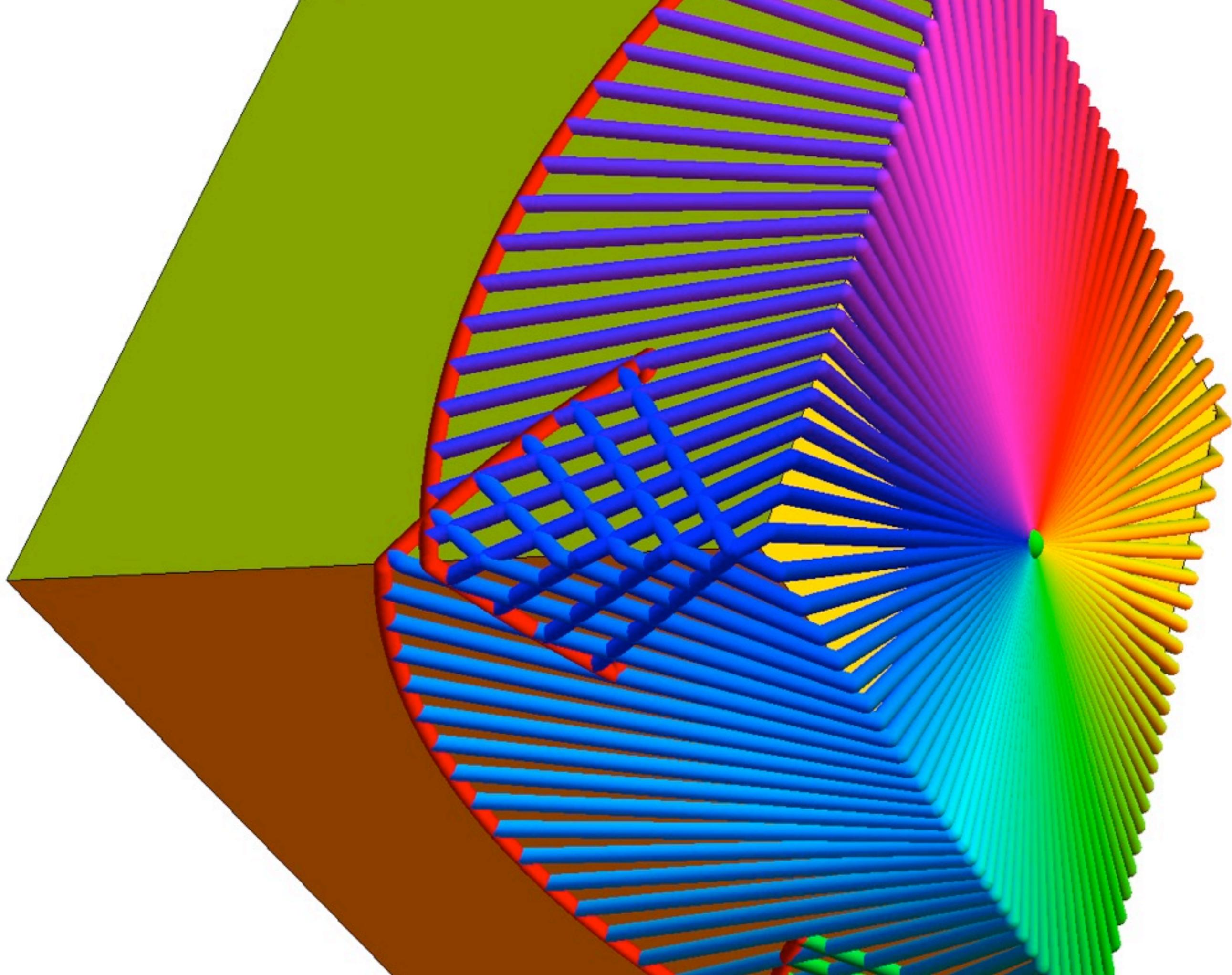


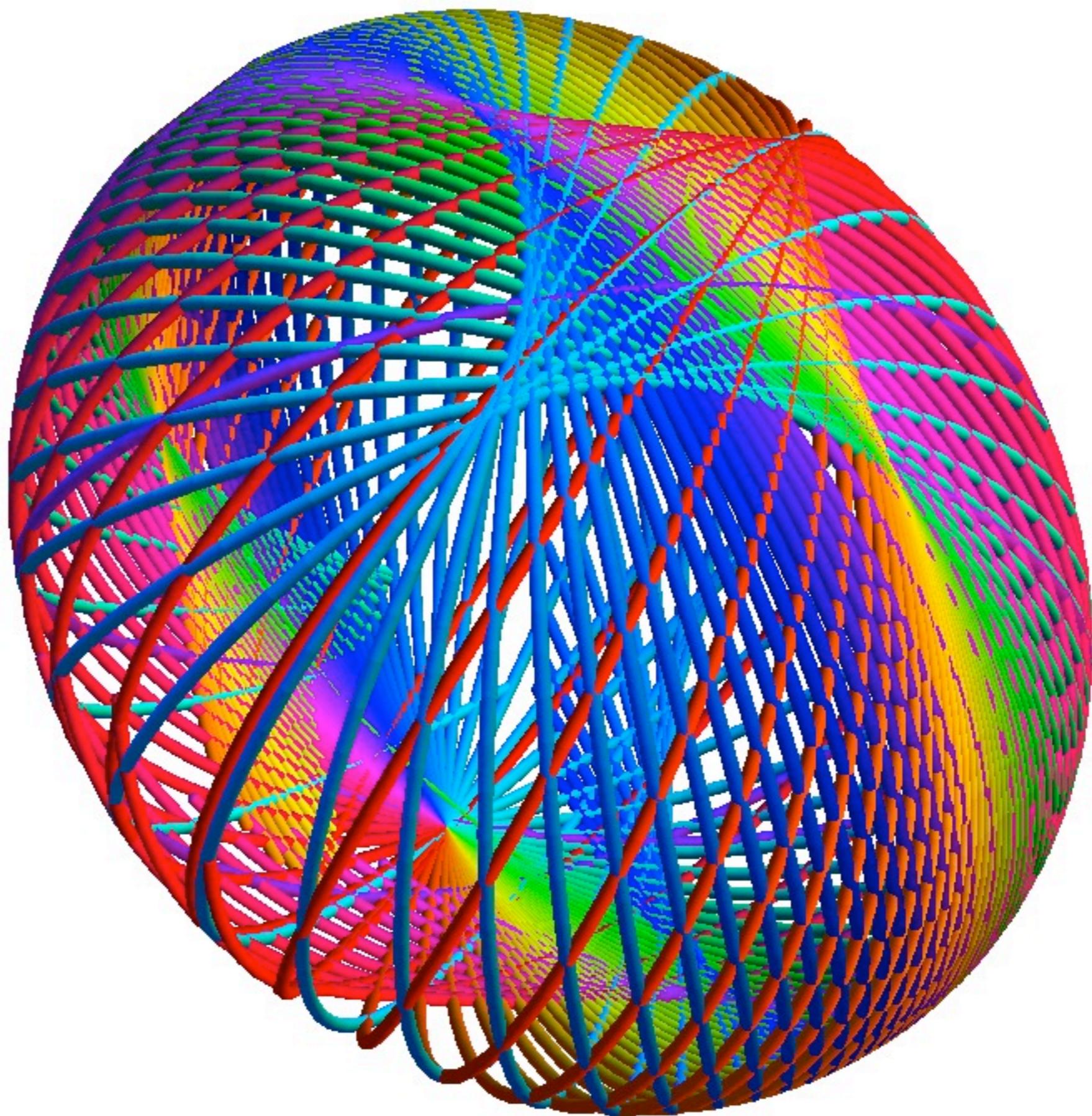
π
base 6



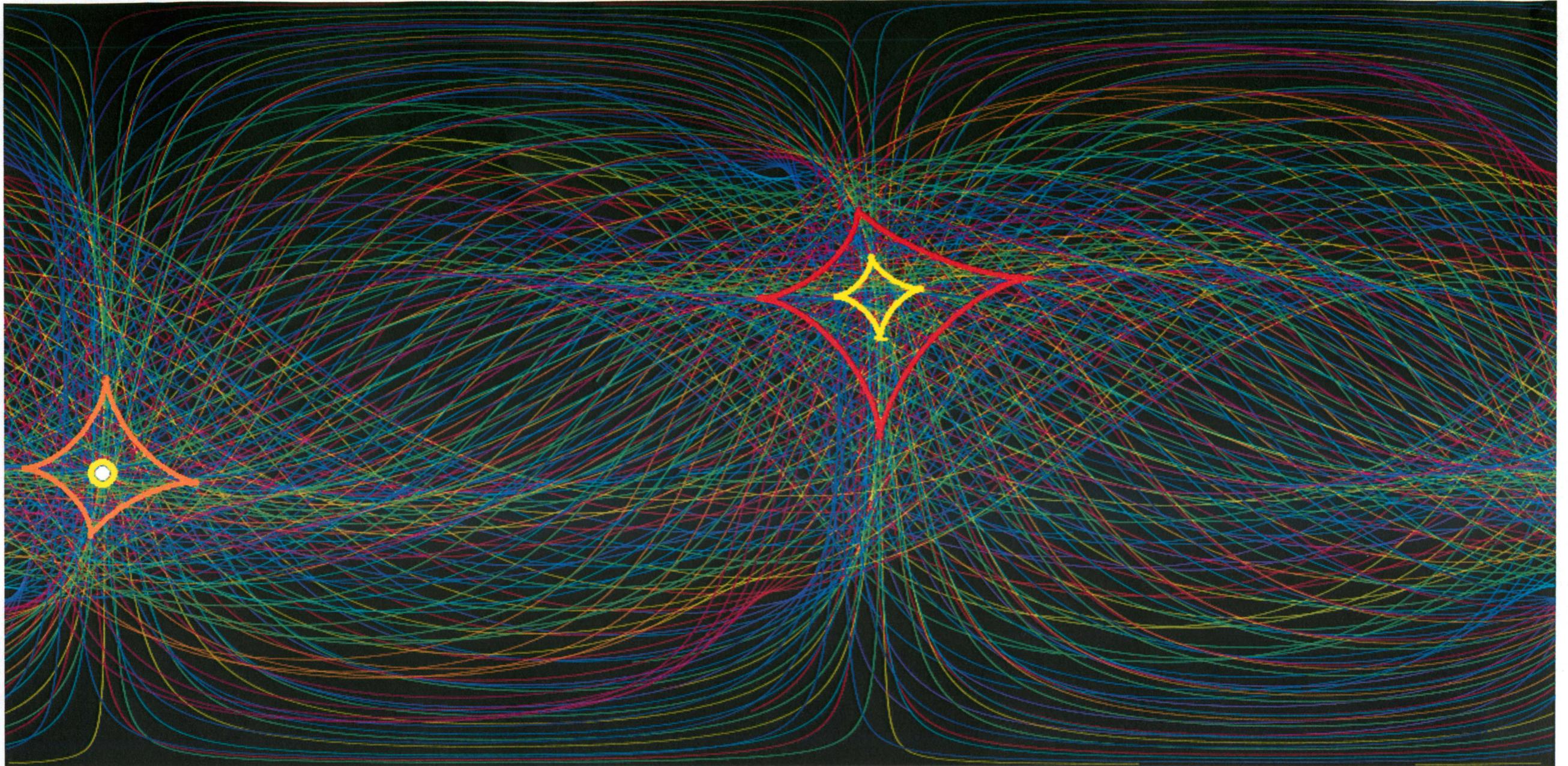








Jacobi last geometric statement

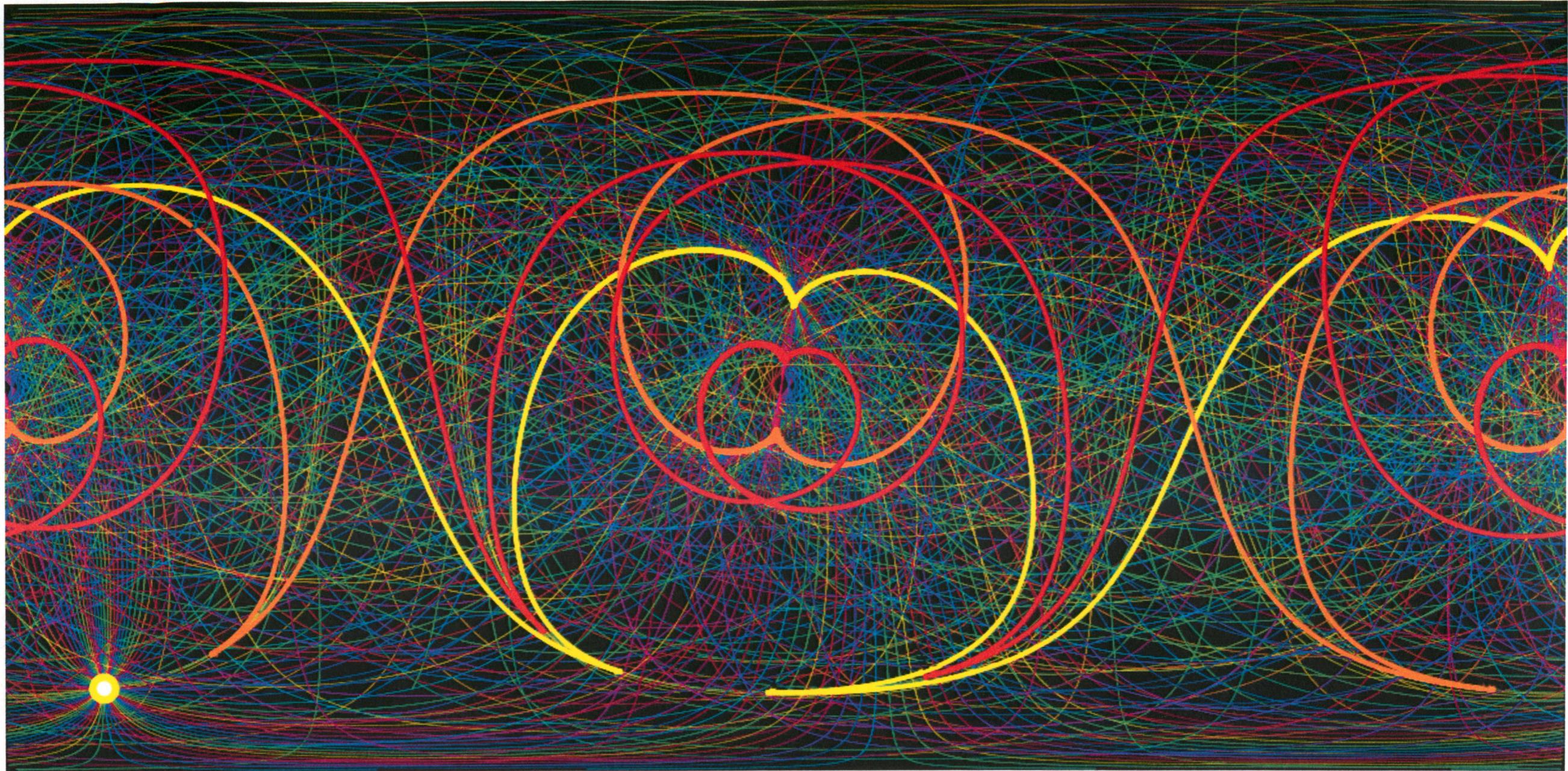


The non-rotationally symmetric ellipsoid

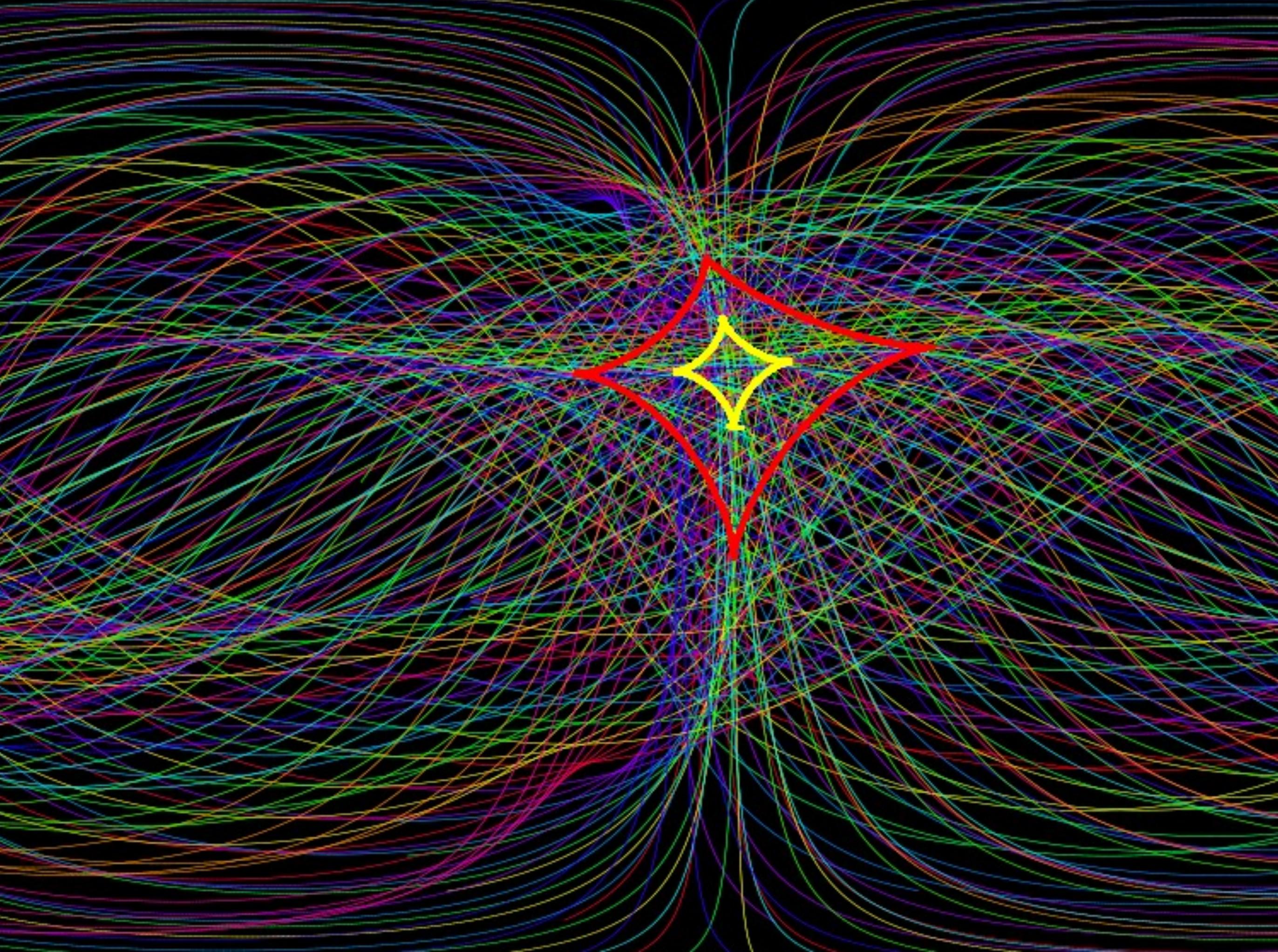
$$\frac{x^2}{1.12} + \frac{y^2}{1.062} + z^2 = 1$$

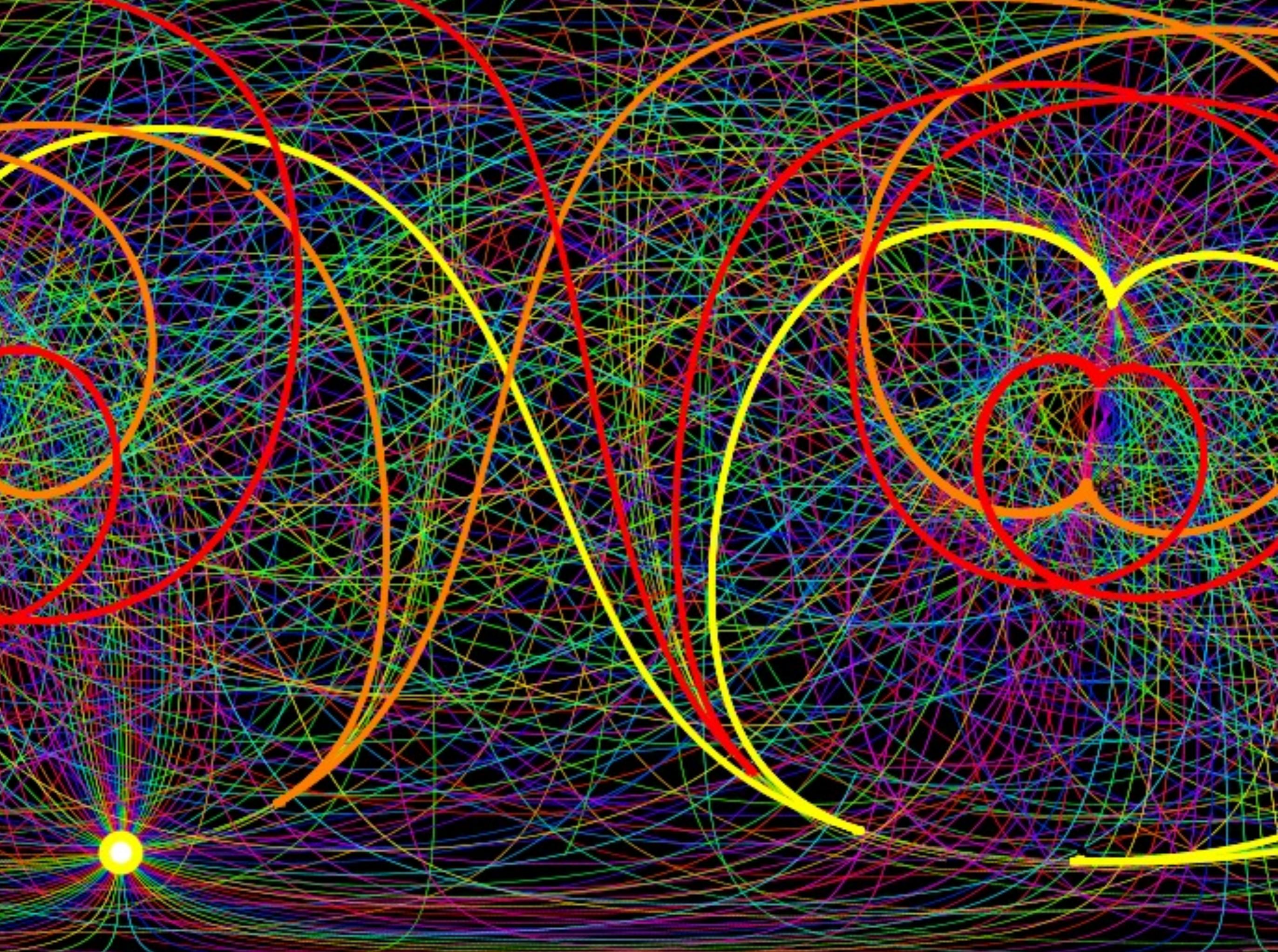
has caustics close to the antipode in the sphere case. We see again the primary, secondary and ternary caustic. 4000 geodesics

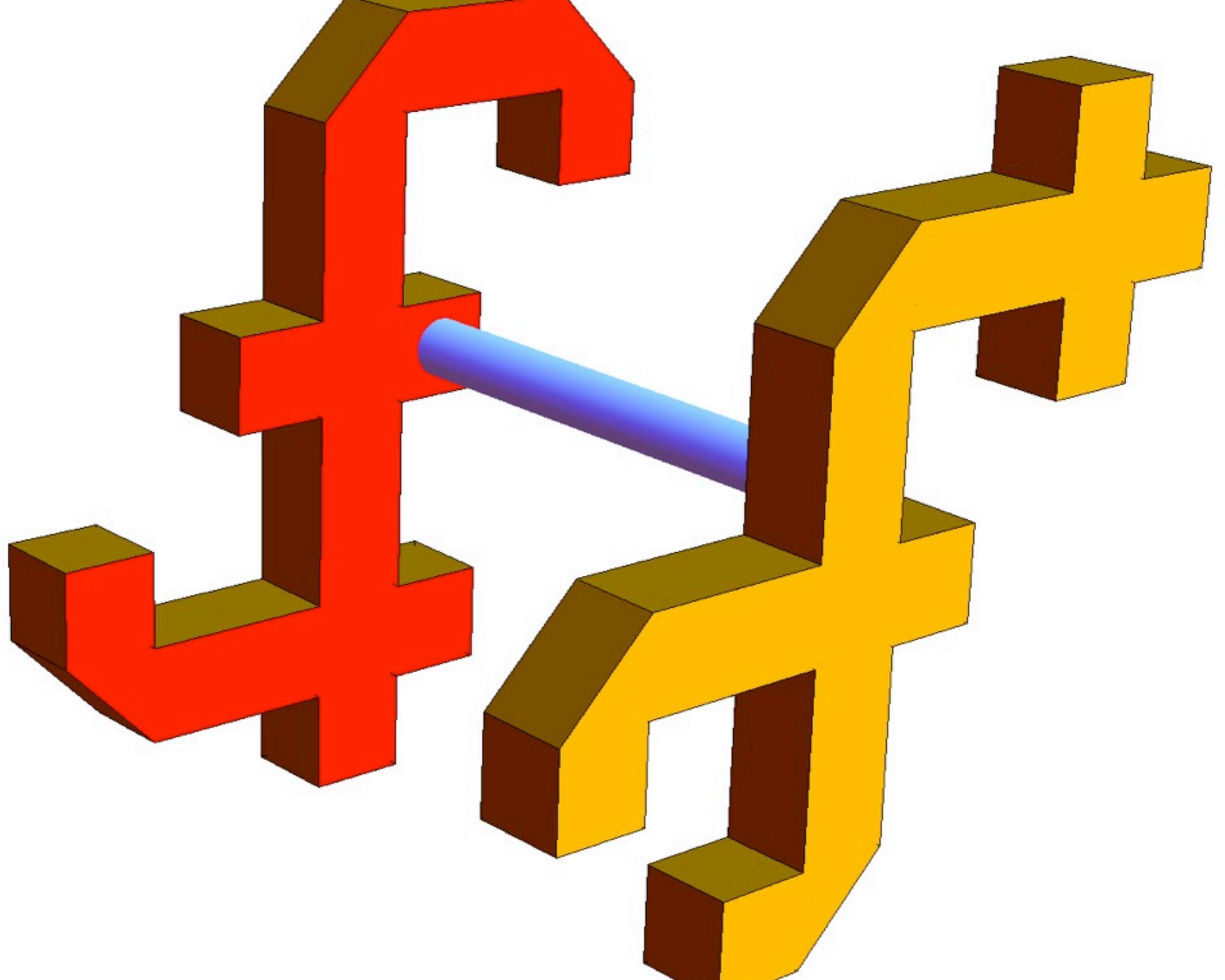
Jacobi last geometric statement

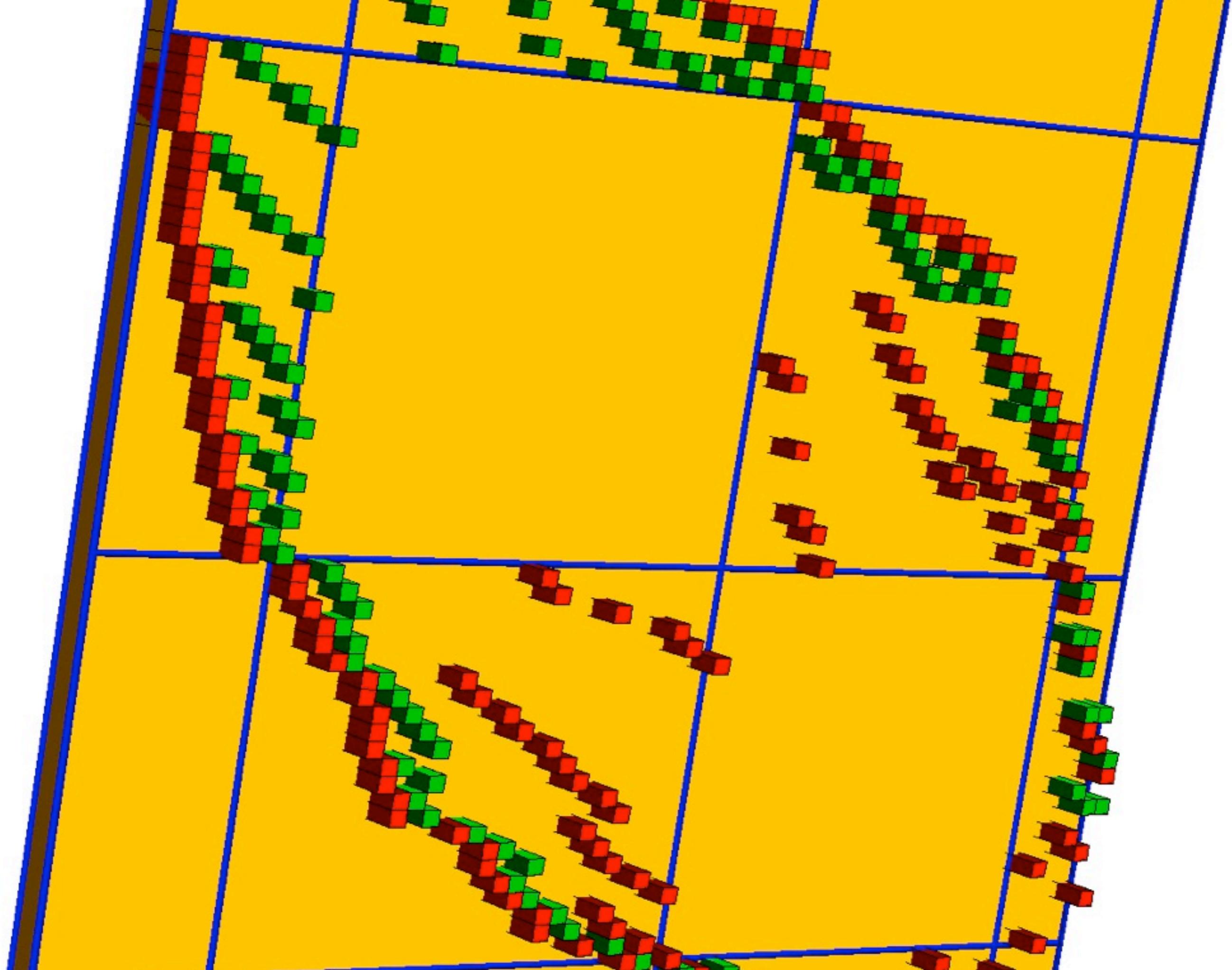


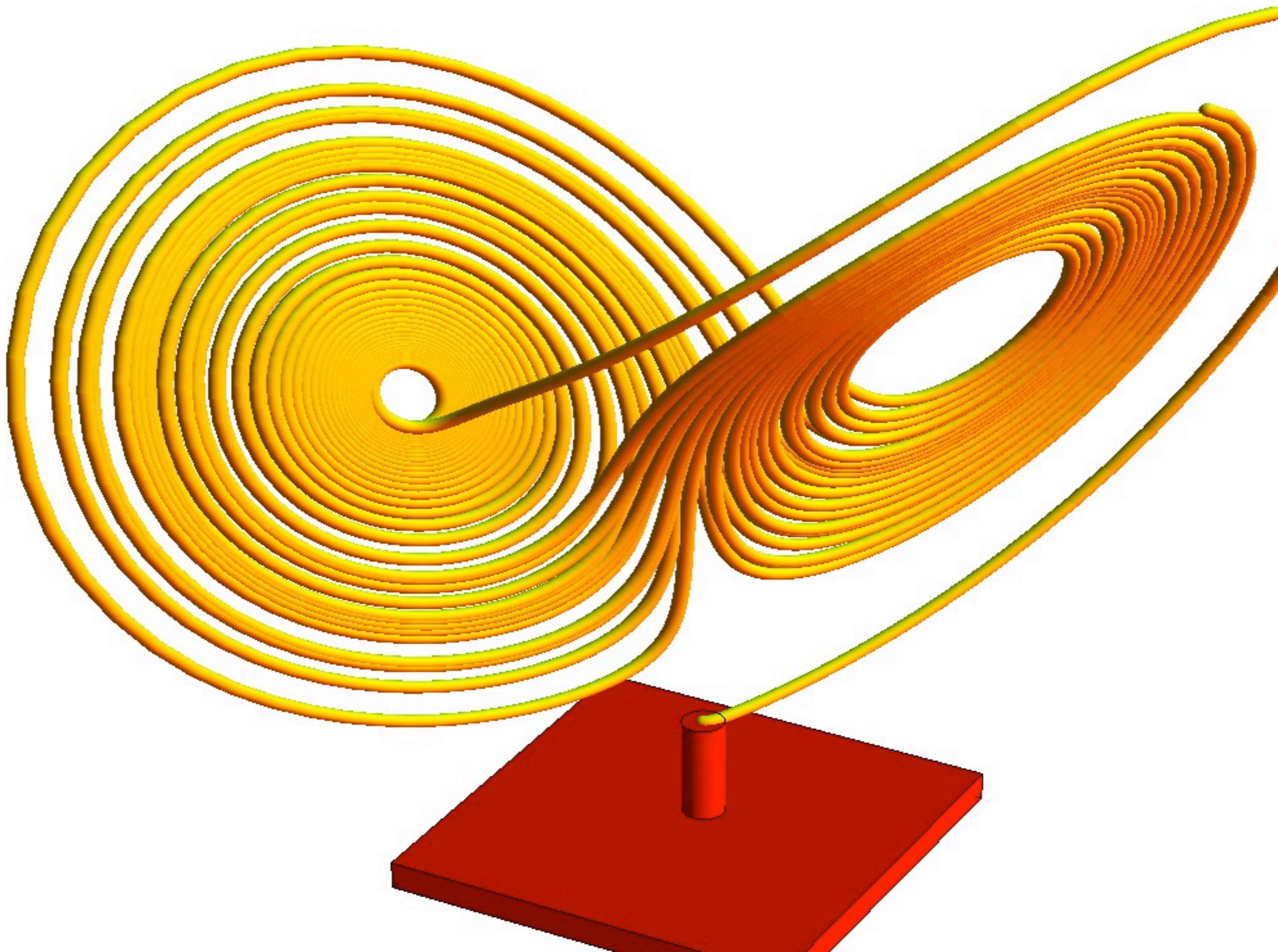
The exponential map on a spheroid $x^2/4+y^2+z^2=1$. The primary caustic (the first root of the Jacobi field f starting at the initial point) is drawn in yellow, the secondary caustic (the second root of $f(g(t))$) orange, the tertiary red. One clearly sees the 4 cusps as the still unproven Jacobi's last geometric statement claims. 100 geodesics of the 6000 computed geodesics have been drawn. The picture was computed by solving the geodesic equations $g''^k = -G(i,j,k) g'^i g'^j$ (where G is the connection using Einstein notation) in conjunction with the Gauss-Jacobi equation $f' = -K(g(t)) f$ (where K is the curvature of the surface) numerically with Mathematica. Since special needs are required (identifications of the map, assuring that we stay on the energy surface, checking whether the Jacobi field f reaches zero), the differential equations were "hand" integrated using Runge-Kutta and not using built-in DSolve routines. 6000 geodesics $g(t)$ were computed on the ellipsoid and drawn in the spherical coordinate plane with (θ, ϕ) coordinate

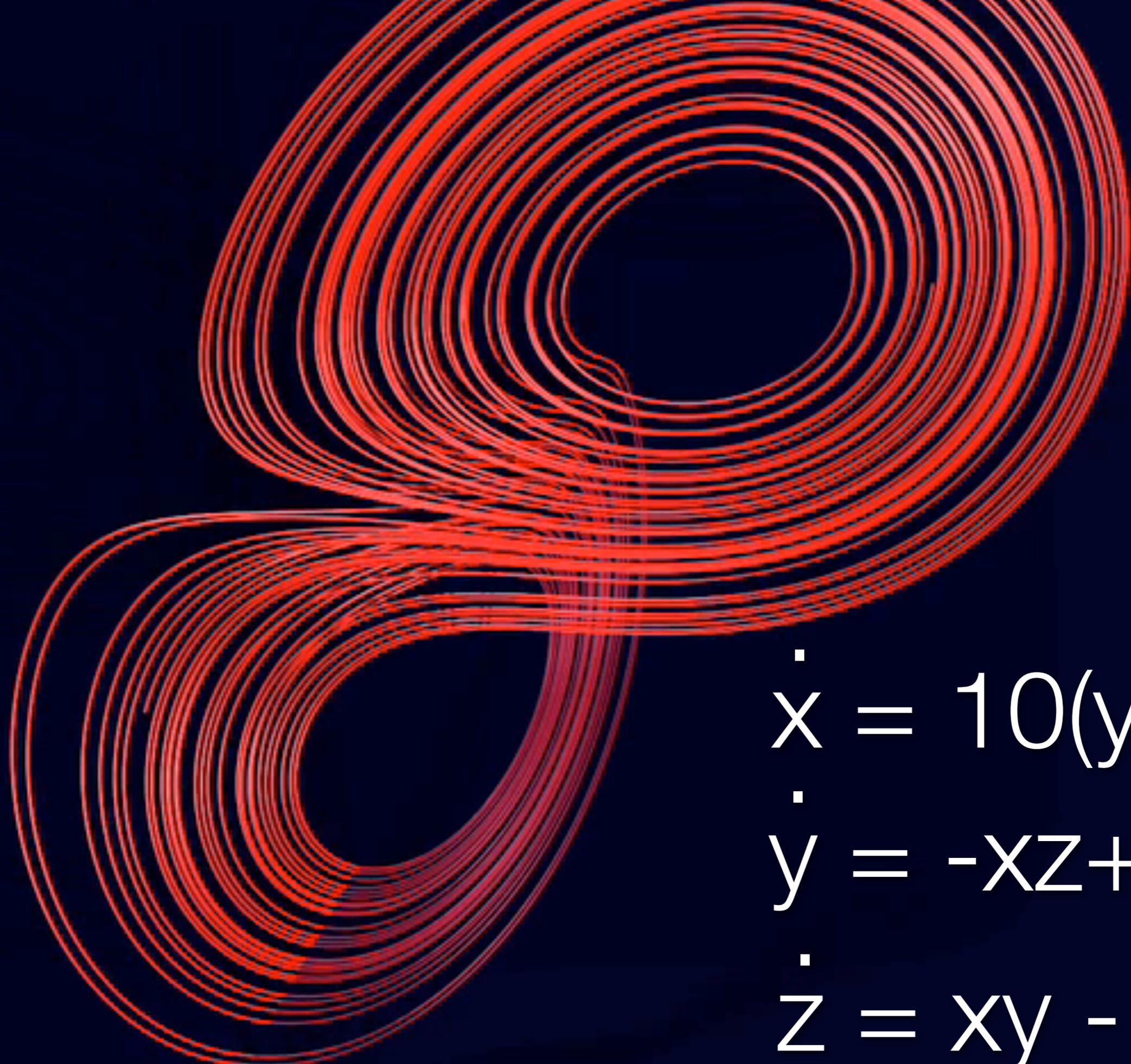








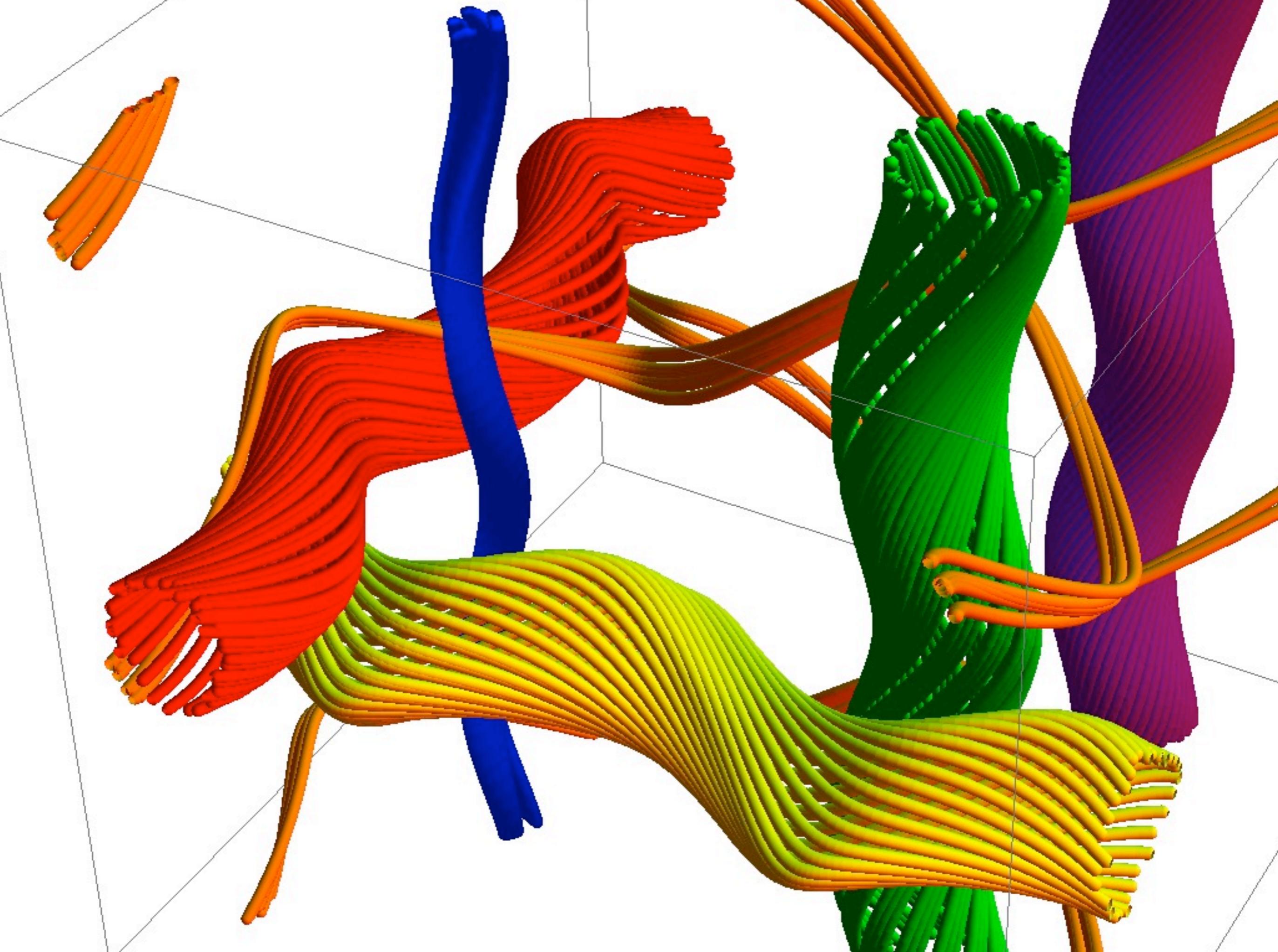


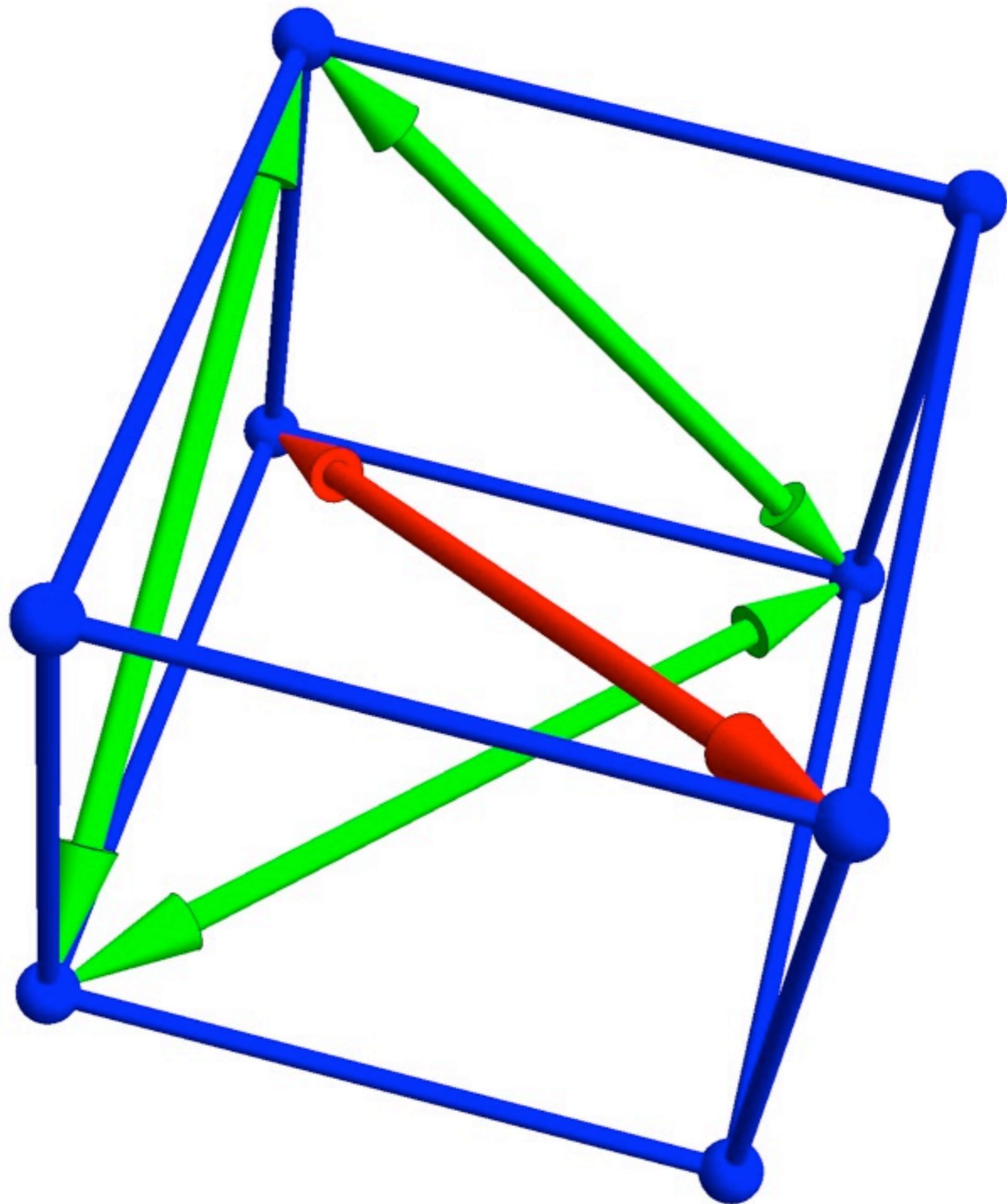


$$\dot{x} = 10(y-x)$$

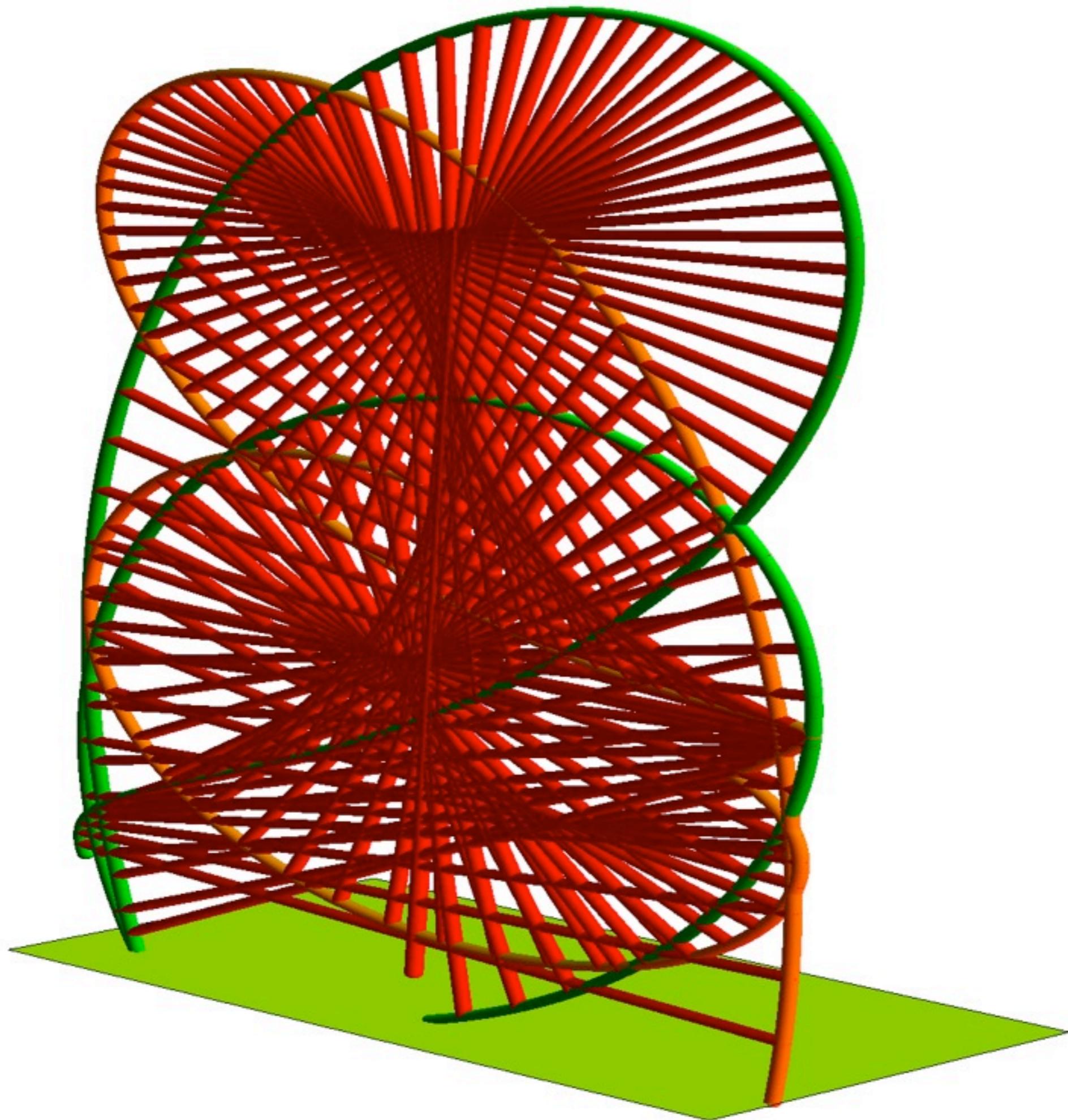
$$\dot{y} = -xz + 28x - y$$

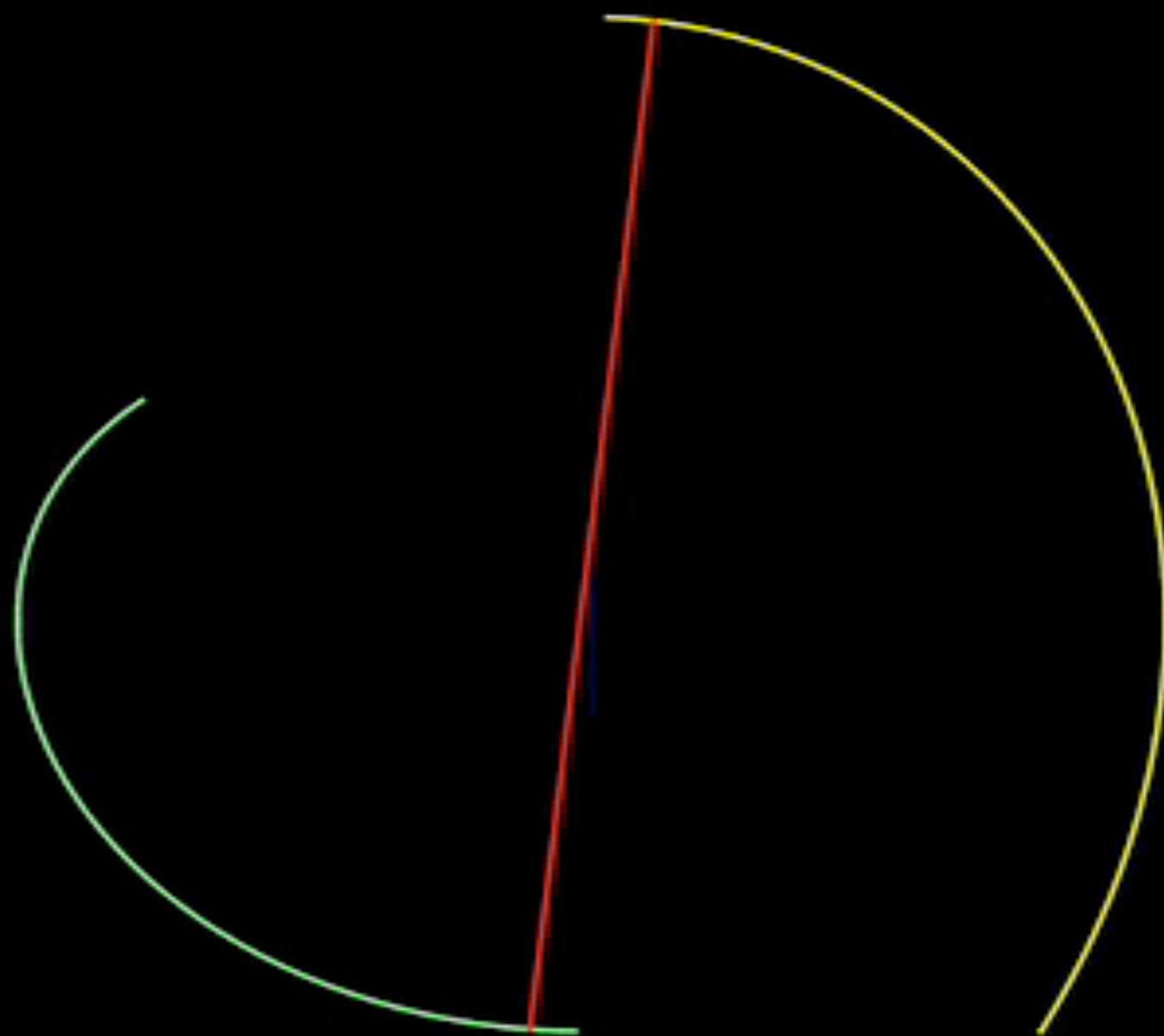
$$\dot{z} = xy - z \frac{8}{3}$$

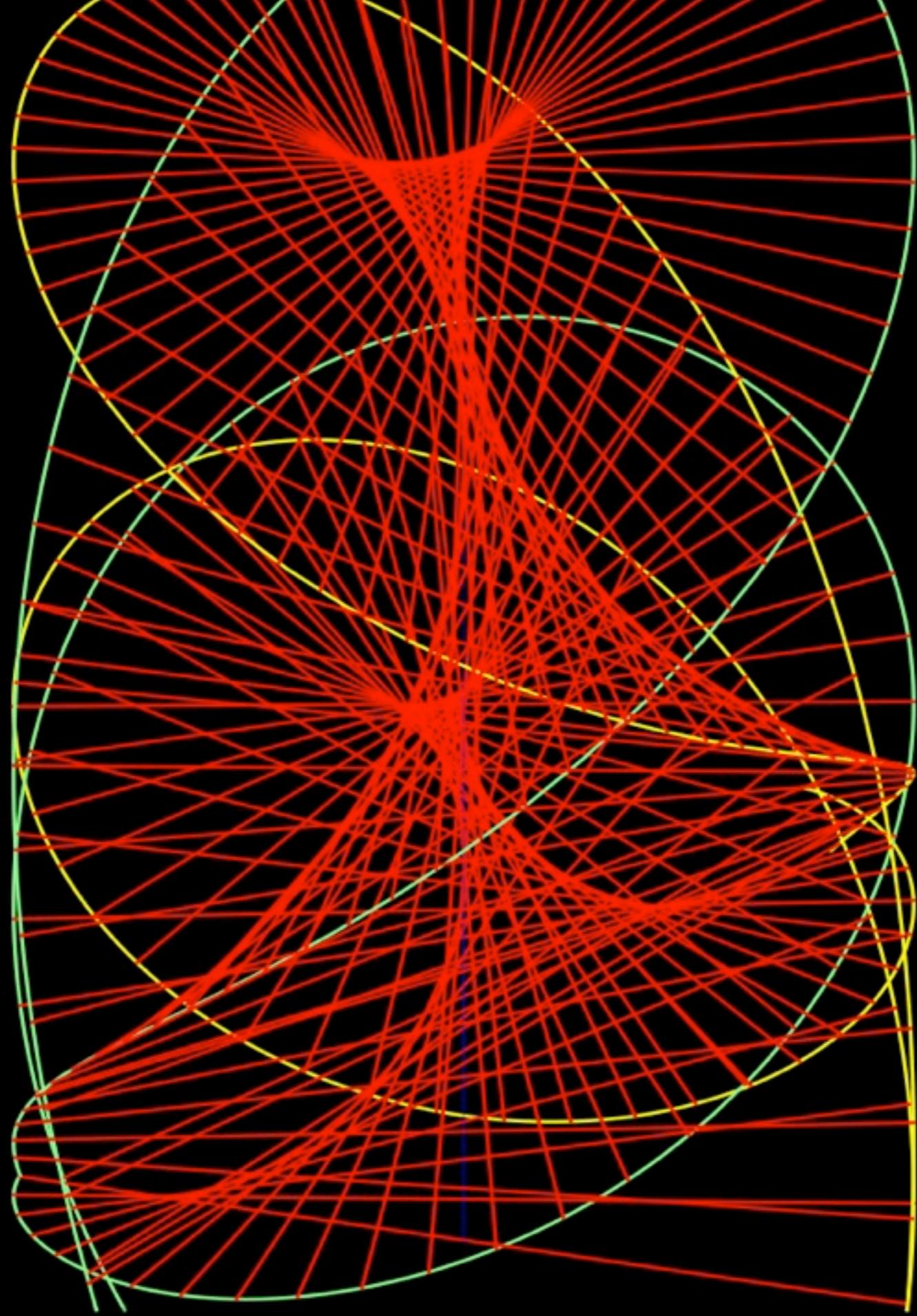


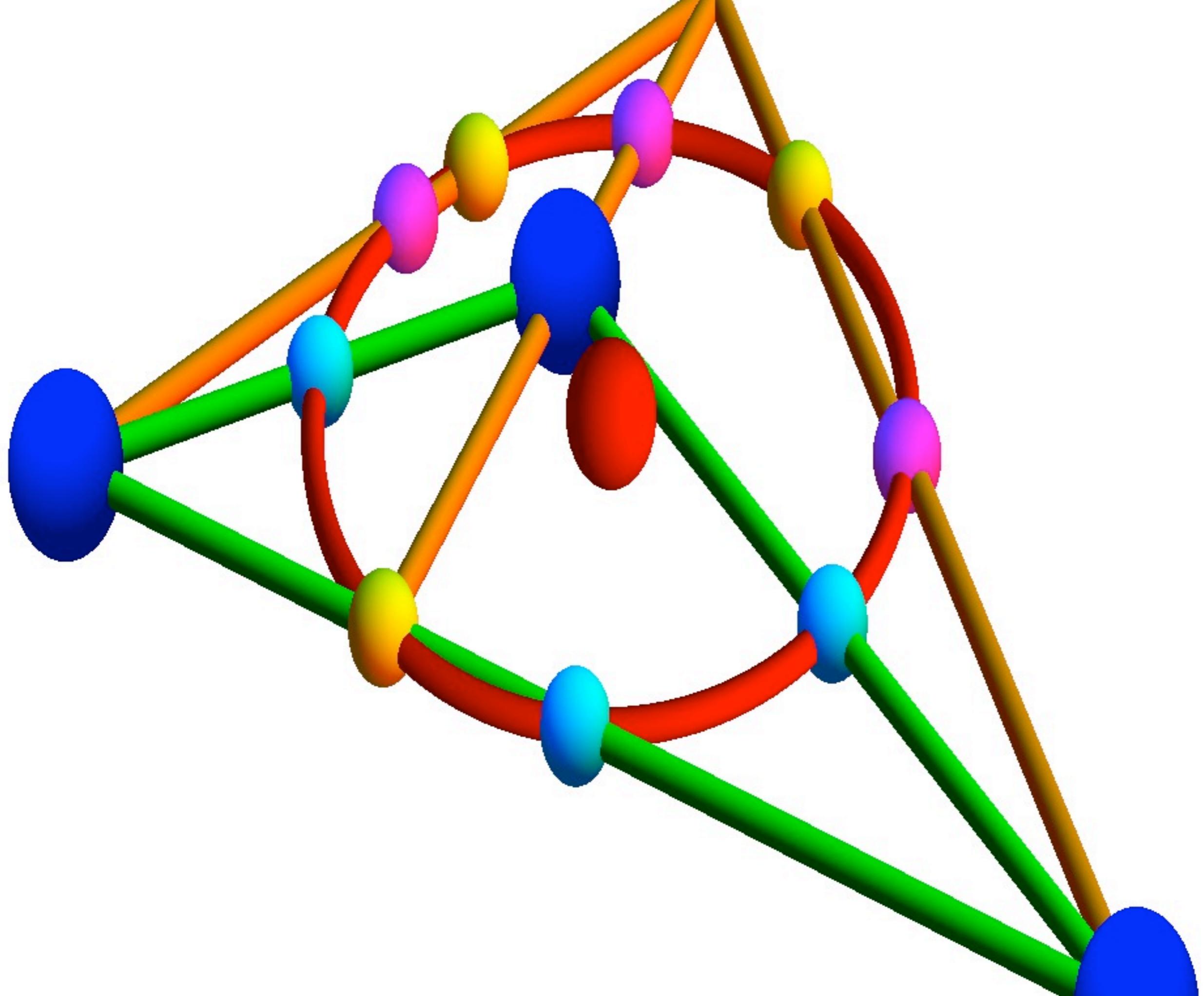


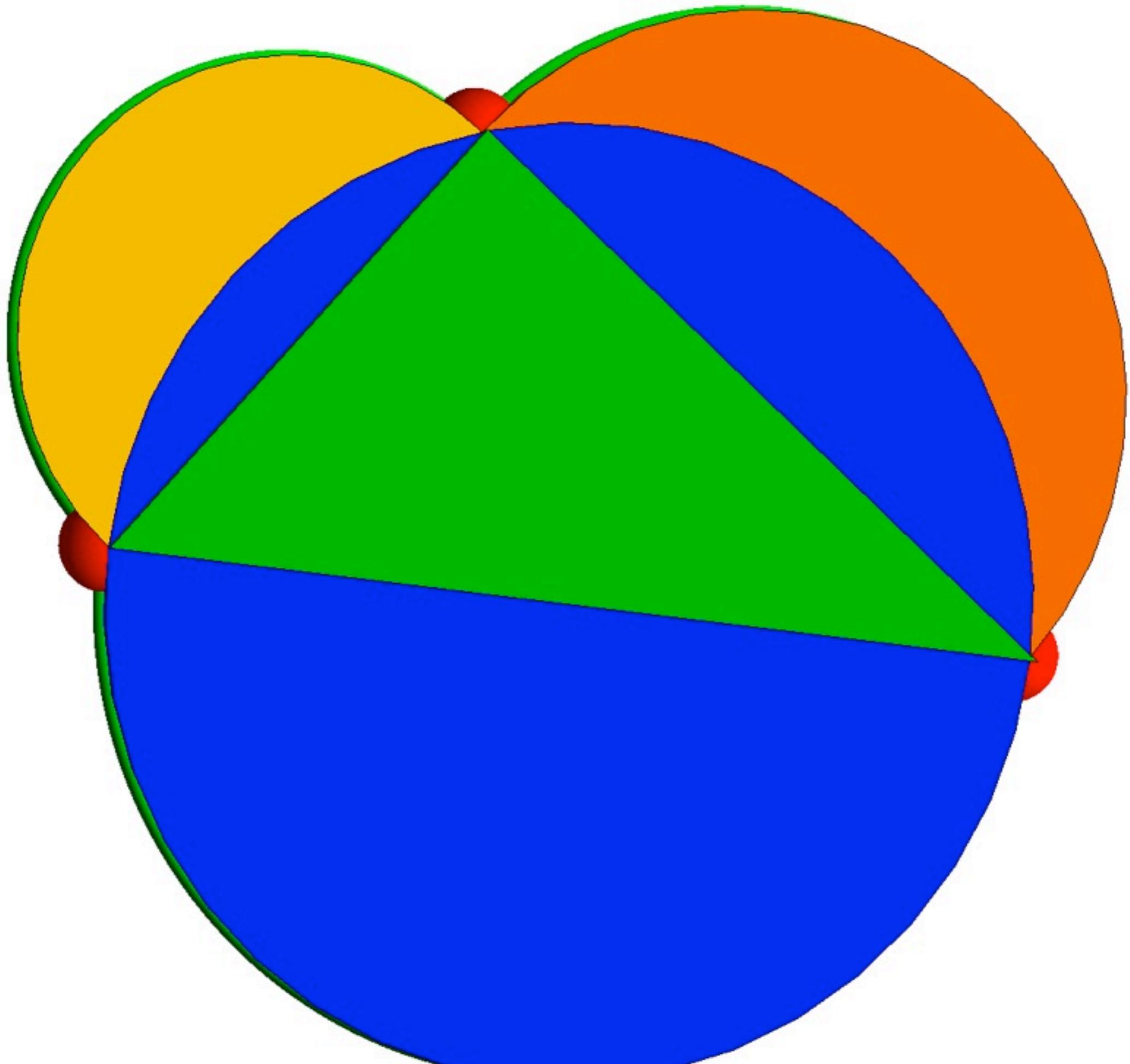


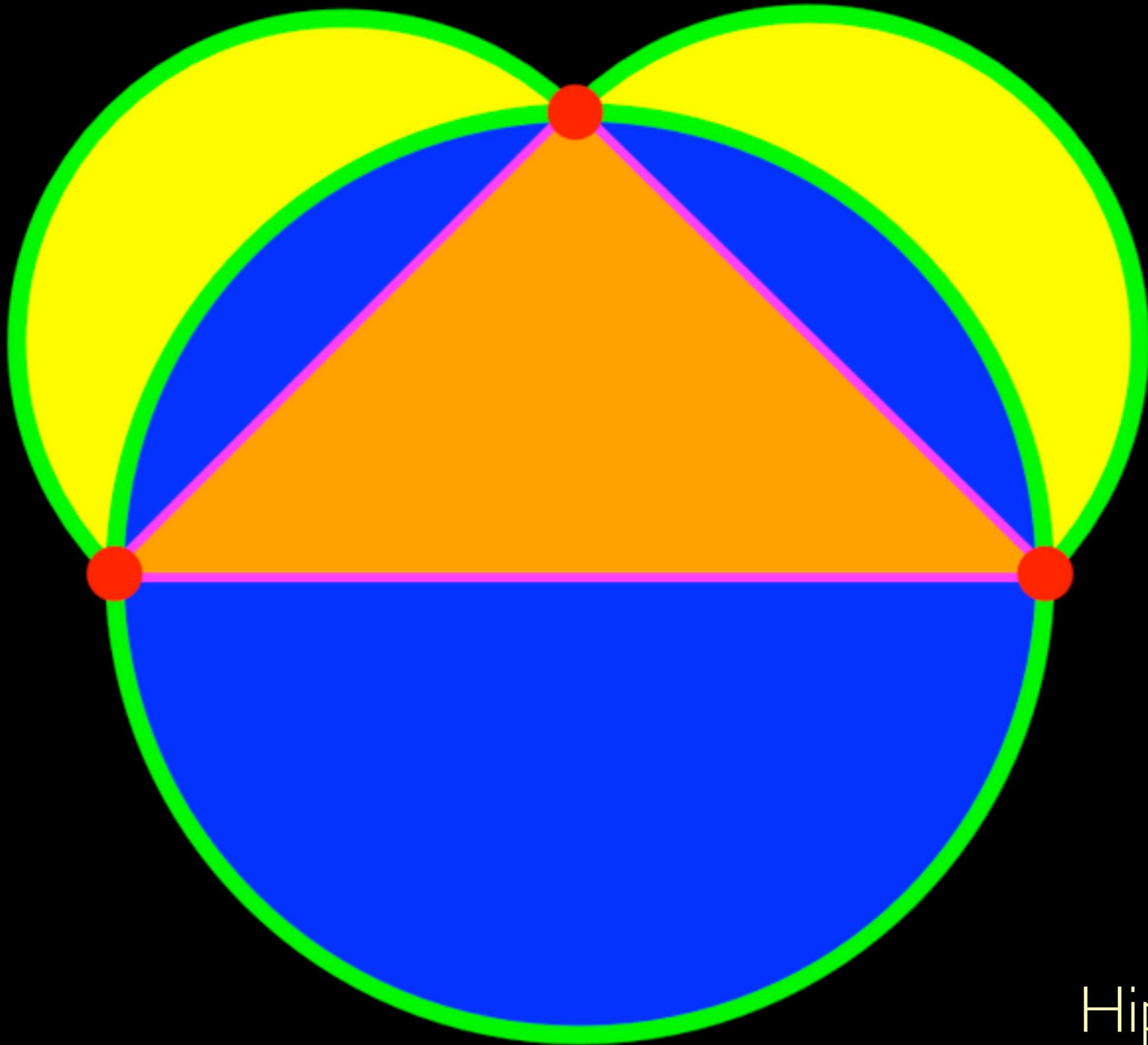




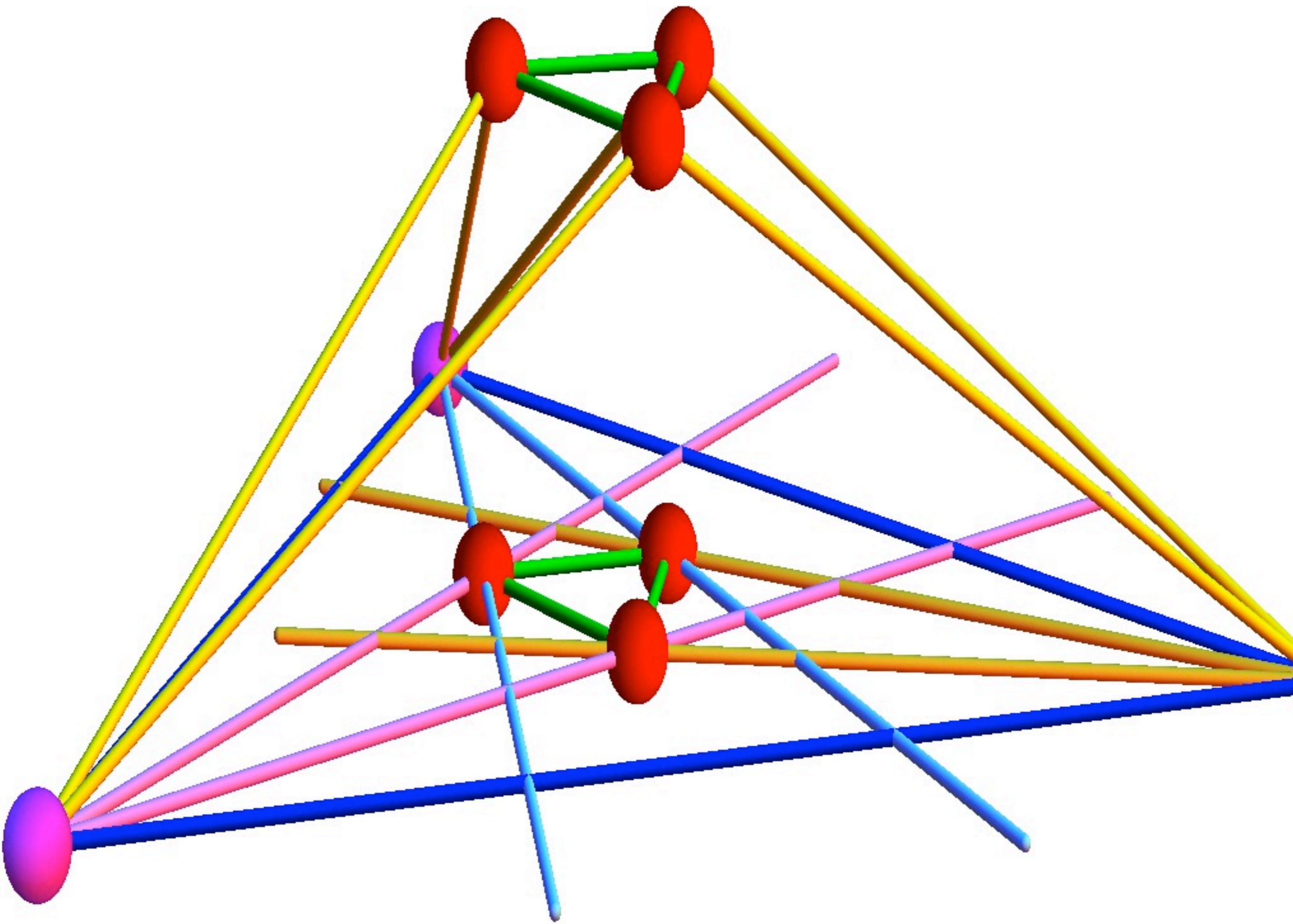


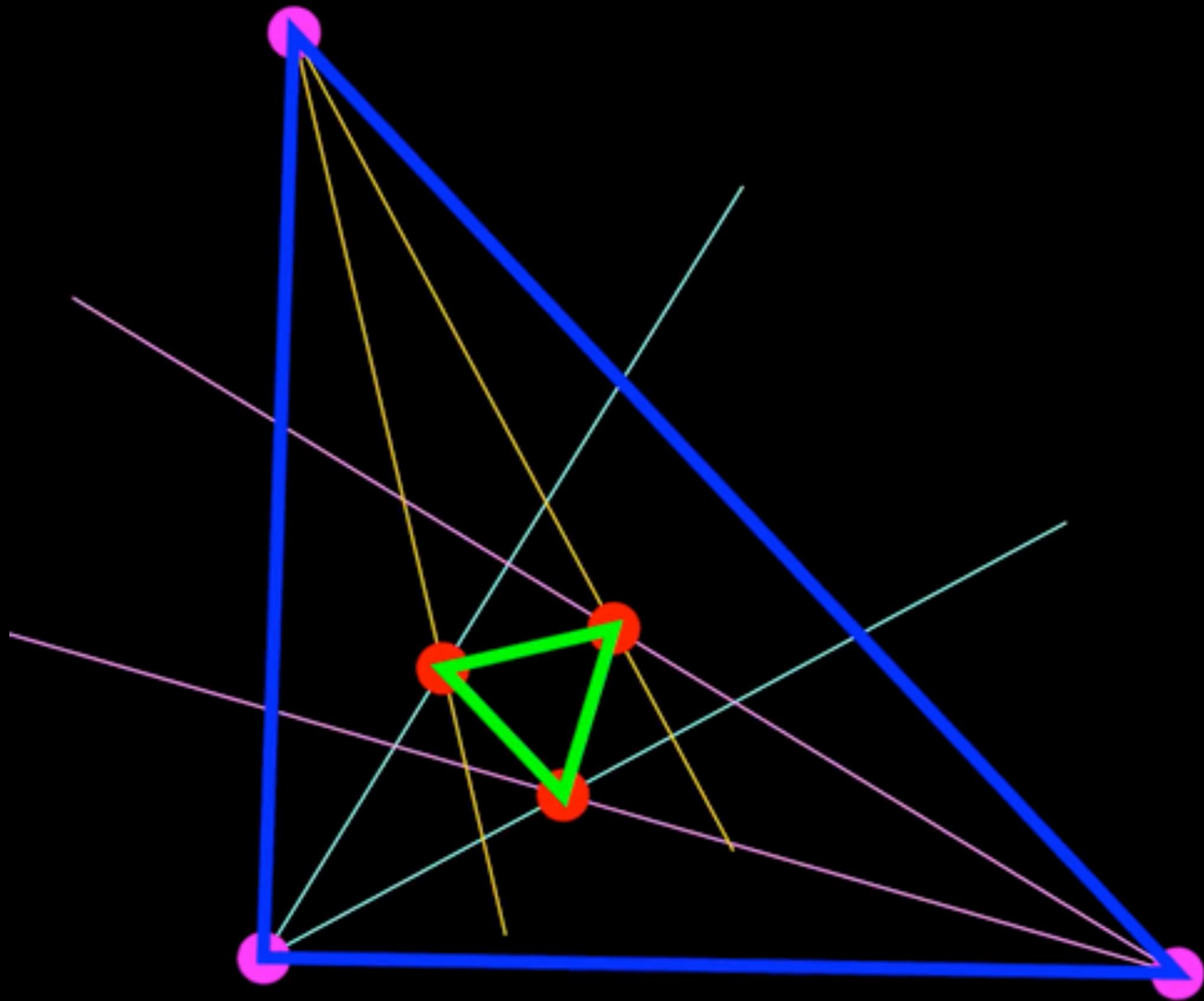


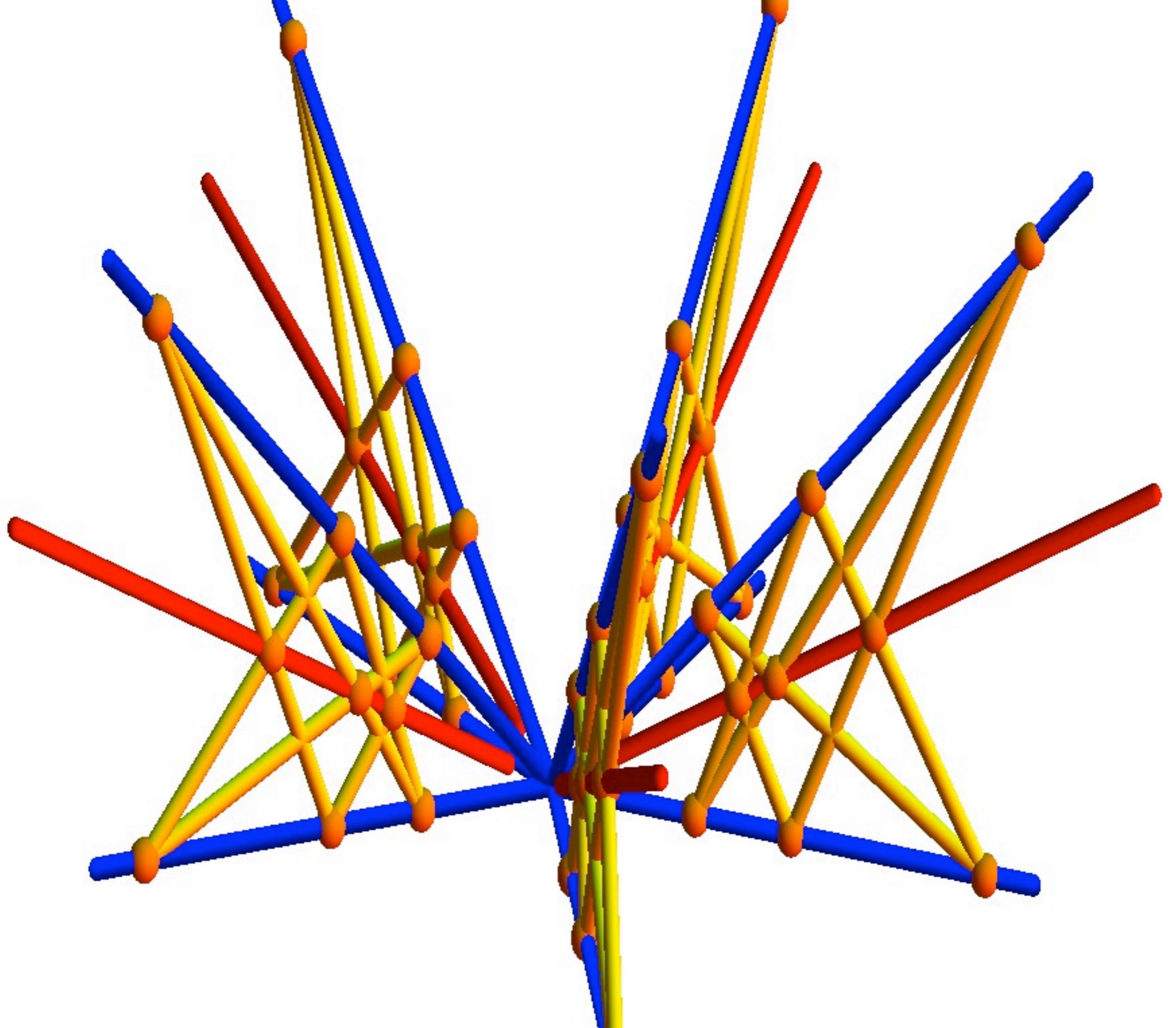


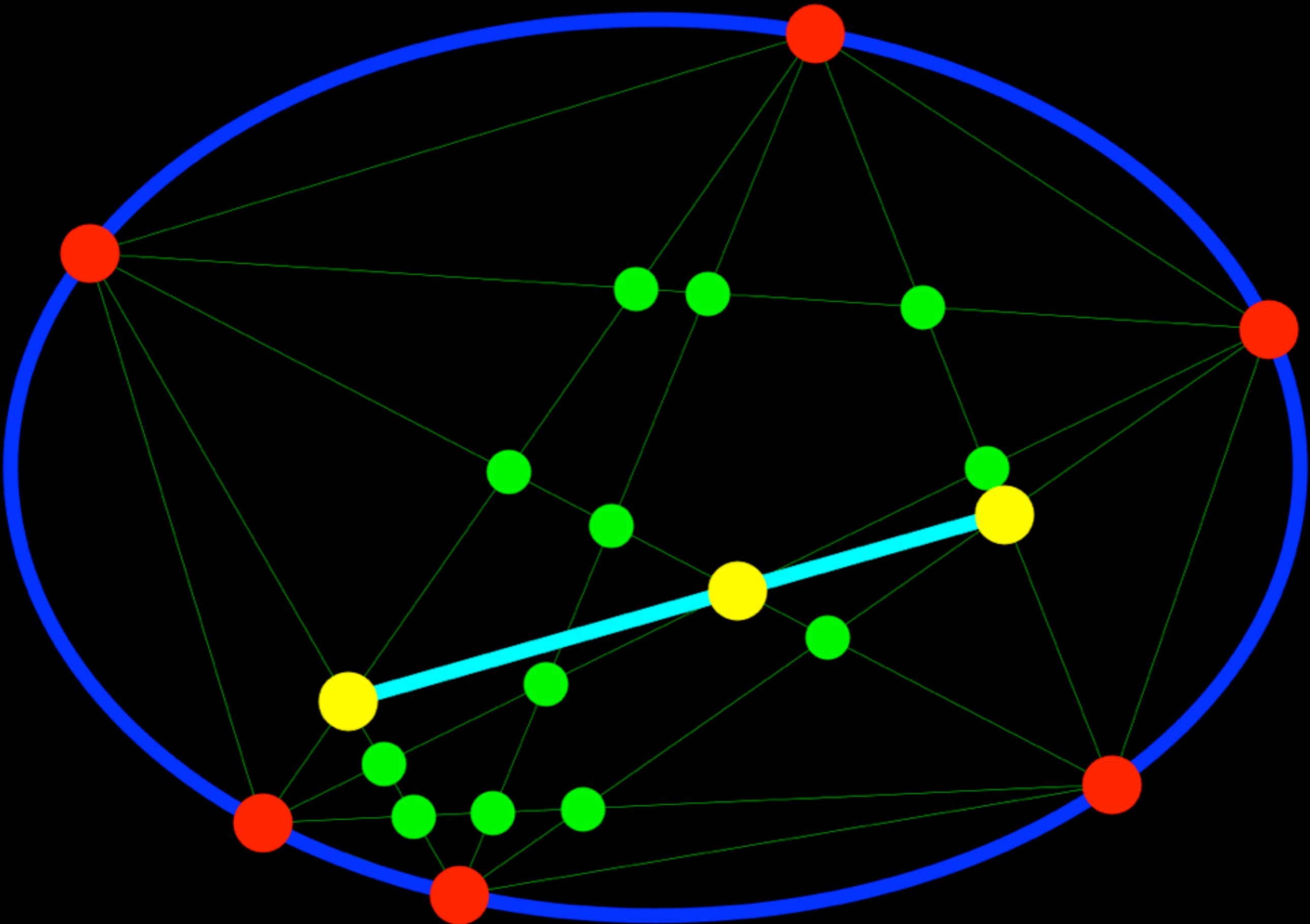


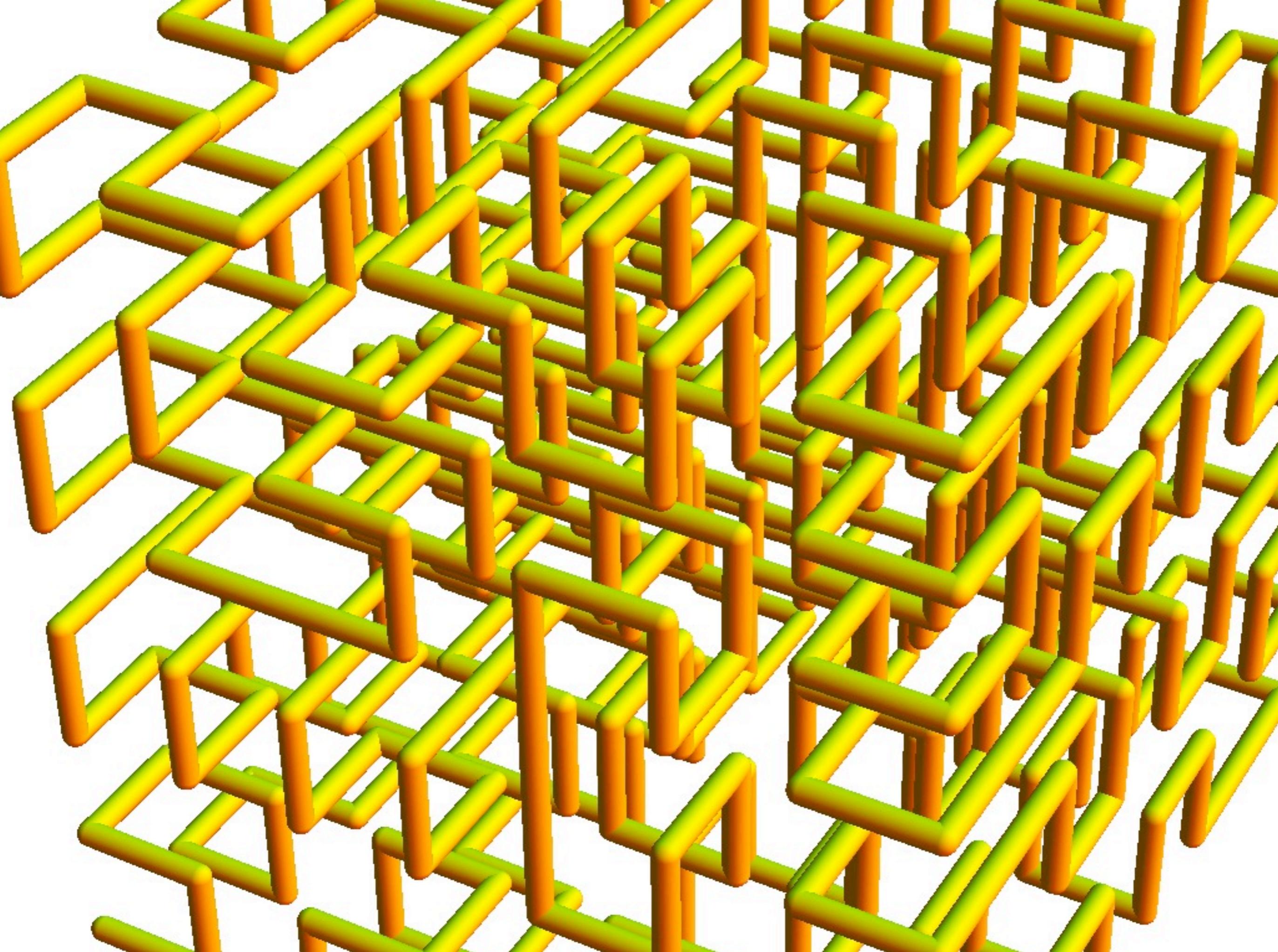
Hipocrates

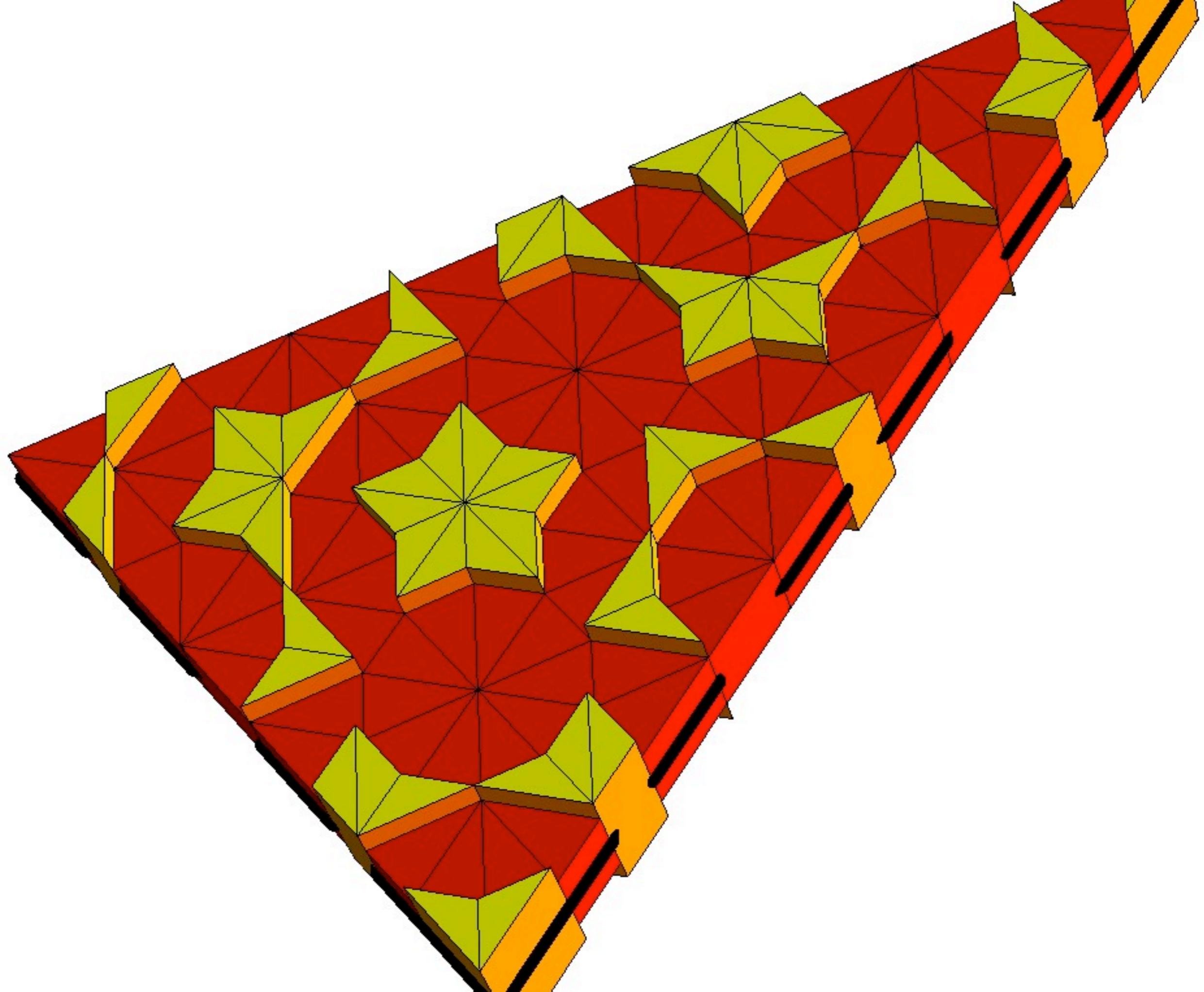


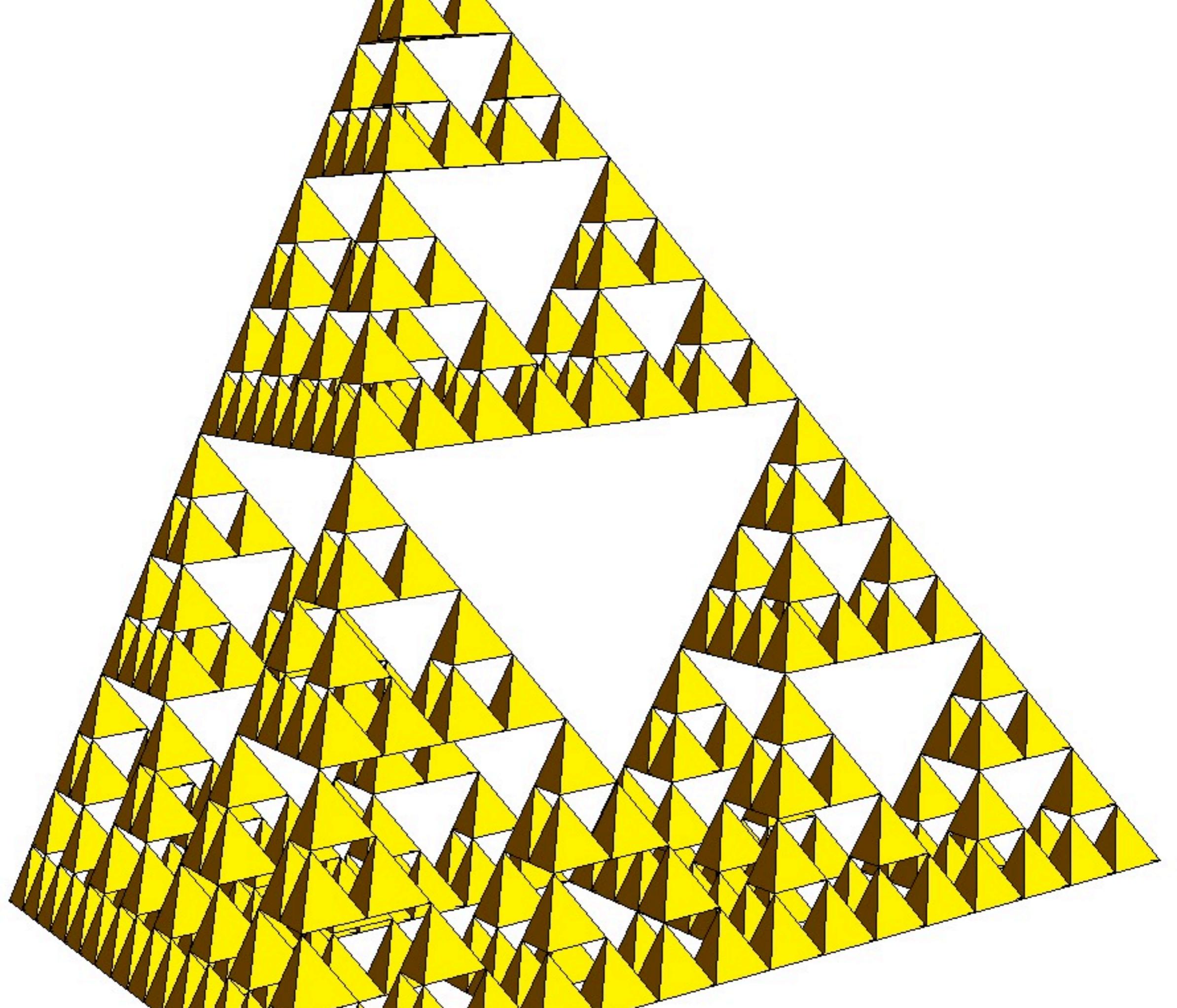


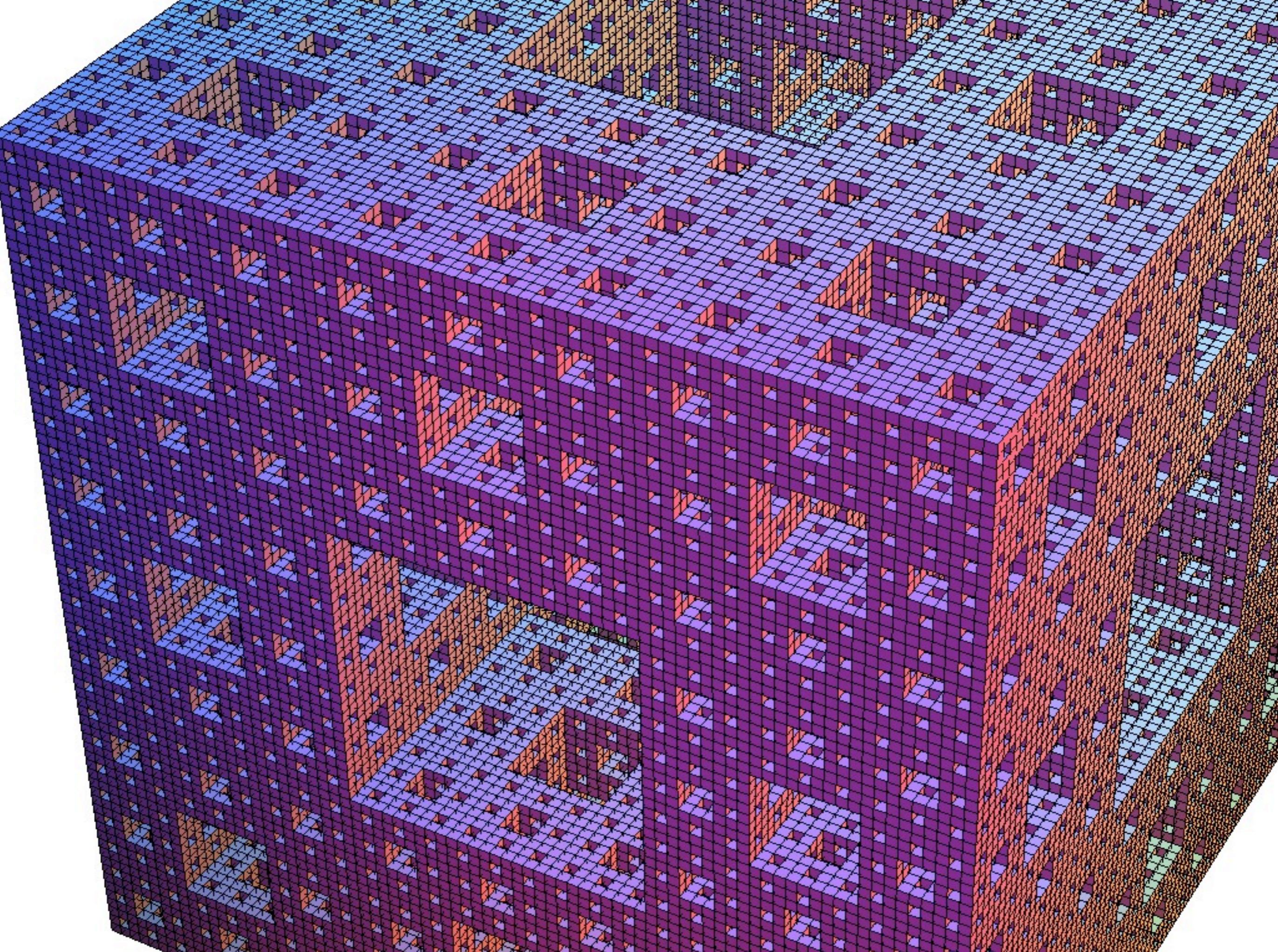


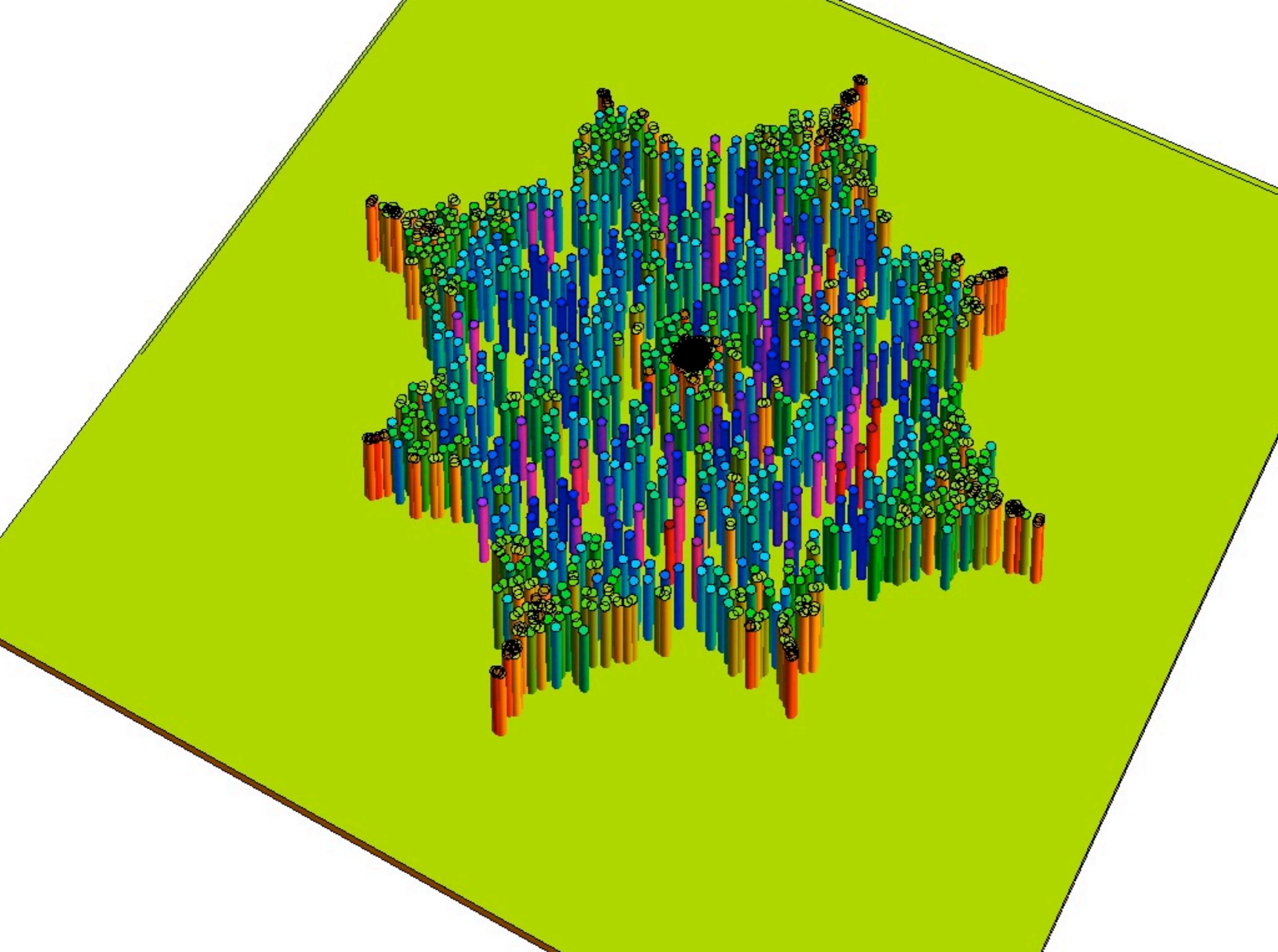


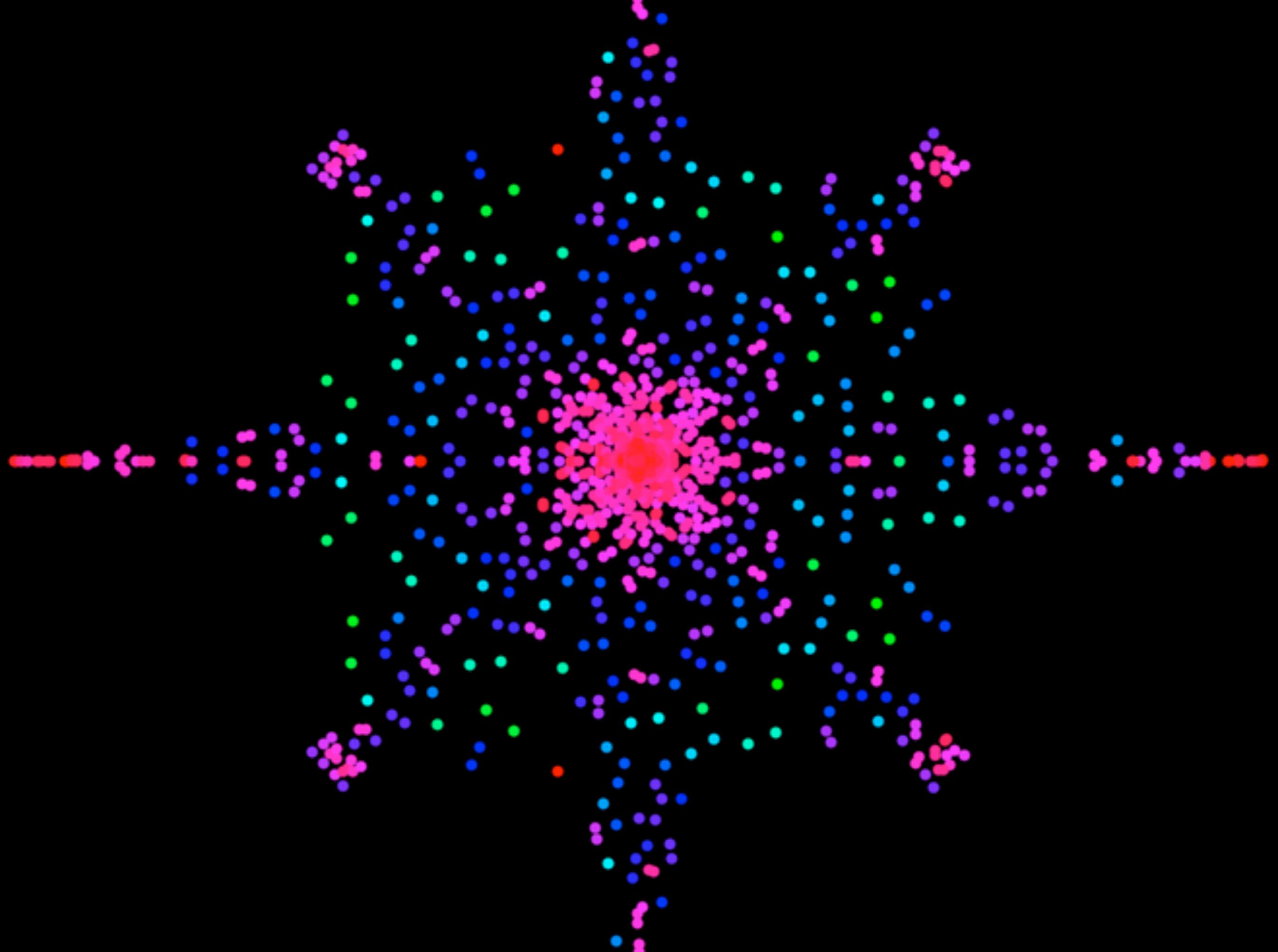


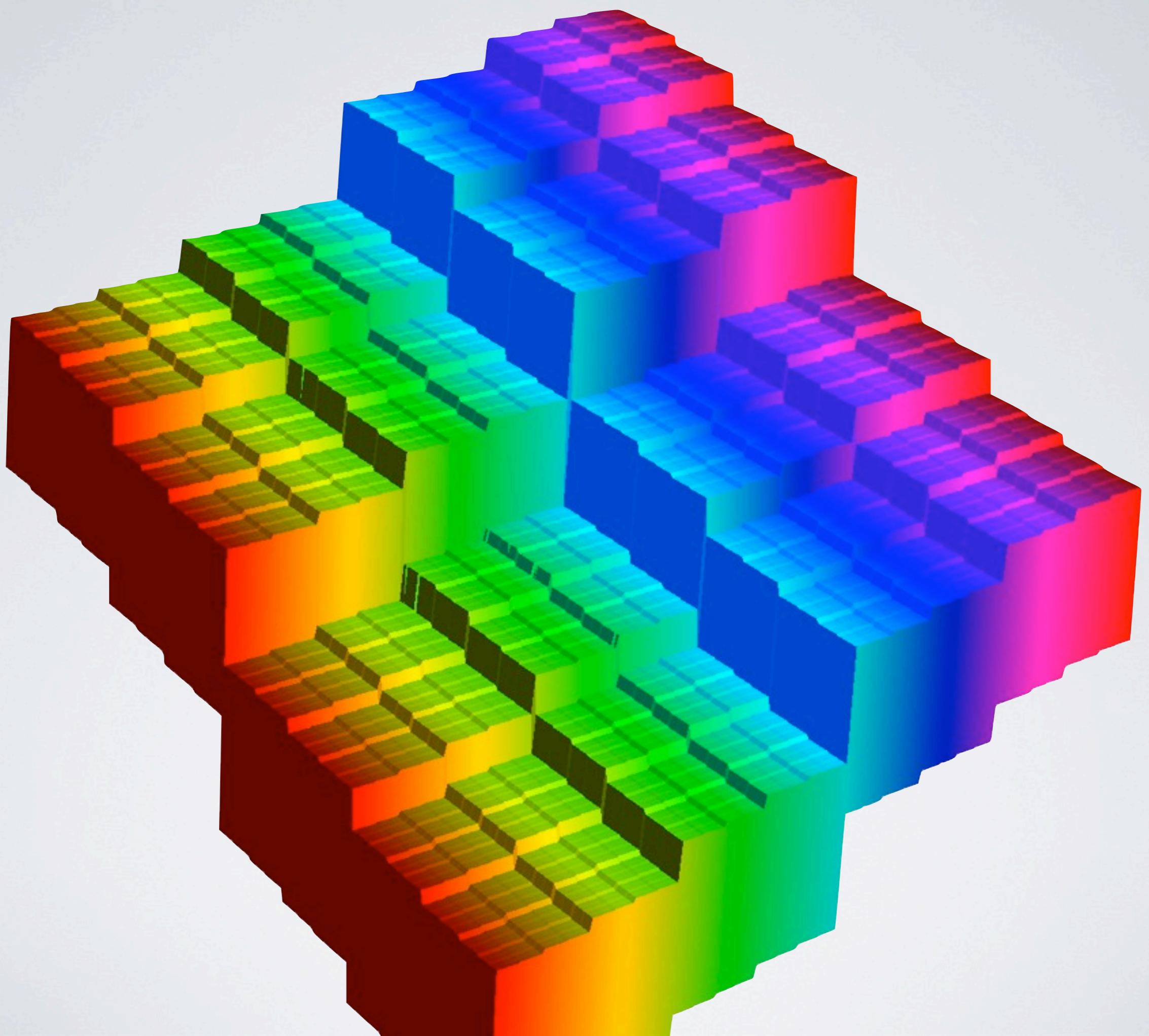


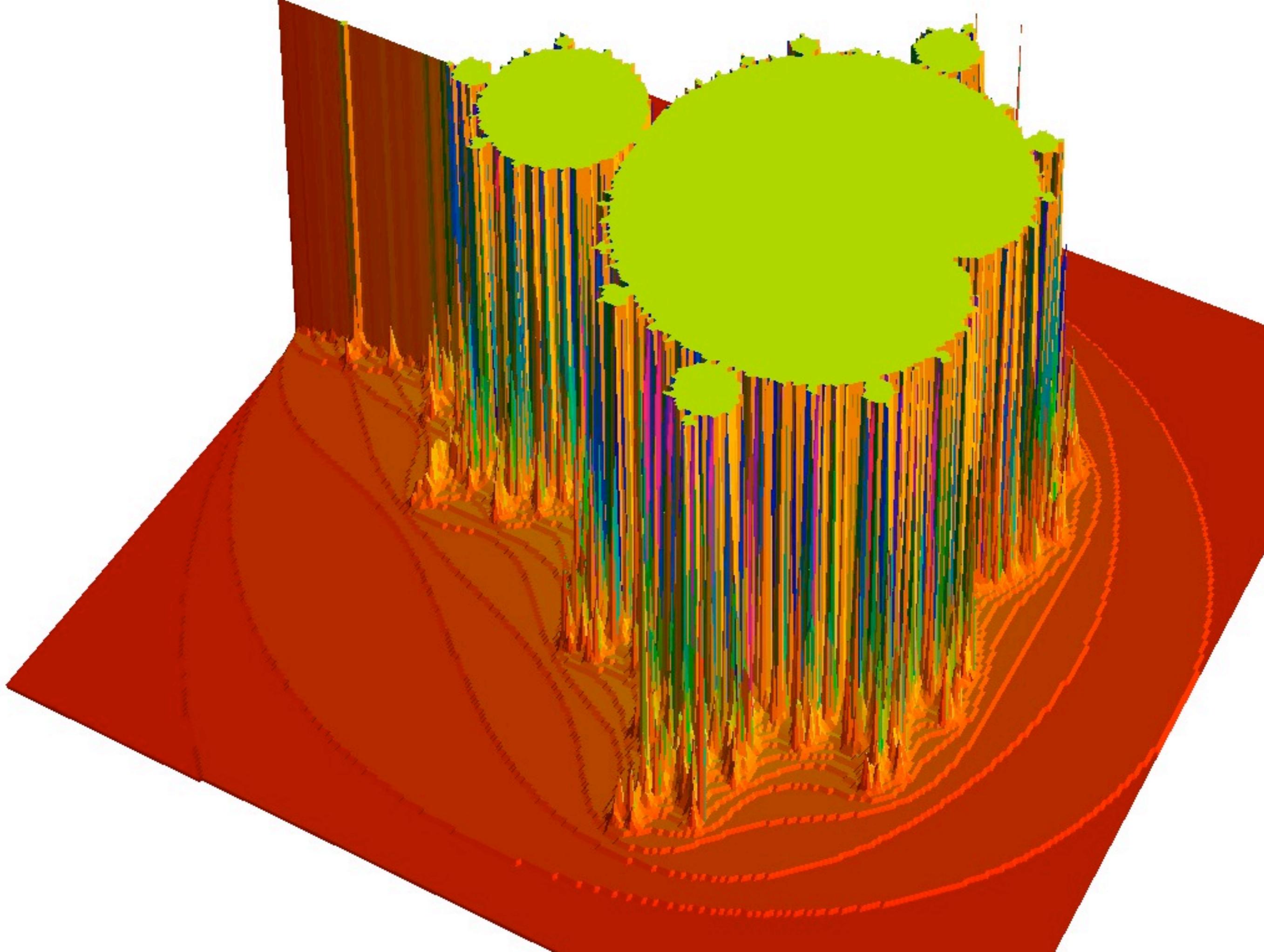




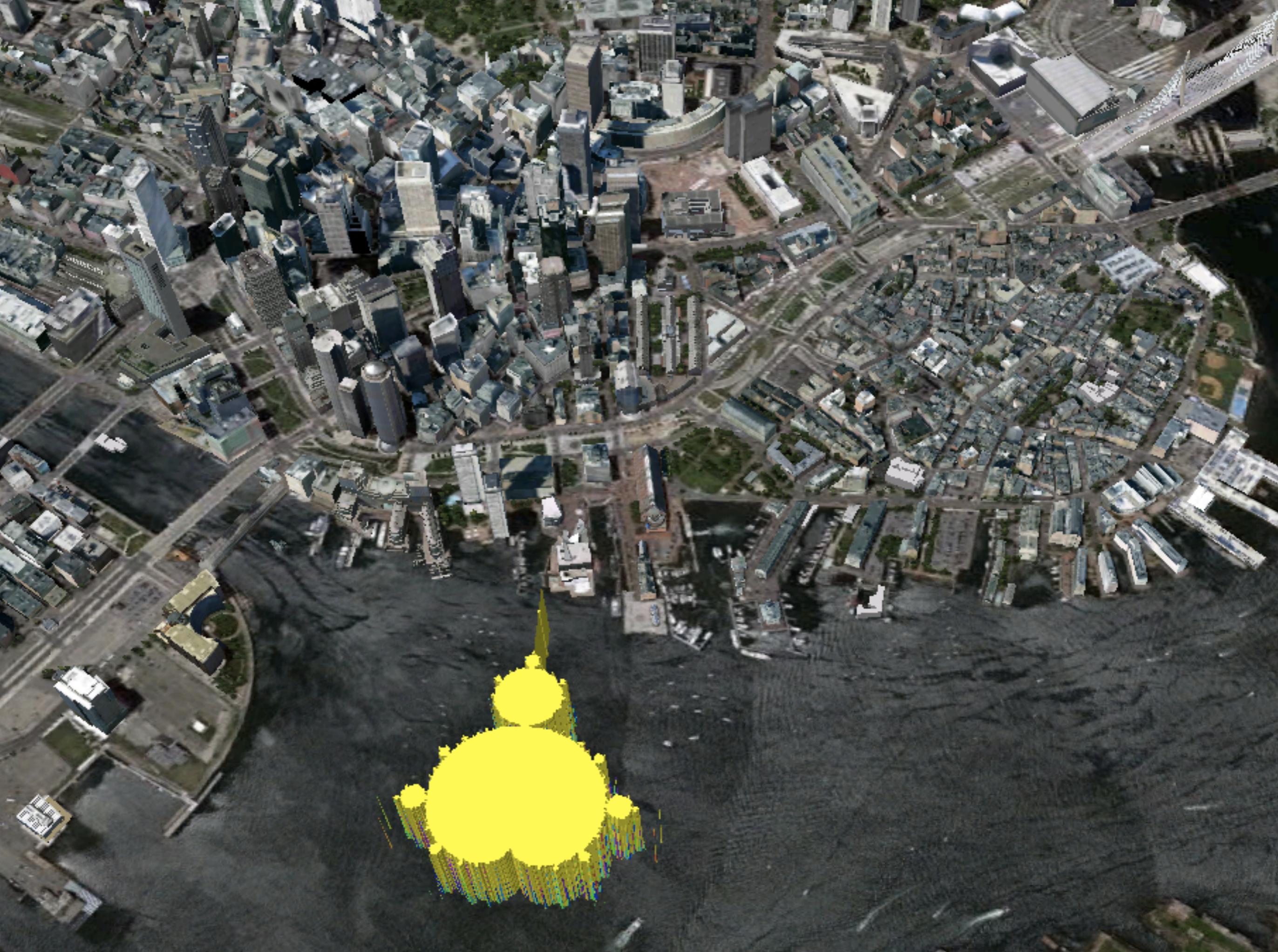


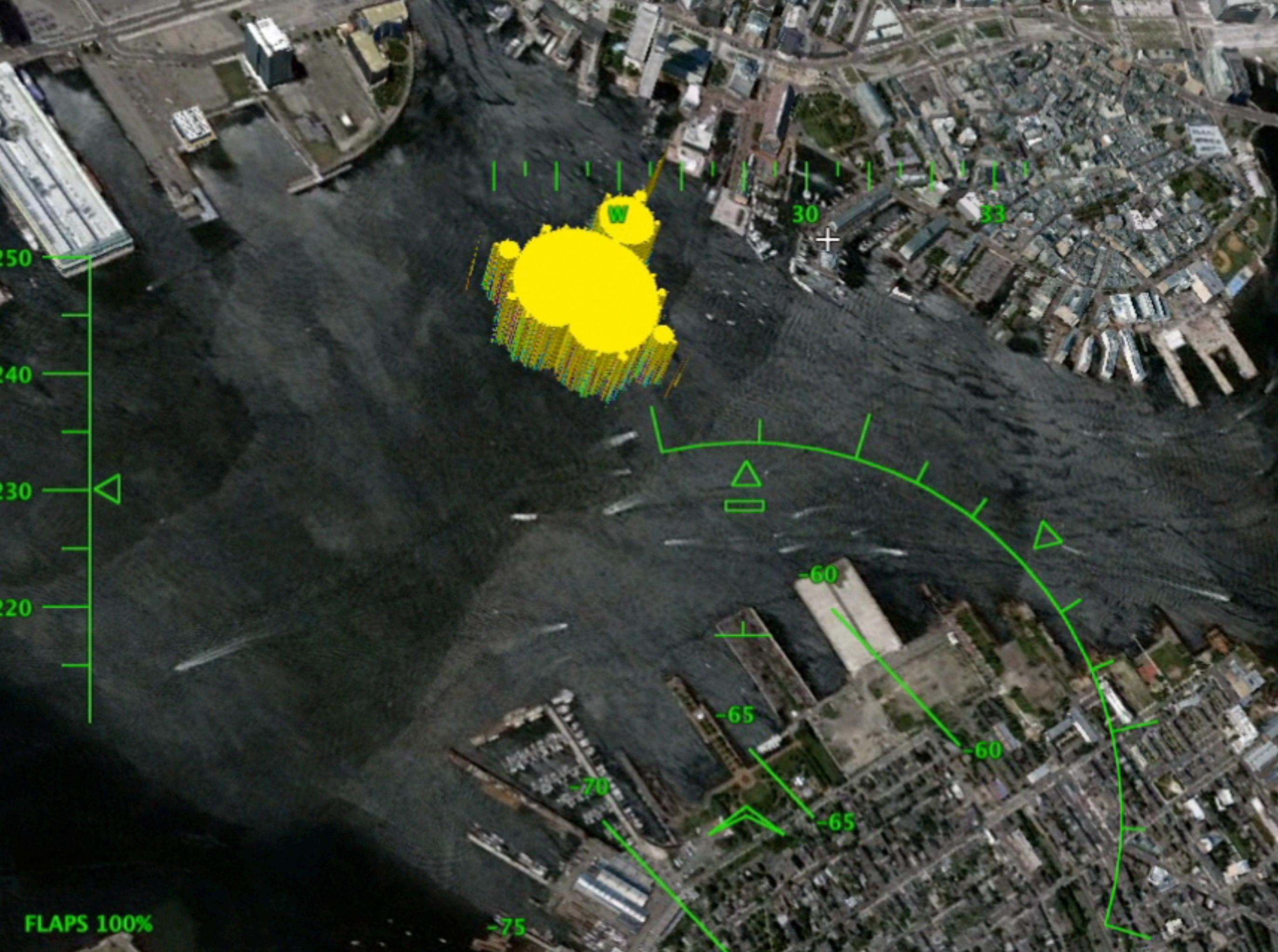












W

30

33

-60

-65

-70

-65

-75

2.50

2.40

2.30

2.20

FLAPS 100%

My Models

Per Page: 12 36 View: [Grid](#) | [List](#)

Sort: [Newest](#) ▾

Displaying:

All Models

Public

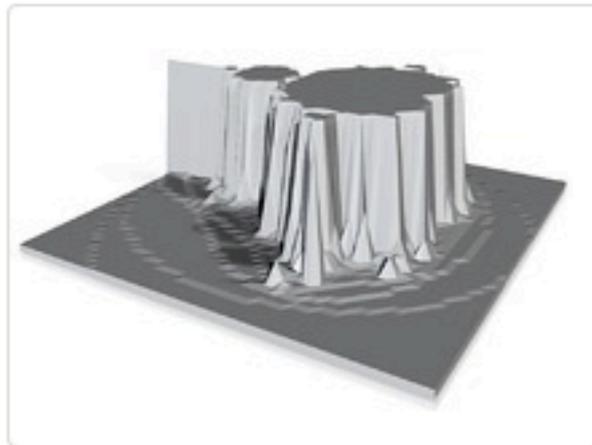
Private

For Sale

Not For Sale

Downloadable

Not Downloadable



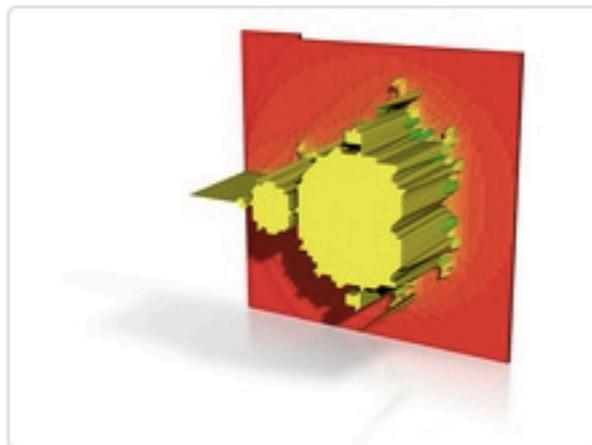
Mandelbrot

Edit Model

Test with OBJ format

Frosted Ultra Detail \$5.00 ▾

\$5.00 [Add to Cart](#)



Mandelbrot

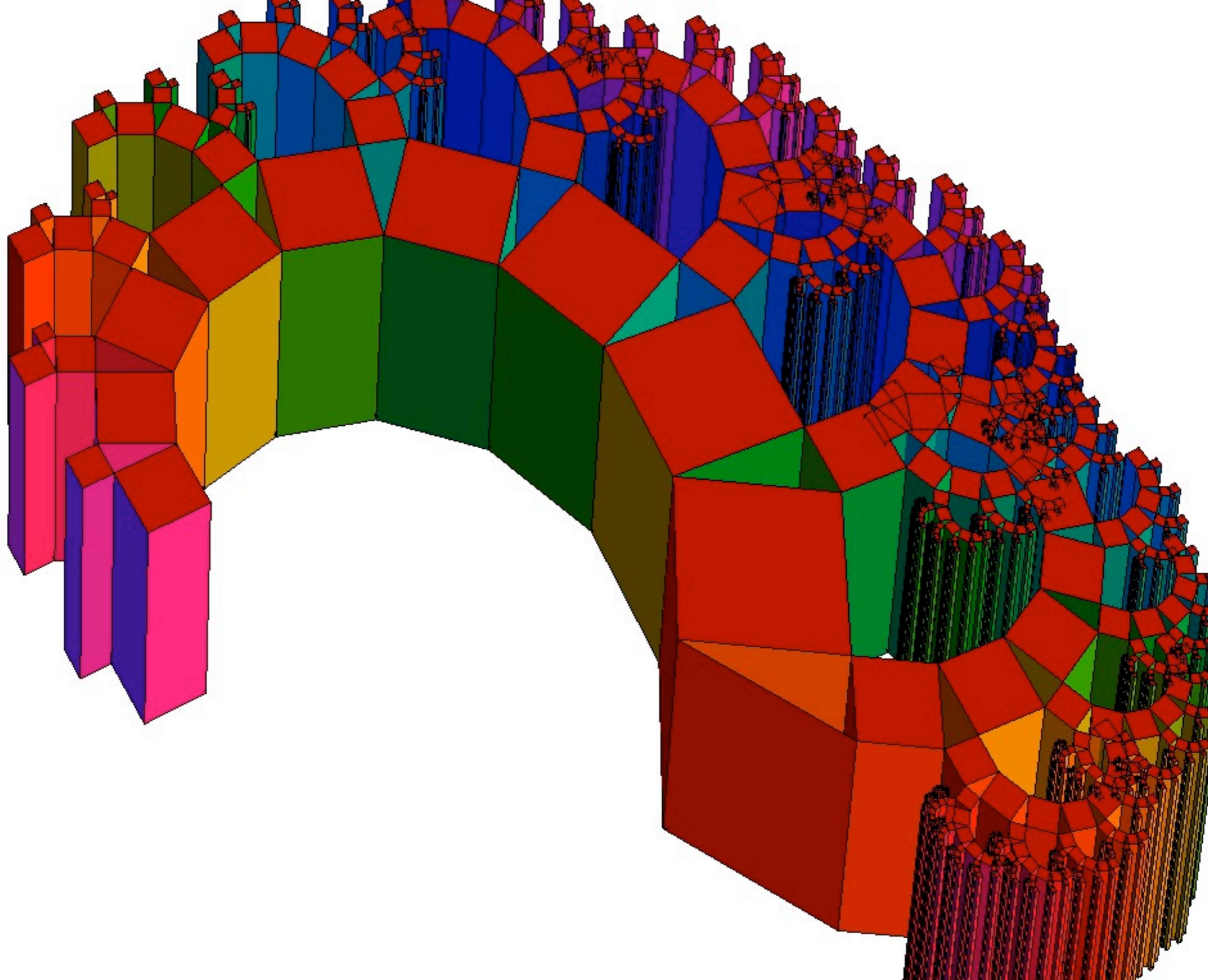
Edit Model

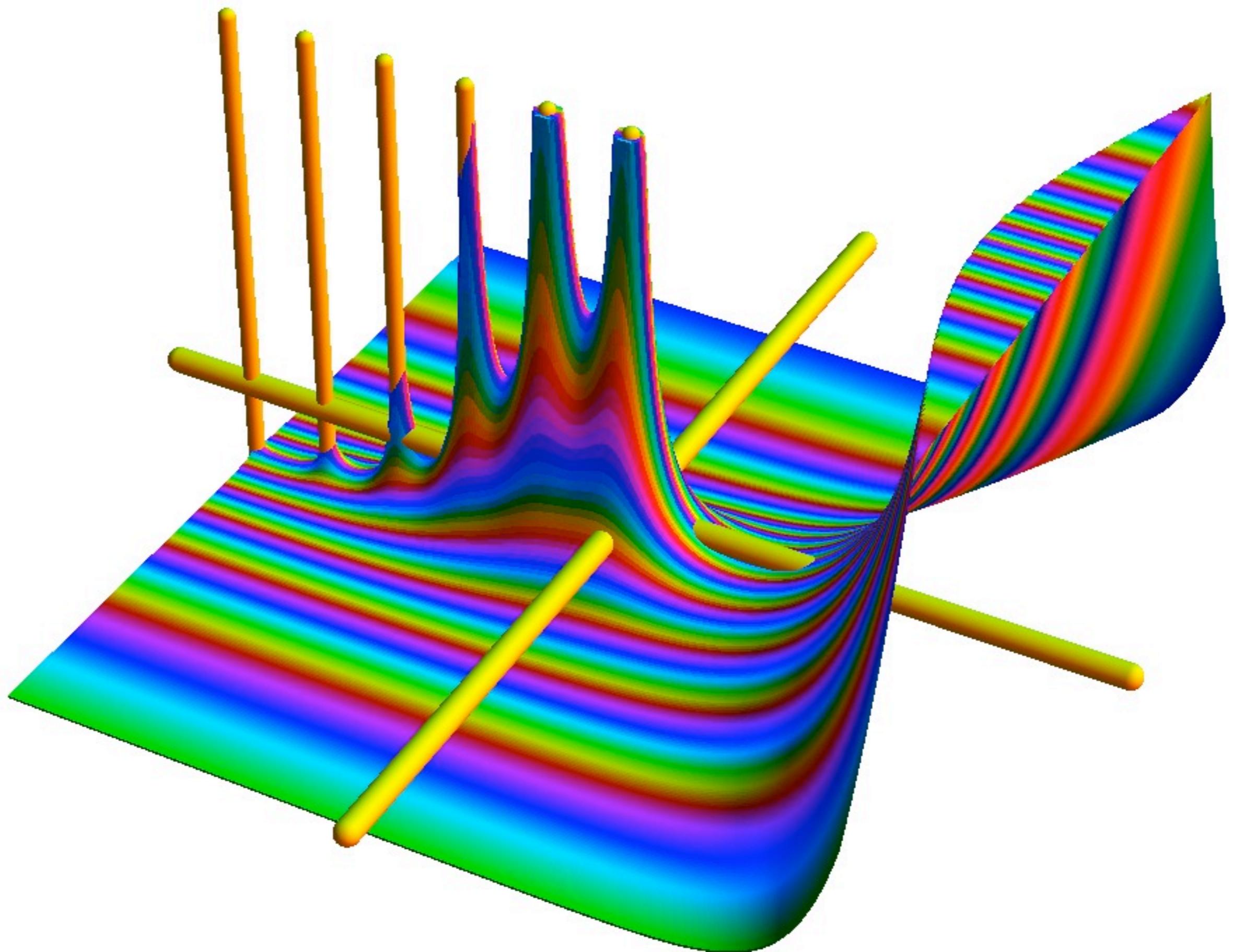
Test with X3D format

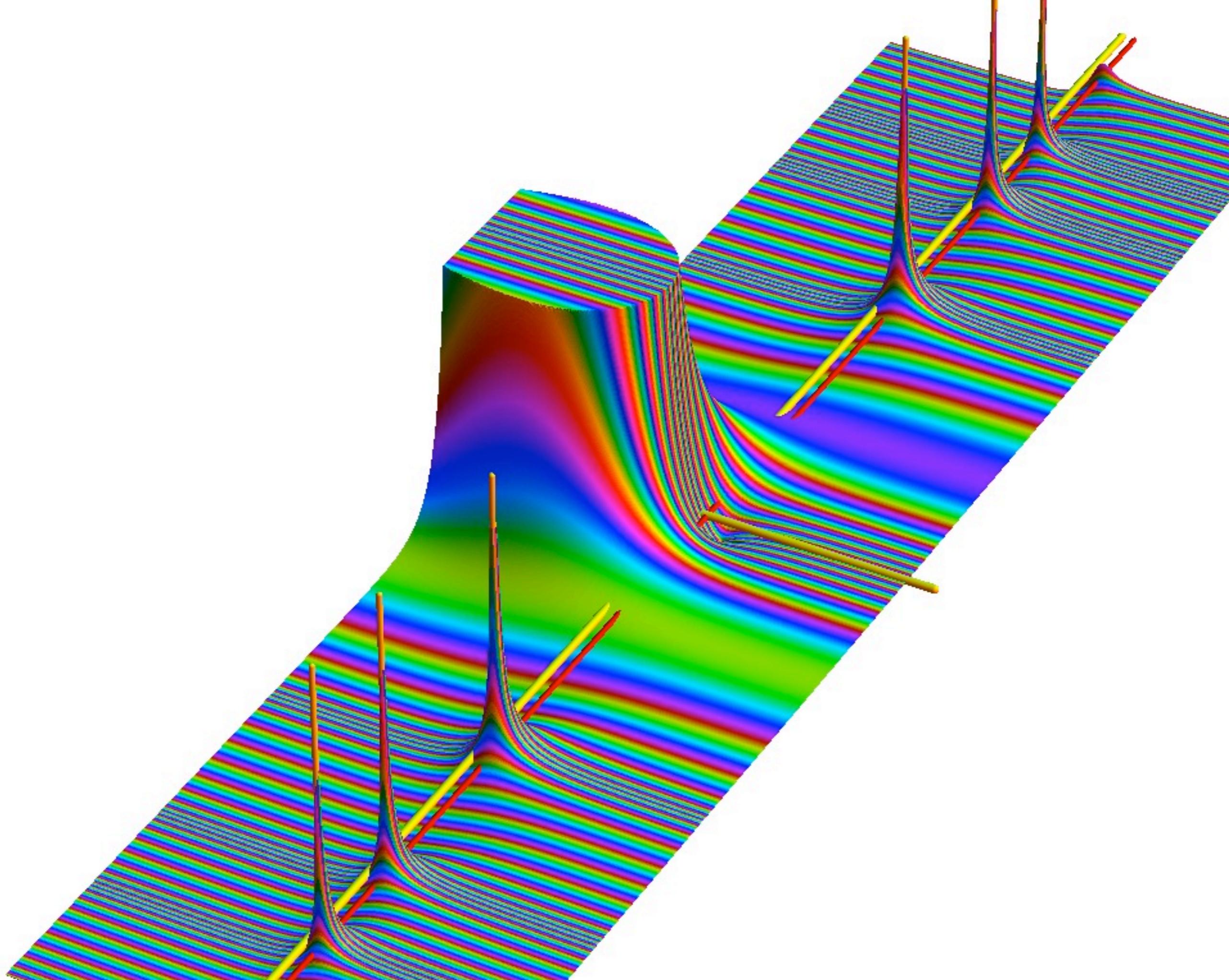
White Strong & Flexible \$11.68 ▾

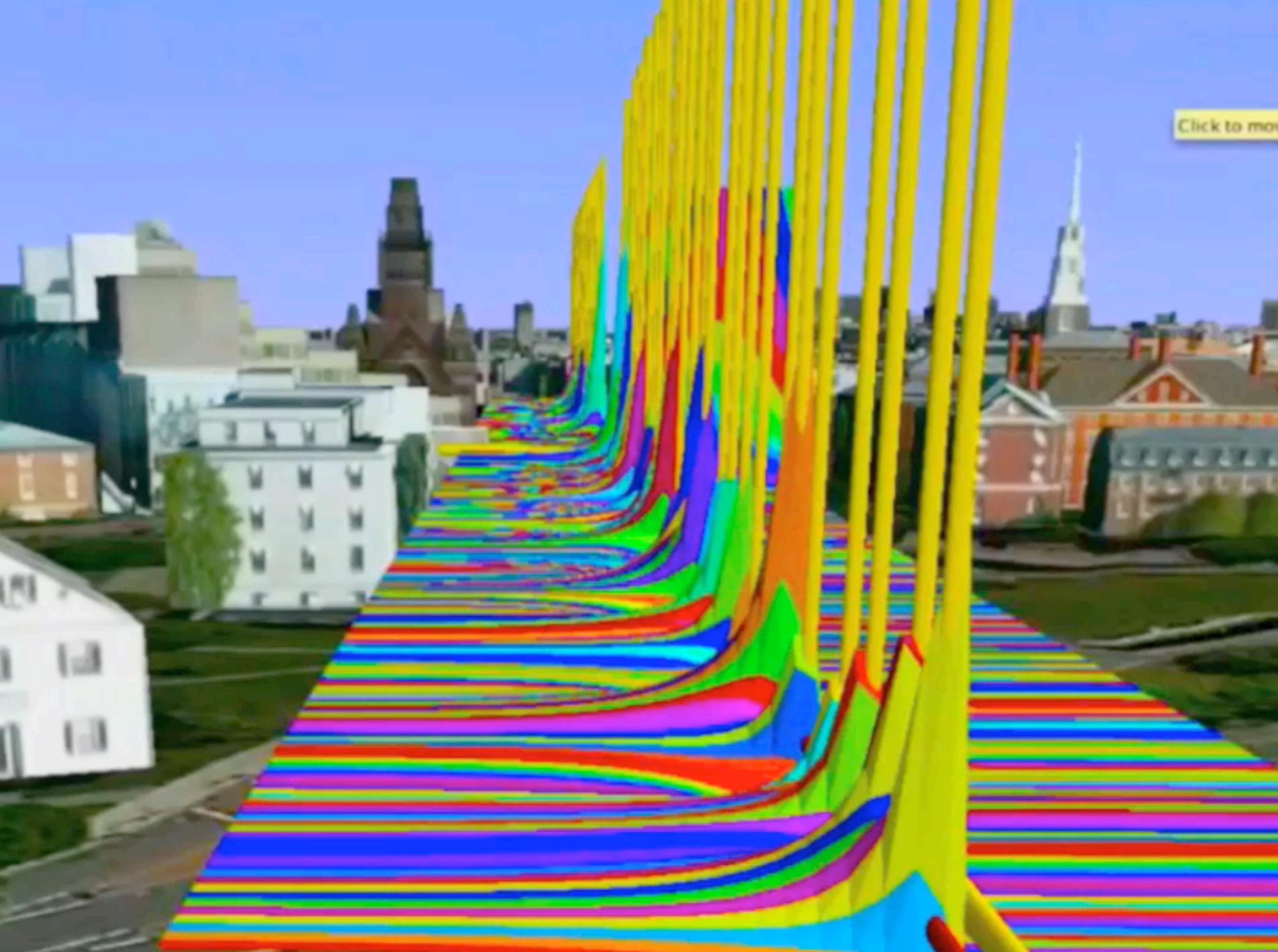
\$11.68 [Add to Cart](#)



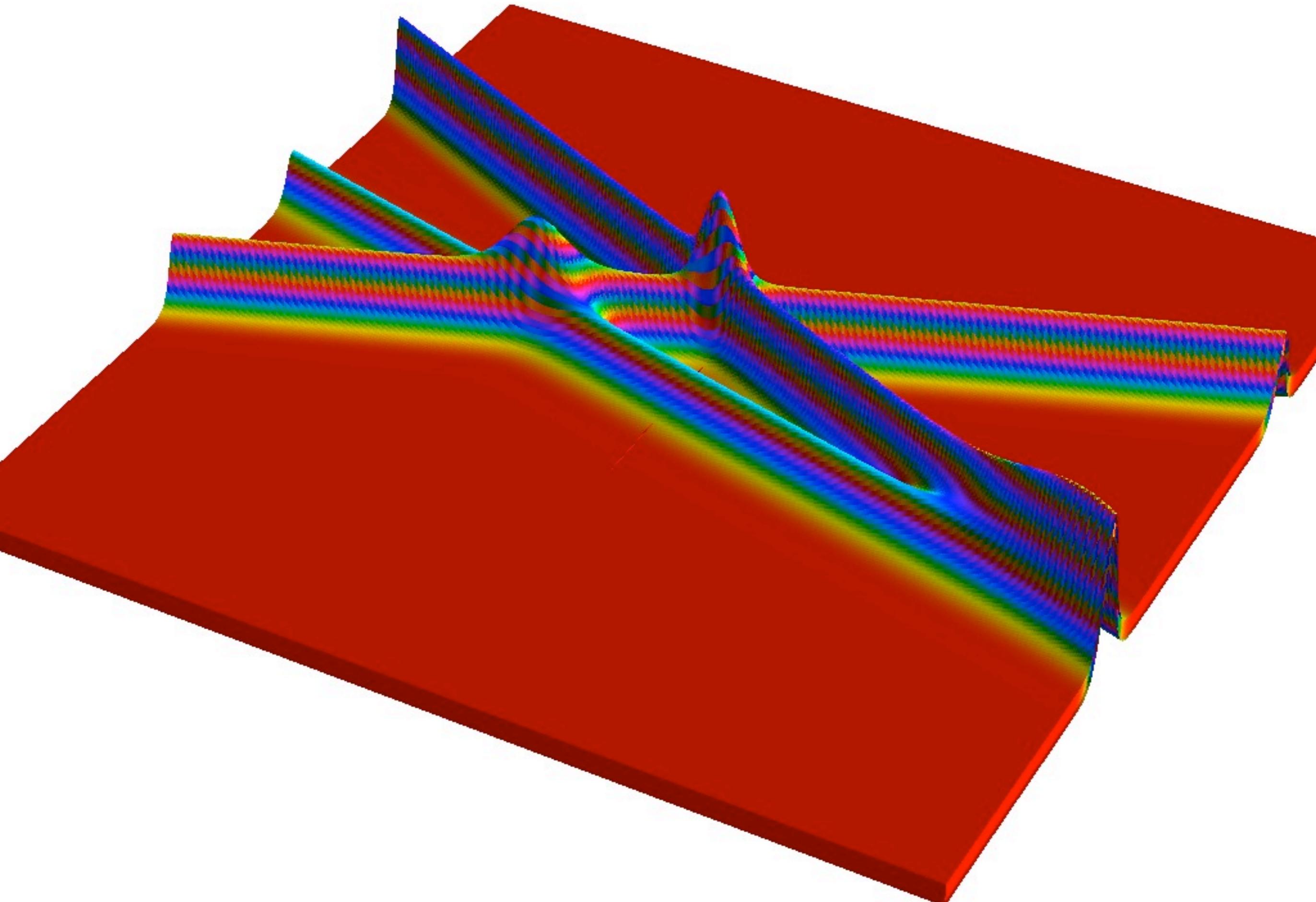


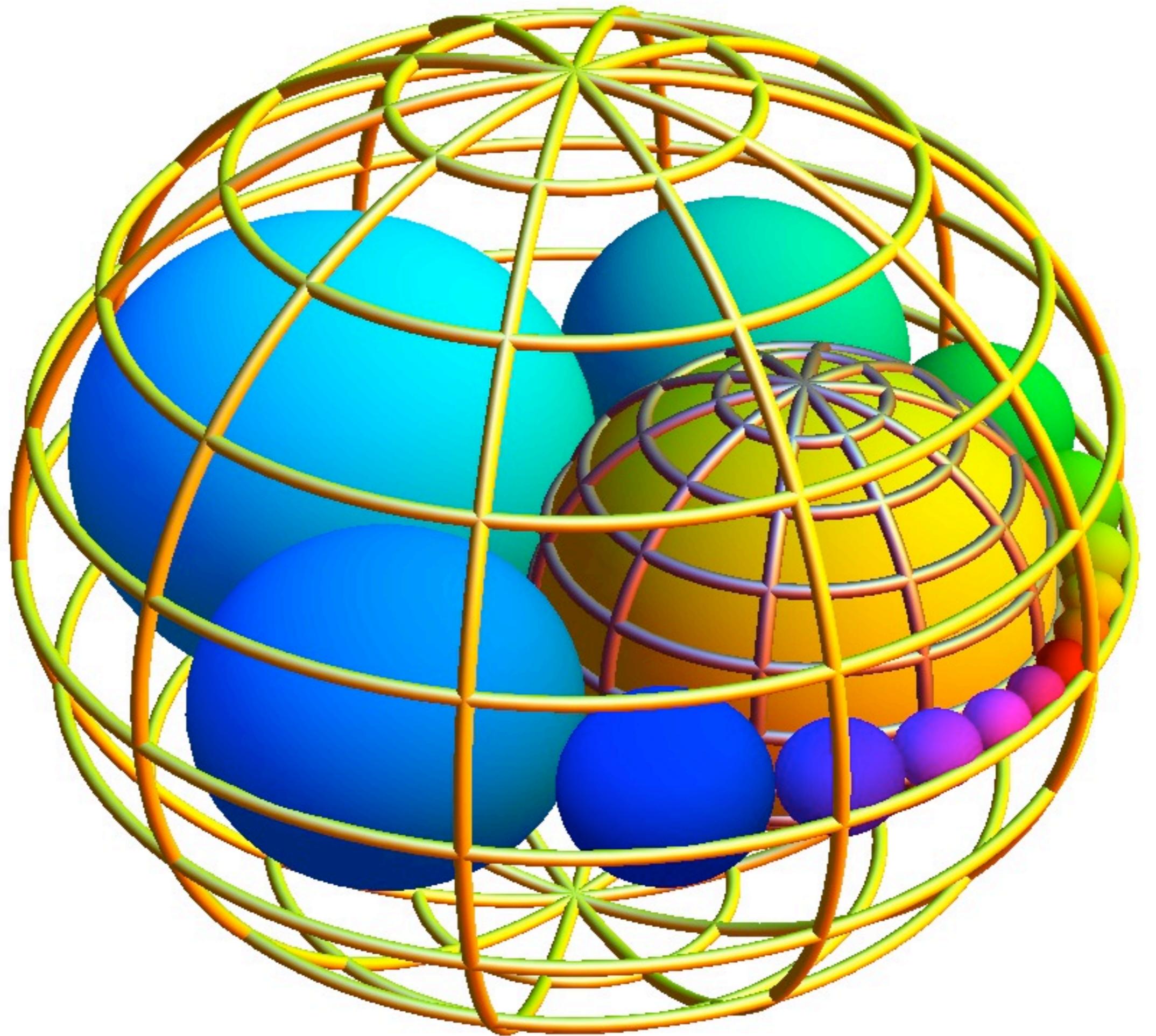




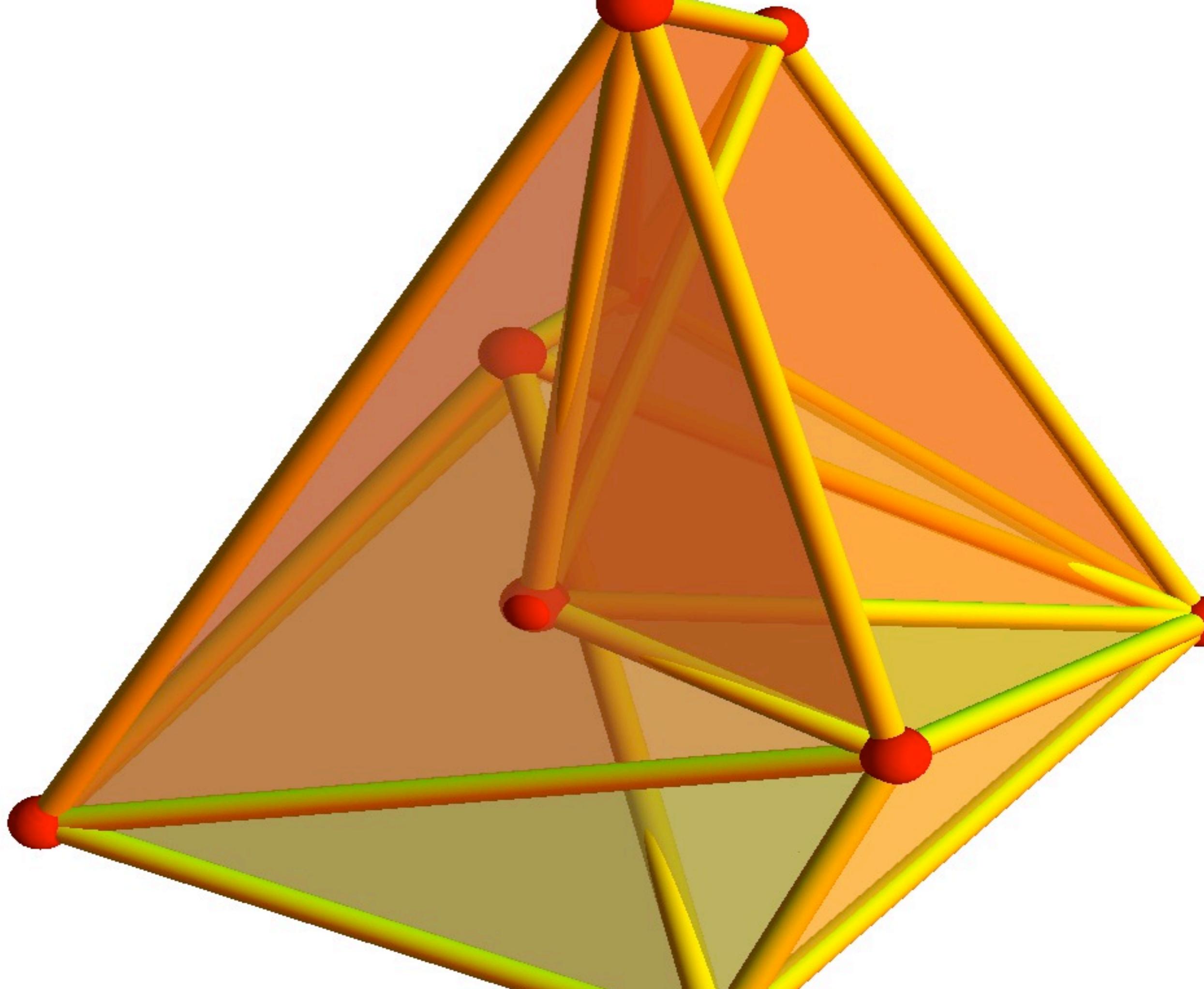


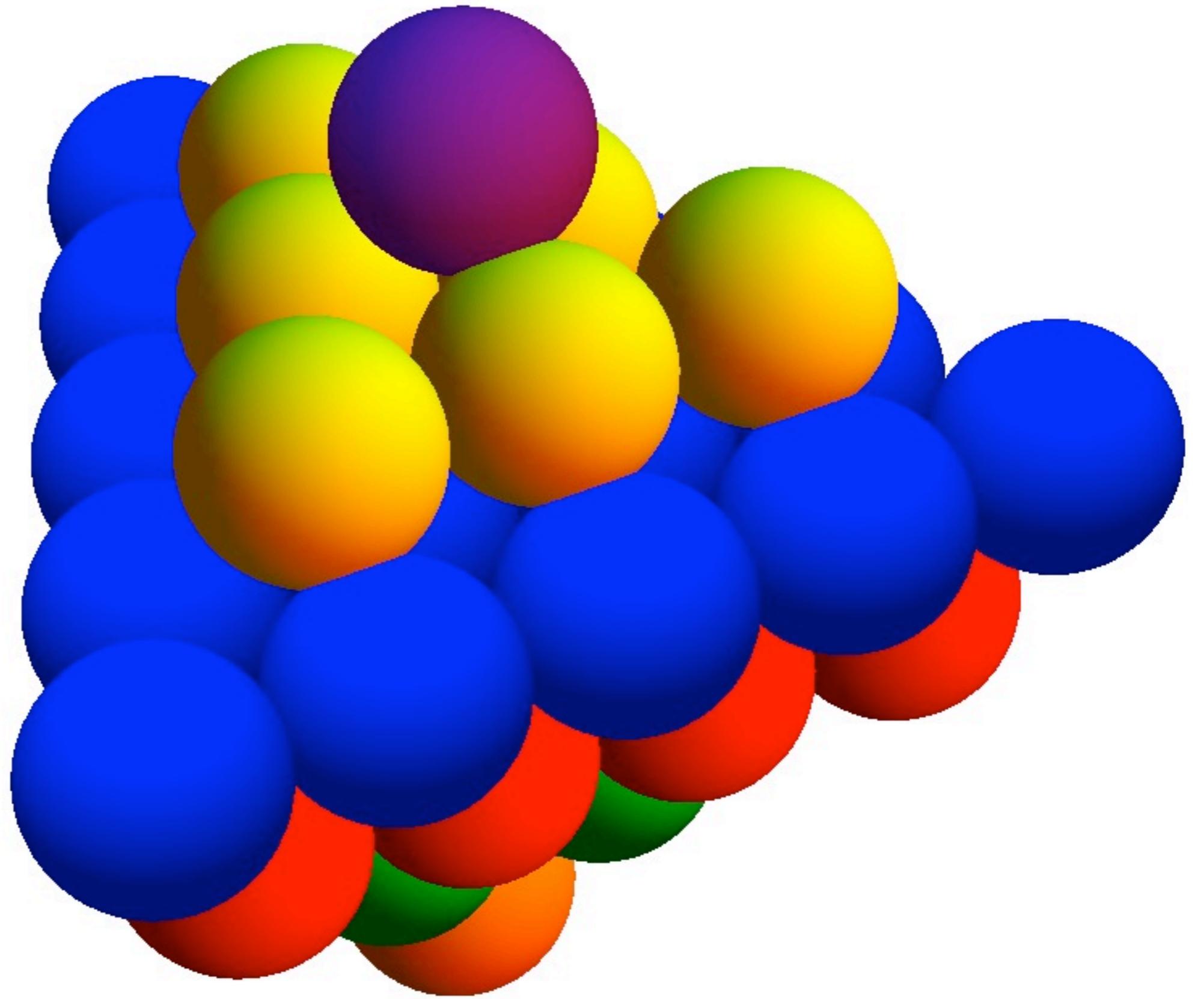
Click to move



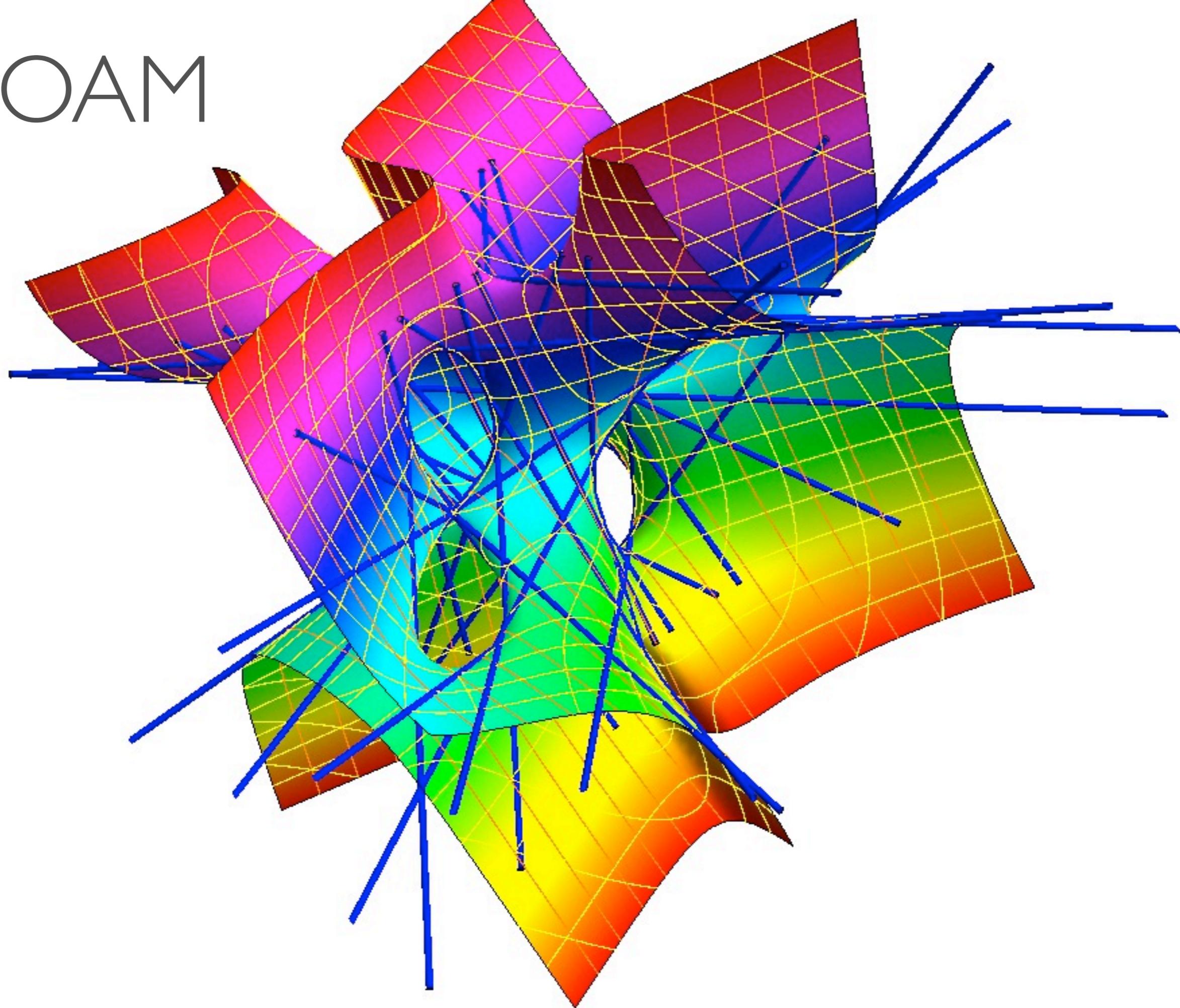


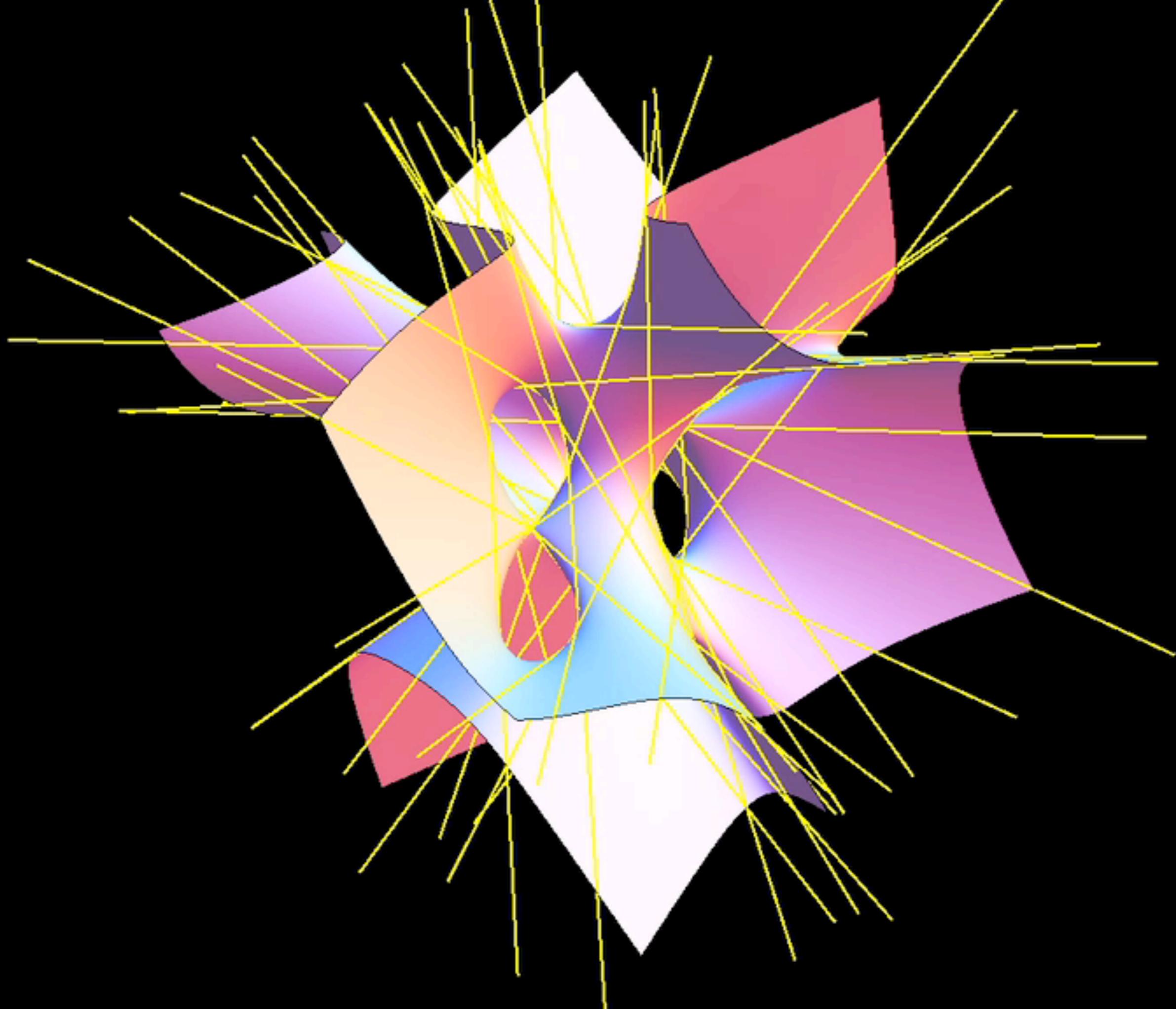


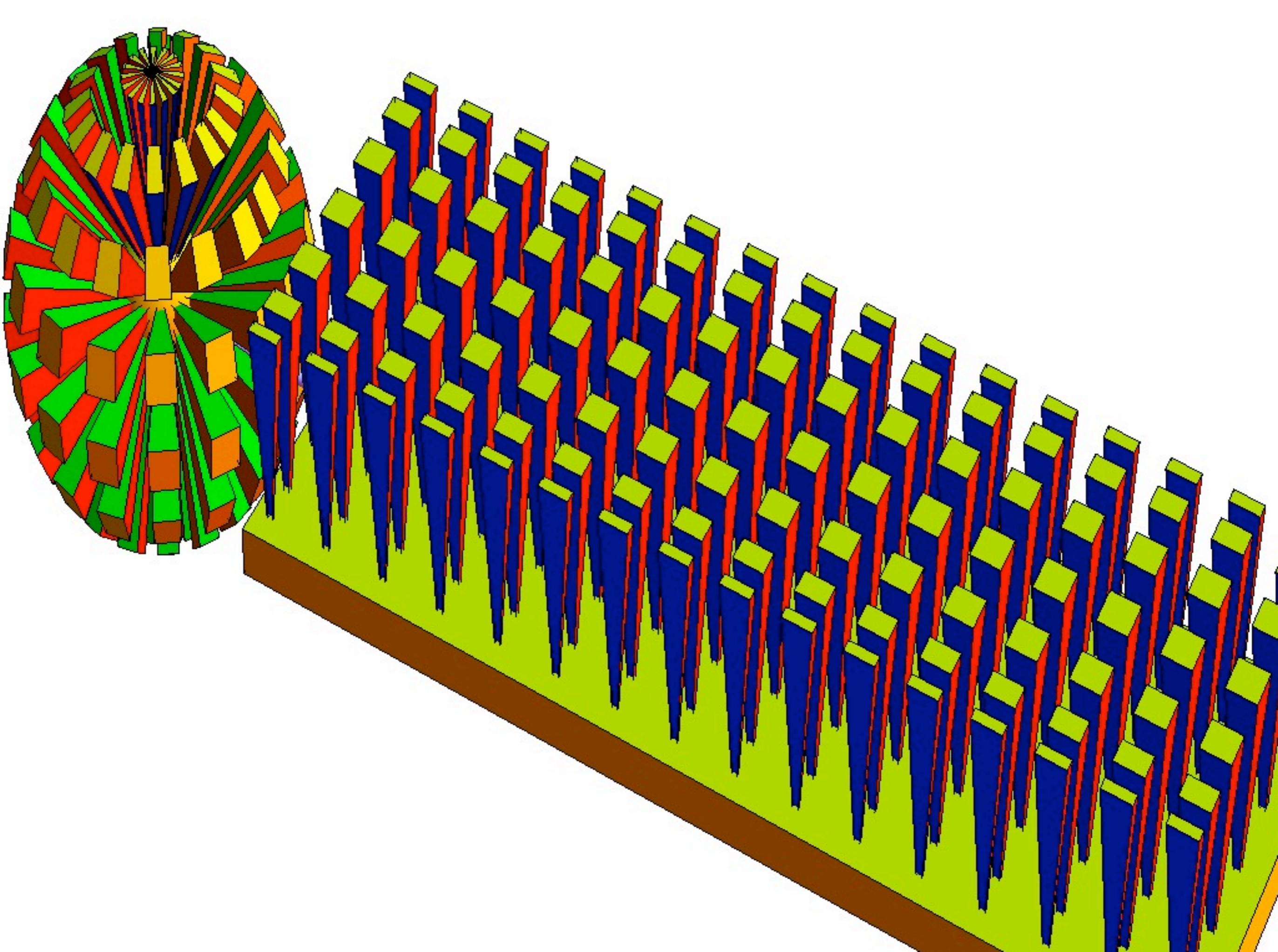


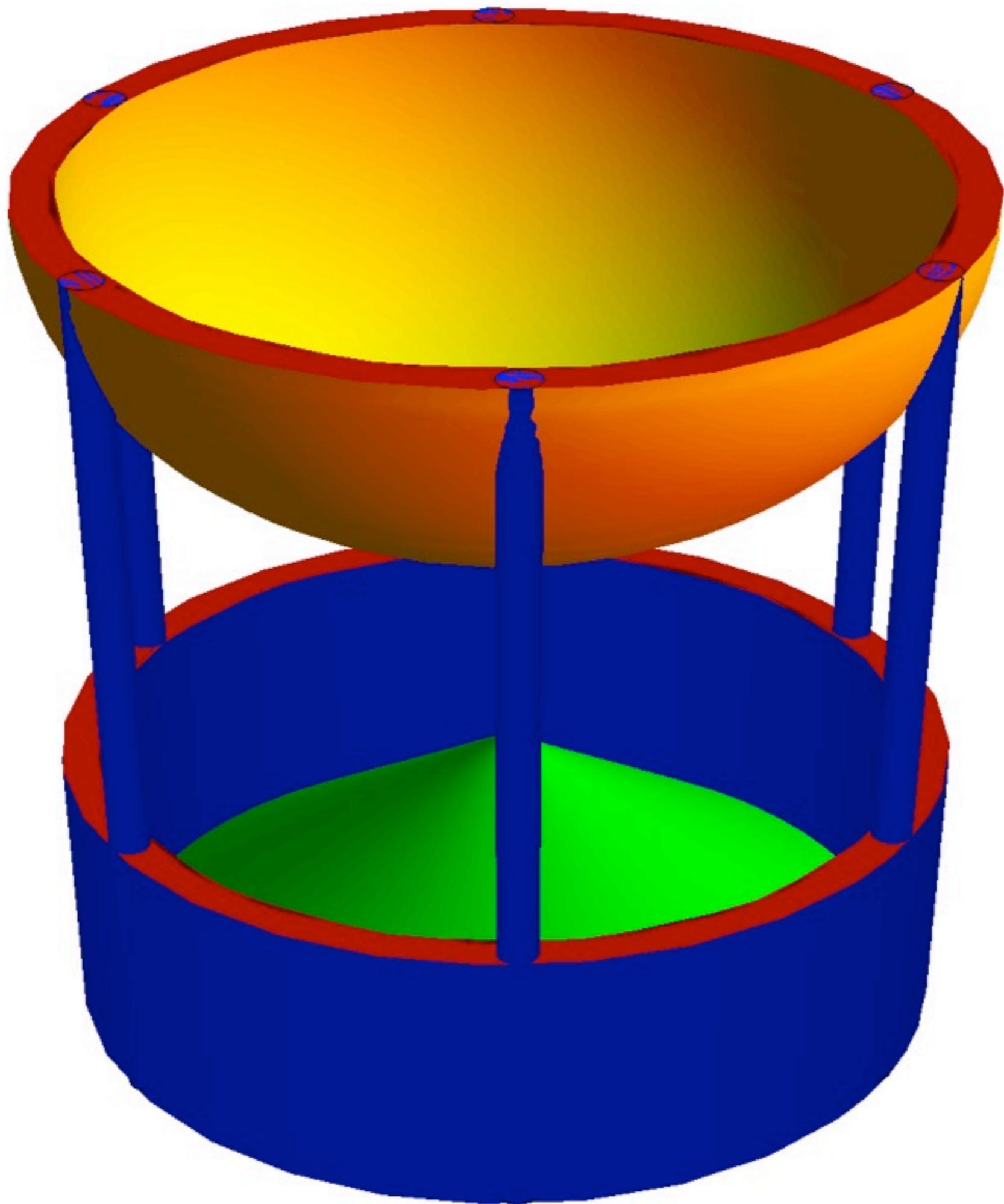


NOAM



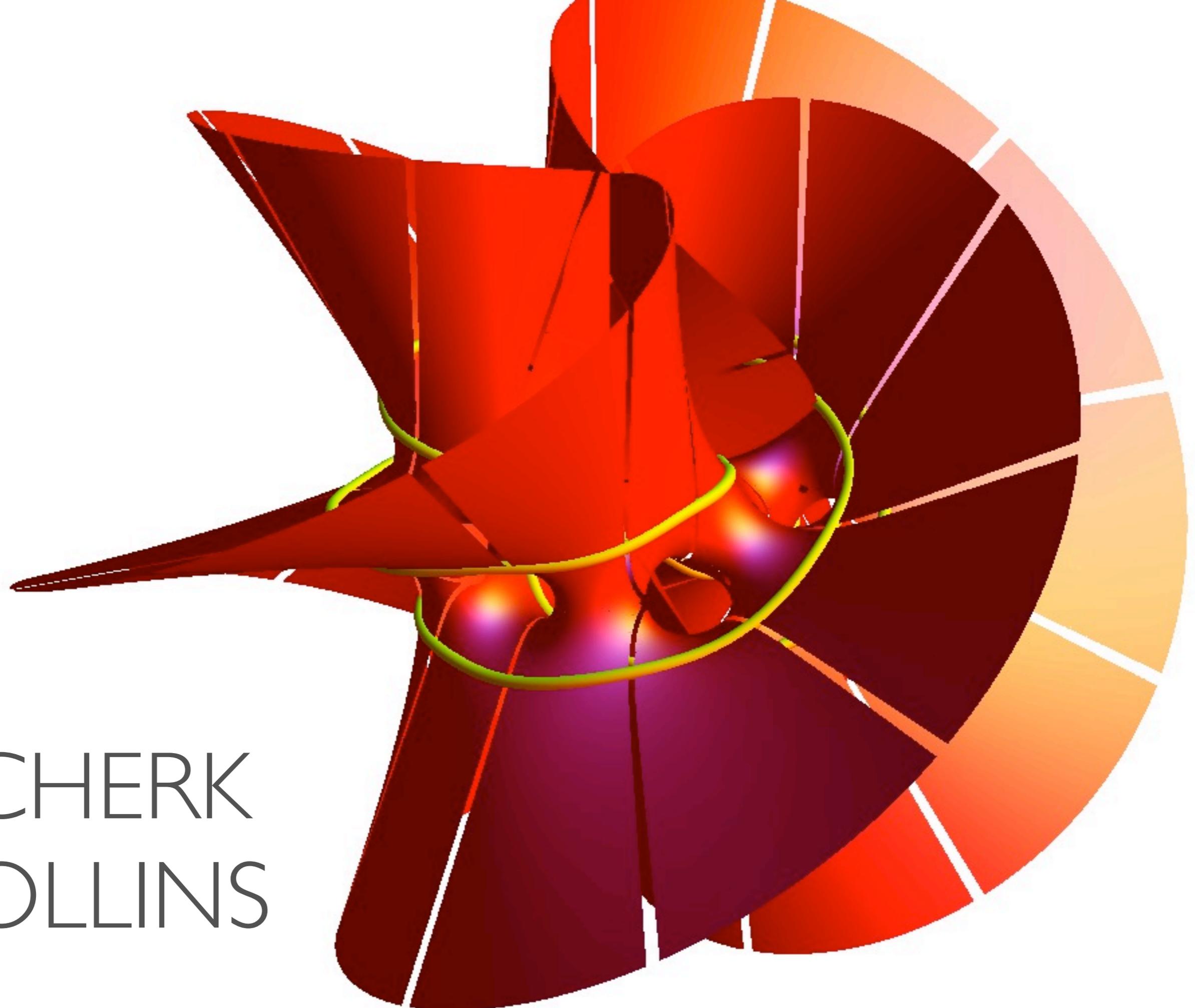




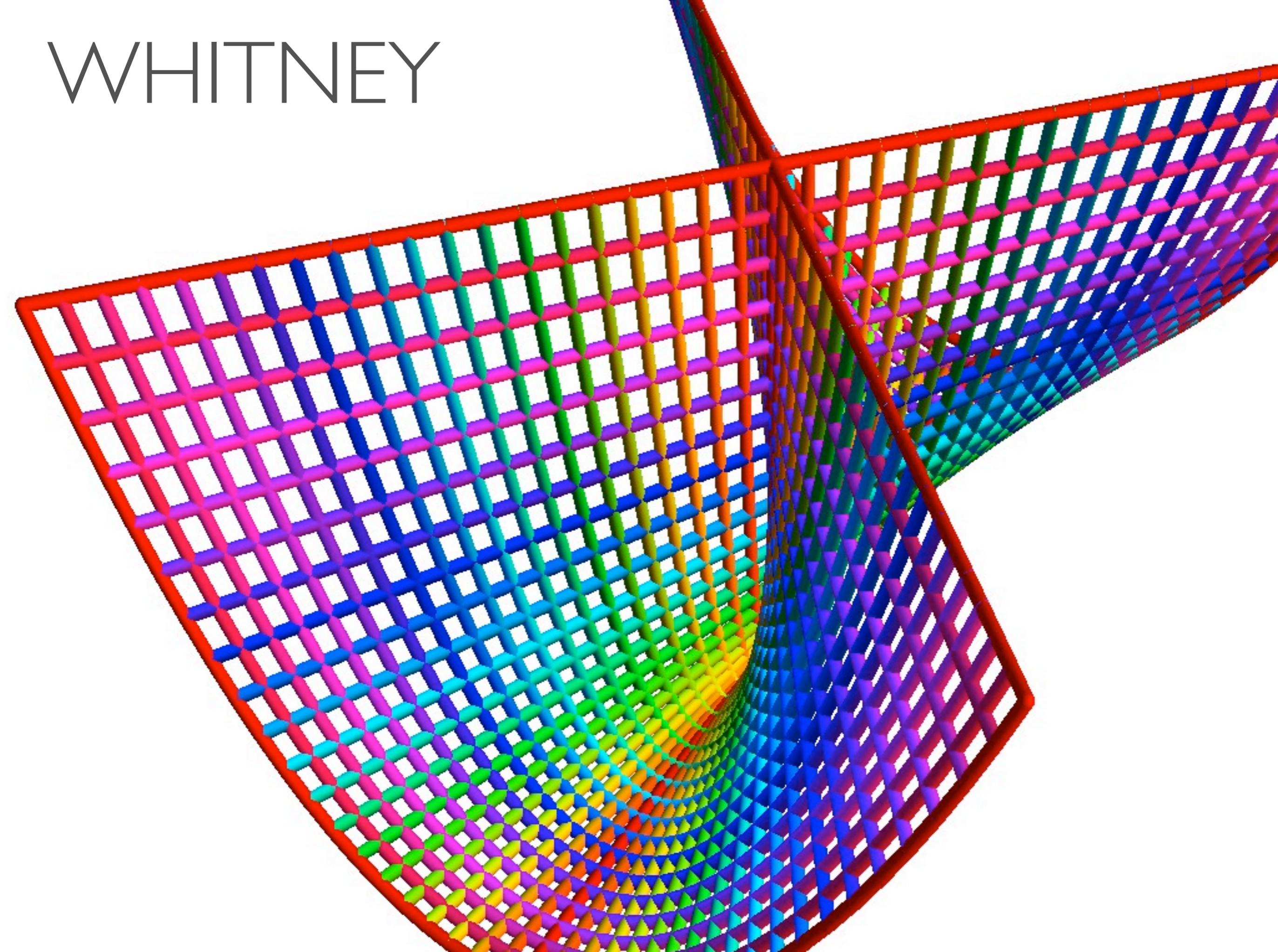


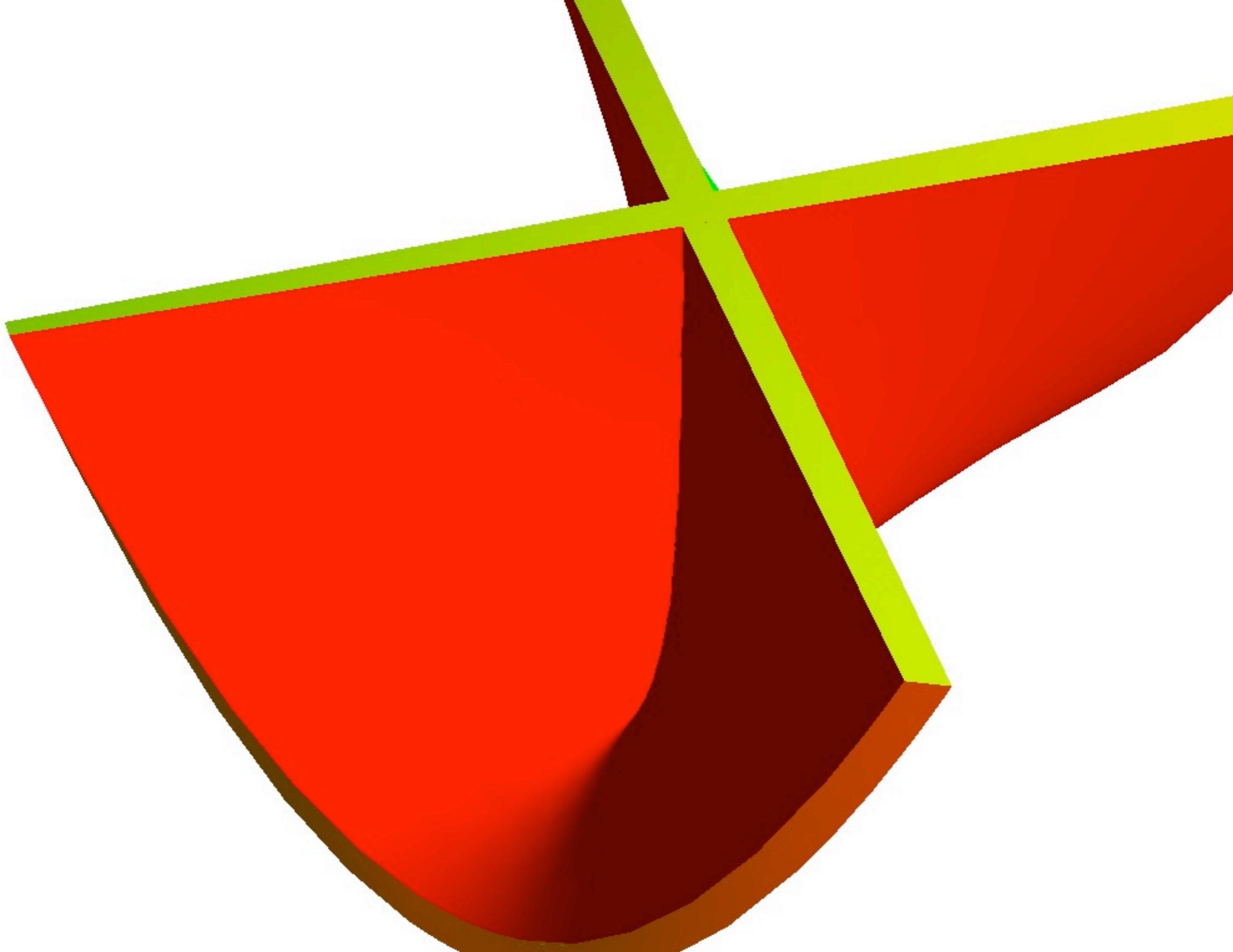


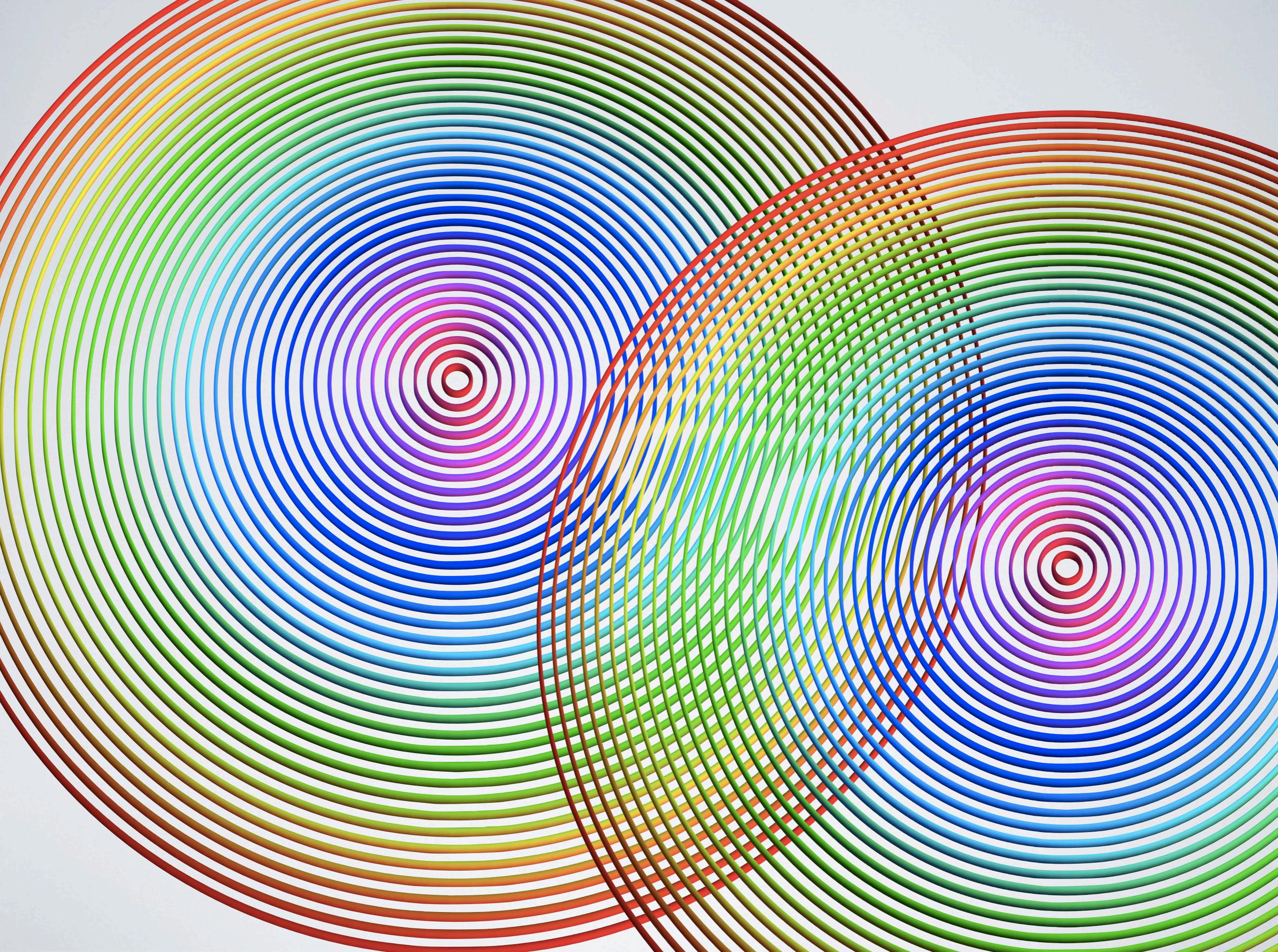
SCHERK
COLLINS

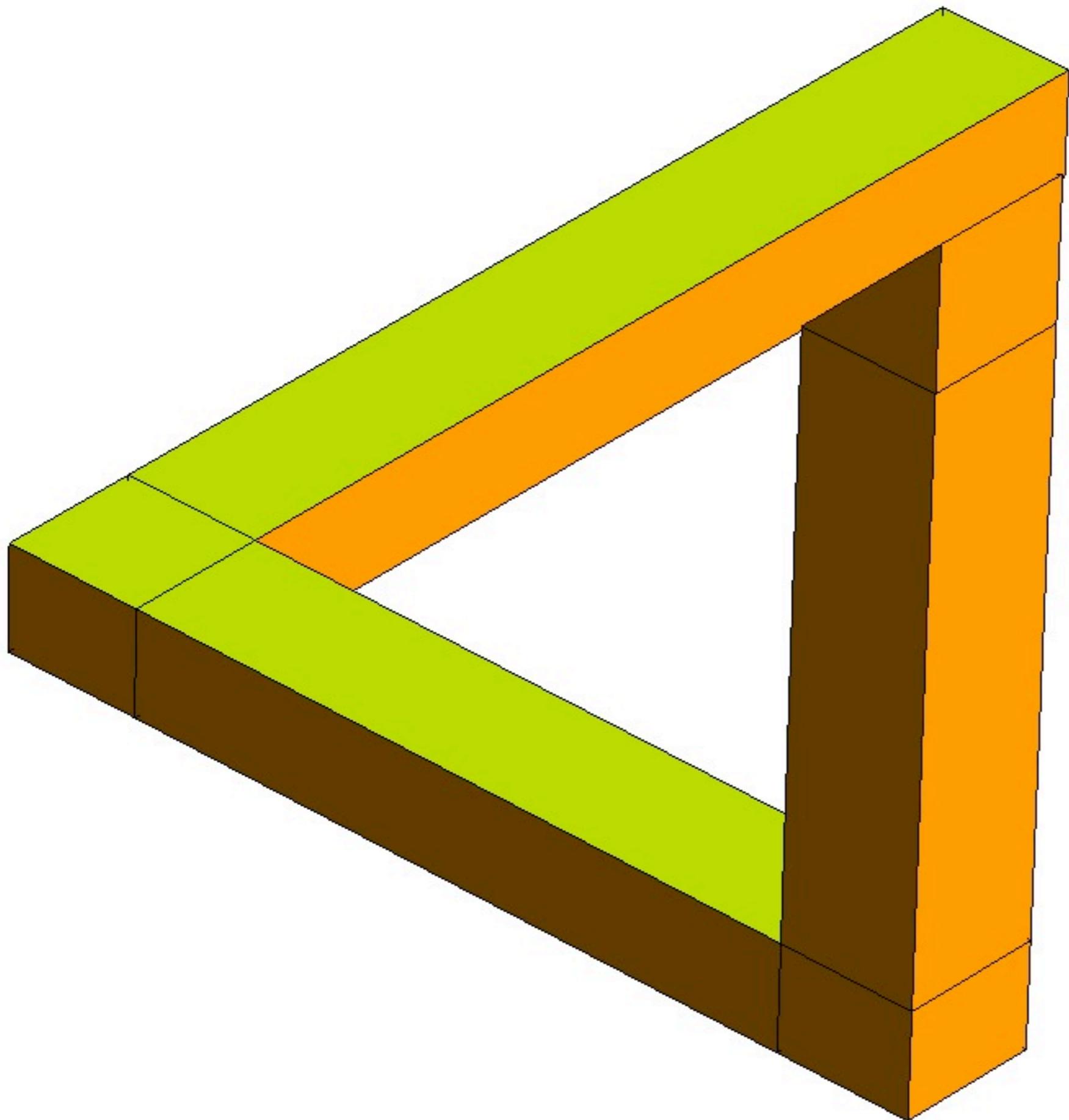


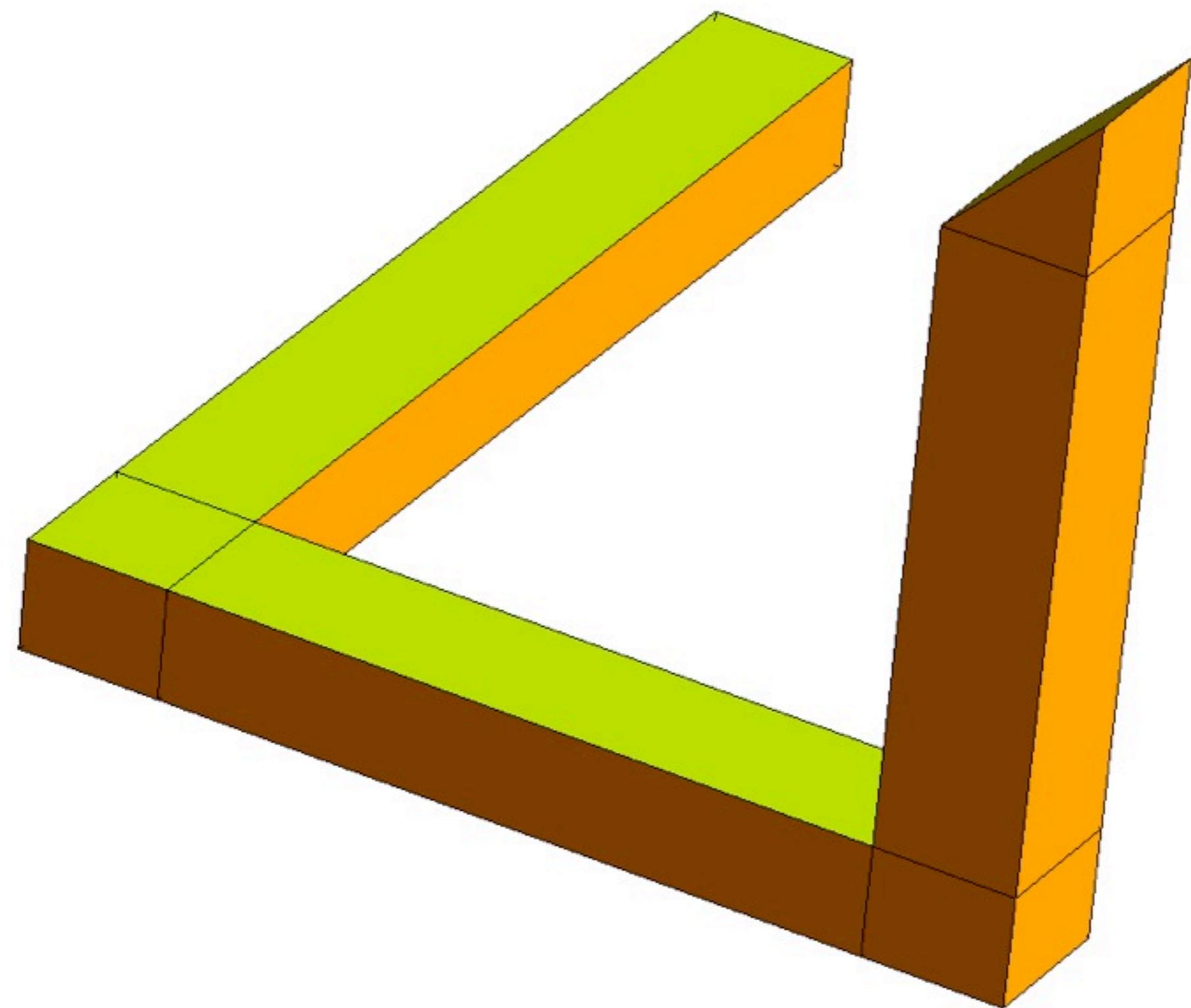
WHITNEY

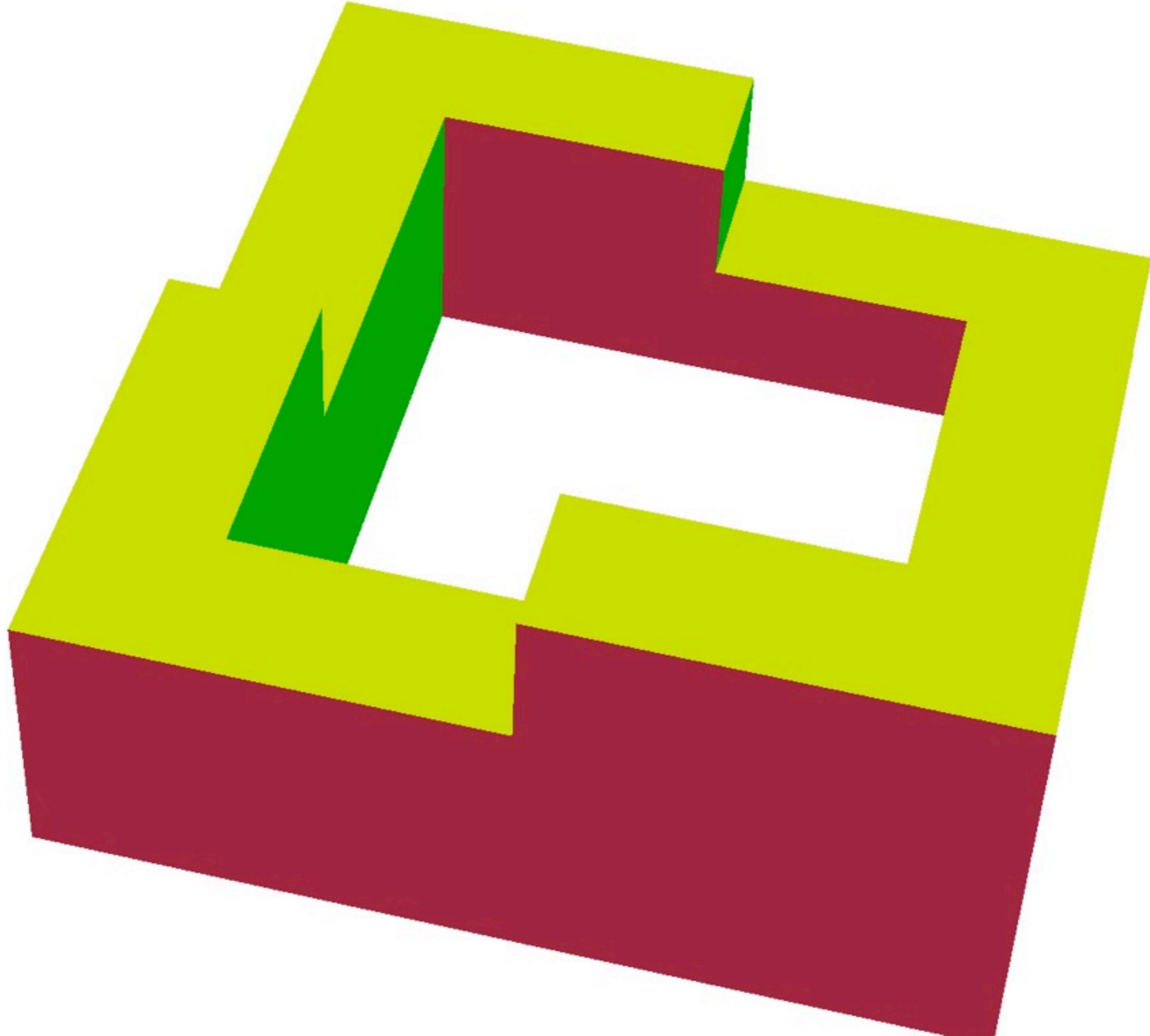


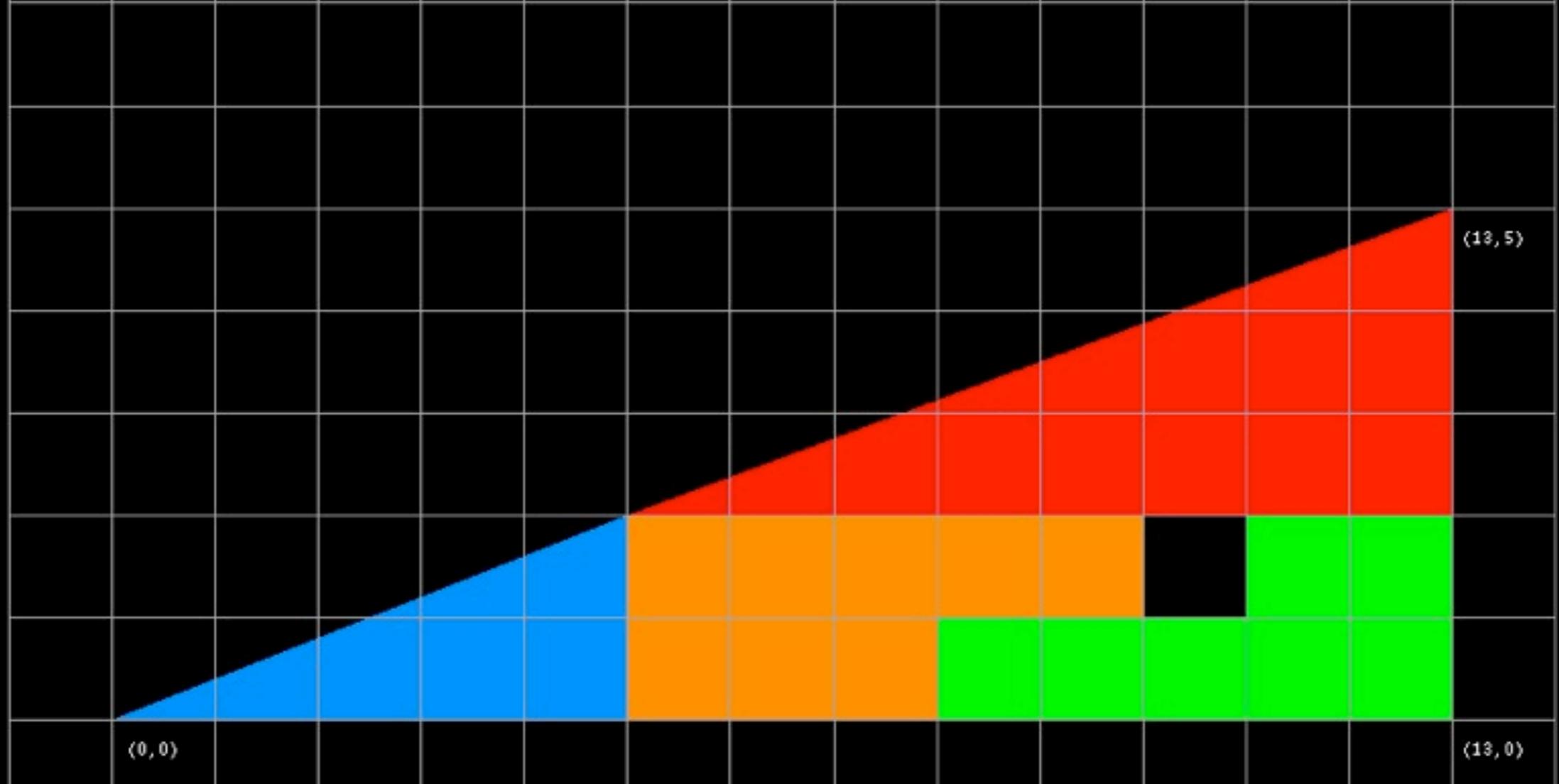
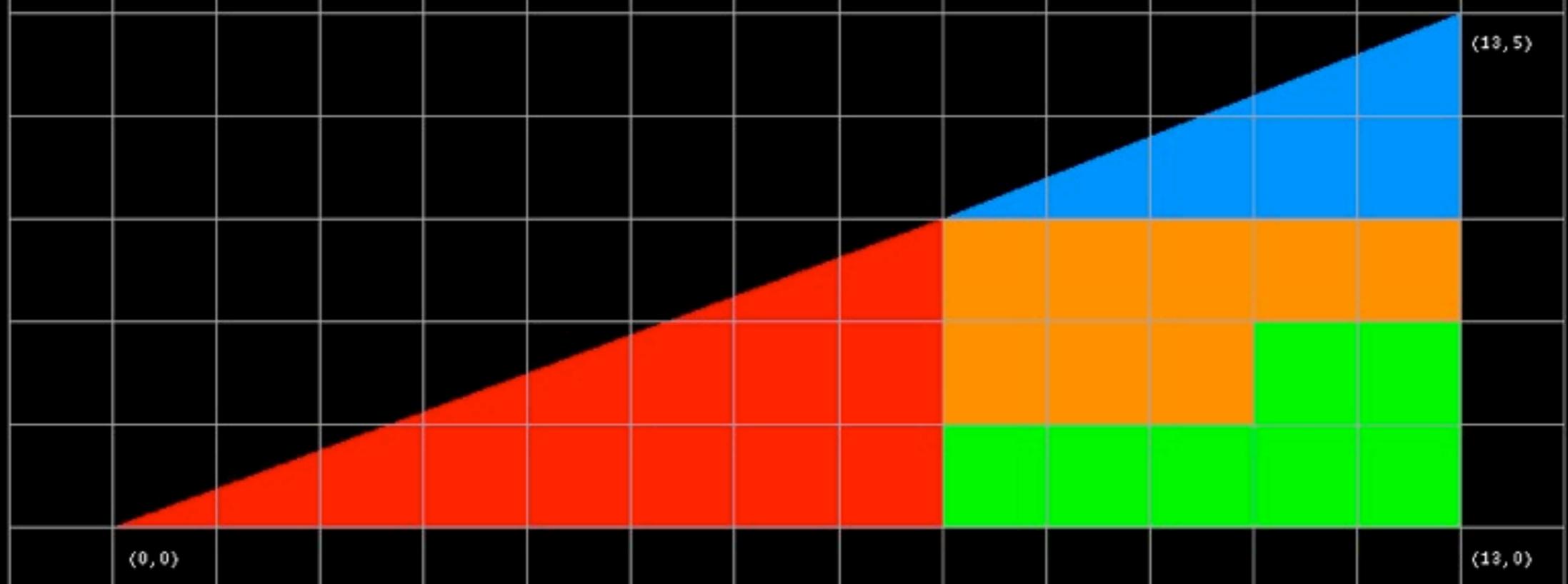


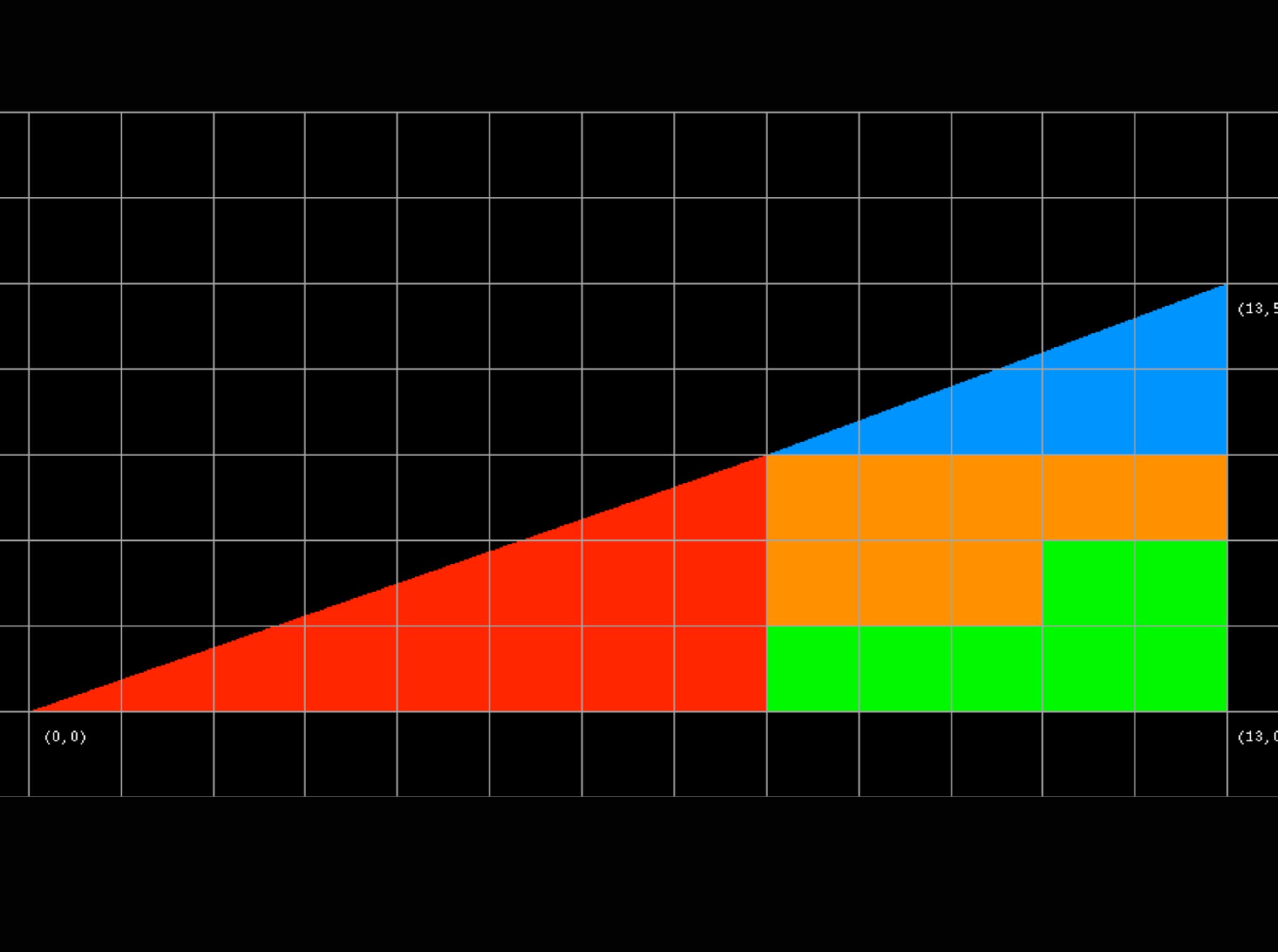












WISH LIST

Wanted: A Print Button for 3-D Objects

A lack of accessible design tools is holding back 3-D printing.

By Antonio Regalado on April 24, 2013



Detail oriented: A metal heat exchanger created using additive manufacturing contains small contours and internal shapes that would be hard to make

The largest companies in 3-D printing are racing to simplify design software so that it can become as easy to make an object as it is to send a document to a printer.

Interest in 3-D printing, also known as additive manufacturing, is exploding due to the falling cost of machines that can lay down finely targeted layers of plastic to make simple products, like jewelry or sculptures, much as a traditional printer sprays ink onto paper. The idea is that 3-D printers could democratize design and eventually manufacturing by letting anyone make physical things in small quantities, without the expense of an assembly line.

The technology still has a ways to go – making objects on consumer printers is slow and expensive. To print a solid plastic apple on MakerBot's \$2,000 consumer printer, for instance, takes seven hours and costs \$50 in supplies, so it's no competition for cheap plastic goods made in China.

[Click here](#)

Invest
Northern
Ireland

WHY IT MATTERS

One day 3-D printing could cheaply create customized objects that would be prohibitively expensive with traditional manufacturing methods.

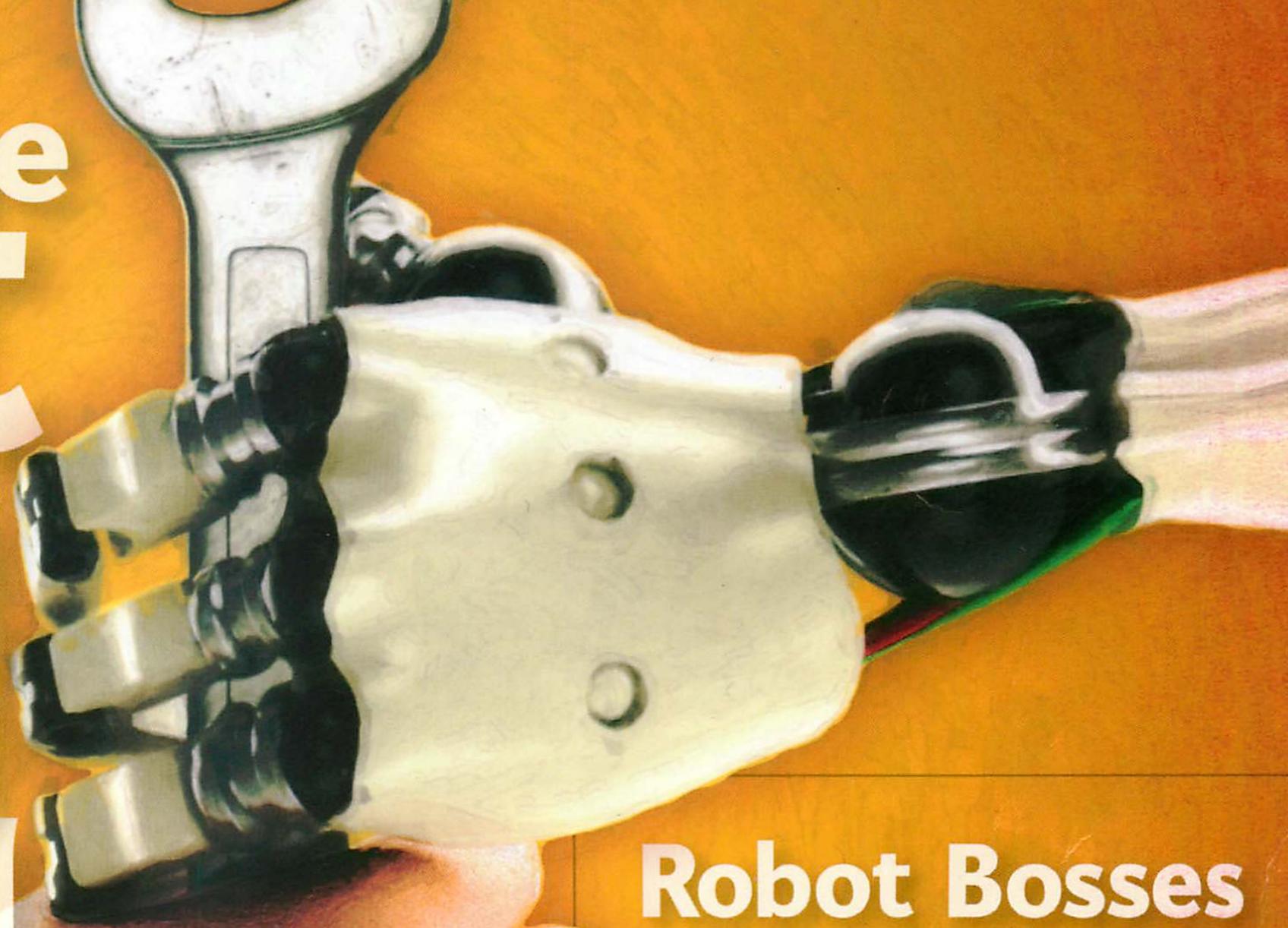


**SPECIAL
REPORT**

**How to
Make the**

Next Big Thing

**The Future of
Manufacturing**



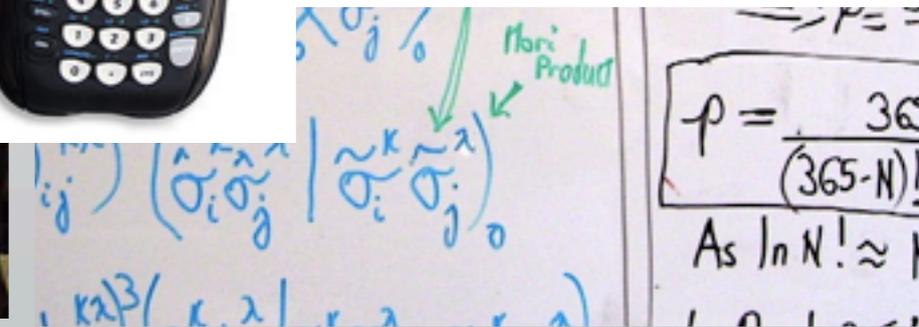
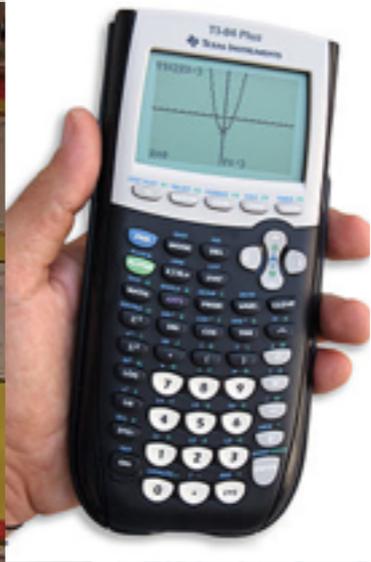
Robot Bosses

Next-Gen Materials

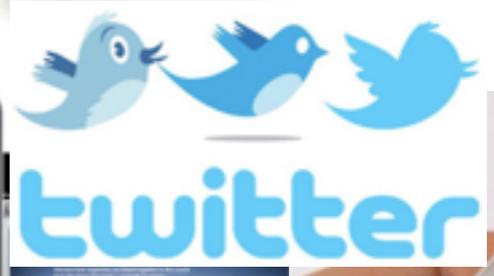
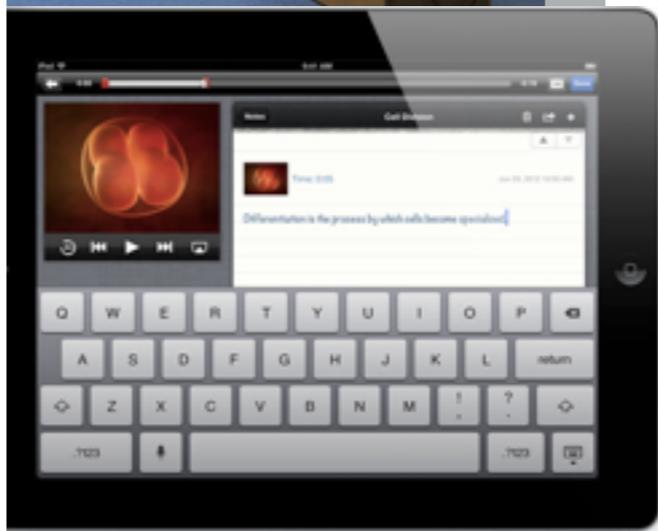
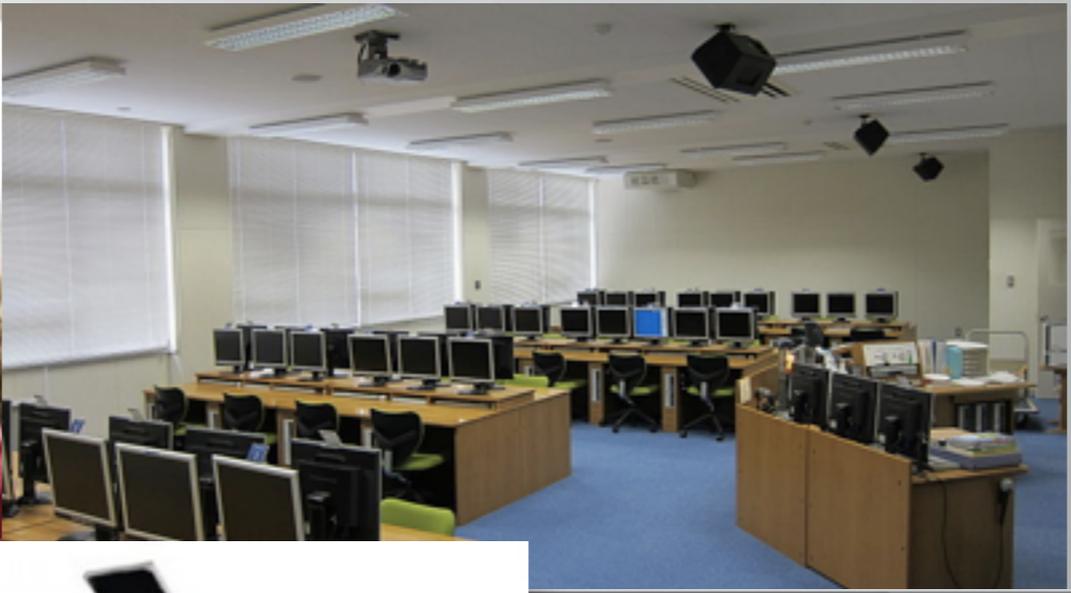
3-D Printing

**Nanotech
Machines**

Code Factories

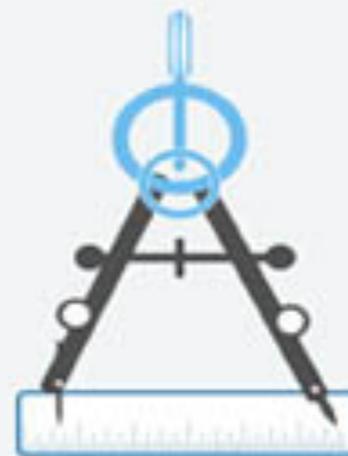


Media in the Classroom



30
YEARS

3D printers have actually been around for about 30 years. Barriers like cost are breaking down, so they're now becoming available to the public.



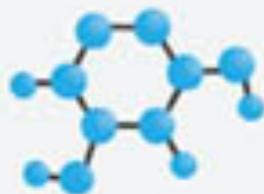
Printed objects can be incredibly intricate. They can also be created with working components, hinges, and parts within parts.

Biology students can study cross-sections of hearts or other organs.



REVOLUTIONIZING *the* **CLASSROOM**

Chemistry students could print out molecules to study.



Auto class students could print replacement or modified car parts.



3D printing has caught the attention of educators who are looking into ways to incorporate it into the classroom.

Using 3D printers in the classroom could mean:



Engineering and design students can print out prototypes of their creations.

Cooking class students could design intricate molds for ices and gelatins.



Graphic design students could create 3D versions of their artwork.



History classes could print out historic artifacts for closer examination.

Students in geography courses could print out maps showing the topography, population or demographics of an area.

How to Make the Next

Scientific american

PHYSICS EXPERIMENTS

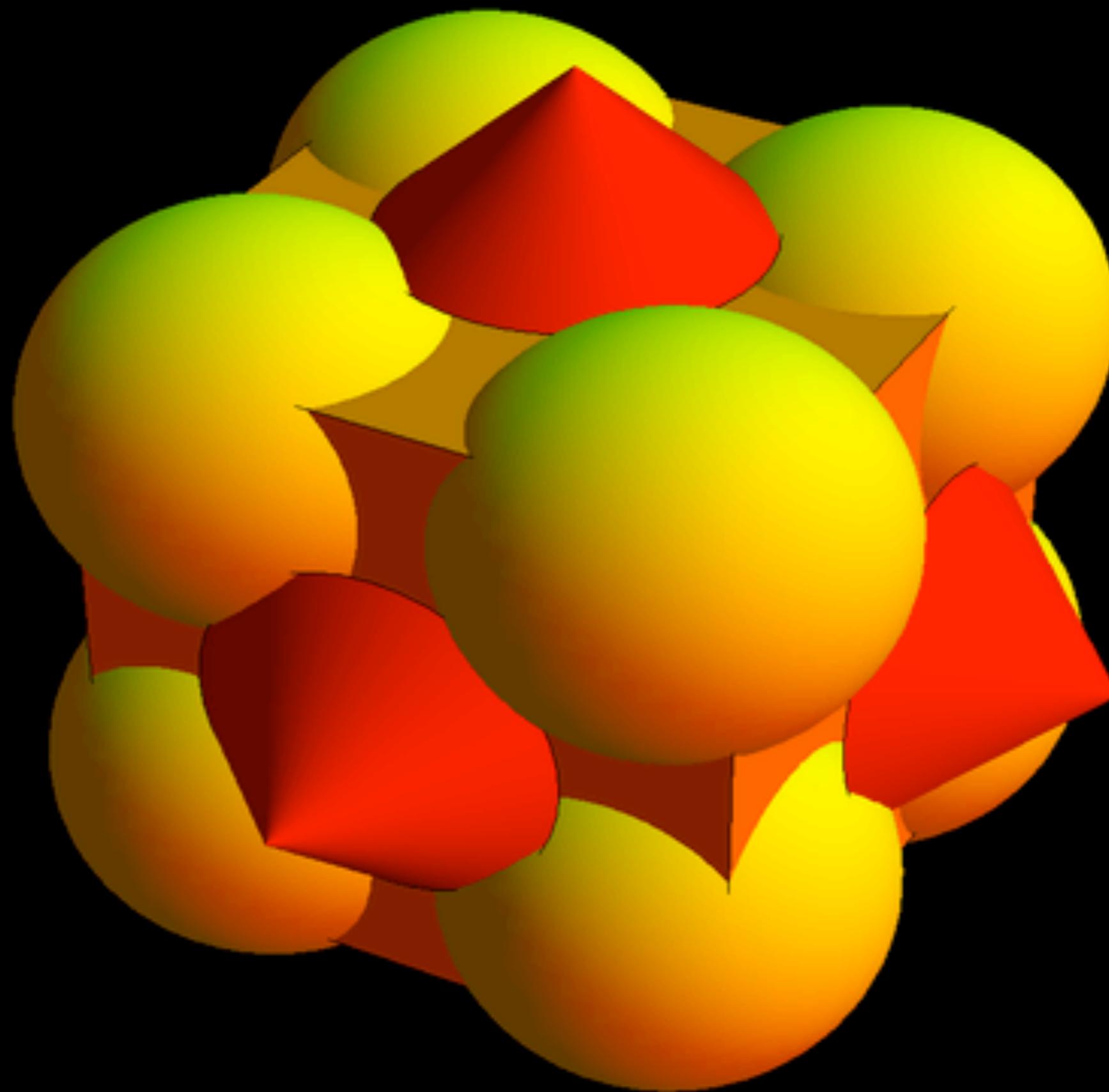




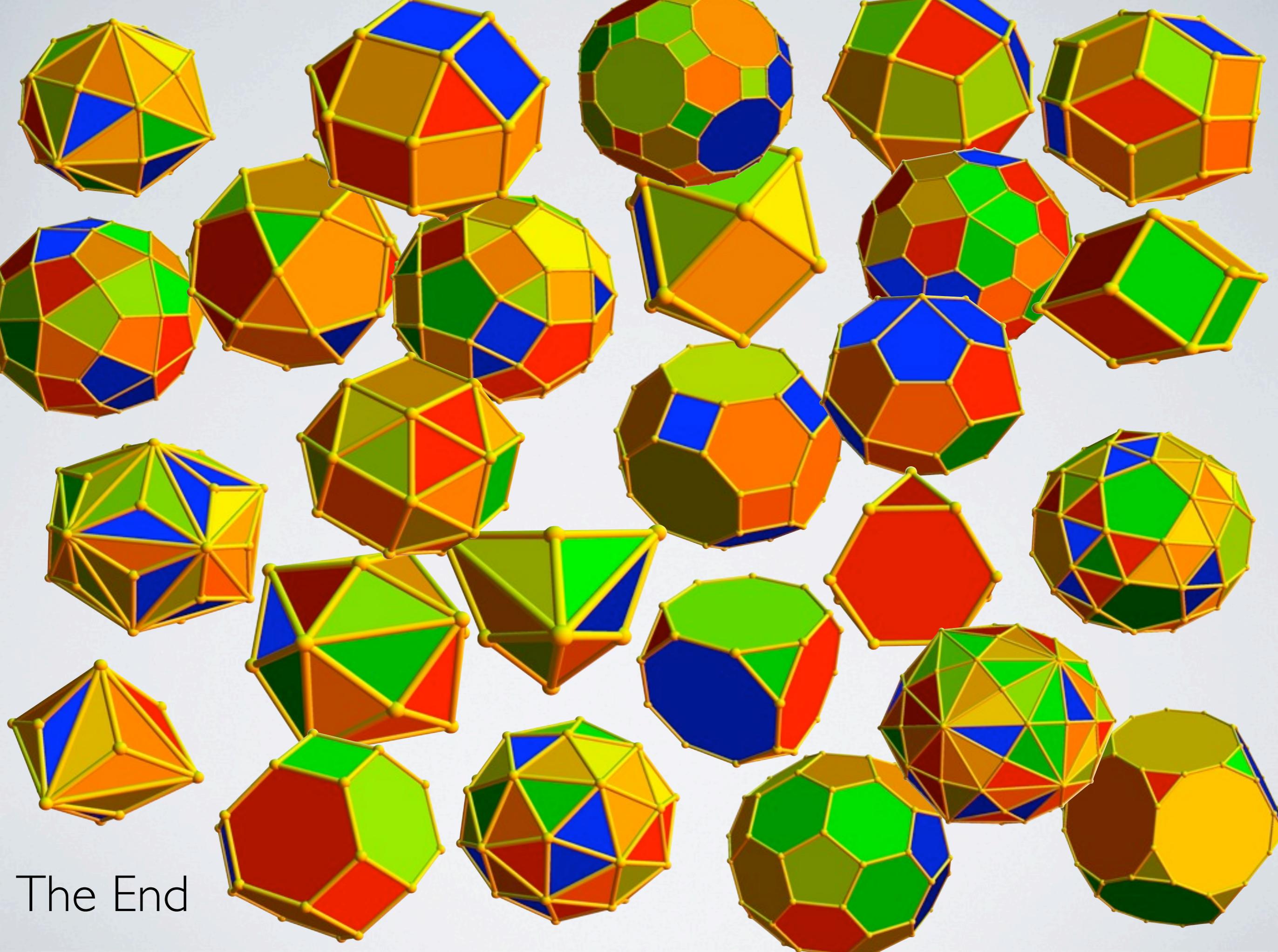


WISH LIST

- More open source conversion systems
- Computer algebra support of 3D printing
- More libraries with objects



Math 21a Class Fall 2012



The End