ENTRY ARTIFICIAL INTELLIGENCE

[ENTRY ARTIFICIAL INTELLIGENCE] Authors: Oliver Knill: March 2000 Literature: Peter Norvig, Paradigns of Artificial Intelligence Programming Daniel Juravsky and James Martin, Speech and Language Processing

Adaptive Simulated Annealing

[Adaptive Simulated Annealing] A language interface to a neural net simulator.

artificial intelligence

[artificial intelligence] (AI) is a field of computer science concerned with the concepts and methods of symbolic knowledge representation. AI attempts to model aspects of human thought on computers. Aspectrs of AI:

- computer vision
- language processing
- pattern recognition
- expert systems
- problem solving
- roboting
- optical character recognition
- artificial life
- grammars
- game theory

Babelfish

[Babelfish] Online translation system from Systran.

Chomsky

[Chomsky] Noam Chomsky is a pioneer in formal language theory. He is MIT Professor of Linguistics, Linguistic Theory, Syntax, Semantics and Philosophy of Language.

Eliza

[Eliza] One of the first programs to feature English output as well as input. It was developed by Joseph Weizenbaum at MIT. The paper appears in the January 1966 issue of the "Communications of the Association of Computing Machinery".

Google

[Google] A search engine emerging at the end of the 20'th century. It has AI features, allows not only to answer questions by pointing to relevant webpages but can also do simple tasks like doing arithmetic computations, convert units, read the news or find pictures with some content.

GPS

[GPS] General Problem Solver. A program developed in 1957 by Alan Newell and Herbert Simon. The aim was to write a single computer program which could solve any problem. One reason why GPS was destined to fail is now at the core of computer science. There are a large set of problems which are NP hard and where finding a solution becomes exponentially hard in dependence of the size of the problem. Nonetheless, GPS has been a useful tool for exploring AI programming.

HAL

[HAL] The HAL 9000 computer was the main character in Stanley Kurbrick's film 2001: a Space Odyssey. HAL is an AI agent capable to understand advanced language processing behavior as speaking and understanding language and even reading lips.

Lisp

[Lisp] Lisp is one of the oldest programming languages still in widespread use today. "Common Lisp" is the most widely accepted standard. Other dialects like "Franz Lisp" MacLisp, InterLisp, ZetaLisp or "Standard Lisp" are considered obsolete. Lisp is the most popular language for AI programming. Lisp programs are concise and are uncluttered by low-level detail.

Loebner Prize

[Loebner Prize] A competition attempted to put various computer programs to the Turing test. A consistent result over the years has been that even the crudest programs can fool some of the judges some of the time.

MIT ai lab

[MIT ai lab] Massachusetts Institute of Technology AI laboratory.

neural network

[neural network] Artificial neural networks try to simulate biological neural networks as found in the brain. Such a network consists of many simple processors called neurons, each possibly having some local memory. These neurons are connected and evolve depending to their local data and on the inputs they receive via the connections. A neural network can either be an algorithm, or be realized as actual hardware. Neural networks typically allow training. They learn by adjusting the weights of the connections on the basis of presented patterns. The individual neurons are elementary non-linear signal processors. Neural networks are distinguished from other computing devices by a high degree of interconnection allowing parallelism. There is no idle memory containing data and programs. Each neuron is pre-programmed and continuously active.

pattern recognition

[pattern recognition] A branch of artificial intelligence concerned with the classification or description of observations. The classification uses either statistical, syntactic or neural aproches.

pilot

[pilot] Programmed Inquiry Learning Or Teaching.

prolog

[prolog] A popular AI programming language used in Europe and Japan. Prolog shares most of Lisp's advantages in terms of flexibility and conciseness.

regular expression

[regular expression] is a language for specifying text search strings. It is used in UNIX programs like vi, perl, emacs or grep. It is also used in Microsoft word or web search engines.

scheme

[scheme] A dialect of Lisp which is gaining popularity, primarily for teaching and experimenting with programming language design and techniques.

Shrdlu

[Shrdlu] Terry Winograd's SHRDLU system of 1972 simulated a robot embedded in a world of toy blocks. The program was able to accept natural language text commands.

Student

[Student] Student was an early language understanding program written by Daniel Bubrow in 1964. It was designed to read and solve the kind of word problems found in high school algebra books. Unlike Eliza, "Student" must process and understand a great deal of input as well as be able to solve algebraic equations.

toy problem

[toy problem] A deliberately oversimplified case of a challenging problem used to investigate, prototype, or test algorithms for a real problem.

Turing test

[Turing test] A test introduced in 1950 by Alan Turing. There are three participants. Two people and a computer. One person plays the role of an interrogator who has to find out, which of the two others is a machine. This interrogator is connected to the two other participants through teletype. The task of the machine is to fool the interrogator into believing it is a person. The task of the other participant is to convince the interrogator that he is human. Turing predicted that in 2000 a machine with 10 Gig memory would have a 30 percent change of fooling a human interrogator after 5 minutes of questioning.

Weizenbaum

[Weizenbaum] Joseph Weizenbaum was the principal developer of Eliza, one of the first programs to feature English output as well as input.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 22 entries in this file.

Index

ENTRY ABSTRACT ALGEBRA

[ENTRY ABSTRACT ALGEBRA] Authors: started Oliver Knill: September 2003 Literature: Lecture notes

additive

A function $f : G \to H$ from a semigroup G to a semigroup H is [additive] if $f(a + b) = f(a) + f(b)$. A group-valued function on sets is additive if $f(Y \cup Z) = f(Y) + f(Z)$ if Y and Z are disjoint.

algebra

An [algebra] over a field K is a ring with 1 which is also a vector space over K and whose multiplication is bilinear with respect to K. Examples:

- the complex numbers C is an algebra over the field of real numbers $K = R$.
- The quaternion algebra H is an algebra over the field of complex numbers.
- The matrix algebra $M(n, R)$ is an algebra over the field R.

An algebraic number field

[An algebraic number field] is a subfield of the complex numbers that arises as a finite degree algebraic extension field over the field of rationals.

alternating group

The [alternating group] G is the subgroup of the symmetric group of n objects given by the elements which can be written as a product of an even number of transpositions.

Artinian module

An [Artinian module] is a module which satisfies the descending chain condition. Every Artinian module is a Noetherian module but the integers for example are a Noetherian module which is not an Artinian module.

Artinian ring

An [Artinian ring] is a ring which when considered as a R-module is an Artinian module.

Artinian ring

Two elements of an integral domain that are unit-multipliers of each other are called [associate numbers].

Cayley's theorem

[Cayley's theorem] assures that every finite group is isomorphic to a permutation group.

center

The [center] of a group $(G, *)$ is the set of all elements g which satisfy $gh = hg$ for all h in G. The center is a subgroup of G.

commutator

The [commutator] of two elements g, h in a group $(G, *)$ is defined as $[g, h] = g * h * g^{-1} * h^{-1}$.

commutator subgroup

The [commutator subgroup] of a group $(G, *)$ is the set of all commutators $[g, h]$ in G. It is a subgroup of G.

factor group

A [factor group] G/N is defined when N is a normal subgroup of the group G. It is the group, where the elements are equivalent classes gN and operation $(gN)(hN) = (gh)N$ which is defined because N was assumed to be normal. For example, if G is the group of additive integers and $N = kN$ with an integer k, then $G/N = Z_k$ is finite group of integers modulo k.

finite group

A group is called a [finite group] if G is a set with finitely many elements. For example, the set of all permutations of a finite set form a finite group. The set of all operations on the Rubik cube form a finite group.

group

A [group] $(X, +, 0)$ is a set X with a binary operation $+$ and a zero element 0 (also called neutral element or identity) with the following properties

$(a + b) + c = a + (b + c)$	associativity
$a + 0 = a$	zero element
$\forall a \exists b a + b = 0$	inverse

Examples:

- the real numbers form a group under addition $5 + 2.34 = 7.34, 3 - 3 = 0$.
- the set $GL(n, R)$ of real matrices with nonzero determinant form a group under matrix multiplication
- the nonzero integers form a group under multiplication $4 * 7 = 28$.
- all the invertible linear transformations of the plane plane form a group under composition. The "zero element" is the identity transformation $T(x) = x$.
- all the continuous functions on the unit interval form a group with addition $(f + g)(x) = f(x) + g(x)$.
- all the permutations on a finite set form a group under composition.
- the set of subsets Y of a set X with the operation $A \Delta B = (A \cup B) \setminus (A \cap B)$ form a group. The inverse of A is A itself because $A \Delta A = \emptyset$, the zero element is $\emptyset$.

normal subgroup

a [normal subgroup] of a group $(G, *)$ is a subgroup $(H, *)$ of $(G, *)$ which has the property that for all g in H and all g in G one has $g^{-1}hg$ is in H. For an abelian group all subgroups are normal. The subgroup $Sl(n, R)$ of $Gl(n, R)$ is a normal subgroup.

ring

A [ring] $(X, +, *, 0)$ is a set X with a binary operation $+$ and a binary operation $*$ such that $(X, +, 0)$ is a commutative group and $(X, *)$ is a semigroup and such that the distributivity laws $a * (b + c) = a * b + a * c$, $(a + b) * c - a * c + b * c$ hold. Examples:

- the integers Z form a ring with addition and multiplication
- the set of rational numbers Q, the set of real numbers R or the complext numbers C form a ring with addition and multiplication.
- the set of 3x3 matrices with real entries form a ring with addition and matrix multiplication.
- the set P of polynomials with real coefficients form a ring with addition and multiplication.
- the set of subsets Y of a set X with addition Δ and multiplication $\cap$ forms a ring.
- the set of continuous functions on an interval $[0, 1]$ with addition $(f+g)(x) = f(x)+g(x)$ and multiplication $f * g(x) = f(x)g(x)$.

commutative group

A [commutative group] is a group $(X, +, 0)$ which is commutative: $a + b = b + a$.

- the set of real numbers R forms a commutative group under addition.
- the set of permutations S of a set X form a noncommutative group under composition.

commutative ring

A [commutative ring] is a ring $(X, +, *, 0)$ for which the multiplicative semigroup $(X, *)$ is commutative: $a * b = b * a$. Examples:

- the integers form a commutative ring.
- the set of 2×2 matrices form a noncommutative ring
- the set of polyomials with real coefficients $(x^2 + \pi x + 2) * (x + 5x) = 6x^3 + 6\pi x^2 + 12x$.

function field

A [function field] is a finite extension of the field $C(z)$ of rational functions in the variable z.

homomorphism

An [homomorphism] ϕ between two groups G, H is a map $f : G \to H$ which has the property $\phi(g*h) = \phi(g)*\phi(h)$ and $\phi(0) = 0$ for all elements $g, h \in G$. Examples:

- if G is the multiplicative group $(R^+, *)$ of positive real numbers and H is the additive group $(R, +)$ of all positive real numbers then $\phi(x) = \log(x)$ is a homomorphism:
- if G is the group of matrices with nonzero determinant and H is the group of nonzero real numbers and $\phi(A) = \det(A)$, we have $\phi(x * y) = \phi(x)\phi(y)$.

isomorphism

An [isomorphism] ϕ between two groups G, H is a homomorphism between groups which is also invertible.

number field

A [number field] is a finite extension of Q, the field of rational numbers. It is a field extension of Q which is also a vector space of finite dimension over Q. Since the elements of a number field are algebraic numbers, roots of a fixed polyonomial $a_0 + a_1 z + ... + z^n$ with integer coefficitients, one calls them also algebraic number fields. The study of algebraic number fields is part of algebraic number theory.
Examples:

- quadratic fields: $Q(\sqrt{d})$, where d is a rational number. It is in general a field extension of degree 2 over the field of rational number.
- cyclotimic fields: $Q(\xi)$, where ξ is a n'th root of 1. It is a field extension of degree $\phi(n)$, where $\phi(n)$ is the Euler function.

octonions

The [octonions] can be written as linear combinations of elements $e_0, e_1, e_2, ..., e_7$. The multiplication is determined by the multiplication table

*	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
1	1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	-1	e_4	e_7	-e_2	e_6	-e_5	-e_3
e_2	e_2	-e_4	-1	e_5	e_1	-e_3	e_7	-e_6
e_3	e_3	-e_7	-e_5	-1	e_6	e_2	-e_4	e_1
e_4	e_4	e_2	-e_1	-e_6	-1	e_7	e_3	-e_5
e_5	e_5	-e_6	e_3	-e_2	-e_7	-1	e_1	e_4
e_6	e_6	e_5	-e_7	e_4	-e_3	-e_1	-1	e_2
e_7	e_7	e_3	e_6	-e_1	e_5	-e_4	-e_2	-1

Octonions are also called Cayley numbers. The multiplication of octonions is not associative. Octonions have been discovered by John T. Graves in 1843 and independently by Arthur Cayley.

order

The [order] of a finite group is the set of elements in the group.

p-group

A [p-group] is a finite group with order p^n, where p is a prime integer and $n > 0$.

quaterions

The [quaterions] can be written as linear combinations of elements $1, i, j, k$. The multiplication is determined by the multiplication table

*	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

Quaternions are useful to compute rotations in three dimensions.

semigroup

A [semigroup] $(X, +)$ is a set X with a binary operation $+$ which satisfies the associativity law $(a + b) + c = a + (b + c)$. Examples:

- a group is a semigroup.
- the set of finite words in an alphabet with composition form a semigroup $word1 + word2 = word1word2$
- the natural numbers form a semigroup under addition.

commutative semigroup

A [commutative semigroup] is a semigroup $(X, +)$ which is commutative. $a + b = b + a$.

- the natural numbers form a commutative semigroup under addition.
- composition of words over a finite alphabet form a noncommutative semigroup

kernel

The [kernel] of a homomorphism between two groups G, H is the set of all elements in G which are maped to the zero element of H. For example, $SL(n, R)$ is the kernel of the homomorphism from $GL(n, R)$ to $R \setminus \{0\}$ defined by $\phi(A) = \det(A)$.

subgroup

A [subgroup] of a group G is a subset of G which is also a group. Examples:

- the set of $n \times n$ matrices with determinant 1 is a subgroup of the set of $n \times n$ matrices with nonzero determinant.
- the trivial subgroup $\{0\}$ is always a subgroup of a group $(G, *, 0)$.

Theorem of Cauchy

The [Theorem of Cauchy] in group theory states that every finite group whose order is divisible by a prime number p contains a subgroup of order p.

sedenions

[sedenions] form a zero Divisor Algebra. By a theorem of Frobenius (1877), there are three and only three associative finite division algebras: the real numbers R, the complex numbers C and the quaternions Q. Similar algebras in higher dimensions have zero divisors: sedenions are examples.

field

A [field] is a commutative ring $(R, +, *, 0, 1)$ such that $(R, +, 0)$ and $(R \setminus 0, *, 1)$ are both commutative groups.

theorem of Zorn

By a [theorem of Zorn] (1933), every alternative, quadratic, real non-associative algebra without zero divisors is isomorphic to the 8-dimensional octonions O.

Theorem of Hurwitz

[Theorem of Hurwitz]: the normed composition algebras with unit are: real numbers, complex numbers, quaternions; and octonions.

Theorem of Kervaire and Milnor

[Theorem of Kervaire and Milnor] In 1958, Kervaire and Milnor proved independently of each other that the finite-dimensional real division algebras have dimensions $1, 2, 4$, or 8.

Theorem of Adams

[Theorem of Adams] In 1960, Adams proved that a continuous multiplication in R^{n+1} with two-sided unit and with norm product exists only for $n + 1 = 1, 2, 4$, or 8.

Theorem of Hurwitz

[Theorem of Hurwitz]: the normed composition algebras with unit are:

- real numbers
- complex numbers
- quaternions
- octonions

Theorems of Sylov

[Theorems of Sylov] If G is a finite group of order $|G| = p^n q$, where p is a prime number, then G has a subgroup of order p^n. Such groups are called Sylov groups and all of them are isomorphic. Furthermore, the number N of different p-Sylov groups in G satisfies $N = 1 \bmod(p)$.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 39 entries in this file.

Index

AMS FIELDS

[AMS FIELDS] Authors: Oliver Knill: September 2003 Literature: AMS Website

AMS CLASSIFICATION

[AMS CLASSIFICATION]

00-xx	General
01-xx	History and biography
03-xx	Mathematical logic and foundations
05-xx	Combinatorics
06-xx	Order, lattices, ordered algebraic structures
08-xx	General algebraic systems
11-xx	Number theory
12-xx	Field theory and polynomials
13-xx	Commutative rings and algebras
14-xx	Algebraic geometry
15-xx	Linear and multilinear algebra; matrix theory
16-xx	Associative rings and algebras
17-xx	Nonassociative rings and algebras
18-xx	Category theory; homological algebra
19-xx	K-theory
20-xx	Group theory and generalizations
22-xx	Topological groups, Lie groups
26-xx	Real functions
28-xx	Measure and integration
30-xx	Functions of a complex variable
31-xx	Potential theory
32-xx	Several complex variables and analytic spaces
33-xx	Special functions
34-xx	Ordinary differential equations
35-xx	Partial differential equations
37-xx	Dynamical systems and ergodic theory
39-xx	Difference and functional equations
40-xx	Sequences, series, summability
41-xx	Approximations and expansions
42-xx	Fourier analysis
43-xx	Abstract harmonic analysis
44-xx	Integral transforms, operational calculus
45-xx	Integral equations
46-xx	Functional analysis
47-xx	Operator theory
49-xx	Calculus of variations and optimal control; optimization
51-xx	Geometry
52-xx	Convex and discrete geometry
53-xx	Differential geometry
54-xx	General topology
55-xx	Algebraic topology
57-xx	Manifolds and cell complexes
58-xx	Global analysis, analysis on manifolds
60-xx	Probability theory and stochastic processes
70-xx	Mechanics of particles and systems
74-xx	Mechanics of deformable solids
76-xx	Fluid mechanics
78-xx	Optics, electromagnetic theory
80-xx	Classical thermodynamics, heat transfer
81-xx	Quantum theory
82-xx	Statistical mechanics, structure of matter
83-xx	Relativity and gravitational theory
85-xx	Astronomy and astrophysics
86-xx	Geophysics
90-xx	Operations research, mathematical programming
91-xx	Game theory, economics, social and behavioral sciences
92-xx	Biology and other natural sciences
93-xx	Systems theory; control
94-xx	Information and communication, circuits
97-xx	Mathematics education

Index

ENTRY MATH CITATIONS

[ENTRY MATH CITATIONS] Collected by Oliver Knill: 2000-2002

solution

[solution] Every problem in the calculus of variations has a solution, provided the word solution is suitably understood. – David Hilbert

enhusiast

[enhusiast] The real mathematician is an enthusiast per se. Without enthusiasm no mathematics. – Novalis

royal

[royal] There is no royal road to geometry. – Euclid

computer

[computer] One may be a mathematician of the first rank without being able to compute. It is possible to be a great computer without having the slightest idea of mathematics – Novalis

analysis

[analysis] Geometry may sometimes appear to take the lead over analysis, but in fact precedes it only as a servant goes before his master to clear the path and light him on the way. – James Joseph Sylvester

freedom

[freedom] The essence of mathematics lies in its freedom. – Georg Cantor

fantasy

[fantasy] Fantasy, energy, self-confidence and self-criticism are the characteristic endowments of the mathematician. – Sophus Lie

magacian

[magacian] Pure mathematics is the magician's real wand. – Novalis

axiomatics

[axiomatics] When a mathematician has no more ideas, he pursues axiomatics. – Felix Klein

turbulence

[turbulence] The paper "On the nature of turbulence" with F. Takens was eventually published in a scientific journal. (Actually, I was an editor of the journal, and I accepted the paper by myself for publication. This is not a recommended procedure in general, but I felt that it was justified in this particular case). – D. Ruelle, in Chance and Chaos

hairy-ball

[hairy-ball] A good topological theorem to mention any time is the theorem which, in essence, states that however you try to comb the hair on a hairy ball, you can never do it smoothly - the so-called 'hairy-ball' theorem. You can make snide comments about the grooming of the hosts' dog or cat in the meantime as you pick hairs off your trouser leg. – R. Ainsley in Bluff your way in Maths, 1988

large

[large] LARGE NUMBERS: (10^n means that 10 is raised to the n'th power)

10^4	One "myriad". The largest numbers, the Greeks were considering.
10^5	The largest number considered by the Romans.
10^{10}	The age of our universe in years.
10^{22}	Distance to our neighbor galaxy Andromeda in meters.
10^{23}	Number of atoms in two gram Carbon (Avogadro).
10^{26}	Size of universe in meters.
10^{41}	Mass of our home galaxy "milky way" in kg.
10^{51}	Archimedes's estimate of number of sand grains in universe.
10^{52}	Mass of our universe in kg.
10^{80}	The number of atoms in our universe.
10^{100}	One "googol". (Name coined by 9 year old nephew of E. Kasner).
10^{153}	Number mentioned in a myth about Buddha.
10^{155}	Size of ninth Fermat number (factored in 1990).
$10^{(10^6)}$	Size of large prime number (Mersenne number, Nov 1996).
$10^{(10^7)}$	Years, ape needs to write "hound of Baskerville" (random typing).
$10^{(10^(33))}$	Inverse is chance that a can of beer tips by quantum fluctuation.
$10^{(10^(42))}$	Inverse is probability that a mouse survives on sun for a week.
$10^{(10^{51}))}$	Inverse is chance to find yourself on Mars (quantum fluctuations)
$10^{(10^{100})}$	One "Gogoolplex", Decimal expansion can not exist in universe.

– from R.E. Crandall, Scient. Amer., Feb. 1997

analytic

[analytic] The statement sometimes made, that there exist only analytic functions in nature, is in my opinion absurd. – F. Klein, Lectures on Mathematics, 1893

violence

[violence] The introduction of numbers as coordinates ... is an act of violence... – H. Weyl, Philosophy of Mathematics and Natural Science 1949

beauty

[beauty] Mathematics possesses not only truth but supreme beauty - a beauty cold and austere, like that of a sculpture – Bertrand Russell

geometry

[geometry] Geometry is magic that works... – R. Thom. Stability Structurelle et Morphogenese, 1972

Zermelo

[Zermelo] Ernst Zermelo, who created a system of axioms for set theory, was a Privatdozent at Goettingen when Herr Geheimrat Felix Klein held sway over the fabled mathematics department. As Pauli told it, ”Zermelo taught a course on mathematical logic and stunned his students by posing the following question: All mathematicians in Goettingen belong to one of two classes. In the first class belong those mathematicians who do what Felix Klein likes, but what they dislike. In the second class are those mathematicians who do what Felix Klein likes, but what they dislike. To what class does Felix Klein belong?” Jordan, having listened intently, broke into roaring laughter. Pauli paused, took a sip of wine and said disapprovingly, ”Herr Jordan, you have laughed too soon”. He continued: ”None of the awed students could solve this blasphemous problem. Zermelo then crowed in his high-pitched voice, ’But, meine Herren, it’s very simple. Felix Klein isn’t a mathematician.’” Jordan laughed again. Pauli drained his wine glass approvingly and concluded with ”Zermelo was not offered a professorship at Goettingen”. – E.L. Schucking, in ’Jordan, Pauli,Politics, Brecht and a variable gravitational constant’ Physics Today, Oct. 1999

Conway

[Conway] In the beginning, everything was void, and J.H.W.H.Conway began to create numbers. Conway said, ”Let there be two rules which bring forth all numbers large and small. This shall be the first rule: Every number corresponds to two sets of previously created numbers, such that no member of the left set is greater than or equal to any member of the right set. And the second rule shall be this: One number is less than or equal to another number if and only if no member of the first number’s left set is greater than or equal to the second number, and no member of the second number ’s right set is less than or equal to the first number.” And Conway examined these two rules he had made, and behold! they were very good. And the first number was created from the void left set and the void right set. Conway called this number ”zero”, and said that it shall be a sign to separate positive numbers from negative numbers. Conway proved that zero was less than or equal to zero, and he saw that it was good. And the evening and the morning were the day of zero. On the next day, two more numbers were created, one with zero as its left set and one with zero as its right set. And Conway called the former number ”one”, and the latter he called ”minus one”. And he proved that minus one is less than but not equal to zero and zero is less than but not equal to one. And the evening... – D. Knuth, Surreal numbers, 1979

obvious

[obvious] Mathematics consists essentially of :
a) proving the obvious
b) proving the not so obvious
c) proving the obviously untrue

For example, it took mathematicians until the 1800'ies to prove that 1+1=2 and not before the late 1970 were they confident of proving that any map requires no more than four colors to make it look nice, a fact known by cartographers for centuries. There are many not-so-obvious things which can be proved true too. Like the fact that for any group of 23 people, there is an even chance two or more of them share birthdays. (With groups of twins this becomes almost certain. Not quite certain as you will of course point out: they might all have been born either side of midnight). Mathematicians are also fond of proving things which are obviously false, like all straight lines being curved, and an engaged telephone being just as likely to be free if you ring again immediately after, as if you wait twenty minutes. – R. Ainsley in Bluff your way in Maths, 1988

infimum

[infimum] There exists a subset of the real line such that the infimum of the set is greater then the supremum of the set. – Gary L. Wise and Eric B. Hall, Counter examples in probability and real analysis, 1993, First Example in book

transcendental

[transcendental] Transcendental number : A number which is not the root of any polynomial equation, like pi and e, and which can only be understood after several hours meditation in the lotus position. – R. Ainsley in Bluff your way in Maths, 1988

illiteracy

[illiteracy] There are great advantages to being a mathematician: a) you do not have to be able to spell b) you do not have to be able to add up The illiteracy of mathematicians is taken for granted. There still persists a myth that mathematics somehow involves numbers. Many fondly believe that university students spend their time long dividing by 173 and learning their 39 times table; in fact, the reverse is true. Mathematicians are renowned for their inability to add up or take away, in much the same way as geographers are always getting lost, and economists are always borrowing money off you. – R. Ainsley in Bluff your way in Maths, 1988

prime

[prime] In this note we would like to offer an elementary 'topological' proof of the infinitude of the prime numbers. We introduce a topology into the space of integers S, by using the arithmetic progressions (from -infinity to +infinity) as a basis. It is not difficult to verify that this actually yields a topological space. In fact, under this topology, S may be shown to be normal and hence metrisable. Each arithmetic progression is closed as well as open, since its complement is the union of the other arithmetic progressions (having the same difference). As a result, the union of any finite number of arithmetic progressions is closed. Consider now the set A which is the union of A(p), where A(p) consists of primes greater or equal to p. The only numbers not belonging to A are -1 and 1, and since the set -1,1 is clearly not an open set, A cannot be closed. Hence A is not a finite union of closed sets, which proves that there is an infinity of primes. – H. Fuerstenberg, On the infinitude of primes, American Mathematical Montly, 62, 1955, p. 353

barber

[barber] The barber in a certain town shaves all the people who don't shave themselves. Who shaves the barber? This is meant to be a clever little paradox with no solution but you can annoy the asker intensely by saying it's easy and that the barber is a women. You can then ask the following (a version of Russell's Paradox, - point this out too): in a library there are some books for the catalogue section which is a list of all books which don't list themselves. Shold he or she include this book in its own list? If so, then it becomes a book which lists itself, so it shouldn't be in the list of books which don't and vice versa. This should keep the most determined assailant at bay while you attack the wine. – R. Ainsley in Bluff your way in Maths, 1988

Hadamard

[Hadamard] Hadamard, trying to find a job in a US university, came to a small university and was received by the chairman of the department of mathematics. He explained who he was and gave his curriculum vitae. The chairman said: 'our means are very limited and I can not promise that we shal take you'. Then Hadamard noticed that among the portraits on he wall was his own. 'That's me!' he said. 'Well, come again in a week, we shal think about this'. On his next visit, the answer was negative and his portrait had been removed. – Vladimir Mazya and Tatyana Shaposhnikova, in Jacques Hadamard, a universal Mathematician, AMS History of Mathematics Volume 14

Cantor

[Cantor] The appropriate object is known as the Cantor set, because it was discovered by Henry Smith in 1875. (The founder of set theory, Georg Cantor, used Smith's invention in 1883. Let's fact it, 'Smith set' isn't very impressive, is it?) – Ian Stewart, in Does God Play Dice, 1989 p. 121

jouissance

[jouissance] ... Thus the erectile organ comes to symbolize the place of jouissance, not in itself, or even in the form of an image, but as a part lacking in the desired image: that is why it is equivalent to the $(-1)^(1/2)$ of the signification produced above, of the Jouissance that it restores by the coefficient of its statement to the function of lack of signifier (-1).
– Lacan, Ecrits, Paris 1966 (cited in 'Fashionable nonsense' by Alan Sokal and Jean Bricmont)

Mandelbrot

[Mandelbrot] Mandelbrot made quite good computer pictures, which seemed to show a number of isolated "islands" (in the Mandelbrot set M). Therefore, he conjectured that the set M has many distinct connected components. (The editors of the journal thought that his islands were specks of dirt, and carefully removed them from the pictures). – John Milnor, in Dynamics in one complex variable, 1991

sin

[sin] sin, cos, tan, cot, sec, cosec - Formulae derived from the sides of triangles but which crop up in completely unexpected places. Sins are extremely common, but rarely do you encounter secs in mathematics. – R. Ainsley in Bluff your way in Maths, 1988

Moser

[Moser] This reminds me of the Hilbert story, which I learned from my teacher Franz Rellich in Goettingen: When Hilbert - who was old and retired - was asked at a party by the newly appointed Nazi-minister of education: "Herr Geheimrat, how is mathematics in Goettingen, now that it has been freed of the Jewish influences" he replied: "Mathematics in Goettingen? That does not EXIST anymore". – Jurgen Moser, in Dynamical Systems-Past and Present, Doc. Math. J. DMV I p. 381-402, 1998

wine

[wine] There are two glasses of wine, one white and one red. A teaspoonful of wine is taken from the red and mixed in with the white. Then a teaspoonful of this mixture is taken and mixed in with the red. Which is bigger, the amount of red in the white or the amount of white in the red? The answer is that the're both the same, because there's the same volume in each glass, so whatever quantity of red is in the white must be equal to the quantity of white in the red. However in practice it is impossible to do this because the white always runs out first at parties and the red always gets spilt on someone's white trousers. – R. Ainsley in Bluff your way in Maths, 1988

Monty-Hall

[Monty-Hall] "Suppose you're on a game show and you are given a choice of three doors. Behind one door is a car and behind the others are goats. You pick a door-say No. 1 - and the host, who knows what's behind the doors, opens another door-say, No. 3-which has a goat. (In all games, the host opens a door to reveal a goat). He then says to you, "Do you want to pick door No. 2?" (In all games he always offers an option to switch). Is it to your advantage to switch your choice?" – The three doors problem, also known as Monty-Hall Problem

sex

[sex] Pure mathematician - Anyone who prefers set theory to sex. – R. Ainsley in Bluff your way in Maths, 1988

mad

[mad] There was a mad scientist (a mad ...social... scientist) who kidnaped three colleagues, an engineer, a physicist, and a mathematician, and locked each of them in separate cells with plenty of canned food and water but no can opener. A month later, returning, the mad scientist went to the engineer's cell and found it long empty. The engineer had constructed a can opener from pocket trash, used aluminum shavings and dried sugar to make an explosive, and escaped. The physicist had worked out the angle necessary to knock the lids off the tin cans by throwing them against the wall. She was developing a good pitching arm and a new quantum theory. The mathematician had stacked the unopened cans into a surprising solution to the kissing problem; his dessicated corpse was propped calmly against a wall, and this was inscribed on the floor in blood:
Theorem: If I can't open these cans, I'll die. Proof: assume the opposite...

induction

[induction] Proof by induction - A very important and powerful mathematical tool, because it works by assuming something is true and then goes on to prove that therefore it is true. Not surprisingly, you can prove almost everything by induction. So long as the proof includes the following phrases:
a) Assume true for n; then also true for n+1 because.. (followed by some plausible but messy working out in which n, n+1 appear prominently).
b) But is true for n=0 (a little more messy working out with lots of zeros sprayed at random through the proof).
c) So is true for all n. Q.E.D.
– R. Ainsley in Bluff your way in Maths, 1988

horse

[horse] LEMMA: All horses are the same color. Proof (by induction): Case n=1: In a set with only one horse, it is obvious that all horses in that set are the same color. Case n=k: Suppose you have a set of k+1 horses. Pull one of these horses out of the set, so that you have k horses. Suppose that all of these horses are the same color. Now put back the horse that you took out, and pull out a different one. Suppose that all of the k horses now in the set are the same color. Then the set of k+1 horses are all the same color. We have k true =¿ k+1 true; therefore all horses are the same color.
THEOREM: All horses have an infinite number of legs. Proof (by intimidation): Everyone would agree that all horses have an even number of legs. It is also well-known that horses have fore-legs in front and two legs in back. But 4 + 2 = 6 legs is certainly an odd number of legs for a horse to have! Now the only number that is both even and odd is infinity; therefore all horses have an infinite number of legs. However, suppose that there is a horse somewhere that does not have an infinite number of legs. Well, that would be a horse of a different color; and by the Lemma, it doesn't exist. QED

dean

[dean] Dean, to the physics department. "Why do I always have to give you guys so much money, for laboratories and expensive equipment and stuff. Why couldn't you be like the maths department - all they need is money for pencils, paper and waste-paper baskets. Or even better, like the philosophy department. All they need are pencils and paper."

astronomer

[astronomer] An astronomer, a physicist and a mathematician were holidaying in Scotland. Glancing from a train window, they observed a black sheep in the middle of a field. "How interesting," observed the astronomer, "all Scottish sheep are black!" To which the physicist responded, "No, no! Some Scottish sheep are black!" The mathematician gazed heavenward in supplication, and then intoned, "In Scotland there exists at least one field, containing at least one sheep, at least one side of which is black." – J. Steward in 'Concepts of Modern Mathematics'

coffee

[coffee] An engineer, a chemist and a mathematician are staying in three adjoining cabins at an old motel. First the engineer's coffee maker catches fire. He smells the smoke, wakes up, unplugs the coffee maker, throws it out the window, and goes back to sleep. Later that night the chemist smells smoke too. He wakes up and sees that a cigarette butt has set the trash can on fire. He says to himself, "Hmm. How does one put out a fire? One can reduce the temperature of the fuel below the flash point, isolate the burning material from oxygen, or both. This could be accomplished by applying water." So he picks up the trash can, puts it in the shower stall, turns on the water, and, when the fire is out, goes back to sleep. The mathematician, of course, has been watching all this out the window. So later, when he finds that his pipe ashes have set the bed-sheet on fire, he is not in the least taken aback. He says: "Aha! A solution exists!" and goes back to sleep.

logs

[logs] Taking logs - Broadly speaking, any equation which looks difficult will look much easier when logs are taken on both sides. Taking logs on one side only is tempting for many equations, but may be noticed. – R. Ainsley in Bluff your way in Maths, 1988

cat

[cat] Theorem: A cat has nine tails. Proof: No cat has eight tails. A cat has one tail more than no cat. Therefore, a cat has nine tails.

chocolate

[chocolate] Prime number - A number with no divisors. Boxes of chocolates always contain a prime number so that, whatever the number of people present, somebody has to have that one left over. – R. Ainsley in Bluff your way in Maths, 1988

aleph

[aleph] Aleph-null bottles of beer on the wall, Aleph-null bottles of beer, You take one down, and pass it around, Aleph-null bottles of beer on the wall.

qed

[qed] At the end of a proof you write Q.E.D, which stands not for Quod Erat Demonstrandum as the books would have you believe, but for Quite Easily Done. – R. Ainsley in Bluff your way in Maths, 1988

1+1

[1+1] $1+1 = 3$, for large values of 1

painting

[painting] Group theory - An exceedingly beautiful branch of pure mathematics used for showing in how many ways blocks of wood can be painted. – R. Ainsley in Bluff your way in Maths, 1988

engeneer

[engeneer]

Mathematician:	3 is prime,5 is prime,7 is prime, by induction - every odd integer higher than 2 is prime.
Physicist:	3 is prime, 5 is prime, 7 is prime, 9 is an experimental error, 11 is prime,...
Engineer:	3 is prime, 5 is prime, 7 is prime, 9 is prime, 11 is prime,...
Programmer:	3's prime, 5's prime, 7's prime, 7's prime, 7's prime,...
Salesperson:	3 is prime, 5 is prime, 7 is prime, 9 – we'll do for you the best we can,...
Software seller:	3 is prime, 5 is prime, 7 is prime, 9 will be prime in the next release,...
Biologist:	3 is prime, 5 is prime, 7 is prime, 9 – results have not arrived yet,...
Advertiser:	3 is prime, 5 is prime, 7 is prime, 11 is prime,...
Lawyer:	3 is prime, 5 is prime, 7 is prime, 9 – there is not enough evidence to prove that it is not prime,...
Accountant:	3 is prime, 5 is prime, 7 is prime, 9 is prime, deducing 10 percent tax and 5 percent other obligations.
Statistician:	Let's try several randomly chosen numbers: 17 is prime, 23 is prime, 11 is prime...
Psychologist:	3 is prime, 5 is prime, 7 is prime, 9 is prime but tries to suppress it,...

pi

[pi] PI= 3.14159265358979323846264338327950288419716939937510582097494459230781640628

e

[e] Euler E= 2.71828182845904523536028747135266249775724709369995957496696762772407663035

cancel

[cancel] THEOREM: The limit as n goes to infinity of sin x/n is 6. PROOF: cancel the n in the numerator and denominator.

coffee

[coffee] A mathematician is a device for turning coffee into theorems. – P. Erdos

stupider

[stupider] Finally I am becoming stupider no more. – Epitaph, P. Erdos wrote for himself

Erdoes

[Erdoes]

epsilon	child
bosses	women
slaves	men
captured	married
liberated	divorced
recaptured	remarried
trivial beings	nonmathematicians
noise	music
poison	alcohol
preaching	giving a lecture
supreme fascist	god
died	stopped doing mathematics
preach	lecture
Joedom	UDSSR
Samland	USA

on the long wave length communists on the short wave length fashists – from the vocabulary of P. Erdos 'the man who loved only numbers'

Chebyshev

[Chebyshev] Chebyshev said it, and I say it again There is always a prime between n and 2n – P. Erdos

Outrage

[Outrage] Outrage, disgust, the characterization of group theory as a plague or as a dragon to be slain - this is not an atypical physist's reaction in the 1930s-50s to the use of group theory in physics. – S. Sternberg

digits

[digits] Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin. – J. von Neumann

poet

[poet] The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics... It may be very hard to define mathematical beauty, but that is just as true of beauty of any kind - we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognizing one when we read it. – G.H. Hardy

melancholy

[melancholy] It is a melancholy experience for a professional mathematician to find himself writing about mathematics. – G.H. Hardy

Hilbert

[Hilbert] There is a much quoted story about David Hilbert, who one day noticed that a certain student had stopped attending class. When told that the student had decided to drop mathematics to become a poet, Hilbert replied, "Good- he did not have enough imagination to become a mathematician". – R. Osserman

refreree

[refreree] Referee's report: This paper contains much that is new and much that is true. Unfortunately, that which is true is not new and that which is new is not true. – H. Eves 'Return to Mathematical Circles', 1988.

weapons

[weapons] Structures are the weapons of the mathematician. – N. Bourbaki

undogmatic

[undogmatic] Mathematics is the only instructional material that can be presented in an entirely undogmatic way. – M. Dehn

solve

[solve] Each problem that I solved became a rule which served afterwards to solve other problems – R. Decartes

tool

[tool] For a physicist mathematics is not just a tool by means of which phenomena can be calculated, it is the main source of concepts and principles by means of which new theories can be created. – F. Dyson

sheet

[sheet] If the entire Mandelbrot set were placed on an ordinary sheet of paper, the tiny sections of boundary we examine would not fill the width of a hydrogen atom. Physicists think about such tiny objects; only mathematicians have microscopes fine enough to actually observe them. – J. Eving

recommendation

[recommendation] Sample letter of recommendation:
Dear Search Committee Chair, I am writing this letter for Mr. Still Student who has applied for a position in your department. I should start by saying that I cannot recommend him too highly. In fact, there is no other student with whom I can adequately compare him, and I am sure that the amount of mathematics he knows will surprise you. His dissertation is the sort of work you don't expect to see these days. It definitely demonstrates his complete capabilities. In closing, let me say that you will be fortunate if you can get him to work for you. Sincerely, A. D. Advisor (Prof.) – from MAA Focus Newsletter

cube

[cube] To divide a cube into two other cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible, and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it. – P. de Fermat

reality

[reality] Mathematics is not only real, but it is the only reality. That is that entire universe is made of matter, obviously. And matter is made of particles. It's made of electrons and neutrons and protons. So the entire universe is made out of particles. Now what are the particles made out of? They're not made out of anything. The only thing you can say about the reality of an electron is to cite its mathematical properties. So there's a sense in which matter has completely dissolved and what is left is just a mathematical structure. – M. Gardner

arithmetic

[arithmetic] God does arithmetic. – K.F. Gauss

hypothesis

[hypothesis] Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis? – P.R. Halmos

dice

[dice] God not only plays dice. He also sometimes throws the dice where they cannot be seen. – S.W. Hawking

wissen

[wissen] 'Wir muessen wissen. Wir werden wissen.' (We have to know. We will know.) – D. Hilbert (engraved in tombstone)

physics

[physics] Physics is much too hard for physicists. – D. Hilbert

Hofstadter

[Hofstadter] Hofstadter's Law: It always takes longer than you expect, even when you take into account Hofstadter's Law. – D.R. Hofstadter, Goedel-Escher-Bach

experience

[experience] The science of mathematics presents the most brilliant example of how pure reason may successfully enlarge its domain without the aid of experience. – E. Kant

doughnut

[doughnut] A topologist is one who doesn't know the difference between a doughnut and a coffee cup. – J. Kelley

Kovalevsky

[Kovalevsky] Say what you know, do what you must, come what may. – S. Kovalevsky

god

[god] God made the integers, all else is the work of man. – L. Kronecker

abstract

[abstract] There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world. – N. Lobatchevsky

medicine

[medicine] Medicine makes people ill, mathematics make them sad and theology makes them sinful. – M. Luther

intelligence

[intelligence] The mathematician who pursues his studies without clear views of this matter, must often have the uncomfortable feeling that his paper and pencil surpass him in intelligence. – E. Mach

flesh

[flesh] I tell them that if they will occupy themselves with the study of mathematics they will find in it the best remedy against the lusts of the flesh. – T. Mann

philosophers

[philosophers] Today, it is not only that our kings do not know mathematics, but our philosophers do not know mathematics and - to go a step further - our mathematicians do not know mathematics. – J.R. Oppenheimer

obvious

[obvious] Mathematics consists of proving the most obvious thing in the least obvious way. – G. Polya

whispers

[whispers] However successful the theory of a four dimensional world may be, it is difficult to ignore a voice inside us which whispers: "At the back of your mind, you know a fourth dimension is all nonsense". I fancy that voice must have had a busy time in the past history of physics. What nonsense to say that this solid table on which I am writing is a collection of electrons moving with prodigious speed in empty spaces, which relative to electronic dimensions are as wide as the spaces between the planets in the solar system! What nonsense to say that the thin air is trying to cursh my body with a load of 14 lbs. to the square inch! What nonsense that the star cluster which I see through the telescope, obviously there NOW, is a glimpse into a past age 50'000 years ago! Let us not be beguiled by this voice. It is discredited... – Sir Arthur Eddington

decimal

[decimal] The first million decimal places of pi are comprised of:

99959	0's
99758	1's
100026	2's
100229	3's
100230	4's
100359	5's
99548	6's
99800	7's
99985	8's
100106	9's

–David Blatner, the joy of pi

historians

[historians] Math historians often state that the Egyptians thought pi = 256/81. In fact, there is no direct evidence that the Egyptians conceived of a constant number pi, much less tried to calculate it. Rather, they were simply interested in finding the relationship between the circle and the square, probably to accomplish the task of precisely measuring land and buildings. –David Blatner, the joy of pi

pi

[pi]

2000 BC	Babilonians use pi=25/8, Egyptians use pi=256/81
1100 BC	Chinese use pi=3
200 AC	Ptolemy uses pi=377/120
450	Tsu Ch'ung-chih uses pi=255/113
530	Aryabhata uses pi=62832/20000
650	Brahmagupta uses pi=sqrt(10)
1593	Romanus finds pi to 15 decimal places
1596	Van Ceulen calculates pi to 32 places
1699	Sharp calculates pi to 72 places
1719	Tantet de Lagny calculates pi to 127 places
1794	Vega calculates pi to 140 decimal places
1855	Richter calculates pi to 500 decimal places
1873	Shanks finds 527 decimal places
1947	Ferguson calculates 808 places
1949	ENIAC computer finds 2037 places
1955	NORC computer computes 3089 places
1959	IBM 704 computer finds 16167 places
1961	Shanks-Wrench (IBM7090) find 100200 places
1966	IBM 7030 computes 250000 places
1967	CDC6600 computes 500000 places
1973	Guilloud-Bouyer (CDC7600) find 1 Mio places
1983	Tamura-Kanada (HITACM-280H) compute 16 Mio places
1988	Kanada (HITAC M-280H) computes 16 Mio digits
1989	Chudnovsky finds 1000 Mio digits
1995	Kanada computes pi to 6000 Mio digits
1996	Chudnovsky computes pi to 8000 Mio digits
1997	Kanada determines pi to 51000 Mio digits

–David Blatner, the joy of pi

FBI

[FBI] The following is a transcript of an interchange between defence attorney Robert Blasier and FBI Special Agent Roger Martz on July 26, 1995, in the courtroom of the O.J. Simpson trial:
Q: Can you calculate the area of a circle with a five-millimeter diameter? A: I mean I could. I don't...math I don't ... I don't know right now what it is. Q: Well, what is the formula for the area of a circle? A: Pi R Squared Q: What is pi? A: Boy, you ar really testing me. 2.12... 2.17... Judge Ito: How about 3.1214? Q: Isn't pi kind of essential to being a scientist knowing what it is? A: I haven't used pi since I guess I was in high school. Q: Let's try 3.12. A: Is that what it is? There is an easier way to do... Q: Let's try 3.14. And what is the radius? A: It would be half the diameter: 2.5 Q: 2.5 squared, right? A: Right. Q: Your honor, may we borrow a calculator? [pause] Q: Can you use a calculator? A: Yes, I think. Q: Tell me what pi times 2.5 squared is. A: 19 Q: Do you want to write down 19? Square millimeters, right? The area. What is one tenth of that? A: 1.9 Q: You miscalculated by a factor of two, the size, the minimum size of a swatch you needed to detect EDTA didn't you? A: I don't know that I did or not. I calculated a little differently. I didn't use this. Q: Well, does the area change by the different method of calculation? A: Well, this is all estimations based on my eyeball. I didn't use any scientific math to determine it. –David Blatner, the joy of pi

beauty

[beauty] To those who do not know Mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty of nature. ... If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in. – Richard Feynman in "The Character of Physical Law"

Bacon

[Bacon] All science requires Mathematics. The knowledge of mathematical things is almost innate in us... This is the easiest of sciences, a fact which is obvious in that no one?s brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon. – Roger Bacon

deductions

[deductions] Pure mathematics consists entirely of such asseverations as that, if such and such a proposition is true of anything, then such and such another proposition is true of that thing... It's essential not to discuss whether the proposition is really true, and not to mention what the anything is of which it is supposed to be true... If our hypothesis is about anything and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. – Bertrand Russell

ambitious

[ambitious] The more ambitious plan may have more chances of success – G. Polya, How To Solve It

fourteen

[fourteen] THEOREM: Every natural number can be completely and unambiguously identified in fourteen words or less. PROOF: 1. Suppose there is some natural number which cannot be unambiguously described in fourteen words or less. 2. Then there must be a smallest such number. Let's call it n. 3. But now n is "the smallest natural number that cannot be unambiguously described in fourteen words or less". 4. This is a complete and unambiguous description of n in fourteen words, contradicting the fact that n was supposed not to have such a description! 5. Since the assumption (step 1) of the existence of a natural number that cannot be unambiguously described in fourteen words or less led to a contradiction, it must be an incorrect assumption. 6.Therefore, all natural numbers can be unambiguously described in fourteen words or less!

1=2

[1=2] THEOREM: 1=2 PROOF:
1. Let $a = b$.
2. Then $a^2 = ab$,
3. $a^2 + a^2 = a^2 + ab$,
4. $2a^2 = a^2 + ab$,
5. $2a^2 - 2ab = a^2 + ab - 2ab$,
6. and $2a^2 - 2ab = a^2 - ab$
7. Writing this as $2(a^2 - ab) = 1(a^2 - ab)$,
8. and cancelling the $(a^2 - ab)$ from both sides gives $1 = 2$.

primes

[primes] II III V VII XI XIII XVII XIX XXIII XXIX ...

Queen

[Queen] "Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?" "I don't know," said Alice, "I lost count.". – Lewis Carrol alias Charles Lutwidge Dodgson, Alice's Adventures in Wonderland

subtraction

[subtraction] "She can't do Subtraction", said the White Queen. "Can you do Division? Divide a loaf by a knife – what's the answer to that?" "I suppose –" Alice was beginning, but the Red Queen answerd for her. "Bread and butter, of course ..." – Lewis Carrol alias Charles Lutwidge Dodgson, Alice's Adventures in Wonderland

subtraction

Theorem: the square root x of 2 is irrational. Proof: x=n/m with $gcd(n,m) = 1$ implies $2 = n^2/m^2$ which is $2m^2 = n^2$ so that n must be even and n^2 a multiple of 4. Therefore m is even. This contradicts gcd(n,m)=1.

blackboard

[blackboard] It is still an unending source of surprise for me to see how a few scribbles on a blackboard or on a sheet of paper could change the course of human affairs. – Stanislaw Ulam.

ephermeral

[ephermeral] Of all escapes from reality, mathematics is the most successful ever. It is a fantasy that becomes all the more addictive because it works back to improve the same reality we are trying to evade. All other escapes- sex, drugs, hobbies, whatever - are ephemeral by comparison. The mathematician's feeling of triumph, as he forces the world to obey the laws his imagimation has created, feeds on its own success. The world is premanently changed by the workings of his mind, and the certainty that his creations will endure renews his confidence as no other pursuit. – Gian-Carlo Rota

joke

[joke] A good mathematical joke is better, and better mathematics than a dozen mediocre papers.
– John Edensor Littlewood

Leibniz

[Leibniz] $pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9....$
– Wilhelm von Leibniz

war

[war] It has been said that the First World War was the chemists' war because mustard gas and chlorine were empolyed for the first time, and that the Second World War was the physicists war, because the atom bomb was detonated. Similarly, it has been argued that the Third World War would be the mathematicians' war, because mathematics will have control over the next great weapon of war - information. – Simon Singh, in 'The code book'

clearly

[clearly] Never speak more clearly than you think. – Jeremy Bernstein

Piaget

[Piaget] What, in effect are the conditions for the construction of formal thought? The child must not only apply operations to objects - in other words, mentally execute possible actions on them - he must also 'reflect' those operations in the absence of the objects which are replaced by pure propositions. Thus 'reflection' is thought raised to the second power. Concrete thinking is the representation of a possible action, and formal thinking is the representation of a representation of possible action... It is not surprising, therefore, that the system of concrete operations must be completed during the last years of childhood before it can be 'reflected' by formal operations. In terms of their function, formal operations do not differ from concrete operations except that they are applied to hypotheses or propositions whose logic is an abstract translation of the system of 'inference' that governs concrete operations. – Jean Piaget

Mersenne

[Mersenne] An integer $2^n - 1$ is called a Mersenne number. If it is prime, it is called a Mersenne prime. In that case, n must be prime. Known examples are n = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377. It is not known whether there are infinitely many Mersenne primes.

Mersenne

A positive integer n is called a perfect number if it is equal to the sum of all of its positive divisors, excluding n itself. Examples are 6=1+2+3, 28=1+2+4+7+14. An integer k is an even perfect number if and only if it has the form $2^(n-1)(2^n - 1)$ and $2^n - 1$ is prime. In that case $2^n - 1$ is called a Mersenne prime and n must be prime. It is unknown whether there exists an odd perfect number.

Wilson

[Wilson] WILSON'S THEOREM: p prime if and only if $(p-1)! == -1 \pmod{p}$ PROOF. $1, 2, ..., p-1$ are roots of $x^(p-1) == 0 \pmod{p}$. A congruence has not more roots then its degree, hence $x^(p-1) - 1 ==$ $(x-1)(x-2)...(x-(p-1))$ mod p. For x=0, this gives $-1 == (-1)^(p-1)(p-1)! == (p-1)!$ which is also true for p=2.
– from P. Ribenboim, 'The new book of prime number records'

twin

[twin] There is keen competition to produce the largest pair of twin primes. On October 9, 1995, Dubner discovered the largest known pair of twin primes $p, p+2$, where $p = 570918348 * 10^{5120} - 1$. It took only one day with 2 crunchers. The expected time would be 150 times longer! What luck! – from P. Ribenboim, 'The new book of prime number records'

lion

[lion] How to catch a lion:

- THE HILBERT METHOD. Place a locked cage in the desert. Set up the following axiomatic system. (i) The set of lions is non-empty
(ii) If there is a lion in the desert, then there is a lion in the cage.
Theorem. There is a lion in the cage
- THE PEANO METHOD. There is a space-filling curve passing through every point of the desert. Such a curve may be traversed in as short a time as we please. Armed with a spear, traverse the curve faster than the lion can move his own length.
- THE TOPOLOGICAL METHOD. The lion has a least the connectivity of a torus. Transport the desert into 4-space. It can now be deformed in such a way as to knot the lion. He is now helples.
- THE SURGERGY METHOD. The lion is an orientable 3-manifold with boundary and so may be rendered contractible by surgery.
- THE UNIVERSAL COVERING METHOD. Cover the lion by his simply-connected covering space. Since this has no holes, he is trapped.
- THE GAME THEORY METHOD. The lion is a big game, hence certainly a game. There exists an optimal strategy. Follow it.
- THE SCHROEDINGER METHOD. At any instant there is a non-zero probability that the lion is in the cage. Wait.
- THE ERASTOSHENIAN METHOD. Enumerate all objects in the desert: examine them one by one; discard all those that are not lions. A refinement will capture only prime lions.
- THE PROJECTIVE GEOMETRY METHOD. The desert is a plane. Project this to a line, then project the line to a point inside the cage. The lion goes to the same point.
- THE INVERSION METHOD. Take a cylindrical cage. First case: the lion is in the cage: Trivial. Second case: the lion is outside the cage. Go inside the cage. Invert at the boundary of the cage. The lion is caught. Caution: Don't stand in the middle of the cage during the inversion!

Euler

[Euler] Euler's formula: A connected plane graph with n vertices, e edges and f faces satisfies n - e + f = 2 Proof. Let T be the edge set of a spanning tree for G. It is a subset of the set E of edges. A spanning tree is a minimal subgraph that connects all the vertices of G. It contains so no cycle. The dual graph G* of G has a vertex in the interior of each face. Two vertices of G* are connected by an edge if the correponding faces have a common boundary edge. G* can have double edges even if the original graph was simple. Consider the collection T* of edges E* in G* that correspond to edges in the complement of T in E. The edges of T* connect all the faces because T does not have a cycle. Also T* does not contain a cycle, since otherwise, it would seperate some vertices of G contradicting that T was a spanning subgraph and edges of T and T* don't intersect. Thus T* is a spanning tree for G*. Clearly e(T)+e(T*)=2. For every tree, the number of vertices is one larger than the number of vertices. Applied to the tree T, this yields n = e(T)+1, while for the tree T* it yields f=e(T*)+1. Adding both equations gives n+f=(e(T)+1)+(e(T*)+1)=e+2. – from M.Aigner, G. Ziegler "Proofs from THE BOOK"

irrational

[irrational] Theorem: e = sum(k) 1/k! is irrational. Proof. e=a/b with integers a,b would imply N = n! (e - sum(k¡n+1) 1/k!) is an integer for n¿b because n! e and n!/k! were both integers. However, $0 < N = \sum_{k>n} n!/k! = 1/(n+1) + 1/(n+1)(n+2) + ... < 1/(n+1) + 1/(n+1)^2 + ... = 1/n$ (second sum is a geometric series) for every n is not possible.
– from M.Aigner, G. Ziegler "Proofs from THE BOOK"

Wiener

[Wiener] After a few years at MIT, the Mathematician Norbert Wiener moved to a larger house. His wife, knowing his nature, figured that he would forget his new address and be unable to find his way home after work. So she wrote the address of the new home on a piece of paper that she made him put in his shirt pocket. At lunchtime that day, the professor had an inspiring idea. He pulled the paper out of his pocket and used it to scribble down some calculations. Finding a flaw, he threw the paper away in disgust. At the end of the day he realized he had thrown away his address, he now had no idea where he lived. Putting his mind to work, he came up with a plan. He would go to his old house and await rescue. His wife would surely realize that he was lost and go to his old house to pick him up. Unfortunately, when he arrived at his old house, there was no sign of his wife, only a small girl standing in front of the house. "Excuse me, little girl" he said "but do you happen to know where the people who used to live here moved to?" "It's okay, Daddy," said the little girl, "Mommy sent me to get you". Moral 1. Don't be surprised if the professor doesn't know your name by the end of the semester. Moral 2. Be glad your parents aren't mathematicians. if your parents are mathematicians, introduce yourself and get them to help you through the course. - From the introduction of "How to ace calculus" by C. Adams, A. Thompson and J. Hass

funeral

[funeral] David Hilbert was one of the great European mathematicians at the turn of the century. One of his students purchased an early automobile and died in one of the first car accidents. Hilbert was asked to speak at the funeral. "Young Klaus" he said, "was one of my finest students. He had an unusual gift for doing mathematics. He was insterested in a great variety of problems, such as..." There was a short pause, follwed by "Consider the set of differentiable functions on the unit interval and take their closure in the ..." Moral 1. Sit near the door. Moral 2. Some mathematicians can be a little out of touch with reality. If your professor falls in this category, look at the bright side. You will have lots of funny stories by the end of the semester. - From the introduction of "How to ace calculus" by C. Adams, A. Thompson and J. Hass

rabbit

[rabbit] In a forest a fox bumps into a little rabbit, and says, "Hi, junior, what are you up to?" "I'm writing a dissertation on how rabbits eat foxes," said the rabbit. "Come now, friend rabbit, you know that's impossible!" "Well, follow me and I'll show you." They both go into the rabbit's dwelling and after a while the rabbit emerges with a satisfied expression on his face. Along comes a wolf. "Hello, what are we doing these days?" "I'm writing the second chapter of my thesis, on how rabbits devour wolves." "Are you crazy? Where is your academic honesty?" "Come with me and I'll show you." ...... As before, the rabbit comes out with a satisfied look on his face and this time he has a diploma in his paw. The camera pans back and into the rabbit's cave and, as everybody should have guessed by now, we see an enormous mean-looking lion sitting next to the bloody and furry remains of the wolf and the fox. The moral of this story is: It's not the contents of your thesis that are important – it's your PhD advisor that counts. - Unknown Usenet Source

poet

[poet] It is true that a mathematician who is not also something of a poet will never be a perfect mathematician. - K. Weierstrass, Quoted in D MacHale, Comic Sections (Dublin 1993)

equilateral

[equilateral] THEOREM: All triangles are equilateral. PROOF: 1) Given an arbitrary triangle ABC. Construct the middle orthogonal on AB in D and cut it with the line dividing the angle at C. Call the intersection E. Form the normal from E to AC in F and from E to BC in G. Draw the lines AE und BE. C * / / *F *G / E* / — / — / —D A*———*————*B

2. The angles ECF and ECG are gleich. The angles EFC and EGC are both right angles. Because the triangles ECF and ECG have also EC common, they must be congruent. Therefore CF=CG and EF=EG.

3. The sides DA and DB are equal. The angle EDA and EDB are both right angles. Because the triangles EDA and EDB have also ED in common, they have to be congruent and EA=EB.

4. The angle EGB and EFA are both right angle. Also, EF=EG and EA=EB. Therefore both triangles EGB and EFA are congruent. Therefore FA=GB.

5. Since CF=CG and FA=GB, addition of the sides gives also CA=CB.

6. Having proved that two arbitrary sides are equal, all are equal.

widow

[widow] I married a widow, who had an adult stepdaughter. My father, a widow and who often visited us, fell in love with my stepdaughter and married her. So, my father became my son-in-law and my stepdaughter became my stepmother. But my wife became the mother-in-law of her father-in-law. My stepmother, stepdaughter of my wife had a son and I therefore a brother, because he is the son of my father and my stepmother. But since he was in the same time the son of our stepdaughter, my wife became his grandmother and I became the grandfather of my stepbrother. My wife gave me also a son. My stepmother, the stepsister of my son, is in the same time his grandmother, because he is the son of her stepson and my father is the brother-in-law of my child, because his sister is his wife. My wife, who is the mother of my stepmother, is therefore my grandmother. My son, who is the child of my grandmother, is the grandchild of my father. But I'm the husband of my wife and in the same time the grandson of my wife. This means: I'm my own grandfather.

dots

[dots] I never could make out what those damned dots meant. – Lord Randolph Churchill (1849-1895) Brittish conservative politician, referring to decimal points.

ladder

[ladder] The mathematician has reached the highest rung on the ladder of human thought. – Havelock Ellis

ignorant

[ignorant] Let no one ignorant of mathematics enter here. – Plato, Inscription written over the entrance to the academy

god

[god] I knew a mathematician, who said 'I do not know as much as God. But I know as much as God knew at my age'. – Milton Shulman, Candian writer

english

[english] English professor: In English, a double negative makes a positive. In other languages such as Russian, a double negative is still a negative. There are, however, no languages in which a double positive makes a negative. Student in back of class: "Yea, right"

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 124 entries in this file.

Index

ENTRY COMPUTABILITY

[ENTRY COMPUTABILITY] Authors: Oliver Knill: nothing real yet Literature: not yet, some lectures of E.Engeler on computation theory

Church's theses

The generally accepted [Church's theses] tells that everything which is computable can be computed using a Turing machine. In that case, the problem to determine, whether a Turing machine will halt, is not computable.

cipher

A [cipher] is a secret mode of writing, often the result of subsituting numbers of letters and then carrying out arithmetic operations on the numbers.

Coding theory

[Coding theory] is the theory of encryption of messages employed for security during the transmission of data or the recovery of information from corrupted data.

Cooks hypothesis

[Cooks hypothesis] $P = NP$. A proof or disproof is one of the millenium problems.

Graph isomorphism problem

[Graph isomorphism problem] It is not known whether graph isomorphism can be decided in deterministic polynomial time. It is an open problem in computational complexity theory.

Inductive structure

[Inductive structure] A set U with a subset A and operations $g_1, ..., g_n$ define an inductive structure $(U, A, g_1, ..., g_n)$ If all elements of U can be generated by repeated applications of the operations g_i on elements of A. Examples:

- $(N, A = \{0, 1\}, g_1(a, b) = a + b, g_2(a, b) = a * b$ defines an inductive structure.

- If $U = N$ is the set of natural numbers, $A = \{1, 2, 3\}$ $g_1(x, y) = 3x - 4, g_2(x, y, z) = 7x + 5y - z$, then (U, A, g_1, g_2) define an inductive structure.

syntactic structure

An inductive structure $(U, A, g_1, ..., g_n)$ is called a [syntactic structure] if it is uniquely readable that is if $g_1(u_1, ..., u_k) = g_2(v_1, ..., v_l)$, then $g_1 = g_2, k = l$ and $u_1 = v_1, ..., u_k = v_k$. Example: if X is the set of finite words in the alphabet $\{p, q, r, K, N\}$ and $A = \{p, q, r\}$. Define $g_1(x, y) = Kxy$ and $g_2(x, y) = Nx$ and U the set of words generated from A. The structure is the language of elementary logic in polnic notation. It is a syntactic structure. Syntactic structures are in general described by grammers.

grammar

A [grammar] (N, T, G) is given by two sets of symbols N, T and a finite set G of pairs (n_i, t_i) which define transitions $n_i \to t_i$. For example: $N = \{S\}, T = \{K, N, p, q, r\}, G = \{S \to p, S \to q, S \to r, S \to KSS, S \to NS\}$. Acoording to Chomsky, one classifies grammers with additional conditions like context sensitivity or regularity.

context sensitive

A grammer (N, T, G) is called [context sensitive] if $(n, t) \in G$ then $|t| \geq |n|$.

context sensitive

Index

ENTRY COMPUTER

[ENTRY COMPUTER] Authors: Oliver Knill: May 2001 Literature: for video stuff: http://www.doom9.org, foldoc

AAC

[AAC] Advanced Audio Coding will be the successor of AC3 audio. It is based on AC3 while adding a number of improvements in various areas. Currently player and hardware support for this upcoming audio format is still very limited.

acrobat

[acrobat] A product from Adobe for manipulating documents stored in the PDF (Portable Document Format).

amd

[amd] Daemon which enables the NFS automount.

AMD

[AMD] Advanced Micro Devices, Chip company.

arpwatch

[arpwatch] Daemon to log and buil a database of Ethernet address/IP address pairings it sees on a LAN interface.

ASCII

[ASCII] American Standard Code for Information Interexchange, an industry standard, which assigns letters, numbers and other characters within the 256 slots available in the 8-bit code.

AC3

[AC3] Initially known as Audio Coding 3 AC3 is a synonym for Dolby Digital these days. Dolby Digital is an advanced audio compression technology allowing to encode up to 6 separate channels at bitrates up to 448kbit/s. For more information please check out the Dolby website.

ASF

[ASF] Advanced Streaming Format. Microsoft's answer to Real Media and streaming media in general.

AT

[AT] keyboard The standard keyboard used with the IBM compatible computer.

backdoor

A [backdoor] is a "mechanism surreptitiously introduced into a computer system to facilitate unauthorized access to the system". An example of a backdoor is "bindshell".

AVI

[AVI] Audio Video Interleave. The video format most commonly used on Windows PC's. It defines how video and audio are attached to each other, without specifying a codec.

Bandwidth

[Bandwidth] Bandwidth measures how much information can be carried in a given time period over a wired or wireless communications link. A typical broadband speed is 1270 Kbps (kilo bit per second) which is 155.6 KBytes/sec

Technology	Speed mbit/s
56k modem	0.056
DSL	varies
cable	varies
T1	1.544
Ethernet	10.000
T3	44.736
OC-3	155.520
OC-12	622.080
OC-48	2,488.320
OC-96	4,976.640
OC-192	9,953.280
OC-255	13,219.200

see http://home.cfl.rr.com/cm3/speedtest7.htm

http://jetstreamgames.co.nz/speed/ADSLdownload1MB.html http://home.cfl.rr.com/eaa/Bandwidth.htm

BUP file

[BUP file] A bup file is a Back UP file of an IFO file. These files are commonly found on DVDs.

Byte

One [Byte] is an information unit of a sequence of 8 bits.

CASE

[CASE]: Computer Aided Software Engineering.

Cell (ID)

[Cell (ID)] A cell is the smallest video unit on a DVD. Normally used to contain a chapter it can also be used to contain a smaller unit in case of multiangles or seamless branching titles.

certificate

A [certificate] is a digital identifcation of a physical or abstract object, a person, business, computer, program or document. A digital certificate is much like a passport. It is issued by a certificate authority, which vouches for its authenticity.

Codec

[Codec] COder/DECoder. A codec is a piece of software that allows to encode something - usually audio or video - to a specific format and can decode media encoded in this specific format again. Popular Codecs are MPEG1, MPEG2, MPEG-4 (=divx=xvid), realvideo, wmv, dv Indeo, etc. MPEG, AVI, ASF, Quicktime is not a codec but a container format - that can be encoded using different codecs. In avi container files, there's mostly mpeg4 video content and mp3 audio content but this is not obligatory. For DVD, the video should be in mpg2, the audio in mp2 and both of these will be in a mpeg-ps (program stream aka "vob") container.

Container

[Container] A container is, like the name says, a construct to contain data - in this case video and audio date and possibly subtitles and navigational information. For instance, you would like to put a soundless video stream and the audio track together in one file. To do that you need a container format. Examples of container formats are: AVI, ASF, OGM, Quicktime, VOB and MPG. In avi container files, there's mostly mpeg4 video content and mp3 audio content but this is not obligatory. For DVD, the video should be in mpg2, the audio in mp2 and both of these will be in a mpeg-ps (program stream aka "vob") container. A [cookie] is a block of information recorded and stored within the client's browser.

CSS

[CSS] Cascading Style Sheets is a simple mechanism for adding style (e.g. fonts, colors, spacing) to Web documents. For example:
body, table font-family: verdana, arial, geneva, sans-serif;

CSS

[CSS] Content Scrambling System. Prioprietary scrambling system for video DVDs. Designed to stop people from making copies of DVDs, most commercial DVDs are encrypted using CSS. During playback, DVDs are then decrypted on the fly. Only parts of the DVD are encrypted (for instance all IFO and BUP files are not encrypted, and VIDEOTS.VOB often isn't encrypted either) and the encryption scheme is rather weak and was quickly defeated. If you want to know what CSS does, insert a DVD video disc into your PC, start playing the disc using a software DVD player, then close the player. Now copy a 0.99GB VOB file from the disc to your harddisk and try to play back that VOB file in your software DVD player. You'll see a lot of funny colored blocks all over the picture making the movie unwatchable. But you'll also see parts of the movie (the parts that are not encrypted).

DAR

[DAR] Display Aspect Ratio. Indicates the dimension of a screen. Most PC screens have a DAR of 4:3, meaning that the horizontal size is 4/3 as large as the vertical size. For TVs we have a lot of old 4:3 displays and more and more 16:9 displays. As you can guess from the numbers 16:9 displays are broader than 4:3 displays having the same diagonal size. 16:9 screens are more suited to display Hollywood movies which are usually shot with an aspect ratio of 1:2.35 or 1:1.85 (meaning that the horizontal size of the picture is 1.85 times as wide as the vertical size).

Deinterlace

[Deinterlace] The process of restoring a progressive video stream out of an interlaced one is called deinterlacing.

Demultiplexing

[Demultiplexing] The opposite of multiplexing. In this process a combined audio/video stream will be separated into the number of streams it consists of (a video stream, at least one audio stream and a navigational stream). Every VOB encoder demultiplexes the VOB files before encoding (FlaskMpeg, mpeg2avi, dvd2mpg, ReMpeg2) and every DVD player does the same (audio and video are being treated by different circuits, or decoded by different filters on a PC).

Descrambling

[Descrambling] DVDs are usually CSS scrambled - imagine you decide to give a number to each letter, starting with 1 for a, etc. A sentence would become a couple of digits - that's what we call scrambled. Of course CSS is much better than that but it's still quite easy to crack. Descrambling means reversing the scrambling process, rendering our digits to a sentence again, or making our movie playable again - you can try to copy a movie to your hard disk when you've authenticated your DVD drive and play it, you'll get a garbled picture because it's still scrambled. Common CSS descramblers either use a pool of known descrambling keys (DeCSS or DODSrip - they contain a large number of keys but not all of them) or try to derive the key by a cryptographic attack (VobDec - that's why it works on most disc since it's not dependent on a pool of discs).

Digital Video

[Digital Video] Digital video is usually compressed. Since standard loss less compression is insufficient for video, the video codecs have to get rid of unimportant information - stuff the human eye won't see or is unlikely to see. Since that is still not enough modern compression algorithms use keyframes, I and P frames in order to save space.

DivX

[DivX] There are 2 flavors of DivX today: DivX is the name of the hacked Microsoft MPEG4 codecs (Windows Media Video V3). Those codecs were developed by Microsoft for use in its proprietary Windows Media architecture and initially supported encoding AVIs and ASFs but all non-beta versions included an AVI lock, making it impossible to use them to encode to the AVI format - and only a few tools support ASF today. What the makers of DivX did is remove that AVI lock making it possible to encode to AVI again, and changed the name to DivX video in order to prevent confusion of codecs, since it's possible to have both the unhacked and hacked codecs on the same computer if you use the Windows Media Encoder. The latest releases of DivX also include a hacked Windows Media Audio Codec called DivX audio - the hack of that codec is not perfect yet and its use is limited for higher bitrates. This codec is also known as DivX3. The other DivX is a brand-new MPEG-4 video codec developed by DivXNetworks. It offers much advanced encoding controls and 2 pass encoding. Furthermore the codec can play the old DivX3 movies. The codec is commonly called DivX4.

DHCPD

[DHCPD] Daemon to service which can dynamically assign IP addresses to its client hosts.

DOM

[DOM] The Document Object Model is a platform- and language-neutral interface that will allow programs and scripts to dynamically access and update the content, structure and style of documents.

DOS

[DOS] is a Disk operating system, based on a command line user interface. MS-DOS 1.0 was released in 1981 for IBM computers. While MS-DOS is not much used by itself today, it still can be accessed from Windows 95, Windows 98 or Windows ME by clicking Start/Run and typing command or CMD in Windows NT, 2000 or XP.

DRC

[DRC] Dynamic Range Compression. AC3 Tracks contain a much larger dynamic range that most audio equipment can handle, therefore most standalone and software DVD player will compress the dynamic range somewhat, according to the actual dynamic range. In layman terms the volume will be augmented dynamically, e.g. explosions won't become louder or only a bit louder, whereas in normal dialogues the volume will be augmented quite a bit. Since your player will do the same this is the way to go to have augmented volume.

DTML

[DTML] document template markup language.

DTP

[DTP] Desktop publishing.

Dynamic HTML

[Dynamic HTML] is a term used by some vendors to describe the combination of HTML, style sheets and scripts that allows documents to be animated.

Elementary Stream (ES)

[Elementary Stream (ES)] An elementary stream is a single (video or audio) stream without container. For instance a basic MPEG-2 video stream (.m2v or .mpv) is an MPEG-2 ES, and on the audio side we have AC3, MP2, etc files that are ES. Most DVD authoring program require ES as input.

EULA

[EULA] End user licence agreement.

FAT

[FAT] File allocation table. Filesystem used by Windows. Example: Windows 95 users rely on the FAT 16, In 1996 Microsoft introduced the FAT 32 file system, which is still very widely used today besides NTFS on the windows platform.

FUD

[FUD] stands for Fear, Uncertainty, Doubt. It is a marketing technique used when a competitor launches a product that is both better than yours and costs less, i.e. your product is no longer competitive. Unable to respond with hard facts, scare-mongering is used via 'gossip channels' to cast a shadow of doubt over the competitors offerings and make people think twice before using it.

GUI

[GUI] - Graphical User Interface; A desktop-like interface usually containing icons, menus and windows. Invented by Xerox, later "borrowed" by Microsoft and Apple.

HTML

[HTML] Hypertext markup language. Will be replaced by XHTML, and XHTML 2.0 in particular.

HTTP

[HTTP] Hypertext Transfer Protocol.

HTTPD

[HTTPD] Daemon to Apache webserver.

Hypertext

[Hypertext] - shortcuts or links between different parts of a document, article, website or world wide web. While early hypertext formats were already Apples Hypercard, it is now common in HTML (Hypertext markup language).

inetd

[inetd] Daemon which is at the heart of providing network services like telnet or ftp.

IFO file

[IFO file] InFOrmation file commonly found on DVDs. Such files contain navigational information for DVD players.

Interlaced

[Interlaced] Interlaced is a video storage mode. An interlaced video stream doesn't contain frames but fields with each field containing either even or odd lines of one frame.

IP

[IP] Internet Protocol. Standard which defines the structure of a message sent between two computers over the network.

IPFW

[IPFW] IP firewall.

ICMP

[ICMP] Internet Control Message Protocol. ICMP messages contain information about communication between two computers.

Java

[Java] is a true compiler-based, low level programming language. [Javascript] is a scripting programming language. It was developed by Netscape and used to create interactive Web sites. JavaScript is a popular client-side scripting language because it is supported by virtually all browsers.

KISS

[KISS] - Keep It Simple Stupid. Rule of thumb for software designers. Keep design small to minimize confusion.

LDAP

[LDAP] Lightweight Directory Access Protocol. A network directory which can substitute DNS and much more. Not to be confused with a database. A directory is mostly looked up and not written often into.

LDIF

[LDIF] LDAP interchange Format is a standard text file for storing LDAP configuration information and directory contents.

MathML

[MathML] is a low-level specification for describing mathematics as a basis for machine to machine communication. It provides a foundation for the inclusion of mathematical expressions in Web pages.

miniDVD

[miniDVD] Basically a DVD on a CD. A miniDVD can contain bitrates up to 10mbit/s (audio and video combined). Video is MPEG2, preferably VBR and audio can be MPEG1 audio layer 2, raw uncompressed PCM or AC3. Video quality can be up to an actual DVD level if a limited playtime is accpted.

MPEG

[MPEG] MPEG means Motion Picture Expert Group and it's the resource for video formats in general. This group defines standards in digital video, among it the MPEG1 standard (used in Video CDs), the MPEG2 standard (used on DVDs and SVCDs), the MPEG4 standard and several audio standards - among them MP3 and AAC. Files containing MPEG-1 or MPEG-2 video often use either the .mpg or .mpeg extension.

MPEG4

[MPEG4] Is pretty much a collection of standards defined by the MPEG Group, and it should become the next standard in digital video. MPEG4 allows the use of different encoding methods, for instance a keyframe can be encoded using ICT or Wavelets resulting in different output qualities.

MPG

[MPG] MPG can be either an abbreviation for MPEG or is used as a file extension for MPEG-1 and MPEG-2 video data. It is a container to contain MPEG-1/2 video stream and MPEG1 layer 2 audio (aka mp2 files). MPG containers are also refered to as program streams (PS).

MM4

[MM4] Multiple MPEG 4: A combination of different bitrate encoded files. For instance you could take a 2000kbit/s encode, a 910kbit/s encode and combine the files together, use the lower bitrate file and replace scenes where the quality gets too bad due to a lot of action with the parts taken from the 2000kbit/s one.

NAT

[NAT] Network Address translation. A typical home user with broadband access and router performs Network Address Translation, or NAT allowing multiple computers to share a single fast Internet connection.

.Net

[.Net] A collectionof technologies pushed by Microsoft. It contains $C\#$ programming language (an alternative to Java). Part of the .Net initiative. It builds on standards like XML and SOAP.

Network layers

[Network layers] Application layer: Client and server programs. Transport layer: TCP and UDP protocols, service ports Network layer: IP packets, IP addresses, ICMP messsages Data link layer: Ethernet frames and MAC addresses Physical layer: Copper wire, fiberoptic cable, radio

Newbie

[Newbie] - (Also n00b and newb) a newcomer to a certain computer topic or program asking help from experienced user.

NFS

[NFS] Network file system.

OGM

[OGM] OGM stands for OGg Media which is the name of the Ogg container implementation by Tobias Waldvogel. OGM can be used as an alternative to the AVI container and it can contain Ogg Vorbis, MP3 and AC3 audio, all kinds of video formats, chapter information and subtitles.

Perl

[Perl] Perl is a high-level programming language. It derives from the C programming language and to a lesser extent from sed, awk, the Unix shell, and at least a dozen other tools and languages. Perl's process, file, and text manipulation facilities make it particularly well-suited for tasks involving quick prototyping, system utilities, software tools, system management tasks, database access, graphical programming, networking, and world wide web programming. These strengths make it especially popular with system administrators and CGI script authors, but mathematicians, geneticists, journalists, and even managers also use Perl.

PHP

[PHP] PHP is a widely-used general-purpose scripting language that is especially suited for Web development and can be embedded into HTML.

Pocket PC

[Pocket PC] Operating system for handhelds. Usually running Microsoft CE or the Palm OS.

PNG

[PNG] is graphics file format for the lossless, portable, well-compressed storage of raster images. Indexed-color, grayscale, and truecolor images are supported, plus an optional alpha channel for transparency. Sample depths range from 1 to 16 bits per component (up to 48bit images for RGB, or 64bit for RGBA).

Python

[Python] is an interpreted, high-level, object-oriented programming language.

QNX

[QNX] is a realtime, microkernel, preemptive, prioritized, message passing, network distributed, multitasking, multiuser, fault tolerant operating system.

Qt

[Qt] ("kjut") Multi platform toolkit and graphics library. Developed by Trolltech. Runs on Windows systems including XP, all unix derivates with X windows as well as Mac OS X.

PCMCIA

[PCMCIA] Personal Computer Memory Card International Association.

RDBM

[RDBM] relational data base manager.

rff/tff

[rff/tff] RFF means repeat first frame, it's a technique used to make the necessary 29.97 frames per second out of a 24 frames per second source - the movie like it was recorded with a traditional movie camera used by Hollywood. The rff flag tells the player to repeat one field of the video stream. Tff means top field first and is also used to perform a telecine to make a 24fps movie into 29.97fps.

Real time operating systems

[Real time operating systems] Operating systems which are used in handhelds, robots, telphone switches. Examples: QNX, VxWorks (as used in Mars rovers), Windows CE (as used in handhelds), Nucleus RTX. Realtime systems must function reliably in event of failures. It is said that the three most important things in Realtime system design are timing, timing and timing.

Ripping

[Ripping] Ripping means copying a DVD movie to the hard disk of the computer. This includes the authentication process for the DVD Drive and the actual CSS Descrambling. CSS (Content Scrambling System) is a copy protection scheme designed to prevent unauthorized copying of DVD movies, although many argue that it was also designed to control where DVD movies can be played since without a CSS license you essentially have to crack the encryption to play a DVD movie. The term "ripping" is also often used to describe the whole process of descrambling a DVD, then convert the audio and video into another format.

RSS

[RSS] is a method of distributing links to content in a web site so that others can use use it. It's a mechanism to "syndicate" the content. The original RSS, version 0.90, was designed by Netscape as a format for building portals of headlines to mainstream news sites. RSS is an acronym for Really Simple Syndication.

RTFM

[RTFM] - Read the fucking manual. Common answer to basic and often repeated questions, that could be avoided in the first place just by looking at the manual.

RSS

Really Simple Syndication [RSS] is an XML-based format for content distribution. For example, News.com offers several RSS feeds with headlines, descriptions and links back to News.com for the full story. [ROUTER] a machine designed to direct packets from their source host to their destination.

RDBMS

[RDBMS] A Relational Database Management System. Stores data related tables. A single database can be spread across several tables unlike flat-file databases where each database is self-contained in a single table.

SBC

[SBC] Smart Bitrate Control. A new kind of DivX encoder called Nandub can modify many internal codec parameters on the fly during compression, giving you better quality and a lot more control over the encoding session. More information can be found in the SBC guide in the DivX guides section.

SGML

[SGML] The standard Generalized Markup Language (SGML) is a meta Markup Languages like XML. They are used for defining markup languages. A markup language defines using SGML or XML has a specific vocabulary.

SMIL

[SMIL] Synchronized Multimedia Integration Language. It enables simple authoring of interactive audiovisual presentations. SMIL is typically used for ”rich media”/multimedia presentations which integrate streaming audio and video with images, text or any other media type. SMIL is an easy-to-learn HTML-like language, and many SMIL presentations are written using a simple text-editor.

SOAP

[SOAP] Simple object access protocol.

SQL

[SQL] structured query language.

Sun ONE

[Sun ONE] Sun Open net initiative. Answer to .Net initiative of Microsoft. Has Java, XML and SOAP as foundation.

SVG

[SVG] The Scalable Vector Graphics is a language for describing two-dimensional graphics in XML. SVG allows for three types of graphic objects: vector graphic shapes, images and text. Graphical objects can be grouped, styled, transformed and composited into previously rendered objects. The feature set includes nested transformations, clipping paths, alpha masks, filter effects and template objects.

TCP

[TCP] Transmission control protocol. An IP message type. Most network services run over TCP. A typical TCP connection is visiting a remote web site.

TCPA

[TCPA] (Trusted Computing Platform Architecture) belongs to DRM (Digital rights management). TCPA aims at integrity of kernel and system components - to assure you that your system can be trusted. Palladium, on the other hand, uses similar technology to make sure that the user does not do anything else than what is allowed by content owners.

TLD

[TLD] Top level domain. The last entry in a webaddress. The TLD of www.w3c.org is ”org”. In the 1980s, seven TLDs (.com, .edu, .gov, .int, .mil, .net, and .org) were created. Later four of the new TLDs (.biz, .info, .name, and .pro) as well as sponsered TLD’s .aero, .coop, and .museum) were created. TLDs with two letters (such as .de, .mx, and .jp) have been established for over 240 countries and external territories and are referred to as ”country-code” TLD.

UDP

[UDP] User Datagramm Protocol. Sendes transport-level data between two network-based programs. For example, internet-time servers are assigned UDP services.

UML

[UML] Unified modelling language is a language for specifying, visualizing, constructing, and documenting software systems, as well as for business modeling and modeling of other non-software systems.

VCD

[VCD] Video CD, works on many DVD players, there are software players on almost every operating systems, doesn't need a fast computer but the image is VHS-like. Video is MPEG1 at 1150kbit/s and audio MPEG1 audio layer 2 at 224kbit/s.

VLDB

[VLDB] Very large data base.

VML

[VML] Vector Markup Language

W3C

[W3C] The World Wide Web Consortium (W3C) develops interoperable technologies (specifications, guidelines, software, and tools) to lead the Web to its full potential. W3C is a forum for information, commerce, communication, and collective understanding.

Wavelets

[Wavelets] Wavelets are an alternative basis space. There are infinitely many wavelet bases (Daubechies, Haar, Mexican Hat, "Spline", Zebra, etc), but their primary feature is that they are localized. Fourier basis functions span all space (from negative to positive infinity). Wavelets are basically individual pulses of waves (at various positions and scales). Their value in compression stems from factors like the grouping which generally shows that a good 90filters, with the high-pass filters generally showing very small values that are mostly details. (of course, this is not true if the source is noisy in the first place). For images, the greatest value comes from localization of the basis, which means that we can model discontinuities (e.g. edges) VERY well with wavelets. You will NOT get those weird JPEG halos if you use wavelets.

WebDAV

[WebDAV] Web-based Distributed Authoring and Versioning. A set of extensions to the HTTP protocol which allows users to collaboratively edit and managa files on remote web servers.

Widget

[Widget]- objects that make up interfaces, i.e. mouse, menus, textbox, buttons; basic tools and objects.

Windows Media

[Windows Media] Microsoft's proprietary architecture for audio and video on the PC. It's based on a collection of codecs which can be used by the WindowsMedia Player to play files encoded in any supported format. WindowsMedia 7.0 offers a new set of codecs, among them a fully ISO compliant MPEG4 codec (called MS Windows Video V1), an improved MPEG-4 codec called MS Video V7 (although I did not notice any improvement compared with MS Windows Video V3 on which DivX is based), an encoder that supports Deinterlacing and Inverse Telecine.

Win FS

[Win FS] Windows Future Storage file system, planned in Windows Longhorn, the successor of Windows XP.

WYSIWYG

[WYSIWYG]- What You See Is What You Get. Usually to distinguish document authoring tools. Writing a Latex file as a text document is not WYSIWYG, while authoring a word document is. Writing a HTML document with a text editor is not WYSIWYG, while writing it with an authoring tool is.

WORM

[WORM] a program that connects to other machines and replicates itself. Worms have the potential to both damage infected machines and to interfere with networks and services due to congestion caused by the spread of the worm.

WORM

XHTML 2 is a general purpose markup language designed for representing documents for a wide range of purposes across the World Wide Web. To this end it does not attempt to be all things to all people, supplying every possible markup idiom, but to supply a generally useful set of elements. Here is an element of a XHTML 2.0 document: ¡?xml version="1.0" encoding="UTF-8"?¿ ¡!DOCTYPE html PUBLIC "-//W3C//DTD XHTML 2.0//EN" "TBD"¿ ¡html xmlns="http://www.w3.org/2002/06/xhtml2" xml:lang="en"¿ ¡head¿ ¡title¿Virtual Library¡/title¿ ¡/head¿ ¡body¿ ¡p¿Moved to ¡a href="http://vlib.org/"¿vlib.org¡/a¿.¡/p¿ ¡/body¿ ¡/html¿

XML

[XML] The Extensible textbased Markup Language is a format for structured documents and data on the Web. It is derived from SGML (ISO 8879). XML is also playing an increasingly important role in the exchange of a wide variety of data on the Web.

XviD

[XviD] XviD is a word play, read it the reverse way and you might find a familiar term. XviD is an open source MPEG-4 codec which depending on whom you're asking yields even better quality than the best DivX codec.

zombie

[zombie] A unix process that has died but has not yet relinquished its process table slot. The parent process hasn't executed a "wait" for it yet).

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 108 entries in this file.

Index

FromContinuedFractionTablek,k,0,100

N[FromContinuedFraction[Table[k,k,0,100]]]

ENTRY CONSTANTS

[ENTRY CONSTANTS] Authors: Oliver Knill: March 2000 - March 2004 Literature: Some from Mario Livio "The golden ratio", www.mathworld.com David Wells: "The Penguin Dictionary of Curious and Interesting Numbers".

Archimedes Constant, pi

The [Archimedes Constant, pi] $\pi = 3.14159$ is the length of a half circle with radius 1. It is the area of a disc of radius 1.

Bruns constant

[Bruns constant] is the sum of the reciprocals of all twin primes. Brun has proven that this sum converges evenso it is unknown whether there are infinitely many twin primes.

Catalan constant

The [Catalan constant] is defined as the sum $(-1)^n/(2n+1)^2 = 0.91596$.

Champernown's number

[Champernown's number] is 0.12345678910111213... whose digits are those of all natural numbers in succession.

Continued fraction constant

[Continued fraction constant] is the number with continued fraction $(0, 1, 2, 3, 4, 5, 6, ...)$ it is about 0.697774658.

Euler Mascheroni constant

[Euler Mascheroni constant] is defined as the limit of $(1 + 1/2 + 1/3 + ... + 1/n) - \log(n)$ as n goes to infinity.

Aperi constant

[Aperi constant] It is an irrational number $\zeta(3) = 1.20206$, the value of the zeta function at 3. [Feigenbaum constant] When iterating maps $f(x) = ax(1-x)$ on the unit interval the stable periodic orbits bifurcate when varing a. If a_n are the bifurcation values, then $\delta = \lim(a_n - a_{n-1})/(a_{n+1} - a_n)$ is a Feigenbaum constant.

golden ratio

The [golden ratio] is $\tau = (1+\sqrt{5})/2 = 0.618...$ If $1,1,2,3,5,8,13,21...$ are the Fibonnachi numbers (the next number is always the sum of the two previous once), then the ratio of neighboring entries approaches the golden mean. $13/21 = 0.61904$ is already quite close to the golden mean. The Golden ratio has the continued fraction expansion $[1,1,1,1...]$ which means that the number can be written as $\tau = 1+1/(1+1/(1+...))$. The golden mean is an example of a Diophantine number, a number which can not be approximated well by rational numbers. Especially, it is irrational. The golden ratio is also called "golden mean" or "divine constant".

golden mean

The [golden mean] see golden ratio.

Khinchin constant

The [Khinchin constant] is defined as the limit $(a_1 a_2 ... a_n)^{1/n}$ where $[a_1, a_2, ...]$ is the continued fraction of a random number in the sense that the limit is known to exist for almost all real numbers. It is not known for example, if π is a typical number in the sense that it produces the Khinchin constant.

natural logarithmic base

The [natural logarithmic base] $e = 2.7182818...$ can be defined as $\exp(1) = 1 + 1/1! + 1/2! + 1/3! + ...$ or $\lim_{n\to\infty}(1+1/n)^n$.

number of the beast

The [number of the beast] is the integer 666. The "beast" is associated with the "antichrist". The origin of the association is the bible: the book of revelations (13:18) reads: "this calls for wisdom: let anyone with understanding calculate the number of the beast, for it is the number of a man. Its number is six hundred and sixty six."

Pythagoras constant

The [Pythagoras constant] is the square root of 2 $x = \sqrt{2} = 1.41421....$ It is the length of the diagonal of the unit square. It is irrational because $x = p/q$ would imply $2q^2 = x^2q^2 = p^2$, which is impossible because the prime factorization on the left contains an odd number of 2's, while it contains an even number of 2's on the right.

Smith number

[Smith number] Smith numbers are integers n such that the sum of its digits in the decimal exapansion of n is qual to the sum of the digits of its prime factorization, excluding 1. Smith numbers were defined by A. Wilansky. He called it Smith numbers after his brother in law H. Swmith, whose telephone number $4937775 = 3*5*5*65'837$ is a Smith number. Here are the first Smith numbers: $4, 22, 27, 58, 85, 94, 121, 166, 202, 265....$

Wallis constant

The [Wallis constant] is the real solution to the polynomial x^3-2x+5 which is 2.0945514815.... This equation was solved by the English mathematicaian John Wallis [1616-1703] to illustrate Newton's method for the numerical solution of equations. It has since served as a test for many subsequent methods of approximation.

Zero

[Zero] The integer zero 0 is the neutral element in the additive group of integers $n + 0 = n$.

Index

ENTRY CURVES

[ENTRY CURVES] Authors: Oliver Knill, Andrew Chi, 2003 Literature: www.mathworld.com, www.2dcurves.com

astroid

An [astroid] is the curve $t \mapsto (cos^3(t), a\sin^3(t))$ with $a > 0$. An asteroid is a 4-cusped hypocycloid. It is sometimes also called a tetracuspid, cubocycloid, or paracycle.

Archimedes spiral

An [Archimedes spiral] is a curve described as the polar graph $r(t) = at$ where $a > 0$ is a constant. In words: the distance $r(t)$ to the origin grows linearly with the angle.

bowditch curve

The [bowditch curve] is a special Lissajous curve $r(t) = (asin(nt + c), bsin(t))$.

brachistochone

A [brachistochone] is a curve along which a particle will slide in the shortest time from one point to an other. It is a cycloid.

Cassini ovals

[Cassini ovals] are curves described by $((x + a) + y^2)((x - a)^2 + y^2) = k^4$, where $k^2 < a^2$ are constants. They are named after the Italian astronomer Goivanni Domenico Cassini (1625-1712). Geometrically Cassini ovals are the set of points whose product to two fixed points $P = (-a, 0), Q = (0, 0)$ in the plane is the constant k^2. For $k^2 = a^2$, the curve is called a Lemniscate.

cardioid

The [cardioid] is a plane curve belonging to the class of epicycloids. The fact that it has the shape of a heart gave it the name. The cardioid is the locus of a fixed point P on a circle roling on a fixed circle. In polar coordinates, the curve given by $r(\phi) = a(1 + \cos(\phi))$.

catenary

The [catenary] is the plane curve which is the graph $y = c\cosh(x/c)$. It was discovered by Jacques Bernoulli. It has the shape of a uniform flexible chain hung from two points.

catenoid

The [catenoid] is the surface obatained by rotating the catenary about the x-axis. The minimal surface bounded by two coaxial rings can be a catenoid.

circle

A [circle] in the plane is the curve $r(t) = (r\cos(t), r\sin(t))$ where the radius r is a constant. It is the set of points which have a fixed distance r from the origin. More generally, a circle is the set of points in a metric space which have a fixed distance from a given point.

cissoid

A [cissoid] is a plane curve given in Euclidean coordinates by $y^2(2a - x) = x^3$. In polar coordinates, it satisfies $r(t) = 2a\tan(t)\sin(t)$ or in Euclidean coordinates $r(t) = (2a\sin^2(t), 2a\sin^3(t)/\cos(t))$. The curve has a cusp at the origin. It was first mentioned by Diocles in 180 B.C.

conic section

A [conic section] is a nondegenerate curve generated by intersecting a plane with one or two nappes of a cone. The three congruence classes of conic sections are the ellipse, the parabola, and the hyperbola.

ellipse

An [ellipse] is the locus of all points in the plane the sum of whose distances from two fixed points is a positive constant. It is also the conic section which results from a plane which intersects only one nappe of the cone. The general formula for an ellipse up to rotation and translation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

hyperbola

A [hyperbola] is the set of points in the plane for which the difference of the distances from two fixed points is a constant. It is also the conic section which results from a plane intersecting a cone. The general formula for a hyperbola up to rotation and translation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

curve

A [curve] is a continuous map from the real line to an the plane or to space. The word "curve" is often used to mean the image of this map. Curves can be represented parametrically by $r(t) = (x(t), y(t))$ or implicitely as $f(x, y) = 0$.

Airy function

The [Airy function] is commonly found as a solution to boundary value problems in quantum mechanics and electromagnetism. It is the solution to the differential equation: $y'' = xy$. The two independent solutions are (without constants):

$$Ai(x) = \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt$$

$$Bi(x) = \int_0^\infty \left(e^{-\frac{t^3}{3}+xt} + \sin\left(\frac{t^3}{3} + xt\right)\right) dt$$

algebraic curve

A plane curve is an [algebraic curve] if it is given by $g(x, y) = 0$ where g is algebraic a polynomial in x and y. An algebraic curve with degree greater than 2 is called a higher plane curve. The circle $g(x, y) = x^2 + y^2 - 1 = 0$ is an example of an algebraic curve, the catenary $g(x, y) = y - c\cosh(x/c) = 0$ is an example of a nonalgebraic curve.

cubic curve

A [cubic curve] is an algebraic curve of order three. Newton showed that all cubics can be generated as projections of the five divergent cubic parabolas. Examples include the cissoid of Diocles and ellptic curves.

ampersand curve

The [ampersand curve] is a quartic curve with implicit equation $(y^2 - x^2)(x - 1)(2x - 3) = 4(x^2 + y^2 - 2x)^2$. It looks like an ampersand.

bean curve

The [bean curve] is a quartic curve given by the implicit equation: $x^4 + x^2y^2 + y^4 = x(x^2 + y^2)$. It looks like a bean.

bicorn

The [bicorn] is the name of a collection of quartic curves studied by Sylvester in 1864 and Cayley in 1867. It is given by $y^2(a^2 - x^2) = (x^2 + 2ay - a^2)^2$.

bicuspid

The [bicuspid] is the quartic curve given by the implicit equation: $(x^2 - a^2)(x - a)^2 + (y^2 - a^2)^2 = 0$.

bow

The [bow] is a quartic curve with the implicit equation: $x^4 = x^2y - y^3$.

cartesian oval

A [cartesian oval] is a quartic curve consisting of two ovals. It is the locus of a point P whose distances from two foci F_1 and F_2 in two-center bipolar coordinates satisfy

$$mr \pm nr' = k$$

where m and n are positive integers, k is a positive real, and r and r' are the distances from F_1 and F_2. If $m = n$, then the oval becomes an ellipse.

Cassini oval

A [Cassini oval] is one of a family of quartic curves, also called Cassini ellipses, described by a point such that the product of its distances from two fixed points a distance $2a$ apart is constant b^2. The shape of the curve depends on b/a. The Cassini ovals are defined in two-center bipolar coordinates by the equation $r_1r_2 = b^2$ where b is a positive constant.

cruciform

A [cruciform] is a plane quartic curve also called the cross curve or policeman on point duty curve. It is given by the implicit equation: $x^2y^2 - b^2x^2 - a^2y^2 = 0$.

lemniscate

The [lemniscate], also known as the lemniscate of Bernoulli, is a polar curve whose most common form is the locus of points the product of whose distances from two fixed points a distance $2a$ away is the constant a^2. The usual polar coordinate form is as follows: $r^2 = a^2 \cos(2\theta)$.

natural equation

A [natural equation] is an equation which specifies a curve independent of any choice of coordinates or parametrization. This arose in the solution to the following problem: given two functions of one parameter, find the space curve for which the functions are the curvature and torsion. Often, the natural equation will be in terms of integrals.

polynomial curve

A [polynomial curve] is a curve obtained by fitting polynomials to a sequence of points. To fit curves better, splines like Bezier curve are more suited.

quadrifolium

A [quadrifolium] is a rose curve with $n = 2$. It has polar equation

$$r = a\sin(2\theta).$$

sextic curve

A [sextic curve] is an algebraic curve of degree 6. Examples include the atriphtaloid and the butterfly curve $y^6 = x^2 - x^6$.

atriphtaloid

The [atriphtaloid] is a sextic curve also known as atriphtothlassic curve and given by the equation: $x^4(x^2 + y^2) - (ax^2 - b)^2 = 0$.

butterfly curve

The [butterfly curve] is a sextic plane curve given by the implicit equation $y^6 = x^2 - x^6$.

trifolium

A [trifolium] is the 3-petalled rose given in polar form as $r(t) = a|\cos(3t)|$.

spiral

A [spiral], in general, is a curve with $\tau(s)/\kappa(s)$ constant for

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 34 entries in this file.

Index

ENTRY FUNCTIONAL ANALYSIS

[ENTRY FUNCTIONAL ANALYSIS] Authors: Oliver Knill: 2002 Literature: various notes

adjoint

The [adjoint] of a bounded linear operator A on a Hilbert space is the unique operator B which satisfies $(Ax, y) = (x, By)$ for all $x, y \in H$. One calls the adjoint A^*. An bounded linear operator is selfadjoint, if $A = A^*$.

Alaoglu's theorem

[Alaoglu's theorem] (=Banach-Alaoglu theorem): the closed unit ball in a Banach space is weak-* compact.

angle

The [angle] ϕ between two vectors v and w of a Hilbert space is a solution ϕ of the equation $\cos(\phi)||v||||w|| = (v, w)$, usually the smaller of the two solutions.

balanced

A subset Y of a vector space X is called [balanced] if tx is in Y whenever x is in Y and $t < 1$.

B*-algebra

A [B*-algebra] is a Banach algebra with a conjugate-linear anti-automorphic involution $*$ satisfying $||xx^*|| = ||x||^2$.

Banach algebra

A [Banach algebra] is an algebra X over the real numbers or complex numbers which is also a Banach space such that $||xy|| \leq ||x||||y||$ for all $x, y \in X$.

Banach limit

A [Banach limit] is a translation-invariant functional f on the Banach space of all bounded sequence such that $f(c) = c_1$ for constant sequences.

Banach space

A [Banach space] is a complete normed space.

barrel

A [barrel] is a closed, convex, absorbing, balanced subset of a topological vector space.

basis

A [basis] (= Schauder basis) of a separable normed space is a sequence of vectors x_j such that every vector x can uniquely be written as $y = \sum_j y_j x_j$.

basis

A [basis] in a vector space is a linearly independent subset that generates the space.

baralled space

A [baralled space] is a topological vector space in which every barrel contains a neighborhood of the origin.

biorthogonal

Two sequences a_n and b_n in a Hilbert space are called [biorthogonal] if $A_{nm} = (a_n, b_m)$ is an unitary operator.

Bergman space

The [Bergman space] for an open subset G of the complex plane C is the collection of all anlytic function f on G for which $\int \int_G |f(x+iy)|^2 \, dxdy$ is finite. It is an example of a Hilbert space.

Buniakovsky inequality

The [Buniakovsky inequality] (=Cauchy-Schwarz inequality) in a Hilbert space tells that $|(a,b)| \leq ||a|| \, ||b||$.

Cauchy-Schwartz inequality

The [Cauchy-Schwartz inequality] in a Hilbert space H states that $|(f,g)| \leq ||f|| \, ||g||$. It is also called Buniakovsky inequality or CBS inequality.

compact operator

A [compact operator] is a bounded linear operator A on a Hilbert space, which has the property that the image $A(B)$ of the unit ball B has compact closure in H.

compact operator

A bounded operator A on a separable Hilbert space is called [diagonalizable] if there exists a basis in H such that $Hv_i = \lambda_i v_i$ for every basis vector v_i. Compact normal operators are diagonalizable.

dimension

The [dimension] of a Hilbert space H is the cardinality of a basis of H. A Hilbert space is called seperable, if the cardinality of the basis is the cardinality of the integers.

Egorov's theorem

[Egorov's theorem] Let (X, S, m) be a measure space, where $m(S)$ has finite measure. If a sequence f_n of measurable functions converges to f almost everywhere, then for every d¿0, there is a set $E_d \subset X$ such that $f_n \to f$ uniformly on $E \setminus E_d$ and $m(E_d) < d$.

finite rank operator

A bounded linear operator A on a Hilbert space H is called a [finite rank operator] if the rank of A is finite dimensional. Finite rank operators are examples of compact operators.

Hilbert space

A [Hilbert space] H is a vector space equiped with an inner product (x, y) for which the corresponding metric $d(x, y) = ||x - y|| = (x - y, x - y)$ makes (H, d) into a complete metric space. Examples:

- $l^2(N)$ is the collection of sequences a_n such that $\sum_n |a_n| < \infty$ is a Hilbert space with inner product $(a, b) = \sum_n a_n b_n$.
- $L^2(G)$ the space of all analytic functions on an open subset of the complex plane which are also in $L^2(G, \mu)$, where μ is the Lebesgue measure on G.
- All vectors of a finite dimensional vector space, where the inner product is the usual dot product.
- All square integrable functions $L^2(X, \mu)$ on a measure space (X, S, μ).

idempotent

A bounded linear operator A on a Hilbert space is called [idempotent] if $A^2 = A$. Projections P are examples of idempotent operators.

Lusin's theorem

[Lusin's theorem] If (X, S, m) is a measure space and f is a measureable function on S. For every $d > 0$, there is a set E_d with $m(E_d) < d$ and a measurable function g such that g is continuous on E_d.

linear operator

A [linear operator] is a linear map between two Hilbert spaces or two Banach spaces. Important examples are bounded linear operators, linear operators which also continuous maps. Linear operators are also called linear transformations.

norm

The [norm] of a bounded linear operator A on a Hilbert space H is defined as $||A|| = \sup_{||x|| \leq 1, x \in H} ||Ax||$.

normal

A bounded linear operator A is called [normal] if $AA^* = A^*A$, where A^* is the adjoint of A. Examples:

- selfadjoint operators are normal.
- unitary operators are normal

Open mapping theorem

[Open mapping theorem] If a map A from X to Y is a surjective continuous linear operator between two Banach spaces X and Y, and U is an open set in X, then $A(U)$ is open in Y.
The proof of the theorem which is also called the Banach-Schauder theorem uses the Baire category theorem. implications:

- A bijective continuous linear operator between the Banach spaces X and Y has a continuous inverse.
- Closed graph theorem: if for every sequence $x_n \in X$ with $x_n \to 0$ and $Ax_n \to y$ follows $y = 0$, then A is continuous.

Riesz representation theorem

[Riesz representation theorem] If f is a bounded linear functional on a Hilbert space H, then there exists a vector $y \in H$ such that $f(x) = (x, y)$ for all $x \in H$.

Sturm Liouville operator

A [Sturm Liouville operator] L is an unbounded operator on the Hilbert space $L^2[a, b]$ defined by $L(f) = -f'' + gf$, where g is a continuous function on $[a, b]$.

unitary

A bounded linear operator A on a Hilbert space is called [unitary] if $AA^* = A^*A = 1$ if 1 is the identity operator $1(x) = x$.

unit ball

The [unit ball] B in a Hilbert space H is the set of all points $x \in H$ satisfiying $||x| \leq 1$.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 32 entries in this file.

Index

ENTRY FUNCTIONS

[ENTRY FUNCTIONS] Authors: Oliver Knill: 2003, Literature: no

real-valued function

A [real-valued function] is usually assumed to be map to the reals.

abscissa

[abscissa] The x-coordinate in an (x,y) graph of a function. The y-coordinates is called ordinate.

ordinate

[ordinate] The y-coordinate in an (x,y) graph of a function. The x-coordinates is called abscissa.

Airy function

The [Airy function] is defined as the solution of the differential equation $y'' - xy = 0$.

Briggsian logarithm

The [Briggsian logarithm] also called common logarithm is the logarithm to the base 10.

Bessel function

THe [Bessel function] is a special function. Bessel function of the first kind of order zero is defined as $J_0 = \sum_{k=0}^{\infty} (-1)^k (x/2)^{2k}/(k!)^2$.

Sin

The [Sin] is a trigonometric function. It can be defined by its series $\sin(x) = x - x^3/3! + x^5/5! - ...$, where $5! = 54321$ is the factorial of 5. The sine function can also be defined as the imaginary part of $\exp(ix) = \cos(x) + i\sin(x)$, where $i = (-1)^(1/2)$ is the imaginary unit. Examples of values $\sin(0) = 0, \sin(\pi/2) = 1, \sin(\pi) = 0, \sin(3\pi/2) = -1$.

Csc

[Csc] The cosecant is defined as $\csc(x) = 1/\sin(x)$.

Arcsin

[Arcsin] The arcsin is the inverse of sin. It is also denoted by $\sin^(-1)(x)$ or $\text{asin}(x)$. One has the identities $\arcsin(\sin(x)) = x$, or $\sin(\arcsin(x)) = x$.

Sinh

[Sinh] The hyperbolic sine can be defined as $\sinh(x) = (\exp(x) - \exp(-x))/2$. Examples: $\sinh(0) = 0$.

ArcSinh

[ArcSinh] The inverse of sinh is called arcsinh.

Cos

The trigonometric function [Cos] can be defined by its series $\cos(x) = 1 - x^2/2! + x^4/4! - x^6/6! - ...$, where $4! = 4321$ is the factorial of 4. It can also be defined as the real part of $\exp(ix)$, where $i = (-1)^1/2$ is the imaginary unit, the square root of -1. Examples: $\cos(0) = 1, \cos(pi/2) = 0 \sin(\pi) = -1, \sin(3\pi/2) = 0$.

Arccos

[Arccos] The inverse of the function cos is written $\arccos(x)$, also denoted by $\cos^(-1)(x)$ or $\text{acos}(x)$. We have the identities $\arccos(\cos(x)) = x$, or $\cos(\arccos(x)) = x$.

Sec

[Sec] The secant is defined as $\sec(x) = 1/\cos(x)$.

Cosh

[Cosh] The hyperbolic cosine can be defined as $\sinh(x) = (\exp(x) + \exp(-x))/2$. Examples: $\cosh(0) = 1$.

ArcCosh

[ArcCosh] The inverse of cosh is called arccosh.

Tan

The [Tan] is a trigonometric function. It can be defined as $\tan(x) = \sin(x)/\cos(x)$. Examples: $\tan(0) = 0, \tan(\pi/4) = 1$.

Arctan

[Arctan] The inverse of tan is the function $arctan(x)$. It is also called $tan^(-1)(x)$. One has $\arctan(\tan(x)) = x$ and $\tan(\arctan(x)) = x$. Examples: $\arctan(1) = \pi/2$.

Cot

[Cot] is a trigonometric function. It can be defined as $\cot(x) = \cos(x)/\sin(x)$. It can also be defined geometrically as the relation of two sides in a right angle triangle if x is one of the angles. Examples: $\cot(\pi/2) = 0, \cot(\pi/4) = 1$.

Exp

[Exp] is the exponential function. It can be defined by its series $\exp(x) = 1 + x + x^2/2! + x^3/3! + x^4/4! + ...$ where $4! = 4321$ is the factorial of 4. Examples: $\exp(0) = 1$, $\exp(1) = e = 2.712...$.

Sqr

[Sqr] The square of a number is the product of the number by itself. For example, the square of 4 is 16. The square of a function $\sin(x)$ is denoted by $\sin^2(x)$.

Zeta

[Zeta] $\zeta(s)$ is the Riemann zeta function. It is defined for complex numbers s which have $\mathrm{Re}(s) > 1$ as $\zeta(s) = 1+1/2^s+1/3^s+....$ The function can be continued to the entire complex plane except at $s = 1$, where the function has a singularity. The zeta function has zeros at $-2, -4, -6$ and also zeros on the real line $\mathrm{Re}(s) = 1/2$. The famous Riemann hypothesis claims that all the nontrivial zeros are on this line. This conjecture remains unproven until today and is considered one of the most important open problems in mathematics.

Log

[Log] The logarithm is the inverse to the exponential function: $\log(\exp(x)) = x$ and $\exp(\log(x)) = x$. For example: $\log(1) = 0$, $\log(e) = 1$. The logarithm function satisfies for example the laws $\log(xy) = \log(x)+\log(y)$, $\log(x/y) = \log(x) - \log(y)$, $\log(x^y) = y \log(x)$.

Sqrt

[Sqrt] The square root of a number x is the number which

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 24 entries in this file.

Index

ENTRY GROUP THEORY

[ENTRY GROUP THEORY] Authors: started Mark Lezama: October 2003 Literature: "Algebra" by Michael Artin, Mathworld

Group theory

[Group theory] is studies algebraic objects called groups. The German mathematician Karl Friedrich Gauss (1777-1855) developed but did not publish some of the mathematics of group theory. The French mathematician Evariste Galois (1811-1832) is generally credited with being the first to develop the theory, which he did by developing new techniques to study the solubility of equations. Group theory is a powerful method for analyzing abstract and physical systems in which symmetry –the intrinsic property of an object to remain invariant under certain classes of transformations– is present because the mathematical study of symmetry is systematized and formalized in group theory. Consequently, group theory is an important tool in physics particularly in quantum mechanics.

group

A [group] is an object consisting of a set G and a law of composition (or binary operation) L on G satisfying:

- L is associative.
- L has an identity in G.
- Every element of G has an inverse.

The study of groups is known as group theory. If a group G has n elements where n is a positive integer, then G is a finite group with order n. If a group is not finite it is infinite. Examples:

- Z^+, the integers under addition;
- $R^+ = (R, +)$, the real numbers under addition;
- $R^\times = (R - \{0\}, \cdot)$, the real numbers without zero under multiplication;
- $GL_n(C)$, the $n \times n$ general linear group under matrix multiplication;
- S_n, the symmetric group on n objects under composition.

law of composition

A [law of composition] or, binary operation, on a set S is a function from $S \times S$ into S. That is, a law of composition on S prescribes a rule for combining pairs of elements in S to get an element in S. For convenience, functional notation is not used; that is, if a law of composition f sends (a, b) to c, one does not usually write $f(a, b) = c$. It is customary to instead use notation that resembles that used for multiplication or addition of real numbers, such as $ab = c$, $a \cdot b = c$, $a \circ b = c$, $a + b = c$, and so on.
An example of a law of composition is multiplication on the real numbers, R. If $m: R \times R \to R$ defines multiplication on R then $m(x, y) = x \cdot y$. For example $m(2, 5) = 2 \cdot 5 = 10$.

binary operation

A [binary operation], or law of composition, on a set S is a function from $S \times S$ into S. That is, a binary operation on S prescribes a rule for combining pairs of elements in S to get an element in S. For convenience, functional notation is not used; that is, if a binary operation f sends (a, b) to c, one does not usually write $f(a, b) = c$. It is customary to instead use notation that resembles that used for multiplication or addition of real numbers, such as $ab = c$, $a \cdot b = c$, $a \circ b = c$, $a + b = c$, and so on.
An example of a binary operation is multiplication on the real numbers, R. If $m\colon R \times R \to R$ defines multiplication on R then $m(x, y) = x \cdot y$. For example $m(2, 5) = 2 \cdot 5 = 10$.

associative

A law of composition on a set S is [associative] if for all $a, b, c \in S$, $(ab)c = a(bc)$. The informal intuition behind associativity (the property of being associative) is that if one has an expression in which there are many parentheses and the only operation performed in this expression is that defined by an associative law of composition, then one may ignore the parentheses. For example, if $\cdot$ is an associative law of composition on S and $a, b, c, d \in S$, then $((a \cdot (b \cdot c)) \cdot d = ((a \cdot b) \cdot c) \cdot d = (a \cdot b) \cdot (c \cdot d)$ and so on; thus one may write $a \cdot b \cdot c \cdot d$ without being ambiguous.
An example of an associative law of composition is addition on the integers, Z. That is, for all $a, b, c \in Z$, $(a + b) + c = a + (b + c)$.

identity

An [identity] for a law of composition on a set S is an element e such that, for all $a \in S$, $ea = a$ and $ae = a$. Note that a law of composition has at most one identity. The symbols e, 0 and 1 are commonly used to denote the identity element of a group. The number 0 is an identity for addition on the real numbers.

identity

Suppose a set S has a law of composition with identity 1. For every element $a \in S$, if there exists an element $b \in S$ such that $ab = 1$ and $ba = 1$ then b is the [inverse] of a. When using multiplicative notation for the law of composition, the inverse of a can be written as a^{-1}. As an example, the inverse of any integer n is $-n$ where the law of composition is addition and the identity is 0. As another example, the inverse of any nonzero real number x is $\frac{1}{x}$, where the law of composition is multiplication and the identity is 1.

general linear group

The $n \times n$ [general linear group] $GL_n(F)$ is the set of $n \times n$ matrices with entries in the field F and nonzero determinant, under the law of composition of matrix multiplication. Thus $GL_n(F)$ is the group of $n \times n$ invertible matrices with entries in F. If F is a finite field of field order q then sometimes the general linear group $GL_n(F)$ is denoted by $GL_n(q)$. The general linear group often appears with respect to the real numbers, R, or the complex numbers, C; that is, the general linear group often appears as $GL_n(R)$ or $GL_n(C)$.
The special linear group $SL_n(F)$ is the subgroup of $GL_n(F)$ whose elements have determinant equal to 1.

special linear group

The $n \times n$ [special linear group] $SL_n(F)$ is the set of $n \times n$ matrices with entries in the field F and determinant equal to 1, under the law of composition of matrix multiplication.
If F is a finite field of field order q then sometimes the special linear group $SL_n(F)$ is denoted by $SL_n(q)$.
$SL_n(F)$ is a subgroup of the general linear group $GL_n(F)$.

trivial

A group is [trivial] if it contains exactly one element. The one element in the group is the identity element. As all trivial groups are isomorphic, one usually refers to a trivial group as *the* trivial group. A group that is not trivial is nontrivial.

trivial

The group containing exactly one element (the identity) is unique up to isomorphism and is therefore called the [trivial group]. The trivial group is a normal subgroup of every group.

nontrivial

A group is [nontrivial] if it is not trivial.

abelian

A group is [abelian] if its law of composition is commutative. Examples of abelian groups include the following:

- $R^+ = (R, +)$, the real numbers under addition;
- $R^\times = (R - \{0\}, \cdot)$, the real numbers without zero under multiplication;
- any cyclic group.

Examples of nonabelian groups, i.e. groups that are not abelian:

- $GL_n(C)$, the general linear group;
- The symmetric group on n objects, where n is a positive integer greater than 2.

commutative

A law of composition on a set S is [commutative] if for all $a, b \in S$ $ab = ba$. An example of a commutative law of composition is addition on the real numbers: for example, $3.2 + 4 = 7.2 = 4 + 3.2$.

cancellation Law

The [cancellation Law] states that if a, b, and c are elements of a group and if $ab = ac$ then $b = c$. Similarly, if $ba = ca$ then $b = c$. The Cancellation Law follows from the fact that every element of a group has an inverse.

permutation

If S is a set, then a [permutation] of S is a bijective map from S into S. The intuition underlying the definition of a permutation is that a permutation determines a reordering of the elements in a list or the rearrangement of objects. For example, the permutation $\sigma : \{1, 2, 3\} \to \{1, 2, 3\}$ defined by $\sigma(1) = 2$, $\sigma(2) = 1$, and $\sigma(3) = 3$ can be thought to represent the reordering of the list 1,2,3 that results in the list 2,1,3. There is an important kind of permutation called a transposition. A transposition of a set S is a permutation $\sigma\colon S \to S$ satisfying the following: there exist $s_1, s_2 \in S$ such that

- $\sigma(s_1) = s_2$
- $\sigma(s_2) = s_1$
- and for all $s \in S$, if $s \neq s_1$ and $s \neq s_2$, then $\sigma(s) = s$.

Every permutation of a finite set can be written as the composition of a finite number of transpositions of that set. For example, the permutation $\sigma : \{1, 2, 3\} \to \{1, 2, 3\}$ defined by $\sigma(1) = 2$, $\sigma(2) = 3$, and $\sigma(3) = 1$ is equivalent to the composition of two transpositions. Define $\sigma_1 : \{1, 2, 3\} \to \{1, 2, 3\}$ by $\sigma_1(1) = 2$, $\sigma_1(2) = 1$, and $\sigma(3) = 3$, and define $\sigma_2 : \{1, 2, 3\} \to \{1, 2, 3\}$ by $\sigma_2(1) = 3$, $\sigma_1(2) = 2$, and $\sigma(3) = 1$. Then σ_1 and σ_2 are transpositions and $\sigma = \sigma_2 \circ \sigma_1$.
The sign of a permutation σ is $(-1)^n$ where n is a finite positive integer such that there exist n transpositions whose composition equals σ. If a permutation has sign 1, then it is called an even permutation. If a permutation has sign -1 then it is called an odd permutation. Thus the identity permutation is an even permutation (since it is equal to the composition of any transposition with itself), and any transposition is an odd permutation (since it is equal to one transposition). The sign map encapsulates the notion of the sign of a permutation of a finite set.
The set of permutations on a set forms a group where the law of composition is composition of functions. One example of a group of permutations that appears frequently in group theory is the symmetric group on n objects, i.e. the group of permutations of the set $\{1, 2, \ldots, n\}$.

transposition

A [transposition] of a set S is a permutation $\sigma\colon S \to S$ satisfying the following: there exist $s_1, s_2 \in S$ such that

- $\sigma(s_1) = s_2$
- $\sigma(s_2) = s_1$
- and for all $s \in S$, if $s \neq s_1$ and $s \neq s_2$, then $\sigma(s) = s$.

Every permutation of a finite set can be written as the composition of a finite number of transpositions of that set. For example, the permutation $\sigma : \{1, 2, 3\} \to \{1, 2, 3\}$ defined by $\sigma(1) = 2$, $\sigma(2) = 3$, and $\sigma(3) = 1$ is equivalent to the composition of two transpositions. Define $\sigma_1 : \{1, 2, 3\} \to \{1, 2, 3\}$ by $\sigma_1(1) = 2$, $\sigma_1(2) = 1$, and $\sigma(3) = 3$, and define $\sigma_2 : \{1, 2, 3\} \to \{1, 2, 3\}$ by $\sigma_2(1) = 3$, $\sigma_1(2) = 2$, and $\sigma(3) = 1$. Then σ_1 and σ_2 are transpositions and $\sigma = \sigma_2 \circ \sigma_1$. Every transposition is an odd permutation.

sign

The [sign] of a permutation σ is $(-1)^n$ where n is a finite positive integer such that there exist n transpositions whose composition equals σ. If a permutation has sign 1, then it is called an even permutation. If a permutation has sign -1 then it is called an odd permutation. Thus the identity permutation is an even permutation (since it is equal to the composition of any transposition with itself), and any transposition is an odd permutation (since it is equal to one transposition).

even permutation

An [even permutation] is a permutation that has sign 1. That is, an even permutation is the composition of an even number of transpositions. Thus the identity permutation is an even permutation.

odd permutation

An [odd permutation] is a permutation that has sign -1. That is, an odd permutation is the composition of an odd number of transpositions. Thus every tranposition is an odd permutation.

symmetric group

The [symmetric group] on n objects, denoted S_n, is the group of permutations of the set $\{1, 2, \ldots, n\}$; the law of composition is composition of functions. The order of S_n is $n!$ for all positive integers n. For example, $S_2 = \{e, \sigma\}$, where e is the identity permutation, and σ is a transposition. That is, e is the identity element of S_2 and is defined by $e(1) = 1$ and $e(2) = 2$; σ is defined by $\sigma(1) = 2$ and $\sigma(2) = 1$.

sign map

The [sign map], denoted sign, is a group homomorphism from the symmetric group, S_n, into the group $\{1, -1\}$ (under multiplication). The sign map is defined by $\text{sign}(\sigma) = (-1)^k$ where σ is equal to the composition of k transpositions. The sign map is well-defined because it is a standard result that if σ is any permutation (of any set), and if σ is equal to the composition of k transpositions and is also equal to the composition of m transpositions, then $(-1)^k = (-1)^m$. The kernel of the sign map is the alternating group, A_n; that is, A_n is the group of even permutations on n objects.

alternating group

The [alternating group] is the kernel of the sign map. In other words, the alternating group on n objects, usually denoted A_n, is a normal subgroup of the symmetric group on n objects: $A_n = \{\sigma \in S_n \mid \text{sign}(\sigma) = 1\}$. Thus A_n is the group of even permutations in S_n. For $n \geq 5$, A_n is a simple group, i.e. a group that has no proper normal subgroup.

simple group

A group G is a [simple group] if every normal subgroup N of G is not a proper subgroup. That is, G is simple if its only normal subgroups are G and the trivial group. The alternating group A_n is simple for $n \geq 5$. Any cyclic group of prime order is simple. Any cyclic group of prime order is simple. In fact, any simple abelian group is a cyclic group of prime order.

subgroup

A subset H of a group G is a [subgroup] of G if it satisfies the following properties:

- If $a \in H$ and $b \in H$, then $ab \in H$.
- $1 \in H$, where 1 is the identity element of G.
- If $a \in H$ then the inverse of a, a^{-1}, is also in H.

When it is clear that G is a group, sometimes $H \subseteq G$ is used to denote that H is a subgroup of G (as opposed to merely being a subset of G).
Every nontrivial group G has at least two subgroups: the whole group G and the subgroup $\{1\}$ consisting exactly of the identity element of G. If G is trivial then these two subgroups are the same and G has exactly one subgroup. A subgroup is a proper subgroup if it is neither the whole group nor the trivial group.
As an example, the integers under addition are a subgroup of the real numbers under addition.
By Lagrange's Group Theorem, if H is a subgroup of a finite group G, the order of H divides the order of G.

proper subgroup

A [proper subgroup] of a group G is a nontrivial subgroup of G that is not equal to G.

order

A finite group G is said to have [order] n if G has n elements. More generally, the order of a group G is the cardinality of the set G, both of which are often denoted $|G|$.
For any given element x of a given group, if there exists a positive integer k such that $x^k = 1$, then x is said to have order m, where m is the least positive integer satisfying $x^m = 1$. If $x^k \neq 1$ for all positive integers k, then x is said to have infinite order.

cyclic group

If G is a group and if x is an element of G, the [cyclic group] $\langle x \rangle$ generated by x is the set of all powers of x: $\langle x \rangle = \{\dots, x^{-2}, x^{-1}, 1, x, x^2, \dots\}$.
Note that $\langle x \rangle$ is the smallest subgroup of G which contains x. Further note that any cyclic group is abelian. If x has infinite order then $\langle x \rangle$ is said to be infinite cyclic. Note that if $\langle x \rangle$ is infinite cyclic then $\langle x \rangle$ is isomorphic to Z^+, the integers under addition. As a result, one sometimes refers to any infinite cyclic group as *the* infinite cyclic group, denoted Z^+.
If x has order n, then $\langle x \rangle$ has order n and is called a cyclic group of order n: $\langle x \rangle = \{1, x, x^2, \dots, x^n\}$.
If $\langle x \rangle$ is a cyclic group of order n, then $\langle x \rangle$ is isomorphic to Z/n, where Z/n is the group satisfying the following properties (we use additive notation as opposed to multiplicative notation for the law of composition of Z/n): 1. $Z/n = \{0, 1, 2, \dots, n-1\}$. 2. for any $x, y \in Z/n$, $x +_1 y$ is the unique element in Z/n which is congruent modulo n to $x + y$, where $+_1$ denotes the law of composition on Z/n and $+$ denotes conventional integer addition. Normally $+$ is used to denote the law of composition on Z/n, but $+_1$ is used here to distinguish it from conventional addition. Two integers a and b are congruent modulo n, written $a \equiv b(\text{modulo } n)$, if n divides $b - a$.
As a result, one sometimes refers to any cyclic group of order n as *the* cyclic group of order n, often denoted Z/n.

cyclic group

Let G_1 and G_2 be groups. A map $\varphi \colon G_1 \to G_2$ is a group [homomorphism] if $\varphi(ab) = \varphi(a)\varphi(b)$ for all $a, b \in G_1$. Here we use the same multiplicative notation for the laws of composition of G_1 and G_2, even though there is no requirement that their laws of composition be the same.
Note that $\varphi(1_{G_1}) = 1_{G_2}$ and $\varphi(a^{-1}) = \varphi(a)^{-1}$ for all $a \in G$, where 1_{G_i} is the identity of G_i.
The kernel of φ, sometimes denoted ker φ, is the set $\{x \in G_1 \mid \varphi(x) = 1_{G_2}\}$. Note that ker φ is a normal subgroup of G_2.
Another subgroup of G_2 determined by φ is the image of φ, sometimes denoted im φ. The image of φ is im $\varphi = \{x \in G_2 \mid x = \varphi(a) \text{ for some } a \in G_1\}$. Sometimes the image of φ is denoted $\varphi(G_1)$. The following are examples of homomorphisms:

- The inclusion map $i \colon H \to G$ defined by $i(a) = a$, where H is a subgroup of G. ker $i = \{1_G\}$ and im $i = H$.
- For a fixed $a \in G$, the map $\varphi \colon Z^+ \to G$ defined by $\varphi(n) = a^n$, where Z^+ denotes the integers with addition. ker $\varphi = \{n \mid a^n = 1_G\}$ and im $\varphi = \langle a \rangle$ (the cyclic subgroup generated by a).
- The determinant map $\det \colon GL_n(R) \to R^\times$, where $GL_n(R)$ denotes the general linear group and $R^\times$ denotes the real numbers without zero under multiplication. $kerdet = SL_n(R)$, the special linear group, and $imdet = R^\times$.
- The sign map on permutations sign: $S_n \to \{1, -1\}$, where S_n denotes the symmetric group on n objects. ker sign $= A_n$, the alternating group, and im sign $= \{1, -1\}$.

Let R_1 and R_2 be rings. A map $\varphi \colon R_1 \to R_2$ is a ring homomorphism if

- $\varphi(a + b) = \varphi(a) + \varphi(b)$,
- $\varphi(ab) = \varphi(a)\varphi(b)$, and
- $\varphi(1_{R_1}) = 1_{R_2}$, for all $a, b \in R_1$.

Here we use the same additive and multiplicative notation for the laws of composition of R_1 and R_2, even though there is no requirement that their laws of composition be the same.

kernel

The [kernel] of a group homomorphism $\varphi\colon G_1 \to G_2$ is the set $\ker\varphi = \{x \in G_1 \mid \varphi(x) = 1_{G_2}\}$, where 1_{G_2} denotes the identity of G_2. The kernel of a homomorphism is an important example of a normal subgroup. There are many results involving the kernel of a homomorphism.

image

The [image] of a map $\varphi\colon G_1 \to G_2$ is the set $\{x \in G_2 \mid x = \varphi(a) \text{ for some } a \in G_1\}$. In general, the image of φ is often denoted $\varphi(G_1)$. If φ is a group homomorphism, then the image of φ is a subgroup of G_2 and is sometimes denoted $\operatorname{im}\varphi$.

isomorphism

A group [isomorphism] is a bijective group homomorphism.
A ring isomorphism is a bijective ring homomorphism.

isomorphic

Two groups G_1 and G_2 are [isomorphic] if there exists a group isomorphism from G_1 into G_2. Sometimes $G_1 \cong G_2$ is used to denote 'G_1 and G_2 are isomorphic.' Note that $\cong$ is an equivalence relation on the set of all groups. When one speaks of classifying groups, hat is usually referred to is the classification of isomorphism classes. Thus one might say that there are two groups of order 6 *up to isomorphism*, meaning that there are two isomorphism classes of groups of order 6.
One sometimes says that 'G_1 is isomorphic to G_2' instead of saying 'G_1 and G_2 are isomorphic.'

automorphism

An [automorphism] of a group G is an isomorphism from G into G. The identity map is a simple example of an automorphism. Conjugation by an element of the group is an important example of an automorphism. That is, for a fixed element $b \in G$, conjugation by b is the map $\varphi\colon G \to G$ defined by $\varphi(a) = bab^{-1}$. Here we use multiplicative notation for the group law of composition. Note that if G is abelian, then conjugation by any element is the identity map. However, if G is not abelian, then there exists a nontrivial conjugation (i.e. a conjugation not equal to the identity map) of G.

automorphism

Let G be a group and let $b \in G$. The map $\varphi\colon G \to G$ defined by $\varphi(a) = bab^{-1}$ is [conjugation] by b. Note that conjugation is an automorphism of G. Further note that if G is abelian, then conjugation by any element is the identity map. However, if G is not abelian, then there exists a nontrivial conjugation (i.e. a conjugation not equal to the identity map) of G.

conjugation

Let G be a group and let H be a subgroup of G. H is a [normal subgroup] of G (sometimes written $H \triangleleft G$) if for all $a \in H$ and for all $x \in G$, $xax^{-1} \in H$. Note that it follows that any subgroup of an abelian group is normal.
Normal subgroups appear often in group theory. Every group G has at least one normal subgroup, called the center of G, denoted by Z or $Z(G)$. The center of G is the set of elements that commute with every element of G: $Z(G) = \{z \in G \mid zx = xz \text{ for all } x \in G\}$. Another important example of a normal subgroup is the kernel of a group homomorphism.

center

The [center] of a group G, denoted by Z or $Z(G)$ is the set of elements that commute with every element of G: $Z(G) = \{z \in G \mid zx = xz \text{ for all } x \in G\}$. Note that if G is abelian then $Z(G) = G$.

coset

Given a subgroup H of a group G, a [coset] of H is a subset H' of G such that there exists an $a \in G$ such that (1) $H' = aH = \{ah \mid h \in H\}$, in which case H' is said to be a left coset; or (2) $H' = Ha = \{ha \mid h \in H\}$, in which case H' is said to be a right coset.
Given $a \in G$, aH is not necessarily equal to Ha. However, one can show that the subgroup H of G is a normal subgroup if and only if $aH = Ha$ for every $a \in G$.
In what follows, only left cosets will be discussed, though similar statements may be made about right cosets.
The left cosets of H are the equivalence classes of the equivalence relation $\sim$ defined by $a \sim b$ if there exists $h \in H$ such that $a = bh$. Since equivalence classes form a partition, the left cosets of H partition G.
The cardinality of the set of left cosets of H is called the index of H in G and is denoted by $[G : H]$. Given $a \in G$, $h \mapsto ah$ defines a bijective map from H into aH. If G is finite, it follows that $|G| = |H|[G : H]$, where $|G|$ denotes the order of G. A very important result follows: if G is finite, then the order of H divides the order of G. Moreover, since the order of any element of G is the order of the cyclic subgroup it generates, if G is finite then the order of an element of G divides the order of G. These results follow from a special case of what is known as Lagrange's Group Theorem: if G is a group, H is a subgroup of G and K is a sugroup of H, then $[G : K] = [G : H][H : K]$, where the products are taken as products of cardinals.
An important result that follows from Lagrange's Theorem is that if the order of G is a prime number then $G = \langle a \rangle$ for any $a \in G$ such that a is not the identity, where $\langle a \rangle$ denotes the cyclic group generated by a.
Note that if $\varphi\colon G \to G'$ is a group [homomorphism], then $[G : ker\varphi] = |im\varphi|$. Thus another result of Langrange's Theorem is that $|G| = |ker\varphi| \cdot |im\varphi|$.

left coset

Given a subgroup H of a group G, a [left coset] of H is a subset H' of G such that there exists an $a \in G$ such that $H' = aH = \{ah \mid h \in H\}$.
Given $a \in G$, aH is not necessarily equal to Ha. However, one can show that the subgroup H of G is a normal subgroup if and only if $aH = Ha$ for every $a \in G$.
In what follows, only left cosets will be discussed, though similar statements may be made about right cosets.
The left cosets of H are the equivalence classes of the equivalence relation $\sim$ defined by $a \sim b$ if there exists $h \in H$ such that $a = bh$. Since equivalence classes form a partition, the left cosets of H partition G.
The cardinality of the set of left cosets of H is called the index of H in G and is denoted by $[G : H]$. Given $a \in G$, $h \mapsto ah$ defines a bijective map from H into aH. If G is finite, it follows that $|G| = |H|[G : H]$, where $|G|$ denotes the order of G. A very important result follows: if G is finite, then the order of H divides the order of G. Moreover, since the order of any element of G is the order of the cyclic subgroup it generates, if G is finite then the order of an element of G divides the order of G. These results follow from a special case of what is known as Lagrange's Group Theorem: if G is a group, H is a subgroup of G and K is a sugroup of H, then $[G : K] = [G : H][H : K]$, where the products are taken as products of cardinals.
An important result that follows from Lagrange's Theorem is that if the order of G is a prime number then $G = \langle a \rangle$ for any $a \in G$ such that a is not the identity, where $\langle a \rangle$ denotes the cyclic group generated by a.
Note that if $\varphi: G \to G'$ is a group [homomorphism], then $[G : ker(\varphi)] = |im(\varphi)|$. Thus another result of Langrange's Theorem is that $|G| = |ker(\varphi)| \cdot |im(\varphi)|$.

right coset

Given a subgroup H of a group G, a [right coset] of H is a subset H' of G such that there exists an $a \in G$ such that $H' = Ha = \{ha \mid h \in H\}$.
Given $a \in G$, aH is not necessarily equal to Ha. However, one can show that the subgroup H of G is a normal subgroup if and only if $aH = Ha$ for every $a \in G$.
In what follows, only left cosets will be discussed, though similar statements may be made about right cosets.
The left cosets of H are the equivalence classes of the equivalence relation $\sim$ defined by $a \sim b$ if there exists $h \in H$ such that $a = bh$. Since equivalence classes form a partition, the left cosets of H partition G.
The cardinality of the set of left cosets of H is called the index of H in G and is denoted by $[G : H]$. Given $a \in G$, $h \mapsto ah$ defines a bijective map from H into aH. If G is finite, it follows that $|G| = |H|[G : H]$, where $|G|$ denotes the order of G. A very important result follows: if G is finite, then the order of H divides the order of G. Moreover, since the order of any element of G is the order of the cyclic subgroup it generates, if G is finite then the order of an element of G divides the order of G. These results follow from a special case of what is known as Lagrange's Group Theorem: if G is a group, H is a subgroup of G and K is a sugroup of H, then $[G : K] = [G : H][H : K]$, where the products are taken as products of cardinals.
An important result that follows from Lagrange's Theorem is that if the order of G is a prime number then $G = \langle a \rangle$ for any $a \in G$ such that a is not the identity, where $\langle a \rangle$ denotes the cyclic group generated by a.
Note that if $\varphi: G \to G'$ is a group homomorphism, then $[G : ke(\varphi)] = |im(\varphi)|$. Thus another result of Langrange's Theorem is that $|G| = |ker(\varphi)| \cdot |im(\varphi)|$.

index

The [index] of subgroup H of a group G is the cardinality of the set of left cosets of H in G. The index of H in G is denoted $[G : H]$.

quotient group

Given a group G and a normal subgroup N of G, the [quotient group] of N in G, written G/N and read "G mod(ulo) N", is the set of cosets of N in G, under the law of composition that is defined as follows: $(aN)(bN) = abN$, where $xN = \{xn \mid n \in N\}$. Note that since N is normal, $aN = Na$ for all $a \in G$, so it is not necessary to define this law of composition in terms of left cosets instead of right cosets.
The order of G/N is the index $[G : N]$ of N in G.
Quotient groups can be identified by the First Isomorphism Theorem: if $\varphi\colon G \to G'$ is a surjective group homomorphism and if $N = ke(\varphi)$ then $\psi\colon G/N \to G'$ is an isomorphism, where ψ is defined by $\psi(aN) = \varphi(a)$.

First Isomorphism Theorem

The [First Isomorphism Theorem]. Suppose $\varphi\colon G \to G'$ is a surjective group homomorphism, and let N denote the kernel of φ. Then the quotient group G/N is isomorphic to G' by the map ψ defined by $\psi(aN) = \varphi(a)$.
The First Isomorphism Theorem is the principle method of identifying quotient groups. As an example, consider the group homomorphism φ from $C^{\times}$, the nonzero complex numbers under multiplication, into $R^{\times}$, the nonzero real numbers under multiplication, defined by $\varphi(z) = |z|$, where $|z|$ denotes the absolute value of z. The kernel of φ is the unit circle, U, and the image of φ is the group of positive real numbers. So $C^{\times}/U$ is isomorphic to the multiplicative group of positive real numbers.

operation

Given a group G and a set S, an [operation] of a G on S is a map from $G \times S$ into S - often written using multiplicative notation: $(g, s) \mapsto gs$ - satisfying:

- $1s = s$ for all $s \in S$, where 1 is the identity of G; and
- $(gg')s = g(g's)$, for all $g, g' \in G$ and for all $s \in S$.

There are some terms that are sometimes associated with a group operation: S is often called a G-set; G is sometimes called a transformation group; and the group operation is often also called a group action.
Mathworld: "Historically, the first group action studied was the action of the Galois group on the roots of a polynomial. However, there are numerous examples and applications of group actions in many branches of mathematics, including algebra, topology, geometry, number theory, and analysis, as well as the sciences, including chemistry and physics."

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 44 entries in this file.

Index

ENTRY HARVARD

[ENTRY HARVARD] Authors: Oliver Knill: 2000, Literature: no

Harvard

[Harvard]

Science center

[Science center] The science center is the Polaroid camera shape building near Harvard square. The actual building is close to Memorial Hall. I actually live and think in the science center. You can virtually walk into the science center

president of Harvard

[president of Harvard] The President of Harvard is currently Lawrence H. Summers. To find out more about him, visit the website

chairman of math department

[chairman of math department] The chairman of the Mathematics department is currently Joe Harris.

number people math department

[number people math department] Currently, there are over 190 people at the Math department.

preceptor

[preceptor] Preceptors work alongside other faculty on teaching, developing and supporting sections of entry level courses at the Harvard Mathematics department.

ca

[ca] A course assistant (CA) is an undergraduate student who assists the teaching fellow (TF) with grading, running problem sessions and tutoring in the question center.

tf

[tf] TF stands for teaching fellow. Everybody who is teaching is called a TF. It can be a senior or junior faculty, a visiting fellow or a graduate student.

concentrator

[concentrator] A (math) concentrator is a sophomore or senior undergraduate student. A math concentrator would also be called a math major. There are about one hundred math concentrators. Each year, there are about 30 new concentrators.

head tutor

[head tutor] The head tutor is a professor who is the chief undergraduate advisor.

question center

[question center] The question center QC is a place to work on homework problems or exam preparation. Tutors (both course assistants or teaching fellows) are available for questions.

qc

[qc] see question center

math table

[math table] The math table is a "dinner seminar" which takes place in one of the student houses. A faculty or student presents a half an hour talk just after a dinner.

grade

[grade] Grades are an important issue for most students. Unfortunately, I have no access to your grades.

where harvard

[where harvard] Harvard University is located in Cambridge, Massachusetts USA. We are on the east cost. One can see from here Boston. The Campus is quite close to the Charles River. To look up something specific, it is best to start online with the Harvard search page.

get into Harvard

[get into Harvard] Work hard, have lots of interests. You need of course some luck.

life at Harvard

[life at Harvard] Fun. Beside a great academic environment, there are a lot of things to see. You can spend weeks at Harvard square and still find new things.

grade inflation

[grade inflation] Grade inflation is when most students get A's. One of the problems with grade inflation is that grades start losing their purpose.

computer type

[computer type] At the Mathematics department, people use all kind of operating systems

- Sun work stations running Solaris
- Macintoshs running OSX
- PC's running Linux
- PC's running Windows.

software

[software] Have a look at http://www.math.harvard.edu/computing.

mathematics

[mathematics] Mathematics is both a science and an art. It is also a language for other sciences. Quite many topics in Mathematics have turned out to be useful. Examples is the theory of operators which provided the framework for Quantum mechanics. An other example is number theory which provides the foundation for many encryption algorithms.

afread of math

[afread of math] You probably had some bad experiences in the past. You should chat more with me!

learn math

[learn math] Just do it! There are hundreds of nice books about Math, many resources on the internet. Take a math class with a good teacher.

physics and math

[physics and math] There are not many differences. Indeed, there are branches of Mathematics like computational number theory where people do a lot of experiments and there are branches of physics, where people do very abstract theory which presumably never can be tested in laboratories.

charles river

[charles river] A great place to row, run bike and relax.

harvard yard

[harvard yard] the center of the Harvard campus

math professors

[math professors] They are all excellent Mathematicians.

pi day

[pi day]

student number

[student number] There are more than 18'000 degree candidates at Harvard.

Harvard founded

[Harvard founded] Harvard College was established in 1636, already 16 years after the arrival of the pilgrims at Plymouth.

people at harvard

[people at harvard] There are over 14'000 people at Harvard including more than 2'000 faculty. There are also 7,000 faculty appointments in affiliated teaching hospitals.

Nobel Laureates

[Nobel Laureates] Harvard produced nearly 40 Nobel Laureates.

US presidents

[US presidents] Harvard produced seven presidents of the United States: John Adams, John Quincy Adams, Theodore and Franklin Delano Roosevelt, Rutherford B. Hayes, John Fitzgerald Kennedy and George W. Bush.

Millenium prize problems

[Millenium prize problems]

- P versus NP
- Hodge Conjecture
- Poincare Conjecture
- Riemann Hypothesis
- Yang-Mills Existence and Mass Gap
- Navier-Stokes Existence and Smoothness
- Birch-Swinnerton-Dyer Conjecture

Core course

[Core course] In 1978, Harvard adopted a 'core' of courses in fields of inquiry that spanned domains, including historical study, moral reasoning, social analysis, science, music and art, literature and so on. These courses are designed to introduce 'approaches to knowledge' rather than specific information and thus legitimized a trend throughout education toward ways of knowing rather than knowledge.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 35 entries in this file.

Index

ENTRY JOKES

[ENTRY JOKES] Authors: Oliver Knill: 2002 Literature: not yet

JOKE GARAGE SALE

[JOKE GARAGE SALE] Pride is what you feel when your kids net 143 dollars from a garage sale. Panic is what you feel when you realize your car is missing.

JOKE DOORBELL

[JOKE DOORBELL] A priest was walking down the street when he saw a little boy jumping up and down to try to reach a doorbell. So the priest walked over and pressed the button for the youngster. "And now what, my little man?" he asked. "Now." said the boy, "run like hell!"

JOKE FAMOUS LAST WORDS

[JOKE FAMOUS LAST WORDS]

postman:	"good doggy, nice doggy"
butcher:	"could you throw me the big knife, please?"
computer:	"are you sure? (yes/no)"
stuntman:	"what? reality TV?"
doorman:	"only over my dead body"
detective"	"clear case: you are the murderer"
muchroom picker:	"I never saw this one"
boss:	"nice present, a lighter which looks like a revolver"
submarine crew:	"I need some fresh air, open the window"
sysadmin:	"I recently had a fresh backup"
student:	"I'm going to eat in the mensa, anybody coming?"
bungee jumper:	"hurrey!"
PC:	"loading windows - please wait"

JOKE PAINT JOB

[JOKE PAINT JOB] There was a college student trying to earn some pocket money by going from house to house offering to do odd jobs. He explained this to a man who answered one door. "How much will you charge to paint my porch?" asked the man. "Forty dollars." "Fine" said the man, and gave the student the paint and brushes. Three hours later the paint-splattered lad knocked on the door again. "All done!", he says, and collects his money. "By the way," the student says, "That's not a Porsche, it's a Ferrari."

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 4 entries in this file.

Index

ENTRY K12

[ENTRY K12] Authors: Oliver Knill: 2000 Literature: not yet

abacus

An [abacus] is an acient mechanical computing device. It is made of beads arranged on a frame.

absolute value

The [absolute value] $|n|$ of a real number n is the maximum of n and its negative $-n$. For example, the absolute value of -6 is $|-6| = 6$. The absolute value is the distance from 0.

adjacent angles

Two angles that share a ray are called [adjacent angles].

affine cipher

An [affine cipher] uses affine functions to scramble the letters in an alphabet of a secret message. For example, with an alphabet of 26 letters, $f(x) = bx + a = 5x + 2 \text{mod}(26)$ produces a new alphabet of the same size if b has no common multiple with 26. The simplest example is the Caesar cipher, where $b = 2$. It rotates the letters in an alphabet $x \mapsto x + a \text{mod}(26)$. For example, for $a = 1$, we get

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a

This cipher changes the word *hello* to the word $ifmmp$. A requently used Caesar cipher is "rot13" defined by $f(x) = x + 13 \text{mod}(13)$. It has the property that encryption and decryption are the same. For example, applying rot13 on the word "decryption" produces *qrpelcgvba* and applying rot13 on that word again gives back "decryption. More complicated versions of affine cyphers can be obtained by writing the to encoded text as a sequence of vectors x and then applying $Ax + b$ on each vector. Affine cyphers are very easy to crack. They are only used to illustrate the concept like for educational purposes.

algebra

[algebra] is a branch of elementary mathematics that generalizes arithmetic by using variables. An example of an algebraic identity is $x * (y + z) = x * y + x * z$.

acute

An angle is called [acute], if is smaller than 90 degrees. An angle which is 90 degrees is called a right angle.

addition

[addition] is a basic operation for numbers. The result is called the sum of the two numbers. Examples: $5+3=8$.

$$\begin{array}{ccccc} & 2 & 3 & 4 & 5 \\ + & 9 & 2 & 3 & 5 \\ \hline 1 & 1 & 5 & 8 & 0 \end{array}$$

More generally, a group operation in a commutative group is often called addition. Examples of groups are integers, real numbers, vectors or matrices.

alternate exterior angles

[alternate exterior angles] are angles located outside a set of two parallel lines and on opposite sides of the transversal line. They are equal.

alternate interior angles

[alternate interior angles] are angles located inside a set of parallel lines and on opposite sides of the transversal.

angle bisector

A ray that divides an angle into two equal angles is called an [angle bisector]. The bisector can be constructed with ruler and compass. An angle trisector on the other hand, a ray which splits an angle into three equal parts can not be constructed by ruler and compass.

apex

The [apex] is the highest vertex in a given orientation of a polygon.

Arabic numerals

[Arabic numerals]: symbols $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ that represent successive entries of words representing numbers in the decimal system. For example, 2347 is the number $2000 + 300 + 40 + 7$.

area

The [area] of a surface is a measure for the number of square units needed to cover the surface. For example, the sphere of radius 1 has the surface area 4π.

arithmetic mean

[arithmetic mean] Given two numbers a, b, the arithmetic mean is defined as $(a+b)/2$. It is sometimes also called the mean. Other means are the geometric mean $\sqrt{ab}$ or the harmonic mean $1/(1/a+1/b)$.

average

The [average] of a few numbers is the sum of all the numbers divided by the number of numbers. For example, the average of $2, 4, 6$ is $(2+4+6)/3 = 4$, the average of the numbers $1, 5, 8, 4$ is $(1+5+8+5)/3 = 19/3$ The average is also called the mean. The average of two numbers is also called the arithmetic mean.

base

A [base] is the number of distinct single-digit numbers in a counting system. Example: the binary system has base 2. The decimal system has base 10. The base is also called radix. Numbers can be represented in any base $r > 1$. Because humans have 10 fingers, the decimal system is the one favoured by this species. Because computers work with circuits which are based on the principle "on" or "off", they like the base 2. The hexadecimal system (base 16) or octal system (base 8) are also used a lot by computers. Modern computers can even work directly with numbers written in base 32 or 64.

bell curve

The [bell curve] is an other term for graph of the normal distribution $f(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$. It is also called the Gauss distribution. The bell curve is often seen in probability distributions. There is a reason for that called the central limit theorem which assures that if we average independent data with some distribution, we approach the normal distribution.

billion

A [billion] is one thousand millions in the American or French system, it is a million millions in the English or German system. In other words

One billion in UK,Germany:	10^{12}	1'000'000'000'000
One billion in US,France:	10^9	1'000'000'000

binary number

A [binary number] is a number expressed in place-value notation to the base 2. For example: 101101 represents the decimal number $1+0+4+8+0+32 = 45$.

cipher

[cipher] Ciphers are codes used to encrypt "secret" messages.

coefficient

The word [coefficient] is used to denote numbers in the front of the variables in an algebraic formula. For example: $4x + 5y = 3$ has coefficients $4, 5$.

combinatorics

[combinatorics] The science of counting things. Combinatorics is an important part of probability and statistics.

common factor

A [common factor] of two integers n and m is a number which is a factor of both. A common factor is also called a common divisor. Examples: 3 is a common factor of 18 and 27. Also, 9 is the greatest common factor of 18 and 27: we write $9 = \gcd(18, 27)$, where *gcd* stands for the greatest common divisor.

complex numbers

[complex numbers] can be written as a pair of real numbers $z = x+iy$, where i is a symbol which satisfies $i^2 = -1$. One can add and subtract complex numbers by adding their coefficients x, y. For example $4+5i+5-7i = 9-2i$.

complementary angles

Two angles whose sum is 90 degrees form [complementary angles]. For example, the two non-right angles in a right triangle form complementary angles.

concave up

A graph of a function f is [concave up] if f has the property $f((x + y)/2) \leq (f(x) + f(y))/2$. If f is concave up, then $-f$ is concave down. For example, the graph of the function $f(x) = x^2$ is concave up, the graph of the function $f(x) = -x^4$ is concave down.

conditional probability

The [conditional probability] is the probability that an event A happens provided a second event B occurs. One writes $P[A|B]$. It satisfies $P[A|B] = P[A \cap B]/P[B]$, where $P[B]$ is the probability of the event B and $P[A \cap B]$ probability of the intersection of A and B. For example, if we throw 2 coins and we know one of the coins is head H, then the probability that the there is also a coin with tail is 2/3. Proof: The probability space is $X = \{HT, TH, TT, HH\}$. The event that one of the coins is head is $A = \{HT, TH, HH\}$. The event B that one of the coins shows tail is $B = \{HT, TH, TT\}$. The intersection of B and A is $\{TH, HT\}$. We have $P[B|A] = P[B \cap A]/P[A] = (1/2)/(3/4) = 2/3$.

congruent

Two figures are called [congruent] if one can move one to an other by translation and rotation.

constant

A quantity that does not change in an equation is called a [constant].

constant function

A [constant function] is a function which takes the same value whatever input we enter to it.

coordinate

A [coordinate] is an entry in a collection of numbers identifiing the point in coordinate space.

continuous graph

The graph of a continuous function is called a [continuous graph]. Roughly speaking, a graph of a function defined on some interval $[a, b]$ is a continuous graph if one can draw the graph using a pencil without having to the lift the pencil. Examples:

- $1/x$ is not continuous on $[-1, 1]$.
- $x^2 + 1$ is continuous on $[-1, 1]$.
- $1/x^2$ is not continuous on $[-1, 1]$.
- $\sin(1/x)$ is not continuous on $[-1, 1]$.
- $x \sin(1/x)$ is continuous on $[-1, 1]$.
- $f'(x)$ is not continuous on $[-1, 1]$ if $f(x) = |x|$.

corresponding angles

[corresponding angles] are two angles in the same relative position on two straight lines when those lines are intersected by a a transversal straight line.

decimal number

[decimal number] is a fraction, where the denominator is a power of 10. It can be expressed using a decimal point. For example: 0.872 is the decimal equivalent of 872/1000.

degrees

An angle is often measured in [degrees]. The entire circle has 360 degrees, a half a circle is 180 degrees, a quarter circle is a right angle and has 90 degrees. A more natural unit is the length unit where the entire circle has angle 2π and the right angle is the angle $\pi/2$.

denominator

The [denominator] is the integer q below the fraction in a rational number p/q. The other number p is called the nominator.

discontinuous graph

A [discontinuous graph] is the graph of a function which is not continuous. Discontinuities can occur in different ways. The function can jump from one value to an other. The function can also be infinite at some point or the function can oscillate infinitely much at some point. Examples:

- The graph of the function $f(x) = 1/x$ on $[-1, 1]$.
- The graph of the function $f(x) = \sin(1/x)$ on $[-1, 1]$.
- The graph of the function $f(x) = sign(x)$, which is 1 for $x > 0$ equal to 0 for $x = 0$ and -1 for $x = -1$.

disjoint events

Two events are called [disjoint events] if they have no common elements.

division

The inverse operation of multiplication is called [division].

domain

The [domain] of a function f is the set of numbers x for which $f(x)$ is defined. For example, the domain of the function $f(x) = 1/x$ is the entire real line except the point 0.

element

An [element] of a set is is a member of that set. For example *table* is an element of the set $\{table, chair, floor\}$.

empty set

The [empty set] $\emptyset$ is the set which does not contain any elements.

equally likely

If two events have the same probability they are called [equally likely]. For example, the event of throwing an even number with one dice is equally likely than throwing an odd number.

event

An [event] is a subset of the entire probability space. For example, if $X = \{HH, HT, TH, TT\}$ is the probability space of all throwing of two coins, then $A = \{HH, HT\}$ is the event that in the second throw one had a head.

exponent

The [exponent] of an expression a^x is part x. One can get the exponent of $y = a^x$ by the formula $x = \log(y)/\log(a)$.

Fibonacci numbers

[Fibonacci numbers] are numbers obtained in the Fibonacci sequence defined by starting the numbers $0, 1$ and defining the next element as the sum of the two previous ones: $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...$ The sequence is named after Leonardo of Pisa, who called himself Fibonacci, short for Filius Bonacci (= Son of Bonacci). The original problem he investigated in 1202 A.D. was the growth of rabbits. Explicit expressions for the n'th term of the sequence can be obtained using linear algebra. More generally, one can find explicit formulas for the n'th term in a linear recursion of the form $a_{n+1} = \sum_{j=0}^{k} c_j a_{n-j}$.

fractal

A [fractal] is a set which has non-integer dimension. The term was coined by Benoit Mandelbrot in 1975. Many objects in nature appear to be fractals, like coast lines, trees, mountains. One can mathematically define fractals using iterative constructions. Examples are the Koch curve, the snow flake, the Menger Sponge, the Shirpinsky carpet, the Cantor set.

fraction

A [fraction] is a rational number written in the form a/b, where a is called the numerator and b is called the denominator.

function

A [function] f of a variable x is a rule that assigns to each number x in the function's domain a single number $f(x)$. For example $f(x) = x^2$ is a function which assigns to each number its square like $f(4) = 16$.

geometric sequence

The [geometric sequence] is a sequence where each element is a multiple of the previous element. For example: $1, 2, 4, 8, 16, 32, 64, ...$ is a geometric sequence.

graph of the function

The [graph of the function] is the set of all points $(x, f(x))$ in the plane, where x in the domain of f.

greatest common factor

The [greatest common factor] of two numbers n,m is the larest common factor of both. One denotes the greatest common factor with "gcd". Examples:

6 is the greatest common factor of 12 and 18	6=gcd(12,18)
8 is the greatest common factor of 8 and 80.	8=gcd(8,80)
1 is the greatest common factor of 7 and 11	1=gcd(7,11)

greatest common divisor

[greatest common divisor] see greatest common factor.

histogram

A [histogram] is a bar graph in which area over each range of values is proportional to the relative frequency of the data in this interval.

hypotenuse

The [hypotenuse] of a right triangle is the opposite side to the right angle.

independent events

Two events A and B are called [independent events] if the probability that both happen is the product of the probabilities that each occurs alone: $P[A \cap B] = P(A)P(B)$. Using conditional probability one can write this as $P[A|B] = P[A]$. Knowing A under the condition B is the same as knowing A without knowing B.

infinity

[infinity] is a "number" which is larger than any other number. One writes ∞. One should rather treat of it as a symbol evenso some computations can be extended to the real numbers including ∞ like $\infty + x = \infty, \infty + \infty = \infty, x * \infty = \infty$ for $x > 0$, $x * \infty = -\infty$ for $x < 0$ or $\infty * \infty = \infty$, $(-\infty) * \infty = -\infty$. One can not define $\infty - \infty$ in a consistent way nor can one do that with $0 * \infty$. Also the expression $1/0 = \infty$ is ill defined because $1/x$ takes near $x = 0$ arbitrary large and arbitrary small values.

integer

An [integer] is a number of the form n or $-n$, where n is a natural number. Examples of integers are $... - 3, 2, 1, 0, 1, 2, 3, 4....$ The fraction $2/5$ is not an integer.

intersection

The [intersection] of two or more sets is the set of elements which are in both sets. One writes $A \cap B$ for the intersection of A and B.

isosceles triangle

An [isosceles triangle] is a triangle which has at least two congruent sides. A special case is the isocline triangle in which all sides are congruent.

least common multiple

The [least common multiple] of two numbers n, m is the least common multiple of both. One denotes the least common multiple with "lcm". Examples:

18 is the least common multiple of 9 and 6	18=lcm(9,6)
77 is the least common multiple of 7 and 11	77=gcd(7,11)

limit

The [limit] of a sequence of numbers is the limiting value the sequence converges to. It needs not to exist. For example, the sequence $a_n = 1/n$ converges to 0. One sais that 0 is the limit of that sequence. The sequence $a_n = n$ has no finite limit. One could assign infinity as a limit. The sequence $1, -1, 1, -1, 1, -1, ...$ has no limit.

logarithm

The [logarithm] of b is the exponent to which one has to rize a base number to get b. For example, 2 is the logarithm of 100 to the base 10 or 10 is the logarithm of 1024 to the base 2.

mean

The [mean] of a list of numbers is their sum divided by the total number of members in the list. It is also called arithmetic mean.

median

The [median] is the "middle value" of a list. If the list has an odd number $2m+1$ elements, the median is the number in the list such that m scores are smaller or equal and m scores are bigger or equal. If the list has an even number of elements, one usually takes the algebraic average between the middle to elements Examples: $\text{med}(1,1,2,2) = 3/2$, $\text{med}(1,2,3,4,7) = 3$, $\text{med}(1,2,3,4,5,6) = 7/2$.

multiplication table

[multiplication table] A table of products of numbers which has to be memorized.

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

The diagonal contains squares. All numbers between 11 and 99 which do not appear in this table are prime numbers, numbers only divisible by 1 and itself.

obtuse angle

An angle whose measure is greater than 90 degrees is called an [obtuse angle].

optical illusion

An [optical illusion] is a drawing of an object that makes certain things appear which it does not have.

palindrome

A [palindrome] is a word or number that is the same when read backwards. Examples: "otto", "anna", "racecar", "78777787".

paradox

A [paradox] is a statement that appears to contradict itself. For example, the statement "I always lie" is a paradox. If I tell the truth, then I lie, if I lie, then I tell the truth.

parallel

Two lines which do not intersect are called [parallel].

parallelogram

A [parallelogram] is a quadrilateral that contains two pairs of parallel sides.

pattern

A [pattern] is a characteristic observed in one item that may be repeated in other items. For example, the sequence $3, 4, 5, 4, 3, 4, 5, 4, 3, 4, 5, 4, 3, ...$ has a pattern which is also visible in a similar way in the sequence $1, 3, 4, 3, 1, 3, 4, 3, 1, 3, 4, 3, 1, ...$

percent

A [percent] is one hunderth. The symbol for percent is %. For example 0.1 is 10 percent. 2 is two hundred percent.

perimeter

The [perimeter] of a polygon is the sum of the lengths of all the sides of the polygon.

permutation

A [permutation] is a rearrangement of objects in a set. There are for example 6 permutations of the set $A = (a, b, c)$. They are $(a, b, c), (a, c, b), (b, a, c), (b, c, a), (c, a, b), (c, b, a)$.

polygon

A [polygon] is a closed plane figure formed by connecting a finite set of points in such a way that they do not cross each other.

polyhedra

[polyhedra] A solid figure for which the outer surface is composed of polygons.

prime number

A [prime number] is a number which is divisible only by 1 and itself. The first prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37....$ The number 33 for example is no prime number because it is divisible by 3.

quadrant

A [quadrant] is one of the regions in the plane obtained when cutting the plane along the coordinate axes.

- The first quadrant contains all the points with positive x and positive y coordinates.
- The second quadrant contains all the points with negative x and positive y coordinates.
- The third quadrant contains all the points with negative x and negative y coordinates.
- The fourth quadrant contains all the points with positive x and negative y coordinates.

quadratic function

A function of the form $f(x) = ax^2 + bx + c$ is called a [quadratic function]. For example $f(x) = x^2 + 2$ is a quadratic function. The graph of a quadratic function is a parabola if a is not zero. If a is zero it is a linear function which has as the graph a line.

quotient

The [quotient] of two numbers n and m is the largest integer smaller or equal to n/m. for example the quotient of 11 and 4 is 2 with areminder of 3.

smallest common multiple

[smallest common multiple] see least common multiple.

subtraction

[subtraction] is a basic operation for numbers. Examples: $5 - 3 = 2$.

	2	3	5	2
-	1	4	5	6
		8	9	6

More generally in any commutative group, the addition of the inverse -b of an element b to an element a, denoted by a-b is called a subtraction.

multiplication

[multiplication] is a basic operation for numbers. Example: $4 * 5 = 20$.

1	2	4	x	1	2	3	3	4
+				4	9	3	3	6
+			2	4	6	6	8	
+		1	2	3	3	4		
		1	5	2	9	4	1	6

More generally, in any group the group operation is called multiplication. Example: The composition of two transformations in the plane is a multiplication, matrix multiplication is a multiplication.

fraction

[fraction]: The representation of a rational number r as p/q, where p and $q \neq 0$ are integers is called a fraction. Fractions can be added, subtracted, multiplied and one can divide by nonzero fractions.

Addition of fractions:	$5/3 + 7/5 = 25/15 + 21/15 = 46/15$
Subtraction of fractions:	$5/3 - 7/5 = 25/15 - 21/15 = 4/15$
Multiplication of fractions:	$5/3x7/5 = 35/15$
Division of fractions:	$5/3 : 7/5 = 5/3x5/7 = 25/21$

Not all numbers are rational numbers. Non rational numbers are called irrational. For example, sqr(2) can not be written as p/q because $\sqrt{(2)} = p/q$ would imply $2 = p^2/q^2$ or $2q^2 = p^2$. But by the uniqueness of prime factorisation of integers the left hand side has an odd number of factors 2, the right hand side an even number. $\sqrt{(2)}$ is an example of an irrational number.

hexagon

A [hexagon] is a polygon with six sides.

parallelogram

A [parallelogram] is a quadrilateral whose opposite sides are parallel and congruent.

polygon

A [polygon] is a closed curve in the plane formed by three or more line segments. One usually assumes that the segments don't intersect. Examples:

3 sides:	triangle
4 sides:	quadrilateral, (i.e. rectangle, rhombus, rhombus)
5 sides:	pentagon
6 sides:	hexagon
7 sides:	septagon
8 sides:	octogon

quadrilateral

A [quadrilateral] is a polygon with four sides.

parallel

Two lines in the plane are called [parallel] if they do not intersect. Two parallel lines can be translated into each other. Two lines in space are called [parallel] if they can be translated into each other. Unlike in the plane, two lines in space which are not parallel do not need to intersect.

triangle

A [triangle] is a polygon defined by three points in the plane. The three points form the edges of the triangles, the three connections of the points form the sides of the triangle.

random number generator

A [random number generator] is a device used to produce random numbers. In daily life like for gambling, one often uses dice or coin tossing to find random numbers. Computers often use pseudo random number generators, which are deterministic sequences which look random. Computers can also access hardware internal staes of the computer to improve randomness.

range of a function

The [range of a function] is the set of all values $f(x)$, where x is in the domain of f.

ratio

[ratio] A rational number of the form a/b where a is called the numerator and b is called the denominator.

rectangle

A [rectangle] is a parallelogram with four right angles. It is a quadrilateral, a polygon with four points in the plane. All angles have to be right angles. In a rectangle, opposite sides are parallel. A rectangle is therefore a special parallelogram.

regular polygon

A [regular polygon] is a polygon which has sides of equal length and equal angles. Squares, equilateral triangles or regular hexagons are examples of regular polygons.

remainder

The [remainder] of a division p/q is the amount left after subtracting the maximal integer multiple of q from p. For example $7/3$ has the reminder 1 because $7 - 2 * 3 = 1$. $-11/5$ has the reminder -1.

rhombus

A [rhombus] is a parallelogram with four congruent sides. A special case is the square.

right angle

An angle of 90 degrees is called a [right angle].

right triangle

A triangle which has a right angle is also called a [right triangle].

sequence

An ordered list of elements is called a [sequence] For example, $(1, 3, 2, 1)$ is a list of elements which form a finite sequence. The list $(1, 2, 3, 4, 5, 6, ...)$ forms an infinite sequence.

set

A [set] is a collection of things, without regard to their order.

slope

The [slope] of a line $y = mx + b$ is the number m. One often measures it in percentages. $m = 1$ means 100 percent. If a street has a slope of 10 percent, one climbes for every 10 meters going forwards 1 meter up.

square

A [square] is a polygonal shape in the plane with four sides where each side has the same length and all sides are perpendicular on each other. A square is also a number of the form n*n like 65=8*8. A square with integer side length has a square number as the area.

subset

A [subset] of a set is a set of elements which are all contained in that set. For example the set $A = \{1, 2, 3, 8, 4\}$ has $B = \{2, 3\}$ as a subset.

subtraction

[subtraction] is the operation of taking the difference between two numbers. For example $7 - 2 = 5$.

tessellation

A [tessellation] is a cover of the plane using a finite set of polygons without leaving gaps or overlaps. Examples are regular tessellation into triangles or squares or regular hexagons. Semiregular tesselations allow to cover the plane with different types of shapes. Tesselations are also called tilings and can be defined also in higher dimensions.

trapezoid

A [trapezoid] is a quadrilateral with one pair of parallel sides.

union of sets

The [union of sets] is the set which contains the elements of all sets. One writes $A \cup B$. For example, if $A = \{1, 2, 3, 4\}$ and $B = \{0, 2, 4, 6\}$, then $A \cup B = \{0, 1, 2, 3, 4, 6\}$.

Venn Diagram

In a [Venn Diagram] sets are represented as simple geometric shapes. It visualizes intersections and unions of sets. For example if A is the set of all even numbers between 0 and 10 and B is the set of all numbers divisible by 3 between 0 and 10 one can visualize this with two circles, one of which contains $2, 5, 8, 6$, the other $3, 6, 9$. The circles intersect in a region which has the single element 6.

whole number

A [whole number] is one of the numbers $0, 1, 2, 3, 4, ...$ A whole number is also called natural number or nonnegative integer.

Index

you sound tired

[you sound tired] I worked all night

This is great

[This is great] this is fantastic

ok

[ok] not ok

switzerland

[switzerland] country center europe

Germany

[Germany] country in Europe

Austria

[Austria] country in Europe

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 5 entries in this file.

Index

Famous Literature

[Famous Literature] List from http://www.literaturepage.com

A Christmas Carol

[A Christmas Carol] Charles Dickens

A Connecticut Yankee in King Arthur's Court

[A Connecticut Yankee in King Arthur's Court] Mark Twain

A Dream Within a Dream

[A Dream Within a Dream] Edgar Allan Poe

A Midsummer Night's Dream

[A Midsummer Night's Dream] William Shakespeare

A Princess of Mars

[A Princess of Mars] Edgar Rice Burroughs

A Room With a View

[A Room With a View] E. M. Forster

A Study in Scarlet

[A Study in Scarlet] Sir Arthur Conan Doyle

A Tale of Two Cities

[A Tale of Two Cities] Charles Dickens

A Thief in the Night

[A Thief in the Night] E. W. Hornung

A Treatise on Government

[A Treatise on Government] Aristotle

A Woman of No Importance

[A Woman of No Importance] Oscar Wilde

A Woman of Thirty

[A Woman of Thirty] Honore de Balzac

Adventures of Sherlock Holmes

[Adventures of Sherlock Holmes] Sir Arthur Conan Doyle

Aesop's Fables

[Aesop's Fables] Aesop

Agnes Grey

[Agnes Grey] Anne Bronte

Alice's Adventures in Wonderland

[Alice's Adventures in Wonderland] Lewis Carroll

All's Well That Ends Well

[All's Well That Ends Well] William Shakespeare

Allan Quatermain

[Allan Quatermain] H. Rider Haggard

Allan's Wife

[Allan's Wife] H. Rider Haggard

An Ideal Husband

[An Ideal Husband] Oscar Wilde

An Occurrence at Owl Creek Bridge

[An Occurrence at Owl Creek Bridge] Ambrose Bierce

Andersen's Fairy Tales

[Andersen's Fairy Tales] Hans Christian Andersen

Anna Karenina

[Anna Karenina] Leo Tolstoy

Annabel Lee

[Annabel Lee] Edgar Allan Poe

Anne of Green Gables

[Anne of Green Gables] L. M. Montgomery

Antony and Cleopatra

[Antony and Cleopatra] William Shakespeare

Apology

[Apology] Plato

Around the World in Eighty Days

[Around the World in Eighty Days] Jules Verne

As You Like It

[As You Like It] William Shakespeare

At the Earth's Core

[At the Earth's Core] Edgar Rice Burroughs

Barnaby Rudge

[Barnaby Rudge] Charles Dickens

Berenice

[Berenice] Edgar Allan Poe

Black Beauty

[Black Beauty] Anna Sewell

Bleak House

[Bleak House] Charles Dickens

Buttered Side Down

[Buttered Side Down] Edna Ferber

Cousin Betty

[Cousin Betty] Honore de Balzac

Crime and Punishment

[Crime and Punishment] Fyodor Dostoevsky

Daisy Miller

[Daisy Miller] Henry James

David Copperfield

[David Copperfield] Charles Dickens

Dead Men Tell No Tales

[Dead Men Tell No Tales] E. W. Hornung

Discourse on the Method

[Discourse on the Method] Rene Descartes

Dorothy and the Wizard in Oz

[Dorothy and the Wizard in Oz] L. Frank Baum

Dracula

[Dracula] Bram Stoker

Eight Cousins

[Eight Cousins] Louisa May Alcott

Emma

[Emma] Jane Austen

Essays and Lectures

[Essays and Lectures] Oscar Wilde

Essays of Francis Bacon

[Essays of Francis Bacon] Sir Francis Bacon

Essays, First Series

[Essays, First Series] Ralph Waldo Emerson

Essays, Second Series

[Essays, Second Series] Ralph Waldo Emerson

Ethan Frome

[Ethan Frome] Edith Wharton

Cousin Betty

[Cousin Betty] Honore de Balzac

Lady Windermere's Fan

[Lady Windermere's Fan] Oscar Wilde

Lincoln's First Inaugural Address

[Lincoln's First Inaugural Address] Abraham Lincoln

Lincoln’s Second Inaugural Address

[Lincoln’s Second Inaugural Address] Abraham Lincoln

Little Men

[Little Men] Louisa May Alcott

Little Women

[Little Women] Louisa May Alcott

Macbeth

[Macbeth] William Shakespeare

Main Street

[Main Street] Sinclair Lewis

Mansfield Park

[Mansfield Park] Jane Austen

Memoirs of Sherlock Holmes

[Memoirs of Sherlock Holmes] Sir Arthur Conan Doyle

Middlemarch

[Middlemarch] George Eliot

Moby Dick

[Moby Dick] Herman Melville

Moll Flanders

[Moll Flanders] Daniel Defoe

Much Ado About Nothing

[Much Ado About Nothing] William Shakespeare

Night and Day

[Night and Day] Virginia Woolf

Northanger Abbey

[Northanger Abbey] Jane Austen

Nostromo

[Nostromo] Joseph Conrad

O Pioneers!

[O Pioneers!] Willa Cather

Of Human Bondage

[Of Human Bondage] W. Somerset Maugham

Oliver Twist

[Oliver Twist] Charles Dickens

On the Decay of the Art of Lying

[On the Decay of the Art of Lying] Mark Twain

On the Duty of Civil Disobedience

[On the Duty of Civil Disobedience] Henry David Thoreau

Othello, Moor of Venice

[Othello, Moor of Venice] William Shakespeare

Ozma of Oz

[Ozma of Oz] L. Frank Baum

Pandora

[Pandora] Henry James

Paradise Lost

[Paradise Lost] John Milton

Persuasion

[Persuasion] Jane Austen

Poems of Edgar Allan Poe

[Poems of Edgar Allan Poe] Edgar Allan Poe

Pollyanna

[Pollyanna] Eleanor H. Porter

Pride and Prejudice

[Pride and Prejudice] Jane Austen

Pygmalion

[Pygmalion] George Bernard Shaw

Raffles: Further Adventures of the Amateur Cracksman

[Raffles: Further Adventures of the Amateur Cracksman] E. W. Hornung

Rebecca Of Sunnybrook Farm

[Rebecca Of Sunnybrook Farm] Kate Douglas Wiggin

Robinson Crusoe

[Robinson Crusoe] Daniel Defoe

Romeo and Juliet

[Romeo and Juliet] William Shakespeare

Rose in Bloom

[Rose in Bloom] Louisa May Alcott

Sense and Sensibility

[Sense and Sensibility] Jane Austen

Silas Marner

[Silas Marner] George Eliot

Tarzan of the Apes

[Tarzan of the Apes] Edgar Rice Burroughs

Tess of the d'Urbervilles

[Tess of the d'Urbervilles] Thomas Hardy

The Adventures of Huckleberry Finn

[The Adventures of Huckleberry Finn] Mark Twain

The Adventures of Pinocchio

[The Adventures of Pinocchio] Carlo Collodi

The Adventures of Tom Sawyer

[The Adventures of Tom Sawyer] Mark Twain

The Aeneid

[The Aeneid] Virgil

The Age of Innocence

[The Age of Innocence] Edith Wharton

The Amateur Cracksman

[The Amateur Cracksman] E. W. Hornung

The Analysis of Mind

[The Analysis of Mind] Bertrand Russell

The Ballad of Reading Gaol

[The Ballad of Reading Gaol] Oscar Wilde

The Bells

[The Bells] Edgar Allan Poe

The Call of the Wild

[The Call of the Wild] Jack London

The Cask of Amontillado

[The Cask of Amontillado] Edgar Allan Poe

The Categories

[The Categories] Aristotle

The Chessmen of Mars

[The Chessmen of Mars] Edgar Rice Burroughs

The Comedy of Errors

[The Comedy of Errors] William Shakespeare

The Count of Monte Cristo

[The Count of Monte Cristo] Alexandre Dumas

The Emerald City of Oz

[The Emerald City of Oz] L. Frank Baum

The Fall of the House of Usher

[The Fall of the House of Usher] Edgar Allan Poe

The Four Million

[The Four Million] O. Henry

The Gambler

[The Gambler] Fyodor Dostoevsky

The Gettysburg Address

[The Gettysburg Address] Abraham Lincoln

The Gods of Mars

[The Gods of Mars] Edgar Rice Burroughs

The Happy Prince and Other Tales

[The Happy Prince and Other Tales] Oscar Wilde

The History of Tom Jones, a foundling

[The History of Tom Jones, a foundling] Henry Fielding

The History of Troilus and Cressida

[The History of Troilus and Cressida] William Shakespeare

The Hound of the Baskervilles

[The Hound of the Baskervilles] Sir Arthur Conan Doyle

The Hunchback of Notre Dame

[The Hunchback of Notre Dame] Victor Hugo

The Hunting of the Snark

[The Hunting of the Snark] Lewis Carroll

The Idiot

[The Idiot] Fyodor Dostoevsky

The Iliad

[The Iliad] Homer

The Importance of Being Earnest

[The Importance of Being Earnest] Oscar Wilde

The Innocence of Father Brown

[The Innocence of Father Brown] G. K. Chesterton

The Innocents Abroad

[The Innocents Abroad] Mark Twain

The Invisible Man

[The Invisible Man] H. G. Wells

The Island of Doctor Moreau

[The Island of Doctor Moreau] H. G. Wells

The Jungle Book

[The Jungle Book] Rudyard Kipling

The Last Days of Pompeii

[The Last Days of Pompeii] Edward Bulwer-Lytton

The Last of the Mohicans

[The Last of the Mohicans] James Fenimore Cooper

The Legend of Sleepy Hollow

[The Legend of Sleepy Hollow] Washington Irving

The Life and Adventures of Nicholas Nickleby

[The Life and Adventures of Nicholas Nickleby] Charles Dickens

The Life and Death of King Richard III

[The Life and Death of King Richard III] William Shakespeare

The Life of King Henry V

[The Life of King Henry V] William Shakespeare

The Lost Continent

[The Lost Continent] Edgar Rice Burroughs

The Lost World

[The Lost World] Sir Arthur Conan Doyle

The Man in the Iron Mask

[The Man in the Iron Mask] Alexandre Dumas

The Man Upstairs and Other Stories

[The Man Upstairs and Other Stories] P. G. Wodehouse

The Man Who Knew Too Much

[The Man Who Knew Too Much] G. K. Chesterton

The Man with Two Left Feet

[The Man with Two Left Feet] P. G. Wodehouse

The Masque of the Red Death

[The Masque of the Red Death] Edgar Allan Poe

The Merchant of Venice

[The Merchant of Venice] William Shakespeare

The Merry Adventures of Robin Hood

[The Merry Adventures of Robin Hood] Howard Pyle

The Merry Wives of Windsor

[The Merry Wives of Windsor] William Shakespeare

The Moon and Sixpence

[The Moon and Sixpence] W. Somerset Maugham

The Moonstone

[The Moonstone] Wilkie Collins

The Murders in the Rue Morgue

[The Murders in the Rue Morgue] Edgar Allan Poe

The Mysterious Affair at Styles

[The Mysterious Affair at Styles] Agatha Christie

The Mystery of Edwin Drood

[The Mystery of Edwin Drood] Charles Dickens

The Mystery of the Yellow Room

[The Mystery of the Yellow Room] Gaston Leroux

The Odyssey

[The Odyssey] Homer

The Origin of Species means of Natural Selection

[The Origin of Species] means of Natural Selection] Charles Darwin

The Phantom of the Opera

[The Phantom of the Opera] Gaston Leroux

The Picture of Dorian Gray

[The Picture of Dorian Gray] Oscar Wilde

The Pit and the Pendulum

[The Pit and the Pendulum] Edgar Allan Poe

The Portrait of a Lady

[The Portrait of a Lady] Henry James

The Purloined Letter

[The Purloined Letter] Edgar Allan Poe

The Raven

[The Raven] Edgar Allan Poe

The Red Badge of Courage

[The Red Badge of Courage] Stephen Crane

The Republic

[The Republic] Plato

The Return of Sherlock Holmes

[The Return of Sherlock Holmes] Sir Arthur Conan Doyle

The Rime of the Ancient Mariner

[The Rime of the Ancient Mariner] Samuel Taylor Coleridge

The Scarecrow of Oz

[The Scarecrow of Oz] L. Frank Baum

The Scarlet Letter

[The Scarlet Letter] Nathaniel Hawthorne

The Sonnets

[The Sonnets] William Shakespeare

The Strange Case of Dr. Jekyll and Mr. Hyde

[The Strange Case of Dr. Jekyll and Mr. Hyde] Robert Louis Stevenson

The Taming of the Shrew

[The Taming of the Shrew] William Shakespeare

The Tell-Tale Heart

[The Tell-Tale Heart] Edgar Allan Poe

The Tempest

[The Tempest] William Shakespeare

The Tenant of Wildfell Hall

[The Tenant of Wildfell Hall] Anne Bronte

The Three Musketeers

[The Three Musketeers] Alexandre Dumas

The Time Machine

[The Time Machine] H. G. Wells

The Tin Woodman of Oz

[The Tin Woodman of Oz] L. Frank Baum

The Tragedy of Coriolanus

[The Tragedy of Coriolanus] William Shakespeare

The Tragedy of King Lear

[The Tragedy of King Lear] William Shakespeare

The Tragedy of King Richard the Second

[The Tragedy of King Richard the Second] William Shakespeare

The Voyage Out

[The Voyage Out] Virginia Woolf

The War in the Air

[The War in the Air] H. G. Wells

The War of the Worlds

[The War of the Worlds] H. G. Wells

The Wind in the Willows

[The Wind in the Willows] Kenneth Grahame

The Wisdom of Father Brown

[The Wisdom of Father Brown] G. K. Chesterton

The Wonderful Wizard of Oz

[The Wonderful Wizard of Oz] L. Frank Baum

Three Men in a Boat

[Three Men in a Boat] Jerome K. Jerome

Through the Looking Glass

[Through the Looking Glass] Lewis Carroll

Thus Spake Zarathustra

[Thus Spake Zarathustra] Friedrich Nietzsche

Thuvia, Maid of Mars

[Thuvia, Maid of Mars] Edgar Rice Burroughs

To Helen

[To Helen] Edgar Allan Poe

Treasure Island

[Treasure Island] Robert Louis Stevenson

Twelfth Night

[Twelfth Night] William Shakespeare

Twenty Thousand Leagues Under the Seas

[Twenty Thousand Leagues Under the Seas] Jules Verne

Twenty Years After

[Twenty Years After] Alexandre Dumas

Typee

[Typee] Herman Melville

Ulalume

[Ulalume] Edgar Allan Poe

Uneasy Money

[Uneasy Money] P. G. Wodehouse

Up From Slavery: An Autobiography

[Up From Slavery: An Autobiography] Booker T. Washington

Vanity Fair

[Vanity Fair] William Makepeace Thackeray

Walden

[Walden] Henry David Thoreau

War and Peace

[War and Peace] Leo Tolstoy

Warlord of Mars

[Warlord of Mars] Edgar Rice Burroughs

White Fang

[White Fang] Jack London

Wuthering Heights

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 198 entries in this file.

Index

ENTRY SINGLE VARIABLE CALCULUS I

[ENTRY SINGLE VARIABLE CALCULUS I] Authors: Oliver Knill: 2001 Literature: not yet

Abel's partial summation formula

[Abel's partial summation formula] is a discrete version of the partial integration formula: with $A_n = \sum_{k=1}^n a_k$ one has $\sum_{k=m}^n a_k b_k = \sum_{k=m}^n A_k(b_k - b_{k+1}) + A_n b_{n+1} - A_{m-1} b_m$.

Abel's test

[Abel's test]: if a_n is a bounded monotonic sequence and b_n is a convergent series, then the sum $\sum_n a_n b_n$ converges.

absolute value

The [absolute value] of a real number x is denoted by $|x|$ and defined as the maximum of x and $-x$. We can also write $|x| = +\sqrt{x^2}$. The absolute value of a complex number $z = x + iy$ is defined as $\sqrt{x^2 + y^2}$.

accumulation point

An [accumulation point] of a sequence a_n of real numbers is a point a which the limit of a subsequence a_{n_k} of a_n. A sequence a_n converges if and only if there is exactly one accumulation point. Example: The sequence $a_n = \sin(\pi n)$ has two accumulation points, $a = 1$ and $a = -1$. The sequence $a_n = \sin(\pi n)/n$ has only the accumulation point $a = 0$. It converges.

Achilles paradox

The [Achilles paradox] is one of Zenos paradoxon. It argues that motion can not exist: "set up a race between Achilles A and tortoise T. At the initial time $t_0 = 0$, A is at the spot $s = 0$ while T is at position $s_1 = 1$. Lets assume A runs twice as fast. The reace starts. When A reaches s_1 at time $t_1 = 1$, its opponent T has already advanced to a point $s_2 = 1 + 1/2$. Whenever A reaches a point s_k at time t_k, where T has been at time t_{k-1}, T has already advanced further to location s_{k+1}. Because an infinite number of timesteps is necessary for A to reach T, it is impossible that A overcomes T." The paradox exploits a misunderstanding of the concept of summation of infinite series. At the finite time $t = \sum_{n=1}^\infty (t_n - t_{n-1}) = 2$, both A and T will be at the same spot $s = \lim_{n\to\infty} s_n = 2$.

addition formulas

The [addition formulas] for trigonometric functions are

$$\begin{aligned} \cos(a+b) &= \sin(a)\cos(b) + \cos(a)\sin(b) \\ \sin(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \end{aligned}$$

alternating series

An [alternating series] is a series in which terms are alternatively positive and negative. An example is $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n/n = -1 + 1/2 - 1/3 + 1/4 - ....$ An alternating series with $a_n \to 0$ converges by the alternating series test.

alternating series test

Leibniz's [alternating series test] assures that an alternating series $\sum_n a_n$ with $|a_n| \to 0$ is a convergent series.

acute

An angle is [acute], if it is smaller than a right angle. For example $\alpha = \pi/3 = 60^\circ$ is an acute angle. The angle $\alpha = 2\pi/3 = 120^\circ$ is not an acute angle. The right angle $\alpha = \pi/2 = 90^\circ$ does not count as an acute angle. The angle $\alpha = -\pi/6 = -30^\circ$ is an acute angle.

antiderivative

The [antiderivative] of a function f is a function $F(x)$ such that the derivative of F is f that is if $d/dxF(x) = f(x)$. The antiderivative is not unique. For example, every function $F(x) = \cos(x) + C$ is the antiderivative of $f(x) = \sin(x)$. Every function $F(x) = x^{n+1}/(n+1) + C$ is the antiderivative of $f(x) = x^n$.

Arithmetic progression

[Arithmetic progression] A sequence of numbers a_n for which $b_n = a_{n+1} - a_n$ is constant, is called an arithmetic progression. For example, $3, 7, 11, 15, 19, ...$ is an arithmetic progression. The sequence $0, 1, 2, 4, 5, 6, 7$ is not an arithmetic progression.

arrow paradox

The [arrow paradox] is a classical Zeno paradox with conclusion that motion can not exist: "an object occupies at each time a space equal to itself, but something which occupies a space equal to itself can not move. Therefore, the arrow is always at rest."

asymptotic

Two real functions are called [asymptotic] at a point a if $\lim_{x\to a} f(x)/g(x) = 1$. For example, $f(x) = \sin(x)$ and $g(x) = x$ are asymptotic at $a = 0$. The point a can also be infinite: for example, $f(x) = x$ and $g(x) = \sqrt{x^2+1}$ are asymptotic at $a = \infty$.

Bernstein polynomials

The [Bernstein polynomials] of a continuous function f on the unit interval $0 \leq x \leq 1$ are defined as $B_n(x) = \sum_{k=1}^{n} f(k/n)x^k(1-x)^{n-k}n!/(k!(n-k!)$.

Binomial coefficients

[Binomial coefficients] The coefficients $B(n,k)$ of the polynomial $(x+1)^n$ for integer n are called Binomial coefficients. Explicitly one has $B(n,k) = n!/(k!(n-k)!)$, where $k! = k(k-1)!$, $0! = 1$ is the factorial of k. The function $B(n,k)$ can be defined for any real numbers n,k by writing $n! = \Gamma(n+1)$, where Γ is the Gamma function. If p is a positive real number and k is an integer, one has one has $B(p,k) = p(p-1)...(p-k+1)/k!$. For example, $B(1/2,0) = B(1/2,1) = 1/2, B(1/2,2) = -1/8$. Indeed, $(1+x)^{1/2} = 1 + x/2 - x^2/8 + ...$

Binominal theorem

The [Binominal theorem] tells that for a real number $|z| < 1$ and real number p, one has $(1+z)^p = \sum_{k=0}^{\infty} B(p,k)z^k$, where $B(p,k)$ is called the Binomial coefficient. If p is a positive integer, then $(1+z)^p$ is a polynomial. For example:

$$(1+z)^4 = 1 + 4z + 6z^2 + 4z^3 + z^4 \ .$$

If p is a noninteger or negative, then $(1+z)^p$ is an infinite sum. For example

$$(1+z)^{-1/2} = 1 - x/2 + 3x^2/8 - 5x^3/16 + ...$$

bisector

A [bisector] is a straight line that bisects a given angle or a given line segment. For example, the y-axis $x = 0$ in the plane bisects the line segment connecting $(-1,0)$ with $(1,0)$. The line $x = y$ bisects the angle $\angle(CAB)$ where $C = (0,1), A = (0,0), B = (1,0)$ at the point A.

Bolzano's theorem

[Bolzano's theorem] also called intermediate value theorem says that a continuous function on an interval (a,b) takes each value between $f(a)$ and $f(b)$. For example, the function $f(x) = \sin(x)$ takes any value between -1 and 1 because f is continuous and $f(-\pi/2) = -1$ and $f(\pi/2) = 1$.

Fermat principle

The [Fermat principle] tells that if f is a function which is differentiable at z and $f(x) > f(z)$ for all points in an interval $(z-a, z+a)$ with $a > 0$, then $f'(z) = 0$.

fundamental theorem of calculus

The [fundamental theorem of calculus]: if f is a differentiable function on $a \leq x \leq b$ where $a < b$ are real numbers, then $f(b) - f(a) = \int_a^b f'(x)\, dx$.

integration rules

[integration rules]:

- $\int af(x)\, dx = a \int f(x)\, dx$.
- $\int f(x) + g(x)\, dx = \int f(x)\, dx + \int g(x)\, dx$.
- $\int fg\, dx = fG - \int f'G$, where $G' = g$. This is called integration by parts.

intermediate value theorem

The [intermediate value theorem] also called Bolzno theorem assures that a continuous function on an interval $a \leq z \leq b$ takes each each value between $f(a)$ and $f(b)$. For example $f(x) = \cos(x) + \cos(3x) + \cos(5x)$ takes any value between $[-3, 3]$ on $[0, \pi]$ because $f(0) = 3$ and $f(\pi) = -3$.

Cauchy's convergence condition

[Cauchy's convergence condition]: a sequence a_n converges, if and only if it is a Cauchy sequence that is if for every constant $c > 0$, we can find n, such that for all $k > n, m > n$ one has $|a_m - a_k| < c$. For example, the sequence $a_n = 1/\log(n)$ converges because for all $c > 0$, and the integer n closest to $e^{2c} + 1$ one has for $k > n$ and $m > n$ $|a_m - a_k| \leq c$.

Cauchy's convergence test

[Cauchy's convergence test]. Given a series $\sum_{k=1}^{\infty} a_k$ with positive summands a_k. If $r = \lim_{n\to\infty} a_n^{1/n} < 1$ then the series is a convergent series. If $r > 1$, then the series diverges.

continuous

A function f is called [continuous] at a point x if for every open interval V around $f(x)$ there exists an open interval U around x such that $f(U)$ is a subset of V. A function f is continuous in a set Y if it is continous at every point in Y. This definition is equivalent to: for every sequence $x_n \to x$, the sequence $f(x_n)$ converges to $f(x)$. Examples:

- Any polynomial like $x^5 + 5x^3 + 3x$ is continuous on the entire line.
- The sum and product of continuous functions is continuous.
- the composition of two continuous functions is continuous.

Discontinuities can happen in different ways: the function can become infinite like $f(x) = 1/x$ at 0 or $\tan(x)$ at $x = \pi/2$, the function can jump like $f(x) = sign(x)$ which is 1 if $x > 0$, -1 if $x < 0$ and 0 if $x = 0$. A function can also become too oscillatory at a point like $f(x) = \sin(1/x)$ at $x = 0$. Note that $f(x) = x \sin(1/x)$ is continuous on the entire real line. There are functions which are discontinuous at every point. An example is $f(x) = 1$ if x is rational and $f(x) = -1$ if x is irrational.
Note that be restricting the domain of a function, one an make it continuous. For example: $f(x) = 1/x$ is continuous on the positive real axes.

converges

A function $f(x)$ [converges] to a value z at x if the function g which agrees with f away from x and satisfies $g(x) = z$ is continouous at x. The value z is called the limit of f at x. For example the function $f(x) = (1-x^2)/(1-x)$ has the limit $z = 2$ at $x = 1$. The function $g(x)$ which is defined bo be $f(x)$ for $x \neq 1$ and $g(1) = 2$ is indeed continuous. One writes $z = \lim_{y \to x} f(y)$. One has

- $\lim_{x \to z}(f(x) + g(x)) = \lim_{x \to z} f(x) + \lim_{x \to z} g(x)$.
- $\lim_{x \to z}(f(x)g(x)) = \lim_{x \to z} f(x) + g(x)$.
- $\lim_{x \to z} f(g(x)) = f(\lim_{x \to z} g(x))$.

A series a_n is a [convergent series], if the partial sum sequence $b_n = \sum_{k=1}^{n} a_k$ converges to a finite limit a.

absolutely convergent series

A series $\sum_n a_n$ is called an [absolutely convergent series] if $\sum_n |a_n|$ is a convergent series.

series

Summing up a sequence is called a [series]. An important example is the geometric series $1 + 1/2 + 1/4 + 1/8 + 1/16 + ...$, which sums up to 2. An other example is the harmonic series $1 + 1/2 + 1/3 + 1/4 + 1/5 + ...$ which has no finite limit.

change of variables

The [change of variables] in integration theory is the formula $\int f(x)dx = \int f(g(u))g'(u)du$ if $x = g(u)$. For example, $\int \sqrt{1-x^2}\, dx$ becomes with $x = g(u) = \sin(u)$ and $dx = g'(u)du = \cos(u)du$ the integral $\int \cos 2(u)\, du$.

differentiable

A function f is called [differentiable] at z if there exists a function g which is continuous at z such that $f(x) = f(z) + (x - z)g(x)$. The derivative of f at z is $g(z)$ and also denoted $f'(x)$. By solving for $g(x)$ and letting $x \to z$ one can write $g(z) = \lim_{x\to z}(f(x) - f(z))/(x - z)$. The quotient is called the differential quotient.

- The sum of two at z differentiable functions is differentiable at z and $(f + g)'(z) = f'(z) + g'(z)$. This is called the sum rule.
- The prduct of two at z differentiable functions is differentiable at z and $(fg)' = f'g + fg'$. This is called the product rule.
- The composition of two differentiable functions is differentiable and $(f \circ g)' = (f' \circ g)g'$. This is called the chain rule.

Functions can be continuous without being differentiable. For example $f(x) = |x|$ is continuous at 0 but not differentiable at 0. There are functions which are continuous everywhere but not differentiable at most points. An example is the Weierstrass function $f(x) = \sum_{k=1}^{\infty} \cos(k^2 x)/k^2$.

Extended mean value theorem

[Extended mean value theorem]. If $f(x)$ and $g(x)$ are differentiable on the interval (a, b) and are continuous on the closed interval $I = \{a \leq x \leq b\}$ then there exists a point $x \in I$ for which

$$f'(x)/g'(x) = (f(b) - f(a))/(g(b) - g(a)) \; .$$

Proof. Otherwise one would have one of the following two possibilities:

$$\begin{array}{ll} f'(x)(g(b) - g(a)) < g'(x)(f(b) - f(a)) & \text{for all } x \text{ in } (a, b) \text{ or} \\ f'(x)(g(b) - g(a)) < g'(x)(f(b) - f(a)) & \text{for all } x \text{ in } (a, b). \end{array}$$

Integration of these expressions using the fundamental theorem of calculus gives

$$\begin{array}{ll} (f(b) - f(a))(g(b) - g(a)) < (g(b) - g(a))(f(b) - f(a)) & \text{or} \\ (f(b) - f(a))(g(b) - g(a)) > (g(b) - g(a))(f(b) - f(a)) & \text{which both are not possible.} \end{array}$$

The special case $g(x) = x$ is called the mean value theorem.

factorial

The [factorial] of a positive integer n is defined recursively by $n! = n(n - 1)!$ and $0! = 1$. For example, $5! = 120$. The factorial function can be extended to the real line and is then called the Γ function: $n! = \Gamma(n + 1)$, where

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} \, dt \; .$$

which is finite everywhere except at $z = 0, -1, -2, ...$

limit

The [limit] of a sequence of numbers a_n is a number a such that a_n converges to a in the following sense: for every $c > 0$ there exists an integer m such that $|a_n - a| < c$ for $n > m$. Limits can be defined in any metric space and more generally in any topological space.

maximum-value theorem

The [maximum-value theorem] assures that a continuous function on an interval $a < z < b$ has a maximum on that interval.

parabola

A [parabola] is the graph of the function $f(x) = x^2$. More general parabolas can be obtained as graphs of $f(x) = a(x-b)^2 + c$ where a, b, c are constants or curves obtained by rotating such a curve in the plane. For example the set of points in the plane satisfying $x = y^2$ form a parabola. Parabolas are examples of conic sections, intersections of a plane with a cone.

L'Hopital rule

The [L'Hopital rule] tells that if f and g are differentiable functions at x and $f(x) = g(x)$ and $g'(x) \neq 0$, then $\lim_{x \to z} f(x)/g(x) = \lim_{x \to z} f'(x)/g'(x)$.
For example, $\lim_{x \to 0} \sin(3x)/x = \lim_{x \to 0} 3\cos(3x) = 3$. The rule essentially tells that one can replace the functions by their linear approximation near a point to find the limit. The proof follows immediatly from the definition of differentiability: there exist continuous functions F, G such that $f(x) = f(z) + (x-z)F(x)$ and $g(x) = g(z) + (x-z)G(x)$. Because $G(z) \neq 0$, the function $F(x)/G(x)$ is continuous at z with value $F(z)/G(z)$. Now: $\lim_{x \to z} f(x)/g(x) = \lim_{x \to z}(f(z) + (x-z)F(x))/(g(z) + (x-z)G(x)) == \lim_{x \to z} f(z)/g(z)$.

Hopital rule

[Hopital rule] see L'Hopital rule.

hyperbola

A [hyperbola] is curve in the plane which can be described as the graph of the function $f(x) = 1/x$. Also translated, scaled and rotated versions of this curve is called a hyperbola. For example, the set of points (x, y) in the plane which satisfy $(x-1)^2 - (y-2)^2 = 5$ is a hyperbola.

Mean value theorem

[Mean value theorem]. If $f(x)$ is a continuous function on an interval $I = \{a \leq x \leq b\}$ which is differentiable on the open interval (a, b), then there exists a point $x \in I$ for which $f'(x) = C = (f(b) - f(a))/(b-a)$. Proof. Otherwise, $f'(x) < C$ on (a, b) or $f'(x) > C$ on (a, b). Integration gives using the fundamental theorem of calculus

$$\begin{array}{ll} f(x) - f(a) = \int_a^x f'(t)dt < C(x-a) & \text{or} \\ f(x) - f(a) = \int_a^x f'(t)dt > C(x-a) & \end{array}$$

especially

$$\begin{array}{ll} f(x) - f(a) = \int_a^b f'(t)dt < C(b-a) & \text{or} \\ f(x) - f(a) = \int_a^b f'(t)dt > C(b-a) & \text{which is a contradiction} \end{array}$$

The mean value theorem is a special case of the extended mean value theorem.

Rolle's theorem

[Rolle's theorem] If $f(x)$ is a continuous function on the interval $I = \{a \leq x \leq b\}$ which is differentiable on the open interval (a, b) and $f(a) = f(b)$, then there exists a point $x \in (a, b)$, for which $f'(x) = 0$.
Proof. f takes both its maximum and minimum on I. If the maximum is equal to the minimum, then $f(x)$ is constant on I, otherwise, either the minium or the maximum is a point x in (a, b). At that point $f'(x) = 0$. qed. Rolle's theorem is a special case of the mean value theorem

rule of three

The [rule of three] ia a rough rule of thumb when solving calculus problems or teaching calculus:

> Look at a calculus problem graphically, numerically and analytically.

In other words, one should try to understand a calculus problem geometrically, algebraically and computationally. For example, the notion of the derivative of a function of one variable can be understood geometrically as a slope, can be understood through algebraic manipulations like $(x^n)' = nx^{n-1}$ or computationally by plugging in numbers or doing things on a computer.

Weierstrass function

A [Weierstrass function] is an example of a function which is continuous but almost nowhere differentiable. An example is $f(x) = \sum_{k=1}^{\infty} \cos(k^2 x)/k^2$.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 42 entries in this file.

Index

ENTRY MULTIVARIABLE CALCULUS

[ENTRY MULTIVARIABLE CALCULUS] Author: Oliver Knill: March 2000 -March 2004 Literature: Standard glossary of multivariable calculus course as taught at the Harvard mathematics department.

acceleration

The [acceleration] of a parametrized curve $r(t) = (x(t), y(t), z(t))$ is defined as the vector $r''(t)$. It is the rate of change of the velocity $r'(t)$. It is significant, because Newtons law relates the acceleration $r''(t)$ of a mass point of mass m with the force F acting on it: $mr''(t) = F(r(t))$. This ordinary differential equation determines completely the motion of the particle.

advection equation

The [advection equation] $u_t = cu_x$ is a linear partial differential equation. Its general solution is u(t,x)=f(x+ct), where f(x)=u(0,x). The advection equation is also called transport equation. In higher dimensions, it generalizes to the gradient flow $u_t = c\text{grad}(u)$.

Archimedes spiral

The [Archimedes spiral] is the plane curve defined in polar coordinates as $r(t) = ct$, where c is a constant. In Euclidean coordinates, it is given by the parametrization $r(t) = (ct\cos(t), ct\sin(t))$.

axis of rotation

The [axis of rotation] of a rotation in Euclidean space is the set of fixed points of that rotation.

Antipodes

Two points on the sphere of radius r are called [Antipodes] (=anti-podal points) if their Euclidean distance is maximal $2r$. If the sphere is centered at the origin, the antipodal point to (x, y, z) is the point $(-x, -y, -z)$.

boundary

The [boundary] of a geometric object. Examples:

- The boundary of an interval $I = \{a \leq x \leq b\}$ is the set with two points $\{a, b\}$. For example, $\{0 \leq x \leq 1$ the boundary $\{0, 1\}$.
- The boundary of a region G in the plane is the union of curves which bound the region. The unit disc has as a boundary the unit circle. The entire plane has an empty boundary.
- The boundary of a surface S in space is the union of curves which bound the surface. For example: A semisphere has as the boundary the equator. The entire sphere has an empty boundary.
- The boundary of a region G in space is the union of surfaces which bound the region. For example, the unit ball has the unit sphere as a boundary. A cube has as a boundary the union of 6 faces.
- The boundary of a curve $r(t), t \in [a, b]$ consists of the two points $r(a), r(b)$.

The boundary can be defined also in higher dimensions where surfaces are also called manifolds. The dimension of the boundary is always one less then the dimension of the object itself. In cases like the half cone, the tip of the cone is not considered a part of the boundary. It is a singular point which belongs to the surface. While the boundary can be defined for far more general objects in a mathematical field called "topology", the boundaries of objects occuring in multivariable calculus are assumed to be of dimension one less than the object itself.

Burger's equation

The [Burger's equation] $u_t = uu_t$ is a nonlinear partial differential equation in one dimension. It is a simple model for the formation of shocks.

Cartesian coordinates

[Cartesian coordinates] in three-dimensional space describe a point P with coordinates x, y and z. Other possible coordinate systems are cylindrical coordinates and spherical coordinates. Going from one coordinate system to an other is called a coordinate change.

Cavalieri principle

[Cavalieri principle] tells that if two solids have equal heights and their sections at equal distances have have areas with a given ratio, then the volumes of the solids have the same ratio.

change of variables

A [change of variables] is defined by a coordinate transformation. Examples are changes between cylindrical coordinates, spherical coordinates or Cartesian coordinates. Often one uses also rotations, allowing to use a convenient coordinate system, like for example, when one puts a coordinate system so that a surface of revolution has as the symmetry axes the z-axes.

circle

A [circle] is a curve in the plane whose distance from a given point is constant. The fixed point is called the center of the circle. The distance is the radius of the circle. One can parametrize a circle by r(t) = (cos(t),sin(t) or given as an implicit equation $g(x, y) = x^2+y^2 = 1$. The circle is an example of a conic section, the intersection of a cone with a plane to which ellipses, hyperbola and parabolas belong to.

cone

A [cone] in space is the set of points $x^2 + y^2 = z^2$ in space. Also translates, scaled and rotated versions of this set are still called a cone. For example $2x^2 + 3y^2 = 7z^2$ is an elliptical cone.

conic section

A [conic section] is the intersection of a cone with a plane. Hyperbola, ellipses and parabola lines and pairs of intersecting lines are examples of conic sections.

continuity equation

The [continuity equation] is the partial differential equation $\rho_t + div(\rho v) = 0$, where ρ is the density of the fluid and v is the velocity of the fluid. The continuity equation is the consequence of the fact that the negative change of mass in a small ball is equal to the amount of mass which leaves the ball. The later is the flux of the current $j = v\rho$ through the surface and by the divergence theorem the integral of div(j).

cos theorem

The [cos theorem] relates the length of the edges a,b,c in a triangle ABC with one of the angles α: $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$ Especially, if $\alpha = \pi/2$, it becomes the theorem of Pythagoras.

critical point

A [critical point] of a function $f(x, y)$ is a point (x_0, y_0), where the gradient $\nabla f(x_0, y_0)$ vanishes. Critical points are also called stationary points. For functions of two variables $f(x, y)$, critical points are typically maxima, minima or saddle points realized by $f(x, y) = -x^2 - y^2, f(x, y) = x^2 + y^2$ or $f(x, y) = x^2 - y^2$.

chain rule

The [chain rule] expresses the derivative of the composition of two functions in terms of the derivatives of the functions. It is $(fg)'(x) = f'(g(x))g'(x)$. For example, if $r(t)$ is a curve in space and F a function in three variables, then $(d/dt)f(r(t)) = \text{grad}(f)\cdot r'(t)$. Example. If T and S are maps on the plane, then $(TS)' = T'(S)S'$, where T' is the Jacobean of T and S' is the Jacobean of S.

change of variables

A [change of variables] on a region R in Euclidean space is given by an invertible map $T : R \to T(R)$. The change of variables formula $\int_{T(R)} f(x)\ dx = \int_R f(Tx) det(T'(x))\ dx$ allows to evaluate integrals of a function f of several variables on a complicated region by integrating on a simple region R. In one dimensions, the change of variable formula is the formula for substitution. Example: (2D polar coordinates) $T(r, \phi) = (x \cos(\theta), y \sin(\theta)$. with det(T')=r maps the rectangle $[0, s] \times [0, 2\pi]$ into the disc. Example of 3D spherical coordinates are $T(r, \theta, \phi) = (r \cos(\theta) \sin(\phi), r \sin(\theta) \sin(\phi), r \cos(\phi))$, $det(T') = r^2 \sin(\phi)$ maps the rectangular region $(0, s) \times (0, 2\pi 0 \times (0, \pi)$ onto a sphere of radius s.

curl

The [curl] of a vector field $F = (P, Q, R)$ in space is the vector field $(R_y - Q_z, P_z - R_x, Q_x - P_y)$. It measures the amount of circulation = vorticity of the vector field. The curl of a vector field F=(P,Q) in the plane is the scalar field $(Q_x - P_y)$. It measures the vorticity of the vector field in the plane.

curvature

The [curvature] of a parametrized curve r(t) = (x(t),y(t),z(t)) is defined as $k(t) = |r'(t) \times r''(t)|/|r'(t)|^3$. Examples:

- The curvature of a line is zero.
- The curvature of a circle of radius r is 1/r.

curve

A [curve] in space is the image of a map $X : t- > r(t) = (x(t), y(t), z(t))$, where $x(t), y(t), z(t)$ are three piecewise smooth functions. For general continuous maps $x(t), y(t), z(t)$, the length or the velocity of the curve would no more be defined.

cross product

The [cross product] of two vectors $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$ is the vector $(v_2w_3 - w_2v_3, v_3w_1 - w_3v_1, v_1w_2 - w_2v_1)$.

curve

A [curve] in three-dimensional space is the image of a map $r(t) = (x(t), y(t), z(t))$, where $x(t), y(t), z(t)$ are three continuous functions. A curve in two-dimensional space is the image of a map $r(t) = (x(t), y(t))$.

cylinder

A [cylinder] is a surface in three dimensional space such that its defining equation f(x,y,z)=0 does not involve one of the variables. For example, $z = 2\sin(y)$ defines a cylinder. A cylinder usually means the surface $x^2 + y^2 = r$ or a translated rotated version of this surface.

derivative

The [derivative] of a function f(x) of one variable at a point x is the rate of change of the function at this point. Formally, it is defined as $lim_{dx\to 0}(f(x+dx) - f(x))/dx$. One writes f'(x) for the derivative of f. The derivative measures the slope of the graph of f(x) at the point. If the derivative exists for all x, the function is called differentiable. Functions like $\sin(x)$ or $\cos(x)$ are differentiable. One has for example $f'(x) = \cos(x)$ if $f(x) = sin(x)$. An example of a function which is not differentiable everywhere is f(x)=—x—. The derivative at 0 is not defined.

cylindrical coordinates

[cylindrical coordinates] in three dimensional space describe a point P by the coordinates $r = (x^2 + y^2 + z^2)^{1/2}$, phi=arctan(y/x),z, where P=(x,y,z) are the Cartesian coordinates of P. Other coordinate systems are Cartesian coordinates or spherical coordinates.

determinant

The [determinant] of a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ad - bc$. The determinant of a matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ is $aei + bfg + cdh - ceg - fha - ibd$. The determinant is relevant when changing variables in integration.

directional derivative

The [directional derivative] of f(x,y,z) in the direction v is the dot product of the gradient of f with v. It measures the rate of change of f at a point P when moving trough the point (x,y,z) with velocity v.

distance

The [distance] of two points P=(a,b,c) and Q=(u,v,w) in three dimensional Euclidean space is the square root of $(a-u)^2 + (b-v)^2 + (c-w)^2$. The distance of two points P=(a,b) and $Q = (u, v)$ in the plane is the square root of $(a-u)^2 + (b-v)^2$.

distance

The [distance] between two nonparallel lines in three dimensional Euclidean space is given by the forumla $d = |(v \times w) \cdot u|/|(v \times w)|$, where v and w are arbitrary nonzero vectors in each line and u is an arbitrary vector connecting a point on the first line to a point of the second line.

divergence

The [divergence] of a vector field F=(P,Q,R) is the scalar field $div(F) = P_x + Q_y + R_z$. The value $div(F)(x, y, z)$ measures the amount of expansion of the vector field at the point (x, y, z).

dot product

The [dot product] of two vectors $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$ is the scalar $v_1w_1 + v_2w_2 + v_3w_3$.

ellipse

An [ellipse] is the set of points in the plane which satisfy an equation $(x - a)^2/A^2 + (y - b)^2/B^2 = 1$. It is inscribed in a rectangle of length A and width B centered at (a, b). Ellipses can also be defined as the set of points in the plance whose sum of the distances to two points is constants. The two fixed points are called the foci of the ellipse. The line through the foci of a noncircular ellipse is called the focal line, the points where focal axes and a noncircular ellipse cross, are called vertices of the ellipse. The major axis of the ellipse is the line segment connecting the two vertices, the minor axis is the symmetry line of the ellipse which mirrors the two focal points or the two vertices. Ellipses are examples of conic sections, the intersection of a cone with a plane.

ellipsoid

An [ellipsoid] is the set of points in three dimensional Euclidean space, which satisfy an equation $(x - a)^2/A^2 + (y - b)^2/B^2 + (z - c)^2/C^2 = 1$. It is inscribed in a box of length, width and height A,B,C centered at (a, b, c).

equation of motion

The [equation of motion] of a fluid is the partial differential equation $\rho Dv/dt = -grad(p) + F$, where F are external forces like gravity rho g, or magnetic force $j \times B$ and Dv/dt is the total time derivative $Dv/dt = v_t + vgrad(v)$. The term - grad(p) is the pressure force. Together with an incompressibility assumption div(v)=0, these equations of motion are called Navier Stokes equations.

flux integral

The [flux integral] of a vector field F through a surface S=X(R) is defined as the double integral of X(F).n over R, where $n = X_u \times X_v$ is the normal vector of the surface as defined through the parameterization $X(u, v)$.

Fubinis theorem

[Fubinis theorem] tells that $\int_a^b \int_c^d f(x, y)\, dxdy = \int_c^d \int_a^b f(x, y)\, dydx$.

gradient

The [gradient] of a function f at a point P=(x,y,z) is the vector $(f_x(x,y,z), f_y(x,y,z), f_z(x,y,z))$ where f_x denotes the partial derivative of f with respect to x.

Hamilton equations

The [Hamilton equations] to a function $f(x,y)$ is the system of ordinary differential equations $x'(t) = f_y(x,y), y'(t) = -f_x(x,y)$. which is called Hamilton system. Solution curves of this system are located on level curves $f(x,y) = c$ because by the chain rule one has $d/dt f(x(t), y(t)) = f_x x' + f_y y' = f_x f_y - f_x f_y = 0$. The preservation of f is in physics called energy conservation.

heat equation

The [heat equation] is the Partial differential equation $u_t = mu\Delta(u)$, where mu is a constant, and Δu is the Laplacian of u. The heat equation is also called the diffusion equation.

Hessian

The [Hessian] is the determinant of the Hessian matrix.

Hessian matrix

The [Hessian matrix] of a function f(x,y,z) at a point (u,v,w) is the $3x3$ matrix $f''(u,v,w) = H(u,v,w) = \begin{pmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{pmatrix}$. The Hessian matrix of a function $f(x,y)$ at a point (u,v) is the 2x2 matrix $f'' = H(u,v,w) = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$. The Hessian matrix is useful to classify critical points of $f(x,y)$ using the second derivative test.

hyperbola

A [hyperbola] is a plane curve which can be defined as the level curve $g(x,y) = x^2/a^2 + y^2/b^2 = 1$ or given as a parametrized curve $r(t) = (a\cosh(t), b\sinh(t))$. A hyperbola can geometrically also be defined as the set of points whose distances from two fixed points in the plane is constant. The two fixed points are called the focal points of the hyperbola. The line through the focal points of a hyperbola is called the focal axis. The points, where the focal axis and the hyperbola cross are called vertices. A hyperbola is an example of a conic section, the intersection of a cone with a plane.

hyperboloid

A [hyperboloid] is the set of points in three dimensional Euclidean space, which satisfy an equation $(x-u)^2/a^2 - (y-v)^2/b^2 - (z-w)^2/c^2 = I$, where $I = 1$ or $I = -1$. For a=b=c=1, the hyperboloid is obtained by rotating a hyperbola $x^2 - y^2 = 1$ around the x-axes. It is two-sided for I=-1 (the intersection of the plane z=c with the hyperboloid is then empty) and one-sided for I=1.

incompressible

A vector field F is called [incompressible] if its divergence is zero $\text{div}(F) = 0$. The notation has its origins from fluid dynamics, where velocity fields F of fluids, gases or plasma often are assumed to be incompressible. If a vector field is incompressible and is a velocity field, then the corresponding flow preserves the volume.

continuity equation

The [continuity equation] $\rho_t + \text{div}(i) = 0$ links density ρ and velocity field i. It is an infinitesimal description which is equivalent to the preservation of mass by the theorem of Gauss. The change of mass $M(t) \int\int\int_R \rho \, dV$ inside a region R in space is the minus the flux of mass through the boundary S of R.

interval

An [interval] is a subset of the real line defined by two points a,b. One can write $I = \{a \leq x \leq b\}$ for a closed interval, $I = \{a < x < b\}$ for an open interval and $I = \{a \leq x < b\}, I = \{a < x \leq b\}$ for half open intervals. If $a = -\infty$ and $b = \infty$, then the interval is the entire real line. If $a = 0, b - \infty$, then $I = (a, b)$ is the set of positive real numbers. Intervals can be characterized as the connected sets in the real line.

integral

An [integral] of f(x) over an interval I on the line is the limit $(1/n)\sum_{i=1}^{n} f(i/n)$ for $n \to \infty$ over the integers and the sum is taken over all i such that i/n is in I. An integral of $f(x, y)$ over a region R in the plane is the limit $(1/n^2)\sum_{(i/n,j/n)\in R} f(i/n, j/n)$ for $n \to \infty$. Such an integral is also called double integral. Often, double integrals can be evaluated by iterating two one-dimensional integrals. An integral of f(x,y,z) over a domain R in space is the limit $(1/n^3)\sum_{(i/n,j/n,k/n)\in R} f(i/n, j/n, k/n)$ for n to infinity. Such an integral is also called a triple integral. Often, triple integrals can be evaluated by iterating three one-dimensional integrals.

intercept

An [intercept] is the intersection of a surface with a coordinate axes. Like traces, intercepts are useful for drawing surfaces by hand. For example, the two sheeted hyperboloid $x^2 + y^2 - z^2 = -1$ has the intercepts $x^2 - z^2 = -1$ and $y^2 - z^2 = -1$ (hyperbola) and an empty intercept with the z axes.

jerk

The [jerk] of a parametrized curve r(t)=(x(t),y(t),z(t)) is defined as r"'(t). It is the rate of change of the acceleration. By Newtons law, the jerk measures the rate of change of the force acting on the body.

Lagrange multiplier

A [Lagrange multiplier] is an additional variable introduced for solving extremal problems under constraints. To extremize $f(x, y, z)$ on a surface $g(x, y, z) = 0$ then an extremum satisfies the equations $f' = Lg', g = 0$, where L is the Lagrange multiplier. These are four equations for four unknowns x,y,z,l. Additionally, one has to check for solutions of $g'(x, y, z) = 0$.
Example. If we want to extremize $F(x, y, z) = -x \log(x) - y \log(y) - z \log(z)$ under the constraint $G(x, y, z) = x + y + z = 1$, we solve the equations $-1 - \log(x) = \lambda 1$ $-1 - \log(y) = \lambda 1$ $-1 - \log(z) = \lambda 1$ $x + y + z = 1$, the solution of which is $x = y = z = 1/3$.

Lagrange method

The [Lagrange method] to solve extremal problems under constraints:
1) in order that a function f of several variables is extremal on a constraint set $g = c$, we either have $\nabla g = 0$ or the point is a solution to the Lagrange equations $\nabla f = \lambda \nabla g, g = c$.
2) in order to extremize a function f of several variables under the contraint set $g = c, h = d$, we have to solve the Lagrange equations $\nabla f = \lambda \nabla g + \mu \nabla h, g = c, h = d$ or solve $\nabla g = \nabla h = 0$.

Laplacian

The [Laplacian] of a function $f(x, y, z)$ is defined as $\Delta(f) = f_x x + f_y y + f_z z$. One can write it as $\Delta = \text{divgrad}(f)$. Functions for which the Laplacian vanish are called harmonic. Laplacian appear often in PDE's Examples: the Laplace equation $\Delta(f) = 0$, the Poisson equation $\Delta(f) = \rho$, the Heat equation $f_t = \mu \Delta(f)$ or the wave equation $f_t t = c^2 \Delta f$. The [length] of a curve r(t)=(x(t),y(t),z(t)) from t=a to t=b is the integral of the speed —r'(t)— over the interval a,b. Example. the length of the curve r(t)=(cos(t),sin(t)) from t=0 to $t = \pi$ is π because the speed $|r'(t)|$ is 1.

length

The [length] of a vector $v = (a, b, c)$ is the square root of $v \cdot v = a^2 + b^2 + c^2$. An other word for length is norm. If a vector has length 1, it is called normalized or a unit vector.

level curve

A [level curve] of a function $f(x, y)$ of two variables is the set of points which satisfy the equation $f(x, y) = c$. For example, if $f(x, y) = x^2 - y^2$, then its level curves are hyperbola. Level curves are orthogonal to the gradient vector field grad(f).

level surface

A [level surface] of a scalar function $f(x, y, z)$ is the set of points which satisfy $f(x, y, z) = c$. For example, if $f(x, y, z) = x^2 + y^2 + 3z^2$, then its level surfaces are ellipsoids. Level surfaces are orthogonal to the gradient field grad(f).

linear approximation

The [linear approximation] of a function f(x,y,z) at a point (u, v, w) is the linear function $L(x, y, z) = f(u, v, w) + \nabla f(u, v, w) \cdot (x - u, y - v, z - w)$.

line

A [line] in three-dimensional space is a curve in space given by $r(t) = P + tv$, where P is a point in space and v is a vector in space. The representation r(t)=P+tv is called a parameterization of the line. Algebraically, a line can also be given as the intersection of two planes: $ax+by+cz = d$, $ux+vy+wz = q$. The corresponding vector v in the line is the cross product of (a, b, c) and (u, v, w). A point $P = (x, y, z)$ on the line can be obtained by fixing one of the coordinates, say z=0 and solving the system $ax + by = d$, $ux + vy = q$ for the unknowns x and y.

line integral

The [line integral] of a vector field $F(x, y)$ along a curve $C : r(t) = (x(t), y(t)), t \in [a, b]$ in the plane is defined as

$$\int_C F \cdot ds = \int_a^b F(r(t)) \cdot r'(t)\ dt\ ,$$

where $r'(t) = (x'(t), y'(t))$ is the velocity. The definition is similar in three dimensions where $F(x, y, z)$ is a vector field and $C : r(t) = (x(t), y(t), z(t)), t \in [a, b]$ is a curve in space.

Maxwell equations

The [Maxwell equations] are a set of partial differential equations which determine the electric field E and magnetic field B, when the charge density ρ and the current density j are given. There are 4 equations:

$\text{div}(B) = 0$	no magnetic monopoles
$\text{curl}(E) = -B_t/c$	Faradays law, change of magnetic flux produces voltage
$\text{curl}(B) = E_t/c + (4\pi/c)j$	Ampere's law, current or E change produce magnetism
$\text{div}(E) = 4\pi\rho$	Gauss law, electric charges produce an electric field

nabla

[nabla] is a mathematical symbol used when writing the gradient ∇f of a function $f(x, y, z)$. Nabla looks like an upside down Δ. Etymologically, the name has the meaning of an Egyption harp.

nabla calculus

The [nabla calculus] introduces the vector $\nabla = (\partial_x, \partial_y, \partial_z)$. It satisfies $\nabla(f) = \text{grad}(f)$, $\nabla x F = curl(F)$, $\nabla \cdot F = div(F)$. Using basic vector operation rules and differentiation rules like $\nabla(fg) = (\nabla f)g + f(\nabla g)$ one can verify identities: like for example
$\text{div}(\text{curl})F = 0$, $\text{curl}(\text{grad})f = 0$, $\text{curl}(\text{curl}F) = grad(divF) - \Delta(F)$, $\text{div}(E \times F) = F \cdot curl(E) - E \cdot curl(F)$.

nonparallel

Two vectors v and w are called [nonparallel] if they are not parallel. Two vectors in space are parallel if and only if their cross product $v \times w$ is nonzero.

normal vector

A [normal vector] to a parametrized surface $X(u, v) = (x(u, v), y(u, v), z(u, v))$ at a point P=(x,y,z) is the vector $X_u x X_v$. It is orthogonal to the tangent plane spanned by the two tangent vectors X_u and X_v.

normalized

A vector is called [normalized] if its length is equal to 1. For example, the vector $(3/5, 4/5)$ is normalized. The vector $(2, 1)$ is not normalized.

octant

An [octant] is one of the 8 regions when dividing three dimensional space with coordinate planes. It is the analogue of quadrant in two dimensions.

open set

An [open set] R in the plane or in space is a set for which every point P is contained in a small disc U which is still contained in R. The disc $x^2 + y^2 < 1$ is an example of an open set. The set $x^2 + y^2 \leq 1$ is not open because the point $(1, 0)$ for example has no neighborhood disc contained in R.

open

A set is called [open], if it is an open set. It means that every point in the set is contained in a neighborhood which still is in the set. The complement of open sets are called closed.

ordinary differential equation

An [ordinary differential equation] (ODE) is an equation for a function or curve $f(t)$ which relates derivatives f,f',f".... of f. An example is f'=c f which has the solution $f(t) = Ce^(ct)$, where C is a constant. Only derivatives with respect to one variable may appear in an ODE. In most cases, the variable t is associated with time. Examples:

$f' = cf$	population model $c > 0$.
$f' = -cf$	radioactive decay $c > 0$
$f' = cf(1-f)$	logistic equation
$f'' = -cf$	harmonic oscillator
$f'' = F(f)$	general form of Newton equations

By increasing the dimension of the phase space, every ordinary differential equation can be written as a first order autonomous system $x' = F(x)$. For example, $f'' = -f$ can be written with the vector $x = (x_1, x_2) = (f, f')$ as $(x_1', x_2') = (f', f'') = (f', f) = (x_2', -x_2')$. There is a 2×2 matrix such that $x' = Ax$.

orthogonal

Two vectors v and w are called [orthogonal] if $v \cdot w = 0$. An other word for orthogonal is perpendicular. The zero vector 0 is orthogonal to any other vector.

parabola

A [parabola] is a plane curve. It can be defined as the set of points which have the same distance to a line and a point. The line is called the directrix, the point is called the focus of the parabola. One can parametrize a parabola as $r(t) = (t, t^2)$. It is also possible to give a parabola as a level curve $g(x, y) = y - x^2 = 0$ of a function of two variables. A parabola is an example of a conic section, to which also circles, ellipses and hyperbola belong.

parallelogram

A [parallelogram] E can be defined as the image of the unit square under a map $T(s, t) = sv + tw$, where u and v are vectors in the plane. One says, E is spanned by the vectors v and w. The area of a parallelogram is $|v \times w|$.

parallelepiped

A [parallelepiped] E can be defined as the image of the unit cube under a linear map $T(r, s, t) = ru+sv+tv$, where u,v,w are vectors in space. One says, E is spanned by the vectors u, v and w. The volume of a parallelepiped is $|u \cdot (v \times w)|$.

perpendicular

Two vectors v and w are called [perpendicular] if $v \cdot w = 0$. An other word for perpendicular is orthogonal. The zero vector $v = 0$ is perpendicular to any other vector.

quadratic approximation

The [quadratic approximation] of a function f(x,y,z) at a point (u, v, w) is the quadratic function $Q(x, y, z) = L(x, y, z) + [H(u, v, w)(x - u, y - v, z - w)] \cdot (x - u, y - v, z - w)/2$, where $H(u, v, w)$ is the Hessian matrix of f at (u, v, w) and where $L(x, y, z)$ is the linear approximation of $f(x, y, z)$ at (u, v, w). For example, the function $f(x, y) = 3 + \sin(x + y) + \cos(x + 2y)$ has the linear approximation $L(x, y) = 4 + x + y$ and the quadratic approximation $Q(x, y) = 4 + x + y + (x + 2y)^2/2$.

quadrant

A [quadrant] is one of the 4 regions when dividing the two dimensional space using coordinate axes. It is the analogue of octant in three dimensions. For example, the set $\{x > 0, y > 0\}$ is the open upper right quadrant. The set $\{x \geq 0, y \geq 0\}$ is the closed upper right quadrant.

parallel

Two vectors v and w are called [parallel] if there exists a real number λ such that $v = \lambda w$. Two vectors in space are parallel if and only if their cross product $v \times w$ is zero.

parametrized surface

A [parametrized surface] is defined by a map

$$X(u, v) = (x(u, v), y(u, v), z(u, v))$$

from a region R in the uv-plane to xyz-space. Examples

- Sphere: $X(u, v) = (r\cos(u)\sin(v), r\sin(u)\sin(v), r\cos(v))$ $R = [0, 2\pi) \times [0, \pi]$, u and v are called Euler angles.
- Plane $X(u, v) = P + uU + vV$, where P is a point, U,V are vectors and R is the entire plane.
- Surface of revolution is parametrized by $X(u, v) = (f(v)\cos(u), f(v)\sin(u), v)$ where u is an angle measuring the rotation round the z axes and $f(v)$ is a nonnegative function giving the distance to the z-axes at the height v.
- A graph of a function $f(x, y)$ is parametrized by $X(u, v) = (u, v, f(u, v))$.
- A torus is parametrized by $X(u, v) = (a + b\cos(v))\cos(u), (a + b\cos(v))\sin(u), sin(v))$ on $R = [0, 2\pi) \times [0, 2\pi)$.

parametrized curve

A [parametrized curve] in space is defined by a map $r(t) = (x(t), y(t), z(t))$ from an interval I to space. Examples are

- Circle in the xy-plane $r(t) = (\cos(t), \sin(t), 0)$ with $t \in [0, 2\pi]$.
- Helix $r(t) = (\cos(t), \sin(t), t)$ with $t \in [a, b]$.
- Line $r(t) = P + tV$, where V is a vector and P is a point and $-\infty < t < \infty$.
- A line segment connecting P with Q $r(t) = P + t(Q - P)$, where $t \in [0, 1]$.

partial derivative

The [partial derivative] of a function of several variables f is the derivative with respect to one variable assuming the other variables are constants. One writes for example $f_y(x, y, z)$ for the partial derivative of f(x,y,z) with respect to y.

partial differential equation

A [partial differential equation] is an equation for a function of several variables in which partial derivatives with respect to different variables appear. Examples:

$u_t = cu_x$	Advection equation
$u_t = \mu\Delta(u)$	Heat equation
$u_tt = c^2\Delta(u)$	Wave equation
$u_tt = c^2\Delta(u) - m^2u$	Klein Gordon equation
$\Delta(u) = 0$	Laplace equation
$\Delta(u) = \rho$	Poisson equation
$u_t + u_{xxx} + 6uu_x = 0$	KdV equation
$u_t = uu_x$	Burger equation
$\mathrm{div}(B) = \mathrm{div}(E) = 0$ $B_t = -c\mathrm{curl}(E)$ $E_t = c\mathrm{curl}(B)$	Maxwell equation (vacuum)
$ihu_t = h^2/2m\Delta u + Vu$	Schroedinger equation
$\mathrm{curl}(A) = F$	Vector potential equation

plane

A [plane] in three dimensional space is the set of points (x, y, z) which satisfy an equation $ax + by + cz = d$. A parametrization of a plane is given by the map $(s, t) \mapsto X(u, v) = sv + tw$, where v, w are two vectors. If three points P_1, P_2, P_3 are given in space, then $X(s, t) = P_1 + s(P_2 - P_1) + t(P_3 - P_1)$ is a parametrisation of the plane which contains all three points.

polar coordinates

[polar coordinates] in the plane describe a point P=(x,y) with the coordinate (r, t) where $r = (x^2 + y^2)^{1/2}$ is the distance to the origin and t is the angle between the line OP and the x axes. The angle $t = \arctan(y/x) \in (-\pi/2, \pi/2]$ has to be augmented by π if $x < 0$ or $x = 0, y < 0$. The Cartesian coordinates of P are obtained from the Polar coordinates as $x = r\cos(t), y = r\sin(t)$.

potential

A function $U(x, y, z)$ is called a [potential] to a vector field $F(x, y, z)$ if $grad(U) = F$ at all points. The vector field F is then called conservative or a potential field. Not every vector field is conservative. If $curl(F) = 0$ everywhere in space, then F has a potential.

projection

The [projection] of a vector v onto a vector w is the vector $w(v \cdot w)/|w|^2$. The scalar projection is the length of the projection.

right handed

A coordinate system in space is [right handed] if it can be rotated into the situation such that if the z axes points to the observer of the xy plane, then a 90 degree rotation brings the x axes to the y axes. Otherwise the coordinate system is called left handed. If u is a vector on the positive x axes, v is a vector on the positive y axes and w is a vector on the positive z axes, then the coordinate system is right handed if and only if the triple product $u \cdot (v \times w)$ is positive.

second derivative test

The [second derivative test]. If the determinant of the Hessian matrix $\det(f''(x, y)) < 0$ then (x, y) is a saddle point. If $f''(x, y) > 0$ and and $f_{xx}(x, y) < 0$ then (x, y) is a local maximum. If $\det(f''(x, y)) < 0$ and $f_{xx}(x, y) > 0$ then (x, y) is a local minimum.

Space

[Space] is usually used as an abbreviation for three dimensional Euclidean space. In a wider sense, it can mean linear space a vector space in which on can add and scale.

speed

The [speed] of a curve $r(t) = (x(t), y(t), z(t))$ at time t is the length of the velocity vector $r'(t) = (x'(t), y'(t), z'(t))$.

sphere

A [sphere] is the set of points in space, which have a given distance r from a point P=(a,b,c). It is the set $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$. For a=b=c=0,r=1 one obtains the unit sphere: $x^2 + y^2 + z^2 = 1$. Spheres can be define in any dimenesions. A sphere in two dimensions is a circle. A sphere in 1 dimension is the union of two points. The unit sphere in 4 dimensions is the set of points $(x, y, z, w) \in R^4$ which satisfy $x^2 + y^2 + z^2 + w^2 = 1$ Spheres can be defined in any space equiped with a distance like $d((x, y), (u, v)) = |x - u| + |y - v|$ in the plane.

superformula

The [superformula] describes a class of curves with a few parameters m, n_1, n_2, n_3, a, b. It is the polar graph

$$r(t) = (|\cos(mt/4)|^{n_1}/a + |\sin(mt/4)|^{n_2}/b)^{-1/n_3} \, .$$

It had been proposed by the Belgian Biologist Johan Gielis in 1997.

superposition

The principle of [superposition] tells that the sum of two solutions of a linear partial differential equation (PDE) is again a solution of the PDE. For example, $f(x, y) = \sin(x - t)$ and $g(x, y) = e^{x-t}$ are both solutions to the transport equation $f_t(t, x) + f_x(t, x) = 0$. Therefore also the sum $\sin(x - t) + e^{x-t}$ is a solution. For nonlinear partial differential equations the superposition principle is no more true which is one of the reasons for the difficulty with dealing with nonlinear systems.

surface

A [surface] can either be described as a parametrized surface or implicitely as a level surface $g(x, y, z) = 0$. In the first case, the surface is given as the image of a map $X : (u, v) \mapsto (x(u, v), y(u, v), z(u, v))$ where u,v ranges over a parameter domain R in the plane. In the second case, the surface is determined by a function of three variables. Sometimes, one can describe a surface in both ways like in the following examples:

Sphere:	$X(u, v) = (r\cos(u)\sin(v), r\sin(u)\sin(v), r\cos(v))$, $g(x, y, z) = x^2 + y^2 + z^2 = r^2$
Graphs:	$X(u, v) = (u, v, f(u, v))$, $g(x, y, z) = z - f(x, y) = 0$
Planes:	$X(u, v) = P + uU + vV$, $g(x, y, z) = ax + by + cz = d$, $(a, b, c) = UxV$.
Surface of revolution:	$X(u, v) = (f(v)\cos(u), f(v)\sin(u), v)$, $g(x, y, z) = f((x^2 + y^2)^(1/2)) - z = 0$

surface of revolution

A [surface of revolution] is a surface which is obtained by rotating a curve around a fixed line. If that line is the z-axes, the surface can be given in cylindrical coordinates as r = f(z). A parametrization is $X(t, z) = (f(z)\cos(t), f(z)\sin(t), z)$.

surface area

The [surface area] of surface $S = X(R)$ is defined as the integral of $\int\int_R |X_u \times X_v(u, v)| \, dudv$. For example, for $X(u, v) = (r\cos(u)\sin(v), r\sin(u)\sin(v), r\cos(v))$ on $R = \{0 \leq u \leq 2\pi, 0 \leq v \leq \pi\}$, where $S = X(R)$ is the sphere of radius r, one has $X_u \times X_v = r\sin(v)X$ and $|X_u \times X_v| = \sin(v)r^2$. The surface area is $\int_0^{2\pi}\int_0^{\pi} r^2 \sin(v) \, dudv = 4\pi r^2$.

surface integral

A [surface integral] of a function $f(x, y, z)$ over a surface $S = X(R)$ is defined as the integral of $f(X(u, v))|X_u \times X_v(u, v)|$ over R. In the special case when $f(x, y, z) = 1$, the surface integral is the surface area of the surface S.

tangent plane

The [tangent plane] to an implicitely defined surface $g(x, y, z) = c$ at the point (x_0, y_0, z_0) is the plane $ax + by + cz = d$, where $(a, b, c) = \nabla f(x_0, y_0, z_0)$ is the gradient of g at (x_0, y_0, z_0) and $d = ax_0 + by_0 + cz_0$.

tangent line

The [tangent line] to an implicitely defined curve $g(x, y) = c$ at the point (x_0, y_0) is the line $ax + by = d$, where (a, b) is the gradient of $g(x, y)$ at the point (x_0, y_0) and $d = ax_0 + by_0$.

theorem of Clairot

The [theorem of Clairot] assures that one can interchange the order of differentiation when taking partial derivatives. More precicely, if $f(x, y)$ is a function of two variables for which both $f_{xy} = f_{yx}$ are continuous, then $f_{xy} = f_{yx}$.

theorem of Gauss

The [theorem of Gauss] states that the flux of a vector field F through the boundary S of a solid R in three-dimensional space is the integral of the divergence div(F) of F over the region R:

$$\int\int\int_R \mathrm{div}(F)\; dV = \int\int_S F \cdot dS\ .$$

theorem of Green

The [theorem of Green] states that the integral of the $curl(F) = Q_x - P_y$ of a vector field $F = (P, Q)$ over a region R in the plane is the same as the line integral of F along the boundary C of R.

$$\int\int_R \mathrm{curl}(F)\; dA = \int_C F\; ds\ .$$

The boundary C is traced in such a way that the region is to the left. The boundary has to be piecewise smooth. The theorem of Green can be derived from the theorem of Stokes.

Green's theorem

[Green's theorem] see theorem of Green.

Green's theorem

The determinant of the Jacobean matrix is often called Jacobean or Jacobean determinant.

Jacobean matrix

[Jacobean matrix] If $T(u,v) = (f(u,v), g(u,v))$ is a transformation from a region R to a region S in the plane, the Jacobean matrix dT is defined as $\begin{pmatrix} f_u(u,v) & f_v(u,v) \\ g_u(u,v) & g_v(u,v) \end{pmatrix}$. It is the linearization of T near (u,v). Its determinant called the Jacobean determiant measures the area change of a small area element $dA = dudv$ when maped by T. For example, if $T(r,\theta) = (r\cos(\theta), r\sin(\theta)) = (x,y)$ is the coordinate transformation which maps $R = \{r \geq 0, \theta \in [0, 2\pi)\}$ to the plane, then dT is the matrix $\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -r\sin(\theta) & r\cos(\theta) \end{pmatrix}$ which has determinant r.

theorem of Stokes

The [theorem of Stokes] states that the flux of a vector field F in space through a surface S is equal to the line integral of F along the boundary C of S:

$$\int\int_S \text{curl}(F) \ \cdot dS = \int_C F \ ds \ .$$

three dimensional space

The [three dimensional space] consists of all points (x,y,z) where x,y,z ranges over the set of real numbers. To distinguish it from other three-dimensional spaces, one calls it also Euclidean space.

torus

A [torus] is a surface in space defined as the set of points which have a fixed distance from a circle. It can be parametrized by $X(u,v) = (a + b\cos(v))\cos(u), (a + b\cos(v))\sin(u), sin(v))$ on $R = [0, 2\pi) \times [0, 2\pi)$, where a, b are positive constants.

trace

The [trace] of a surface in three dimensional space is the intersection of the surface with one of the coordinate planes x=0 or y=0 or z=0. Traces help to draw a surface when given the task to do so by hand. Other marking points are intercepts, the intersection of the surface with the coordinate axes.

triangle

A [triangle] in the plane or in space is defined by three points P, Q, R. If $v = PQ, w = PR$, then $|v \times w|/2$ is the area of the triangle.

triple product

The [triple product] between three vectors u, v, w in space is defined as the scalar $u \cdot (v \times w)$. The absolute value $|u \cdot (v \times w)|$ is the volume of the paralelepiped spanned by u, v and w.

triple dot product

[triple dot product] (see triple product).

unit sphere

The [unit sphere] is the sphere $x^2 + y^2 + z^2 = 1$. It is an example of a two-dimensional surface in three dimensional space.

unit tangent vector

The [unit tangent vector] to a parametrized curve r(t)=(x(t),y(t),z(t)) is the normalized velocity vector $T(t) = r'(t)/|r'(t)|$. Together with the normal vector $N(t) = T'(t)/|T'(t)|$ and the binormal vector $B(t) = T(t)xN(t)$, it forms a triple of mutually orthogonal vectors.

vector

A [vector] in the plane is defined by two points P, Q. It is the line segment v pointing from P to Q. If $P = (a, b)$ and $Q = (c, d)$ then the coordinates of the vector are $v = (c - a, d - b)$. Points P in the plane can be identified by vectors pointing from 0 to P. A vector in space is defined by two points P,Q in space. If $P = (a, b, c)$ and $Q = (d, e, f)$, then the coordinates of the vector are $v = (d - a, e - b, f - c)$. Points P in space can be identified by vectors pointing from 0 to P. Two vectors which can be translated into each other are considered equal. Remarks.

- One could define vectors more precisely as affine vectors and introduce an equivalence relation among them: two vectors are equivalent if they can be translated into each other. The equivalence classes are the vectors one deals with in calculus. Since the concept of equivalence relation would unnessesarily confuse students, the more fuzzy definition above is prefered.
- One should avoid definitions like "Vectors are objects which have length and direction" given in some Encyclopedias. The zero vector $(0, 0, 0)$ is an example of an object which has length but no direction. It nevertheless is a vector.

vector field

A [vector field] in the plane is a map $F(x, y) = (P(x, y), Q(x, y))$ which assigns to each point (x, y) in the plane a vector $F(x, y)$. An example of a vector field in the plane is $F(x, y) = (-y, x)$. An other example is the gradient field $F(x, y) = \nabla f(x, y)$ where $f(x, y)$ is a function. A vector field in space is a map $F(x, y, z) = (P(x, y, z), Q(x, y, z), R(x, y, z))$ which assigns to each point (x, y, z) in space a vector $F(x, y, z)$. An example is the vector field $F(x, y, z) = (x^2, yz, x - y)$. An other example is the gradient field $F(x, y, z) = \nabla f(x, y, z)$ of a function $f(x, y, z)$.

velocity

The [velocity] of a parametrized curve r(t)=(x(t),y(t),z(t)) at time t is the vector $r'(t) = (x'(t), y'(t), z'(t))$. It is tangent to the curve at the point $r(t)$.

volume

The [volume] of a body G is defined as the integral of the constant function f(x,y,z)=1 over the body G.

wave equation

The [wave equation] is the partial differential equation $u_{tt} = c^2\Delta(u)$, where $\Delta(u)$ is the Laplacian of u. Light in vacuum satisfies the wave equation. This can be derived from the Maxwell equations: the identity $\Delta(B) = \text{grad}(\text{div}(B) - \text{curl}(\text{curl}(B))$ gives together with $\text{div}(B) = 0$ and $\text{curl}(B) = E_t/c$ the relation $\Delta(B) = -d/dt\text{curl}(E)/c$ which leads with the Maxwell equation $B_t = -c\text{curl}(E)$ to the wave equation $\Delta B = B_{tt}/c^2$. The equation $E_{tt} = c^2\Delta E$ is derived in the same way.

zero vector

The [zero vector] is the vector for which all components are zero. In the plane it is $v = (0, 0)$, in space it is $v = (0, 0, 0)$. The zero vector is a vector. It has length 0 and no direction. Definitions like "a vector is a quantity which has both length and direction" are misleading.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 119 entries in this file.

Index

ENTRY LINEAR ALGEBRA

[ENTRY LINEAR ALGEBRA] Author: Oliver Knill: Spring 2002-Spring 2004 Literature: Standard glossary of multivariable calculus course as taught at the Harvard mathematics department.

adjacency matrix

The [adjacency matrix] of a graph is a matrix A_{ij}, where $A_{ij} = 1$ whenever there is an edge from node i to node j in the graph. Otherwise, $A_{ij} = 0$. Example: the graph with three nodes with the shape of a V has the adjacency matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, where node 2 is conneced to both node 1 and 3 and node 1 and 3 are not connected to each other.

affine transformation

An [affine transformation] is the composition of a linear transformation with a shift like for example: $T(x, y) = (2x + y, 3x + 4y) + (2, 3)$.

Algebra

[Algebra] was originally the art of solving equations and systems of equations. The word comves from the Arabic "al-jabr" meaning "restauration". The term was used by Mohammed al-Khowarizmi, who worked in Bhagdad.

algebraic multiplicity

The [algebraic multiplicity] of a root y of a polynomial p is the maximal integer k for which $p(x) = (x-y)^k q(x)$. The algebraic multiplicity is bigger or equal than the geometric multiplicity.

angle

The [angle] between two vectors v and w is $\arccos((x \cdot y)/(||x||||y||))$, where $x\dot{y}$ is the dot product between x and y and $||x|| = \sqrt{x \cdot x}$ is the length of x. The inverse of cos gives two angles in $[0, 2\pi]$. One usually choses the smaller angle.

argument

The [argument] of a complex number $z = x + iy$ is ϕ if $z = re^{i\phi}$. The argument is determined only up to addition of 2π. It can be determined as $\phi = \arctan(y/x) + A$, where $A = 0$ if $x > 0$ or $x = 0, y > 0$ and $A = \pi$ if $x < 0$ or $x = 0$ and $y < 0$. For example, $\arg(i) = \pi/2$ and $\arg(-i) = 3pi/2$. The argument is the imaginary part of $\log(z)$ because $\log(re^{iphi}) = \log(r) + i\phi$.

associative law

The [associative law] is $(AB)C = A(BC)$. It is an identity which some mathematical operations satisfy. For example, the matrix multiplication satisfies the associative law. One says also, that the operation is associative. An example of a product which is not associative is the cross product $v \times w$: if i, j, k are the standard basis vectors, then $i \times (i \times j) = i \times k = -j$ and $(i \times i) \times j = 0 \times j = 0$.

augmented matrix

The [augmented matrix] of a linear equation $Ax = b$ is the $n \times (n+1)$ matrix $\begin{pmatrix} A & b \end{pmatrix}$. One considers the augmented matrix when solving a linear system $Ax = b$. The reduced row echelon form rref $\begin{pmatrix} A & b \end{pmatrix}$ contains the solution vector x in the last column, if a solution exists. More generally, a matrix which contains a given matrix as a submatrix is called an augmented matrix.

basis

A [basis] of a linear space is a finite set of vectors $v_1, ..., v_n$, which are linearly independent and which span the linear space. If the basis contains n vectors, the vector space has dimension n.

basis theorem

The [basis theorem] states that d linearly independent vectors in a vector space of dimension d forms a basis.

block matrix

A [block matrix] is a matrix A, where the only non-zero elements are contained in a sequence of smaller square matrices arranged along the main diagonal of A. Such matrices are also called block diagonal matrices. The matrix $A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 6 & 7 \\ 0 & 0 & 0 & 8 & 9 \end{pmatrix}$. is an example of a block-diagonal matrix containing a two $2x2$, and a $1x1$ matrix in its diagonal.

Cauchy-Schwarz inequality

The [Cauchy-Schwarz inequality] tells that $|x \cdot y|$ is smaller or equal to $||x||\ ||y||$. Equality holds if and only if x and y are parallel vectors.

Cayley-Hamilton theorem

The [Cayley-Hamilton theorem] assures that every square matrix A satisfies $p(A) = 0$, where $p(x) = \det(A - x)$ is the characteristic polynomial of A and the right hand side 0 is the zero matrix.

change of basis

A [change of basis] from an old basis
v_j to a new basis
w_j is described by an invertible matrix S which relates the coordinates $(a_1, ..., a_n)$ of a vector $a = \sum_i a_i v_i$ in the old v-basis with the coordinates $(b_1, ..., b_n)$ of the same vector $b = \sum_i b_i w_i$ in the new w-basis. The relation of the coordinates is $b = Sa$ In that case, one has $v_j = \sum_j S^T_{ij} w_j$, where S^T is the transpose of S. For example if $v_1 = (1, 0), v_2 = (0, 1), w_1 = (3, 4) w_2 = (2, 3)$, then $a = (a_1, a_2) = (5, 7)$ in the v-basis has the coordinates $b = (b_1, b_2) = (1, 1)$ in the w-basis. With $S = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$ and $S^T = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ we have $b = Sa$ and $w_1 = 3v_1 + 4v_2, w_2 = 2v_1 + 3v_2$.

characteristic matrix

The [characteristic matrix] of a square matrix A is the matrix $A(x) = (xI - A)$, where I is the identity matrix. The characteristic matrix is a function of the free variable x.

characteristic polynomial

The [characteristic polynomial] of a matrix A is the polynomial $p(x) = \det(xI - A)$, where I is the identity matrix. It has the form $p(x) = x^n - \mathrm{tr}(A)x^(n-1) + ... + (-1)^n \det(A)$ where tr(A) is the trace of A and $\det(A)$ is the determinant of the matrix A. The eigenvalues of A are the roots of the characteristic polynomial of A.

Cholesky factoriztion

The [Cholesky factoriztion] of a symmetric and positive definite matrix A is $A = R^T R$, where R is upper triangular with positive diagonal entries.

circulant matrix

A [circulant matrix] is a square matrix, where the entries in each diagonal are constant. If S is the shift matrix which has 1 in the side diagonal and 0 everywhere else like in the 3x3 case: $S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then a circular matrix can be written as $A = a_0 + a_1 S + ... + a_{n-1} S^{n-1}$. A general $3x3$ circulant matrix has the form $A = a + bS + cS^2$ which is $S = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}$.

classical adjoint

The [classical adjoint] $\mathrm{adj}(A)$ of a $n \times n$ matrix A is the $n \times n$ matrix whose entry a_{ij} is $a_{ij} = (-1)^(i+j)\det(A_{ji})$, where A_{ji} is a minor of A. The classical adjoint plays a role in Cramer's rule $A^(- 1) = \mathrm{adj}(A)/\det(A)$. The name "adjoint" comes from the fact that we have a change indices like in the adjoint. However, the classical adjoint has nothing to do with the adjoint.

codomain

The [codomain] of a linear transformation $T : X \to Y$ is the target space Y. The name has its origin from naming X the domain of A.

cofactor

A [cofactor] C_{ij} of a $n \times n$ matrix A is the determinant of the $(n-1) \times (n-1)$ matrix obtained by removing row i and column j from A and multiplying the result with $(-1)^{i+j}$.

coefficient

A [coefficient] of a matrix A is an entry A_{ij} in the i'th row and the j'th column. For a real matrix, all entries are real numbers, for a complex matrix, the entries can be complex numbers.

column

A [column] of a matrix is one of the vectors $\begin{pmatrix} A_{11} \\ A_{21} \\ A_{31} \\ \dots \\ A_{m1} \end{pmatrix}, \begin{pmatrix} A_{1n} \\ A_{2n} \\ A_{3n} \\ \dots \\ A_{mn} \end{pmatrix}$ of a $m \times n$ matrix A. Column vectors are in the image of the transformation $x \mapsto A(x)$.

column

The matrix A defining a linear equation Ax=b or

$$\begin{aligned} A_{11}x_1 + \dots A_{1n}x_n &= b_1 \\ \dots &= \dots \\ A_{m1}x_1 + \dots A_{mn}x_n &= b_m \end{aligned}$$

is called the [coefficient matrix] of the system. The augmented matrix is the $m \times (n+1)$ matrix $\begin{pmatrix} A & b \end{pmatrix}$, where b forms an additional column.

column picture

The [column picture] of a linear equation $Ax = b$ is that the vector b becomes a linear combination of the columns of A. The linear equation is solvable if the vector b is in the column space of A.

column space

The [column space] of a matrix A is the linear space spanned by the columns of A.

commuting matrices

Two [commuting matrices] A, B satisfy $AB = BA$. In that case, if A is diagonalizable, then also B is diagonalizable and both A and B share the same n eigenvectors.

commutative law

The [commutative law] $A * B = B * A$ for some operation $*$ is an identity which holds for certain operations like the addition of matrices. Other operations like the multiplication of matrices does not satisfy the commutative law. One says: matrix multiplication is not commutative.

complex conjugate

The [complex conjugate] of a complex number $z = x + iy$ is the complex number $x - iy$. It has the same length $|z|$ as z.

Complex numbers

[Complex numbers] form an extension of the real numbers. They are obtained by introducing $i = (-1)^{1/2}$ and extending the rules of addition $(a + ib) + (c + id) = (a + c) + i(b + d)$ and multiplication $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$. The absolute value $r = |x + iy|$ is the length of the vector (a, b). The argument of z, $\phi = \arg(z)$ is defined as the angle in $[0, 2\pi)$ between the x axes and the vector (a, b). Using these polar coordinates one can see the Euler identity $z = r\exp(i\phi) = r\cos(\phi) + ir\sin(\phi)$.

consistent

A system of linear equations $Ax = b$ is called [consistent], if there exists for every vector b a solution vector x to the equation Ax=b. If the system has no solution, the system is called inconsistent.

continuous dynamical system

A [continuous dynamical system] is defined by an ordinary differential equation $d/dtu = f(u)$ where $u = u(t)$ is a vector valued function and $f(u)$ is a vector field. If $f(u)$ is linear, the equation has the form $d/dtu = Au$. The name "continuous" comes from the fact that the time variable t is taken in the continuum. This distinguishes the system from discrete dynamical systems $u(t+1) = f(u(t))$ determined by a map f and where t is an integer. For linear continuous dynamical systems, the origin 0 is invariant. The origin is called asymptotically stable if $x(t) \to 0$ for all initial conditions $x(0)$. For continuous dynamical systems $u_t = Au$, this is equivalent with the requirement that all eigenvalues of A have a negative real part. In two dimensions, where the trace and the determinant determine the eigenvalues, linear stability is characterized by $\det(A) > 0, \mathrm{tr}(A) < 0$ (stability quadrant).

covariance matrix

A [covariance matrix] A of two finite dimensional random variables x, y with expectation $E[x] = E[y] = 0$ is defined as $A_{ij} = E[x_i y_j]$, where $E[x] = (x_1 + ... + x_n)/n$ is the mean or expectation of x. The covariance matrix is always symmetric. If the covariance matrix is diagonal, the random variables x, y are called uncorrelated.

Cramer's rule

[Cramer's rule] tells that a solution x of a linear equation $Ax = b$ can be obtained as $x_i = \det(A_i)/\det(A)$, where A_i is the matrix obtained by replacing the column i of A with the vector b.

de Moivre formula

The [de Moivre formula] is $(\cos(x)+i\sin(x))^n = \cos(nx)+i\sin(nx)$. It is useful to derive trigonometric identities like $cos(x)^3 - 3\sin(x)^2\cos(x) = \cos(3x)$.

determinant

The [determinant] of a $n \times n$ square matrix A is the sum over all products $A[1, \pi(1)]...A[n, \pi(n)](-1)^\pi$, where π runs over all permutations of $\{1, 2, ..., n\}$ and $(-1)^\pi$ is the sign of the permutation π. Example: for a 2×2 matrix $A = \left(\begin{array}{cc} a & b \\ c & d \end{array}\right)$ the determinant is $\det(A) = ad - bc$. Example: For a $3x3$ matrix $A = \left(\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & i \end{array}\right)$, the determinant is $\det(A) = aei + bfg + cdh - ceg - bfg - cdh$. Properties of the determinant are $\det(AB) = \det(A)\det(B)$, $\det(A^T) = \det(A)$, $\det(A^{-1}) = 1/\det(A)$.

differential equation

A [differential equation] is an equation for a function f in one or several variables which involves derivatives with respect to these variables. An ordinary differential equation is a differential equation, where derivatives appear only with respect to one variable. By adding new variables if necessary (for example for t, or derivatives $u_t, u_t t$ etc, one can write an ordinary differential equation always in the form $x_t = f(x)$.

dilation

A [dilation] is a linear transformation $x \to bx$. Dilations scale each vector v by the factor b but leave the direction of v invariant.

dimension

The [dimension] of a vector space X is the number of basis vectors in a basis of X.

distributive law

The [distributive law] is $A * (B + C) = A * B + A * C$. The set of matrices with matrix multiplication * and addition + is an example where the distributive law applies.

dot product

The [dot product] $v \cdot w$ of two vectors v and w is the sum of the products $v_i w_i$ of their components v_i, w_i. For complex vectors, the dot product is defined as $\sum_i \overline{v}_i w_i$. Examples:

- $(3, 2, 1) \cdot (1, 2, -1) = 6$.
- if $v \cdot w = 0$, then the vectors are orthogonal.
- the length of the vector $|v|$ is the square root of $v \cdot v$.
- $v \cdot w = |v||w| \cos(\alpha)$, where α is the angle between v and w.
- if A, B are two $n \times n$ matrices, then $(AB)_{ij}$ is the dot product of the i'th row of A with the j'th column of B

echelon matrix

The [echelon matrix] of a matrix A is a matrix $\text{rref}(A)$, where the pivot in each row comes after the pivot in the previous row. The pivot is the first nonzero entry in each row. The echelon matrix is also called a matrix in reduced row echelon form.

eigenbasis

An [eigenbasis] to a matrix A is a basis which consists of eigenvectors of A.

eigenvalue

An [eigenvalue] λ of a matrix A is a number for which there exists a vector v such that $Av = \lambda v$. Example: $A = \begin{pmatrix} 3 & 2 \\ 0 & 4 \end{pmatrix}$ has the eigenvector $v = (0, 1)$ with eigenvalue $x = 4$.

eigenvector

An [eigenvector] v of a matrix A is a nonzero vector v for which $Av = \lambda v$ with some number λ (called eigenvalue). Example: $\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ has the eigenvector $v = (1, 1)$ with eigenvalue $\lambda = 0$.

Elimination

[Elimination] is a process which reduces a matrix A to its echelon matrix rref(A). See row reduced echelon form.

ellipsoid

An [ellipsoid] can be written as the set of points x which satisfy $x\dot{A}x = 1$, where A is a positive definit matrix. The axes v_i of the ellipse are the eigenvectors of A have the length $1/x_i$, where x_i are the eigenvalues of A.

entry

An [entry] or coefficient of a matrix is the number or the variable $A(i,j)$ of a matrix.

expansion factor

The [expansion factor] of a linear map is the absolute value of the determinant of A. It is the volume of the parallelepiped obtained as the image of the unit cube under A.

exponential

The [exponential] $\exp(A)$ of a matrix A is defined as the sum $\exp(A) = 1 + A + A^2/2! + A^3/3! + ...$ The linear system of differential equations $x' = Ax$ for $x(t)$ has the solution $x(t) = \exp(At)x(0)$.

factorization

The [factorization] of a polynomial $p(x)$ is the representation $p(x) = a(\lambda_1 - x)...(\lambda_n - x)$, where $lambda_i$ are the n roots of the polynomials whose existence is assured by the fundamental theorem of algebra.

Fourier coefficients

The [Fourier coefficients] of a 2π periodic function $f(x)$ on $[-\pi,\pi]$ is $c_n = (1/2\pi)\int_{-pi}^{p} if(x)exp(-inx)$. One has $f(x) = \sum_n c_n \exp(inx)$. By writing $f(x) = g(x) + h(x)$, where $g(x) = [f(x) + f(-x)]/2$ is even and $h(x) = [f(x) - f(-x)]/2$ is odd one can obtain real versions: the even function can be written as a cos-series $g(x) = sum_{n=0}^{\infty} a_n \cos(nx)$, where $a_n = (2/\pi)\int_0^{\pi} g(x)\cos(nx)dx$ for $n > 0$ and $a_0 = (1/\pi)\int_0^{\pi} g(x)\, dx$. The odd function can be written as the sin-series $h(x) = sum_{n=1}^{\infty} b_n \sin(nx)$, where $b_n = (2/\pi)\int_0^{\pi} h(x)\sin(nx)dx$. The complex Fourier coefficients c_n are coordinates of $f(x)$ with respect to the orthonormal basis $\exp(inx)$. The real Fourier coefficients are the coordinates of $f(x)$ with respect to orthogonal basis $1, \cos(nx), \sin(nx), n > 0$. The [Fourier series] of a function f is $f(x) = \sum_{n=-\infty}^{\infty} c_n \exp(inx)$ or $f(x) = \sum_n a_n \cos(nx)$ for even functions or $f(x) = sum_{n=1}^{\infty} b_n \sin(nx)$ for odd functions.

fundamental theorem of algebra

The [fundamental theorem of algebra] states that a polynomial $p(x) = x^n + ...a_1 x + a_0$ of degree n has exactly n roots.

Gauss Jordan elimination

The [Gauss Jordan elimination] is a method for solving linear equations. It was already known by the Chinese 2000 years ago. Gauss called it "eliminatio vulgaris". The method does linear combinations of the rows of a $n \times (n+1)$ matrix until the system is solved. Example:

$$\left| \begin{array}{ccc} 2\text{ x} + 4\text{y} & = & 2 \\ 3\text{ x} + \text{y} & = & 12 \end{array} \right|$$

$$\left| \begin{array}{ccc} \text{x} + 2\text{y} & = & 1 \\ 3\text{ x} + \text{y} & = & 13 \end{array} \right|$$

$$\left| \begin{array}{ccc} \text{x} + 2\text{y} & = & 1 \\ \text{- 5y} & = & 10 \end{array} \right|$$

$$\left| \begin{array}{ccc} \text{x} + 2\text{y} & = & 1 \\ \text{y} & = & \text{-2} \end{array} \right|$$

$$\left| \begin{array}{ccc} \text{x} & = & 5 \\ \text{y} & = & \text{-2} \end{array} \right|$$

First, the top equation was scaled, then three times the first equation was subtracted from the second equation. Then the the second equation was scaled. Finally, twice the the second equation was subtracted from the first.

geometric multiplicity

The [geometric multiplicity] of an eigenvalue λ is the dimension of $\ker(\lambda - A)$. The geometric multiplicity is smaller or equal to the algebraic multiplicity.

Gibbs phenomenon

The [Gibbs phenomenon] describes the error when doing a Fourier approximation of the discontinuous Heavyside function $f(x) = \left\{ \begin{array}{c} -1 \text{if} -\pi \leq x \leq 0 \\ 1 \text{if} 0 < x < \pi \end{array} \right.$ The Fourier approximation is $s_n(x) = (4/\pi) \sum_{k=1}^n \sin((2k-1)t)/(2k-1)$. The derivative $d/dx s_n$ can be computed: (differentiate and sum up the geometric series): $(2/\pi)\sin(2nx)/\sin(x)$ which vanishes at $x = \pm\pi/2n$. These are the extrema for s_n. Now, $s_n(\pi/2n) = (4/\pi)\sum \sin((2k-1)(\pi/2n))/((2k-1)(\pi/2n))$ is a Riemann sum approximation for $(2/\pi)\int_0^\pi \sin(t)/t dt = \pi(1-\pi^2/(3!3)+\pi^4/(5!5)-...) = 1.1793...$ This overshoot is called the Gibbs phenomenon. It was first discoverd by Wilbraham in 1848 then by Gibbs in 1899. The human eye can recognize the Gibbs phenomenon as "ghosts" on a TV screen, unless it is corrected for.

Gramm-Schmidt process

The [Gramm-Schmidt process] is an algorithm which constructs from a basis $v_1, ..., v_n$ an orthonormal basis $w_1, ..., w_n$. The procedure goes by induction: if $w_1, ..., w_{k-1}$ are orthonormal, then the next vector w_k is $w_k = u_k/||u_k||$, where $u_k = v_k - (w_1 \cdot v_k)w_1 - (w_2 \cdot v_k)w_2 \cdots - (w_{k-1} \cdot v + k)w_{k-1}$.

graph

A [graph] is a set of n nodes connected by m edges. It is completely defined by its adjacency matrix. In a directed graph, the nodes are oriented. Examples:

- a complete graph has all nodes connected. There are $n(n-1)/2$ edges. Its adjacency matrix is $E - I$, where E is the $n \times n$ matrix with all entries equal to 1 and I is the identity matrix.
- a tree has $m = n - 1$ edges and no closed loops. An example is the graph with the adjecency matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.
- The directed graph with two edges $1 \to 2$ has the adjacency matrix $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

Hankel matrix

A [Hankel matrix] is a square matrix A, where the entries are constant along the antidiagonal. In other words, each entry A_{ij} depends only on $i + j$. A general 3×3 Hankel matrix is of the form $A = \begin{pmatrix} a & b & c \\ b & c & d \\ c & d & e \end{pmatrix}$.

heat equation

The [heat equation] is the linear partial differential equation $u_t = \mu u_{xx}$. The heat equation on a finite interval $[0, \pi]$ with boundary conditions $u(0, t) = 0$, $u(\pi, t) = 0$ and initial conditions $u(x, 0) = f(x)$ can be solved with the Fourier series $u(x, t) = \sum_{n>0} a_n \sin(nx) e^{-\mu n^2 t}$, where $a_n = (2/\pi) \int_0^p i f(x) \sin(nx)\, dx$ are the Fourier coefficients.

Hermitian matrix

A [Hermitian matrix] satisfies $A^* = \overline{A}^T = A$, where A^T is the transpose of A and $\overline{A}$ is the complex conjugate matrix, where all entries are replaced by their complex conjugates.

Hessenberg matrix

A [Hessenberg matrix] A is an upper triangular matrix with only one extra nonzero adjacent diagonal below the diagonal. Example: a general 3×3 Hessenberg matrix is $A = \begin{pmatrix} a & b & c \\ d & e & f \\ 0 & g & h \end{pmatrix}$.

Hilbert matrix

A [Hilbert matrix] is a symmetric square matrix, where $A_{ij} = 1/(i+j-1)$. It is an example of a Hankel matrix and positive definit. Hilbert matrices are examples of matrices which are difficult to invert, because their determinant is small. For example, for $n = 3$, the Hilbert matrix $A = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}$ has determinant 1/2160. A [hyperplane] in n-dimensional space V is a $(n-1)$-dimensional linear subspace of V.

identity matrix

The [identity matrix] is the matrix I which has 1 in the diagonal and zero everywhere else. The identity matrix I satisfies $IA = A$ for any matrix A. The $3x3$ identity matrix is $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

incidence matrix

The [incidence matrix] of a directed graph with n nodes and m edges is a $m \times n$ matrix which has a row for each edge connecting nodes i and j with entries -1 and 1 in columns i, j. Example. The directed graph $1 \Rightarrow 2 \Leftarrow 3$, $1 \rightarrow 4$ with 3 edges and 4 nodes has the 3×4 incidence matrix $A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$.

inconsistent

A system of linear equations $Ax = b$ is called [inconsistent] if the system has no solutions.

indefinite matrix

An [indefinite matrix] is a matrix, whith eigenvalues of different sign. A positive definite matrix is an example of a matrix which is not indefinite.

Independent vectors

[Independent vectors]. If no linear combination $a_1 v_1 + ... + a_n v_n$ is zero unless all a_i are zero, then the vectors $v_1, ..., v_n$ are called independent. If A is the matrix which contains the vectors v_i as columns, then the kernel of A is trivial. A basis consists of independent vectors.

Independent vectors

The linear transformation corresponding to the identity matrix is called the [identity transformation].

image

The [image] of a linear transformation $T : X \to Y, T(x) = Ax$ is the subset of all vectors $y = Ax, xinX$ in Y. The image is denoted by $\text{im}(T)$ or $\text{im}(A)$ and is a subset of the codomain Y of T. The image is also called the range. The dimension of the image of T is equal to the rank of A and the dimension satisfies $\dim(\ker(A)) + \dim(\text{im}(A)) = n$, where n is the dimension of the linear space X.

index

The [index] of a linear map T is defined as $\text{ind}(A) = \dim(\ker A) - \dim(\text{coker} A)$, where $\text{coker}(A)$ is the orthogonal complement of the image of A. Examples are:

- The index of a $n \times n$ matrix A is $\dim(\ker A) - \dim(\text{coker} A) = 0$.
- The index of the differential operator $Df = f'$ acting on smooth functions on the real line is $1 - 0 = 1$ because D has a one dimensional kernel (the constant functions) and a zero dimensional cokernel (all functions can be obtained as the image of D). The index of D^n is n.
- The index of the differential operator $Df = f'$ acting on smooth functions on the circle is $1 - 1 = 0$ because D has a one dimensional kernel (the constant functions) and a one-dimensional cokernel (the constant functions, one can not find a periodic function g such that g'=1).
- The Atiyah-Singer index theorem relates topological properties of a surface M with the index of a "Dirac operator" T on it. The previous two examples exemply that. $T = D$ is a Dirac operator and the topology of the circle or the line are different.

inverse

The [inverse] of a square matrix A is a matrix B satisfying $AB = I$ and $BA = I$ where I is the identity matrix. For example, the inverse of the transformation $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the transformation $B = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} /(ad - bc)$.

invertible

A square matrix A is [invertible] if there exists a matrix B such that $AB = I$. A matrix A is invertible if and only if the determinant of A is different from zero.

Jordan normal form

The [Jordan normal form] $J = S^{-1}AS$ of a square matrix A is a block matrix $J = \text{diag}(J_1, ..., J_k)$, where each block is of the form $J_k = x_k I_k + N_k$, where x_k is an eigenvalue of A, I_k is an identity matrix and N_k is a matrix with 1 in the first sidediagonal. If all eigenvalues of A are different, then the Jordan normal form is a diagonal matrix.

kernel

The [kernel] of a linear transformation $T : X \to Y$, $T(x) = Ax$ is the linear space $\{x in X such that Ax = 0\}$. The kernel is denoted by $\ker(T)$ or ker(A) and is a subset of the domain X of T. The dimension of the kernel $\dim(\ker(A))$ and the dimension of the image $\dim(\mathrm{im}(A))$ are related by $\dim(\ker(A)) + \dim(\mathrm{im}(A)) = n$, where n is the dimension of the linear space X. The kernel of a transformation is computed by building $\mathrm{rref}(A)$, the reduced row echelon form of A. Echelon = "series of steps". Every vector in the kernel of $\mathrm{rref}(A)$ is also in the kernel of A.

Leontief

Wassily [Leontief]. A Russian-born US economist who was working also at Harvard University. Leontief was a winner of the 1973 Economics Nobel prize for the development of the input-output method and for its application to important economic problems. Linear algebra students find the following problem in textbooks: two industries A and B produce output with value x and y (in millions of dollars). Assume that the consumer demand is a for the product of A and b for the product of B. Assume also an industry demand: p x is transfered from A to B, and q y is transfered from B to A. For which x and y are both the industry and conumer demand satisfied? The problem is equivalent to solving the linear system

$$\begin{aligned} x - qy &= a \\ -px + y &= b \end{aligned}$$

Laplace equation

The [Laplace equation] in a region G is the linear partial differential equation $u_{xx} + u_{yy} = 0$. A solution is determined if $u(x, y)$ is prescribed on the boundary of G. On the square $[0, \pi] \times [0, \pi]$ with boundary conditions 0 except at the side $y = \pi$, where one has $u(x, \pi) = f(x)$, one can find a solution via Fourier series: $u(x, y) = \sum_{n>0} a_n \sin(nx) \sinh(ny)/sinh(n\pi)$, where $a_n = (2/\pi) \int_0^\pi f(x) \sin(nx)\ dx$. The case with general boundary conditions can be solved by adding corresponding solutions $u(x, y), u(y, x), u(x, \pi - y), u(\pi - y, x)$ for the other 3 sides of the square.

Laplace expansion

The [Laplace expansion] is a formula for the determinant of A: $\det(A) = (-1)^{i+1} a_{i1} \det(A_{i1}) + ... + (-1)^{i+n} a_{in} \det(A_{in})$.

leading one

A [leading one] is an entry of a matrix in reduced row echelon form which is contained in a row with this element as the first nonzero entry.

leading variable

A [leading variable] is a variable which corresponds to a leading one in rref(A).

least-squares solution

A vector $x \in R^n$ is called a [least-squares solution] of the system $Ax = b$ where A is a $m \times n$ matrix, if $||b - Ay||$ is less or equal then $||b - Ax||$ for all $y \in R^n$. If x is the least-squares solution of $Ax = b$ then Ax is the orthogonal projection of b onto the image im(A). The explicit formula is $x = (A^T A)^(-1) A^T b$ and derived from that $A^T(Ax - b) = 0$ which itself just means that $Ax - b$ is orthogonal to the image of A.

length

The [length] of a vector v is $||v|| = (v \cdot v)^{1/2} = (v_1^2 + ... + v_n^2)^(1/2)$. The length of a complex number $x + iy$ is the length of (x, y). The length of a vector depends on the basis, usually it is understood with respect to the standard basis.

linear combination

A [linear combination] of n vectors $v_1, ..., v_n$ is a vector $a_1 v_1 + ... + a_n v_n$.

Linearly dependent vectors

[Linearly dependent vectors]. If there exist $a_1, ..., a_n$ which are not all zero such that $a_1 v_1 + ... + a_n v_n = 0$, then the vectors $v_1, ..., v_n$ are called linearly dependent.

Linearity

[Linearity] is a property of maps between linear spaces: it means that lines are maped into lines and the image of the sum of two vectors is the same as sum of the images. For example: $T(x, y, z) = (2x + z, y - x)$ is linear. $T(x, y) = (x^2 - y, x)$ is nonlinear.

linear dynamical system

A [linear dynamical system] is defined by a linear map $x \mapsto Ax$. The orbits of the dynamical system are $x, Ax, A^2x, ...$

linear space

A [linear space] is the same as a vector space. It is a set which is closed under addition and multiplication with real numbers.

linear combination

A sum $a_1 v_1 + ... + a_n v_n$ is called a [linear combination] of the vectors $v_1, ..., v_n$.

linear subspace

A [linear subspace] of a vector space V is a subset of V which is also a vector space. In particular, it is closed under addition, scalar multiplication and contains a neutral element.

linear system of equations

A [linear system of equations] is an equation of the form Ax=b, where A is a $m \times n$ matrix, x is a n-vector and b is a m-vector. There are three possibilities:

- consistent with one solution: no row vector (0...0‖1) in rref$(A|b)$. There is exactly one solution if there is a leading one in each column of rref(A).
- consistent with infinitely many solutions: there are columns with no leading one.
- Inconsistent with no solutions: there is a row (0...0‖1) in rref$(A\|b)$.

logarithm

The [logarithm] $\log(z)$ of a complex number $z = x + iy \neq 0$ is defined as $\log|z| + i\arg(z)$, where $\arg(z)$ is the argument of z. The imaginary part of the logarithm is only defined up to a multiple of 2π.

Markov matrix

A [Markov matrix] is a square matrix, where all entries are nonnegative and the sum of each column is 1. One of the eigenvalues of a Markov matrix is 1 because A^T has the eigenvector $(1, 1, ..., 1)$. If all entries of a Markov matrix are positive, then $A^k v$ converges to the eigenvector v with eigenvalue 1. This vector is called the "steady state" vector.

matrix

A [matrix] is a rectangular array of numbers. The following 3×4 matrix for example consists of three rows and four columns: $A = \begin{pmatrix} 2 & 8 & 4 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & -1 & 1 & 2 \end{pmatrix}$. The first index addresses the row, the second the column of the matrix. A $n \times m$ matrix maps the m-dimensional space to the n-dimensional space.

Matrix multiplication

[Matrix multiplication] is an operation defined between a $(n \times m)$ matrix A and a $(m \times p)$ matrix B. $(AB)_{ij}$ is the dot product between the i'th row of A with the j'th column of B. Example: $(n = 2, m = 3, p = 4)$ $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 3 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 1 & 2 \\ 3 & 4 & 1 & 1 \end{pmatrix}$.

minor

A [minor] of a matrix A is a matrix $A(i,j)$ which is obtained from A by deleting row i and column j.

nilpotent matrix

A [nilpotent matrix] is a matrix A for which some power A^k is the zero matrix. A nilpotent matrix has only zero eigenvalues. he matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ for example satisfies $A^3 = 0$ and is therefore nilpotent.

non-leading coefficient

A [non-leading coefficient] is an entry in the row reduced echelon form of a matrix A which is nonzero and which comes after the leading 1. The relevance of this definition comes from the fact that the number of columns with non-leading coefficients is the dimension of the kernel of the map.

normal equation

The [normal equation] to the linear equation Ax=b is the consistent system $A^T Ax = A^T b$.

normal matrix

A matrix A is called a [normal matrix], if $AA^T = A^T A$. A normal matrix has orthonormal possibly complex eigenvectors.

null space

The [null space] of a matrix A is the same as the kernal of A. It is spanned by the solutions $Av = 0$. The dimension of the null space is $n - r$, where n is the number of columns of A and r is the rank of A.

ordinary differential equation

An [ordinary differential equation] is a differential equation, where derivatives appear only with respect to one variable. By adding new variables if necessary (for example for t, or derivatives u_t, u_{tt} etc. one can write such an equation always in the form $x_t = f(x)$. An ordinary differential equation defines a continuous dynamical system. The initial condition $x(0)$ determines the trajectories $x(t)$. An ordinary differential equation of the form $u_t = Au$, where A is a matrix is called a linear ordinary differential equation.

orthogonal

Two vectors v,w are [orthogonal] if their dot product $v \cdot w$ vanishes.

orthogonal basis

An [orthogonal basis] is a basis such that all vectors in the basis are orthogonal.

orthonormal basis

An [orthonormal basis] is a basis such that all vectors are orthogonal and normed.

orthonormal complement

The [orthonormal complement] of a linear subspace V in R^n is the set of vectors which are orthogonal to V.

orthogonal projection

The [orthogonal projection] onto a linear space V is $\text{proj}_V(x) = (x \cdot v_1)v_1 + ... + (x \cdot v_n)v_n$, where the v_j form an orthonormal basis in V. Despite the name, an orthogonal projection is not an orthogonal transformation. It has a kernel. In an eigenbasis, a projection has the form $(x, y) \to (x, 0)$.

Euclidean space

[Euclidean space] is the linear space of all vectors $= 1 \times n$ matrices. R^0 is the space 0. The space R^1 is the real linear space of all real numbers, the R^2 is the plane, the R^3 the Euclidean three dimensional space.

parallel

Two vectors v and w are called [parallel] if v both are nonzero and one is a multiple of the other.

parallelepiped

A set in R^n is a [parallelepiped] E if it is the linear image $A(Q)$ of the unit cube Q. The volume of a n-dimensional parallelepiped E in R^n satisfies $\text{vol}(E) = |\det(A)|$, in general, $\text{vol}(E) = (\det(A^T A))^{1/2}$.

partial differential equation

A [partial differential equation] (PDE) is an equation for a multi-variable function which involves partial derivatives. It is called linear if $(u + v)$ and rv are solutions whenever u and v are solutions. Examples of linear PDEs:

$u_t = cu_x$	transport equation
$u_t = bu_{xx}$	heat equation
$u_t t = au_{xx}$	wave equation
$u_x x + u_y y = 0$	Laplace equation
$u_x x + u_y y = f(x, y)$	Poisson equation
$ihu_t = -u_x x + V(x)$	Schroedinger equation

Examples of nonlinear PDE:

$u_t + uu_x = au_{xx}$	Burger equation
$u_t + uu_x = -u_{xxx}$	Korteweg de Vries equation
$u_{tt} - u_{xx} = \sin(x)$	Sine Gordon equation
$u_{tt} - u_{xx} = f(x)$	Nonlinear wave equation
$ihu_t = -u_x x - \|x\|^2 x$	Nonlinear Schroedinger equation
$u_t + u_x(x,t)^2/2 + V(x) = 0$	Hamilton Jacobi equation

permutation matrix

A [permutation matrix] A is a square matrix with entries $A_{ij} = I_{i\pi(j)}$ where π is a permutation of $1, ..., n$ and where I is the identiy matrix. There are $n!$ permutation matrices. Example: for $n = 3$ the permutation $\pi(1,2,3) = (2,1,3)$ defines the permutation matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

pivot

A [pivot] d is the first nonzero diagonal entry when a row is used in Gaussian elimination.

pivot column

A column of a matrix is called a [pivot column] if the corresponding column of $\text{rref}(A)$ contains a leading one. The pivot columns are important because they form a basis for the image of A.

polar decomposition

The [polar decomposition] of a matrix A is $A = OB$, where O is orthogonal and where B is positive semidefinit.

positive definite matrix

A [positive definite matrix] is a symmetric matrix which satisfies $v \cdot Av > 0$, for every nonzero vector v.

power

The n'th [power] of a matrix A is defined as $A^n = AA^(n-1) = AA....A$. The eigenvalues of A^n are λ_i^n, where λ_i are the eigenvalues of A.

QR decomposition

The [QR decomposition] of a matrix A is obtained during the Gramm-Schmitt orthogonalization process. It is $A = QR$, where Q is an orthogonal matrix and where R is an upper triangular matrix.

rank

The [rank] of a linear matrix A is the set of leading 1's in the matrix rref(A).

orientation

The [orientation] of n vectors $v_1, ..., v_n$ in the n-dimensional Euclidean space is defined as the sign of det(A), where A is the matrix with v_i in the columns.

orthogonal

A square matrix A is [orthogonal] if it preserves length: $||Av|| = ||v||$ for all vectors v.

perpendicular

Two vectors v and w are called [perpendicular] if their dot product vanishes: $v \cdot w = 0$. A synonym of perpendicular is orthogonal.

projection matrix

A [projection matrix] is a matrix P which satisfies $P^2 = P = P^T$. It has eigenvalues 1 or 0. The image is a linear subspace S. The vectors in S are eigenvectors to the eigenvalues 1. The vectors in the orthogonal complement of S are eigenvectors to the eigenvalue 0. If A is the matrix which contains the basis of S as the columns, then $P = A(A^TA)^{-1}A^T$ is the projection onto S.

pseudoinverse

The [pseudoinverse] of a $(m \times n)$ matrix A is the $(n \times m)$ matrix A^+ that maps the image of A to the image of A^T. The kernel of A^+ is the kernel of A^T and the rank of A^+ is equal to the rank of A. A^+A is the projection on the image of A^T and AA^+ is the projection on the image of A. Especially, if A is an invertible $(n \times n)$ matrix, then A^+ is the inverse of A. The pseudoinverse is also called Moore-Penrose inverse.

rank

The [rank] of a matrix A is the dimension of the image of A.

Rayleigh quotient

The [Rayleigh quotient] of a symmetric matrix A is defined as the function $q(v) = (v \cdot Av)/(v \cdot v)$. The maximal value of $q(v)$ is the maximal eigenvalue of A and the minimal value of $q(v)$ is the minimal eigenvalue of A.

rotation

A [rotation] is a linear transformation which preserves the angle between two vectors as well as their lengths. A rotation in three dimensional space is determined by the axis of rotation as well as the rotation angle.

rotation matrix

A [rotation matrix] is the matrix belonging to a rotation. A rotation matrix is an example of an orthogonal matrix. Example: In two dimensions, a rotation matrix has the form $A = \begin{pmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{pmatrix}$.

rotation-dilation matrix

A [rotation-dilation matrix] is a 2x2 matrix of the form $A = \begin{pmatrix} p & -q \\ q & p \end{pmatrix}$. It has the eigenvalues $p \pm iq$. The action of A represents the complex multiplication with the complex number p+iq in the complex plane.

row

A [row] of a matrix is formed by horizontal lines $A_{1j}, j = 1, ..n$ of a $m \times n$ matrix A.

shear

A [shear] is a linear transformation in the plane which has in a suitable basis the form $T(x, y) = (x, y + ax)$. More generally, in n dimensions, one can define as shear along a m-dimensional plane. If a basis is chosen so that the plane has the from $(x, 0)$ then a shear is $T(x, y) = (x, y + ax)$. Shears have determinant 1 and preserve therefore volume.

singular

A square matrix A is called [singular] if it has no inverse. A matrix A is singular if and only if $\det(A) = 0$.

singular value decomposition

The [singular value decomposition] (SVD) of a matrix writes a matrix A in the form $A = UDV^T$, where U, V are orthogonal and D is diagonal. The first r columns of U form an orthonormal basis of the image of A and the first r columns of V form an orthonormal basis of the image of A^T. The last columns of U form an orthonormal basis of the kernel of A^T and the last columns of V form a basis of the kernel of A.

skew symmetric

A matrix A is [skew symmetric] if it is minus its transpose that is if $A^T = -A$. The eigenvalues of a skew-symmetric matrix are purely imaginary. The eigenvectors are orthogonal. If A is skew symmetric, then $B = \exp(At)$ is an orthogonal matrix, because $B^T B = \exp(-At)exp(At) = 1$. For example $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ gives the rotation matrix $exp(At)$.

span

The [span] of a set of vectors $v_1, ...v_n$ is the set of all linear combinations of $v_1, ..., v_n$.

spectral theorem

The [spectral theorem] tells that a real symmetric matrix A can be diagonalized $A = UDU^T$, where U is an orthonormal matrix containing an orthonormal eigenbasis in the columns and where D is a diagonal matrix $D = \text{diag}(x_1, ..., x_n)$, where x_i are the eigenvalues of A.

square matrix

A [square matrix] is a matrix which has the same number of rows than columns.

asymptotically stable

A linear dynamical system is called [asymptotically stable] if $A^n x \to 0$ for all initial values x, where A^n is the n'th power of the matrix A. This is equivalent to the fact that all eigenvalues λ of A satisfy $|\lambda| < 1$.

Stability triangle

[Stability triangle]. A discrete dynamical system in the plane is asymptotically stable if and only if the trace and determinant are in the stability triangle $|\text{tr}(A)| - 1 < \det(A) < 1$. A rotation-dilation A is asymptotically stable if and only if $\det(A) < 1$.

reduced row echelon form

The [reduced row echelon form] rref(A) of a m x n matrix A is the end product of Gauss-Jordan elimination. The matrix rref(A) has the following properties:

- if a row has nonzero entries, then the first nonzero entry is 1, called leading 1.
- if a columns contains a leading 1, then all other entries in that column are 0.
- if a row contains a leading 1, then every row above contains a leading 1 further left.

The algorithm to produce rref(A) from A is obtained by putting the cursor to the upper left corner and repeating the following steps until nothing changes anymore

1. if the cursor entry is zero swap the cursor row with the first row below that has a nonzero entry in that column
2. divide the cursor row by the cursor entry to make the cursor entry = 1
3. eliminate all other entries in cursor column by subtracting suitable multiples of the cursor row from the other row
4. move the cursor down one row and and to the right one column. If the cursor entry is zero and all entries below are zero, move the cursor to the next column.
5. repeat 4 if as long as necessary and move then to 1

reflection

A [reflection] is a linear transformation T different from the identity transformation which satisfies $T^2 = 1$. The eigenvalues of T are -1 or 1. In an eigenbasis, the reflection has the form $T(x, y) = (x, -y)$. The determinant of a reflection is 1 if and only if the dimension of the eigenspace to -1 is even. For example, a reflection at a line in the plane has the matrix $A = \begin{pmatrix} \cos(2x) & sin(2x) \\ \sin(2x) & -cos(2x) \end{pmatrix}$ which has determinant -1. A reflection at the origin in the plane is $-I$ with determinant 1.

root

A [root] of a polynomial $p(x)$ is a complex value z such that $p(z) = 0$. According to the fundamental theorem of algebra, a polynomial of degree n has exactly n roots.

symmetric

A matrix A is [symmetric] if it is equal to its transpose. The spectral theorem for symmetric matrices tells that they have real eigenvalues and symmetric matrices can always be diagonalized with an orthogonal matrix S.

span

The [span] of a finite set of vectors $v_1, ..., v_n$ is the set of all possible linear combinations $c_1 v_1 + c_2 v_2 + ... + c_n v_n$ where c_i are real numbers. For example, if $v_1 = (1, 0, 0)$ and $v_2 = (0, 1, 0)$, then the span of v_1, v_2 in three dimensional space is the xy-plane. The span is a linear space.

spectral theorem

The [spectral theorem] for a symmetric matrix A assures that A can be diagonalized: there exists an orthogonal matrix S such that $A^(-1)AS$ is diagonal and contains the eigenvalues of A in the diagonal.

standard basis

The [standard basis] of the n-dimensional Euclidean space consists of the columns of the identity matrix I.

symmetric

A matrix A is called [symmetric] if $A^T = A$. A symmetric matrix has to be a square matrix. Real symmetric matrices can be diagonalized.

Toepitz matrix

A [Toepitz matrix] is a square matrix A, where the entries are constant along the diagonal. In other ways A_{ij} depends only on $j - i$. Example: a 3×3 Toeplitz matrix is of the form $A = \begin{pmatrix} c & d & e \\ b & c & d \\ a & b & c \end{pmatrix}$.

trace

The [trace] of a matrix A is the sum of the diagonal entries of A. The trace is independent of the basis and is equal to the sum of the eigenvalues of A.

transpose

The [transpose] A^T of a matrix A is the matrix with entries A_{ij} if A has the entries A_{ji}. The rank of A^T is equal to the rank of A. For square matrices, the eigenvalues of A^T and A agree because A and A^T have the same eigenvalues. Transposition satisfies $(A^T)^T = A$, $(AB)^T = B^T A^T$ and $(A^(-1))^T = (A^T)^(-1)$.

triangle inequality

The [triangle inequality] tells that in a linear space, $||v + w|| \leq ||v|| + ||w||$. One has equality if and only if the vectors v and w are orthogonal.

eigenvalues of a two times two matrix

The [eigenvalues of a two times two matrix] $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are $\lambda_1 = \mathrm{tr}(A)/2 + ((\mathrm{tr}(A)/2)^2 - \det(A))^{1/2}$ and $\lambda_2 = \mathrm{tr}(A)/2 - ((\mathrm{tr}(A)/2)^2 - \det(A))^{1/2}$. The eigenvectors are $v_i = \begin{pmatrix} \lambda_i - d \\ c \end{pmatrix}$ if $c \neq 0$ or $v_i = \begin{pmatrix} b \\ \lambda_i - a \end{pmatrix}$ if $b \neq 0$. (If $b = 0, c = 0$, then the standard vectors are eigenvector.)

unit vector

A [unit vector] is a vector of length 1. A given nonzero vector can be made a unit vector by scaling: $v/||v||$ is a unit vector.

Vandermonde Matrix

A [Vandermonde Matrix] is a square matrix with entries $A_{ij} = x_i^{j-1}$, where $x_1, ..., x_n$ are some real numbers. The determinant of a Vandermonde Matrix is $\prod_{j>i}(x_j - x_i)$. Example: $x_1 = 2, x_2 = 3, x_3 = -1$ defines the Vandermonde Matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & -1 & 1 \end{pmatrix}$ which has the determinant $\det(A) = (3-2)(-1-2)(-1-3) = 12$.

vector

A [vector] is a matrix with one column. The entries of the vector are called coefficients.

vector space

A [vector space] X is a set equipped with addition and scalar multiplication. A vector space is also called a linear space. The addition operation is a group:

$f + g = g + f$	Commutativity
$(f + g) + h = f + (g + h)$	Associativity
$f + 0 = 0$	Existence of a neutral element
$f + x = 0$	Existence of a unique inverse

The scalar multiplication satisfies:

$r(f + g) = rf + rg$ Distributivity
$(r + s)f = rf + sf$ Distributivity
$r(sf) = (rs)f$ Associativity
$1f = f$ One element

wave equation

The [wave equation] is the linear partial differential equation $u_{tt} = c^2 u_{xx}$ where c is a constant. The wave equation on a finite interval $0 < x < 1$ with boundary conditions $u(0,t) = 0$, $u(\pi,t) = 0$ and initial conditions $u(x,0) = f(x), u_t(x,0) = g(x)$ can be solved with the Fourier series: $u(x,t) = \sum_n > 0 a_n \sin(nx)\cos(nct) + b_n \sin(nx)\sin(nct)$ where $a_n = (2/\pi)\int_0^\pi f(x)\sin(nx)\ dx$, and $b_n = (2/\pi) int_0^\pi g(x)\sin(nx)\ dx/(cn)$ are Fourier coefficients.

zero matrix

Index

MATHEMATICIANS

[MATHEMATICIANS] Authors: Oliver Knill: 2000 Literature: Started from a list of names with birthdates grabbed from mactutor in 2000.

Abbe

[Abbe] Abbe Ernst (1840-1909)

Abel

[Abel] Abel Niels Henrik (1802-1829) Norwegian mathematician. Significant contributions to algebra and analysis, in particular the study of groups and series. Famous for proving the insolubility of the quintic equation at the age of 19.

AbrahamMax

[AbrahamMax] Abraham Max (1875-1922)

Ackermann

[Ackermann] Ackermann Wilhelm (1896-1962)

AdamsFrank

[AdamsFrank] Adams J Frank (1930-1989)

Adams

[Adams] Adams John Couch (1819-1892)

Adelard

[Adelard] Adelard of Bath (1075-1160)

Adler

[Adler] Adler August (1863-1923)

Adrain

[Adrain] Adrain Robert (1775-1843)

Aepinus

[Aepinus] Aepinus Franz (1724-1802)

Agnesi

[Agnesi] Agnesi Maria (1718-1799)

Ahlfors

[Ahlfors] Ahlfors Lars (1907-1996) Finnish mathematician working in complex analysis, was also professor at Harvard from 1946, retiring in 1977. Ahlfors won both the Fields medal in 1936 and the Wolf prize in 1981.

Ahmes

[Ahmes] Ahmes (1680BC-1620BC)

Aida

[Aida] Aida Yasuaki (1747-1817)

Aiken

[Aiken] Aiken Howard (1900-1973)

Airy

[Airy] Airy George (1801-1892)

Aitken

[Aitken] Aitken Alec (1895-1967)

Ajima

[Ajima] Ajima Naonobu (1732-1798)

Akhiezer

[Akhiezer] Akhiezer Naum Ilich (1901-1980)

Albanese

[Albanese] Albanese Giacomo (1890-1948)

Albert

[Albert] Albert of Saxony (1316-1390)

AlbertAbraham

[AlbertAbraham] Albert A Adrian (1905-1972)

Alberti

[Alberti] Alberti Leone (1404-1472)

Albertus

[Albertus] Albertus Magnus Saint (1200-1280)

Alcuin

[Alcuin] Alcuin of York (735-804)

Alexander

[Alexander] Alexander James (1888-1971)

AlexanderArchie

[AlexanderArchie] Alexander Archie (1888-1958)

Aleksandrov

[Aleksandrov] Alexandroff Pave (1896-1982)

AleksandrovAleksandr

[AleksandrovAleksandr] Alexandroff Alexander

Ampere

[Ampere] Ampère André-Marie (1775-1836)

Amsler

[Amsler] Amsler Jacob (1823-1912)

Anaxagoras

[Anaxagoras] Anaxagoras of Clazomenae (499BC-428BC)

Anderson

[Anderson] Anderson Oskar (1887-1960)

Andreev

[Andreev] Andreev Konstantin (1848-1921)

Angeli

[Angeli] Angeli Stephano degli (1623-1697)

Anstice

[Anstice] Anstice Robert (1813-1853)

Anthemius

[Anthemius] Anthemius of Tralles (474-534)

Antiphon

[Antiphon] Antiphon the Sophist (480BC-411BC)

Apastamba

[Apastamba] Apastamba (600BC-540BC)

Apollonius

[Apollonius] Apollonius of Perga (262BC-190BC)

Appell

[Appell] Appell Paul (1855-1930)

Arago

[Arago] Arago Francois (1786-1853)

Arbogast

[Arbogast] Arbogast Louis (1759-1803)

Arbuthnot

[Arbuthnot] Arbuthnot John (1667-1735)

Archimedes

[Archimedes] Archimedes of Syracuse (287BC-212BC)

Archytas

[Archytas] Archytas of Tarentum (428BC-350BC)

Arf

[Arf] Arf Cahit (1910-1997)

Argand

[Argand] Argand Jean (1768-1822)

Aristaeus

[Aristaeus] Aristaeus the Elder (360BC-300BC)

Aristarchus

[Aristarchus] Aristarchus of Samos (310BC-230BC)

Aristotle

[Aristotle] Aristotle (384BC-322BC)

Arnauld

[Arnauld] Arnauld Antoine (1612-1694)

Aronhold

[Aronhold] Aronhold Siegfried (1819-1884)

Artin

[Artin] Artin Emil (1898-1962)

AryabhataII

[AryabhataII] Aryabhata II

AryabhataI

[AryabhataI] Aryabhata I (476-550)

Atiyah

[Atiyah] Atiyah Michael

Atwood

[Atwood] Atwood George (1745-1807)

Auslander

[Auslander] Auslander Maurice (1926-1994)

Autolycus

[Autolycus] Autolycus of Pitane (360BC-290BC)

Bezout

[Bezout] Bézout Etienne (1730-1783) French geometer and analyst.

Bocher

[Bocher] Bocher Maxime (1867-1918)

Burgi

[Burgi] Bürgi Joost (1552-1632)

Babbage

[Babbage] Babbage Charles (1791-1871)

Bachet

[Bachet] Bachet Claude (1581-1638)

Bachmann

[Bachmann] Bachmann Paul (1837-1920)

Backus

[Backus] Backus John

Bacon

[Bacon] Bacon Roger (1219-1292)

Baer

[Baer] Baer Reinhold (1902-1979)

Baire

[Baire] Baire René-Louis (1874-1932)

BakerAlan

[BakerAlan] Baker Alan

Baker

[Baker] Baker Henry (1866-1956)

Ball

[Ball] Ball Walter W Rouse (1850-1925)

Balmer

[Balmer] Balmer Johann (1825-1898)

Banach

[Banach], Stefan, (1892-1945) Polish mathematician who founded functional analysis.

Banneker

[Banneker] Banneker Benjamin (1731-1806)

BanuMusaMuhammad

[BanuMusaMuhammad] Banu Musa Jafar (810-873)

BanuMusa

[BanuMusa] Banu Musa brothers

BanuMusaal-Hasan

[BanuMusaal-Hasan] Banu Musa al-Hasan (810-873)

BanuMusaAhmad

[BanuMusaAhmad] Banu Musa Ahmad (810-873)

Barbier

[Barbier] Barbier Joseph Emile (1839-1889)

Bari

[Bari] Bari Nina (1901-1961)

Barlow

[Barlow] Barlow Peter (1776-1862)

Barnes

[Barnes] Barnes Ernest (1874-1953)

Barocius

[Barocius] Barozzi Francesco (1537-1604)

Barrow

[Barrow] Barrow Isaac (1630-1677)

Bartholin

[Bartholin] Bartholin Erasmus (1625-1698)

Batchelor

[Batchelor] Batchelor George (1920-2000)

Bateman

[Bateman] Bateman Harry (1882-1946)

Battaglini

[Battaglini] Battaglini Guiseppe (1826-1894)

Al-Battani

[Al-Battani] Battani Abu al- (850-929)

Baudhayana

[Baudhayana] Baudhayana (800BC-740BC)

Bayes

[Bayes] Thomas, (1702-1761). English probability theorist and theologian.

Beaugrand

[Beaugrand] Beaugrand Jean (1595-1640)

Bell

[Bell] Bell Eric Temple (1883-1960)

Bellavitis

[Bellavitis] Bellavitis Giusto (1803-1880)

Beltrami

[Beltrami] Beltrami Eugenio (1835-1899)

Bendixson

[Bendixson] Bendixson Ivar Otto (1861-1935)

Benedetti

[Benedetti] Benedetti Giovanni (1530-1590)

Bergman

[Bergman] Bergman Stefan (1895-1977)

Berkeley

[Berkeley] Berkeley George (1685-1753)

Bernays

[Bernays] Bernays Paul Isaac (1888-1977)

BernoulliDaniel

[BernoulliDaniel] Bernoulli Daniel (1700-1782)

BernoulliJohann

[BernoulliJohann] Bernoulli Johann (1667-1748)

BernoulliNicolaus

[BernoulliNicolaus] Bernoulli Nicolaus

BernoulliJacob

[BernoulliJacob] Bernoulli Jakob (1654-1705) Swiss analyst, probability theorist and physicist.

Bernstein

[Bernstein] Sergei Natanovich (1880-1968). Russian analyst.

BernsteinFelix

[BernsteinFelix] Bernstein Felix (1878-1956)

Bers

[Bers] Bers Lipa (1914-1993)

Bertini

[Bertini] Bertini Eugenio (1846-1933)

Bertrand

[Bertrand] Bertrand Joseph (1822-1900)

Berwald

[Berwald] Berwald Lugwig (1883-1942)

Berwick

[Berwick] Berwick William (1888-1944)

Besicovitch

[Besicovitch] Besicovitch Abram (1891-1970)

Bessel

[Bessel] Friedrich Wilhelm, (1784-1846) calculated orbit of Halley's orbit as 20 year old. Made accurate measurements of stellar positions. Professor of Astronomy at Koenigsberg.

Betti

[Betti] Betti Enrico (1823-1892)

Beurling

[Beurling] Beurling Arne (1905-1986)

BhaskaraI

[BhaskaraI] Bhaskara I (600-680)

BhaskaraII

[BhaskaraII] Bhaskaracharya (1114-1185)

Bianchi

[Bianchi] Bianchi Luigi (1856-1928)

Bieberbach

[Bieberbach] Bieberbach Ludwig (1886-1982)

Bienayme

[Bienayme] Bienaymé Iréneé-Jules (1796-1878)

Binet

[Binet] Binet Jacques (1786-1856)

Bing

[Bing] Bing R (1940-1986)

Biot

[Biot] Biot Jean-Baptiste (1774-1862)

Birkhoff

[Birkhoff] Birkhoff George (1884-1944)

BirkhoffGarrett

[BirkhoffGarrett] Birkhoff Garrett (1911-1996)

Al-Biruni

[Al-Biruni] Biruni Abu al

BjerknesVilhelm

[BjerknesVilhelm] Bjerknes Vilhelm (1862-1951)

BjerknesCarl

[BjerknesCarl] Bjerknes Carl (1825-1903)

Black

[Black] Black Max (1909-1988)

Blaschke

[Blaschke] Blaschke Wilhelm (1885-1962)

Blichfeldt

[Blichfeldt] Blichfeldt Hans (1873-1945)

Bliss

[Bliss] Bliss Gilbert (1876-1951)

Bloch

[Bloch] Bloch André (1893-1948)

Bobillier

[Bobillier] Bobillier Etienne (1798-1840)

Bochner

[Bochner] Bochner Salomon (1899-1982)

Boethius

[Boethius] Boethius Anicus (475-524)

Boggio

[Boggio] Boggio Tommaso (1877-1963)

Bohl

[Bohl] Bohl Piers (1865-1921)

BohrHarald

[BohrHarald] Bohr Harald (1887-1951)

BohrNiels

[BohrNiels] Bohr Niels (1885-1962)

Boltzmann

[Boltzmann] Boltzmann Ludwig (1844-1906)

BolyaiFarkas

[BolyaiFarkas] Bolyai Farkas (1775-1856)

Bolyai

[Bolyai] Bolyai Jänos (1802-1860)

Bolza

[Bolza], Oscar (1857-1943), German-born American analyst.

Bolzano

[Bolzano] Bolzano Bernhard (1781-1848)

Bombelli

[Bombelli] Bombelli Rafael (1526-1573)

Bombieri

[Bombieri] Bombieri Enrico

Bonferroni

[Bonferroni] Bonferroni Carlo (1892-1960)

Bonnet

[Bonnet] Bonnet Pierre (1819-1892)

Boole

[Boole], George (1815-1864), English Logician, who made also contributions to analysis and probability theory.

Boone

[Boone] Boone Bill (1920-1983)

Borchardt

[Borchardt] Borchardt Carl (1817-1880)

Borda

[Borda] Borda Jean (1733-1799)

Borel

[Borel], Felix Edouard Justin Emile, (1871-1956) French measure theorist and probability theorist.

Borgi

[Borgi] Borgi Piero (1424-1484)

Born

[Born] Born Max (1882-1970)

Borsuk

[Borsuk] Borsuk Karol (1905-1982)

Bortkiewicz

[Bortkiewicz] Bortkiewicz Ladislaus (1868-1931)

Bortolotti

[Bortolotti] Bortolotti Ettore (1866-1947)

Bosanquet

[Bosanquet] Bosanquet Stephen (1903-1984)

Boscovich

[Boscovich] Boscovich Ruggero (1711-1787)

Bose

[Bose] Bose Satyendranath (1894-1974)

Bossut

[Bossut] Bossut Charles (1730-1814)

Bouguer

[Bouguer] Bouguer Pierre (1698-1758)

Boulliau

[Boulliau] Boulliau Ismael (1605-1694)

Bouquet

[Bouquet] Bouquet Jean Claude (1819-1885)

Bour

[Bour] Bour Edmond (1832-1866)

Bourbaki

[Bourbaki], Nicolas (1939-) Collective pseudonym of a group of mostly French mathematicians.

Bourgain

[Bourgain] Bourgain Jean

Boutroux

[Boutroux] Boutroux Pierre Léon (1880-1922)

Bowditch

[Bowditch] Bowditch Nathaniel (1773-1838)

Bowen

[Bowen] Bowen Rufus (1947-1978)

Boyle

[Boyle] Boyle Robert (1627-1691)

Boys

[Boys] Boys Charles (1855-1944)

Bradwardine

[Bradwardine] Bradwardine Thomas (1290-1349)

Brahe

[Brahe] Brahe Tycho (1546-1601)

Brahmadeva

[Brahmadeva] Brahmadeva (1060-1130)

Brahmagupta

[Brahmagupta] Brahmagupta (598-670)

Braikenridge

[Braikenridge] Braikenridge William (1700-1762)

Bramer

[Bramer] Bramer Benjamin (1588-1652)

Brashman

[Brashman] Brashman Nikolai (1796-1866)

BrauerAlfred

[BrauerAlfred] Brauer Alfred (1894-1985)

Brauer

[Brauer] Brauer Richard (1901-1977)

Brianchon

[Brianchon] Brianchon Charles (1783-1864)

Briggs

[Briggs] Briggs Henry (1561-1630) English mathematician producing tables of common logarithms up to 15 digits.

Brillouin

[Brillouin] Brillouin Marcel (1854-1948)

Bring

[Bring] Bring Erland (1736-1798)

Brioschi

[Brioschi] Brioschi Francesco (1824-1897)

Briot

[Briot] Briot Charlese (1817-1882)

Brisson

[Brisson] Brisson Barnabé (1777-1828)

Britton

[Britton] Britton John (1927-1994)

Brocard

[Brocard] Brocard Henri (1845-1922)

Brodetsky

[Brodetsky] Brodetsky Selig

Bromwich

[Bromwich] Bromwich Thomas (1875-1929)

Bronowski

[Bronowski] Bronowski Jacob (1908-1974)

Brouncker

[Brouncker] Brouncker William (1620-1684)

Brouwer

[Brouwer] Brouwer Luitzen Egbertus Jan (1881-1966) Dutch matematician and philosopher.

Brown

[Brown] Brown Ernest (1866-1938)

Browne

[Browne] Browne Marjorie (1914-1979)

Bruno

[Bruno] Bruno Giuseppe (1828-1893)

Bruns

[Bruns] Bruns Heinrich (1848-1919)

Bryson

[Bryson] Bryson of Heraclea (450BC-390BC)

Buffon

[Buffon] Buffon Georges Comte de (1707-1788)

Bugaev

[Bugaev] Bugaev Nicolay (1837-1903)

Bukreev

[Bukreev] Bukreev Boris (1859-1962)

Bunyakovsky

[Bunyakovsky] Bunyakovsky Viktor (1804-1889)

Burchnall

[Burchnall] Burchnall Joseph (1892-1975)

Burkhardt

[Burkhardt] Burkhardt Heinrich (1861-1914)

Burkill

[Burkill] Burkill John (1900-1993)

Burnside

[Burnside] Burnside William (1852-1927)

Caccioppoli

[Caccioppoli] Caccioppoli Renato (1904-1959)

Cajori

[Cajori] Cajori Florian (1859-1930)

Calderon

[Calderon] Calderón Alberto (1920-1998)

Callippus

[Callippus] Callippus (370BC-310BC)

Campanus

[Campanus] Campanus of Novara (1220-1296)

Campbell

[Campbell] Campbell John (1862-1924)

Camus

[Camus] Camus Charles (1699-1768)

Cannell

[Cannell] Cannell Doris (1913-2000)

Cantelli

[Cantelli] Cantelli Francesco (1875-1966)

CantorMoritz

[CantorMoritz] Cantor Moritz (1829-1920)

Cantor

[Cantor] Cantor Georg (1845-1918) [Cantor] Georg, (1845-1918) German mathematician. Precise definition of infinite set.

Caramuel

[Caramuel] Caramuel Juan (1606-1682)

Caratheodory

[Caratheodory] Carathéodory Constantin (1873-1950)

Cardan

[Cardan] Cardano Girolamo (1501-1576) Italian mathematicaian, physician and astrologer. First publication for the solution of the general cubic equation (solution found by Tartaglia).

Carlitz

[Carlitz] Carlitz Leonard (1907-1999)

Carlyle

[Carlyle] Carlyle Thomas (1795-1881)

Carnot

[Carnot] Carnot Lazare (1753-1823) French mathematician and politican best known for his work on the foundations of calculus and modern geometry.

CarnotSadi

[CarnotSadi] Carnot Sadi (1796-1832) French mathematical physisist working on the foundations of thermodynamics. Carnot's work led directly to the discovery of the second law of thermodynamics.

Carslaw

[Carslaw] Carslaw Horatio (1870-1954)

Cartan

[Cartan] Cartan Elie (1869-1951)

CartanHenri

[CartanHenri] Cartan Henri

Cartwright

[Cartwright] Cartwright Dame Mary (1900-1998)

Casorati

[Casorati] Casorati Felice (1835-1890)

Cassels

[Cassels] Cassels John

Cassini

[Cassini] Cassini Giovanni (1625-1712)

Castel

[Castel] Castel Louis (1688-1757)

Castelnuovo

[Castelnuovo] Castelnuovo Guido (1865-1952)

Castigliano

[Castigliano] Castigliano Alberto (1847-1884)

Castillon

[Castillon] Castillon Johann (1704-1791)

Catalan

[Catalan] Catalan Eugène (1814-1894)

Cataldi

[Cataldi] Cataldi Pietro (1548-1626)

Cauchy

[Cauchy] Cauchy Augustin-Louis (1789-1857) French mathematician who introduced modern notions of continuity limit, convergence and differentiability, proved Cauchy's theorem in group theory, contributed to the calculus of variations, probability theory and the study of differential equations.

Cavalieri

[Cavalieri] Cavalieri Bonaventura (1598-1647) Italian mathematician. Introduced method of indivisibles, a forerunner of integral calculus to determine the area enclosed by certain curves.

Cayley

[Cayley] Cayley Arthur (1821-1895) English mathematician working in the theory of matrices, abstract groups and algebraic geometry.

Cech

[Cech] Cech Eduard (1893-1960)

Cesaro

[Cesaro] Cesàro Ernesto (1859-1906)

CevaGiovanni

[CevaGiovanni] Ceva Giovanni (1647-1734)

CevaTommaso

[CevaTommaso] Ceva Tommaso (1648-1737)

Chatelet

[Chatelet] Chatelet Gabrielle du (1706-1749)

Chandrasekhar

[Chandrasekhar] Chandrasekhar Subrah. (1910-1995)

Chang

[Chang] Chang Sun-Yung Alice

Chaplygin

[Chaplygin] Chaplygin Sergi (1869-1942)

Chapman

[Chapman] Chapman Sydney (1888-1970)

Chasles

[Chasles] Chasles Michel (1793-1880)

Chebotaryov

[Chebotaryov] Chebotaryov Nikolai (1894-1947)

Chebyshev

[Chebyshev] Chebyshev Pafnuty (1821-1894)

Chern

[Chern] Chern Shiing-shen

Chernikov

[Chernikov] Chernikov Sergei (1912-1987)

Chevalley

[Chevalley] Chevalley Claude (1909-1984)

Ch'in

[Ch'in] Chiu-Shao Ch'in (1202-1261)

Chowla

[Chowla] Chowla Sarvadaman (1907-1995)

Christoffel

[Christoffel] Christoffel Elwin (1829-1900)

Chrysippus

[Chrysippus] Chrysippus (280BC-206BC)

Chrystal

[Chrystal] Chrystal George (1851-1911)

Chuquet

[Chuquet] Chuquet Nicolas (1445-1500)

Church

[Church] Church Alonzo (1903-1995) American mathematical logician.

Clairaut

[Clairaut] Clairaut Alexis (1713-1765) French mathematician and physicist who worked on the problem of geodesic flows, celestial mechanics and cubic curves.

Clapeyron

[Clapeyron] Clapeyron Emile (1799-1864)

Clarke

[Clarke] Clarke Samuel (1675-1729)

Clausen

[Clausen] Clausen Thomas (1801-1885)

Clausius

[Clausius] Clausius Rudolf (1822-1888)

Clavius

[Clavius] Clavius Christopher (1537-1612)

Clebsch

[Clebsch] Clebsch Alfred (1833-1872)

Cleomedes

[Cleomedes] Cleomedes Cleomedes (10AD-70)

Clifford

[Clifford] Clifford William (1845-1879)

Coates

[Coates] Coates John

Coble

[Coble] Coble Arthur (1878-1966)

Cochran

[Cochran] Cochran William (1909-1980)

Cocker

[Cocker] Cocker Edward (1631-1675)

Codazzi

[Codazzi] Codazzi Delfino (1824-1873)

Cohen

[Cohen] Paul Joseph (1934-) American mathematician who resolved the status of the continuum hypothesis.

Cole

[Cole] Cole Frank (1861-1926)

Collingwood

[Collingwood] Collingwood Edward (1900-1970)

Collins

[Collins] Collins John (1625-1683)

Condorcet

[Condorcet] Condorcet Marie Jean (1743-1794)

Connes

[Connes] Connes Alain

Conon

[Conon] Conon of Samos (280BC-220BC)

ConwayArthur

[ConwayArthur] Conway Arthur (1875-1950)

Conway

[Conway] Conway John

Coolidge

[Coolidge] Coolidge Julian (1873-1954)

Cooper

[Cooper] Cooper Lionel (1915-1977)

Copernicus

[Copernicus] Copernicus Nicolaus (1473-1543)

Copson

[Copson] Copson Edward (1901-1980)

Cosserat

[Cosserat] Cosserat Eugène (1866-1931)

Cotes

[Cotes] Cotes Roger (1682-1716)

Courant

[Courant] Courant Richard (1888-1972)

Cournot

[Cournot] Cournot Antoine (1801-1877)

Couturat

[Couturat] Couturat Louis (1868-1914)

Cox

[Cox] Cox Gertrude (1900-1978)

Coxeter

[Coxeter] Coxeter Donald

Craig

[Craig] Craig John (1663-1731)

CramerHarald

[CramerHarald] Cramér Harald (1893-1985)

Cramer

[Cramer] Cramer Gabriel (1704-1752)

Crank

[Crank] Crank John

Crelle

[Crelle] Crelle August (1780-1855)

Cremona

[Cremona] Cremona Luigi (1830-1903)

Crighton

[Crighton] Crighton David (1942-2000)

Cunha

[Cunha] Cunha Anastäcio da (1744-1787)

Cunningham

[Cunningham] Cunningham Ebenezer (1881-1977)

Curry

[Curry] Curry Haskell (1900-1982)

Cusa

[Cusa] Cusa Nicholas of (1401-1464)

Durer

[Durer] Dürer Albrecht (1471-1528)

Dandelin

[Dandelin] Dandelin Germinal (1794-1847)

Danti

[Danti] Danti Egnatio (1536-1586)

DantzigGeorge

[DantzigGeorge] Dantzig George

Darboux

[Darboux] Darboux Gaston (1842-1917)

Darwin

[Darwin] Darwin George (1845-1912)

Dase

[Dase] Dase Zacharias (1824-1861)

Davenport

[Davenport] Davenport Harold (1907-1969)

Davidov

[Davidov] Davidov August (1823-1885)

Davies

[Davies] Davies Evan Tom (1904-1973)

Dechales

[Dechales] Dechales Claude (1621-1678)

Dedekind

[Dedekind] Dedekind Richard (1831-1916)

Dee

[Dee] Dee John (1527-1608)

Dehn

[Dehn] Dehn Max (1878-1952)

Delamain

[Delamain] Delamain Richard (1600-1644)

Delambre

[Delambre] Delambre Jean Baptiste (1749-1822)

Delaunay

[Delaunay] Delaunay Charles (1816-1872)

Deligne

[Deligne] Deligne Pierre

Delone

[Delone] Delone Boris (1890-1973)

Delsarte

[Delsarte] Delsarte Jean (1903-1968)

Democritus

[Democritus] Democritus of Abdera (460BC-370BC)

Denjoy

[Denjoy] Denjoy Arnaud (1884-1974)

Deparcieux

[Deparcieux] Deparcieux Antoine (1703-1768)

Desargues

[Desargues] Desargues Girard (1591-1661)

Descartes

[Descartes] Descartes René (1596-1650)

Dickson

[Dickson] Dickson Leonard (1874-1954)

Dickstein

[Dickstein] Dickstein Samuel (1851-1939)

Dieudonne

[Dieudonne] Dieudonné Jean (1906-1992)

Digges

[Digges] Digges Thomas (1546-1595)

Dilworth

[Dilworth] Dilworth Robert

Dinghas

[Dinghas] Dinghas Alexander (1908-1974)

Dini

[Dini] Dini Ulisse (1845-1918)

Dinostratus

[Dinostratus] Dinostratus (390BC-320BC)

Diocles

[Diocles] Diocles (240BC-180BC)

Dionis

[Dionis] Dionis du Séjour A (1734-1794)

Dionysodorus

[Dionysodorus] Dionysodorus (250BC-190BC)

Diophantus

[Diophantus] Diophantus of Alexandria (200-284)

Dirac

[Dirac] Dirac Paul (1902-1984)

Dirichlet

[Dirichlet] Dirichlet Lejeune (1805-1859)

DixonArthur

[DixonArthur] Dixon Arthur Lee (1867-1955)

Dixon

[Dixon] Dixon Alfred (1865-1936)

Dodgson

[Dodgson] Dodgson Charles (1832-1898)

Doeblin

[Doeblin] Doeblin Wolfang (1915-1940)

Domninus

[Domninus] Domninus of Larissa (420-480)

Donaldson

[Donaldson] Donaldson Simon

Doob

[Doob] Doob Joseph

Doppelmayr

[Doppelmayr] Doppelmayr Johann (1671-1750)

Doppler

[Doppler] Doppler Christian (1803-1853)

Douglas

[Douglas] Douglas Jesse (1897-1965)

Dowker

[Dowker] Dowker Clifford (1912-1982)

Drach

[Drach] Drach Jules (1871-1941)

Drinfeld

[Drinfeld] Drinfeld Vladimir

Dubreil

[Dubreil] Dubreil Paul (1904-1994)

Dudeney

[Dudeney] Dudeney Henry (1857-1931)

Duhamel

[Duhamel] Duhamel Jean-Marie (1797-1872)

Duhem

[Duhem] Duhem Pierre (1861-1916)

Dupin

[Dupin] Dupin Pierre (1784-1873)

Dupre

[Dupre] Dupré Athanase (1808-1869)

Dynkin

[Dynkin] Dynkin Evgenii

EckertJohn

[EckertJohn] Eckert J Presper (1919-1995)

EckertWallace

[EckertWallace] Eckert Wallace J (1902-1971)

Eckmann

[Eckmann] Eckmann Beno

Eddington

[Eddington] Eddington Arthur (1882-1944)

Edge

[Edge] Edge William (1904-1997)

Edgeworth

[Edgeworth] Edgeworth Francis (1845-1926)

Egorov

[Egorov] Egorov Dimitri (1869-1931)

Ehrenfest

[Ehrenfest] Ehrenfest Paul (1880-1933)

Ehresmann

[Ehresmann] Ehresmann Charles (1905-1979)

Eilenberg

[Eilenberg] Eilenberg Samuel (1913-1998)

Einstein

[Einstein] Einstein Albert (1879-1955)

Eisenhart

[Eisenhart] Eisenhart Luther (1876-1965)

Eisenstein

[Eisenstein] Eisenstein Gotthold (1823-1852)

Elliott

[Elliott] Elliott Edwin (1851-1937)

Empedocles

[Empedocles] Empedocles (492BC-432BC)

Engel

[Engel] Engel Friedrich (1861-1941)

Enriques

[Enriques] Enriques Federigo (1871-1946)

Enskog

[Enskog] Enskog David (1884-1947)

Epstein

[Epstein] Epstein Paul (1871-1939)

Eratosthenes

[Eratosthenes] Eratosthenes of Cyrene (276BC-197BC)

Erdelyi

[Erdelyi] Erdélyi Arthur (1908-1977)

Erdos

[Erdos] Erdös Paul (1913-1996)

Erlang

[Erlang] Erlang Agner (1878-1929)

Escher

[Escher] Escher Maurits (1898-1972)

Esclangon

[Esclangon] Esclangon Ernest (1876-1954)

Euclid

[Euclid] Euclid of Alexandria (325BC-265BC)

Eudemus

[Eudemus] Eudemus of Rhodes (350BC-290BC)

Eudoxus

[Eudoxus] Eudoxus of Cnidus (408BC-355BC)

Euler

[Euler] Euler Leonhard (1707-1783) Swiss mathematician, worked on practially all fields of mathematics.

Eutocius

[Eutocius] Eutocius of Ascalon (480-540)

Evans

[Evans] Evans Griffith (1887-1973)

Ezra

[Ezra] Ezra Rabbi Ben (1092-1167)

FaadiBruno

[FaadiBruno] Faà di Bruno Francesco (1825-1907)

Faber

[Faber] Faber Georg

Fabri

[Fabri] Fabri Honoré (1607-1688)

FagnanoGiovanni

[FagnanoGiovanni] Fagnano Giovanni (1715-1797)

FagnanoGiulio

[FagnanoGiulio] Fagnano Giulio (1682-1766)

Faltings

[Faltings] Faltings Gerd

Fano

[Fano] Fano Gino (1871-1952)

Faraday

[Faraday] Faraday Michael (1791-1867)

Farey

[Farey] Farey John (1766-1826)

Fatou

[Fatou] Fatou Pierre (1878-1929)

Faulhaber

[Faulhaber] Faulhaber Johann (1580-1635)

Fefferman

[Fefferman] Fefferman Charles

Feigenbaum

[Feigenbaum] Feigenbaum Mitchell

Feigl

[Feigl] Feigl Georg (1890-1945)

Fejer

[Fejer] Fejér Lipót (1880-1959)

Feller

[Feller] Feller William (1906-1970)

Fermat

[Fermat] Fermat Pierre de (1601-1665)

Ferrar

[Ferrar] Ferrar Bill (1893-1990)

Ferrari

[Ferrari] Ferrari Lodovico (1522-1565)

Ferrel

[Ferrel] Ferrel William (1817-1891)

Ferro

[Ferro] Ferro Scipione del (1465-1526)

Feuerbach

[Feuerbach] Feuerbach Karl (1800-1834)

Feynman

[Feynman] Feynman Richard (1918-1988)

Fields

[Fields] Fields John (1863-1932)

Finck

[Finck] Finck Pierre-Joseph (1797-1870)

Fincke

[Fincke] Fincke Thomas (1561-1656)

FineHenry

[FineHenry] Fine Henry (1858-1928)

Fine

[Fine] Fine Oronce (1494-1555)

Finsler

[Finsler] Finsler Paul (1894-1970)

Fischer

[Fischer] Fischer Ernst (1875-1959)

Fisher

[Fisher] Fisher Sir Ronald (1890-1962)

Fiske

[Fiske] Fiske Thomas (1865-1944)

FitzGerald

[FitzGerald] FitzGerald George (1851-1901)

Flugge-Lotz

[Flugge-Lotz] Flügge-Lotz Irmgard (1903-1974)

Flamsteed

[Flamsteed] Flamsteed John

Fomin

[Fomin] Fomin Sergei (1917-1975)

FontainedesBertins

[FontainedesBertins] Fontaine des Bertins A (1704-1771)

Fontenelle

[Fontenelle] Fontenelle Bernard de (1657-1757)

Forsyth

[Forsyth] Forsyth Andrew (1858-1942)

Burali-Forti

[Burali-Forti] Forti Cesare Burali- (1861-1931)

Fourier

[Fourier] Fourier Joseph (1768-1830)

Fowler

[Fowler] Fowler Ralph (1889-1944)

Fox

[Fox] Fox Charles (1897-1977)

Frechet

[Frechet] Fréchet Maurice (1878-1973)

Fraenkel

[Fraenkel] Fraenkel Adolf (1891-1965)

FrancaisJacques

[FrancaisJacques] Francais Jacques (1775-1833)

FrancaisFrancois

[FrancaisFrancois] Francais Francois (1768-1810)

Francoeur

[Francoeur] Francoeur Louis (1773-1849)

Frank

[Frank] Frank Philipp (1884-1966)

Franklin

[Franklin] Franklin Philip (1898-1965)

FranklinBenjamin

[FranklinBenjamin] Franklin Benjamin (1706-1790)

Frattini

[Frattini] Frattini Giovanni (1852-1925)

Fredholm

[Fredholm] Fredholm Ivar (1866-1927)

Freedman

[Freedman] Freedman Michael

Frege

[Frege] Frege Gottlob (1848-1925)

Freitag

[Freitag] Freitag Herta (1908-2000)

Frenet

[Frenet] Frenet Jean (1816-1900)

FrenicledeBessy

[FrenicledeBessy] Frenicle de Bessy B (1605-1675)

Frenkel

[Frenkel] Frenkel Jacov (1894-1952)

Fresnel

[Fresnel] Fresnel Augustin (1788-1827)

Freudenthal

[Freudenthal] Freudenthal Hans (1905-1990)

Freundlich

[Freundlich] Freundlich Finlay (1885-1964)

Friedmann

[Friedmann] Friedmann Alexander (1888-1925)

Friedrichs

[Friedrichs] Friedrichs Kurt (1901-1982)

Frisi

[Frisi] Frisi Paolo (1728-1784)

Frobenius

[Frobenius] Frobenius Georg (1849-1917)

Fubini

[Fubini] Fubini Guido (1879-1943)

Fuchs

[Fuchs] Fuchs Lazarus (1833-1902)

Fueter

[Fueter] Fueter Rudolph (1880-1950)

Fuller

[Fuller] Fuller R Buckminster (1895-1983)

Fuss

[Fuss] Fuss Nicolai (1755-1826)

Godel

[Godel] Gödel Kurt (1906-1978)

Gopel

[Gopel] Göpel Adolph (1812-1847)

Galerkin

[Galerkin] Galerkin Boris (1871-1945)

Galileo

[Galileo] Galileo Galilei (1564-1642)

Gallarati

[Gallarati] Gallarati Dionisio

Galois

[Galois] Galois Evariste (1811-1832)

Galton

[Galton] Galton Francis (1822-1911)

Gassendi

[Gassendi] Gassendi Pierre (1592-1655)

Gauss

[Gauss] Gauss Carl Friedrich (1777-1855)

Gegenbauer

[Gegenbauer] Gegenbauer Leopold (1849-1903)

Geiser

[Geiser] Geiser Karl (1843-1934)

Gelfand

[Gelfand] Gelfand Israil

Gelfond

[Gelfond] Gelfond Aleksandr (1906-1968)

Gellibrand

[Gellibrand] Gellibrand Henry (1597-1636)

Geminus

[Geminus] Geminus (10BC-60AD)

GemmaFrisius

[GemmaFrisius] Gemma Frisius Regnier (1508-1555)

Genocchi

[Genocchi] Genocchi Angelo (1817-1889)

Gentzen

[Gentzen] Gentzen Gerhard (1909-1945)

Gergonne

[Gergonne] Gergonne Joseph (1771-1859)

Germain

[Germain] Germain Sophie (1776-1831)

Gherard

[Gherard] Gherard of Cremona (1114-1187)

Ghetaldi

[Ghetaldi] Ghetaldi Marino (1566-1626)

Gibbs

[Gibbs] Gibbs J Willard (1839-1903)

GirardAlbert

[GirardAlbert] Girard Albert (1595-1632)

GirardPierre

[GirardPierre] Girard Pierre Simon (1765-1836)

Glaisher

[Glaisher] Glaisher James (1848-1928)

Glenie

[Glenie] Glenie James (1750-1817)

Gohberg

[Gohberg] Gohberg Israel

Goldbach

[Goldbach] Goldbach Christian (1690-1764)

Goldstein

[Goldstein] Goldstein Sydney (1903-1989)

Gompertz

[Gompertz] Gompertz Benjamin (1779-1865)

Goodstein

[Goodstein] Goodstein Reuben (1912-1985)

Gordan

[Gordan] Gordan Paul (1837-1912)

Gorenstein

[Gorenstein] Gorenstein Daniel (1923-1992)

Gosset

[Gosset] Gosset William (1876-1937)

Goursat

[Goursat] Goursat Edouard (1858-1936)

Govindasvami

[Govindasvami] Govindasvami (800-860)

Graffe

[Graffe] Gräffe Karl (1799-1873)

Gram

[Gram] Gram Jorgen (1850-1916)

Grandi

[Grandi] Grandi Guido (1671-1742)

Granville

[Granville] Granville Evelyn

Grassmann

[Grassmann] Grassmann Hermann (1808-1877)

Grave

[Grave] Grave Dmitry (1863-1939)

Green

[Green] Green George (1793-1841)

Greenhill

[Greenhill] Greenhill Alfred (1847-1927)

Gregory

[Gregory] Gregory James (1638-1675)

GregoryDuncan

[GregoryDuncan] Gregory Duncan (1813-1844)

GregoryDavid

[GregoryDavid] Gregory David (1659-1708)

DeGroot

[DeGroot] Groot Johannes de (1914-1972)

Grosseteste

[Grosseteste] Grosseteste Robert (1168-1253)

Grossmann

[Grossmann] Grossmann Marcel (1878-1936)

Grothendieck

[Grothendieck] Grothendieck Alexander

Grunsky

[Grunsky] Grunsky Helmut (1904-1986)

Guarini

[Guarini] Guarini Guarino (1624-1683)

Guccia

[Guccia] Guccia Giovanni (1855-1914)

Gudermann

[Gudermann] Gudermann Christoph (1798-1852)

Guenther

[Guenther] Guenther Adam (1848-1923)

Guinand

[Guinand] Guinand Andy (1912-1987)

Guldin

[Guldin] Guldin Paul (1577-1643)

Gunter

[Gunter] Gunter Edmund (1581-1626)

Hajek

[Hajek] Häjek Jaroslav (1926-1974)

Herigone

[Herigone] Hérigone Pierre (1580-1643)

Holder

[Holder] Hölder Otto (1859-1937)

Hormander

[Hormander] Hörmander Lars

Haar

[Haar] Haar Alfréd (1885-1933)

Hachette

[Hachette] Hachette Jean (1769-1834)

Hadamard

[Hadamard] Hadamard Jacques (1865-1963)

Hadley

[Hadley] Hadley John (1682-1744)

Hahn

[Hahn] Hahn Hans (1879-1934)

Hall

[Hall] Hall Philip (1904-1982)

HallMarshall

[HallMarshall] Hall Marshall Jr. (1910-1990)

Halley

[Halley] Halley Edmond (1656-1742)

Halmos

[Halmos] Halmos Paul

Halphen

[Halphen] Halphen George (1844-1889)

Halsted

[Halsted] Halsted George (1853-1922)

Hamill

[Hamill] Hamill Christine

Hamilton

[Hamilton] Hamilton William R (1805-1865)

HamiltonWilliam

[HamiltonWilliam] Hamilton William (1788-1856)

Hamming

[Hamming] Hamming Richard W (1915-1998)

Hankel

[Hankel] Hankel Hermann (1839-1873)

HardyClaude

[HardyClaude] Hardy Claude (1598-1678)

Hardy

[Hardy] Hardy G H (1877-1947)

Harish-Chandra

[Harish-Chandra] Harish-Chandra (1923-1983)

Harriot

[Harriot] Harriot Thomas (1560-1621)

Hartley

[Hartley] Hartley Brian (1939-1994)

Hartree

[Hartree] Hartree Douglas (1897-1958)

Hasse

[Hasse] Hasse Helmut (1898-1979)

Hausdorff

[Hausdorff] Hausdorff Felix (1869-1942)

Hawking

[Hawking] Hawking Stephen

Al-Haytham

[Al-Haytham] Haytham Abu Ali al

Heath

[Heath] Heath Thomas (1861-1940)

Heaviside

[Heaviside] Heaviside Oliver (1850-1925)

Heawood

[Heawood] Heawood Percy (1861-1955)

Hecht

[Hecht] Hecht Daniel (1777-1833)

Hecke

[Hecke] Hecke Erich (1887-1947)

Hedrick

[Hedrick] Hedrick Earle (1876-1943)

Heegaard

[Heegaard] Heegaard Poul (1871-1948)

Heilbronn

[Heilbronn] Heilbronn Hans (1908-1975)

Heine

[Heine] Heine Eduard (1821-1881)

Heisenberg

[Heisenberg] Heisenberg Werner (1901-1976)

Hellinger

[Hellinger] Hellinger Ernst (1883-1950)

Helly

[Helly] Helly Eduard (1884-1943)

Heng

[Heng] Heng Zhang (78AD-139)

Henrici

[Henrici] Henrici Olaus (1840-1918)

Hensel

[Hensel] Hensel Kurt (1861-1941)

Heraclides

[Heraclides] Heraclides of Pontus (387BC-312BC)

Herbrand

[Herbrand] Herbrand Jacques (1908-1931)

Hermann

[Hermann] Hermann Jakob (1678-1733)

Hermite

[Hermite] Hermite Charles (1822-1901)

Heron

[Heron] Heron of Alexandria (10AD-75)

HerschelCaroline

[HerschelCaroline] Herschel Caroline (1750-1848)

Herschel

[Herschel] Herschel John (1792-1871)

Herstein

[Herstein] Herstein Yitz (1923-1988)

Hesse

[Hesse] Hesse Otto (1811-1874)

Heyting

[Heyting] Heyting Arend (1898-1980)

Higman

[Higman] Higman Graham

Hilbert

[Hilbert] Hilbert David (1862-1943)

Hill

[Hill] Hill George (1838-1914)

Hille

[Hille] Hille Einar (1894-1980)

Hindenburg

[Hindenburg] Hindenburg Carl (1741-1808)

Hipparchus

[Hipparchus] Hipparchus of Rhodes (190BC-120BC)

Hippias

[Hippias] Hippias of Elis (460BC-400BC)

Hippocrates

[Hippocrates] Hippocrates of Chios (470BC-410BC)

Hironaka

[Hironaka] Hironaka Heisuke

Hirsch

[Hirsch] Hirsch Kurt (1906-1986)

Hirst

[Hirst] Hirst Thomas (1830-1891)

Gnedenko

[Gnedenko] Hniedenko Boris (1912-1995)

Houel

[Houel] Hoüel Jules (1823-1886)

Hobbes

[Hobbes] Hobbes Thomas (1588-1679)

Hobson

[Hobson] Hobson Ernest (1856-1933)

Hodge

[Hodge] Hodge William (1903-1975)

Hollerith

[Hollerith] Hollerith Herman (1860-1929)

Holmboe

[Holmboe] Holmboe Bernt (1795-1850)

Honda

[Honda] Honda Taira (1932-1975)

Hooke

[Hooke] Hooke Robert (1635-1703)

HopfEberhard

[HopfEberhard] Hopf Eberhard (1902-1983)

Hopf

[Hopf] Hopf Heinz (1894-1971)

Hopkins

[Hopkins] Hopkins William (1793-1866)

Hopkinson

[Hopkinson] Hopkinson John (1849-1898)

Hopper

[Hopper] Hopper Grace (1906-1992)

Horner

[Horner] Horner William (1786-1837)

Householder

[Householder] Householder Alston (1904-1993)

Hsu

[Hsu] Hsu Pao-Lu (1910-1970)

Hubble

[Hubble] Hubble Edwin (1889-1953)

Hudde

[Hudde] Hudde Johann (1628-1704)

HumbertPierre

[HumbertPierre] Humbert Pierre (1891-1953)

HumbertGeorges

[HumbertGeorges] Humbert Georges (1859-1921)

Huntington

[Huntington] Huntington Edward (1874-1952)

Hurewicz

[Hurewicz] Hurewicz Witold (1904-1956)

Hurwitz

[Hurwitz] Hurwitz Adolf (1859-1919)

Hutton

[Hutton] Hutton Charles (1737-1823)

Huygens

[Huygens] Huygens Christiaan (1629-1695)

Hypatia

[Hypatia] Hypatia of Alexandria (370-415)

Hypsicles

[Hypsicles] Hypsicles of Alexandria (190BC-120BC)

Ibrahim

[Ibrahim] Ibrahim ibn Sinan (908-946)

Ingham

[Ingham] Ingham Albert (1900-1967)

Ito

[Ito] Ito Kiyosi

Ivory

[Ivory] Ivory James (1765-1842)

Iwasawa

[Iwasawa] Iwasawa Kenkichi (1917-1998)

Iyanaga

[Iyanaga] Iyanaga Skokichi

JabiribnAflah

[JabiribnAflah] Jabir ibn Aflah (1100-1160)

Jacobi

[Jacobi] Jacobi Carl (1804-1851)

Jacobson

[Jacobson] Jacobson Nathan (1910-1999)

Jagannatha

[Jagannatha] Jagannatha Samrat

James

[James] James Ioan

Janiszewski

[Janiszewski] Janiszewski Zygmunt (1888-1920)

Janovskaja

[Janovskaja] Janovskaja Sof'ja (1896-1966)

Jarnik

[Jarnik] Jarnik Vojtech (1897-1970)

Al-Jawhari

[Al-Jawhari] Jawhari al-Abbas al (800-860)

Al-Jayyani

[Al-Jayyani] Jayyani Abu al

Jeans

[Jeans] Jeans Sir James (1877-1946)

Jeffrey

[Jeffrey] Jeffrey George (1891-1957)

Jeffreys

[Jeffreys] Jeffreys Sir Harold (1891-1989)

Jensen

[Jensen] Jensen Johan (1859-1925)

Jerrard

[Jerrard] Jerrard George (1804-1863)

Jevons

[Jevons] Jevons William (1835-1882)

Joachimsthal

[Joachimsthal] Joachimsthal Ferdinand (1818-1861)

John

[John] John Fritz

JohnsonBarry

[JohnsonBarry] Johnson Barry

Johnson

[Johnson] Johnson William (1858-1931)

JonesBurton

[JonesBurton] Jones F (1910-1999)

Jones

[Jones] Jones William (1675-1749)

JonesVaughan

[JonesVaughan] Jones Vaughan

Jonquieres

[Jonquieres] Jonquières Ernest de (1820-1901)

Jordan

[Jordan] Jordan Camille (1838-1922)

Jordanus

[Jordanus] Jordanus Nemorarius (1225-1260)

Jourdain

[Jourdain] Jourdain Philip (1879-1921)

Juel

[Juel] Juel Christian (1855-1935)

Julia

[Julia] Julia Gaston (1893-1978)

Jungius

[Jungius] Jungius Joachim (1587-1657)

Jyesthadeva

[Jyesthadeva] Jyesthadeva (1500-1575)

KonigJulius

[KonigJulius] König Julius (1849-1913)

KonigSamuel

[KonigSamuel] König Samuel (1712-1757)

Konigsberger

[Konigsberger] Königsberger Leo (1837-1921)

Kurschak

[Kurschak] Kürschäk József (1864-1933)

Kac

[Kac] Kac Mark (1914-1984)

Kaestner

[Kaestner] Kaestner Abraham (1719-1800)

Kagan

[Kagan] Kagan Benjamin (1869-1953)

Kakutani

[Kakutani] Kakutani Shizuo

Kalmar

[Kalmar] Kalmär Läszló (1905-1976)

Kaluza

[Kaluza] Kaluza Theodor (1885-1945)

Kaluznin

[Kaluznin] Kaluznin Lev (1914-1990)

Al-Farisi

[Al-Farisi] Kamal al-Farisi (1260-1320)

Kamalakara

[Kamalakara] Kamalakara (1616-1700)

AbuKamil

[AbuKamil] Kamil Abu Shuja (850-930)

Kantorovich

[Kantorovich] Kantorovich Leonid (1912-1986)

Kaplansky

[Kaplansky] Kaplansky Irving

Al-Karaji

[Al-Karaji] Karkhi al

Karp

[Karp] Karp Carol (1926-1972)

Al-Kashi

[Al-Kashi] Kashi Ghiyath al (1390-1450)

Katyayana

[Katyayana] Katyayana (200BC-140BC)

Keill

[Keill] Keill John (1671-1721)

Kelland

[Kelland] Kelland Philip (1808-1879)

Kellogg

[Kellogg] Kellogg Oliver (1878-1957)

Kemeny

[Kemeny] Kemeny John (1926-1992)

Kempe

[Kempe] Kempe Alfred (1849-1922)

KendallMaurice

[KendallMaurice] Kendall Maurice (1907-1983)

Kendall

[Kendall] Kendall David

Kepler

[Kepler] Kepler Johannes (1571-1630)

Kerekjarto

[Kerekjarto] Kerékjärtó Béla (1898-1946)

Keynes

[Keynes] Keynes John Maynard (1883-1946)

Al-Khalili

[Al-Khalili] Khalili Shams al (1320-1380)

Al-Khazin

[Al-Khazin] Khazin Abu Jafar al (900-971)

Khinchin

[Khinchin] Khinchin Aleksandr (1894-1959)

Al-Khujandi

[Al-Khujandi] Khujandi Abu al

Al-Khwarizmi

[Al-Khwarizmi] Khwarizmi Abu al- (790-850)

Killing

[Killing] Killing Wilhelm (1847-1923)

Al-Kindi

[Al-Kindi] Kindi Abu al (805-873)

Kingman

[Kingman] Kingman John

Kirchhoff

[Kirchhoff] Kirchhoff Gustav (1824-1887)

Kirkman

[Kirkman] Kirkman Thomas (1806-1895)

Klugel

[Klugel] Klügel Georg (1739-1812)

Kleene

[Kleene] Kleene Stephen (1909-1994)

KleinOskar

[KleinOskar] Klein Oskar (1894-1977)

Klein

[Klein] Klein Felix (1849-1925)

Klingenberg

[Klingenberg] Klingenberg Wilhelm

Kloosterman

[Kloosterman] Kloosterman Hendrik (1900-1968)

Kneser

[Kneser] Kneser Adolf (1862-1930)

KneserHellmuth

[KneserHellmuth] Kneser Hellmuth (1898-1973)

Knopp

[Knopp] Knopp Konrad (1882-1957)

Kober

[Kober] Kober Hermann (1888-1973)

Kochin

[Kochin] Kochin Nikolai (1901-1944)

Kodaira

[Kodaira] Kodaira Kunihiko (1915-1997)

Koebe

[Koebe] Koebe Paul (1882-1945)

Koenigs

[Koenigs] Koenigs Gabriel (1858-1931)

Kolmogorov

[Kolmogorov] Kolmogorov Andrey (1903-1987) Russian probabilist who established in 1933 the mathematical foundation of probability theory and did important work also in other fields like Hamiltonian dynamics (KAM theorem) or turbulence Kolmogorov scaling.

Kolosov

[Kolosov] Kolosov Gury (1867-1936)

KonigDenes

[KonigDenes] Konig Denes (1884-1944)

Korteweg

[Korteweg] Korteweg Diederik (1848-1941)

Kotelnikov

[Kotelnikov] Kotelnikov Aleksandr (1865-1944)

Kovalevskaya

[Kovalevskaya] Kovalevskaya Sofia (1850-1891)

Kramp

[Kramp] Kramp Christian (1760-1826)

Krawtchouk

[Krawtchouk] Krawtchouk Mikhail (1892-1942)

Krein

[Krein] Krein Mark (1907-1989)

Kreisel

[Kreisel] Kreisel Georg

Kronecker

[Kronecker] Kronecker Leopold (1823-1891)

Krull

[Krull] Krull Wolfgang (1899-1971)

KrylovAleksei

[KrylovAleksei] Krylov Aleksei (1863-1945)

KrylovNikolai

[KrylovNikolai] Krylov Nikolai (1879-1955)

Kulik

[Kulik] Kulik Yakov (1783-1863)

Kumano-Go

[Kumano-Go] Kumano-Go Hitoshi (1935-1982)

Kummer

[Kummer] Kummer Eduard (1810-1893)

Kuratowski

[Kuratowski] Kuratowski Kazimierz (1896-1980)

Kurosh

[Kurosh] Kurosh Aleksandr (1908-1971)

Kutta

[Kutta] Kutta Martin (1867-1944)

Kuttner

[Kuttner] Kuttner Brian (1908-1992)

Leger

[Leger] Léger Emile (1795-1838)

LevyPaul

[LevyPaul] Lévy Paul (1886-1971)

Lowenheim

[Lowenheim] Löwenheim Leopold (1878-1957)

Loewner

[Loewner] Löwner Karl (1893-1968)

DeL'Hopital

[DeL'Hopital] L'Hopital Guillaume de (1661-1704)

LaHire

[LaHire] La Hire Philippe de (1640-1718)

LaFaille

[LaFaille] La Faille Charles de (1597-1652)

LaCondamine

[LaCondamine] La Condamine Charles de (1701-1774)

Lacroix

[Lacroix] Lacroix Sylvestre (1765-1843)

Lagny

[Lagny] Lagny Thomas de (1660-1734)

Lagrange

[Lagrange] Lagrange Joseph-Louis (1736-1813)

Laguerre

[Laguerre] Laguerre Edmond (1834-1886)

Lakatos

[Lakatos] Lakatos Imre (1922-1974)

Lalla

[Lalla] Lalla (720-790)

Lame

[Lame] Lamé Gabriel (1795-1870)

Lamb

[Lamb] Lamb Horace (1849-1934)

Lambert

[Lambert] Lambert Johann (1728-1777)

HermannofReichenau

[HermannofReichenau] Lame Hermann the (1013-1054)

Lamy

[Lamy] Lamy Bernard (1640-1715)

Lanczos

[Lanczos] Lanczos Cornelius (1893-1974)

Landau

[Landau] Landau Edmund (1877-1938)

LandauLev

[LandauLev] Landau Lev (1908-1968)

Landen

[Landen] Landen John (1719-1790)

Landsberg

[Landsberg] Landsberg Georg (1865-1912)

Langlands

[Langlands] Langlands Robert

Laplace

[Laplace] Laplace Pierre-Simon (1749-1827)

Larmor

[Larmor] Larmor Sir Joseph (1857-1942)

Lasker

[Lasker] Lasker Emanuel (1868-1941)

Kramer

[Kramer] Lassar Edna Kramer (1902-1984)

LaurentHermann

[LaurentHermann] Laurent Hermann (1841-1908)

LaurentPierre

[LaurentPierre] Laurent Pierre (1813-1854)

Lavanha

[Lavanha] Lavanha Joao Baptista (1550-1624)

Lavrentev

[Lavrentev] Lavrentev Mikhail (1900-1980)

Lax

[Lax] Lax Gaspar (1487-1560)

LeFevre

[LeFevre] Le Fèvre Jean (1652-1706)

Lebesgue

[Lebesgue] Lebesgue Henri (1875-1941)

Ledermann

[Ledermann] Ledermann Walter

Leech

[Leech] Leech John (1926-1992)

Lefschetz

[Lefschetz] Lefschetz Solomon (1884-1972)

Legendre

[Legendre] Legendre Adrien-Marie (1752-1833)

Lemoine

[Lemoine] Lemoine Emile (1840-1912)

Leray

[Leray] Leray Jean (1906-1998)

Lerch

[Lerch] Lerch Mathias (1860-1922)

Leshniewski

[Leshniewski] Leshniewski Stanislaw (1886-1939)

Leslie

[Leslie] Leslie John (1766-1832)

Leucippus

[Leucippus] Leucippus (480BC-420BC)

Levi

[Levi] Levi ben Gerson (1288-1344)

Levi-Civita

[Levi-Civita] Levi-Civita Tullio (1873-1941)

Levinson

[Levinson] Levinson Norman (1912-1975)

LevyHyman

[LevyHyman] Levy Hyman (1889-1975)

Levytsky

[Levytsky] Levytsky Volodymyr (1872-1956)

Lexell

[Lexell] Lexell Anders (1740-1784)

Lexis

[Lexis] Lexis Wilhelm (1837-1914)

Lhuilier

[Lhuilier] Lhuilier Simon (1750-1840)

Libri

[Libri] Libri Guglielmo (1803-1869)

Lie

[Lie] Lie Sophus (1842-1899)

Lifshitz

[Lifshitz] Lifshitz Evgenii (1915-1985)

Lighthill

[Lighthill] Lighthill Sir James (1924-1998)

Lindelof

[Lindelof] Lindelöf Ernst (1870-1946)

Linnik

[Linnik] Linnik Yuri (1915-1972)

Lions

[Lions] Lions Pierre-Louis

Liouville

[Liouville] Liouville Joseph (1809-1882)

Lipschitz

[Lipschitz] Lipschitz Rudolf (1832-1903)

Lissajous

[Lissajous] Lissajous Jules (1822-1880)

Listing

[Listing] Listing Johann (1808-1882)

Littlewood

[Littlewood] Littlewood John E (1885-1977)

LittlewoodDudley

[LittlewoodDudley] Littlewood Dudley (1903-1979)

Livsic

[Livsic] Livsic Moshe

Llull

[Llull] Llull Ramon (1235-1316)

Lobachevsky

[Lobachevsky] Lobachevsky Nikolai (1792-1856)

Loewy

[Loewy] Loewy Alfred (1873-1935)

Lopatynsky

[Lopatynsky] Lopatynsky Yaroslav (1906-1981)

Lorentz

[Lorentz] Lorentz Hendrik (1853-1928)

Love

[Love] Love Augustus (1863-1940)

Lovelace

[Lovelace] Lovelace Augusta Ada (1815-1852)

Loyd

[Loyd] Loyd Samuel (1841-1911)

Lucas

[Lucas] Lucas F Edouard (1842-1891)

Lueroth

[Lueroth] Lueroth Jacob (1844-1910)

Lukacs

[Lukacs] Lukacs Eugene (1906-1987)

Lukasiewicz

[Lukasiewicz] Lukasiewicz Jan (1878-1956)

Luke

[Luke] Luke Yudell (1918-1983)

Luzin

[Luzin] Luzin Nikolai (1883-1950)

Lyapunov

[Lyapunov] Lyapunov Aleksandr (1857-1918)

Lyndon

[Lyndon] Lyndon Roger (1917-1988)

Meray

[Meray] Méray Charles (1835-1911)

Mobius

[Mobius] Möbius August (1790-1868)

MacCullagh

[MacCullagh] MacCullagh James (1809-1896)

MacLane

[MacLane] MacLane Saunders

MacMahon

[MacMahon] MacMahon Percy (1854-1929)

Macaulay

[Macaulay] Macaulay Francis (1862-1937)

Macdonald

[Macdonald] Macdonald Hector (1865-1935)

Maclaurin

[Maclaurin] Maclaurin Colin (1698-1746)

Madhava

[Madhava] Madhava Sangamagramma (1350-1425)

Al-Maghribi

[Al-Maghribi] Maghribi Muhyi al (1220-1280)

Magnitsky

[Magnitsky] Magnitsky Leonty (1669-1739)

Magnus

[Magnus] Magnus Wilhelm (1907-1990)

Al-Mahani

[Al-Mahani] Mahani Abu al (820-880)

Mahavira

[Mahavira] Mahavira Mahavira (800-870)

MahendraSuri

[MahendraSuri] Mahendra Suri (1340-1410)

Mahler

[Mahler] Mahler Kurt (1903-1988)

Maior

[Maior] Maior John (1469-1550)

Malcev

[Malcev] Malcev Anatoly (1909-1967)

Malebranche

[Malebranche] Malebranche Nicolas (1638-1715)

Malfatti

[Malfatti] Malfatti Francesco (1731-1807)

Malus

[Malus] Malus Etienne Louis (1775-1812)

Manava

[Manava] Manava (750BC-690BC)

Mandelbrot

[Mandelbrot] Mandelbrot Benoit

Mannheim

[Mannheim] Mannheim Amédée (1831-1906)

Mansion

[Mansion] Mansion Paul (1844-1919)

Mansur

[Mansur] Mansur ibn Ali Abu

Marchenko

[Marchenko] Marchenko Vladimir

Marcinkiewicz

[Marcinkiewicz] Marcinkiewicz Jozef (1910-1940)

Marczewski

[Marczewski] Marczewski Edward (1907-1976)

Margulis

[Margulis] Margulis Gregori

Marinus

[Marinus] Marinus of Neapolis (450-500)

Markov

[Markov] Markov Andrei (1856-1922)

Al-Banna

[Al-Banna] Marrakushi al (1256-1321)

Mascheroni

[Mascheroni] Mascheroni Lorenzo (1750-1800)

Maschke

[Maschke] Maschke Heinrich (1853-1908)

Maseres

[Maseres] Maseres Francis (1731-1824)

Maskelyne

[Maskelyne] Maskelyne Nevil (1732-1811)

Mason

[Mason] Mason Max (1877-1961)

Mathews

[Mathews] Mathews George (1861-1922)

MathieuClaude

[MathieuClaude] Mathieu Claude-Louis (1783-1875)

MathieuEmile

[MathieuEmile] Mathieu Emile (1835-1890)

Matsushima

[Matsushima] Matsushima Yozo (1921-1983)

Mauchly

[Mauchly] Mauchly John (1907-1980)

Maupertuis

[Maupertuis] Maupertuis Pierre de (1698-1759)

Maurolico

[Maurolico] Maurolico Francisco (1494-1575)

Maxwell

[Maxwell] Maxwell James Clerk (1831-1879)

MayerAdolph

[MayerAdolph] Mayer Adolph (1839-1903)

MayerTobias

[MayerTobias] Mayer Tobias (1723-1762)

Mazur

[Mazur] Mazur Stanislaw (1905-1981)

Mazurkiewicz

[Mazurkiewicz] Mazurkiewicz Stefan (1888-1945)

McClintock

[McClintock] McClintock John (1840-1916)

McDuff

[McDuff] McDuff Margaret

McShane

[McShane] McShane Edward (1904-1989)

Meissel

[Meissel] Meissel Ernst (1826-1895)

Mellin

[Mellin] Mellin Hjalmar (1854-1933)

Menabrea

[Menabrea] Menabrea Luigi (1809-1896)

Menaechmus

[Menaechmus] Menaechmus (380BC-320BC)

Menelaus

[Menelaus] Menelaus of Alexandria (70AD-130)

Menger

[Menger] Menger Karl

Mengoli

[Mengoli] Mengoli Pietro (1626-1686)

Menshov

[Menshov] Menshov Dmitrii (1892-1988)

MercatorGerardus

[MercatorGerardus] Mercator Gerardus (1512-1592)

MercatorNicolaus

[MercatorNicolaus] Mercator Nicolaus (1620-1687)

Mercer

[Mercer] Mercer James (1883-1932)

Merrifield

[Merrifield] Merrifield Charles (1827-1884)

Merrill

[Merrill] Merrill Winifred (1862-1951)

Mersenne

[Mersenne] Mersenne Marin (1588-1648)

Mertens

[Mertens] Mertens Franz (1840-1927)

Meshchersky

[Meshchersky] Meshchersky Ivan (1859-1935)

Meyer

[Meyer] Meyer Wilhelm (1856-1934)

Miller

[Miller] Miller George (1863-1951)

Milne

[Milne] Milne Edward (1896-1950)

Milnor

[Milnor] Milnor John

Minding

[Minding] Minding Ferdinand (1806-1885)

Mineur

[Mineur] Mineur Henri (1899-1954)

Minkowski

[Minkowski] Minkowski Hermann (1864-1909)

Mirsky

[Mirsky] Mirsky Leon (1918-1983)

Mittag-Leffler

[Mittag-Leffler] Mittag-Leffler Gösta (1846-1927)

Mohr

[Mohr] Mohr Georg (1640-1697)

DeMoivre

[DeMoivre] Moivre Abraham de (1667-1754)

Molin

[Molin] Molin Fedor (1861-1941)

Monge

[Monge] Monge Gaspard (1746-1818)

Monte

[Monte] Monte Guidobaldo del (1545-1607)

Montel

[Montel] Montel Paul (1876-1975)

Montmort

[Montmort] Montmort Pierre Rémond de (1678-1719)

Montucla

[Montucla] Montucla Jean (1725-1799)

MooreJonas

[MooreJonas] Moore Jonas (1627-1679)

MooreRobert

[MooreRobert] Moore Robert (1882-1974)

MooreEliakim

[MooreEliakim] Moore Eliakim (1862-1932)

Morawetz

[Morawetz] Morawetz Cathleen

Mordell

[Mordell] Mordell Louis (1888-1972)

DeMorgan

[DeMorgan] Morgan Augustus De (1806-1871)

Mori

[Mori] Mori Shigefumi

Morin

[Morin] Morin Arthur (1795-1880)

MorinJean-Baptiste

[MorinJean-Baptiste] Morin Jean-Baptiste

Morley

[Morley] Morley Frank (1860-1937)

Morse

[Morse] Morse Harald Marston (1892-1977)

Mostowski

[Mostowski] Mostowski Andrzej (1913-1975)

Motzkin

[Motzkin] Motzkin Theodore (1908-1970)

Moufang

[Moufang] Moufang Ruth (1905-1977)

Mouton

[Mouton] Mouton Gabriel (1618-1694)

Muir

[Muir] Muir Thomas (1844-1934)

Mumford

[Mumford] Mumford David

Mydorge

[Mydorge] Mydorge Claude (1585-1647)

Mytropolshy

[Mytropolshy] Mytropolshy Yurii

Naimark

[Naimark] Naimark Mark (1909-1978)

Napier

[Napier] Napier John (1550-1617)

Narayana

[Narayana] Narayana Pandit (1340-1400)

Al-Nasawi

[Al-Nasawi] Nasawi Abu al (1010-1075)

Nash

[Nash] Nash John

Navier

[Navier] Navier Claude (1785-1836)

Al-Nayrizi

[Al-Nayrizi] Nayrizi Abu'l al (875-940)

Neile

[Neile] Neile William (1637-1670)

Nekrasov

[Nekrasov] Nekrasov Aleksandr (1883-1957)

Netto

[Netto] Netto Eugen (1848-1919)

Neuberg

[Neuberg] Neuberg Joseph (1840-1926)

Neugebauer

[Neugebauer] Neugebauer Otto (1899-1990)

NeumannHanna

[NeumannHanna] Neumann Hanna (1914-1971)

NeumannCarl

[NeumannCarl] Neumann Carl Gottfried (1832-1925)

NeumannFranz

[NeumannFranz] Neumann Franz Ernst (1798-1895)

NeumannBernhard

[NeumannBernhard] Neumann Bernhard

Nevanlinna

[Nevanlinna] Nevanlinna Rolf (1895-1980)

Newcomb

[Newcomb] Newcomb Simon (1835-1909)

Newman

[Newman] Newman Maxwell (1897-1984)

Newton

[Newton] Newton Sir Isaac (1643-1727)

Neyman

[Neyman] Neyman Jerzy (1894-1981)

Nicolson

[Nicolson] Nicolson Phyllis (1917-1968)

Nicomachus

[Nicomachus] Nicomachus of Gerasa (60AD-120)

Nicomedes

[Nicomedes] Nicomedes (280BC-210BC)

Nielsen

[Nielsen] Nielsen Niels (1865-1931)

NielsenJakob

[NielsenJakob] Nielsen Jacob

Nightingale

[Nightingale] Nightingale Florence (1820-1910)

Nilakantha

[Nilakantha] Nilakantha Somayaji (1444-1544)

Niven

[Niven] Niven William (1843-1917)

NoetherMax

[NoetherMax] Noether Max (1844-1921)

NoetherEmmy

[NoetherEmmy] Noether Emmy (1882-1935)

Novikov

[Novikov] Novikov Petr (1901-1975)

NovikovSergi

[NovikovSergi] Novikov Sergi

Oenopides

[Oenopides] Oenopides of Chios (490BC-420BC)

Ohm

[Ohm] Ohm Georg Simon (1789-1854)

Oka

[Oka] Oka Kiyoshi (1901-1978)

Olivier

[Olivier] Olivier Théodore (1793-1853)

Khayyam

[Khayyam] Omar Khayyam (1048-1122)

Oresme

[Oresme] Oresme Nicole d' (1323-1382)

Orlicz

[Orlicz] Orlicz Wladyslaw (1903-1990)

Ortega

[Ortega] Ortega Juan de (1480-1568)

Osgood

[Osgood] Osgood William (1864-1943)

Osipovsky

[Osipovsky] Osipovsky Timofei (1765-1832)

Ostrogradski

[Ostrogradski] Ostrogradski Mikhail (1801-1862)

Ostrowski

[Ostrowski] Ostrowski Alexander (1893-1986)

Oughtred

[Oughtred] Oughtred William (1574-1660)

D'Ovidio

[D'Ovidio] Ovidio Enrico D' (1842-1933)

Ozanam

[Ozanam] Ozanam Jacques (1640-1717)

Peres

[Peres] Pérès Joseph (1890-1962)

Peter

[Peter] Péter Rózsa (1905-1977)

Polya

[Polya] Pólya George (1887-1985)

Pacioli

[Pacioli] Pacioli Luca (1445-1517)

Pade

[Pade] Padé Henri (1863-1953)

Padoa

[Padoa] Padoa Alessandro (1868-1937)

LePaige

[LePaige] Paige Constantin Le (1852-1929)

Painleve

[Painleve] Painlevé Paul (1863-1933)

Paley

[Paley] Paley Raymond (1907-1933)

Paman

[Paman] Paman Roger (1710-1748)

Panini

[Panini] Panini (520BC-460BC)

Papin

[Papin] Papin Denis (1647-1712)

Pappus

[Pappus] Pappus of Alexandria (290-350)

Pars

[Pars] Pars Leopold (1896-1985)

Parseval

[Parseval] Parseval des Chees M-A (1755-1836)

Pascal

[Pascal] Pascal Blaise (1623-1662)

PascalEtienne

[PascalEtienne] Pascal Etienne (1588-1640)

Pasch

[Pasch] Pasch Moritz (1843-1930)

Patodi

[Patodi] Patodi Vijay (1945-1976)

Pauli

[Pauli] Pauli Wolfgang (1900-1958)

Peacock

[Peacock] Peacock George (1791-1858)

Peano

[Peano] Peano Giuseppe (1858-1932)

Pearson

[Pearson] Pearson Karl (1857-1936)

PearsonEgon

[PearsonEgon] Pearson Egon (1895-1980)

PeirceBenjamin

[PeirceBenjamin] Peirce Benjamin (1809-1880)

PeirceCharles

[PeirceCharles] Peirce Charles (1839-1914)

Pell

[Pell] Pell John (1611-1685)

Penney

[Penney] Penney Bill (1909-1991)

Perron

[Perron] Perron Oskar (1880-1975)

Perseus

[Perseus] Perseus (180BC-120BC)

Petersen

[Petersen] Petersen Julius (1839-1910)

Peterson

[Peterson] Peterson Karl (1828-1881)

Petit

[Petit] Petit Aléxis (1791-1820)

Petrovsky

[Petrovsky] Petrovsky Ivan (1901-1973)

Petryshyn

[Petryshyn] Petryshyn Volodymyr

Petzval

[Petzval] Petzval Józeph (1807-1891)

Peurbach

[Peurbach] Peurbach Georg (1423-1461)

Pfaff

[Pfaff] Pfaff Johann (1765-1825)

Pfeiffer

[Pfeiffer] Pfeiffer Georgii (1872-1946)

Philon

[Philon] Philon of Byzantium (280BC-220BC)

PicardEmile

[PicardEmile] Picard Emile (1856-1941)

PicardJean

[PicardJean] Picard Jean (1620-1682)

Pieri

[Pieri] Pieri Mario (1860-1913)

Francesca

[Francesca] Piero della Francesca (1412-1492)

Pillai

[Pillai] Pillai K C Sreedharan (1920-1980)

Pincherle

[Pincherle] Pincherle Salvatore (1853-1936)

Fibonacci

[Fibonacci] Pisano Leonardo Fibonacci (1170-1250)

Pitiscus

[Pitiscus] Pitiscus Bartholomeo (1561-1613)

Plucker

[Plucker] Plücker Julius (1801-1868)

Plana

[Plana] Plana Giovanni (1781-1864)

Planck

[Planck] Planck Max (1858-1947)

Plateau

[Plateau] Plateau Joseph (1801-1883)

Plato

[Plato] Plato (428BC-347BC)

Playfair

[Playfair] Playfair John (1748-1819)

Plessner

[Plessner] Plessner Abraham

Poincare

[Poincare] Poincaré J Henri (1854-1912)

Poinsot

[Poinsot] Poinsot Louis (1777-1859)

Poisson

[Poisson] Poisson Siméon (1781-1840)

Poleni

[Poleni] Poleni Giovanni (1683-1761)

Polozii

[Polozii] Polozii Georgii (1914-1968)

Poncelet

[Poncelet] Poncelet Jean-Victor (1788-1867)

Pontryagin

[Pontryagin] Pontryagin Lev (1908-1988)

Poretsky

[Poretsky] Poretsky Platon (1846-1907)

Porphyry

[Porphyry] Porphyry of Malchus (233-309)

Porta

[Porta] Porta Giambattista Della (1535-1615)

Posidonius

[Posidonius] Posidonius of Rhodes (135BC-51BC)

Post

[Post] Post Emil (1897-1954)

Potapov

[Potapov] Potapov Vladimir (1914-1980)

Prufer

[Prufer] Prüfer Heinz (1896-1934)

Pratt

[Pratt] Pratt John (1809-1871)

Pringsheim

[Pringsheim] Pringsheim Alfred (1850-1941)

Privalov

[Privalov] Privalov Ivan (1891-1941)

PrivatdeMolieres

[PrivatdeMolieres] Privat de Molières Joseph (1677-1742)

Proclus

[Proclus] Proclus Diadochus (411-485)

DeProny

[DeProny] Prony Gaspard de (1755-1839)

Prthudakasvami

[Prthudakasvami] Prthudakasvami (830-890)

Ptolemy

[Ptolemy] Ptolemy (85AD-165)

Puiseux

[Puiseux] Puiseux Victor (1820-1883)

Puissant

[Puissant] Puissant Louis (1769-1943)

Pythagoras

[Pythagoras] Pythagoras of Samos (580BC-520BC)

Al-Qalasadi

[Al-Qalasadi] Qalasadi Abu'l al (1412-1486)

Quetelet

[Quetelet] Quetelet Adolphe (1796-1874)

Al-Quhi

[Al-Quhi] Quhi Abu al

Quillen

[Quillen] Quillen Daniel

Quine

[Quine] Quine Willard Van (1908-2000)

Renyi

[Renyi] Rényi Alfréd (1921-1970)

Rado

[Rado] Radó Tibor (1895-1965)

Rademacher

[Rademacher] Rademacher Hans (1892-1969)

RadoRichard

[RadoRichard] Rado Richard (1906-1989)

Radon

[Radon] Radon Johann (1887-1956)

Rahn

[Rahn] Rahn Johann (1622-1676)

Rajagopal

[Rajagopal] Rajagopal Cadambathur (1903-1978)

Ramanujam

[Ramanujam] Ramanujam Chidambaram (1938-1974)

Ramanujan

[Ramanujan] Ramanujan Srinivasa (1887-1920)

Ramsden

[Ramsden] Ramsden Jesse (1735-1800)

Ramsey

[Ramsey] Ramsey Frank (1903-1930)

Ramus

[Ramus] Ramus Peter (1515-1572)

Rankin

[Rankin] Rankin Robert (1915-2001)

Rankine

[Rankine] Rankine William (1820-1872)

Raphson

[Raphson] Raphson Joseph (1648-1715)

Rasiowa

[Rasiowa] Rasiowa Helena (1917-1994)

Razmadze

[Razmadze] Razmadze Andrei (1889-1929)

Recorde

[Recorde] Recorde Robert (1510-1558)

Rees

[Rees] Rees Mina (1902-1997)

Regiomontanus

[Regiomontanus] Regiomontanus Johann (1436-1476)

Reichenbach

[Reichenbach] Reichenbach Hans (1891-1953)

Reidemeister

[Reidemeister] Reidemeister Kurt (1893-1971)

Reiner

[Reiner] Reiner Irving (1924-1986)

Remak

[Remak] Remak Robert (1888-1942)

Remez

[Remez] Remez Evgeny (1896-1975)

ReyPastor

[ReyPastor] Rey Pastor Julio (1888-1962)

Reye

[Reye] Reye Theodor (1838-1919)

DuBois-Reymond

[DuBois-Reymond] Reymond Paul du Bois- (1831-1889)

Reynaud

[Reynaud] Reynaud Antoine-André (1771-1844)

Reyneau

[Reyneau] Reyneau Charles (1656-1728)

Reynolds

[Reynolds] Reynolds Osborne (1842-1912)

DeRham

[DeRham] Rham Georges de (1903-1990)

Rheticus

[Rheticus] Rheticus Georg Joachim (1514-1574)

Riccati

[Riccati] Riccati Jacopo (1676-1754)

RiccatiVincenzo

[RiccatiVincenzo] Riccati Vincenzo (1707-1775)

RicciMatteo

[RicciMatteo] Ricci Matteo (1552-1610)

Ricci

[Ricci] Ricci Michelangelo (1619-1682)

Ricci-Curbastro

[Ricci-Curbastro] Ricci-Curbastro Georgorio (1853-1925)

RichardLouis

[RichardLouis] Richard Louis (1795-1849)

RichardJules

[RichardJules] Richard Jules (1862-1956)

Richardson

[Richardson] Richardson Lewis (1881-1953)

Richer

[Richer] Richer Jean (1630-1696)

Richmond

[Richmond] Richmond Herbert (1863-1948)

Riemann

[Riemann] Riemann G F Bernhard (1826-1866)

Ries

[Ries] Ries Adam (1492-1559)

RieszMarcel

[RieszMarcel] Riesz Marcel (1886-1969)

Riesz

[Riesz] Riesz Frigyes (1880-1956)

Ringrose

[Ringrose] Ringrose John

Roberts

[Roberts] Roberts Samuel (1827-1913)

Roberval

[Roberval] Roberval Gilles de (1602-1675)

Robins

[Robins] Robins Benjamin (1707-1751)

RobinsonJulia

[RobinsonJulia] Robinson Julia Bowman (1919-1985)

Robinson

[Robinson] Robinson Abraham (1918-1974)

Rocard

[Rocard] Rocard Yves-André (1903-1992)

LaRoche

[LaRoche] Roche Estienne de La (1470-1530)

Rogers

[Rogers] Rogers Ambrose

Rohn

[Rohn] Rohn Karl (1855-1920)

Rolle

[Rolle] Rolle Michel (1652-1719)

Rosanes

[Rosanes] Rosanes Jakob (1842-1922)

Rosenhain

[Rosenhain] Rosenhain Johann (1816-1887)

Rota

[Rota] Rota Gian-Carlo (1932-1999)

Roth

[Roth] Roth Leonard (1904-1968)

RothKlaus

[RothKlaus] Roth Klaus

Routh

[Routh] Routh Edward (1831-1907)

Rudio

[Rudio] Rudio Ferdinand (1856-1929)

Rudolff

[Rudolff] Rudolff Christoff (1499-1545)

Ruffini

[Ruffini] Ruffini Paolo (1765-1822)

Runge

[Runge] Runge Carle (1856-1927)

RussellScott

[RussellScott] Russell John (1808-1882)

Russell

[Russell] Russell Bertrand (1872-1970)

Rutherford

[Rutherford] Rutherford Daniel E (1906-1966)

Rydberg

[Rydberg] Rydberg Johannes (1854-1919)

Saccheri

[Saccheri] Saccheri Giovanni (1667-1733)

Sacrobosco

[Sacrobosco] Sacrobosco Johannes de (1195-1256)

Saks

[Saks] Saks Stanislaw (1897-1942)

Nunez

[Nunez] Salaciense Pedro Nunez (1502-1587)

Salem

[Salem] Salem Raphaël (1898-1963)

Salmon

[Salmon] Salmon George (1819-1904)

Al-Samarqandi

[Al-Samarqandi] Samarqandi Shams al (1250-1310)

Al-Samawal

[Al-Samawal] Samawal Ibn al (1130-1180)

Samoilenko

[Samoilenko] Samoilenko Anatoly

Sang

[Sang] Sang Edward (1805-1890)

Sankara

[Sankara] Sankara Narayana (840-900)

Sasaki

[Sasaki] Sasaki Shigeo

Saurin

[Saurin] Saurin Joseph (1659-1737)

Savage

[Savage] Savage Leonard (1917-1971)

Savart

[Savart] Savart Felix (1791-1841)

Savary

[Savary] Savary Félix (1797-1841)

Abraham

[Abraham] Savasorda (1070-1130)

Savile

[Savile] Savile Sir Henry (1549-1622)

Schonflies

[Schonflies] Schönflies Arthur (1853-1928)

Schatten

[Schatten] Schatten Robert (1911-1977)

Schauder

[Schauder] Schauder Juliusz (1899-1943)

Scheffe

[Scheffe] Scheffé Henry (1907-1977)

Scheffers

[Scheffers] Scheffers Georg (1866-1945)

Schickard

[Schickard] Schickard Wilhelm (1592-1635)

Schlafli

[Schlafli] Schläfli Ludwig (1814-1895)

Schlomilch

[Schlomilch] Schlömilch Oscar (1823-1901)

Schmidt

[Schmidt] Schmidt Erhard (1876-1959)

Schoenberg

[Schoenberg] Schoenberg Isaac (1903-1990)

Schottky

[Schottky] Schottky Friedrich (1851-1935)

Schoute

[Schoute] Schoute Pieter (1846-1923)

Schouten

[Schouten] Schouten Jan (1883-1971)

Schroder

[Schroder] Schröder Ernst (1841-1902)

Schrodinger

[Schrodinger] Schrödinger Erwin (1887-1961)

Schreier

[Schreier] Schreier Otto (1901-1929)

Schroeter

[Schroeter] Schroeter Heinrich (1829-1892)

Schubert

[Schubert] Schubert Hermann (1848-1911)

Schur

[Schur] Schur Issai (1875-1941)

Schwartz

[Schwartz] Schwartz Laurent

SchwarzStefan

[SchwarzStefan] Schwarz Stefan (1914-1996)

Schwarz

[Schwarz] Schwarz Herman (1843-1921)

Schwarzschild

[Schwarzschild] Schwarzschild Karl (1873-1916)

Schwinger

[Schwinger] Schwinger Julian (1918-1994)

Scott

[Scott] Scott Charlotte (1858-1931)

Macintyre

[Macintyre] Scott Sheila (1910-1960)

SegreBeniamino

[SegreBeniamino] Segre Beniamino (1903-1977)

SegreCorrado

[SegreCorrado] Segre Corrado (1863-1924)

Seifert

[Seifert] Seifert Karl (1907-1996)

Selberg

[Selberg] Selberg Atle

Selten

[Selten] Selten Reinhard

Semple

[Semple] Semple Jack (1904-1985)

Serenus

[Serenus] Serenus (300-360)

Serre

[Serre] Serre Jean-Pierre

Serret

[Serret] Serret Joseph (1819-1885)

Servois

[Servois] Servois Francois (1768-1847)

Severi

[Severi] Severi Francesco (1879-1961)

Shanks

[Shanks] Shanks William (1812-1882)

Shannon

[Shannon] Shannon Claude (1916-2001)

Sharkovsky

[Sharkovsky] Sharkovsky Oleksandr

Shatunovsky

[Shatunovsky] Shatunovsky Samuil (1859-1929)

Shen

[Shen] Shen Kua (1031-1095)

Shewhart

[Shewhart] Shewhart Walter (1891-1967)

Shields

[Shields] Shields Allen (1927-1989)

Shnirelman

[Shnirelman] Shnirelman Lev (1905-1938)

Shoda

[Shoda] Shoda Kenjiro (1902-1977)

Shtokalo

[Shtokalo] Shtokalo Josif (1897-1987)

Siacci

[Siacci] Siacci Francesco (1839-1907)

Siegel

[Siegel] Siegel Carl (1896-1981)

Sierpinski

[Sierpinski] Sierpinski Waclaw (1882-1969)

Siguenza

[Siguenza] Siguenza y Gongora (1645-1700)

Al-Sijzi

[Al-Sijzi] Sijzi Abu al

Simplicius

[Simplicius] Simplicius Simplicius (490-560)

Simpson

[Simpson] Simpson Thomas (1710-1761)

Simson

[Simson] Simson Robert (1687-1768)

Avicenna

[Avicenna] Sina ibn

Sinan

[Sinan] Sinan ibn Thabit (880-943)

Sintsov

[Sintsov] Sintsov Dmitrii (1867-1946)

Sitter

[Sitter] Sitter Willem de (1872-1934)

Skolem

[Skolem] Skolem Thoralf (1887-1963)

Slaught

[Slaught] Slaught Herbert (1861-1937)

Sleszynski

[Sleszynski] Sleszynski Ivan (1854-1931)

Slutsky

[Slutsky] Slutsky Evgeny (1880-1948)

Sluze

[Sluze] Sluze René de (1622-1685)

Smale

[Smale] Smale Stephen

Smirnov

[Smirnov] Smirnov Vladimir (1887-1974)

Smith

[Smith] Smith Henry (1826-1883)

Sneddon

[Sneddon] Sneddon Ian (1919-2000)

Snell

[Snell] Snell Willebrord (1580-1626)

Snyder

[Snyder] Snyder Virgil (1869-1950)

Sobolev

[Sobolev] Sobolev Sergei (1908-1989)

Sokhotsky

[Sokhotsky] Sokhotsky Yulian-Karl (1842-1927)

Sokolov

[Sokolov] Sokolov Yurii (1896-1971)

Somerville

[Somerville] Somerville Mary (1780-1872)

Sommerfeld

[Sommerfeld] Sommerfeld Arnold (1868-1951)

Sommerville

[Sommerville] Sommerville Duncan (1879-1934)

Somov

[Somov] Somov Osip (1815-1876)

Sonin

[Sonin] Sonin Nikolay (1849-1915)

Spanier

[Spanier] Spanier Edwin (1921-1996)

Spence

[Spence] Spence William (1777-1815)

Sporus

[Sporus] Sporus of Nicaea (240-300)

Spottiswoode

[Spottiswoode] Spottiswoode William (1825-1883)

Sridhara

[Sridhara] Sridhara Sridhara (870-930)

Sripati

[Sripati] Sripati (1019-1066)

Stackel

[Stackel] Stäckel Paul (1862-1919)

Stampioen

[Stampioen] Stampioen Jan (1610-1690)

Steenrod

[Steenrod] Steenrod Norman (1910-1971)

StefanJosef

[StefanJosef] Stefan Josef (1835-1893)

StefanPeter

[StefanPeter] Stefan Peter (1941-1978)

Steiner

[Steiner] Steiner Jakob (1796-1863)

Steinhaus

[Steinhaus] Steinhaus Hugo (1887-1972)

Steinitz

[Steinitz] Steinitz Ernst (1871-1928)

Steklov

[Steklov] Steklov Vladimir A (1864-1926)

Stepanov

[Stepanov] Stepanov Vyacheslaw V (1889-1950)

Stevin

[Stevin] Stevin Simon (1548-1620)

Stewart

[Stewart] Stewart Matthew (1717-1785)

Stewartson

[Stewartson] Stewartson Keith (1925-1983)

Stieltjes

[Stieltjes] Stieltjes Thomas Jan (1856-1894)

Stifel

[Stifel] Stifel Michael (1487-1567)

Stirling

[Stirling] Stirling James (1692-1770)

Stokes

[Stokes] Stokes George Gabriel (1819-1903)

Stolz

[Stolz] Stolz Otto (1842-1905)

Stone

[Stone] Stone Marshall (1903-1989)

Stott

[Stott] Stott Alicia Boole (1860-1940)

Struik

[Struik] Struik Dirk (1894-2000)

Rayleigh

[Rayleigh] Strutt (1842-1919)

Study

[Study] Study Eduard (1862-1930)

Sturm

[Sturm] Sturm J Charles-Francois (1803-1855)

SturmRudolf

[SturmRudolf] Sturm Rudolf (1841-1919)

Subbotin

[Subbotin] Subbotin Mikhail (1893-1966)

Suetuna

[Suetuna] Suetuna Zyoiti (1898-1970)

Suter

[Suter] Suter Heinrich (1848-1922)

Suvorov

[Suvorov] Suvorov Georgii (1919-1984)

Swain

[Swain] Swain Lorna (1891-1936)

Sylow

[Sylow] Sylow Ludwig (1832-1918)

Sylvester

[Sylvester] Sylvester James Joseph (1814-1897)

Synge

[Synge] Synge John (1897-1995)

Szasz

[Szasz] Szäsz Otto (1884-1952)

Szego

[Szego] Szegö Gäbor (1895-1985)

Tacquet

[Tacquet] Tacquet Andrea (1612-1660)

Al-Baghdadi

[Al-Baghdadi] Tahir ibn

Tait

[Tait] Tait Peter Guthrie (1831-1901)

Takagi

[Takagi] Takagi Teiji (1875-1960)

Seki

[Seki] Takakazu (1642-1708)

Talbot

[Talbot] Talbot Henry Fox (1800-1877)

Taniyama

[Taniyama] Taniyama Yutaka (1927-1958)

TanneryPaul

[TanneryPaul] Tannery Paul (1843-1904)

TanneryJules

[TanneryJules] Tannery Jules (1848-1910)

Tarry

[Tarry] Tarry Gaston (1843-1913)

Tarski

[Tarski] Tarski Alfred (1902-1983)

Tartaglia

[Tartaglia] Tartaglia Niccolo Fontana (1500-1557)

Tauber

[Tauber] Tauber Alfred (1866-1942)

Taurinus

[Taurinus] Taurinus Franz (1794-1874)

Taussky-Todd

[Taussky-Todd] Taussky-Todd Olga

TaylorGeoffrey

[TaylorGeoffrey] Taylor Geoffrey (1886-1975)

Taylor

[Taylor] Taylor Brook (1685-1731)

Teichmuller

[Teichmuller] Teichmüller Oswald (1913-1943)

Temple

[Temple] Temple George (1901-1992)

LeTenneur

[LeTenneur] Tenneur Jacques (1610-1660)

Tetens

[Tetens] Tetens Johannes (1736-1807)

Thabit

[Thabit] Thabit ibn Qurra Abu'l (826-901)

Thales

[Thales] Thales of Miletus (624BC-546BC)

Theaetetus

[Theaetetus] Theaetetus of Athens (415BC-369BC)

Theodorus

[Theodorus] Theodorus of Cyrene (465BC-398BC)

Theodosius

[Theodosius] Theodosius of Bithynia (160BC-90BC)

TheonofSmyrna

[TheonofSmyrna] Theon of Smyrna (70AD-135)

Theon

[Theon] Theon of Alexandria (335-395)

Thiele

[Thiele] Thiele Thorvald (1838-1910)

Thom

[Thom] Thom René

Thomae

[Thomae] Thomae Johannes (1840-1921)

Thomason

[Thomason] Thomason Bob (1952-1995)

ThompsonJohn

[ThompsonJohn] Thompson John

ThompsonD'Arcy

[ThompsonD'Arcy] Thompson D'Arcy W (1860-1948)

Thomson

[Thomson] Thomson W (1824-1907)

Thue

[Thue] Thue Axel (1863-1922)

Thurston

[Thurston] Thurston Bill

Thymaridas

[Thymaridas] Thymaridas (400BC-350BC)

Tibbon

[Tibbon] Tibbon Jacob ben (1236-1312)

Tietze

[Tietze] Tietze Heinrich (1880-1964)

Tilly

[Tilly] Tilly Joseph de (1837-1906)

Tinbergen

[Tinbergen] Tinbergen Jan (1903-1994)

Tinseau

[Tinseau] Tinseau Charles (1748-1822)

Tisserand

[Tisserand] Tisserand Félix (1845-1896)

Titchmarsh

[Titchmarsh] Titchmarsh Edward (1899-1963)

Todd

[Todd] Todd John (1908-1994)

Todhunter

[Todhunter] Todhunter Isaac (1820-1884)

Toeplitz

[Toeplitz] Toeplitz Otto (1881-1940)

Torricelli

[Torricelli] Torricelli Evangelista (1608-1647)

Trail

[Trail] Trail William (1746-1831)

Tricomi

[Tricomi] Tricomi Francesco (1897-1978)

Troughton

[Troughton] Troughton Edward (1753-1836)

Tsu

[Tsu] Tsu Ch'ung Chi (430-501)

Tukey

[Tukey] Tukey John (1915-2000)

Tunstall

[Tunstall] Tunstall Cuthbert (1474-1559)

Turan

[Turan] Turän Paul (1910-1976)

Turing

[Turing] Turing Alan (1912-1954)

Turnbull

[Turnbull] Turnbull Herbert (1885-1961)

Turner

[Turner] Turner Peter (1586-1652)

Al-TusiSharaf

[Al-TusiSharaf] Tusi Sharaf al (1135-1213)

Al-TusiNasir

[Al-TusiNasir] Tusi Nasir al (1201-1274)

Tikhonov

[Tikhonov] Tychonoff Andrey (1906-1993)

UhlenbeckKaren

[UhlenbeckKaren] Uhlenbeck Karen

Uhlenbeck

[Uhlenbeck] Uhlenbeck George (1900-1988)

Ulam

[Ulam] Ulam Stanislaw (1909-1984)

UlughBeg

[UlughBeg] Ulugh Beg (1393-1449)

Al-Umawi

[Al-Umawi] Umawi Abu al (1400-1489)

Upton

[Upton] Upton Francis (1852-1921)

Al-Uqlidisi

[Al-Uqlidisi] Uqlidisi Abu'l al (920-980)

Urysohn

[Urysohn] Urysohn Pavel (1898-1924)

Vacca

[Vacca] Vacca Giovanni (1872-1953)

Vailati

[Vailati] Vailati Giovanni (1863-1909)

DuVal

[DuVal] Val Patrick du (1903-1987)

Valerio

[Valerio] Valerio Luca (1552-1618)

ValleePoussin

[ValleePoussin] Vallée Poussin C de (1866-1962)

Vandermonde

[Vandermonde] Vandermonde Alexandre (1735-1796)

Vandiver

[Vandiver] Vandiver Harry (1882-1973)

Varahamihira

[Varahamihira] Varahamihira Varahamihira (505-587)

Varignon

[Varignon] Varignon Pierre (1654-1722)

Veblen

[Veblen] Veblen Oswald (1880-1960)

Saint-Venant

[Saint-Venant] Venant Adhémar de St- (1797-1886)

Venn

[Venn] Venn John (1834-1923)

Verhulst

[Verhulst] Verhulst Pierre (1804-1849)

Vernier

[Vernier] Vernier Pierre (1584-1637)

Veronese

[Veronese] Veronese Giuseppe (1854-1917)

LeVerrier

[LeVerrier] Verrier Urbain Le (1811-1877)

Vessiot

[Vessiot] Vessiot Ernest (1865-1952)

Viete

[Viete] Viète Francois (1540-1603)

Vijayanandi

[Vijayanandi] Vijayanandi

Saint-Vincent

[Saint-Vincent] Vincent Gregorius Saint- (1584-1667)

Leonardo

[Leonardo] Vinci Leonardo da (1452-1519)

Vinogradov

[Vinogradov] Vinogradov Ivan (1891-1983)

Vitali

[Vitali] Vitali Giuseppe (1875-1932)

Viviani

[Viviani] Viviani Vincenzo (1622-1703)

Vlacq

[Vlacq] Vlacq Adriaan (1600-1667)

VanVleck

[VanVleck] Vleck Edward van (1863-1943)

Volterra

[Volterra] Volterra Vito (1860-1940)

Voronoy

[Voronoy] Voronoy Georgy (1868-1908)

Vranceanu

[Vranceanu] Vranceanu Gheorghe (1900-1979)

VanderWaerden

[VanderWaerden] Waerden Bartel van der (1903-1996)

Abu'l-Wafa

[Abu'l-Wafa] Wafa al-Buzjani Abu'l (940-998)

Wald

[Wald] Wald Abraham (1902-1950)

WalkerJohn

[WalkerJohn] Walker John (1825-1900)

WalkerArthur

[WalkerArthur] Walker Geoffrey (1909-2001)

Wall

[Wall] Wall C Terence

Wallace

[Wallace] Wallace William (1768-1843)

Wallis

[Wallis] Wallis John (1616-1703)

Wang

[Wang] Wang Hsien Chung (1918-1978)

Wangerin

[Wangerin] Wangerin Albert (1844-1933)

Wantzel

[Wantzel] Wantzel Pierre (1814-1894)

Waring

[Waring] Waring Edward (1734-1798)

Watson

[Watson] Watson G N (1886-1965)

WatsonHenry

[WatsonHenry] Watson Henry (1827-1903)

Wazewski

[Wazewski] Wazewski Tadeusz (1896-1972)

Weatherburn

[Weatherburn] Weatherburn Charles (1884-1974)

Weber

[Weber] Weber Wilhelm (1804-1891)

WeberHeinrich

[WeberHeinrich] Weber Heinrich Martin (1842-1913)

Wedderburn

[Wedderburn] Wedderburn Joseph (1882-1948)

Weierstrass

[Weierstrass] Weierstrass Karl (1815-1897)

Weil

[Weil] Weil André (1906-1998)

Weingarten

[Weingarten] Weingarten Julius (1836-1910)

Weinstein

[Weinstein] Weinstein Alexander (1897-1979)

Weisbach

[Weisbach] Weisbach Julius (1806-1871)

Weldon

[Weldon] Weldon Raphael (1860-1906)

Werner

[Werner] Werner Johann (1468-1522)

Wessel

[Wessel] Wessel Caspar (1745-1818)

West

[West] West John (1756-1817)

Weyl

[Weyl] Weyl Hermann (1885-1955)

Weyr

[Weyr] Weyr Emil (1848-1894)

Wheeler

[Wheeler] Wheeler Anna J Pell (1883-1966)

Whiston

[Whiston] Whiston William (1667-1752)

White

[White] White Henry (1861-1943)

Whitehead

[Whitehead] Whitehead Alfred N (1861-1947)

WhiteheadHenry

[WhiteheadHenry] Whitehead J Henry C (1904-1960)

Whitney

[Whitney] Whitney Hassler (1907-1989)

Whittaker

[Whittaker] Whittaker Edmund (1873-1956)

WhittakerJohn

[WhittakerJohn] Whittaker John (1905-1984)

Whyburn

[Whyburn] Whyburn Gordon (1904-1969)

Widman

[Widman] Widman Johannes (1462-1498)

Wielandt

[Wielandt] Wielandt Helmut (1910-2001)

Wien

[Wien] Wien Wilhelm (1864-1928)

WienerNorbert

[WienerNorbert] Wiener Norbert (1894-1964)

WienerChristian

[WienerChristian] Wiener Christian (1826-1896)

Wigner

[Wigner] Wigner Eugene (1902-1995)

Wilczynski

[Wilczynski] Wilczynski Ernest (1876-1932)

Wiles

[Wiles] Wiles Andrew

Wilkins

[Wilkins] Wilkins John (1614-1672)

Wilkinson

[Wilkinson] Wilkinson Jim (1919-1986)

Wilks

[Wilks] Wilks Samuel (1906-1964)

Ockham

[Ockham] William of Ockham (1285-1349)

WilsonEdwin

[WilsonEdwin] Wilson Edwin (1879-1964)

WilsonJohn

[WilsonJohn] Wilson John (1741-1793)

WilsonAlexander

[WilsonAlexander] Wilson Alexander (1714-1786)

Winkler

[Winkler] Winkler Wilhelm (1884-1984)

Wintner

[Wintner] Wintner Aurel (1903-1958)

Wirtinger

[Wirtinger] Wirtinger Wilhelm (1865-1945)

Wishart

[Wishart] Wishart John (1898-1956)

DeWitt

[DeWitt] Witt Johan de (1625-1672)

Witt

[Witt] Witt Ernst (1911-1991)

Witten

[Witten] Witten Edward

Wittgenstein

[Wittgenstein] Wittgenstein Ludwig (1889-1951)

Wolf

[Wolf] Wolf Rudolph (1816-1893)

Wolfowitz

[Wolfowitz] Wolfowitz Jacob (1910-1981)

Wolstenholme

[Wolstenholme] Wolstenholme Joseph (1829-1891)

Woodhouse

[Woodhouse] Woodhouse Robert (1773-1827)

Woodward

[Woodward] Woodward Robert (1849-1924)

Wren

[Wren] Wren Sir Christopher (1632-1723)

Wronski

[Wronski] Wronski Hoëné (1778-1853)

Xenocrates

[Xenocrates] Xenocrates of Chalcedon (396BC-314BC)

Yang

[Yang] Yang Hui (1238-1298)

Yates

[Yates] Yates Frank (1902-1994)

Yativrsabha

[Yativrsabha] Yativrsabha (500-570)

Yau

[Yau] Yau Shing-Tung

Yavanesvara

[Yavanesvara] Yavanesvara (120-180)

Yoccoz

[Yoccoz] Yoccoz Jean-Christophe

Youden

[Youden] Youden William (1900-1971)

Young

[Young] Young William (1863-1942)

YoungAlfred

[YoungAlfred] Young Alfred (1873-1940)

ChisholmYoung

[ChisholmYoung] Young Grace Chisholm (1868-1944)

Yule

[Yule] Yule George (1871-1951)

Yunus

[Yunus] Yunus Abu'l-Hasan ibn

Yushkevich

[Yushkevich] Yushkevich Adolph P (1906-1993)

Ahmed

[Ahmed] Yusuf Ahmed ibn (835-912)

QadiZada

[QadiZada] Zada al-Rumi Qadi (1364-1436)

Vashchenko

[Vashchenko] Zakharchenko M V- (1825-1912)

Zarankiewicz

[Zarankiewicz] Zarankiewicz Kazimierz (1902-1959)

Zaremba

[Zaremba] Zaremba Stanislaw (1863-1942)

Zariski

[Zariski] Zariski Oscar (1899-1986)

Zassenhaus

[Zassenhaus] Zassenhaus Hans (1912-1991)

Zeckendorf

[Zeckendorf] Zeckendorf Edouard (1901-1983)

Zeeman

[Zeeman] Zeeman Chris

Zelmanov

[Zelmanov] Zelmanov Efim

ZenoofSidon

[ZenoofSidon] Zeno of Sidon (150BC-70BC)

ZenoofElea

[ZenoofElea] Zeno of Elea (490BC-430BC)

Zenodorus

[Zenodorus] Zenodorus (200BC-140BC)

Zermelo

[Zermelo] Zermelo Ernst (1871-1951)

Zeuthen

[Zeuthen] Zeuthen Hieronymous (1839-1920)

Chu

[Chu] Zhu Shie-jie (1270-1330)

Zhukovsky

[Zhukovsky] Zhukovsky Nikolay (1847-1921)

Zolotarev

[Zolotarev] Zolotarev Egor (1847-1878)

Zorn

[Zorn] Zorn Max (1906-1993)

Zuse

[Zuse] Zuse Konrad (1910-1995)

Zygmund

[Zygmund] Zygmund Antoni (1900-1992)

D'Alembert

[D'Alembert] d'Alembert Jean (1717-1783)

BudandeBoislaurent

[BudandeBoislaurent] de Boislaurent Budan (1761-1840)

Coulomb

[Coulomb] de Coulomb Charles (1736-1806)

DeBeaune

[DeBeaune] de Beaune Florimond (1601-1652)

Carcavi

[Carcavi] de Carcavi Pierre (1600-1684)

Broglie

[Broglie] de Broglie Louis duc (1892-1987)

Bougainville

[Bougainville] de Bougainville Louis (1729-1811)

Billy

[Billy] de Billy Jacques (1602-1679)

Coriolis

[Coriolis] de Coriolis Gustave (1792-1843)

Hunayn

[Hunayn] ibn Ishaq Hunayn (808-873)

Lansberge

[Lansberge] van Lansberge Philip (1561-1632)

Roomen

[Roomen] van Roomen Adriaan (1561-1615)

Schooten

[Schooten] van Schooten Frans (1615-1660)

Dantzig

[Dantzig] van Dantzig David (1900-1959)

Heuraet

[Heuraet] van Heuraet Hendrik (1633-1660)

VanCeulen

[VanCeulen] van Ceulen Ludolph (1540-1610)

Amringe

[Amringe] van Amringe Howard (1835-1915)

Geiringer

[Geiringer] von Mises Hilda Geiringer (1893-1973)

Helmholtz

[Helmholtz] von Helmholtz Hermann (1821-1894)

Lindemann

[Lindemann] von Lindemann Carl (1852-1939)

Eotvos

[Eotvos] von Eötvös Roland (1848-1919)

Segner

[Segner] von Segner Johann (1704-1777)

Seidel

[Seidel] von Seidel Philipp (1821-1896)

VonNeumann

[VonNeumann] von Neumann John (1903-1957)

Vega

[Vega] von Vega Georg (1754-1802)

Mises

[Mises] von Mises Richard (1883-1953)

VonDyck

[VonDyck] von Dyck Walther (1856-1934)

Koch

[Koch] von Koch Helge (1870-1924)

Tschirnhaus

[Tschirnhaus] von Tschirnhaus E (1651-1708)

Karman

[Karman] von Kärmän Theodore (1881-1963)

Leibniz

[Leibniz] von Leibniz Gottfried (1646-1719)

VonStaudt

[VonStaudt] von Staudt Karl (1798-1867)

VonBrill

[VonBrill] von Brill Alexander (1842-1935)

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 1508 entries in this file.

Index

ENTRY MATH MOVIES

[ENTRY MATH MOVIES] Author: Oliver Knill: March 2000 -March 2004 Literature: actual DVD's and corresponding movie websites

Enigma

[Enigma] is an espionage thriller set during WW II. Most of the story is fictional. The main character Tom Jericho who serves with the British "Government Communication Headquarters" at Bletchley Park and played a significant role in breaking the German "Enigma" codes using a machine called "Collossus" to decipher the Enigma codes. The story is inspired by the lif of the mathematician Alan Turing who indeed contributed to the deciphering of Enigma during WW II.

A beautiful mind

The movie [A beautiful mind] describes the life of the Mathematician John Nash. Nash is introduced while entering Princeton as a young graduate student. The movie shows how Nash was struggling writing his PhD with the title "Non-cooperative games", a work which later would give him the Nobel prize. Nash is described as an impossible college teacher. In a calculus class, he introduces the following problem:

> Find a subset X of three dimensional space which has the property that if V is the set of vector fields F on the complement of X, which satisfy $\text{curl}(F) = 0$ and W is the set of vector fields F which are conservative $F = \text{grad}(f)$. Then, the space V/W should be 8 dimensional.

Good will hunting

The movie [Good will hunting] shows a math prodigy Will Hunting who grew up in a succession of orphanages in South Boston. Working as a janitor at MIT, he has taught himself mathematics. He would anonymously solve complex math problems which were left overnight on blackboards. From an AMS review: "The mathematics referred to later on ranges from basic linear algebra, through simple graph theory, to Parseval's theorem, Fourier analysis, and on to what seem to be some deeper graph theoretical results. Mathematics is referred to constantly, but in no scene is it presented coherently."

Cube

[Cube] Six strangers wake up in a maze of cubes equiped with movie traps and have to find their way out. Each room is equipped with a triple of numbers and colored. If all numbers are simultaneously not prime, then the room is trapped and entering it would kill the person entering it.

Hypercube

[Hypercube] In this horror movie, eight strangers wake up in a bizare cube-shaped room not knowing how they got there or how to escape. They soon learn that their "hypercube" operates in the fourth dimension and shifts into an endless maze of danger and in the end everyone dies. The movie is the sequel to the 1999 cult hit cube. Cube 2 was directed by Andrzej Sekula.

Sneakers

[Sneakers] An espionage thriller with Robert Redford. A hunt for a futuristic device which allows to decrypt secret messages. The device was built by a "genious Mathematician" who appears in the movie giving a pompeous lecture on factorization algorithms. The movie which appeared in 1992 is not totally unrealistic from the mathematical point of view. Shortly after the movie was released, in the year 1993, mathematicians have shown that in principle, a quantum computer could break the factorization difficulty which is the fundament for many modern encryption algorithms. An other interpretation for the device would be that a new algorithm for factoring large integers would be found secretly and be hardwired into a chip.

Pi

[Pi] In the movie Pi, the pursuit of the infinite takes on a deeper meaning. Max Cohen is a number theorist living in New York obsessed with a potentially unsolvable problem. Yet, what the story and the age-old problem uncovers is the deeper link between the mysteries of life and other topics of consciousness as seemingly disparate as the stock market, the Kaballah, technology, the DNA and the stars in the sky.

"11:15 Restate my assumptions:

- Mathematics is the language of nature.
- Everything around us can be represented and understood through numbers.
- If you graph these numbers, patterns emerge. Therefore: There are patterns everywhere in nature."

Max Cohen in Pi

Old School

[Old School] In the college comedy "old school", three men, disenchanted with their life try to recapture their college life and wild youth by opening a frat house. In the movie, some aereal shots of Harvard appear evenso the movie seems have no scenes at all taken in Cambridge. At one point, the fraternaty members have to take a test in which they are asked about Hariotts method to solve cubics.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 8 entries in this file.

Index

ENTRY MEASURE THEORY

[ENTRY MEASURE THEORY] Authors: Oliver Knill: 2003 Literature: measure theory

analytic set

An [analytic set] in a complete seperable metric space is the continuous image of a Borel set. Also called A-set. Any A-set is Lebesgue measurable. Any uncountable A-set topologically contains a perfect Cantor set. Suslins criterium tells that an analytic set is a Borel set if and only if its complement is also an analytic set.

atom

An [atom] is a measurable set Y of positive measure in a measure space such that every subset Z of Y has either zero or the same measure. Often an atom consists of only one point. More generally, an atom is minimal, non-zero element in a Boolean algebra.

atom

A property which holds up to a set of measure zero is said to hold [almost everywhere] (= almost surely).

Banach-Tarski theorem

The [Banach-Tarski theorem]: a ball in Euclidean space of dimension 3 can be decomposed into finitely many sets and rearranged by rigid motion to obtain two balls. The

barycentre

The [barycentre] of a Lebesgue measurable set S in an Euclidean space is the point $\int_S x dx$.

Boolean algebra

A [Boolean algebra] is a set S with two binary operations + and * which are commutative monoids $(S, +, 0)$, $(S, *, 1)$ and satisfy the two distributive laws $(x * (y + z) = x * y + y * z, x + (y * z) = (x + y) * (x + z)$ as well as the complementary laws $x * x = 1, y + y = 0$. A Boolean algebra is especially an algebra. Examples are the algebra of classes, where + is the union and $*$ is the intersection or the algebra of propositions, for which + is *and* and $*$ is *or*.

Boolean ring

A [Boolean ring] is a ring in which every member is idempotent.

Borel-Cantelli lemma

The [Borel-Cantelli lemma]: if Y_n are events in a probability space and the sum of their probabilities is finite, then the probability that infinitely many events occur is zero. If the events are independent and the sum of their probabilities is infinite, then the probability that infinitely many events occur is one.

Borel measure

A [Borel measure] is a measure on the sigma-algebra of Borel sets.

Borel set

A [Borel set] (=Borel measurable set) in a topological space is an element in the smallest sigma-algebra which contains all compact sets. Borel sets are also called B-sets. One can say that a B-set is a set which can be obtained of not more than a countable number of operations of union and intersection of closed open sets in a topological space. Borel sets are special cases of analytic sets.

Borel set

The smallest sigma-algebra A of subsets of a topological space (X,O) containing O is called a Borel sigma-algebra.

absolutely continuous

A measure μ is [absolutely continuous] to a measure ν if $\nu(Y) = 0$ implies $\mu(Y) = 0$.

centre of mass

The [centre of mass] (=barycentre) of a Borel measure μ in a Euclidean space X is the point $\overline{x} = \int_X x\mu(x)$. For example, if μ is supported on finitely many points x_i and $m_i = \mu(x_i)$ then $\overline{x} = \sum_i m_i x_i$. If μ is the mass distribution of a body, then its centre of mass is called the centre of gravity.

abstract integral

[abstract integral]. Denote by L, L^+ the set of measureable maps from a measure space (X, A, μ) to the real line (R, B), where B is the Borel sigma-algebra on R, R^+. For $f \in S = \{f = \sum_{i=1}^n \alpha_i \cdot 1_{A_i} \ \alpha_i \in R\ \}$, define $\int_X f\ d\mu := \sum_{a \in f(X)} a \cdot \mu\{X = a\}$. For $f \in L^+$ define $\int_X f\ d\mu = \sup_{g \in S} \int_X g\ d\mu$. For $f \in L$ finally define $\int f = \int f^+ - \int f^-$, where $f^+(x) = \max(f(x), 0)$ and $f^-(x) = -(-f)^+(x)$.

abstract integral

A sigma-additive function $\mu : A \to [0, \infty]$ on a measurable space (X,A) is called a [measure]. It is called a finite measure if $\mu(X) < \infty$.

measure

A map $f : X \to Y$ where (X, A), (Y, B) are measurable spaces is called [measurable] if $f^{-1}(B) \in A$ for all $B \in B$.

measurable space

The pair (X, A) is called a [measurable space] if

measure space

(X, A, μ) is called a [measure space] if (X, A) is a measurable space and μ is a measure on (X, A).

measure space

[sigma-additive: a real-valued function on a set A of subsets of X is called sigma-additive if for all disjoint $Y_n \in A$, one has $\mu(\bigcup_n Y_n) = \sum_n \mu(Y_n)$.

sigma-algebra

A set A of subsets of a set X is called a [sigma-algebra] if

- X is in A
- $Y \in A$ implies $Y^c \in A$. (iii) $Y_n \in A, n = 1, 2, 3, ...$ implies $\bigcup_{n=1}^{\infty} Y_n \in A$.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 20 entries in this file.

Index

ENTRY NUMBER THEORY

[ENTRY NUMBER THEORY] Authors: Oliver Knill: 2003 Literature: Hua, introduction to number theory

ABC conjecture

[ABC conjecture] If a,b,c are positive integers, let N(a,b,c) be the product of the prime divisors of a,b,c, with each divisor counted only once. The conjecture claims that for every $\epsilon > 0$, there is a constant mu¿1 such that for all coprime a,b and c=a+b, then $max(|a|, |b|, |c|) \leq muN(a, b, c)^(1 + epsilon)$.

irreducible polynomial

A root of an [irreducible polynomial] with integer coefficients is called an [algebraic number].

amicable

Two integers are called [amicable] if each is the sum of the distinct proper factors of the other. For example: 220 and 284 are amicable. Amicable numbers are 2-periodic orbits of the sigma function $\sigma(n)$ which is the sum of the divisors of n.

Apery's theorem

[Apery's theorem]: the value of the zeta function at z=3 is rational.

arithmetic function

An [arithmetic function] is a function $f(n)$ whose domain is the set of positive integers. An important class of arithmetic functions are multiplicative functions $f(nm) = f(n)f(m)$. An example is the Möbius function $\mu(n)$ defined by $\mu(1) = 1$, $\mu(n) = (-1)^r$ if n is the product of r distinct primes and $\mu(n) = 0$ otherwise.

Artinian conjecture

[Artinian conjecture]: a quantitative form of the conjecture that every non-square integer is a primitive root of infinitely many primes. [Beal's conjecture] If $a^x + b^y = c^z$, where a,b,c,x,y,z are positive integers and $x, y, z > 2$, then a, b, c must have a common factor. It is known that for every x, y, z, there are only finitely many solutions. The Beal conjecture is a ageneralization of Fermat's last theorem. The conjecture was announced in December 1997. The prize is now 100'000 Dollars for either a proof or a counterexample. The conjecture was discovered by the Texan number theory enthusiast and banker Andrew Beal.

Bertrands postulate

[Bertrands postulate] tells that for any integer n greater than 3, there is a prime between n and $2n - 2$. The postulate is a theorem, proven by Tchebychef in 1850.

Bezout's lemma

[Bezout's lemma] tells that if f and g are polynomials over a field K and d is the greatest common divisor of f and g, then $d = af + bg$, where a,b are two other polynomials. This generalizes Euclid's theorem for integers.

Brun's constant

The [Brun's constant] is the sum of the reciprocals of all the prime twins. It is estimated to be about 1.9021605824. While one does not know, whether infinitely many prime twins exist, the sum of their reciprocals is known to be finite. This has been proven by the Norwegian Mathematician Viggo Brun (1885-1978) in 1919.

Carmichael numbers

[Carmichael numbers] are natural numbers which are Fermat pseudoprime to any base. Named after R.D. Carmichael who discovered them in 1909. It is known that there are infinitely many Carmichael numbers. The Carmichael numbers under 100'000 are 561, 1105, 1729, 2465, 2821, 6601, 8911, 10585, 15841, 29341, 41041, 46657, 52633, 62745, 63973, and 75361.

Chineese reminder problem

The [Chineese reminder problem] tells that if $n_1, \dots, n_k$ are natural numbers which are pairwise relatively prime and if $a_1, \dots, a_k$ are any integers, then there exists an integer x which solves simultaneously the congruences $x == a_1(mod n_1), \dots, x == a_k(mod n_k)$. All solutions are congruent to a given solution modulo $\prod_{j=1}^{k} n_j$. The theorem was established by Qin Jiushao in 1247.

coprime

Two numbers a,b are called [coprime] if their greatest common divisor is 1.

ElGamal

[ElGamal] The ElGamal cryptosystem is based on the difficulty to solve the discrete logarithm problem modulo a large number $n = p^r$, where p is a prime and and r is a positive integer: solving $g^x = b \mathrm{mod} n$ for x is computationally hard. Suppose you want to send a message encoded as an integer m to Alice. A large integer n and a base g are chosen and public. Alice who has a secrete integer a has published the integer $c = g^k \mathrm{mod}(n)$ as her public key. Everybody knows n, g, c. To send Alice a message m, we chose an integer k at random and send Alice the pair $(A, B) = (g^m, kg^{am})$ modulo n. (We can compute $g^{am} = (g^a)^m = c^m$ using the publically availble information only.) Alice can recover from this the secret message $m = A^a/B$. However, somebody intercepting the message is not able to recover m without knowing a. He would have to find the discrete logarithm of g^m with base g to do so but this is believed to be a computationally difficult problem.

Farey Sequence

The [Farey Sequence] of order n is the finite sequence of rational numbers a/b, with $0 \leq a \leq b \leq n$ such that a, b have no common divisor different from 1 and which are arranged in increasing order.

$F_1 =$	$(0/1, 1/1)$
$F_2 =$	$(0/1, 1/2, 1)$
$F_3 =$	$(0/1, 1/3, 1/2, 2/3, 1/1)$
$F_4 =$	$(0/1, 1/4, 1/3, 1/2, 2/3, 3/4)$
$F_5 =$	$(0/1, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4 4/5)$

Number

[Number]:

N:	natural numbers	e.g. $1, 2, 3, 4, ...$
Z:	integers	e.g. $-1, 0, 1, 2, ...$
Q:	rational numbers	e.g. $5/6, 5, -8/10$
R:	real numbers	e.g. $1, \pi, e, \sqrt(2), 5/4$
C:	complex numbers	e.g. $i, 2, e + i\pi/2, 4\pi/e$

natural numbers

The numbers $1, 2, 3, ...$ are called the [natural numbers].

Fermats last theorem

[Fermats last theorem]. For any ineger n bigger than 2, the equation $x^n + y^n = z^n$ has no solutions in positive integers. This theorem was proven in 1995 by Andrew Wiles with the assistance of Richard Taylor. The theorem has a long history: in an annotation of his copy "Diophantus", Fermat wrote a note: "On the other hand, it is impossible to seperate a cube into two cubes, or a biquadrate into two biquadrates, or generally any power except a squre into two powers with the same exponent. I have discovered a truely marvelous proof of this which however the margin is not large enough to contain."

Fermat-Catalan Conjecture

[Fermat-Catalan Conjecture] There are only finitely many triples of coprime integer powers x^q, y^q, z^r for which $x^p + y^q = z^r$ with $1/p + 1/q + 1/r < 1$.

Fermats little theorem

[Fermats little theorem] If p is prime and a is an integer which is not a multiple of p, then $a^(p-1) = 1 mod p$. Example: $2^4 = 16 = 1 \pmod 5$. Fermats little theorem is a consequence of the Lagrange theorem in algebra, which says that for finite groups, the order of a subgroup divides the order of the group. Fermats theorem is the special case, when the finite group is the cyclic group with $p-1$ elements. Fermats little theorem is somtimes also stated in the form: for every integer a and prime number p, the number $a^p - a$ is a multiple of p.

Fermat numbers

Numbers $F_n = 2^(2^n) + 1$ are called [Fermat numbers]. Examples are

$F_0 = 3$	prime
$F_1 = 5$	prime
$F_2 = 17$	prime
$F_3 = 257$	prime
$F_4 = 65537$	prime
$F_5 = 641 \cdot 6700417$	composite

No other prime Fermat number beside the first 5 had been found so far.

fundamental theorem of arithmetic

The [fundamental theorem of arithmetic] assures that every natural number n has a unique prime factorization. In other words, there is only one way in which one can write a number as a product of prime numbers if the order of the product does not matter. For example, $84 = 2237$ is the prime factorization of 84.

Goldbach's conjecture

[Goldbach's conjecture]: Every even integer n greater than two is the sum of two primes. For example: $8 = 5+3$ or $20 = 13 + 7$. The conjecture has not been proven yet.

greatest common divisor

The [greatest common divisor] of two integers n and m is the largest integer d such that d divides n and d divides m. One writes $d = gcd(n, m)$. For example, $gcd(6, 9) = 3$. There are few recursive algorithms for gcd: one of them is the Euclidean algorithm: $gcd(m, n) = \{k = m \ mod \ n; if \ (k == 0) \ return(n); else \ return(gcd(n, k))\}$.

prime number or prime

A positive integer n is called a [prime number] or [prime], if it is divisible by 1 and n only. The first prime numbers are $2, 3, 5, 7, 11, 13, 17$. An example of a non prime number is 12 because it is divisible by 3. There are infinitely many primes. Every natural number n can be factorized uniquely into primes: for example $42 = 237$.

prime factorization

The [prime factorization] of a positive integer n is a sequence of primes whose product is n. For example: $18 = 3 \cdot 3 \cdot 2$ or $100 = 2 \cdot 2 \cdot 5 \cdot 5$ or $17 = 17$. Every integer has a unique prime factorization.

Pells equation

Fermat claimed first that [Pells equation] $dy^2 + 1 = x^2$, where d is an integer has always integer solutions x and y. The name "Pell equation" was given by Euler evenso Pell seems nothing have to do with the equation. Lagrange was the first to prove the existence of solutions. One can find solutions by performing the Continued fraction expansion of the square root of d.

Fermat number

A [Fermat number] is an integer of the form $F_k = 2^(2^k) + 1$. The first Fermat numbers are $F_0 = 3, F_1 = 5, F_2 = 17, F_3 = 257, F_4 = 65537$. They are all primes and called Fermat primes. Fermat had claimed that all F_k are primes. Euler disproved that showing that 641 divides F_5. The Fermat numbers F_5 until F_9 are known to be not prime and also have been factored. Fermat numbers play a role in constructing regular polygons with ruler and compass. The factorization of Fermat numbers serves as a challenge to factorization algorithms.

Fermat prime

A [Fermat prime] is a Fermat number which is prime.

Mersenne number

An integer $2^n - 1$ is called a [Mersenne number]. If a Mersenne number is prime, it is called a Mersenne prime.

Mersenne number

If a Mersenne number $2^n - 1$ is prime, it is called a [Mersenne prime]. In that case, n must be prime. Known examples are n = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377. It is not known whether there are infinitely many Mersenne primes.

perfect number

An integer n is called a [perfect number] if it is equal to the sum of its proper divisors. For example $6 = 1+2+3$ or $28 = 1+2+4+7+14$ are perfect numbers. Also, if $2^n - 1$ is prime then $2^{n-1}(2^n - 1)$ is perfect because $1+2+4+...+2^(n-1) = 2^n - 1$. This was known already to Euclid. Every even perfect number is of the form $p(p+1)/2$, where $p = 2^n - 1$ is a Mersenne prime. It is not known whether there is an odd perfect number, nor whether there are infinitely many Mersenne primes.

partition

The [partition] of a number n is a decomposition of n into a sum of integers. Examples are $5 = 1+2+2$. The number of partitions of a number n is denoted by $p(n)$ and plays a role in the theory of representations of finite groups. For example: $p(4) = 5$ because of the following partitions $4 = 3+1 = 2+2 = 1+1+2 = 1+1+1+1$ Euler introduced the Power series $f(x) = \sum p(n)x^n$ which is $(1-x)^{-1}(1-x^2)^{-1}....$ The algebra of formal power series leads to powerful identites like $(1+x)^{-1}(1+x^2)^{-1}(1+x^3)^{-1}... = (1+x)(1-x)(1+x^2)(1-x^2).../(1-x)(1-x^2)... = (1-x^2)(1-x^4).../(1-x)(1-x^2)... = (1-x)^{-1}(1-x^3)^{-1}(1-x^5)^{-1}....$ The left hand side is the generating function for $a(n)$, the number of partitions of n into distinct numbers. The right hand side is the generating function of $b(n)$, the number of partitions of n into an odd number of summands. The algebraic identity has shown that $a(n) = b(n)$. For example, for $n = 5$, one has $a(5) = 3$ decomposition $5 = 5 = 4+1 = 3+2$ into different summands and also $b(5) = 3$ decompositions into an odd number of summands $5 = 5 = 2+2+1 = 1+1+1+1+1$.

prime number

A [prime number] is an positive integer which is divisible only by 1 or itself. For example, 12 is not a prime number because it is divisible by 3 but the integer 13 is a prime number. The first prime numbers are $2, 3, 5, 7, 11, 13, 17, 23....$ There are infinitely many prime numbers because if there were only finitely many, their product $p_1 p_2 ... p_k = n$ has the property that $n+1$ is not divisible by any p_i. Therefore, $n+1$ would either be a new prime number of be divisible by a new prime number. This contradicts the assumption that there are only finitely many.

prime twin

[prime twin] Two positive integers $p, p+2$ are called prime twins if both p and $p+2$ are prime numbers. For example $(3, 5)$, $(11, 13)$ and $17, 19$ are prime twins. It is unknown, whether there are infinitely many prime twins. One knows that $\sum_i 1/p_i$, where $(p_i, p_i + 2)$ are prime times is finite.

Pythagorean triple

Three integers x, y, z form a [Pythagorean triple] if $x^2 + y^2 = z^2$. An example is $3^2 + 4^2 = 5^2$. Pythagorean triples define triangles with a right angle and integer side lengths x, y, z. They were known and useful already by the Babylonians and used to triangulate rectangular regions. The Pythagorean triples with even x can be parameterized with $p > q$ and $x = 2pq, y = p^2 - q^2$, $z = p^2 + q^2$. Each Pythagrean triple corresponds to rational points on the unit circle: $X^2 + Y^2 = 1$, where $X = x/z, Y = y/z$.

relatively prime

Two integers n and m are [relatively prime] if their greatest common divisor $gcd(n, m)$ is 1. In other words, one does not find a common factor of n and m other than 1.

Sieve of Eratosthenes

The [Sieve of Eratosthenes] allows to construct prime numbers. By sieving away all multiplies of $2, 3, ..., N$ and listing what is left, one obtains a list of all the prime numbers smaller than N^2. For example:

multiples of 2:	4,6,8,10,12,14,16,18,20,22,24,26,...
multiplis of 3:	6,9,12,15,18,21,24,...
multiples of 5:	10,15,20,25,...

The numbers $2, 3, 5, 7, 11, 13, 17, 19, 23$ do not appear in this list and are all the prime numbers smaller or equal than 5^2. To list all the prime number up to N^2, one would have to list all the multiples of k for $k \leq N$.

quadratic residue

A square modulo m, then n is called a A [quadratic residue] modulo m is an integer n which is a square modulo m. That is one can find an integer x such that $n = x^2$ (modm). If m is not a quadratic residue, it is called a quadratic non-residue modulo n. Examples:

- 2 is a quadratic residue modulo 7 because $3^2 = 2$ modulo 7.
- If p is an odd prime, then there are $(p-1)/2$ quadratic residues and $(p-1)/2$ quadratic nonresidues modulo p.

Legendre symbol

The [Legendre symbol] encodes, whether n is a quadratic residue modulo a prime number p or not: $Legendre(n, p) = 1$ if n is a quadratic residue and $Legendre(n, p) = -1$ if n is not a quadratic residue. If p is a prime number, then $Legendre(-1, p) = (-1)^((p-1)/2)$ and $Legendre(2, p) = (-1)^((p^2-1)/8)$.

law of quadratic reciprocity

The [law of quadratic reciprocity] tells that if p, q are distinct odd prime numbers, then $Legendre(p, q) \cdot Legendre(q, p) = (-1)^n$, where $n = (p-1)(q-1)/4$. Gauss called this result the "queen of number theory". The theorem implies that if $p = 3$ (mod4) and $q = 3$ (mod4), then exactly one of the two congruences $x^2 = p$ (modq) or $x^2 = q$ (modp) is solvable. Otherwise, either both or none is solvable.

Jacobi symbol

[Jacobi symbol] If $m = p_1...p_k$ is the prime factorization of m, define $Jacobi(n, m)$ as the product of $Legendre(n, p_i)$, where $Legendre(n, p)$ denotes the Legendre symbol of n and p and $m = p_1 \cdots p_k$ is the prime factorization of m.

Jacobi symbol

A positive integer a which generates the multiplicative group modulo a prime number p is called a [primitive root of p]. Examples:

- $a = 2$ is a primitive root modulo $p = 5$, because $1 = a^0, 2 = a^1, 4 = a^2, 3 = a^3$ is already the list of elements in the multiplicative group of 5.
- $a = 4$ is not a primitive root modulo $p - 5$ because $4^2 = 1$ mod5.

Liouville

An irrational number a is called [Liouville] if there exists for every integer m a sequence p_n/q_n of irreducible fractions such that $\lim_{n\to\infty} q_n^m |a - p_n/q_n| = 0$. Liouville numbers form a class of irrational numbers which can be approximated well by rational numbers. An example of a Liouville number is 0.1010010000100000001... ,where the number of zeros between the 1's grows exponentially.

Möbius function

The [Möbius function] μ is an example of multiplicative arithmetic function. It is defined as

$$\mu(n) = \begin{cases} 1 & n = 1 \\ (-1)^r & n = p_1 \cdot .. \cdot p_r, p_i < p_{i+1} \\ 0 & \text{otherwise} \end{cases} .$$

sigma function

Orbits of the [sigma function] $\sigma(n)$ giving the sum of the divisors of n is called an [aliquot sequence]. One starts with a number n and forms $\sigma(n), \sigma(\sigma(n))$ etc. Example: $12, 16, 15, 9, 4, 3, 1$. Perfect numbers are fixed points, amicable numbers are periodic orbits. Higher periodic orbits are called sociable chains. The Catalan conjecture states that every aliquot sequence

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 45 entries in this file.

Index

ENTRY PHYSICS

[ENTRY PHYSICS] Authors: Oliver Knill: 2002 Literature: Fundamental formulas in physics (edited by D.H. Menzel) Kneubuehl, Repetitorium in physics

angular momentum

[angular momentum] If $r(t)$ is the position of a mass point of mass m, then the vector $L = mrxr'$ is called the angular momentum.

C14 method

[C14 method] Used to estimate the age of substances containing carbon. It is useful in the range between 500 and 50'000 years. The C14 method is based on the asumption that the atmosphere has a constant C14 isotope conentration. The decay of C14 is compensated by creation of C14 in the stratosphere through cosmic radiation. A living plant has the same C14 concentration as the atmosphere. When it dies, the exchange of air stops and the C14 concentration in the plant will decay.

Gravity

[Gravity] a fundamental force which is responsible for the attraction of different masses like for example the Sun and the earth.

Keplers laws

[Keplers laws]
1. Law: (1609) Planets move on ellipses around the sun. The sun is in a focal point of the ellipse.
2. Law: (1609) The radius vector from the sun to the planet covers equal area in equal time.
3. Law: (1619) The squares of the periods of the planets are proportional to the cubes of the semiaxes of the planets.

Newton laws

[Newton laws] (Newton 1686) established four axioms
1. law) Bodies not subject to forces move along straight lines: $r'' = 0$.
2. law) Force is mass times acceleration $F = mr''$.
3. law) To every action there is a reation: $F_{12} = -F_{21}$.
4. law) Forces add like vectors: two forces F_1, F_2 acting on a body can be replaced by $F_1 + F_2$.

Mass

[Mass] measures amount of material in a body. The SI unit is 1 kilogram = 1kg. One liter of water at temperature $4°C$ has the mass of one kilogram. Typical masses are

- Electron $0.9 \cdot 10^{-30} kg$
- Hydrogen atom $210^{-27} kg$
- Virus $610^{-19} kg$
- Earth $6 \cdot 10^{24} kg$
- Sun $2 \cdot 10^{30} kg$
- Milkyway $10^{41} kg$
- Universe $10^{52} kg$

Length

[Length] measures the position in space. The SI unit is 1 meter = 1m. The meter was originally defined as 1/40 millionth of the meridian of the earth but is sine 1960 defined spectroscopically. Typical lengths are

- Diameter of an atomic nucleus $3 \cdot 10^{-15} m$.
- Wave length of the visible light $5 \cdot 10^{-7} m$.
- Diameter of the earth $1.3 \cdot 10^{7} m$
- Diameter of the sun $1.4 \cdot 10^{9} m$
- Distance to alpha centauri $4 \cdot 10^{16} m$
- Diameter of the Milkyway $7 \cdot 10^{20} m$
- Diameter of universe $10^{26} m$.

Maser

[Maser] Maser stands for Microwave Amplification by Stiumlated Emision of Radiation. Masers are oscillators whose frequency are determined by quantized states of atoms or molecules. The frequencies of a Maser are in the microwave range.
[Power] is work per time or force times space. The SI unit is 1 Watt $1W = 1kgm^2/s^3$.

Time

[Time] measures the position on the time axes. The SI unit is 1 second = 1s. The second was originally defined as a fraction of one tropical year. Since 1967 it is defined as 1s=9'192'631 177 periods of a Cesium 133 Maser oscillation. Typicl times:

- Light passing through kernel $10^{-24}s$.
- Light passes through atom $10^{-19}s$.
- Period of light $10^{-15}s$.
- Period of sound $10^{-3}s$.
- One day $10^5 s$.
- Life of a human $210^9 s$.
- Age of earth $1.3 \cdot 10^{17} s$
- Age of universe $5 \cdot 10^{17} s$

Relativitistic addition of velocities

[Relativitistic addition of velocities] As Poincare has realized first, the Maxwell equations are not invariant under Galilei transformations but under Lorentz transformations. The Michelson-Morely measurements of the speed of lights showed that light has a constant speed. The addition of velocities has therefore to be modified to a relativistic addition $v = (v_1 + v_2)/(1 + v_1 v_2/c^2)$. Using a mass which depends on the velocity $m = m_0/sqrt1 - v^2/c^2$ the Newton laws hold unmodified.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 10 entries in this file.

Index

ENTRY POLYHEDRA

[ENTRY POLYHEDRA] Authors: Oliver Knill: December 2000 Source: Translated into this format from data given in http://netlib.bell-labs.com/netlib

tetrahedron

The [tetrahedron] is a polyhedron with 4 vertices and 4 faces. The dual polyhedron is called tetrahedron.

cube

The [cube] is a polyhedron with 8 vertices and 6 faces. The dual polyhedron is called octahedron.

hexahedron

The [hexahedron] is a polyhedron with 8 vertices and 6 faces. The dual polyhedron is called octahedron.

octahedron

The [octahedron] is a polyhedron with 6 vertices and 8 faces. The dual polyhedron is called cube.

dodecahedron

The [dodecahedron] is a polyhedron with 20 vertices and 12 faces. The dual polyhedron is called icosahedron.

icosahedron

The [icosahedron] is a polyhedron with 12 vertices and 20 faces. The dual polyhedron is called dodecahedron.

small stellated dodecahedron

The [small stellated dodecahedron] is a polyhedron with 12 vertices and 12 faces. The dual polyhedron is called great dodecahedron.

great dodecahedron

The [great dodecahedron] is a polyhedron with 12 vertices and 12 faces. The dual polyhedron is called small stellated dodecahedron.

great stellated dodecahedron

The [great stellated dodecahedron] is a polyhedron with 20 vertices and 12 faces. The dual polyhedron is called great icosahedron.

great icosahedron

The [great icosahedron] is a polyhedron with 12 vertices and 20 faces. The dual polyhedron is called great stellated dodecahedron.

truncated tetrahedron

The [truncated tetrahedron] is a polyhedron with 12 vertices and 8 faces. The dual polyhedron is called triakis tetrahedron.

cuboctahedron

The [cuboctahedron] is a polyhedron with 12 vertices and 14 faces. The dual polyhedron is called rhombic dodecahedron.

truncated cube

The [truncated cube] is a polyhedron with 24 vertices and 14 faces. The dual polyhedron is called triakis octahedron.

truncated octahedron

The [truncated octahedron] is a polyhedron with 24 vertices and 14 faces. The dual polyhedron is called tetrakis hexahedron.

rhombicuboctahedron

The [rhombicuboctahedron] is a polyhedron with 24 vertices and 26 faces. The dual polyhedron is called trapezoidal icositetrahedron.

great rhombicuboctahedron

The [great rhombicuboctahedron] is a polyhedron with 48 vertices and 26 faces. The dual polyhedron is called hexakis octahedron.

snub cube

The [snub cube] is a polyhedron with 24 vertices and 38 faces. The dual polyhedron is called pentagonal icositetrahedron.

icosidodecahedron

The [icosidodecahedron] is a polyhedron with 30 vertices and 32 faces. The dual polyhedron is called rhombic triacontahedron.

truncated dodecahedron

The [truncated dodecahedron] is a polyhedron with 60 vertices and 42 faces. The dual polyhedron is called triakis icosahedron.

truncated icosahedron

The [truncated icosahedron] is a polyhedron with 60 vertices and 32 faces. The dual polyhedron is called pentakis dodecahedron.

rhombicosidodecahedron

The [rhombicosidodecahedron] is a polyhedron with 60 vertices and 62 faces. The dual polyhedron is called trapezoidal hexecontahedron.

great rhombicosidodecahedron

The [great rhombicosidodecahedron] is a polyhedron with 120 vertices and 62 faces. The dual polyhedron is called hexakis icosahedron.

snub dodecahedron

The [snub dodecahedron] is a polyhedron with 60 vertices and 92 faces. The dual polyhedron is called pentagonal hexacontahedron.

triangular prism

The [triangular prism] is a polyhedron with 6 vertices and 5 faces.

pentagonal prism

The [pentagonal prism] is a polyhedron with 10 vertices and 7 faces.

hexagonal prism

The [hexagonal prism] is a polyhedron with 12 vertices and 8 faces.

octagonal prism

The [octagonal prism] is a polyhedron with 16 vertices and 10 faces.

decagonal prism

The [decagonal prism] is a polyhedron with 20 vertices and 12 faces.

square antiprism

The [square antiprism] is a polyhedron with 8 vertices and 10 faces.

pentagonal antiprism

The [pentagonal antiprism] is a polyhedron with 10 vertices and 12 faces.

hexagonal antiprism

The [hexagonal antiprism] is a polyhedron with 12 vertices and 13 faces.

octagonal antiprism

The [octagonal antiprism] is a polyhedron with 16 vertices and 18 faces.

decagonal antiprism

The [decagonal antiprism] is a polyhedron with 20 vertices and 22 faces.

triakis tetrahedron

The [triakis tetrahedron] is a polyhedron with 8 vertices and 12 faces. The dual polyhedron is called truncated tetrahedron.

rhombic dodecahedron

The [rhombic dodecahedron] is a polyhedron with 14 vertices and 12 faces. The dual polyhedron is called cuboctahedron.

triakis octahedron

The [triakis octahedron] is a polyhedron with 14 vertices and 24 faces. The dual polyhedron is called truncated cube.

tetrakis hexahedron

The [tetrakis hexahedron] is a polyhedron with 14 vertices and 24 faces. The dual polyhedron is called truncated octahedron.

trapezoidal icositetrahedron

The [trapezoidal icositetrahedron] is a polyhedron with 26 vertices and 24 faces. The dual polyhedron is called rhombicuboctahedron.

hexakis octahedron

The [hexakis octahedron] is a polyhedron with 26 vertices and 48 faces. The dual polyhedron is called great rhombicuboctahedron.

pentagonal icositetrahedron

The [pentagonal icositetrahedron] is a polyhedron with 38 vertices and 24 faces. The dual polyhedron is called snub cube.

rhombic triacontahedron

The [rhombic triacontahedron] is a polyhedron with 32 vertices and 30 faces. The dual polyhedron is called icosidodecahedron.

triakis icosahedron

The [triakis icosahedron] is a polyhedron with 32 vertices and 60 faces. The dual polyhedron is called truncated dodecahedron.

pentakis dodecahedron

The [pentakis dodecahedron] is a polyhedron with 32 vertices and 60 faces. The dual polyhedron is called truncated icosahedron.

trapezoidal hexecontahedron

The [trapezoidal hexecontahedron] is a polyhedron with 62 vertices and 60 faces. The dual polyhedron is called rhombicosidodecahedron.

hexakis icosahedron

The [hexakis icosahedron] is a polyhedron with 62 vertices and 120 faces. The dual polyhedron is called great rhombicosidodecahedron.

pentagonal hexecontahedron

The [pentagonal hexecontahedron] is a polyhedron with 92 vertices and 60 faces. The dual polyhedron is called snub dodecahedron.

square pyramid

The [square pyramid] is a polyhedron with 5 vertices and 5 faces.

pentagonal pyramid

The [pentagonal pyramid] is a polyhedron with 6 vertices and 6 faces.

triangular cupola

The [triangular cupola] is a polyhedron with 9 vertices and 8 faces.

square cupola

The [square cupola] is a polyhedron with 12 vertices and 10 faces.

pentagonal cupola

The [pentagonal cupola] is a polyhedron with 15 vertices and 12 faces.

pentagonal rotunda

The [pentagonal rotunda] is a polyhedron with 20 vertices and 17 faces.

elongated triangular pyramid

The [elongated triangular pyramid] is a polyhedron with 7 vertices and 7 faces.

elongated square pyramid

The [elongated square pyramid] is a polyhedron with 9 vertices and 9 faces.

elongated pentagonal pyramid

The [elongated pentagonal pyramid] is a polyhedron with 11 vertices and 11 faces.

gyroelongated square pyramid

The [gyroelongated square pyramid] is a polyhedron with 9 vertices and 13 faces.

gyroelongated pentagonal pyramid

The [gyroelongated pentagonal pyramid] is a polyhedron with 11 vertices and 16 faces.

triangular dipyramid

The [triangular dipyramid] is a polyhedron with 5 vertices and 6 faces.

pentagonal dipyramid

The [pentagonal dipyramid] is a polyhedron with 7 vertices and 10 faces.

elongated triangular dipyramid

The [elongated triangular dipyramid] is a polyhedron with 8 vertices and 9 faces.

elongated square dipyramid

The [elongated square dipyramid] is a polyhedron with 10 vertices and 12 faces.

elongated pentagonal dipyramid

The [elongated pentagonal dipyramid] is a polyhedron with 12 vertices and 15 faces.

gyroelongated square dipyramid

The [gyroelongated square dipyramid] is a polyhedron with 10 vertices and 16 faces.

elongated triangular cupola

The [elongated triangular cupola] is a polyhedron with 15 vertices and 14 faces.

elongated square cupola

The [elongated square cupola] is a polyhedron with 20 vertices and 18 faces.

elongated pentagonal cupola

The [elongated pentagonal cupola] is a polyhedron with 25 vertices and 22 faces.

elongated pentagonal rotunds

The [elongated pentagonal rotunds] is a polyhedron with 30 vertices and 27 faces.

gyroelongated triangular cupola

The [gyroelongated triangular cupola] is a polyhedron with 15 vertices and 20 faces.

gyroelongated square cupola

The [gyroelongated square cupola] is a polyhedron with 20 vertices and 26 faces.

gyroelongated pentagonal cupola

The [gyroelongated pentagonal cupola] is a polyhedron with 25 vertices and 32 faces.

gyroelongated pentagonal rotunda

The [gyroelongated pentagonal rotunda] is a polyhedron with 30 vertices and 37 faces.

gyrobifastigium

The [gyrobifastigium] is a polyhedron with 8 vertices and 8 faces.

triangular orthobicupola

The [triangular orthobicupola] is a polyhedron with 12 vertices and 14 faces.

square orthobicupola

The [square orthobicupola] is a polyhedron with 16 vertices and 18 faces.

square gyrobicupola

The [square gyrobicupola] is a polyhedron with 16 vertices and 18 faces.

pentagonal orthobicupola

The [pentagonal orthobicupola] is a polyhedron with 20 vertices and 22 faces.

pentagonal gyrobicupola

The [pentagonal gyrobicupola] is a polyhedron with 20 vertices and 22 faces.

pentagonal orthocupolarontunda

The [pentagonal orthocupolarontunda] is a polyhedron with 25 vertices and 27 faces.

pentagonal gyrocupolarotunda

The [pentagonal gyrocupolarotunda] is a polyhedron with 25 vertices and 27 faces.

pentagonal orthobirotunda

The [pentagonal orthobirotunda] is a polyhedron with 30 vertices and 32 faces.

elongated triangular orthobicupola

The [elongated triangular orthobicupola] is a polyhedron with 18 vertices and 20 faces.

elongated triangular gyrobicupola

The [elongated triangular gyrobicupola] is a polyhedron with 18 vertices and 20 faces.

elongated square gyrobicupola

The [elongated square gyrobicupola] is a polyhedron with 24 vertices and 26 faces.

elongated pentagonal orthobicupola

The [elongated pentagonal orthobicupola] is a polyhedron with 30 vertices and 32 faces.

elongated pentagonal gyrobicupola

The [elongated pentagonal gyrobicupola] is a polyhedron with 30 vertices and 32 faces.

elongated pentagonal orthocupolarotunda

The [elongated pentagonal orthocupolarotunda] is a polyhedron with 35 vertices and 37 faces.

elongated pentagonal gyrocupolarotunda

The [elongated pentagonal gyrocupolarotunda] is a polyhedron with 35 vertices and 37 faces.

elongated pentagonal orthobirotunda

The [elongated pentagonal orthobirotunda] is a polyhedron with 40 vertices and 42 faces.

elongated pentagonal gyrobirotunda

The [elongated pentagonal gyrobirotunda] is a polyhedron with 40 vertices and 42 faces.

gyroelongated triangular bicupola

The [gyroelongated triangular bicupola] is a polyhedron with 18 vertices and 26 faces.

gyroelongated square bicupola

The [gyroelongated square bicupola] is a polyhedron with 24 vertices and 34 faces.

gyroelongated pentagonal bicupola

The [gyroelongated pentagonal bicupola] is a polyhedron with 30 vertices and 42 faces.

gyroelongated pentagonal cupolarotunda

The [gyroelongated pentagonal cupolarotunda] is a polyhedron with 35 vertices and 47 faces.

gyroelongated pentagonal birotunda

The [gyroelongated pentagonal birotunda] is a polyhedron with 40 vertices and 52 faces.

augmented triangular prism

The [augmented triangular prism] is a polyhedron with 7 vertices and 8 faces.

biaugmented triangular prism

The [biaugmented triangular prism] is a polyhedron with 8 vertices and 11 faces.

triaugmented triangular prism

The [triaugmented triangular prism] is a polyhedron with 9 vertices and 14 faces.

augmented pentagonal prism

The [augmented pentagonal prism] is a polyhedron with 11 vertices and 10 faces.

biaugmented pentagonal prism

The [biaugmented pentagonal prism] is a polyhedron with 12 vertices and 13 faces.

augmented hexagonal prism

The [augmented hexagonal prism] is a polyhedron with 13 vertices and 11 faces.

parabiaugmented hexagonal prism

The [parabiaugmented hexagonal prism] is a polyhedron with 14 vertices and 14 faces.

metabiaugmented hexagonal prism

The [metabiaugmented hexagonal prism] is a polyhedron with 14 vertices and 14 faces.

triaugmented hexagonal prism

The [triaugmented hexagonal prism] is a polyhedron with 15 vertices and 17 faces.

augmented dodecahedron

The [augmented dodecahedron] is a polyhedron with 21 vertices and 16 faces.

parabiaugmented dodecahedron

The [parabiaugmented dodecahedron] is a polyhedron with 22 vertices and 20 faces.

metabiaugmented dodecahedron

The [metabiaugmented dodecahedron] is a polyhedron with 22 vertices and 20 faces.

triaugmented dodecahedron

The [triaugmented dodecahedron] is a polyhedron with 23 vertices and 24 faces.

metabidiminished icosahedron

The [metabidiminished icosahedron] is a polyhedron with 10 vertices and 12 faces.

tridiminished icosahedron

The [tridiminished icosahedron] is a polyhedron with 9 vertices and 8 faces.

augmented tridiminished icosahedron

The [augmented tridiminished icosahedron] is a polyhedron with 10 vertices and 10 faces.

augmented truncated tetrahedron

The [augmented truncated tetrahedron] is a polyhedron with 15 vertices and 14 faces.

augmented truncated cube

The [augmented truncated cube] is a polyhedron with 28 vertices and 22 faces.

biaugmented truncated cube

The [biaugmented truncated cube] is a polyhedron with 32 vertices and 30 faces.

augmented truncated dodecahedron

The [augmented truncated dodecahedron] is a polyhedron with 65 vertices and 42 faces.

parabiaugmented truncated dodecahedron

The [parabiaugmented truncated dodecahedron] is a polyhedron with 70 vertices and 52 faces.

metabiaugmented truncated dodecahedron

The [metabiaugmented truncated dodecahedron] is a polyhedron with 70 vertices and 52 faces.

triaugmented truncated dodecahedron

The [triaugmented truncated dodecahedron] is a polyhedron with 75 vertices and 62 faces.

gyrate rhombicosidodecahedron

The [gyrate rhombicosidodecahedron] is a polyhedron with 60 vertices and 62 faces.

parabigyrate rhombicosidodecahedron

The [parabigyrate rhombicosidodecahedron] is a polyhedron with 60 vertices and 62 faces.

metabigyrate rhombicosidodecahedron

The [metabigyrate rhombicosidodecahedron] is a polyhedron with 60 vertices and 62 faces.

trigyrate rhombicosidodecahedron

The [trigyrate rhombicosidodecahedron] is a polyhedron with 60 vertices and 62 faces.

diminished rhombicosidodecahedron

The [diminished rhombicosidodecahedron] is a polyhedron with 55 vertices and 52 faces.

paragyrate diminished rhombicosidodecahedron

The [paragyrate diminished rhombicosidodecahedron] is a polyhedron with 55 vertices and 52 faces.

metagyrate diminished rhombicosidodecahedron

The [metagyrate diminished rhombicosidodecahedron] is a polyhedron with 55 vertices and 52 faces.

bigyrate diminished rhombicosidodecahedron

The [bigyrate diminished rhombicosidodecahedron] is a polyhedron with 55 vertices and 52 faces.

parabidiminished rhombicosidodecahedron

The [parabidiminished rhombicosidodecahedron] is a polyhedron with 50 vertices and 42 faces.

metabidiminished rhombicosidodecahedron

The [metabidiminished rhombicosidodecahedron] is a polyhedron with 50 vertices and 42 faces.

gyrate bidiminished rhombicosidodecahedron

The [gyrate bidiminished rhombicosidodecahedron] is a polyhedron with 50 vertices and 42 faces.

tridiminished rhombicosidodecahedron

The [tridiminished rhombicosidodecahedron] is a polyhedron with 45 vertices and 32 faces.

snub disphenoid

The [snub disphenoid] is a polyhedron with 8 vertices and 12 faces.

snub square antiprism

The [snub square antiprism] is a polyhedron with 16 vertices and 26 faces.

sphenocorona

The [sphenocorona] is a polyhedron with 10 vertices and 14 faces.

augmented sphenocorona

The [augmented sphenocorona] is a polyhedron with 11 vertices and 17 faces.

sphenomegacorona

The [sphenomegacorona] is a polyhedron with 12 vertices and 18 faces.

hebesphenomegacorona

The [hebesphenomegacorona] is a polyhedron with 14 vertices and 21 faces.

disphenocingulum

The [disphenocingulum] is a polyhedron with 16 vertices and 24 faces.

bilunabirotunda

The [bilunabirotunda] is a polyhedron with 14 vertices and 14 faces.

triangular hebesphenorotunda

The [triangular hebesphenorotunda] is a polyhedron with 18 vertices and 20 faces.

tetrahemihexahedron

The [tetrahemihexahedron] is a polyhedron with 7 vertices and 16 faces.

octahemioctahedron

The [octahemioctahedron] is a polyhedron with 13 vertices and 32 faces.

small ditrigonal icosidodecahedron

The [small ditrigonal icosidodecahedron] is a polyhedron with 80 vertices and 72 faces.

dodecadodecahedron

The [dodecadodecahedron] is a polyhedron with 110 vertices and 72 faces.

echidnahedron

The [echidnahedron] is a polyhedron with 92 vertices and 180 faces.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 143 entries in this file.

Index

ENTRY POTENTIAL THEORY

[ENTRY POTENTIAL THEORY] Authors: Oliver Knill: jan 2003 Literature: "T. Ransford","Potential theory in the complex plane".

Analytic

[Analytic] Let $D \subset C$ be an open set. A continuous function $f : D \to C$ is called analytic in D, if for all $z \in D$ the complex partial derivative

$$\frac{\partial f}{\partial z} := \lim_{|h| \to 0} \frac{1}{h}(f(z+h) - f(z))$$

exists and is finite. Analytic functions are also called holomorphic. Properties: the sum and the product of analytic functions are analytic. If f_n is a sequence of analytic maps which converges uniformly on compact subsets of D to a function f, then f is analytic too.

complex partial derivative

Define the [complex partial derivative] of a complex function $f(z) = f(x+iy)$ in the complex plane is defined as $\frac{\partial f}{\partial z} = \frac{1}{2}(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y})f$.

conformal map

A [conformal map] is a differentiable map from the complex plane to the complex plane which preserves angles.

- every conformal map which has continuous partial derivatives is analytic.
- An analytic function f is conformal at every point where its derivative $f'(z)$ is different from 0.

Dirichlet problem

Solution of the [Dirichlet problem]. If D is a regular domain in the complex plane and f is a continous function on the boundary of D, then there exists a unique harmonic function f on D such that $h(z) = f(z)$ for all boundary points of D.

Dirichlet problem

Let K be a compact subset of the complex plane. Let $P(K)$ the set of all Borel probability measure on K. A measure ν maximizing the potential theoretical energy in $P(K)$ is called an [equilibrium measure] of K. Properties:

- every compact K has an equilibrium measure.
- if K is not polar then the equilibrium measure is unique.

fine topology

The [fine topology] on the complex plane is defined as the coarsest topology on the plane which makes all subharmonic functions continuous.

Frostman's theorem

[Frostman's theorem]: If ν is the equilibrium measure on a compact set K, then the potential p_ν of ν is bounded below by $I(\nu)$ everywhere on C. Furthermore, $p_\nu = I(\nu)$ everywhere on K except on a F_sigma polar subset E of the boundary of K.

Frostman's theorem

A function h on the complex plane is called harmonic in a region D if it satisfies the mean value property on every disc contained in D.

harmonic measure

A [harmonic measure] w_D on a domain D is a function from D to the set of Borel probability measures on the boundary of D. The measure for z is defined as the functional $g \mapsto H_D(g)(z)$, where $H_D(g)$ is the Perron function of g on D.

- if the boundary of D is non-polar, there exists a unique harmonic measure for D.
- if $D = Im(z) < 0$, then $w_D(z, a, b) = \arg((z-b)/(z-a))/\pi$

Harnack inequality

The [Harnack inequality] assures that for any positive harmonic function h on the disc $D(w, R)$ and for any $r < R$ and $0 < t < 2Pi$

$$h(w)(R-r)/(R+r) \leq h(w+re^{it} \leq h(w)(R+r)/(R-r)$$

extended Liouville theorem

The [extended Liouville theorem]: if f is subharmonic on the complex plane $C - E$, where E is a closed polar set and f is bounded above then f is constant.

generalized Laplacian

The [generalized Laplacian] $\Delta(f)$ of a subharmonic function f on a domain D is the Radon measure μ on D defined as the linear functional $g \mapsto \int_D u \Delta g \; dA$. The Laplacian of a subharmonic function is also called the Riesz measure. The Laplacian is known to exist and is unique. If p_μ is the potential associated to μ, then $\Delta p_\mu = \mu$.

Hadamard's three circle theorem

[Hadamard's three circle theorem] assures that for any subharmonic function f on the annulus $\{r < |z| < R\}$ the function $M(f,r) = sup_{|z|=r} f(z)$ is an increasing convex function of $\log(r)$.

Jensen formula

[Jensen formula] If f is holomorphic in the disc $D = B(0,R), r < R$ and $a_1, ..., a_n$ are the zeros of f in the closure of D counted with multiplicity, then $\int_0^{2pi} \log|f(rexp(it))|\, dt = \log|f(0)| + N\log(r) - \sum_{j=1}^{n} log|a_j|$.

Jensen formula

If f is a subharmonic function in a neighborhood of a point z in the complex plane, then the limit $\lim_{rto0} M(f,r)/\log(r)$ exists and is called the [Lelong number] of f at z. Here $M(f,r) = \sup_{|z|=r} f(z)$.

hyperbolic domain

An open set in the extended complex plane is a [hyperbolic domain] if there is a subharmonic function on G that is bounded above and not constant on any component of G. A domain which is not hyperbolic is called a parabolic domain. Known facts:

- every bounded region is a hyperbolic domain (take f(z)=Re(z)).
- an open not connected set is hyperbolic.
- the complex plane is not a hyperbolic domain

Perron function

The [Perron function] for a domain D is defined as the functional assigning to a continous function g on the boundary of D the value $H_D(g)$, which is the supremum of all subharmonic functions u satisfying $\sup_{z \to w} u(z) \leq g(w)$.

potential

A subharmonic function f is called a [potential] if $f = p_\mu$, where $\mu = \Delta f$ is the Laplacian of f and $p_m u(z) = -\int_D \log|z-w| d\mu(w)/(2\pi)$ is the potential defined by μ.

logarithmic capacity

The [logarithmic capacity] of a subset E of the complex plane is defined as $c(E) = \sup_\mu \exp(-I(\mu))$, where $I(\mu)$ is the potential theoretical energy of μ and the supremum is taken over all Borel probability measures mu on C whose support is a compact subset of E. Known facts:

- $c(E) = 0$ if and only if E is polar.
- a disc of radius r has capacity r
- a line segment of length h has capacity $h/4$.
- if K has diameter d, then $c(K) \leq d/2$.
- if K has area A, then $c(A) \geq (A/\pi)^{1/2}$.

mean value property

The [mean value property] tells that if h is a harmonic function in the disc $D(w, R)$ and $0 < r < R$, then $h(w) = \int_0^{2pi} h(w + re^{it})dt/(2\pi)$.

polar set

A subset S of the complex plane is called a [polar set] if the potential theoretical energy $I(\mu)$ is $-\infty$ for every finite Borel measure μ with compact support supp(μ) in S. Properties of polar sets:

- every countable union of polar sets is polar.
- every polar set has Lebesgue measure zero.

Poisson integral formula

The [Poisson integral formula]: if h is harmonic on the disk $D(w, R')$, then for all $0 < r < R < R'$ and $0 < t < 2\pi$, $h(w + re^{it}) = \int_0^{2pi} h(w + Re^{is})(R^2 - r^2)/(R^2 - 2Rrcos(s - t) + r^2)ds/(2\pi)$

potential theoretical energy

The [potential theoretical energy] $I(\mu)$ of a finite Borel measure μ of compact support on the complex plane is defined as

$$I(\mu) = \int_C \int_C \int \int log|z - w|d\mu(w)d\mu(z) .$$

potential theoretical energy

A function f on an open subset U of the complex plane is called [subharmonic] if it is upper semicontinous and satisfies the local submean inequality. Examples:

- if g is holomorphic then $f = \log|g|$ is subharmonic
- if μ is a Borel measure of compact support, then $f(z) = \int \log|z - w| \, d\mu(w)$ is subharmonic.
- any harmonic function is subharmonic.
- if g is subharmonic, then $\exp(g)$ is subharmonic.

regular

A boundary point w of a domain D is called [regular] if there exists a barrier at w. A barrier is a subharmonic function f defined in a neighborhood N of w which is negative on $D \cap N$ and such that $\lim_{z \to w} f(z) = 0$. It is known that z is a regular boundary point if and only if the complement of D is non-thin at z.

irregular

A boundary point w of a domain D is called [irregular] if it is not regular. It is known that if z has a neighborhood N such that N intersected with the boundary of D is polar, then z is irregular.

irregular

A domain D for which every point is regular is called a [regular domain]. For example, a simply connected domain D such that the complement of D in the Riemann sphere contains at least two points, is regular.

Riemann mapping Theorem

The [Riemann mapping Theorem]: if D is a simply connected proper subdomain of the complex plane, there exists a conformal map of D onto the unit disc.

Riesz decomposition theorem

The [Riesz decomposition theorem] tells that every subharmonic function f can be written as $f = p_\mu + h$, where $\mu = \Delta f$ is the Laplacian of f, $2\pi p_\mu$ is the potential of μ and where h is harmonic.

submean inequality

The local [submean inequality] for a function in the complex plane tells that there exists $R > 0$ such that for all $0 < r < R$ one has

$$f(w) \leq \int_0^{2pi} f(w + re^{it})\, dt/(2pi)\ .$$

submean inequality

Let f be a subharmonic function on a domain D. The [maximum principle] says that if f attains a global maximum in the interior of D then f is constant.

thin set

A subset S of the complex plane is called a [thin set] if for all w in the closure of $S - w$ and all subharmonic functions f, $\limsup_{z \to w} f(z) = f(w)$. Examples:

- every single point in the interiour of S is thin.
- F_σ polar sets S are thin at every point.
- connected sets of cardinality larger than 1 are non-thin at every point of their closure
- A domain S is thin at a point $z \in S$ if and only if z is regular.

Wiener criterion

The [Wiener criterion] gives a necessary and sufficient condition for a set S to be thin at a point w. Let S be a F_σ subset of C and let w be in S. Let $a < 1$ and define $S_n = zinS, a^n < |z - w| < a^(n-1)$. The criterion says that S is thin at w if and only if $\sum_{n \geq 1} n/log(2/c(S_n)) < infinity$, where

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 33 entries in this file.

Index

Estimation theory

[Estimation theory] part of statistics with the goal of extracting parameters from noise-corrupted observations. Applications of estimation theory are statistical signal processing or adaptive filter theory or adaptive optics which allows for example image deblurring.

Parameter estimation problem

[Parameter estimation problem] determine from a set L of observations a parameter vector. A paremeter estimate is a random vector. The estimation error ϵ is the difference between the estimated parameter and the parameter itself. The mean-squared error is given by the mean squared error matrix $E[\epsilon^T \epsilon]$. It is a correlation matrix.

Biased

[Biased] An estimate is said to be biased, if the expected value of the estimate is different than the actual value.

Asymptotically unbiased

[Asymptotically unbiased] An estimate in statistics is called asymptotically unbiased, if the estimate becomes unbiased in the limit when the number of data points goes to infinity.

Consistent estimate

[Consistent estimate] An estimate in statistics is called consistent if the mean squared error matrix $E[\epsilon^T \epsilon]$ converges to the 0 matrix in the limit when the number of data points goes to infinity.

Mean squared error matrix

The [Mean squared error matrix] is defined as $E[\epsilon^T \epsilon]$, where ϵ is the difference between the estimated parameter and the parameter itself.

efficient

An estimator in statistics is called [efficient] if its mean-squared error satisfies the Cramer-Rao bound.

Cramer-Rao bound

[Cramer-Rao bound] The mean-squared error $E[\epsilon^T \epsilon]$ for any estimate of a parameter has a lower bound which is called the Cramer-Rao bound. In the case of unbiased estimators, the Cramer-Rao bound gives for each error ϵ_i the estimate

$$E[\epsilon_i^2] \geq [F^{-1}]_{ii} \ .$$

Fisher information matrix

The [Fisher information matrix] is defined as the expectation of the Hessian $F = E[H(-\log(p))] = E[grad(log(p))grad(log(p))^T]$ of the conditional probability $p(r|\theta)$.

maximum likelihood estimate

The [maximum likelihood estimate] is an estimation technique in statistics to estimate nonrandom parameters. A maximum likelyhood estimate is a maximizer of the log likelihood function $\log(p(r,\theta)$. It is known that the maximum likelihood estimate is asymptotically unbiased, consistent estimate. Furthermore, the maximum likelihood estimate is distributed as a Gaussian random variable.
Example. If X is a normal distributed random variable with unknown mean θ and variance 1, the likelyhood function is $p(r,\theta) = \frac{1}{\sqrt{2\pi}} e^{-(r-\theta)^2/2}$ and the log-likelyhood function is $\log(p(r,\theta)) = -(r-\theta)^2/2 + C$. The maximum likelyhood estimate is r.
The maximum likelihood estimate is difficult to compute in general for non-Gaussian random variables.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 9 entries in this file.

Index

ENTRY TOPOLOGY

[ENTRY TOPOLOGY] Authors: Oliver Knill 2003, John Carlson 2003-2004 Literature: http://at.yorku.ca/cgi-bin/bell/props.cgi

Alexander compactification

The [Alexander compactification] Y of a Hausdorff space (X, O) is the topological space $(Y = X \cup x, P)$, where x is an additional point. The topology P consists of the elements in O and the complements of closed subsets as neighborhoods of that point. The new topological space Y is compact.

Alexander's subbase theorem

The [Alexander's subbase theorem]: if every open cover of a topological space X has a finite sub-cover then X is compact.

arc-connected

A topological space is called [arc-connected] if any two points can be connected by a path, a continuous image of an interval. Path connected is stronger than connected but not equivalent: the subset $\{(x, \sin(1/x)),\ x \in R^+\} \cup \{(0, y), -1 \leq y \leq 1\ \}$ of the plane with topology induced from the plane is connected but not path connected. Arc-connected is also called path-connected.

Baire category

[Baire category] is a measure for the size of a set in a topological space. Countable unions of nowhere dense sets are called of the first categorie or meager, any other set of second category. Complements of meager sets are called residual. Baire category is used to quantify certain sets. For example it is known that "most" numbers are Liouville numbers in the sense that they form a residual set among all real numbers.

Baire space

A [Baire space] is a topological space with the property that the intersection of countable family of open dense subsets is dense.

Baire category theorem

The [Baire category theorem]: a complete metric space is a Baire space.

ball

A [ball] in a metric space is a set of the form $\{y \mid\ d(x,y) < r\ \}$. The closure of an open ball is a closed ball. To make clear that a ball is open, one sometimes calls it also open ball.

barrier function

A [barrier function] for a set S in a topological space (X,O) is a nonnegative continuous function f defined on X which is zero in S and positive in the complement of S. A barrier function is sometimes also called a penalty function.

basis

A [basis] of a topological space (X,T) is a subset B of T such that

- the empty set is in B,
- arbitrary unions of sets in B are in B,
- the intersection of two sets in B is a union of sets in B.

A basis B defines the topology (X,T). Every $A \in T$ is a union of elements in B. An example: if (X,d) is a metric space then the set of all balls $\{y \mid d(x,y) < 1/k\ \}$, where x is taken from a dense set in X and k is a positive integer form a basis.

bicontinuous

A function is [bicontinuous] if it is continuous invertible and has a continuous inverse. A bicontinuous function is also called a homeomorphism.

bounded

A subset of a metric space is [bounded] if it is contained in some ball of finite radius.

boundary

The [boundary] of a set A in a topological space (X,T) is the the set $C \setminus B$, where C is the closure of A and B is the interior of A. Examples:

- if A is the open unit disc in the plane, then the boundary is the unit circle.
- in a discrete topological space, the boundary of any set is empty.

Cantor set

A [Cantor set] is a topological space which is homeomorphic to the Cantor middle set.

Cantor middle set

The [Cantor middle set] is the subset of the unit interval which is the complement of $\bigcup_{n=1}^{\infty} Y_n$, where $Y_1 = (1/3, 2/3), Y_2 = (1/9, 2/9) \cup (7/9, 8/9)$ etc. are sucessive middle sets. It is a fractal with Hausdorff dimension $\log(2)/\log(3)$.

Cantor middle set

A topological space homeomorphic to a ball in Euclidean space is called a [cell]. Examples of cells are polyhedra in three dimensional space.

closure

The [closure] of a set A in a topological space (X, T) is the intersection of all closed sets in X, which contain A. One writes $\overline{Y}$ for the closure of Y.

dense

A set A is called [dense] in a topological space (X, T), if every open set $Y \in O$ in X contains at least one point in A.

finer

A topology (X, T) is [finer] than a topology (X, S) if S is a subset of T. In that case, (X, S) is called a coarser topology than (X, T). Examples:

- the discrete topology on X is finer than any other topology on X.
- A set S of subsets of X defines a topology, the coarsest topology O which contains S.

topological space

A [topological space] is a pair (X,T) where T is a set of subsets of X satisfying

- $\emptyset \in T$,
- if $A, B \in T$, then $A \cap B \in T$,
- an arbitrary union of subsets in T is in T.

Elements in T are called open sets. The complement of an open set is called a closed set. Examples:

- the discrete topology on X: T is all subsets of X
- the indiscrete topology on X: T contains only X and the empty set,
- the cofinite topology: T is the set of all subsets A such that their complement in X is a finite set.
- (X, d) metric space: T is the set of sets A such that for x in A, also a small ball $B = \{|y - x| < a\}$ is contained in A.

open set

An [open set] of a topological space (X, T) is an element of T.

closed set

A [closed set] is the complement of an open set in a topological space (X, T). A closed set contains all its limit points.

continuous

A map f between two topological spaces (X, T) and (Y, S) is called [continuous] if the inverse image of any open set is open: for all $A \in S$ one has $f^(-1)(A) \in T$. Note that f does not need to be invertible: one defines $f^{-1}(A) = \{x \in X | f(x) \in A\}$. Examples of results known:

- The composition of two continuous maps is continuous.
- Every map on the discrete topological space is continuous.
- A map between metric spaces is continuous if and only if it is sequential continuous: for any $x_n \to x$, one has $f(x_n) \to f(x)$.
- A map between topological spaces is continuous if for every net $x_t \to x$, the net $f(x_t)$ converges to $f(x)$.

homeomorphism

An invertible map f between two topological spaces (X, T) and (Y, S) is called a [homeomorphism] if f and the inverse of f are both continuous.

homeomorphic

[homeomorphic] If there exists a homeomorphism between two topological spaces, the topological spaces are called homeomorphic.

connected

A topological space (X, T) is called [connected], if there are no two disjoint open sets U, V whose union is X. For a connected topological space, the empty set $\emptyset$ or X are the only sets which are both open and closed. A subset A of a topological space is connected if it is connected with the on A induced topology: there are no disjoint open sets U, V whose union contains A.

locally connected

A topological space is [locally connected] if every point has arbitrarily small neighborhoods which are connected. Examples.

- A union of disjoint open intervals on the real line is locally connected but not connected.
- The union of the graphs of $f(x) = 2\sin(1/x)$ and $g(x) = 1$ and the y-axes all intersected with the set $\{y > 1\}$ is connected but not locally connected because small neighborhoods of the point $(0,1)$ are not connected.

Hausdorff

A topological space (X,T) is called [Hausdorff] if for every two points $x, y \in X$, there are disjoint open sets $U, V \in T$ such that $x \in U$ and $y \in V$. This is refined through seperation axioms, $T0, ..., T4$. Hausdorff is also called $T2$. Any metric space is Hausdorff: if d is the distance betwen x and y, then balls of radius $d/3$ around x and y seperate the points. The plane X with semimetric $d(x,y) = |x_1 - y_1|$ is not Hausdorff: the points $x = (0,-1)$ and $y = (0,1)$ can not be seperated by open sets.

seperation axioms

[seperation axioms] define classes of topological spaces with decreasing seperability properties: $T4 \Rightarrow T3 \Rightarrow T2 \Rightarrow T1 \Rightarrow T0$.

T0 space:	for two different points x, y in X one of the points has an open neighborhood U not containing the other point.
T1 space:	for two different points x, y in X there exists an open neighborhood U of x and an open neighborhood V of y. such that x is not in V and y is not in U.
T2 space:	also called Hausdorff" two different points x, y can be seperated with disjoint neighborhoods U, V.
T3 space:	T1 and regular: any point x and any closed set F not containing x can be seperated by two disjoint neighborhood.
T4 space:	T1 and normal: any two disjoint sets F,G can be separated by two disjoint open sets.

It is known that a T4 space with a countable basis is metrizable.

Hausdorff topology

The [Hausdorff topology] is a metric on the set of closed bounded subsets of a complete metric space. The distance between two sets A and B is the infimum over all r for which A is contained in a r-neighborhood of B and B is contained in a r-neighborhood of A.

Lindeloef

A topological space is called [Lindeloef] if every open cover of X contains a countable subcover.

compact

A topological space is called [compact] if every open cover of X contains a finite subcover. Examples of results known about compactnes:

- Heine-Borel theorem: a closed interval in the real line is compact.
- If $f : X \to Y$ is continuous and onto and X is compact, then Y is compact. As a consequence, a continouous function on a compact subspace has both a maximum and a minimum.
- In a Hausdorff space, compact sets are closed.
- In a metric space, compact sets are closed and bounded.
- Closed subsets of compact spaces are compact.
- Tychonof theorem: the product of a collection of compact spaces is compact.

countably compact

A topological space is called [countably compact] if every countable open cover of X contains a finite subcover.

locally compact

A topological space is called [locally compact] if every point has a neighborhood, which has a compact closure. Examples.

- The real line is compact but not locally compact.
- A compact Hausdorff space is locally compact.
- The n-dimensional Euclidean space R^n is lcally compact but not compact.

locally compact

A set U of open sets in a topological space (X, O) is called locally finite if every point $x \in X$ has a neighborhood V, such that V has a nonempty intersection with only finitely many elements in U.

paracompact

A topological space (X,O) is called [paracompact] if every open cover has a countable, locally finite subcover.

relatively compact

A subset A of a topological space (X, T) is called [relatively compact] if the closure of A is compact.

filter

A [filter] on a nonempty set X is a set of subsets F satisfying

- X is in F, but the empty set $\emptyset$ is not in F.
- If A and B are in F, then their intersection is in F.
- If A is in F and B is a subset of A, then B is in F.

Examples:

- Principal filter for a nonempty subset A consists of all subsets of X which contain A.
- Frechet filter for an infinite set consists of all subsets of X such that their complement is finite.
- Neighborhood filter of a point x in a topological space (X,T) is the set of open neighborhoods of x.
- Elementary filter for a sequence x_n in X consists of all sets A in X such that x_n is in A for large enough n.

converges

A sequence x_n in a topological space [converges] to a point x, if for every neighborhood U of x, there exists an integer m, such that for $n > m$ one has $x_n \in U$.

Filter convergence

[Filter convergence] A filter F converges to x in a topological space (X, T) if F contains the neighborhood filter G of x, that is if F contains all neighborhoods of x. For example, an elementary filter to a sequence x_n converges to a point x, if and only if x_n converges to x.

accumulation point

A point y is called an [accumulation point] of a filter F, if there exists a filter G containing F such that G converges to x.

directed

A set M is called [directed] if there exists a partial order $(M, <)$ on M satisfying for every two points $a, b \in M$ there exists c, with $a < c$ and $b < c$.

interior

The [interior] of a set A in a topological space (X, T) is the union of all open sets in X, which are contained in A.

Koch curve

The [Koch curve] is a fractal in the plane. It has Hausdorff dimension $\log(4)/\log(3)$. It is constructed by building an equilateral triangle on the middle third of each side of a given equilateral triangle K_0 leading to a curve K_1 and recursively build K_{n+1} from K_n by replacing each middle third of a line segment in K_n with a triangle. The curve is the limit of K_n, when n goes to infinity.

metrizable

A topological space (X, T) is called [metrizable] if there exists a metric on X such that the topology generated by the metric is T.

metric space

A [metric space] (X, d) is a set X with a function d from $X \times X \to [0, \infty)$ satisfying $d(x, y) = d(y, x), d(x, y) = 0 \Leftrightarrow x = y$ and $d(x, z) \leq d(x, y) + d(y, z))$. The set $T = \{U \subset X \mid \forall x \in X, \exists r > 0, B_r(x) = \{y \mid d(x, y) < r\} \subset U\}$ defines a topological space (X, T).

metric space

A [metric space] (X, d) is a set X with a nonnegative function d from $X \times X$ satisfying $d(x, y) = d(y, x)$,$d(x, y) = 0$ if and only if $x = y$ and $d(x, z) \leq d(x, y) + d(y, z)$. A metric space defines a topological space (X,T): the topology T is the set of subsets A of X such that for all points $x \in A$, there is a small ball $d(y, x) < r$ which is also contained in A.

net

A [net] with values in a topological space X is a function f: D -¿ X, where D is a directed set. For example: if D is the set of natural numbers, then a net is a sequence. A net defines a filter F: it is the set of all sets A such that x_t is eventually in A. A net x_t converges to a point x if and only if the associated filter converges to x.

open cover

[open cover] A subset U of O, where (X, O) is a topological space is called an open cover of X if the union of all elements in U is X. If U and V are open covers and $V \subset U$, then V is called a subcover of U.

product space

The [product space] between topological spaces is defined as $(X \times Y, O \times P)$, where $X \times Y$ is the set of all pairs $(x, y), x \in X, y \in Y$ and $O \times P$ is the coarsest topological space which contains all products $A \times B$, where $A \in O$ and $B \in P$. For example, if $(X, O) = (Y, P)$ are both the real line with the topology generated by $d(x, y) = |x - y|$, then the product space is homeomorphic to the plane with the metric $d(x, y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

second countable

A topological space is called [second countable], if it has a countable basis. Example. Every seperable metric space is second countable. Especially, every finite-dimensional Euclidean space is second countable.

metrizable

A topological space is called [metrizable] if there exists a metric d on the set X that induces the topology of X. Any regular space with a countable basis is metrizable.

homotopic

[homotopic] If f and g are continuous maps from the topological space X to a topological space Y, we say that f is homotopic to g if there is a continuous map F from $X \times I$ to Y, such that $F(x,0) = f(x)$ and $F(x,1) = g(x)$ for all x. For example, the maps $f(x) = x^2$ and $g(x) = sin(x)$ on the real line are homotopic, because we can define $F(x,t) = (1-t)x^2 + t\sin(x)$. The maps $f(x) = x$ and $g(x) = \sin(2\pi x)$ on the circle are not homotopic. While g is homotopic to the constant function $h(x) = 0$, the map $f(x)$ can not be deformed to a constant without breaking continuity.

induced topology

The [induced topology] on a subset A of X, where (X,T) is a topological spoace is the the topological space $(A, \{Y \cap A\}_{Y \in T})$.

path homotopic

[path homotopic] If f and g are and continuous homotopic maps from an interval to a space X, we say f and g are path homotopic if their images have the same end points. For instance, the maps $f(x) = x^2$ and $g(x) = x^3$ are path homotopic on the closed interval from 0 to 1. The maps $f(x) = 2x^2$ and $g(x) = x^3$ are homotopic on the unit interval but not path homotopic.

loop

A [loop] is a path in a topological space that begins and ends at the same point. A loop is also called a closed curve. Loops play a role in definitions like simply connected: a topological space is simply connected if every loop is homotopic to a constant loop which is a fancy way telling that every closed path can be collapsed inside X to a point.

fundamental group

The [fundamental group] of a topological space at a point is the set of homotopy classes of loops based at that point.

Topologist's Sine Curve

The [Topologist's Sine Curve] is the union S of the graph of the function $\sin(1/x)$ on the positive real axes R^+ with the y-axes. It an example of a topological space which is connected but not path-connected. Proof: if S were path-connected, there would exist a path $r(t) = (x(t), y(t))$ connecting the two points $(0, 1)$ and $(0, \pi)$. The set $\{t|r(t) \in S\}$ is closed. Let T be the largest t in that set for which $r(t)$ is in the y-axes. Then x(T)=0 and r(t)=(x(t),sin(1/x(t)) for $t > T$. Because there are times $t_n > t_{n-1} > T, t_n \to T$ for which $y(t_n) = (-1)^n$, the function $r(t)$ can not be continuous at $t = T$.

Urysohn lemma

The [Urysohn lemma] tells that if X is a normal space and A and B are disjoint closed subsets of X, then there exists a continuous map f from X to the unit interval such that $f(x) = 0$ for all $x \in A$, and $f(x) = 1$ for all $x \in B$.
Proof: use the normality of X to construct a family U_p of open sets of X indexed by the rational numbers P in the unit interval so that for $p < q$, the closure of U_p is contained in U_q. These sets are simply ordered in the same way that their subscripts are ordered in the real line. Given some enumeration of the rationals, where 1 and 0 are the first two elements of the enumeration, define $U_1 = X \setminus B$. Because A is a closed set contained in the open set U, there is by normality an open set U_0 such that $A \subset U_0$ and the closure of U_0 is a subset of U_1. In general, let P_n denote the set consisting of the first n rational numbers in the sequence. Suppose that U_p is defined for all rational numbers p in a set P_n, then $p < q$ implies that the closure of U_p is a subset of U_q. If r is the next rational number in the sequence; we define U_r: the set $P_{n+1} = P_n \cup \{r\}$ is a finite subset of the unit interval and has a simple ordering induced by the ordering of the real line. In a finite simply ordered set, every element, other than the largest and smallest, has an immediate predecessor and and immediate successor. 0 is the smallest and 1 is the largest element of the simply ordered set P_{n+1}, and $r \notin \{0, 1\}$. So r has an immediate predecessor $p \in P_{n+1}$ and an immediate successor $q \in P_{n+1}$. The sets U_p and U_q are already defined, and the closure of U_p is contained in U_q by the induction hypothesis. Because X is normal, we can find an open set U_r such that the closure of U_p is contained in U_r and the closure of U_r is contained in U_q. Now the induction condition holds for every pair of elements of P_{n+1}.

This file is part of the Sofia project sponsored by the Provost's fund for teaching and learning at Harvard university. There are 58 entries in this file.

Index