ENTRY CURVES

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astroid

An [astroid] is the curve $t \mapsto (cos^3(t), a\sin^3(t))$ with $a > 0$. An asteroid is a 4-cusped hypocycloid. It is sometimes also called a tetracuspid, cubocycloid, or paracycle.

Archimedes spiral

An [Archimedes spiral] is a curve described as the polar graph $r(t) = at$ where $a > 0$ is a constant. In words: the distance $r(t)$ to the origin grows linearly with the angle.

bowditch curve

The [bowditch curve] is a special Lissajous curve $r(t) = (asin(nt + c), bsin(t))$.

brachistochone

A [brachistochone] is a curve along which a particle will slide in the shortest time from one point to an other. It is a cycloid.

Cassini ovals

[Cassini ovals] are curves described by $((x + a) + y^2)((x - a)^2 + y^2) = k^4$, where $k^2 < a^2$ are constants. They are named after the Italian astronomer Goivanni Domenico Cassini (1625-1712). Geometrically Cassini ovals are the set of points whose product to two fixed points $P = (-a, 0), Q = (0, 0)$ in the plane is the constant k^2. For $k^2 = a^2$, the curve is called a Lemniscate.

cardioid

The [cardioid] is a plane curve belonging to the class of epicycloids. The fact that it has the shape of a heart gave it the name. The cardioid is the locus of a fixed point P on a circle roling on a fixed circle. In polar coordinates, the curve given by $r(\phi) = a(1 + \cos(\phi))$.

catenary

The [catenary] is the plane curve which is the graph $y = c\cosh(x/c)$. It was discovered by Jacques Bernoulli. It has the shape of a uniform flexible chain hung from two points.

catenoid

The [catenoid] is the surface obatained by rotating the catenary about the x-axis. The minimal surface bounded by two coaxial rings can be a catenoid.

circle

A [circle] in the plane is the curve $r(t) = (r\cos(t), r\sin(t))$ where the radius r is a constant. It is the set of points which have a fixed distance r from the origin. More generally, a circle is the set of points in a metric space which have a fixed distance from a given point.

cissoid

A [cissoid] is a plane curve given in Euclidean coordinates by $y^2(2a-x) = x^3$. In polar coordinates, it satisfies $r(t) = 2a\tan(t)\sin(t)$ or in Euclidean coordinates $r(t) = (2a\sin^2(t), 2a\sin^3(t)/\cos(t))$. The curve has a cusp at the origin. It was first mentioned by Diocles in 180 B.C.

conic section

A [conic section] is a nondegenerate curve generated by intersecting a plane with one or two nappes of a cone. The three congruence classes of conic sections are the ellipse, the parabola, and the hyperbola.

ellipse

An [ellipse] is the locus of all points in the plane the sum of whose distances from two fixed points is a positive constant. It is also the conic section which results from a plane which intersects only one nappe of the cone. The general formula for an ellipse up to rotation and translation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

hyperbola

A [hyperbola] is the set of points in the plane for which the difference of the distances from two fixed points is a constant. It is also the conic section which results from a plane intersecting a cone. The general formula for a hyperbola up to rotation and translation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

curve

A [curve] is a continuous map from the real line to an the plane or to space. The word "curve" is often used to mean the image of this map. Curves can be represented parametrically by $r(t) = (x(t), y(t))$ or implicitely as $f(x, y) = 0$.

Airy function

The [Airy function] is commonly found as a solution to boundary value problems in quantum mechanics and electromagnetism. It is the solution to the differential equation: $y'' = xy$. The two independent solutions are (without constants):

$$Ai(x) = \int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt$$

$$Bi(x) = \int_0^\infty \left(e^{-\frac{t^3}{3}+xt} + \sin\left(\frac{t^3}{3} + xt\right)\right) dt$$

algebraic curve

A plane curve is an [algebraic curve] if it is given by $g(x, y) = 0$ where g is algebraic a polynomial in x and y. An algebraic curve with degree greater than 2 is called a higher plane curve. The circle $g(x, y) = x^2 + y^2 - 1 = 0$ is an example of an algebraic curve, the catenary $g(x, y) = y - c\cosh(x/c) = 0$ is an example of a nonalgebraic curve.

cubic curve

A [cubic curve] is an algebraic curve of order three. Newton showed that all cubics can be generated as projections of the five divergent cubic parabolas. Examples include the cissoid of Diocles and ellptic curves.

ampersand curve

The [ampersand curve] is a quartic curve with implicit equation $(y^2 - x^2)(x - 1)(2x - 3) = 4(x^2 + y^2 - 2x)^2$. It looks like an ampersand.

bean curve

The [bean curve] is a quartic curve given by the implicit equation: $x^4 + x^2y^2 + y^4 = x(x^2 + y^2)$. It looks like a bean.

bicorn

The [bicorn] is the name of a collection of quartic curves studied by Sylvester in 1864 and Cayley in 1867. It is given by $y^2(a^2 - x^2) = (x^2 + 2ay - a^2)^2$.

bicuspid

The [bicuspid] is the quartic curve given by the implicit equation: $(x^2 - a^2)(x - a)^2 + (y^2 - a^2)^2 = 0$.

bow

The [bow] is a quartic curve with the implicit equation: $x^4 = x^2y - y^3$.

cartesian oval

A [cartesian oval] is a quartic curve consisting of two ovals. It is the locus of a point P whose distances from two foci F_1 and F_2 in two-center bipolar coordinates satisfy

$$mr \pm nr' = k$$

where m and n are positive integers, k is a positive real, and r and r' are the distances from F_1 and F_2. If $m = n$, then the oval becomes an ellipse.

Cassini oval

A [Cassini oval] is one of a family of quartic curves, also called Cassini ellipses, described by a point such that the product of its distances from two fixed points a distance $2a$ apart is constant b^2. The shape of the curve depends on b/a. The Cassini ovals are defined in two-center bipolar coordinates by the equation $r_1 r_2 = b^2$ where b is a positive constant.

cruciform

A [cruciform] is a plane quartic curve also called the cross curve or policeman on point duty curve. It is given by the implicit equation: $x^2y^2 - b^2x^2 - a^2y^2 = 0$.

lemniscate

The [lemniscate], also known as the lemniscate of Bernoulli, is a polar curve whose most common form is the locus of points the product of whose distances from two fixed points a distance $2a$ away is the constant a^2. The usual polar coordinate form is as follows: $r^2 = a^2 \cos(2\theta)$.

natural equation

A [natural equation] is an equation which specifies a curve independent of any choice of coordinates or parametrization. This arose in the solution to the following problem: given two functions of one parameter, find the space curve for which the functions are the curvature and torsion. Often, the natural equation will be in terms of integrals.

polynomial curve

A [polynomial curve] is a curve obtained by fitting polynomials to a sequence of points. To fit curves better, splines like Bezier curve are more suited.

quadrifolium

A [quadrifolium] is a rose curve with $n = 2$. It has polar equation

$$r = a\sin(2\theta).$$

sextic curve

A [sextic curve] is an algebraic curve of degree 6. Examples include the atriphtaloid and the butterfly curve $y^6 = x^2 - x^6$.

atriphtaloid

The [atriphtaloid] is a sextic curve also known as atriphtothlassic curve and given by the equation: $x^4(x^2 + y^2) - (ax^2 - b)^2 = 0$.

butterfly curve

The [butterfly curve] is a sextic plane curve given by the implicit equation $y^6 = x^2 - x^6$.

trifolium

A [trifolium] is the 3-petalled rose given in polar form as $r(t) = a|\cos(3t)|$.

spiral

A [spiral], in general, is a curve with $\tau(s)/\kappa(s)$ constant for

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