ENTRY K12

[ENTRY K12] Authors: Oliver Knill: 2000 Literature: not yet

abacus

An [abacus] is an acient mechanical computing device. It is made of beads arranged on a frame.

absolute value

The [absolute value] $|n|$ of a real number n is the maximum of n and its negative $-n$. For example, the absolute value of -6 is $|-6| = 6$. The absolute value is the distance from 0.

adjacent angles

Two angles that share a ray are called [adjacent angles].

affine cipher

An [affine cipher] uses affine functions to scramble the letters in an alphabet of a secret message. For example, with an alphabet of 26 letters, $f(x) = bx + a = 5x + 2 \mathrm{mod}(26)$ produces a new alphabet of the same size if b has no common multiple with 26. The simplest example is the Caesar cipher, where $b = 2$. It rotates the letters in an alphabet $x \mapsto x + a \mathrm{mod}(26)$. For example, for $a = 1$, we get

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a

This cipher changes the word *hello* to the word $ifmmp$. A requently used Caesar cipher is "rot13" defined by $f(x) = x + 13 \mathrm{mod}(13)$. It has the property that encryption and decryption are the same. For example, applying rot13 on the word "decryption" produces *qrpelcgvba* and applying rot13 on that word again gives back "decryption. More complicated versions of affine cyphers can be obtained by writing the to encoded text as a sequence of vectors x and then applying $Ax + b$ on each vector. Affine cyphers are very easy to crack. They are only used to illustrate the concept like for educational purposes.

algebra

[algebra] is a branch of elementary mathematics that generalizes arithmetic by using variables. An example of an algebraic identity is $x * (y + z) = x * y + x * z$.

acute

An angle is called [acute], if is smaller than 90 degrees. An angle which is 90 degrees is called a right angle.

addition

[addition] is a basic operation for numbers. The result is called the sum of the two numbers. Examples: $5+3=8$.

$$\begin{array}{cccccc} & 2 & 3 & 4 & 5 \\ + & 9 & 2 & 3 & 5 \\ \hline 1 & 1 & 5 & 8 & 0 \end{array}$$

More generally, a group operation in a commutative group is often called addition. Examples of groups are integers, real numbers, vectors or matrices.

alternate exterior angles

[alternate exterior angles] are angles located outside a set of two parallel lines and on opposite sides of the transversal line. They are equal.

alternate interior angles

[alternate interior angles] are angles located inside a set of parallel lines and on opposite sides of the transversal.

angle bisector

A ray that divides an angle into two equal angles is called an [angle bisector]. The bisector can be constructed with ruler and compass. An angle trisector on the other hand, a ray which splits an angle into three equal parts can not be constructed by ruler and compass.

apex

The [apex] is the highest vertex in a given orientation of a polygon.

Arabic numerals

[Arabic numerals]: symbols $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ that represent successive entries of words representing numbers in the decimal system. For example, 2347 is the number $2000 + 300 + 40 + 7$.

area

The [area] of a surface is a measure for the number of square units needed to cover the surface. For example, the sphere of radius 1 has the surface area 4π.

arithmetic mean

[arithmetic mean] Given two numbers a, b, the arithmetic mean is defined as $(a + b)/2$. It is sometimes also called the mean. Other means are the geometric mean $\sqrt{ab}$ or the harmonic mean $1/(1/a + 1/b)$.

average

The [average] of a few numbers is the sum of all the numbers divided by the number of numbers. For example, the average of $2, 4, 6$ is $(2 + 4 + 6)/3 = 4$, the average of the numbers $1, 5, 8, 4$ is $(1+5+8+5)/3 = 19/3$ The average is also called the mean. The average of two numbers is also called the arithmetic mean.

base

A [base] is the number of distinct single-digit numbers in a counting system. Example: the binary system has base 2. The decimal system has base 10. The base is also called radix. Numbers can be represented in any base $r > 1$. Because humans have 10 fingers, the decimal system is the one favoured by this species. Because computers work with circuits which are based on the principle "on" or "off", they like the base 2. The hexadecimal system (base 16) or octal system (base 8) are also used a lot by computers. Modern computers can even work directly with numbers written in base 32 or 64.

bell curve

The [bell curve] is an other term for graph of the normal distribution $f(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$. It is also called the Gauss distribution. The bell curve is often seen in probability distributions. There is a reason for that called the central limit theorem which assures that if we average independent data with some distribution, we approach the normal distribution.

billion

A [billion] is one thousand millions in the American or French system, it is a million millions in the English or German system. In other words

One billion in UK,Germany:	10^{12}	1'000'000'000'000
One billion in US,France:	10^9	1'000'000'000

binary number

A [binary number] is a number expressed in place-value notation to the base 2. For example: 101101 represents the decimal number $1 + 0 + 4 + 8 + 0 + 32 = 45$.

cipher

[cipher] Ciphers are codes used to encrypt "secret" messages.

coefficient

The word [coefficient] is used to denote numbers in the front of the variables in an algebraic formula. For example: $4x + 5y = 3$ has coefficients $4, 5$.

combinatorics

[combinatorics] The science of counting things. Combinatorics is an important part of probability and statistics.

common factor

A [common factor] of two integers n and m is a number which is a factor of both. A common factor is also called a common divisor. Examples: 3 is a common factor of 18 and 27. Also, 9 is the greatest common factor of 18 and 27: we write $9 = \gcd(18, 27)$, where *gcd* stands for the greatest common divisor.

complex numbers

[complex numbers] can be written as a pair of real numbers $z = x+iy$, where i is a symbol which satisfies $i^2 = -1$. One can add and subtract complex numbers by adding their coefficients x, y. For example $4+5i+5-7i = 9-2i$.

complementary angles

Two angles whose sum is 90 degrees form [complementary angles]. For example, the two non-right angles in a right triangle form complementary angles.

concave up

A graph of a function f is [concave up] if f has the property $f((x + y)/2) \leq (f(x) + f(y))/2$. If f is concave up, then $-f$ is concave down. For example, the graph of the function $f(x) = x^2$ is concave up, the graph of the function $f(x) = -x^4$ is concave down.

conditional probability

The [conditional probability] is the probability that an event A happens provided a second event B occurs. One writes $P[A|B]$. It satisfies $P[A|B] = P[A \cap B]/P[B]$, where $P[B]$ is the probability of the event B and $P[A \cap B]$ probability of the intersection of A and B. For example, if we throw 2 coins and we know one of the coins is head H, then the probability that the there is also a coin with tail is $2/3$. Proof: The probability space is $X = \{HT, TH, TT, HH\}$. The event that one of the coins is head is $A = \{HT, TH, HH\}$. The event B that one of the coins shows tail is $B = \{HT, TH, TT\}$. The intersection of B and A is $\{TH, HT\}$. We have $P[B|A] = P[B \cap A]/P[A] = (1/2)/(3/4) = 2/3$.

congruent

Two figures are called [congruent] if one can move one to an other by translation and rotation.

constant

A quantity that does not change in an equation is called a [constant].

constant function

A [constant function] is a function which takes the same value whatever input we enter to it.

coordinate

A [coordinate] is an entry in a collection of numbers identifiing the point in coordinate space.

continuous graph

The graph of a continuous function is called a [continuous graph]. Roughly speaking, a graph of a function defined on some interval $[a, b]$ is a continuous graph if one can draw the graph using a pencil without having to the lift the pencil. Examples:

- $1/x$ is not continuous on $[-1, 1]$.
- $x^2 + 1$ is continuous on $[-1, 1]$.
- $1/x^2$ is not continuous on $[-1, 1]$.
- $\sin(1/x)$ is not continuous on $[-1, 1]$.
- $x \sin(1/x)$ is continuous on $[-1, 1]$.
- $f'(x)$ is not continuous on $[-1, 1]$ if $f(x) = |x|$.

corresponding angles

[corresponding angles] are two angles in the same relative position on two straight lines when those lines are intersected by a a transversal straight line.

decimal number

[decimal number] is a fraction, where the denominator is a power of 10. It can be expressed using a decimal point. For example: 0.872 is the decimal equivalent of 872/1000.

degrees

An angle is often measured in [degrees]. The entire circle has 360 degrees, a half a circle is 180 degrees, a quarter circle is a right angle and has 90 degrees. A more natural unit is the length unit where the entire circle has angle 2π and the right angle is the angle $\pi/2$.

denominator

The [denominator] is the integer q below the fraction in a rational number p/q. The other number p is called the nominator.

discontinuous graph

A [discontinuous graph] is the graph of a function which is not continuous. Discontinuities can occur in different ways. The function can jump from one value to an other. The function can also be infinite at some point or the function can oscillate infinitely much at some point. Examples:

- The graph of the function $f(x) = 1/x$ on $[-1, 1]$.
- The graph of the function $f(x) = \sin(1/x)$ on $[-1, 1]$.
- The graph of the function $f(x) = sign(x)$, which is 1 for $x > 0$ equal to 0 for $x = 0$ and -1 for $x = -1$.

disjoint events

Two events are called [disjoint events] if they have no common elements.

division

The inverse operation of multiplication is called [division].

domain

The [domain] of a function f is the set of numbers x for which $f(x)$ is defined. For example, the domain of the function $f(x) = 1/x$ is the entire real line except the point 0.

element

An [element] of a set is is a member of that set. For example $table$ is an element of the set $\{table, chair, floor\}$.

empty set

The [empty set] $\emptyset$ is the set which does not contain any elements.

equally likely

If two events have the same probability they are called [equally likely]. For example, the event of throwing an even number with one dice is equally likely than throwing an odd number.

event

An [event] is a subset of the entire probability space. For example, if $X = \{HH, HT, TH, TT\}$ is the probability space of all throwing of two coins, then $A = \{HH, HT\}$ is the event that in the second throw one had a head.

exponent

The [exponent] of an expression a^x is part x. One can get the exponent of $y = a^x$ by the formula $x = \log(y)/\log(a)$.

Fibonacci numbers

[Fibonacci numbers] are numbers obtained in the Fibonacci sequence defined by starting the numbers $0, 1$ and defining the next element as the sum of the two previous ones: $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...$ The sequence is named after Leonardo of Pisa, who called himself Fibonacci, short for Filius Bonacci (= Son of Bonacci). The original problem he investigated in 1202 A.D. was the growth of rabbits. Explicit expressions for the n'th term of the sequence can be obtained using linear algebra. More generally, one can find explicit formulas for the n'th term in a linear recursion of the form $a_{n+1} = \sum_{j=0}^{k} c_j a_{n-j}$.

fractal

A [fractal] is a set which has non-integer dimension. The term was coined by Benoit Mandelbrot in 1975. Many objects in nature appear to be fractals, like coast lines, trees, mountains. One can mathematically define fractals using iterative constructions. Examples are the Koch curve, the snow flake, the Menger Sponge, the Shirpinsky carpet, the Cantor set.

fraction

A [fraction] is a rational number written in the form a/b, where a is called the numerator and b is called the denominator.

function

A [function] f of a variable x is a rule that assigns to each number x in the function's domain a single number $f(x)$. For example $f(x) = x^2$ is a function which assigns to each number its square like $f(4) = 16$.

geometric sequence

The [geometric sequence] is a sequence where each element is a multiple of the previous element. For example: $1, 2, 4, 8, 16, 32, 64, ...$ is a geometric sequence.

graph of the function

The [graph of the function] is the set of all points $(x, f(x))$ in the plane, where x in the domain of f.

greatest common factor

The [greatest common factor] of two numbers n,m is the larest common factor of both. One denotes the greatest common factor with "gcd". Examples:

6 is the greatest common factor of 12 and 18	6=gcd(12,18)
8 is the greatest common factor of 8 and 80.	8=gcd(8,80)
1 is the greatest common factor of 7 and 11	1=gcd(7,11)

greatest common divisor

[greatest common divisor] see greatest common factor.

histogram

A [histogram] is a bar graph in which area over each range of values is proportional to the relative frequency of the data in this interval.

hypotenuse

The [hypotenuse] of a right triangle is the opposite side to the right angle.

independent events

Two events A and B are called [independent events] if the probability that both happen is the product of the probabilities that each occurs alone: $P[A \cap B] = P(A)P(B)$. Using conditional probability one can write this as $P[A|B] = P[A]$. Knowing A under the condition B is the same as knowing A without knowing B.

infinity

[infinity] is a "number" which is larger than any other number. One writes ∞. One should rather treat of it as a symbol evenso some computations can be extended to the real numbers including ∞ like $\infty + x = \infty, \infty + \infty = \infty, x * \infty = \infty$ for $x > 0$, $x * \infty = -\infty$ for $x < 0$ or $\infty * \infty = \infty$, $(-\infty) * \infty = -\infty$. One can not define $\infty - \infty$ in a consistent way nor can one do that with $0 * \infty$. Also the expression $1/0 = \infty$ is ill defined because $1/x$ takes near $x = 0$ arbitrary large and arbitrary small values.

integer

An [integer] is a number of the form n or $-n$, where n is a natural number. Examples of integers are $... -3, 2, 1, 0, 1, 2, 3, 4....$ The fraction $2/5$ is not an integer.

intersection

The [intersection] of two or more sets is the set of elements which are in both sets. One writes $A \cap B$ for the intersection of A and B.

isosceles triangle

An [isosceles triangle] is a triangle which has at least two congruent sides. A special case is the isocline triangle in which all sides are congruent.

least common multiple

The [least common multiple] of two numbers n, m is the least common multiple of both. One denotes the least common multiple with "lcm". Examples:

18 is the least common multiple of 9 and 6	18=lcm(9,6)
77 is the least common multiple of 7 and 11	77=gcd(7,11)

limit

The [limit] of a sequence of numbers is the limiting value the sequence converges to. It needs not to exist. For example, the sequence $a_n = 1/n$ converges to 0. One sais that 0 is the limit of that sequence. The sequence $a_n = n$ has no finite limit. One could assign infinity as a limit. The sequence $1, -1, 1, -1, 1, -1, ...$ has no limit.

logarithm

The [logarithm] of b is the exponent to which one has to rize a base number to get b. For example, 2 is the logarithm of 100 to the base 10 or 10 is the logarithm of 1024 to the base 2.

mean

The [mean] of a list of numbers is their sum divided by the total number of members in the list. It is also called arithmetic mean.

median

The [median] is the "middle value" of a list. If the list has an odd number $2m+1$ elements, the median is the number in the list such that m scores are smaller or equal and m scores are bigger or equal. If the list has an even number of elements, one usually takes the algebraic average between the middle to elements Examples: $\text{med}(1,1,2,2) = 3/2$, $\text{med}(1,2,3,4,7) = 3$, $\text{med}(1,2,3,4,5,6) = 7/2$.

multiplication table

[multiplication table] A table of products of numbers which has to be memorized.

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

The diagonal contains squares. All numbers between 11 and 99 which do not appear in this table are prime numbers, numbers only divisible by 1 and itself.

obtuse angle

An angle whose measure is greater than 90 degrees is called an [obtuse angle].

optical illusion

An [optical illusion] is a drawing of an object that makes certain things appear which it does not have.

palindrome

A [palindrome] is a word or number that is the same when read backwards. Examples: "otto", "anna", "racecar", "78777787".

paradox

A [paradox] is a statement that appears to contradict itself. For example, the statement "I always lie" is a paradox. If I tell the truth, then I lie, if I lie, then I tell the truth.

parallel

Two lines which do not intersect are called [parallel].

parallelogram

A [parallelogram] is a quadrilateral that contains two pairs of parallel sides.

pattern

A [pattern] is a characteristic observed in one item that may be repeated in other items. For example, the sequence $3, 4, 5, 4, 3, 4, 5, 4, 3, 4, 5, 4, 3, ...$ has a pattern which is also visible in a similar way in the sequence $1, 3, 4, 3, 1, 3, 4, 3, 1, 3, 4, 3, 1, ...$

percent

A [percent] is one hunderth. The symbol for percent is %. For example 0.1 is 10 percent. 2 is two hundred percent.

perimeter

The [perimeter] of a polygon is the sum of the lengths of all the sides of the polygon.

permutation

A [permutation] is a rearrangement of objects in a set. There are for example 6 permutations of the set $A = (a, b, c)$. They are $(a, b, c), (a, c, b), (b, a, c), (b, c, a), (c, a, b), (c, b, a)$.

polygon

A [polygon] is a closed plane figure formed by connecting a finite set of points in such a way that they do not cross each other.

polyhedra

[polyhedra] A solid figure for which the outer surface is composed of polygons.

prime number

A [prime number] is a number which is divisible only by 1 and itself. The first prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37....$ The number 33 for example is no prime number because it is divisible by 3.

quadrant

A [quadrant] is one of the regions in the plane obtained when cutting the plane along the coordinate axes.

- The first quadrant contains all the points with positive x and positive y coordinates.
- The second quadrant contains all the points with negative x and positive y coordinates.
- The third quadrant contains all the points with negative x and negative y coordinates.
- The fourth quadrant contains all the points with positive x and negative y coordinates.

quadratic function

A function of the form $f(x) = ax^2 + bx + c$ is called a [quadratic function]. For example $f(x) = x^2 + 2$ is a quadratic function. The graph of a quadratic function is a parabola if a is not zero. If a is zero it is a linear function which has as the graph a line.

quotient

The [quotient] of two numbers n and m is the largest integer smaller or equal to n/m. for example the quotient of 11 and 4 is 2 with areminder of 3.

smallest common multiple

[smallest common multiple] see least common multiple.

subtraction

[subtraction] is a basic operation for numbers. Examples: $5 - 3 = 2$.

	2	3	5	2
-	1	4	5	6
		8	9	6

More generally in any commutative group, the addition of the inverse -b of an element b to an element a, denoted by a-b is called a subtraction.

multiplication

[multiplication] is a basic operation for numbers. Example: $4 * 5 = 20$.

1	2	4	x	1	2	3	3	4
+				4	9	3	3	6
+			2	4	6	6	8	
+		1	2	3	3	4		
		1	5	2	9	4	1	6

More generally, in any group the group operation is called multiplication. Example: The composition of two transformations in the plane is a multiplication, matrix multiplication is a multiplication.

fraction

[fraction]: The representation of a rational number r as p/q, where p and $q \neq 0$ are integers is called a fraction. Fractions can be added, subtracted, multiplied and one can divide by nonzero fractions.

Addition of fractions:	$5/3 + 7/5 = 25/15 + 21/15 = 46/15$
Subtraction of fractions:	$5/3 - 7/5 = 25/15 - 21/15 = 4/15$
Multiplication of fractions:	$5/3x7/5 = 35/15$
Division of fractions:	$5/3 : 7/5 = 5/3x5/7 = 25/21$

Not all numbers are rational numbers. Non rational numbers are called irrational. For example, sqr(2) can not be written as p/q because $\sqrt{(2)} = p/q$ would imply $2 = p^2/q^2$ or $2q^2 = p^2$. But by the uniqueness of prime factorisation of integers the left hand side has an odd number of factors 2, the right hand side an even number. $\sqrt{(2)}$ is an example of an irrational number.

hexagon

A [hexagon] is a polygon with six sides.

parallelogram

A [parallelogram] is a quadrilateral whose opposite sides are parallel and congruent.

polygon

A [polygon] is a closed curve in the plane formed by three or more line segments. One usually assumes that the segments don't intersect. Examples:

3 sides:	triangle
4 sides:	quadrilateral, (i.e. rectangle, rhombus, rhombus)
5 sides:	pentagon
6 sides:	hexagon
7 sides:	septagon
8 sides:	octogon

quadrilateral

A [quadrilateral] is a polygon with four sides.

parallel

Two lines in the plane are called [parallel] if they do not intersect. Two parallel lines can be translated into each other. Two lines in space are called [parallel] if they can be translated into each other. Unlike in the plane, two lines in space which are not parallel do not need to intersect.

triangle

A [triangle] is a polygon defined by three points in the plane. The three points form the edges of the triangles, the three connections of the points form the sides of the triangle.

random number generator

A [random number generator] is a device used to produce random numbers. In daily life like for gambling, one often uses dice or coin tossing to find random numbers. Computers often use pseudo random number generators, which are deterministic sequences which look random. Computers can also access hardware internal staes of the computer to improve randomness.

range of a function

The [range of a function] is the set of all values $f(x)$, where x is in the domain of f.

ratio

[ratio] A rational number of the form a/b where a is called the numerator and b is called the denominator.

rectangle

A [rectangle] is a parallelogram with four right angles. It is a quadrilateral, a polygon with four points in the plane. All angles have to be right angles. In a rectangle, opposite sides are parallel. A rectangle is therefore a special parallelogram.

regular polygon

A [regular polygon] is a polygon which has sides of equal length and equal angles. Squares, equilateral triangles or regular hexagons are examples of regular polygons.

remainder

The [remainder] of a division p/q is the amount left after subtracting the maximal integer multiple of q from p. For example $7/3$ has the reminder 1 because $7 - 2 * 3 = 1$. $-11/5$ has the reminder -1.

rhombus

A [rhombus] is a parallelogram with four congruent sides. A special case is the square.

right angle

An angle of 90 degrees is called a [right angle].

right triangle

A triangle which has a right angle is also called a [right triangle].

sequence

An ordered list of elements is called a [sequence] For example, $(1, 3, 2, 1)$ is a list of elements which form a finite sequence. The list $(1, 2, 3, 4, 5, 6, ...)$ forms an infinite sequence.

set

A [set] is a collection of things, without regard to their order.

slope

The [slope] of a line $y = mx + b$ is the number m. One often measures it in percentages. $m = 1$ means 100 percent. If a street has a slope of 10 percent, one climbes for every 10 meters going forwards 1 meter up.

square

A [square] is a polygonal shape in the plane with four sides where each side has the same length and all sides are perpendicular on each other. A square is also a number of the form n*n like 65=8*8. A square with integer side length has a square number as the area.

subset

A [subset] of a set is a set of elements which are all contained in that set. For example the set $A = \{1, 2, 3, 8, 4\}$ has $B = \{2, 3\}$ as a subset.

subtraction

[subtraction] is the operation of taking the difference between two numbers. For example $7 - 2 = 5$.

tessellation

A [tessellation] is a cover of the plane using a finite set of polygons without leaving gaps or overlaps. Examples are regular tessellation into triangles or squares or regular hexagons. Semiregular tesselations allow to cover the plane with different types of shapes. Tesselations are also called tilings and can be defined also in higher dimensions.

trapezoid

A [trapezoid] is a quadrilateral with one pair of parallel sides.

union of sets

The [union of sets] is the set which contains the elements of all sets. One writes $A \cup B$. For example, if $A = \{1, 2, 3, 4\}$ and $B = \{0, 2, 4, 6\}$, then $A \cup B = \{0, 1, 2, 3, 4, 6\}$.

Venn Diagram

In a [Venn Diagram] sets are represented as simple geometric shapes. It visualizes intersections and unions of sets. For example if A is the set of all even numbers between 0 and 10 and B is the set of all numbers divisible by 3 between 0 and 10 one can visualize this with two circles, one of which contains $2, 5, 8, 6$, the other $3, 6, 9$. The circles intersect in a region which has the single element 6.

whole number

A [whole number] is one of the numbers $0, 1, 2, 3, 4, ...$ A whole number is also called natural number or nonnegative integer.

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