Estimation theory

[Estimation theory] part of statistics with the goal of extracting parameters from noise-corrupted observations. Applications of estimation theory are statistical signal processing or adaptive filter theory or adaptive optics which allows for example image deblurring.

Parameter estimation problem

[Parameter estimation problem] determine from a set L of observations a parameter vector. A paremeter estimate is a random vector. The estimation error ϵ is the difference between the estimated parameter and the parameter itself. The mean-squared error is given by the mean squared error matrix $E[\epsilon^T\epsilon]$. It is a correlation matrix.

Biased

[Biased] An estimate is said to be biased, if the expected value of the estimate is different than the actual value.

Asymptotically unbiased

[Asymptotically unbiased] An estimate in statistics is called asymptotically unbiased, if the estimate becomes unbiased in the limit when the number of data points goes to infinity.

Consistent estimate

[Consistent estimate] An estimate in statistics is called consistent if the mean squared error matrix $E[\epsilon^T\epsilon]$ converges to the 0 matrix in the limit when the number of data points goes to infinity.

Mean squared error matrix

The [Mean squared error matrix] is defined as $E[\epsilon^T\epsilon]$, where ϵ is the difference between the estimated parameter and the parameter itself.

efficient

An estimator in statistics is called [efficient] if its mean-squared error satisfies the Cramer-Rao bound.

Cramer-Rao bound

[Cramer-Rao bound] The mean-squared error $E[\epsilon^T\epsilon]$ for any estimate of a parameter has a lower bound which is called the Cramer-Rao bound. In the case of unbiased estimators, the Cramer-Rao bound gives for each error ϵ_i the estimate

$$E[\epsilon_i^2] \geq [F^{-1}]_{ii} \ .$$

Fisher information matrix

The [Fisher information matrix] is defined as the expectation of the Hessian $F = E[H(-\log(p))] = E[grad(log(p))grad(log(p))^T]$ of the conditional probability $p(r|\theta)$.

maximum likelihood estimate

The [maximum likelihood estimate] is an estimation technique in statistics to estimate nonrandom parameters. A maximum likelyhood estimate is a maximizer of the log likelihood function $\log(p(r, \theta)$. It is known that the maximum likelihood estimate is asymptotically unbiased, consistent estimate. Furthermore, the maximum likelihood estimate is distributed as a Gaussian random variable.
Example. If X is a normal distributed random variable with unknown mean θ and variance 1, the likelyhood function is $p(r, \theta) = \frac{1}{\sqrt{2\pi}} e^{-(r-\theta)^2/2}$ and the log-likelyhood function is $\log(p(r, \theta)) = -(r - \theta)^2/2 + C$. The maximum likelyhood estimate is r.
The maximum likelihood estimate is difficult to compute in general for non-Gaussian random variables.

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