LECTURE 4

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PLAN

1. A poll

2. The derivative

3. Lesson plan reviews

4. A volume problem

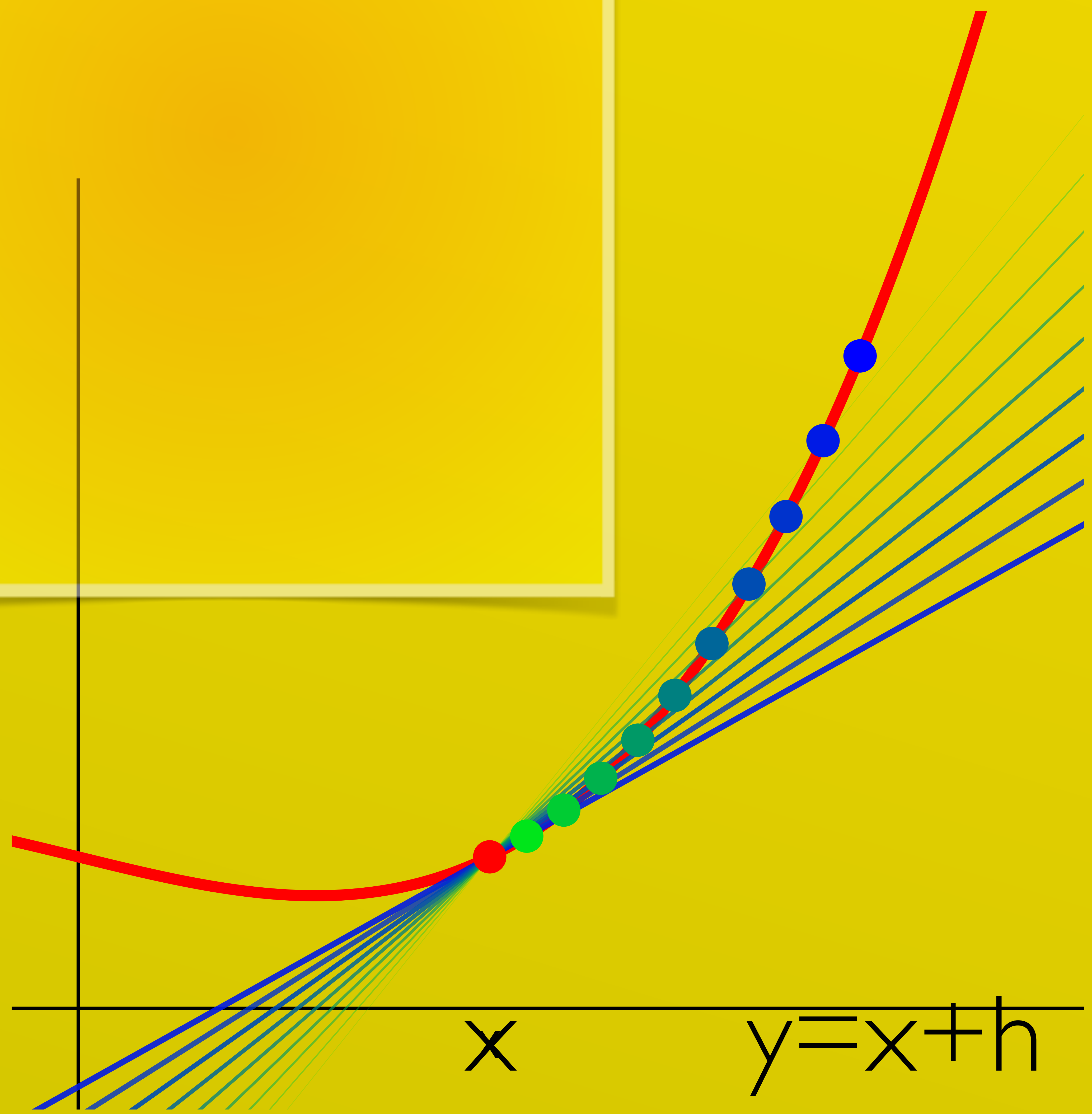
5. Power functions

6. Root function

6.Exponential functions

7 Cupcake and swimming

THE DERIVATIVE



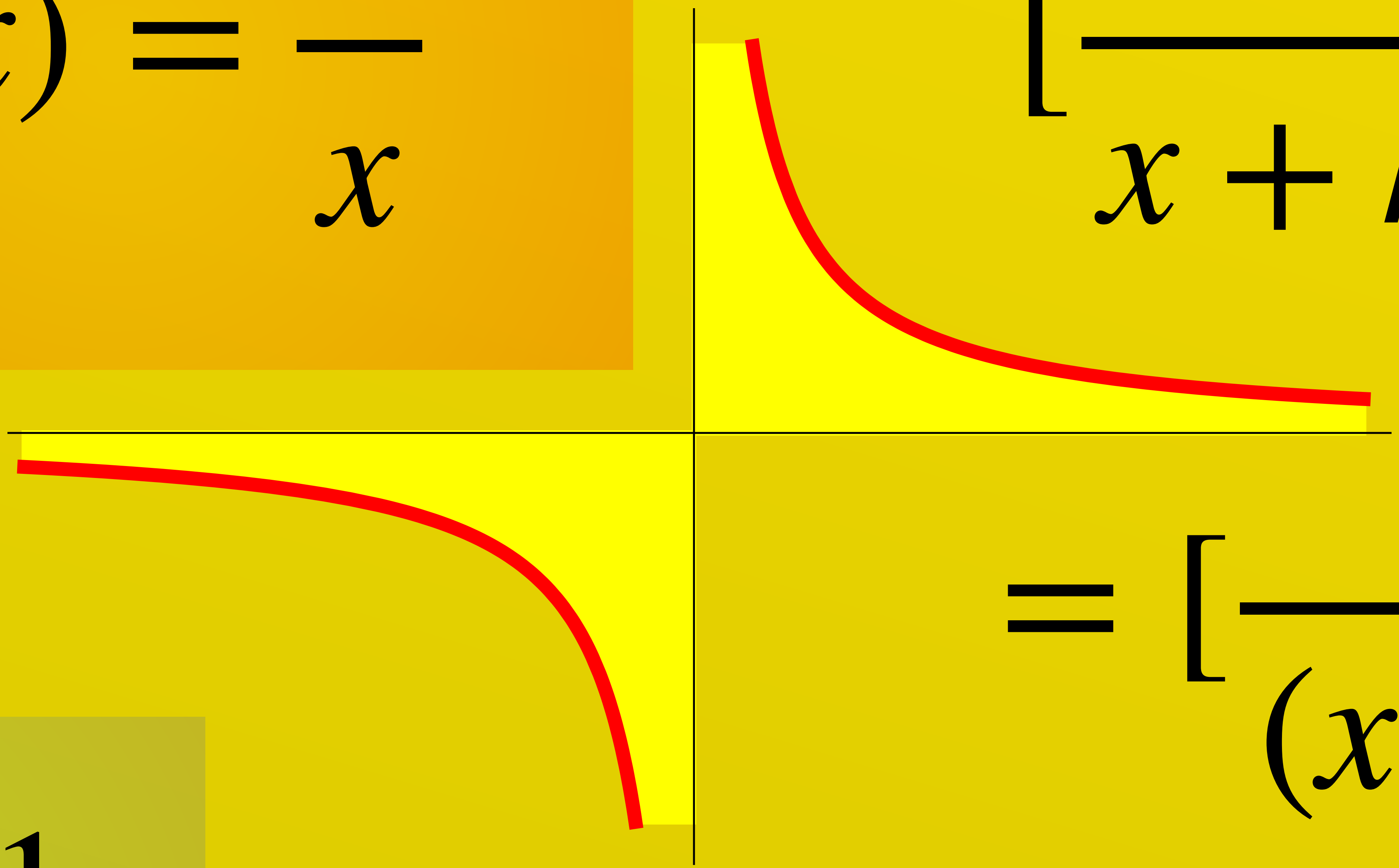
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{y\to x}\frac{f(y)-f(x)}{y-x}$$

EXAMPLES

from lesson problem

$$f(x) = \frac{1}{x}$$



$$\left[\frac{1}{x+h} - \frac{1}{x}\right]\frac{1}{h}$$

$$= \left[\frac{-h}{(x+h)x}\right]\frac{1}{h}$$

$$= \frac{-1}{(x+h)x}$$

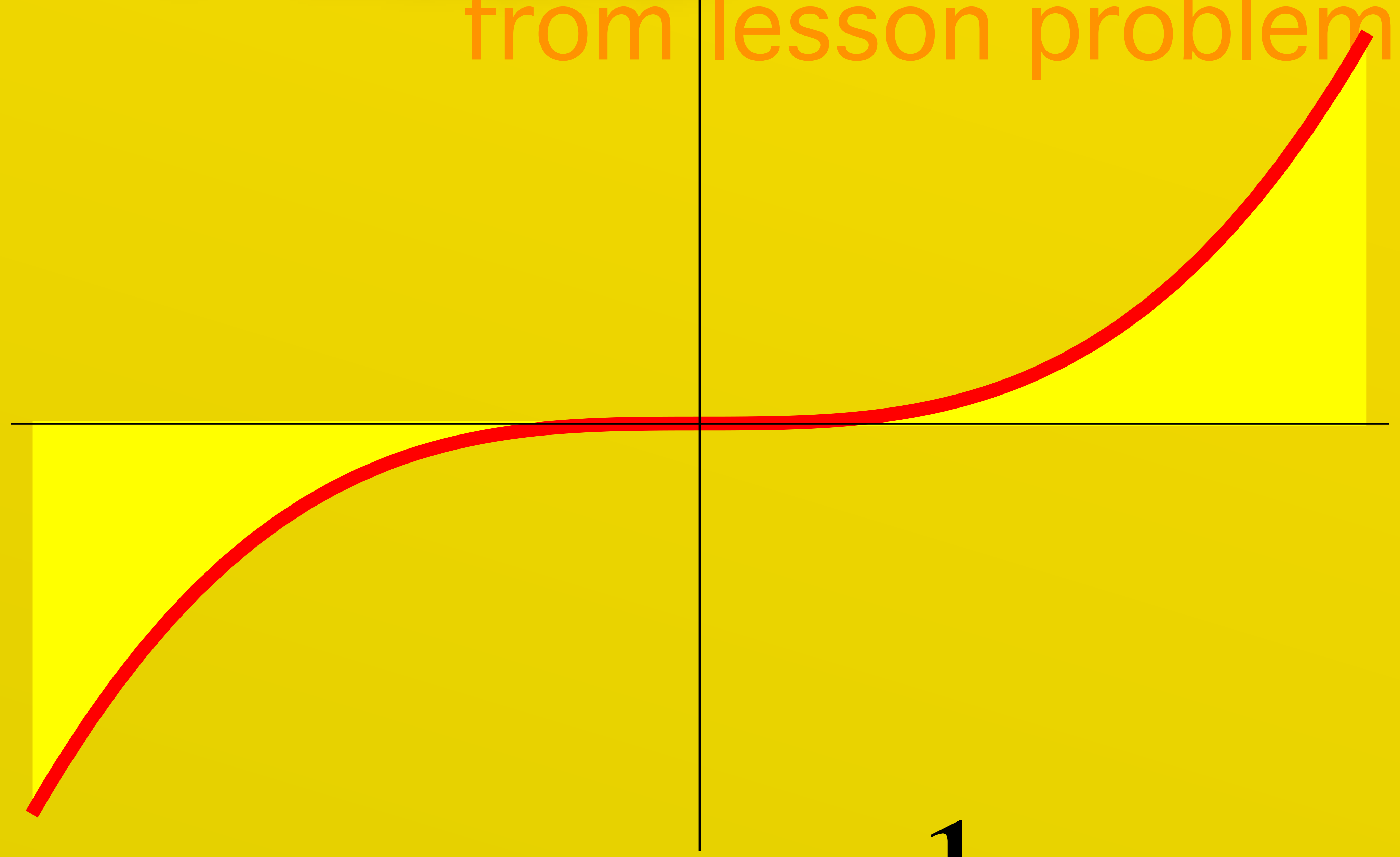
$h \to 0$

$$f'(x) = -\frac{1}{x^2}$$

EXAMPLES

from lesson problem

$$f(x) = x^3$$



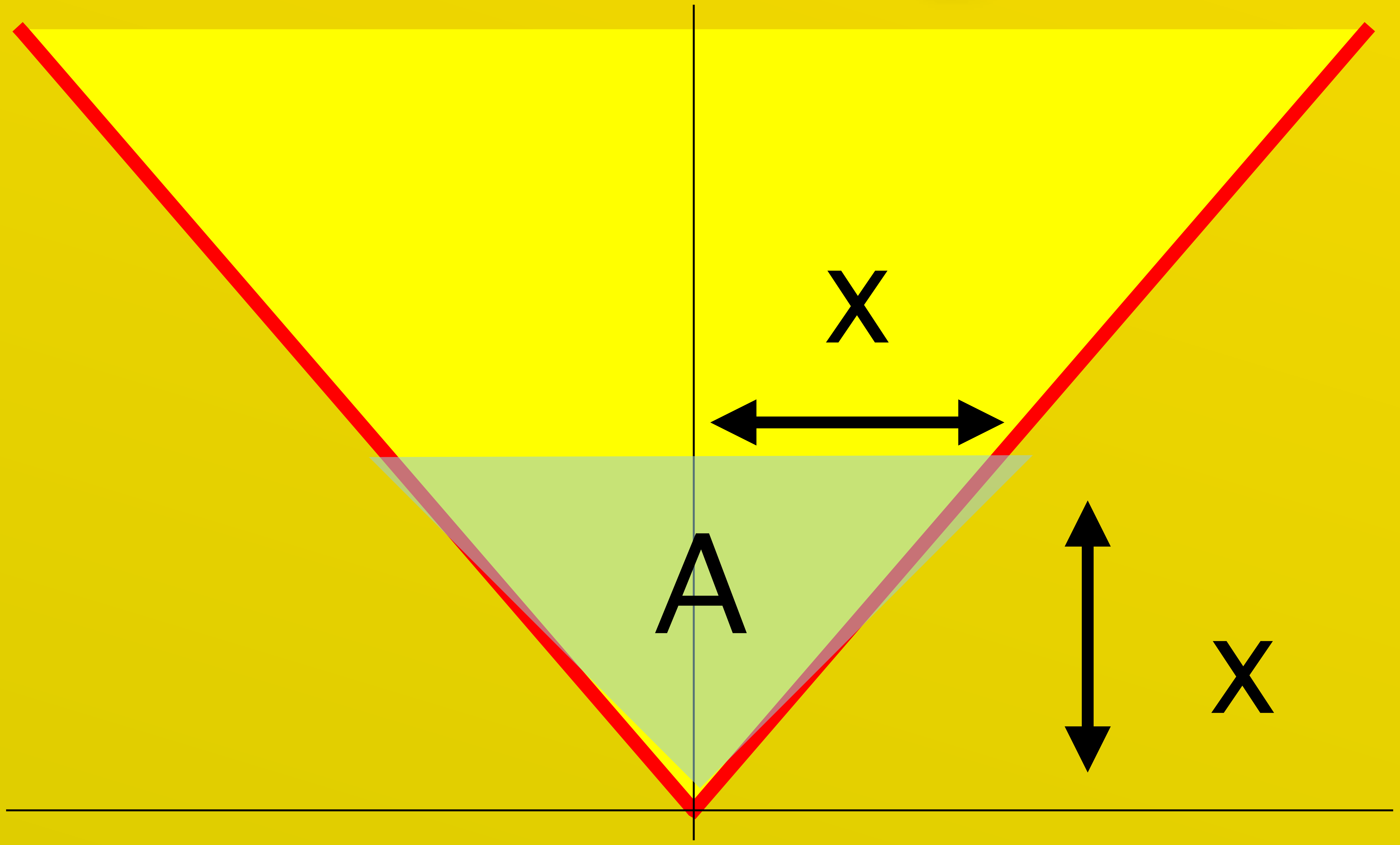
$$[(x+h)^3 - x^3]\frac{1}{h}$$

$$= [x^3 + 3x^2h + 3xh^2 + h^3 - x^3]\frac{1}{h}$$

$$h \to 0$$

$$f'(x) = 3x^2$$

TRIANGLE



$$A = x^2$$

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How fast does x change as
a function of the area A?

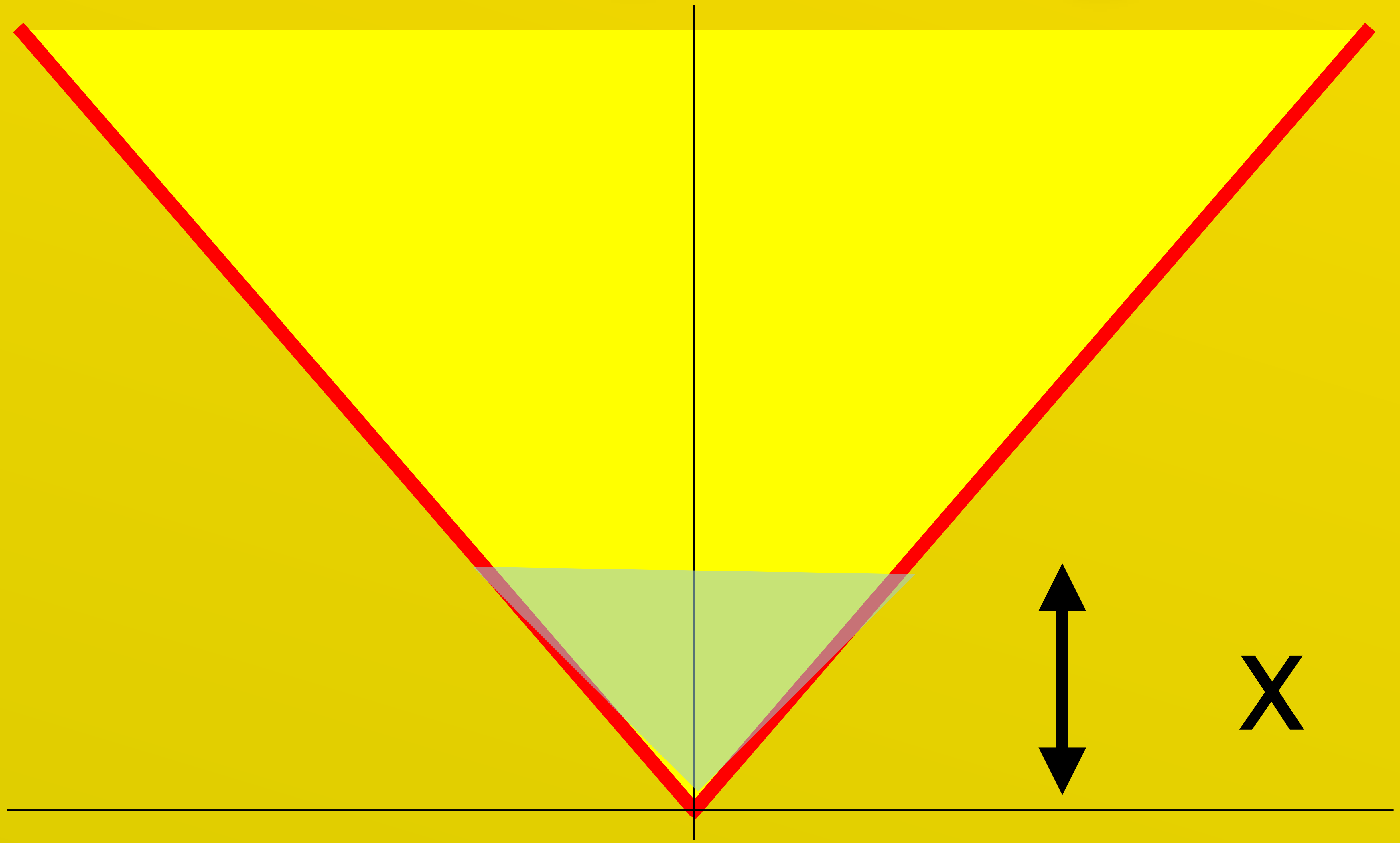
BREAKOUT

$$f(x) = \sqrt{x}$$

$$\sqrt{x+h} - \sqrt{x} = \frac{[\sqrt{x+h} - \sqrt{x}][\sqrt{x+h} + \sqrt{x}]}{\sqrt{x+h} + \sqrt{x}}$$

Can you simplify this?

FILLING A GLASS

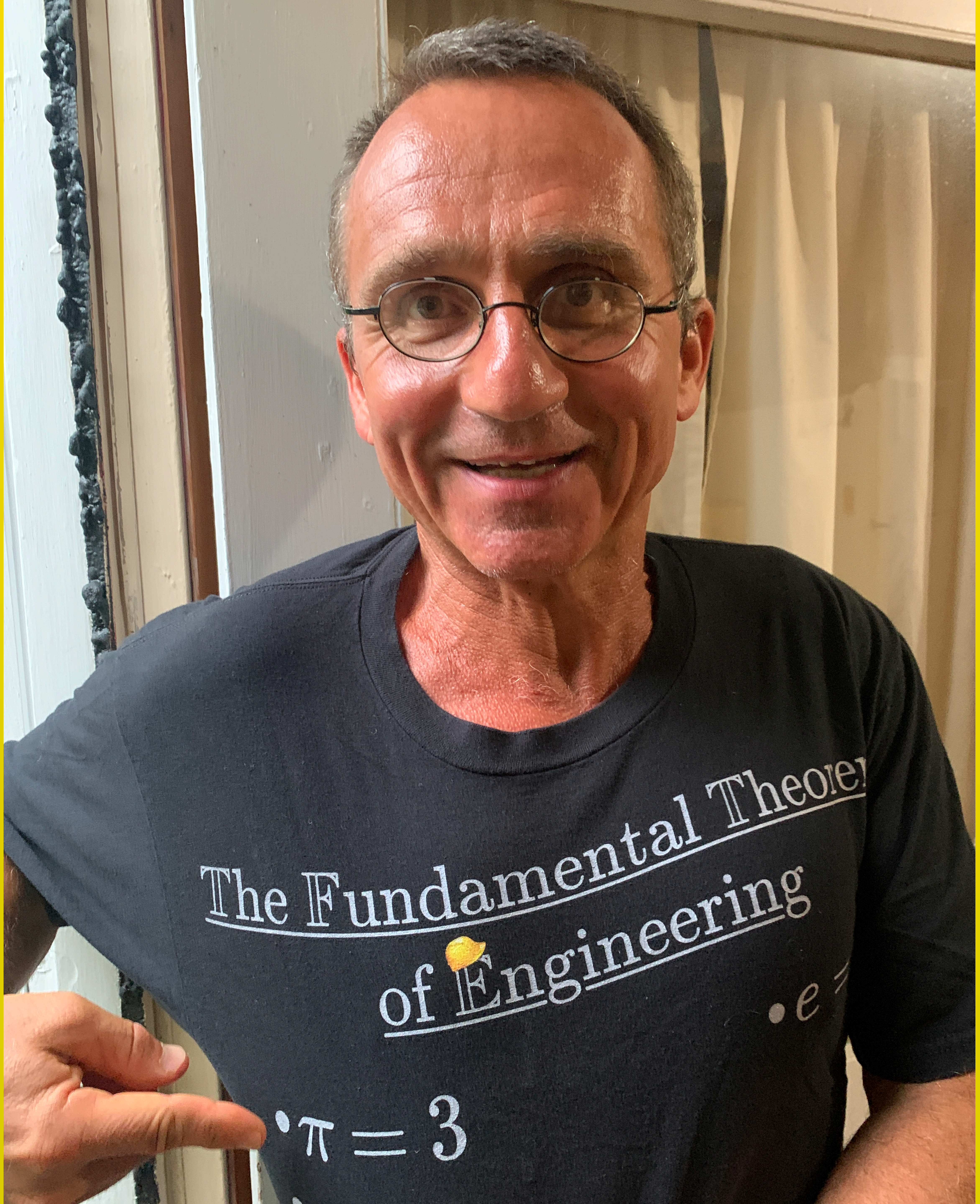


$$V(x) = \frac{x^3\pi}{3} = x^3$$

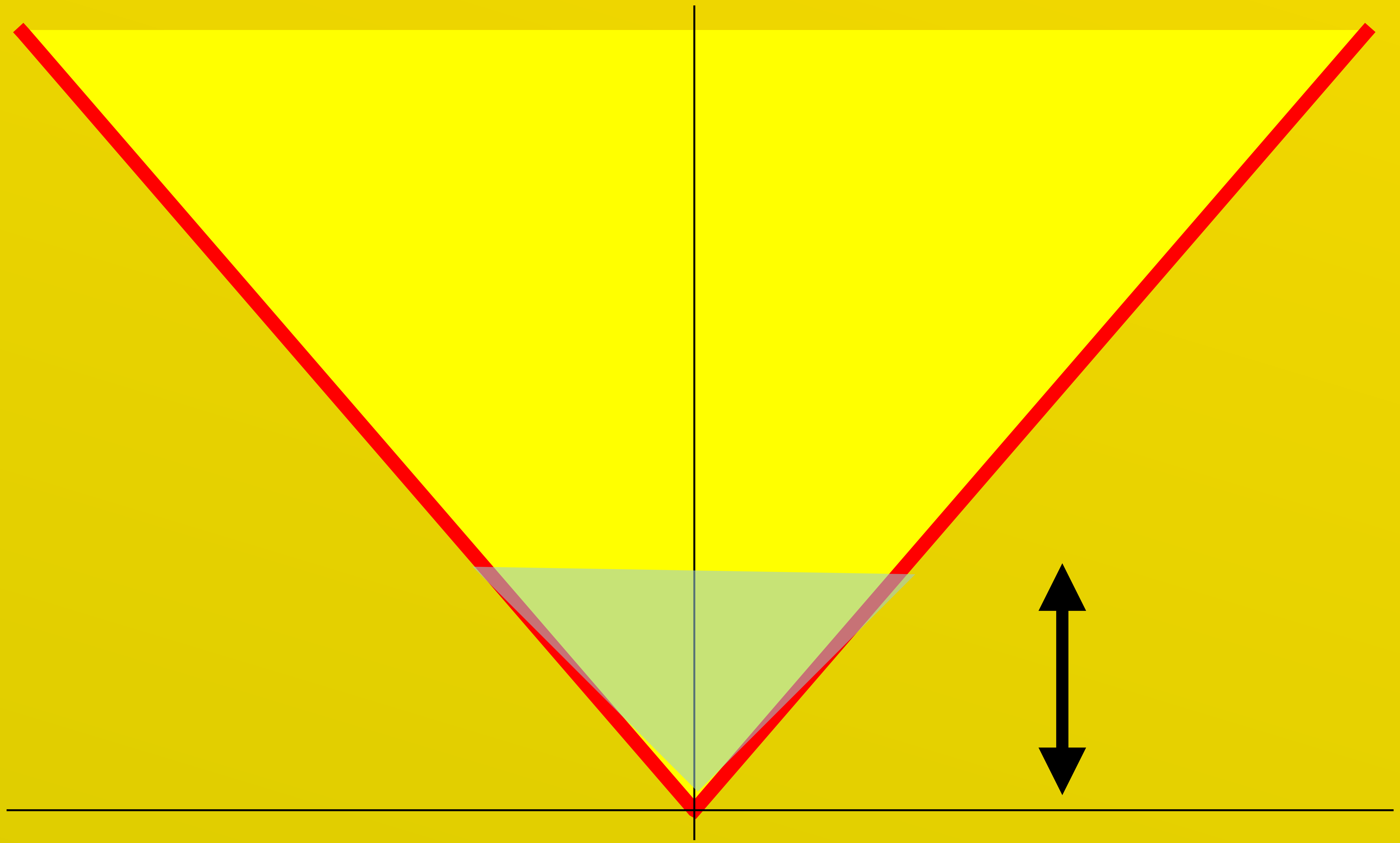
How fast does the volume change depending on x?

YES

$\pi = 3$

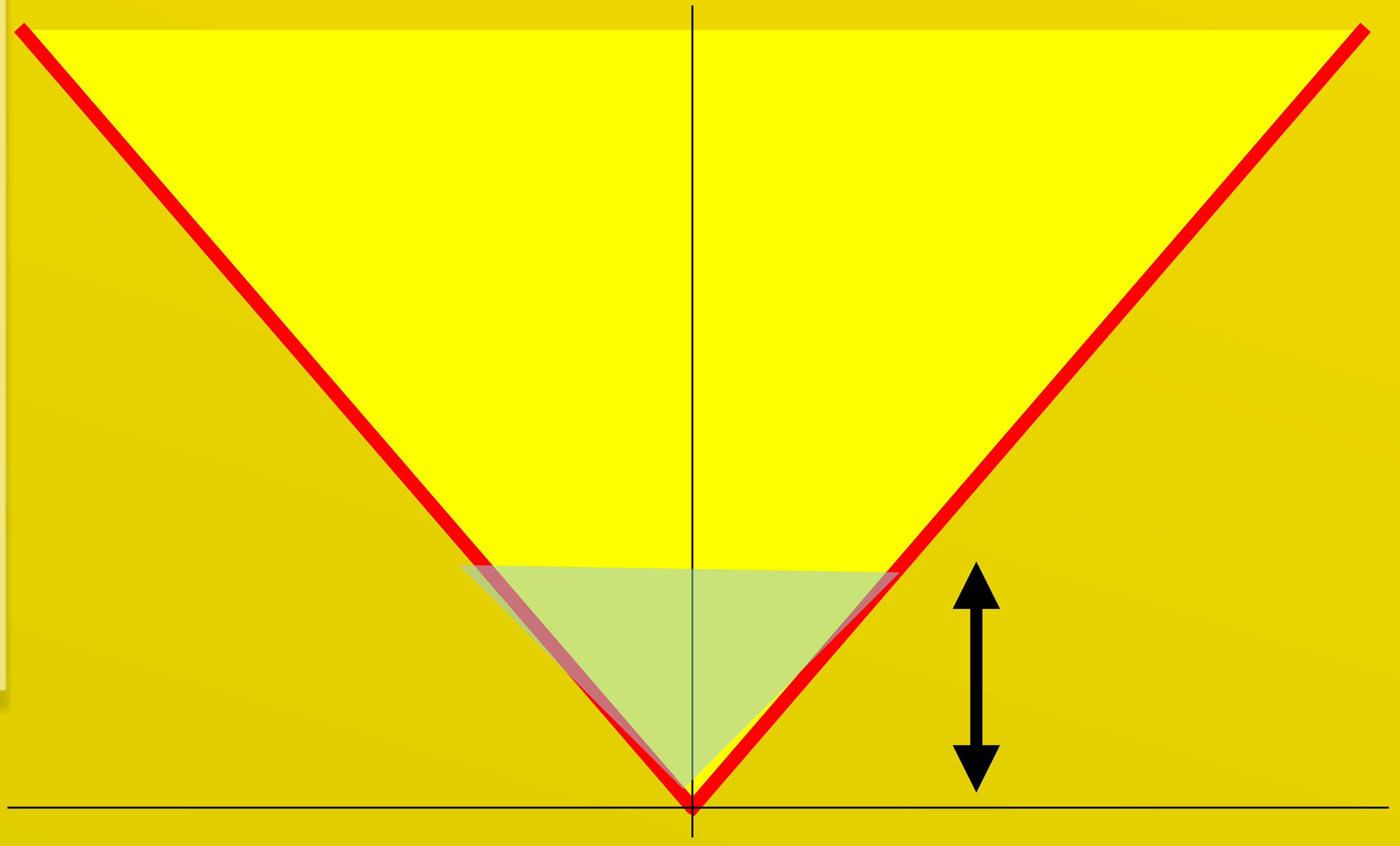


MORE INTERESTING



How fast does the water level x change as a function of the volume?

MORE INTERESTING



Can we compute x'(V)?

$$V(x) = x^3$$

$$x = V^{1/3}$$

$$x'(V) = \frac{1}{3V^{2/3}}$$

Infinite speed at V=0!

HOW TO COMPUTE?

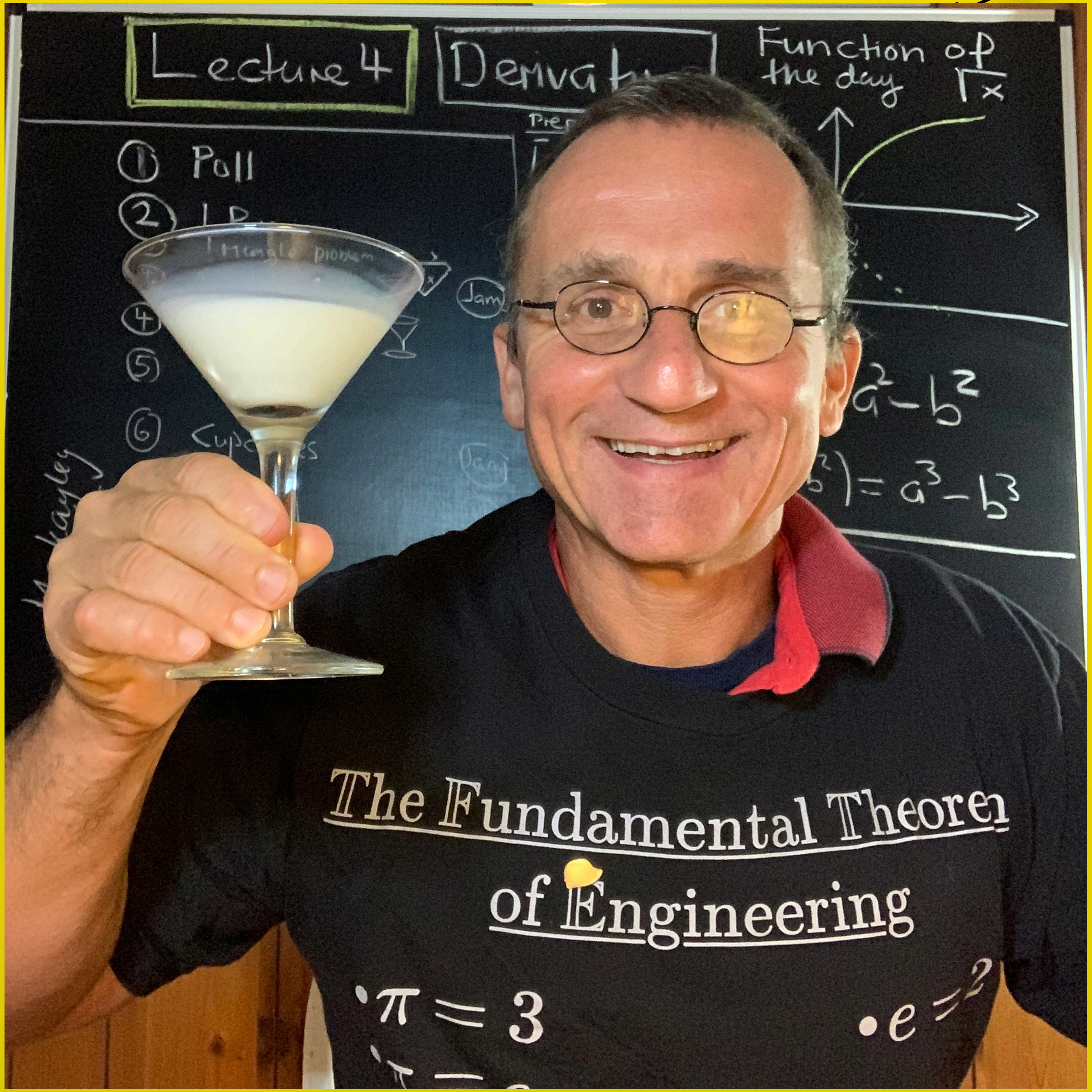
Look at the following identity and note that the nominator is equal to h when multiplying out

$$(x+h)^{1/3} - x^{1/3} =$$

$$\frac{[(x+h)^{1/3} - x^{1/3}][(x+h)^{2/3} + (x+h)^{1/3}x^{1/3} + x^{2/3}]}{[(x+h)^{2/3} + (x+h)^{1/3}x^{1/3} + x^{2/3}]}$$

THAT WAS TOUGH!

$$f(x) = x^{1/3}$$



$$f'(x) = \frac{1}{3x^{2/3}}$$

SEE THE PATTERN?

$$f(x) = x^{-1}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f(x) = x^3$$

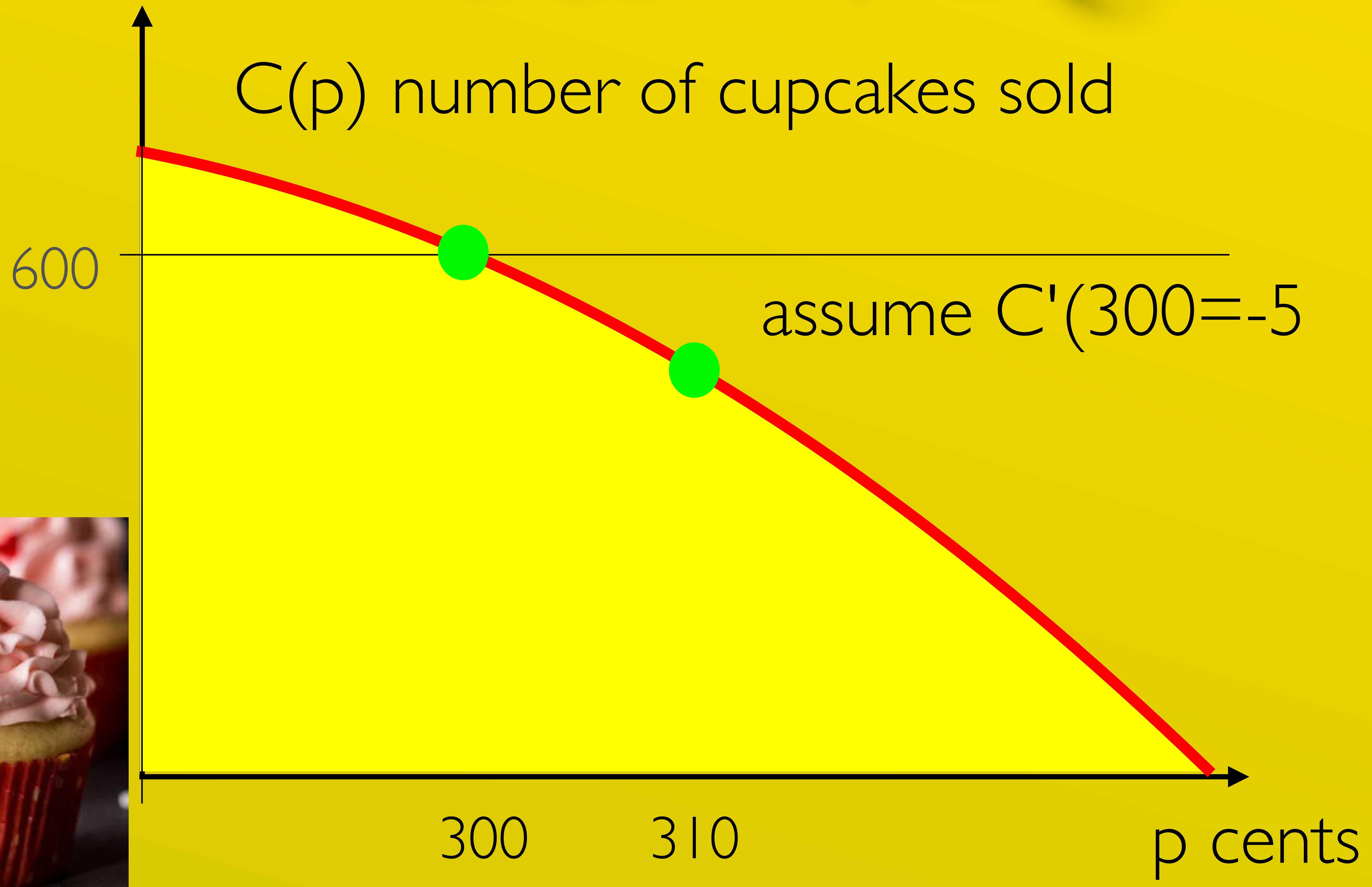
$$f'(x) = 3x^2$$

$$f(x) = x^{1/3}$$

$$f'(x) = \frac{1}{3x^{2/3}}$$

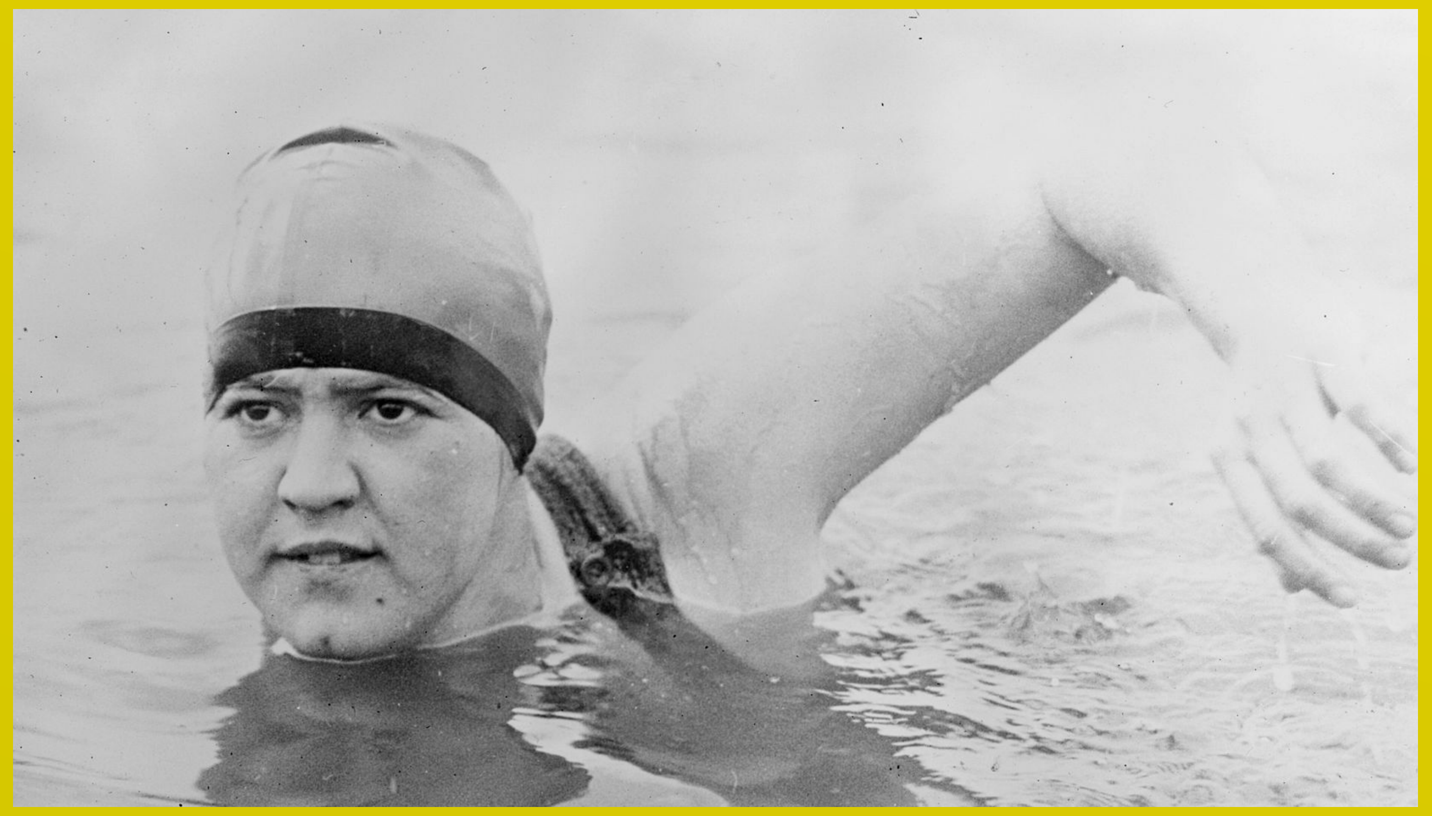
CUPCAKE

Estimate C(310) from this information

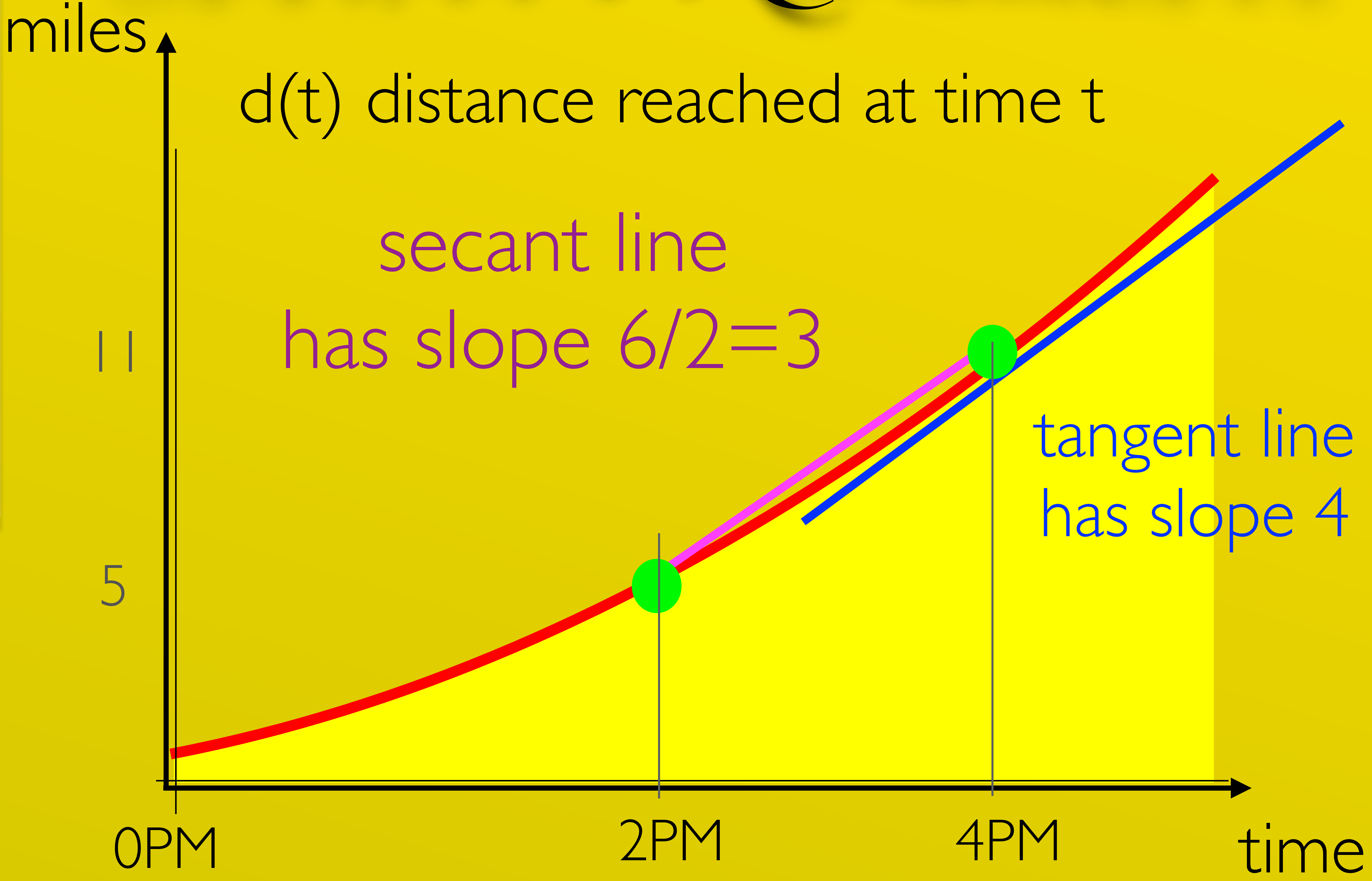


SWIM PROBLEM

estimate
where
the swimmer
is at 3:30,
either with
tangent line
or secand line



Gertrud Ederle 1926



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