

# *LECTURE 6*

## *LIMITS*

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# PLAN

1. Poll: 0 over 0

2. Limits

3. Examples

4. Differential quotients

5.  $\sin(x)/x$

5. Archimedes

6.  $(1 - \cos(x))/x^2$

6. Jam with CA

# LIMITS

A function has a **limit** at  $x=a$  if there exists a value  $b$  such that  $f(x)$  converges to  $b$  when  $x$  approaches  $a$ .

$$\lim_{x \rightarrow a} f(x) = b$$

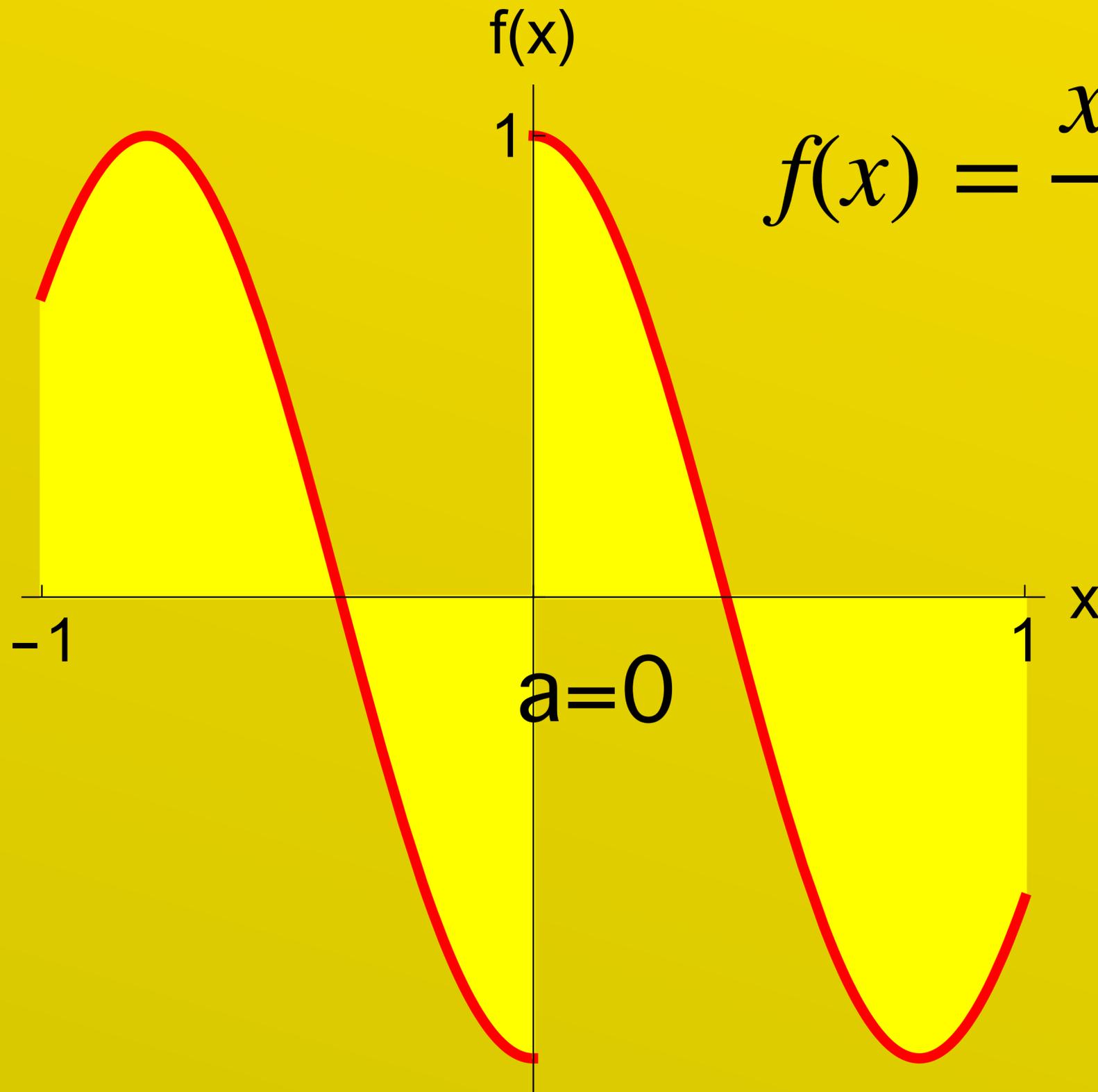
$$\lim_{x \rightarrow a^+} f(x) = b \quad \lim_{x \rightarrow a^-} f(x) = b$$

The limit needs to be **finite**, not like with  $f(x)=1/x$  at  $a=0$

The limit has to be the same from **both sides**, not like  $f(x)=x/|x|$ .

We need convergence to a **unique** value, not like in  $f(x)=\sin(1/x)$ .

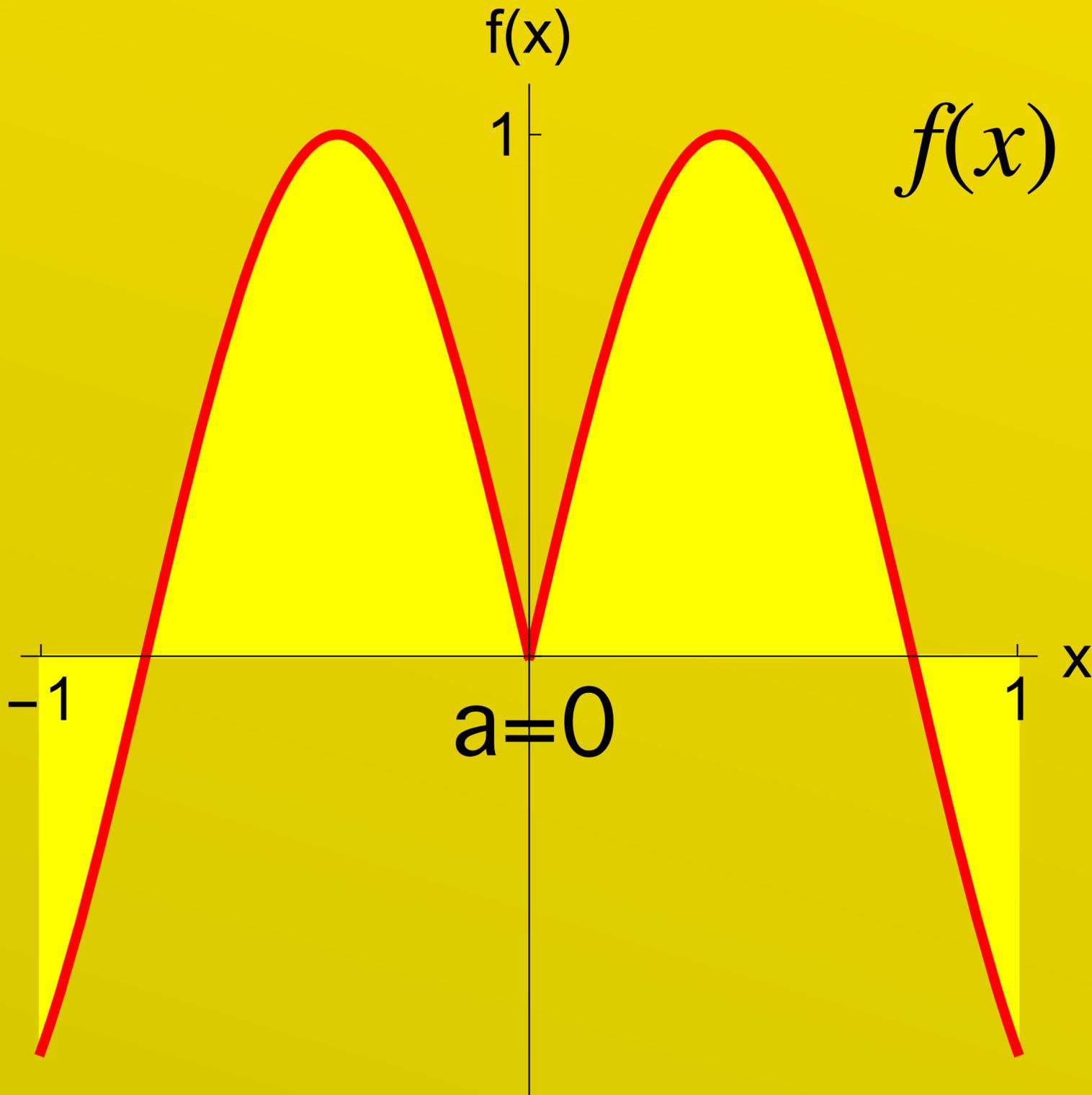
# LIMITS



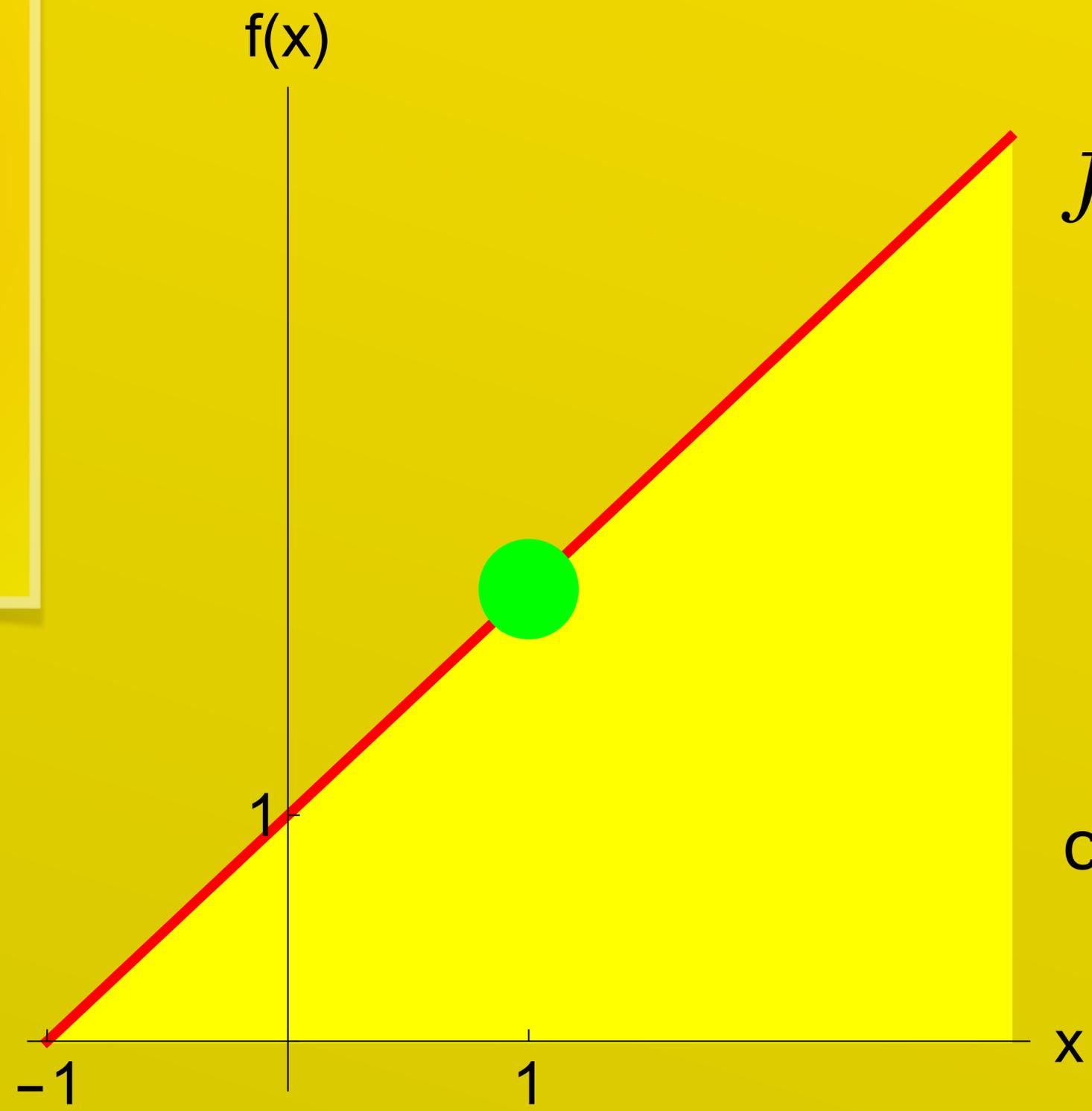
$$f(x) = \frac{x \cos(4x)}{|x|}$$

# LIMITS

$$f(x) = \frac{x \sin(3x)}{|x|}$$



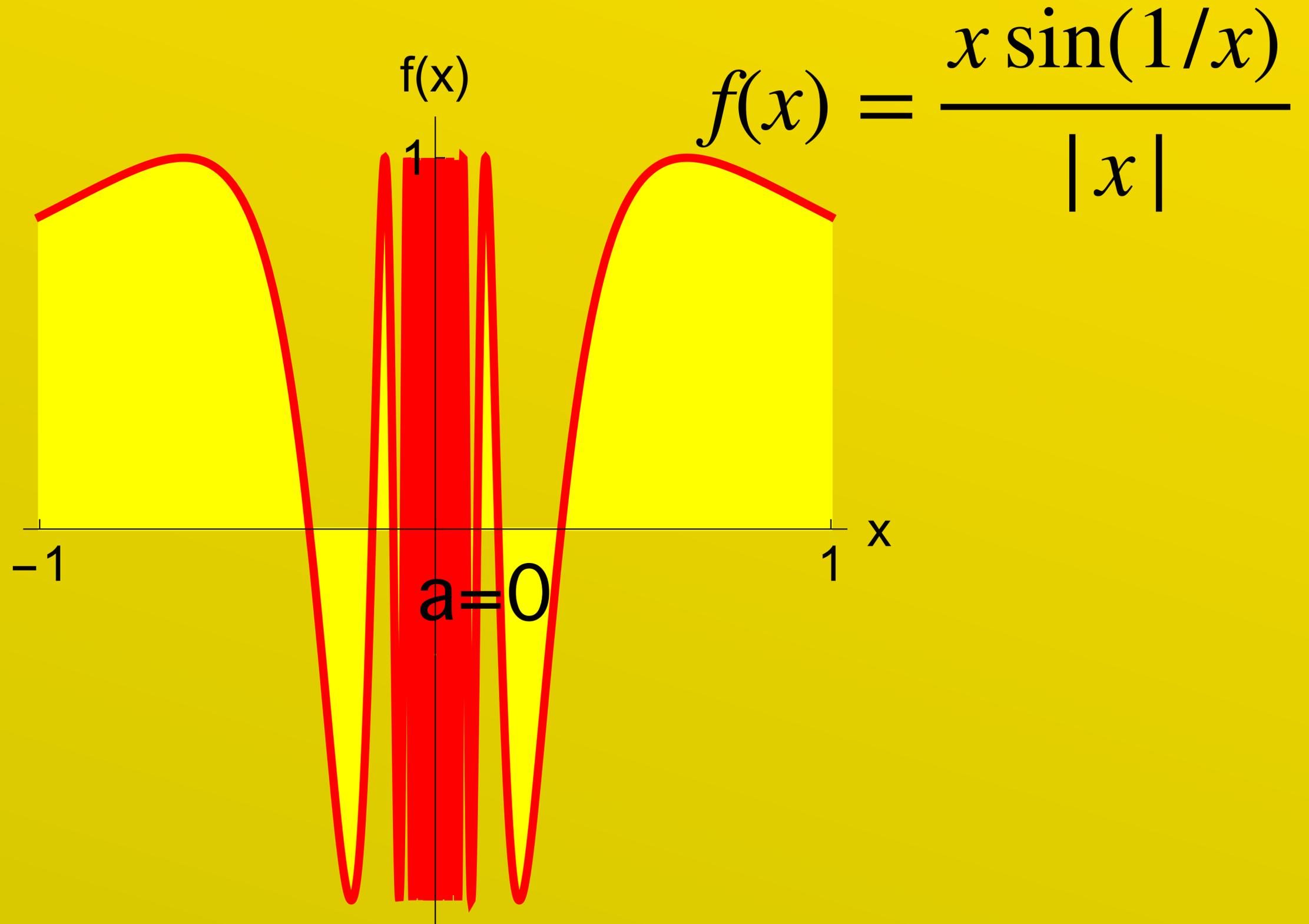
# HEALING



$$f(x) = \frac{x^2 - 1}{x - 1}$$

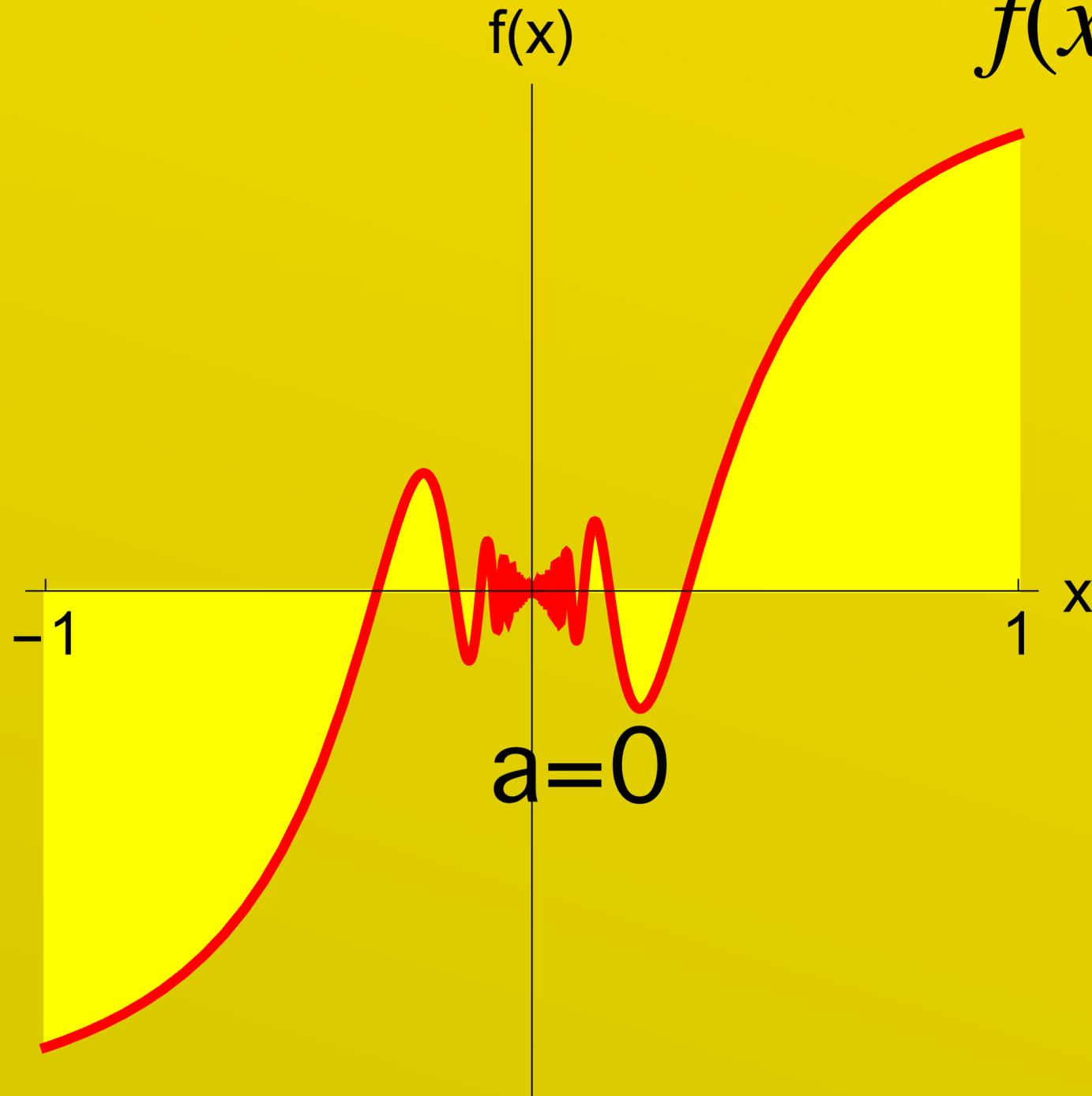
$f$  is not defined at  $x=1$  but we can assign a value to  $x=1$ .

# LIMITS



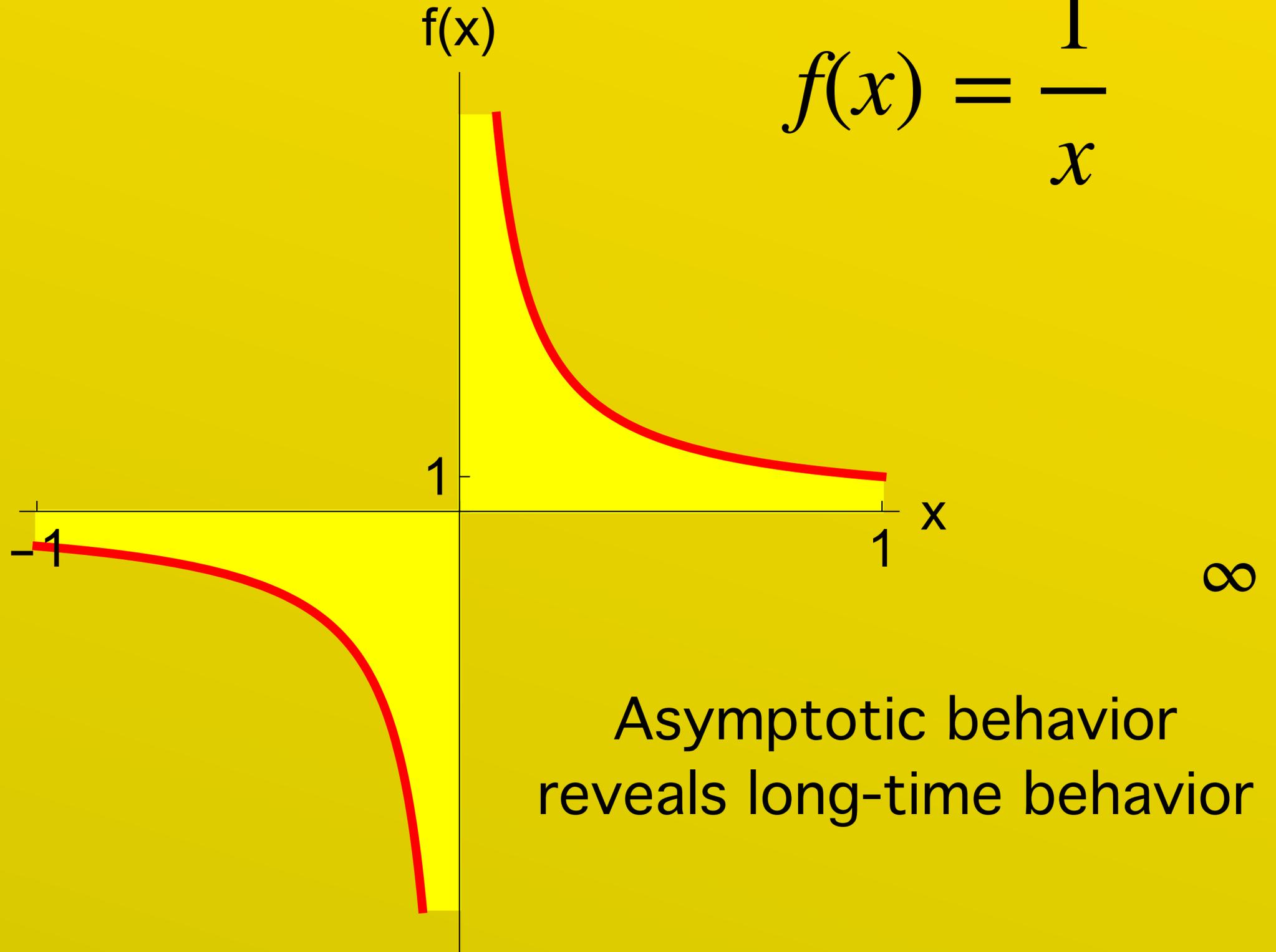
# LIMITS

$$f(x) = \frac{x^2 \sin(1/x)}{|x|}$$



# ASYMPTOTIC

$$f(x) = \frac{1}{x}$$



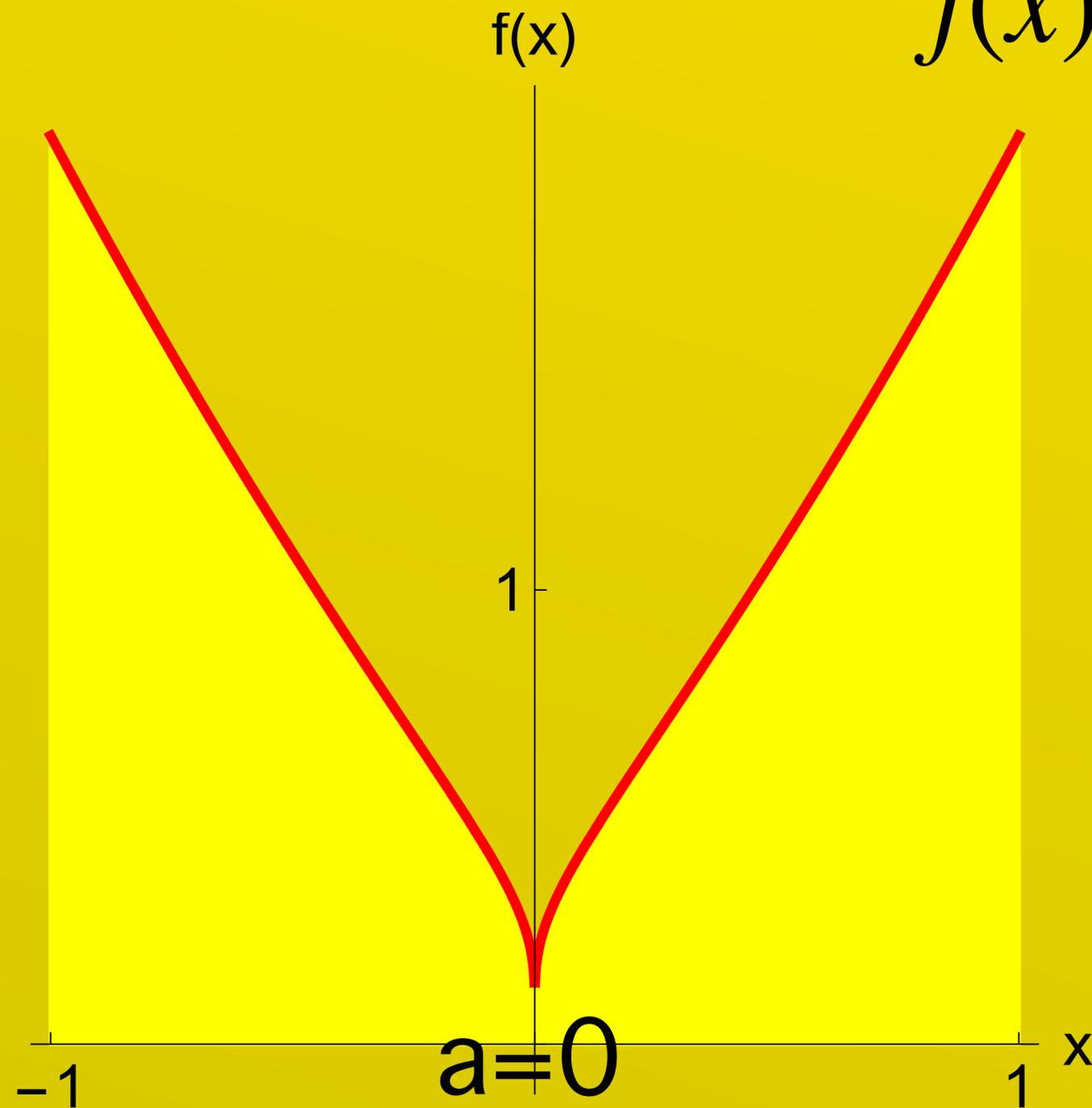
$-\infty$

$\infty$

Asymptotic behavior  
reveals long-time behavior

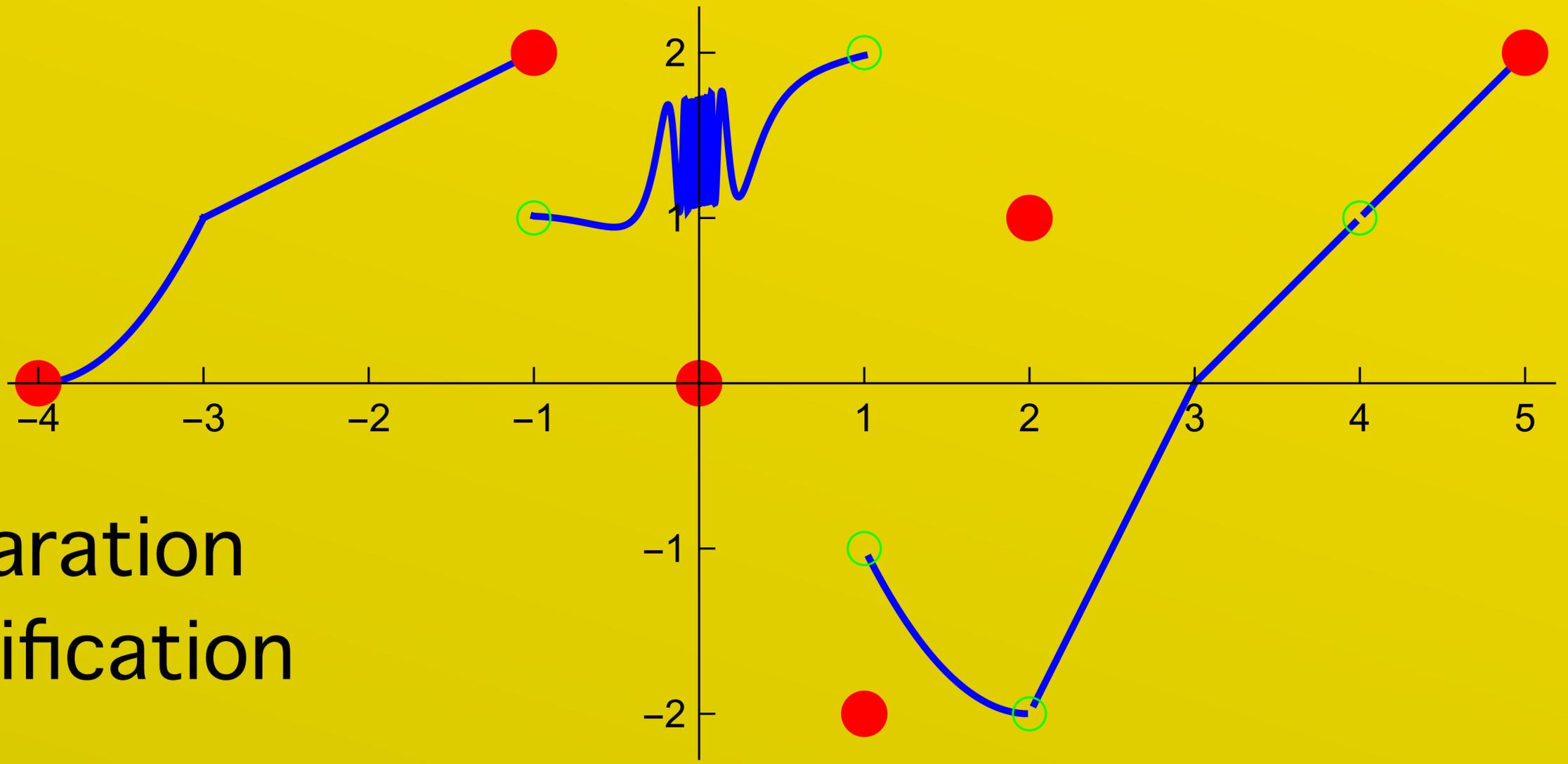
# LIMITS

$$f(x) = \frac{(x^2 - 1)}{\log(|x|)}$$



difficulties  
at  $x=0$  and  $x=1$

# REVIEW



lesson preparation  
problem modification

# *DIFFERENCE QUOTIENT*

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

a=0, is a situation 0/0

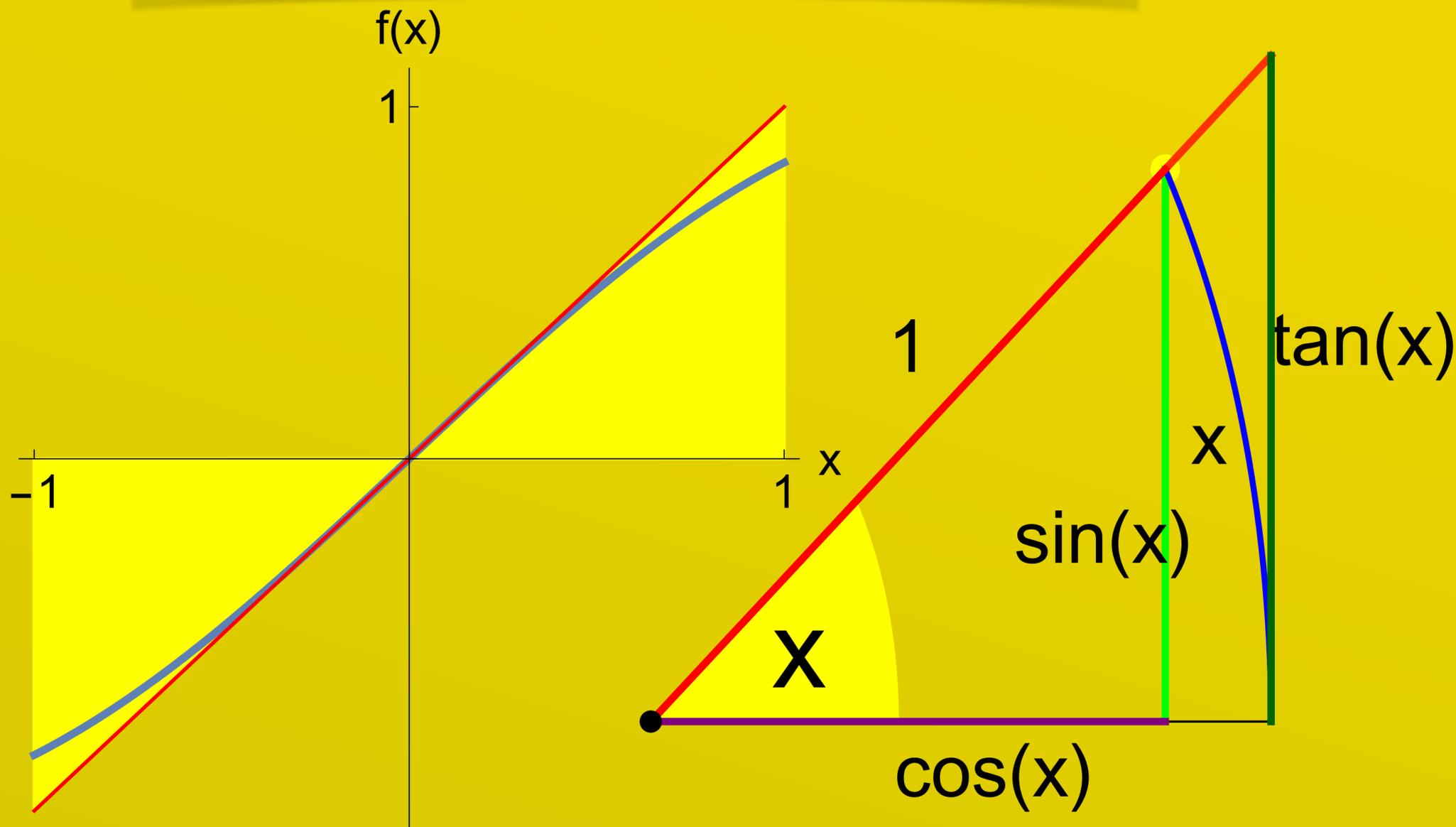
Example:  $f(x) = x^5$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h}$$

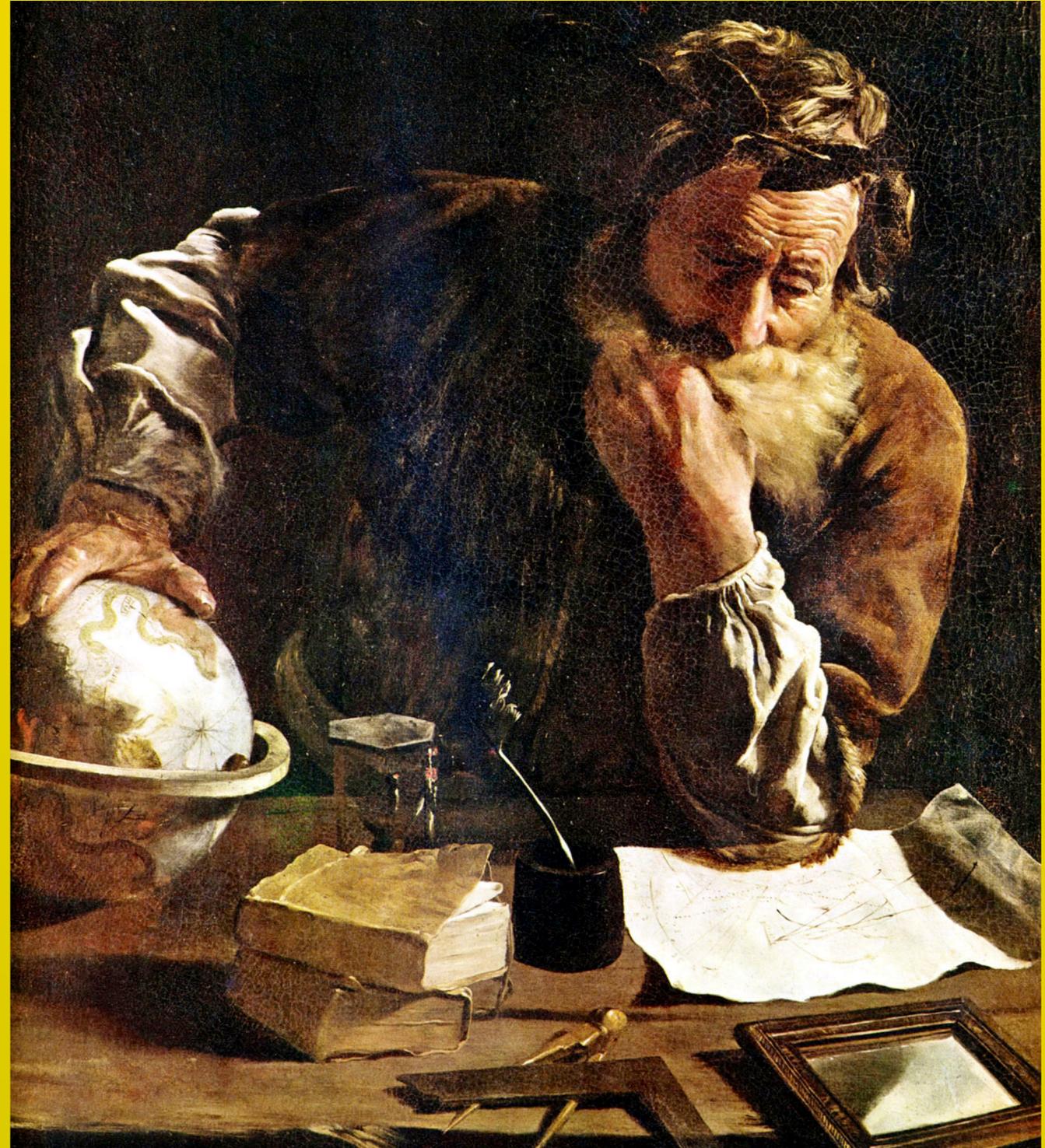
# $\sin(x)/x$

$$\sin'(0) = \frac{\sin(h) - \sin(0)}{h}$$

$$1 = \frac{\sin x}{\sin x} \leq \frac{x}{\sin x} \leq \frac{\tan(x)}{\sin(x)} = \frac{1}{\cos(x)}$$

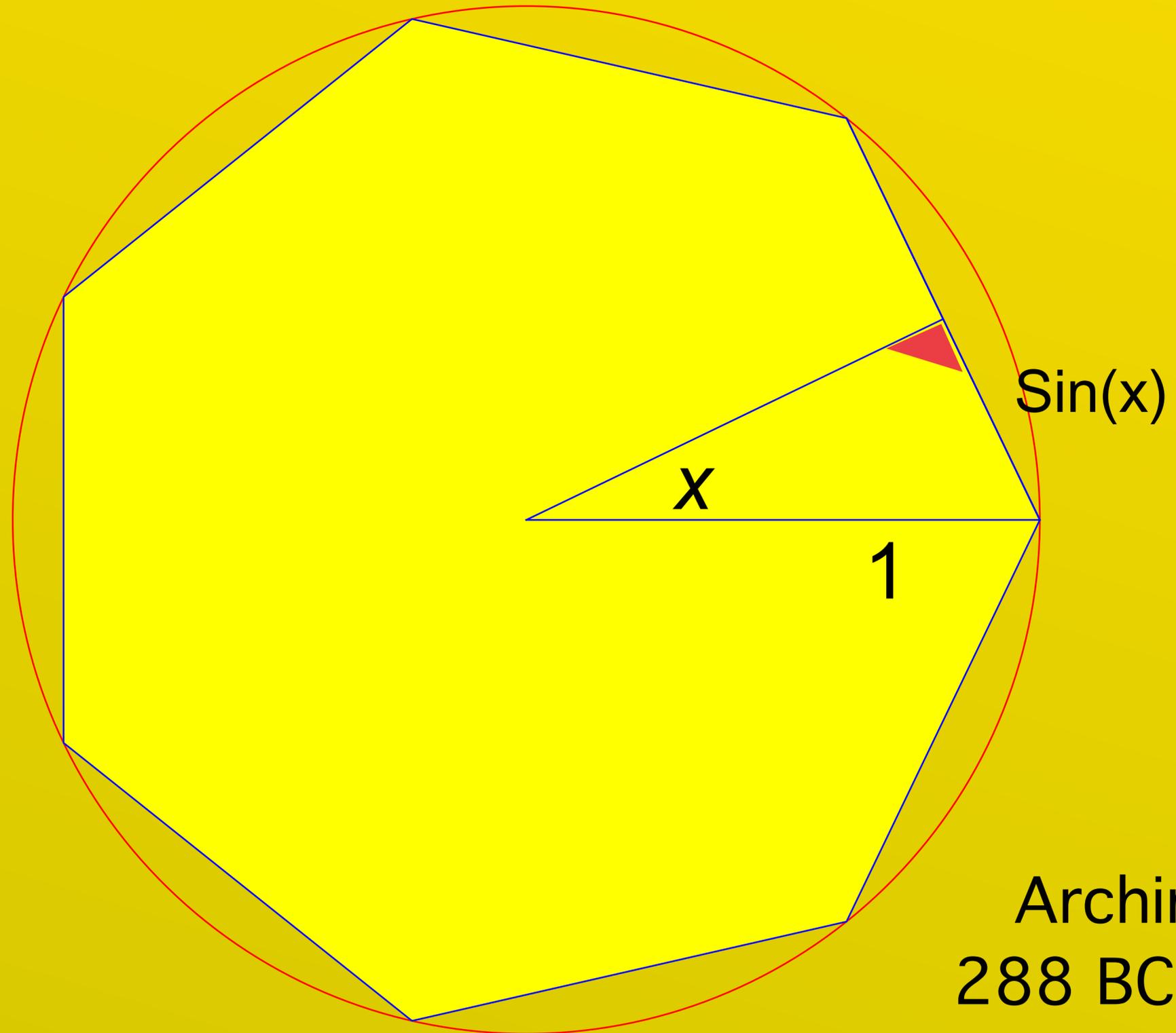


# ARCHIMEDES



Archimedes  
288 BC -212BC

# ARCHIMEDES



$$x = 2\pi/n$$

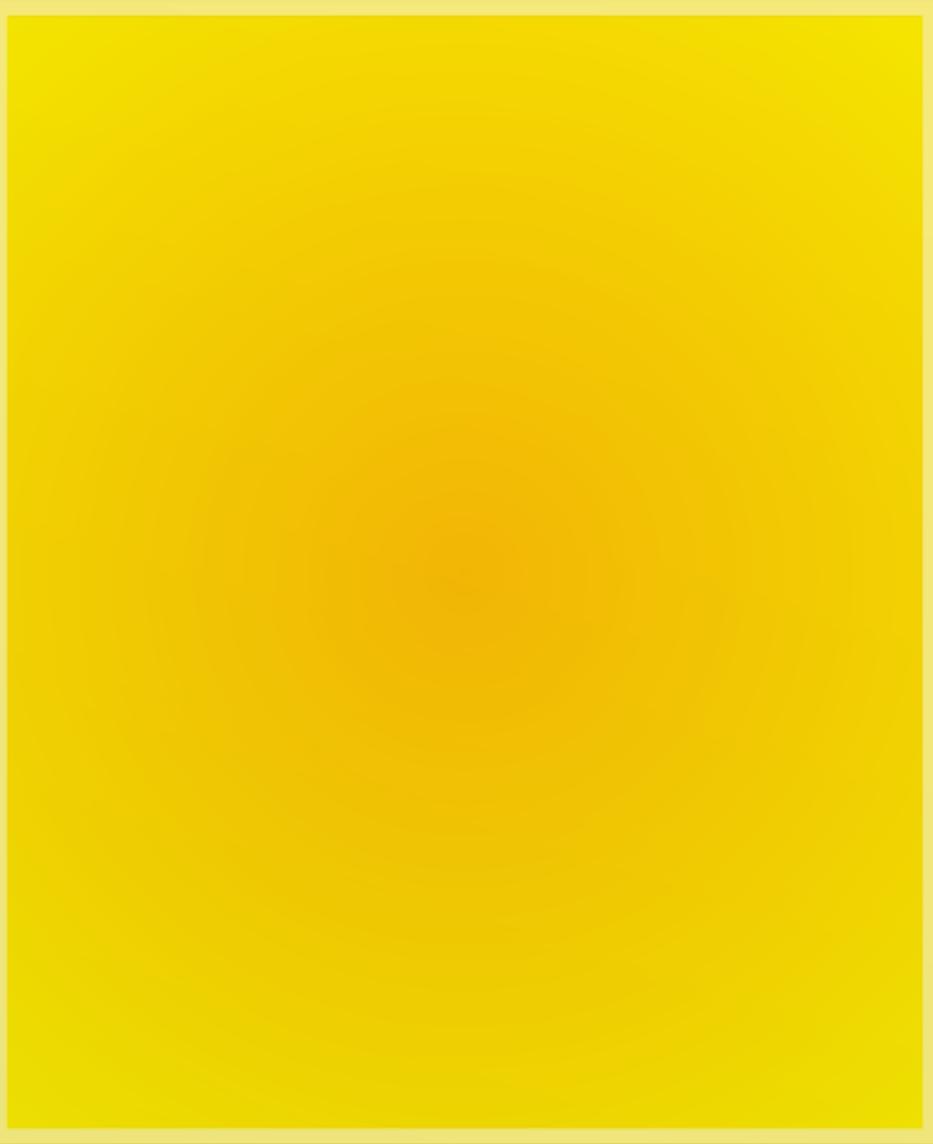
Archimedes  
288 BC -212BC



⊗ Set Marker

👁 Hover Mode

🚀 Boost



*THE END*