

*LECTURE 7*

*CONTINUITY*

Oliver Knill, Harvard University

September 18 2020

# PLAN

1. Continuous graphs or not?

2. Continuity definition

3. Failures of continuity

4.  $\sin(x)/x$  again

6. Jam

5. Why continuity?

6. In applications

7. Jam with CA

# CONTINUITY

A function is **continuous** at  $x=a$  if it has a defined at  $x=a$  and if  $\lim_{x \rightarrow a} f(x) = f(a)$

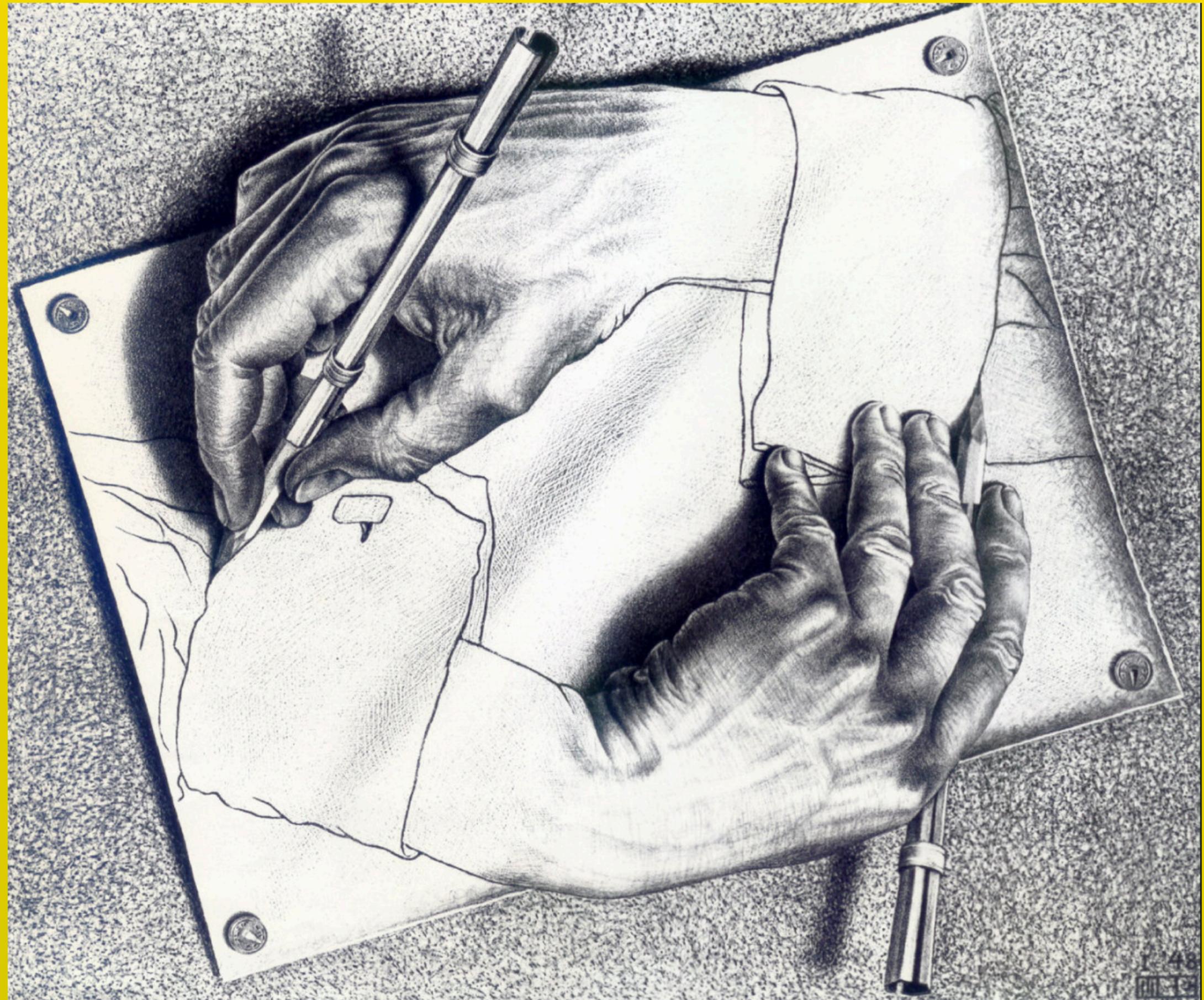
A function is **continuous** on  $[a,b]$  if it is continuous at every point in the interval.

Intuition:

A function is **continuous** on  $[a,b]$  if you can draw the graph with a pen without lifting the pen.



Escher,  
drawing  
hands



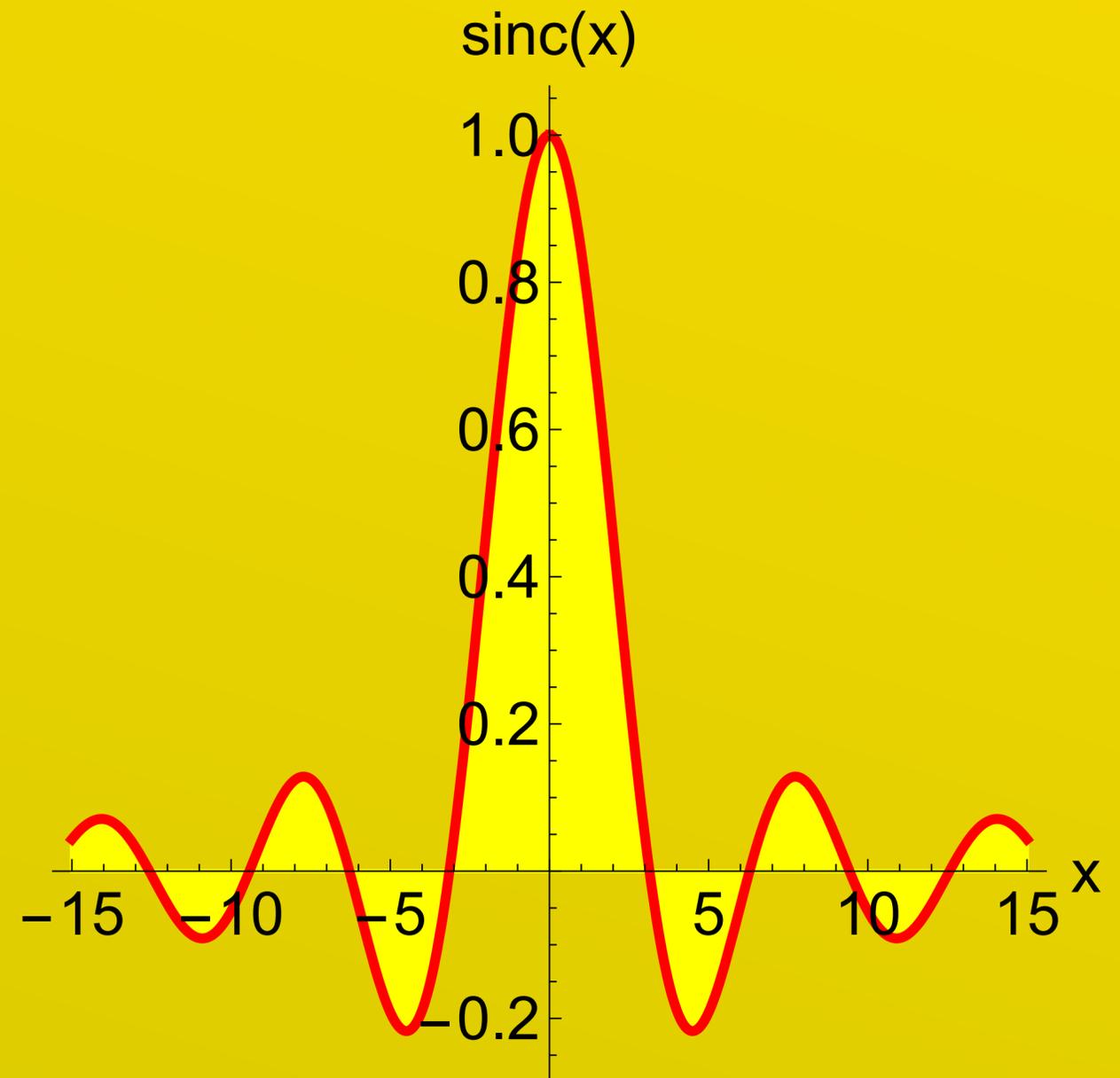
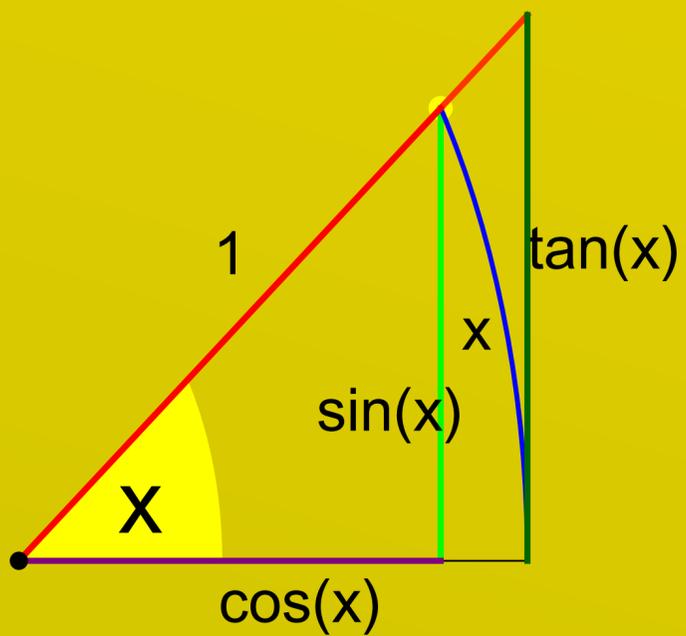
# *IMPORTANCE*

Continuity means **stability**. A small change in  $x$  produces a small change in  $f(x)$ .

Discontinuity can mean **unpredictability**. It can mean catastrophes, fast changes.

With continuity of the derivatives one can even do more and **control** things

$$\text{SIN}(x)/x$$



$$1 = \frac{\sin x}{\sin x} \leq \frac{x}{\sin x} \leq \frac{\tan(x)}{\sin(x)} = \frac{1}{\cos(x)}$$

*JAM*

$$\sin(\arcsin(x))$$

$$\arcsin(\sin(x))$$

$$\arctan(\tan(x))$$

$$\tan(\arctan(x))$$

$$\log |\exp(x)|$$

$$\exp(\log |x|)$$

# *PREDICTION*

Continuity can mean also to be able to  
**predict** things.

## Examples:

you grow continuously



There was a time in your life when you were  
exactly 1 meter high!

you run continuously



If you make a run and cross the Charles river  
once, there has to be a time, when you cross it  
again.

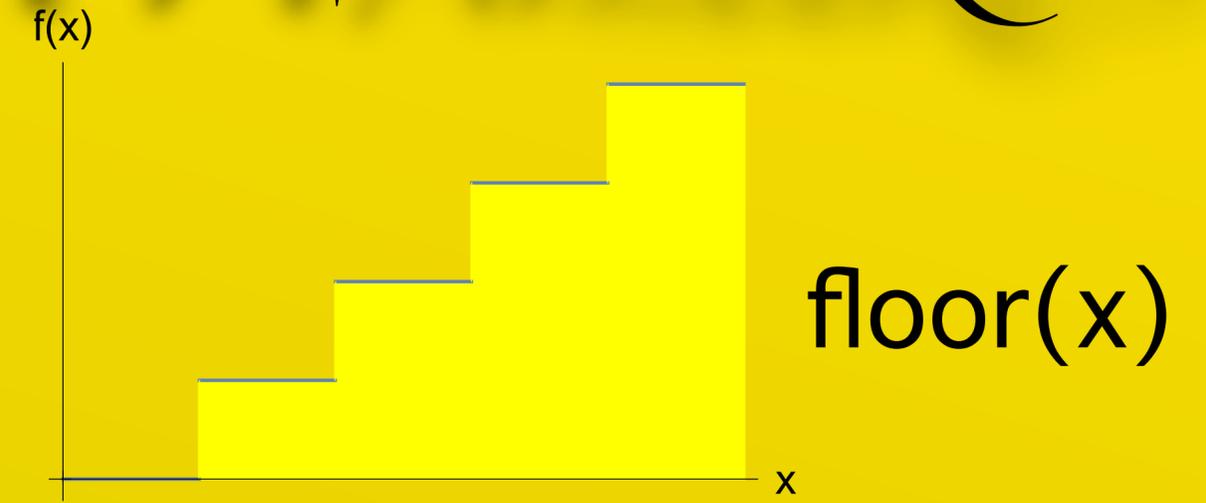
# *APROPOS CHARLES*



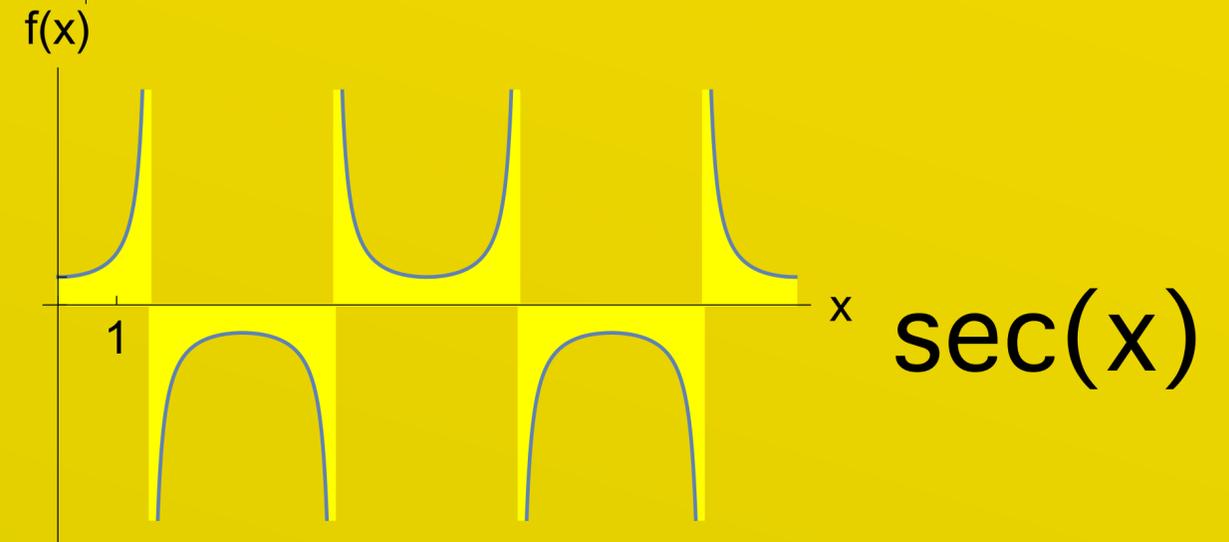
Oliver crossing the Charles in 2002

# CONTINUITY FAILURES

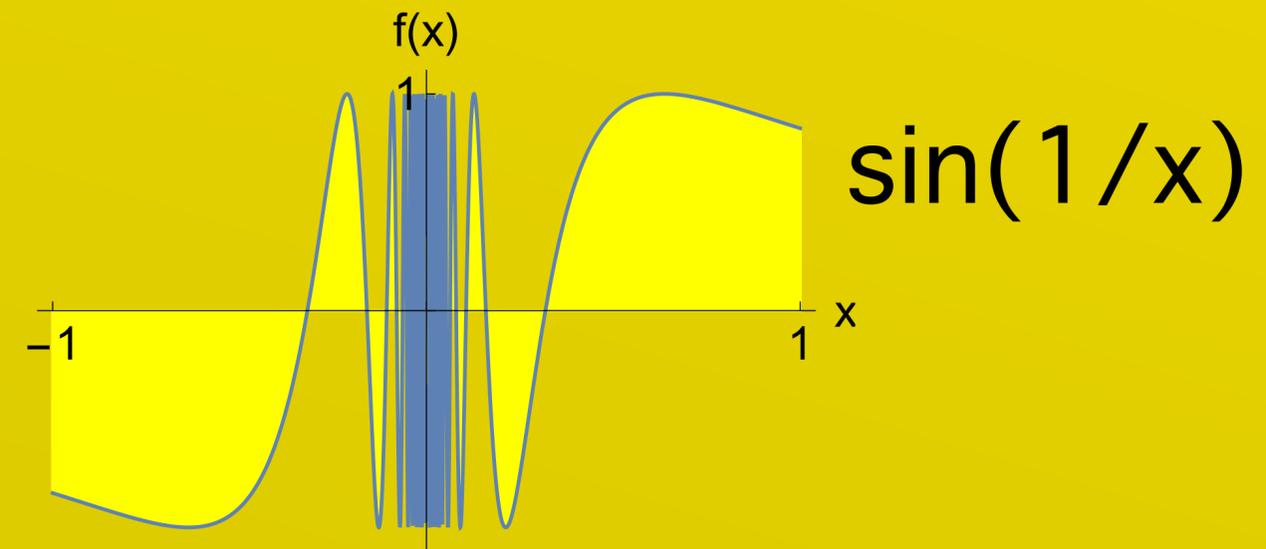
Jumps



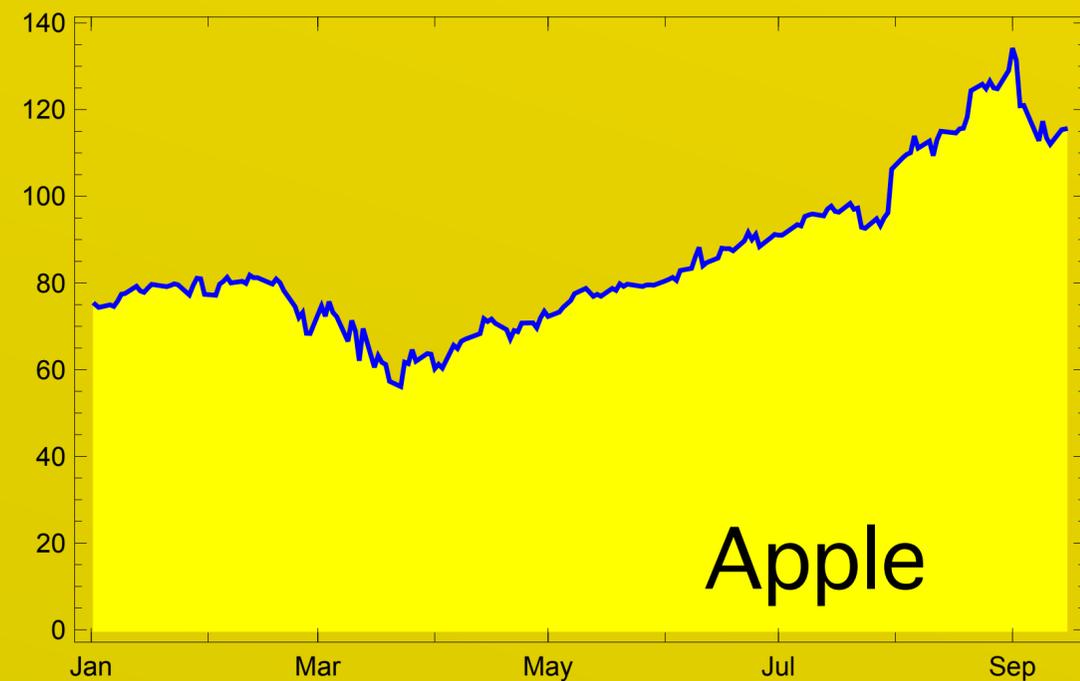
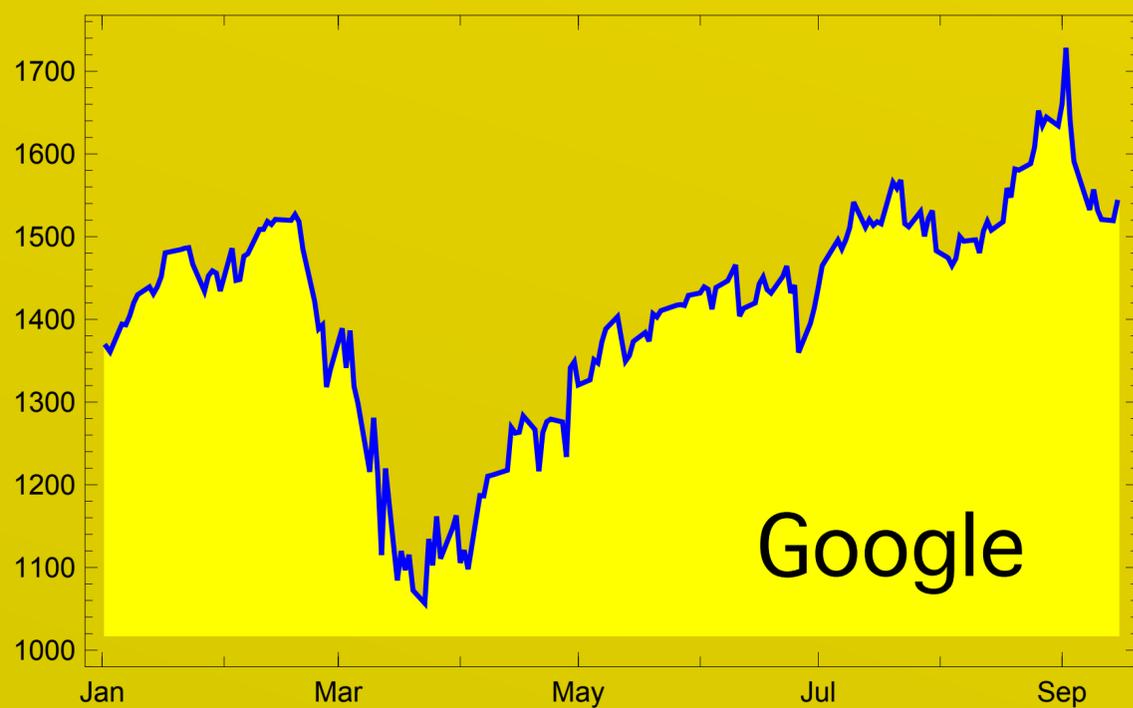
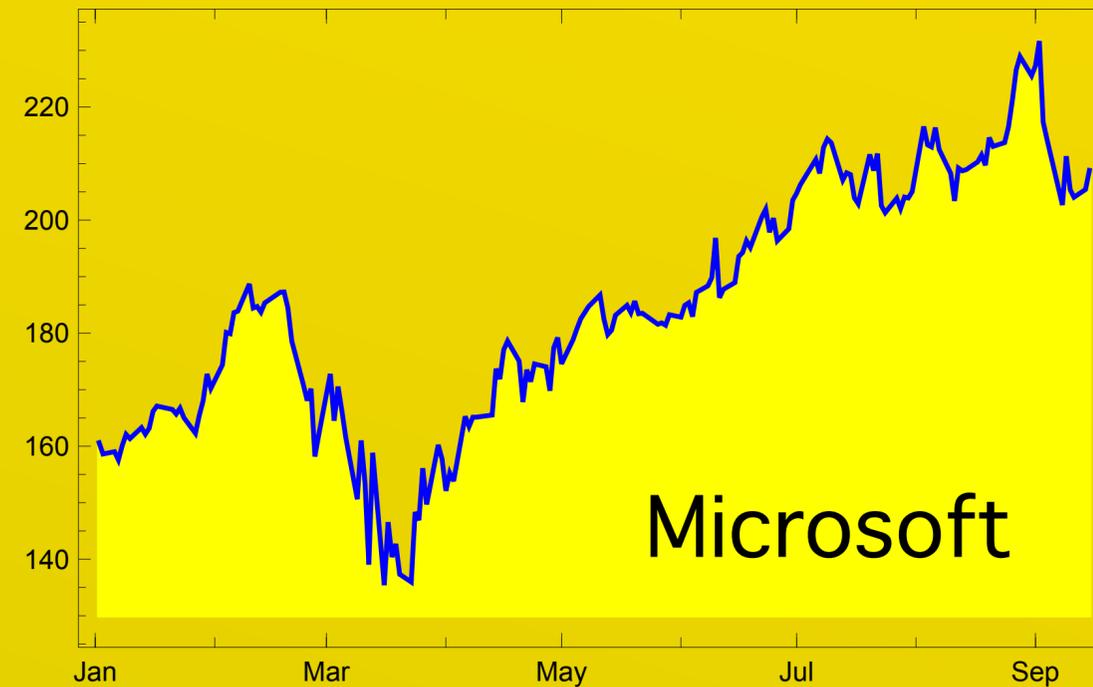
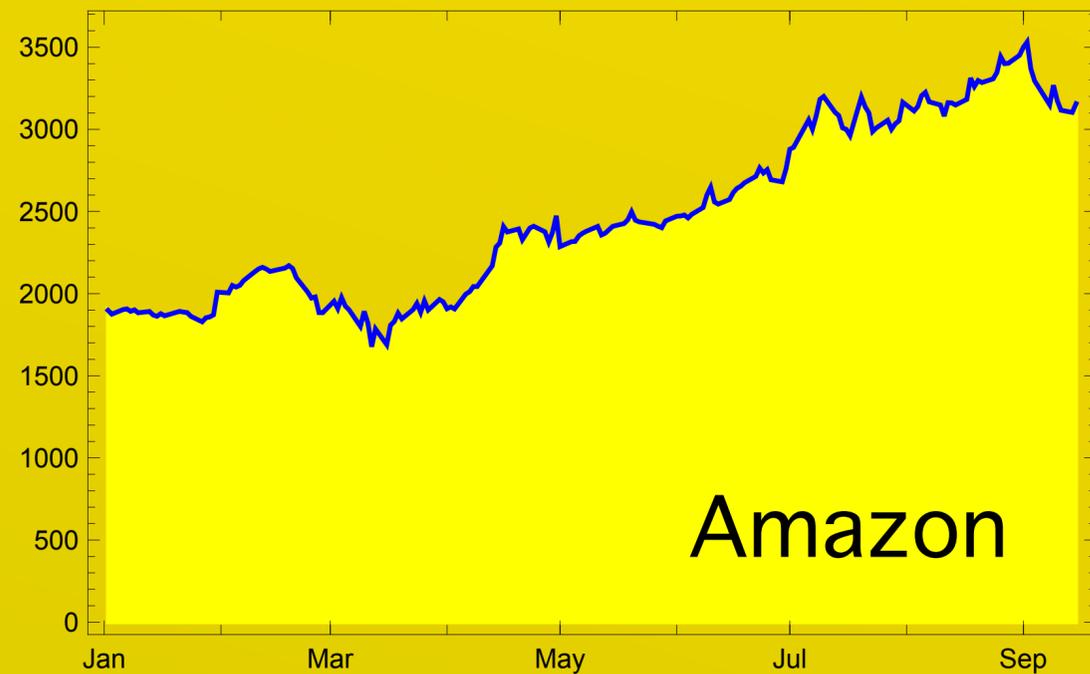
Infinity



Oscillation



# FINANCE



*THE END*