LECTURE 8
COMPUTING LIMITS

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PLAN

1. Do the limits exist or not?

2. Healing

3. Beasts

4. Adding subtracting

6. Multiplying

5. Composing

6. Division

7. Jam with CA

TERMINOLOGY

Limit from the left exists

Limit from the right exists

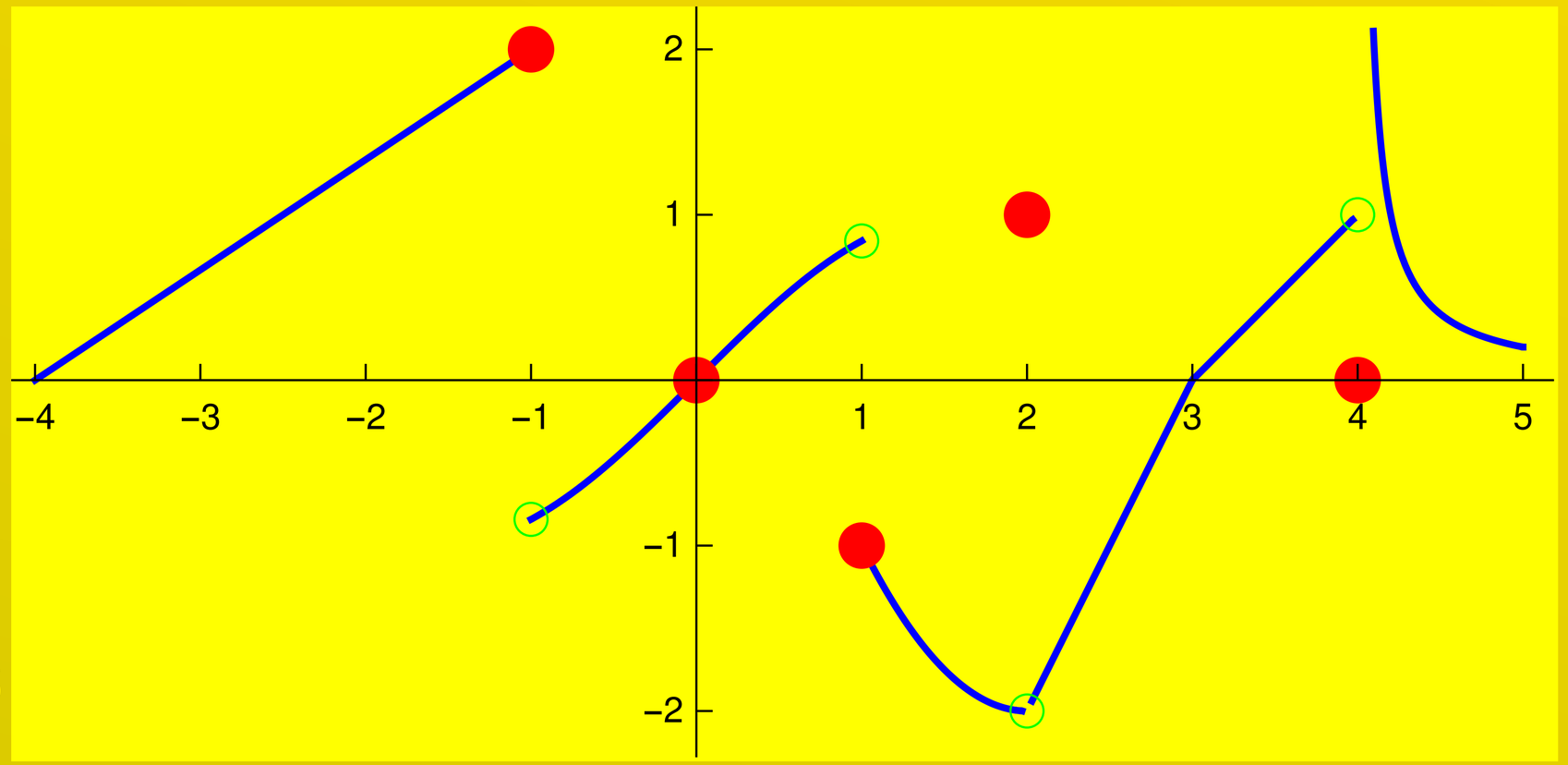
Limit does not exist

Limit exists

Limit is + infinity

Limit is - infinity

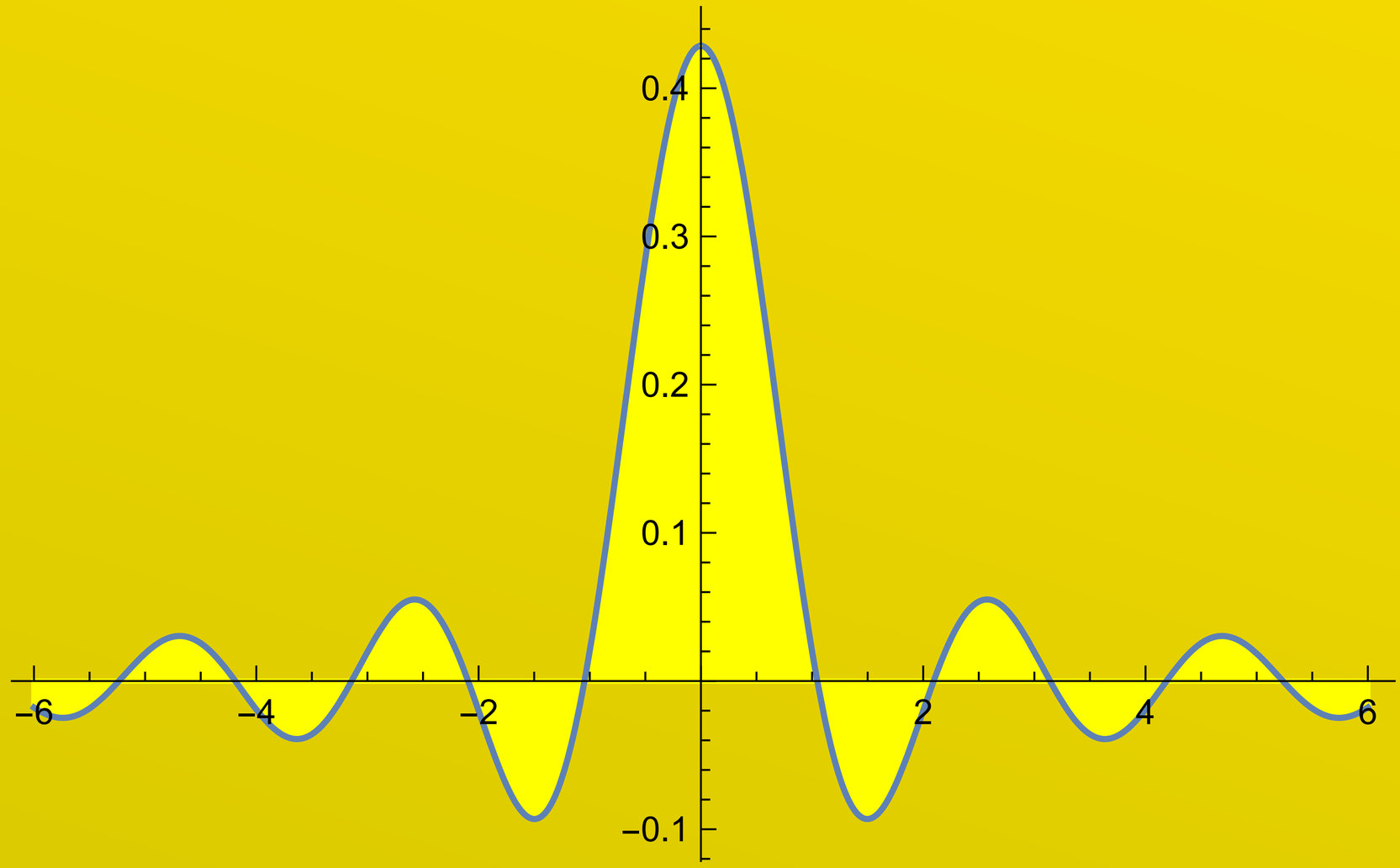
POLL



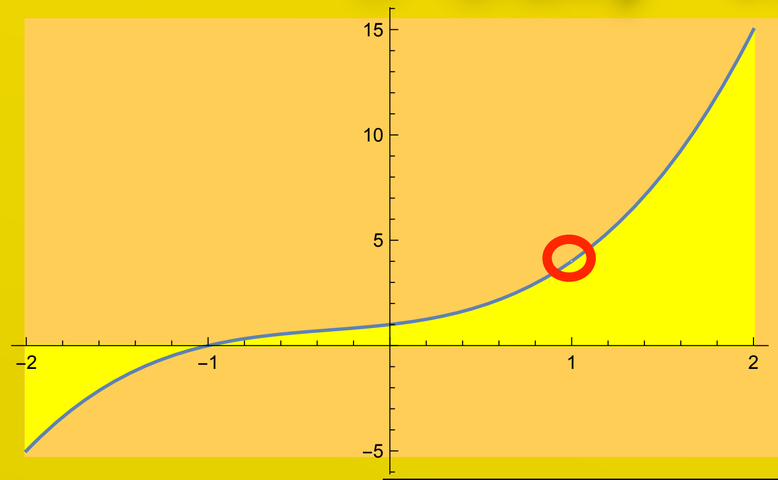
where limit
from the right exists
but not the limit
from the left

POLL

$$\lim_{x \to 0} \frac{\sin(3x)}{7x}$$

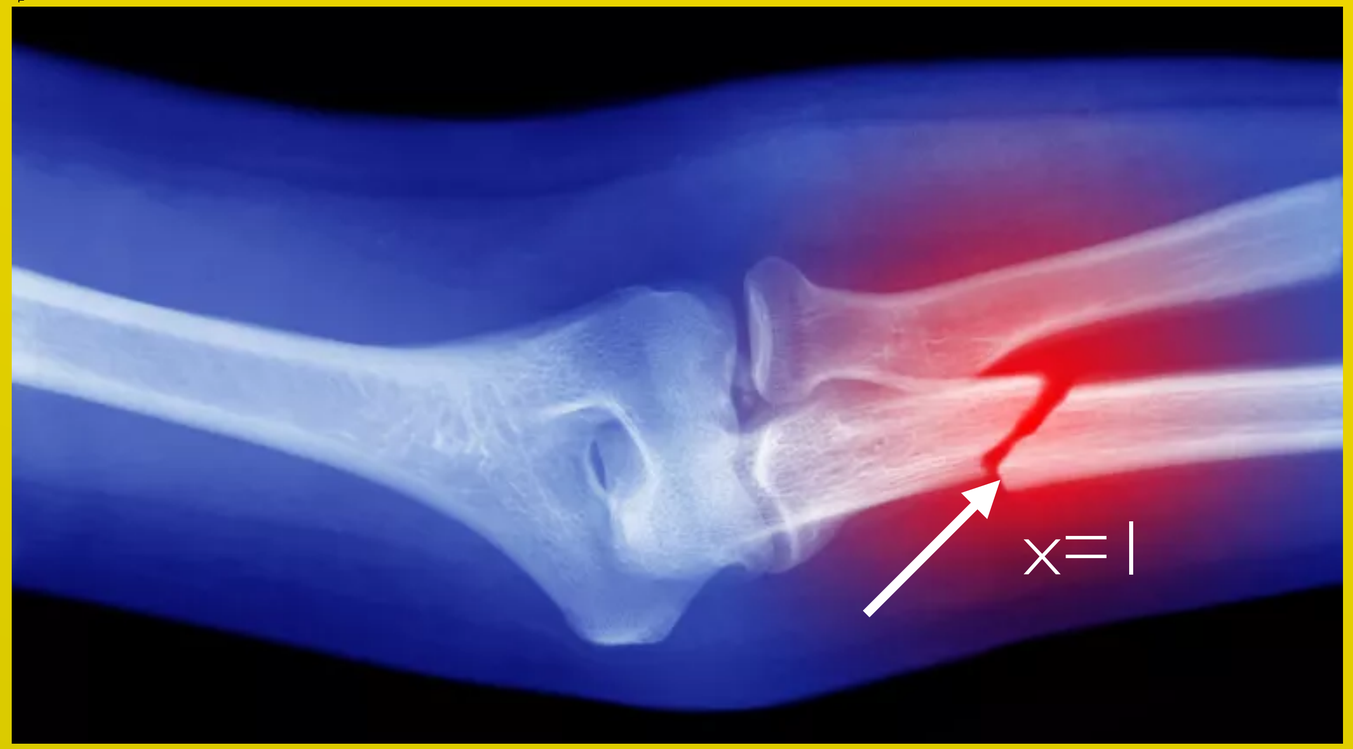


HEALING

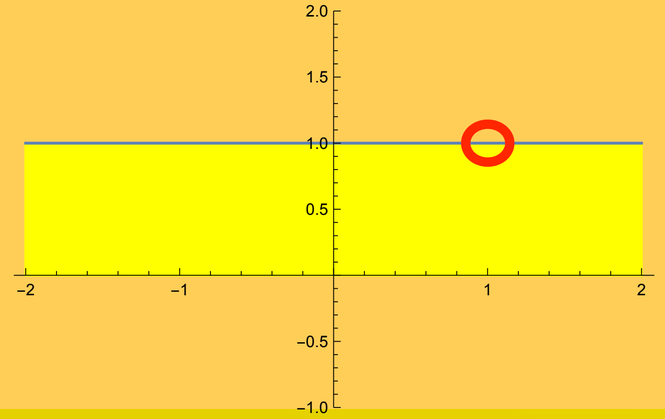


$$\lim_{x \to 1} \frac{x^4 - 1}{x - 1}$$

This is a situation where we divide 0/0. The function is broken at x=1.



SILLY



$$\lim_{x \to 1} \frac{x-1}{x-1}$$

Often these factorizations might look silly but it is important to be able to extend the function to places where it is a priori not defined.

STEPS

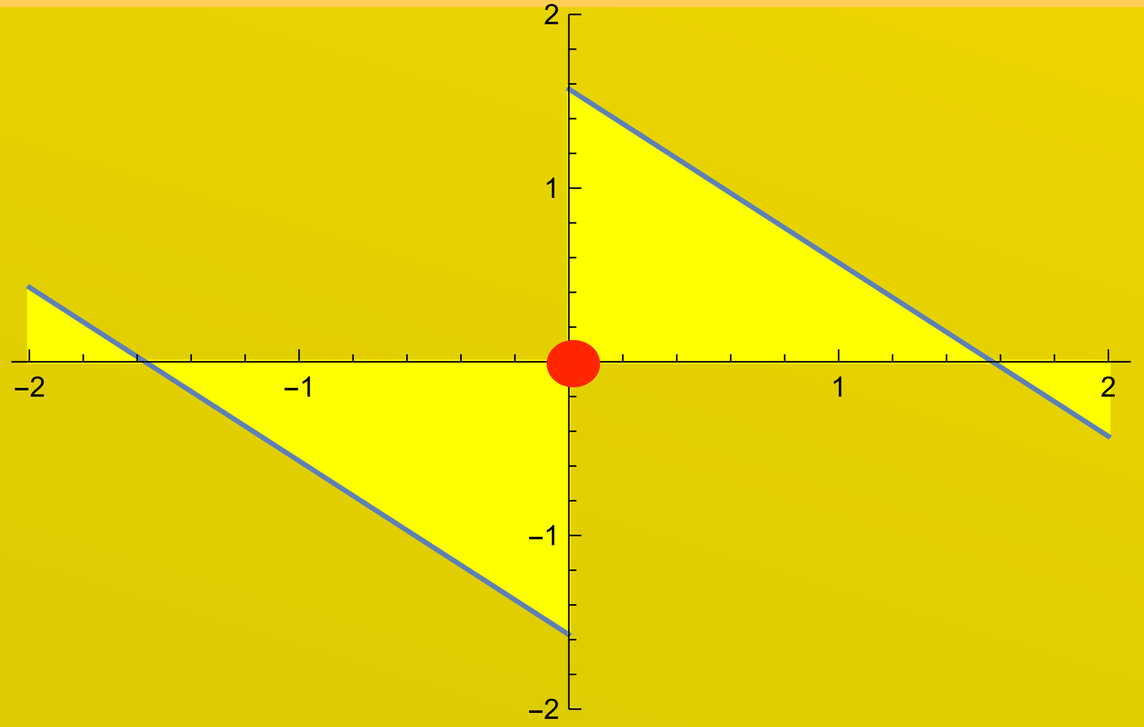
Factor

Divide out

Evaluate

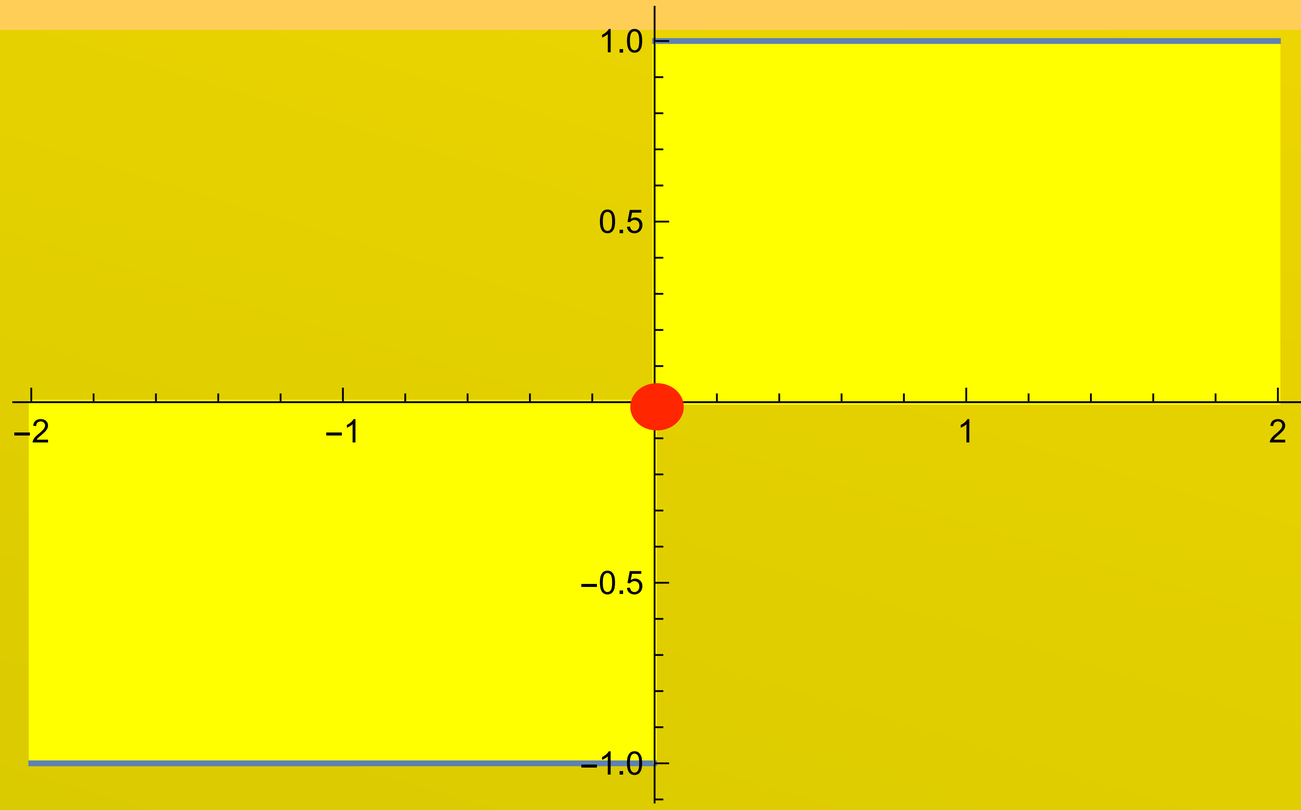
JUMPS

$$\lim_{x \to 0} \operatorname{arccot}(\tan(x))$$



SIGN

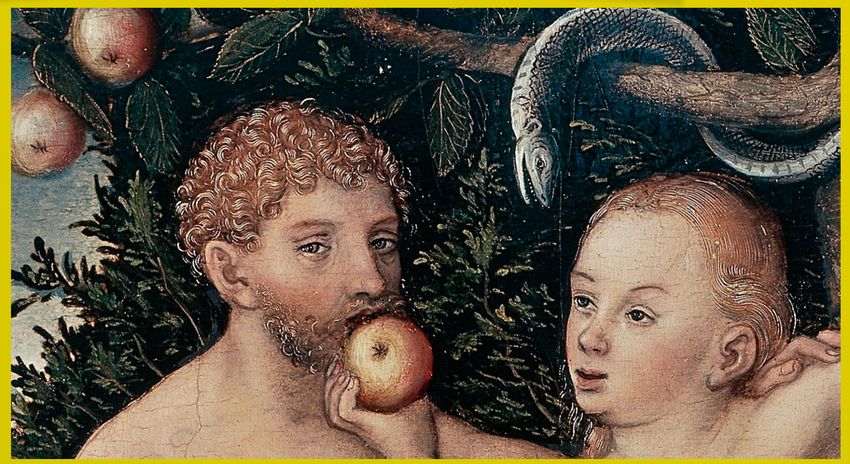
$$\lim_{x \to 0} \frac{x}{|x|}$$

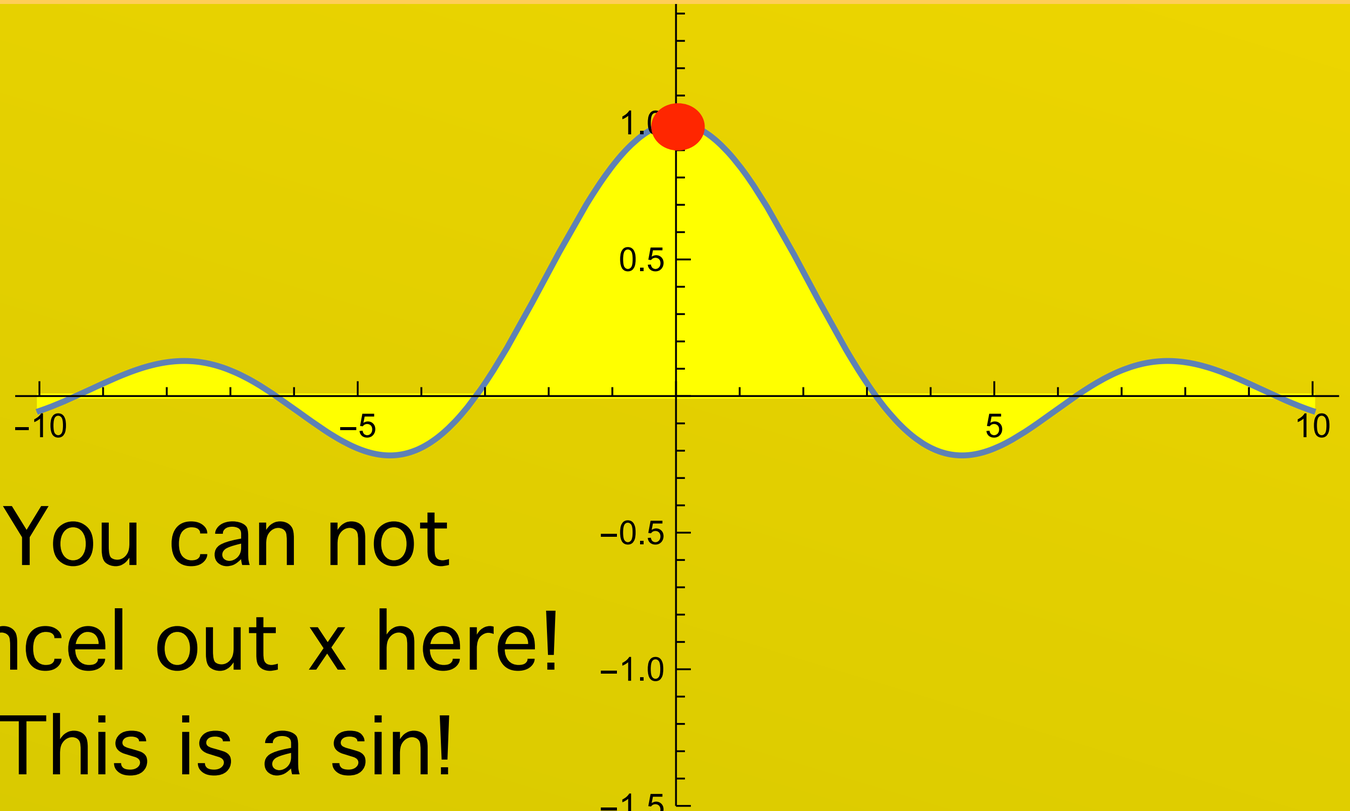


You can not
cancel out x here!

A SIN

$$\lim_{x \to 0} \frac{\sin x}{x} = \sin$$

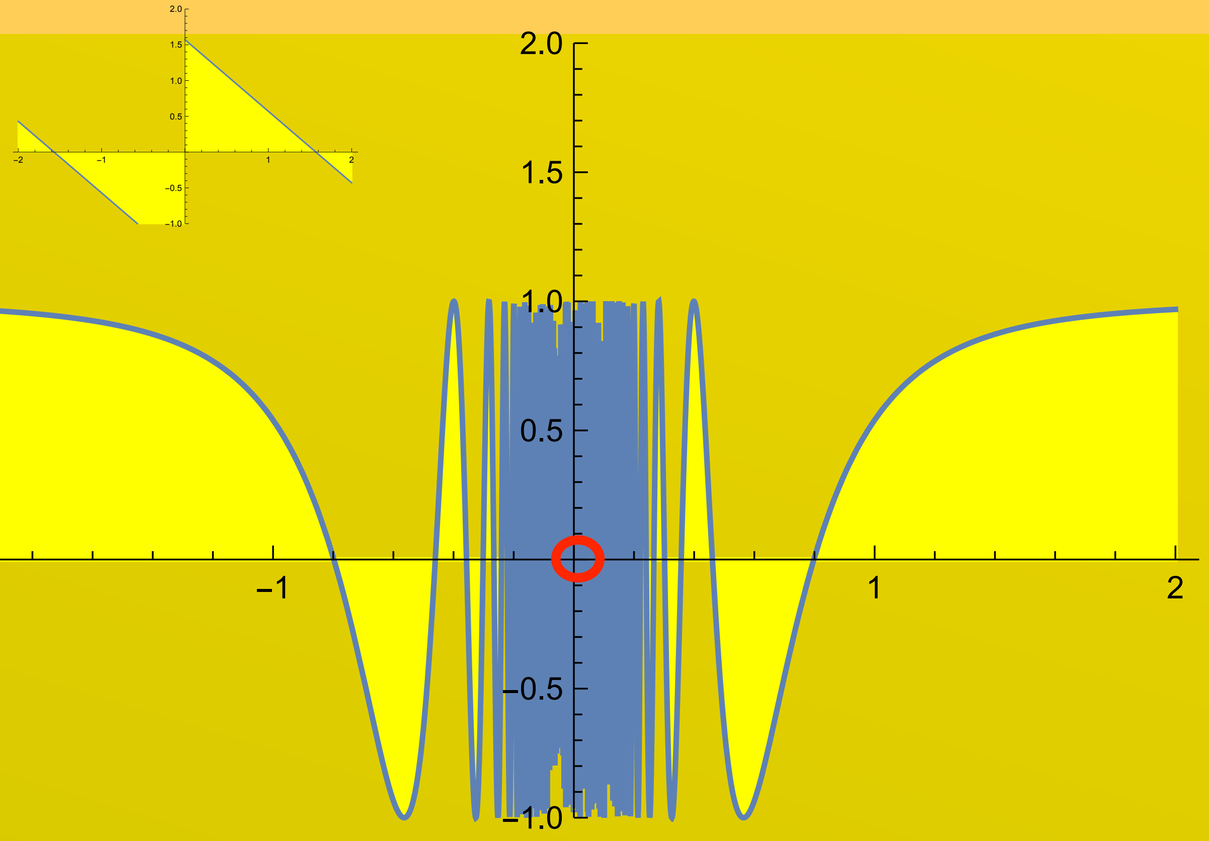




You can not
cancel out x here!
This is a sin!

BEASTS

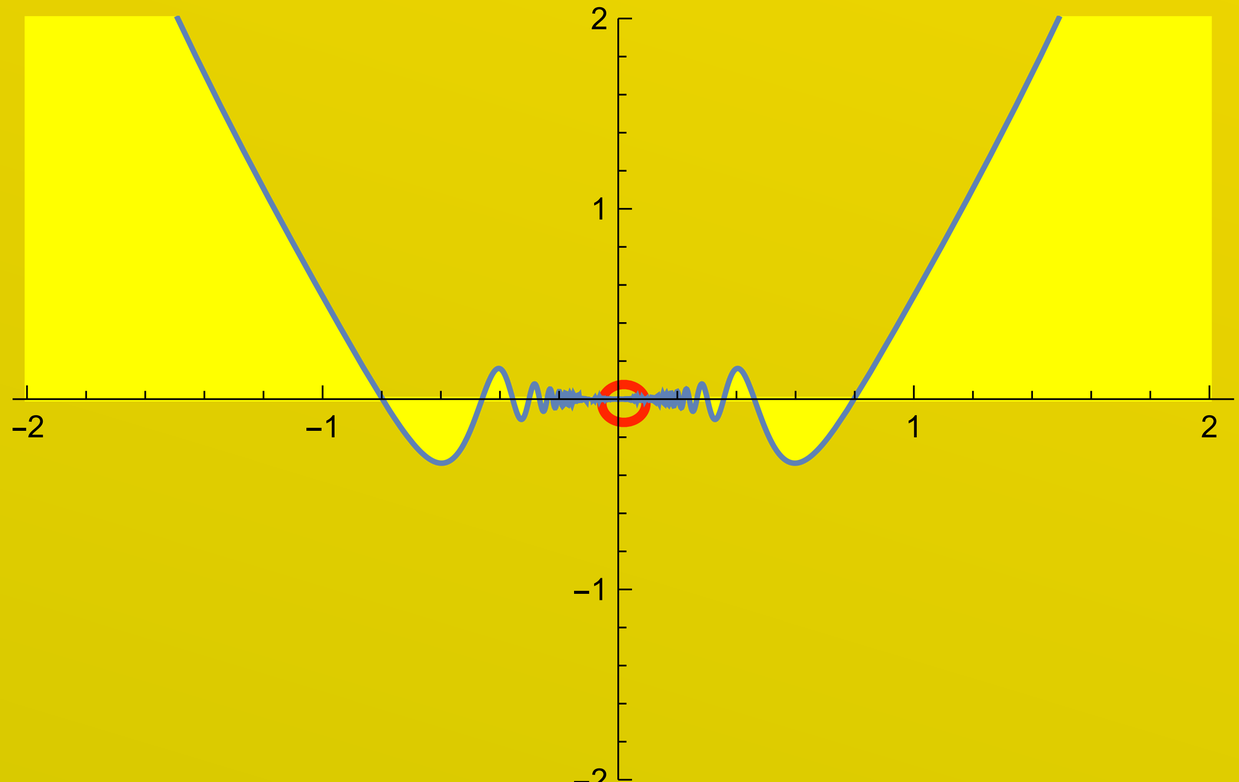
$$\lim_{x \to 0} \cos\left(\frac{1}{x^2}\right)$$



Dare Devil

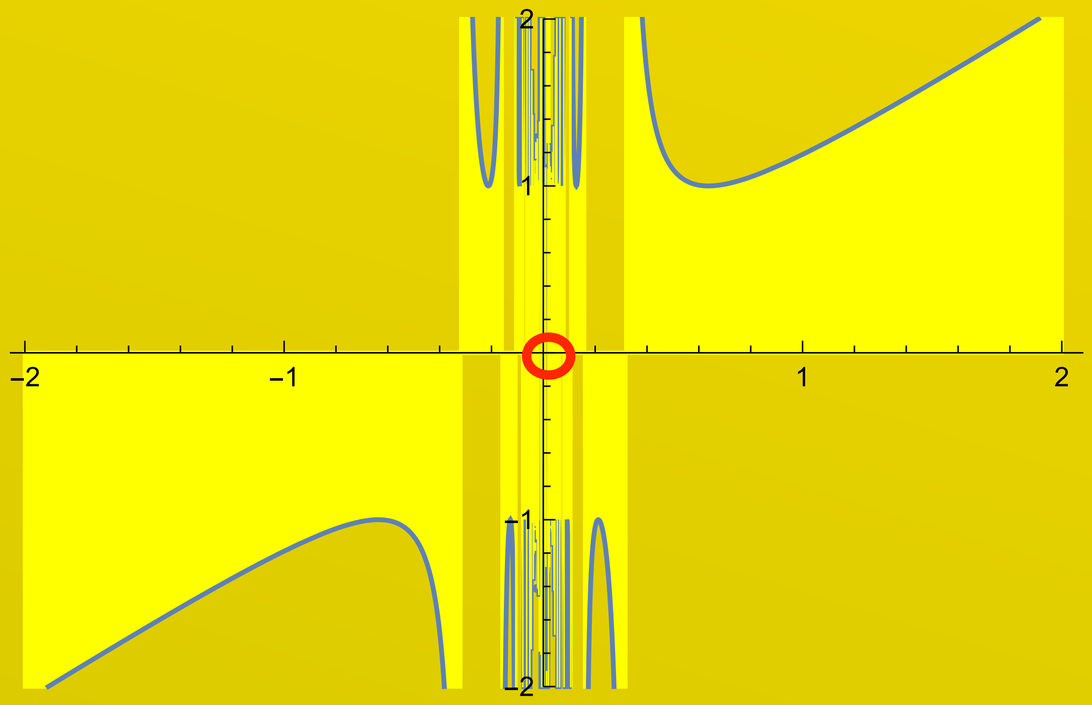
TAMED BEAST

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$$



SUPER BEAST

$$\lim_{x \to 0} 1/\sin(1/x)$$



Beyond Good and Evil!

JAM

$$\lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3}$$

$$\lim_{x \to 0} \frac{1 - \cos^2(x)}{\sin(x)}$$

$$\lim_{x \to \infty} \frac{3x^5 - 2x}{2x^5 + 3x^4}$$

$$\lim_{x \to 1} \frac{\log(x^4)}{\log(x^2)}$$

$$\lim_{x \to 0} \frac{e^{2x} - 1}{e^x - 1}$$

$$\lim_{x \to 0} \frac{\sin(6x)}{5x}$$

RULES

$$\lim_{x\to a} f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a} f(x) \cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

if the g-limit is nonzero →

$$\lim_{x\to a} f(x)/g(x) = \lim_{x\to a} f(x)/\lim_{x\to a} g(x)$$

If f is continuous

$$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$$

THE END