LECTURE 9
TO INFINITY AND BEYOND

Oliver Knill, Harvard University

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horizontal
asymptote

PLAN

1. Guts reaction

2. Limits to infinity

3. Examples

4. Asymptotes

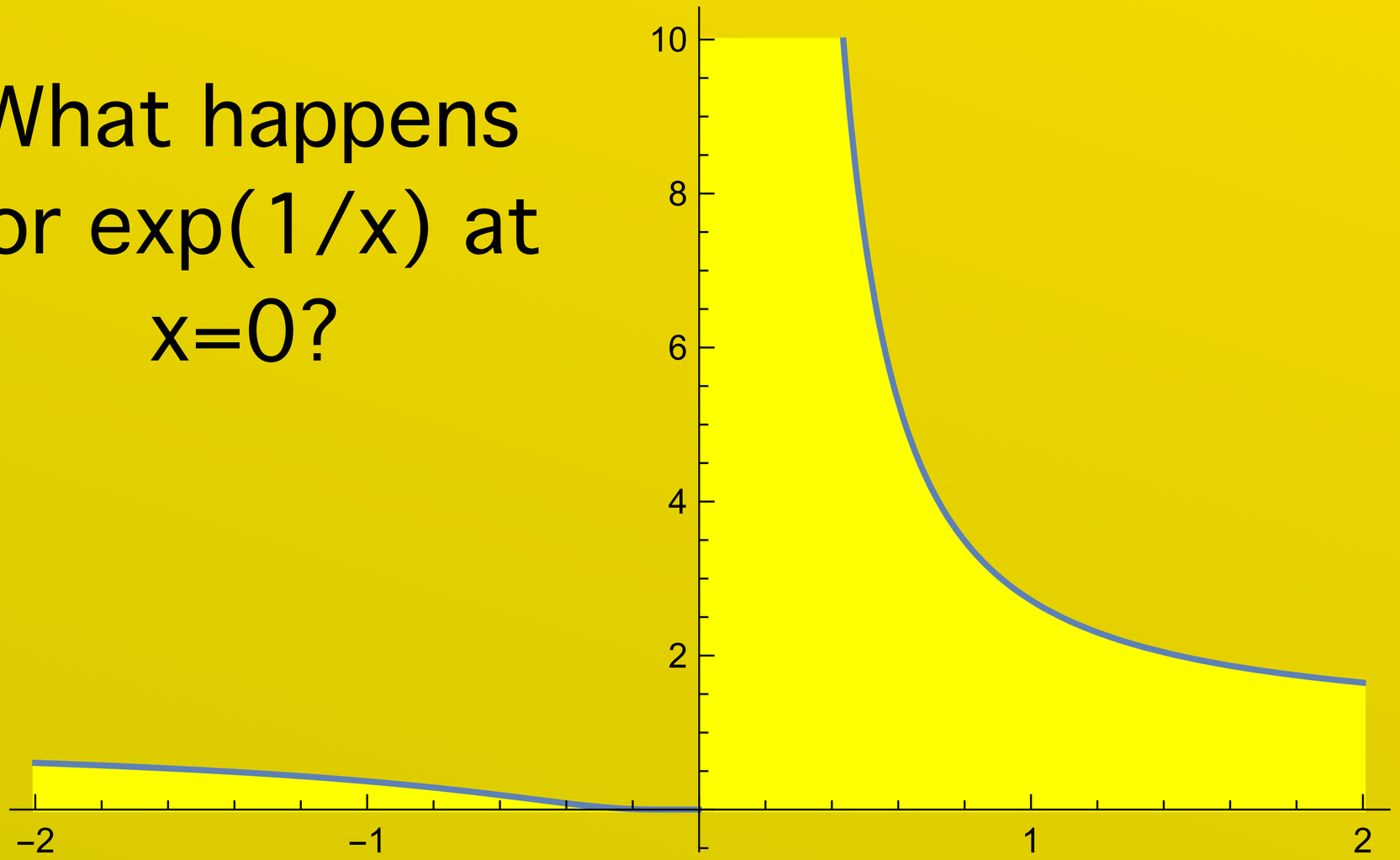
4. Jam

5. Discussion

7. Jam with CA

POLL

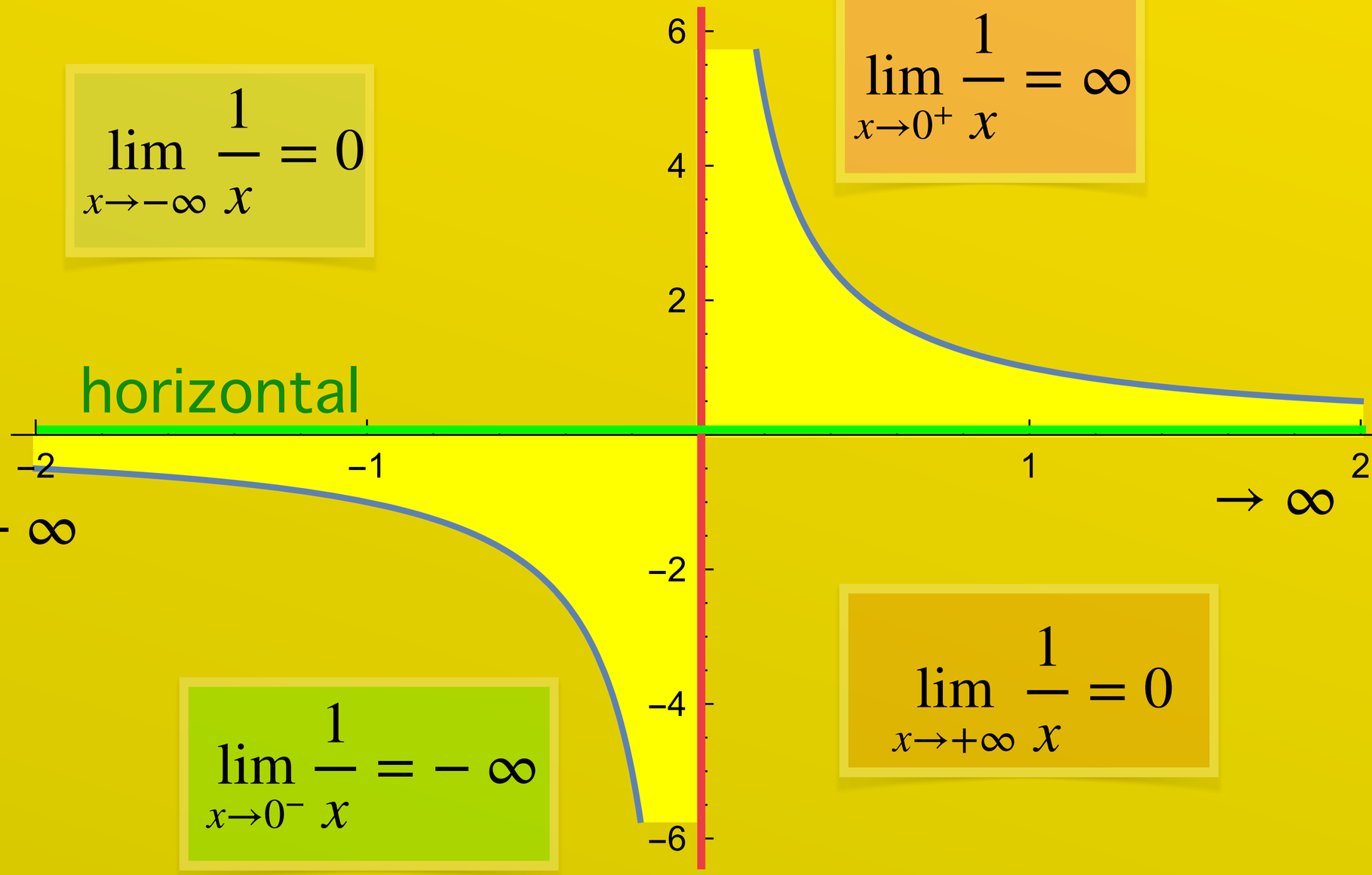
What happens for exp(1/x) at x=0?



INFINITY

y=1/x

vertical



$$\lim_{x\to-\infty}\frac{1}{x}=0$$

$$\lim_{x\to0^+}\frac{1}{x}=\infty$$

$\to -\infty$

$\to \infty$

$$\lim_{x\to0^-}\frac{1}{x}=-\infty$$

$$\lim_{x\to+\infty}\frac{1}{x}=0$$

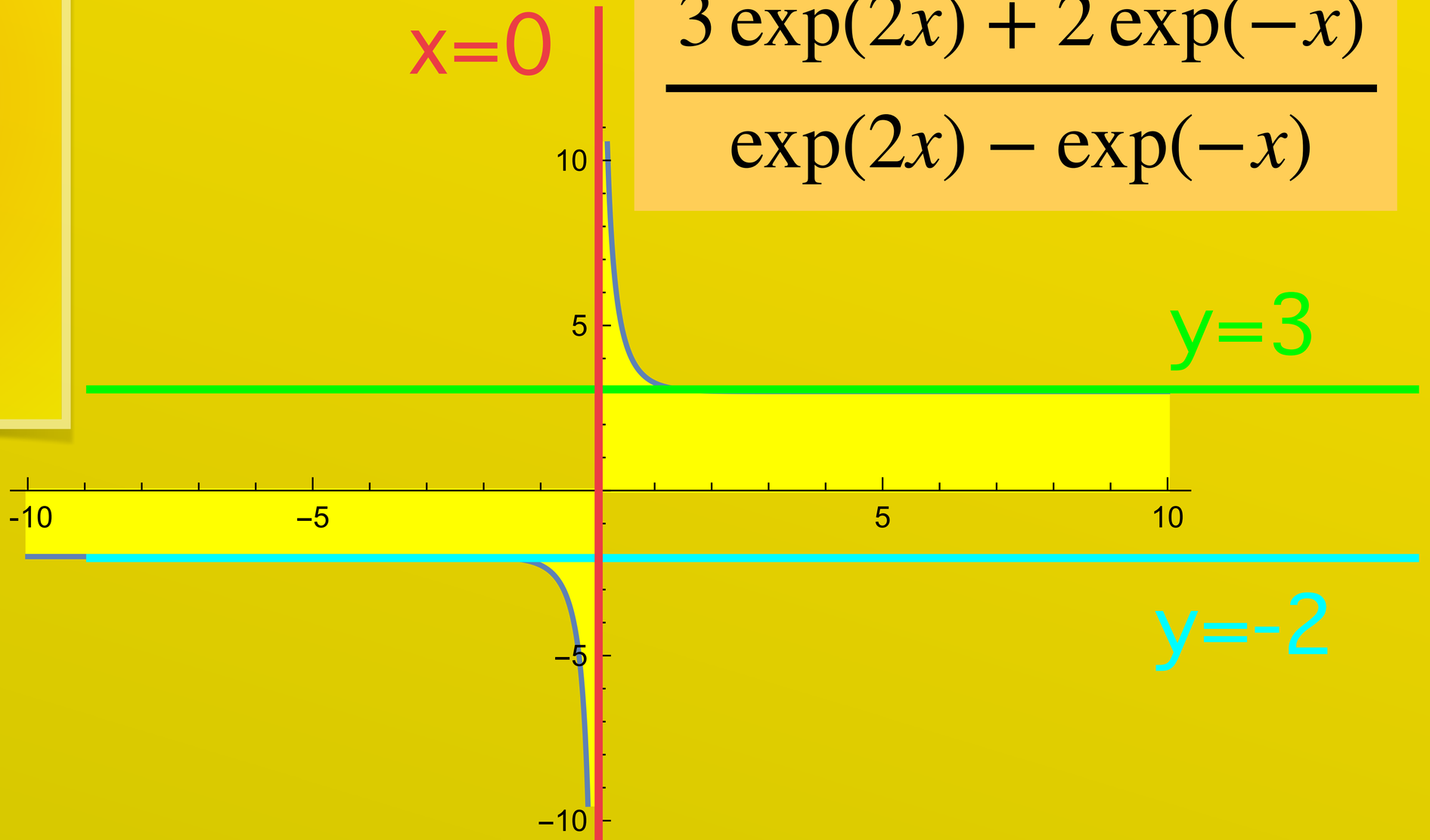
ASYMPTOTES

Find the horizontal and vertical asymptotes

$$\frac{3\exp(2x) + 2\exp(-x)}{\exp(2x) - \exp(-x)}$$

ASYMPTOTES

$$\frac{3\exp(2x)+2\exp(-x)}{\exp(2x)-\exp(-x)}$$



WHICH ONE IS RIGHT?

$$\lim_{x \to \infty} \frac{1}{x - 8} = 8$$

$$\lim_{x \to 8} \frac{1}{x - \infty} = 8$$

$$\lim_{x \to 8} \frac{1}{x - 8} = \infty$$

A CLASSIC!

$$\lim_{x \to 8} 1/(x-8) = \infty$$

so

$$\lim_{x \to 5} 1/(x-5) = \text{ت}$$

A RACE

$$\lim_{x \to +\infty} \frac{10^x}{x^{1000}}$$

What happens at + infinity

What happens at - infinity?

DICHOTOMY

polynomial

x^3

exponential

3^x

P - NP problematic, can we do things in polynomial time?

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fractional power

$x^{1/3}$

logarithmic

$\log(x)$

super exponential

$3^{(x^2)}$

HIGHEST ORDER

$$\lim_{x\to\infty} \frac{5x^4 - 20000}{3x^4 + 1000}$$

divide out:

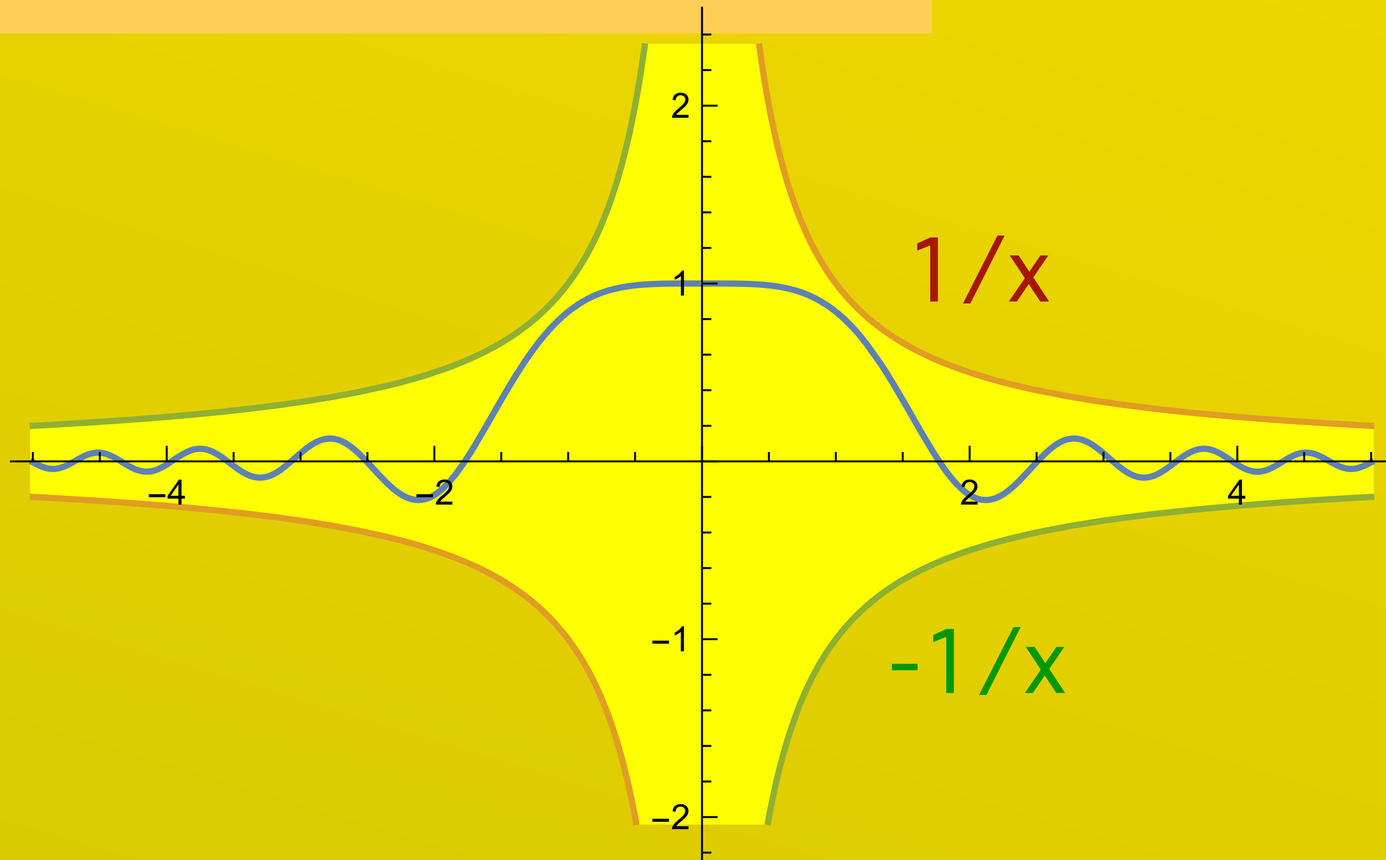
$$\lim_{x\to\infty} \frac{5 - 20000/x^4}{3 + 1000/x^4}$$

SQUEEZE!

$$\lim_{x \to \infty} \frac{\sin(x^{10})}{x}$$

SQUEEZE AT ∞

$$\lim_{x \to \infty} \frac{\sin(x^{10})}{x}$$



Function is bound above by 1/x and bound below by -1/x. Both go to 0.

JAM

$$\lim_{x\to\infty} \frac{x^7 - 5x + 6}{x^8 - 3}$$

$$\lim_{x\to\infty} \frac{x^4 \sin(x)}{\exp(x)}$$

$$\lim_{x\to+\infty} \frac{\sqrt{16x^6 - x^3}}{5x^3 + \sin(x)}$$

$$\lim_{x\to\infty} \frac{\log|x| + x}{x^2 + \exp(-x)}$$

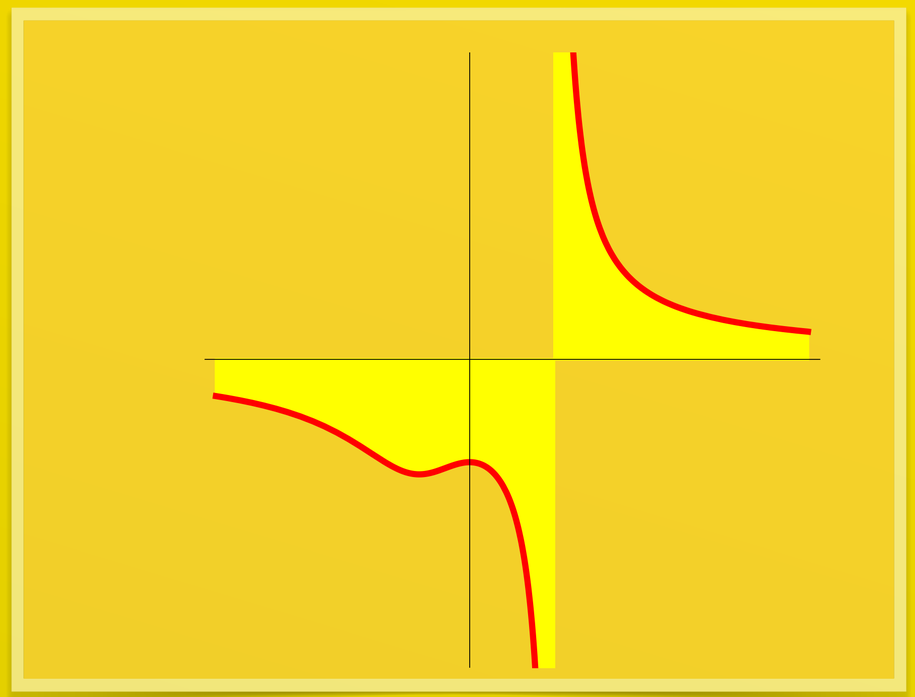
$$\lim_{x\to\infty} \frac{3e^{3x} - e^x}{5e^{3x} + e^{2x}}$$

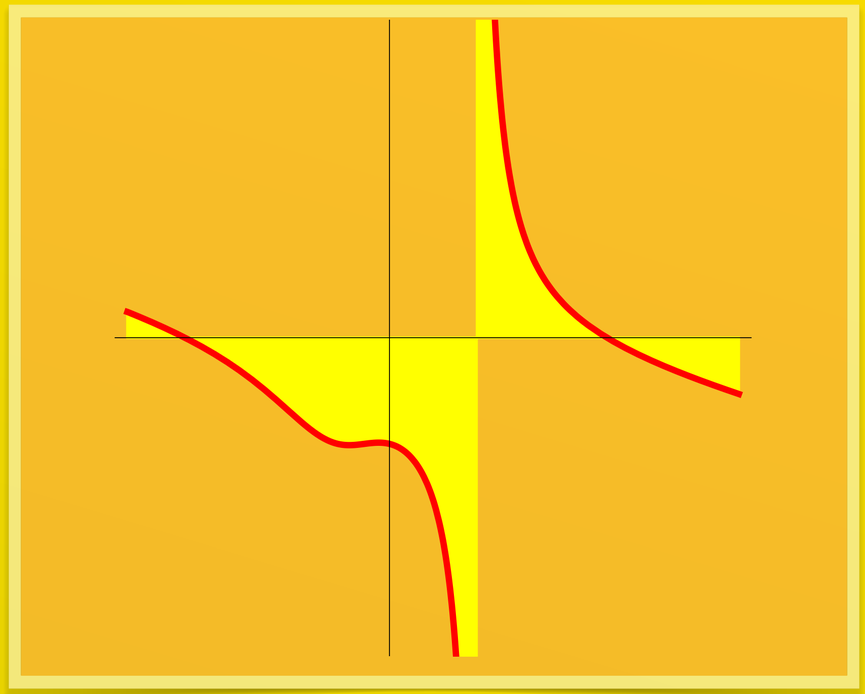
$$\lim_{x\to\infty} \frac{x + \sin(6x)}{5x + 17}$$

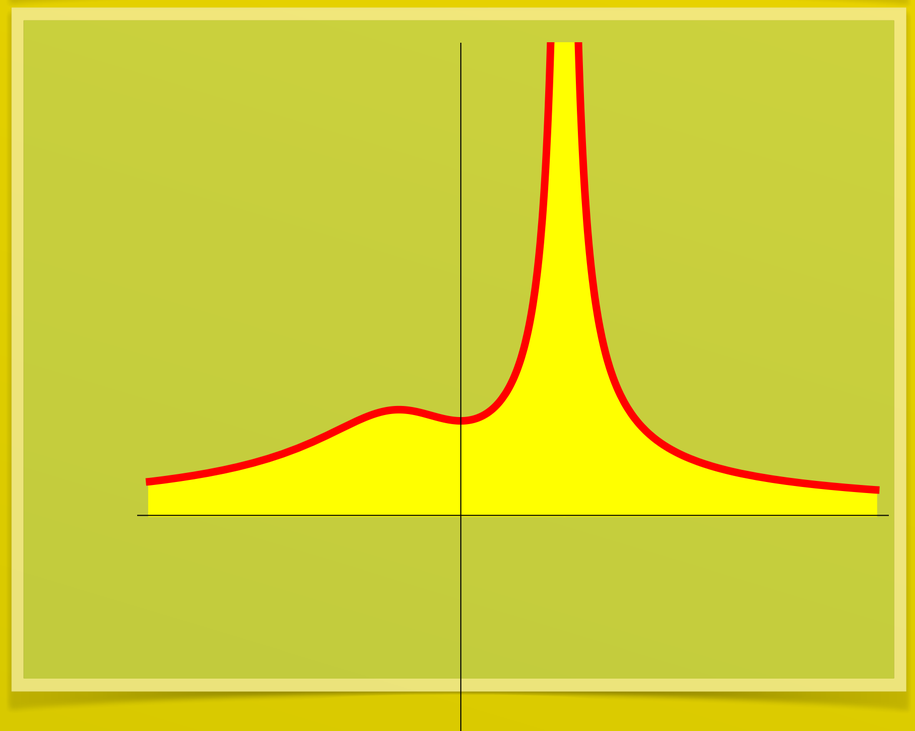
JAM

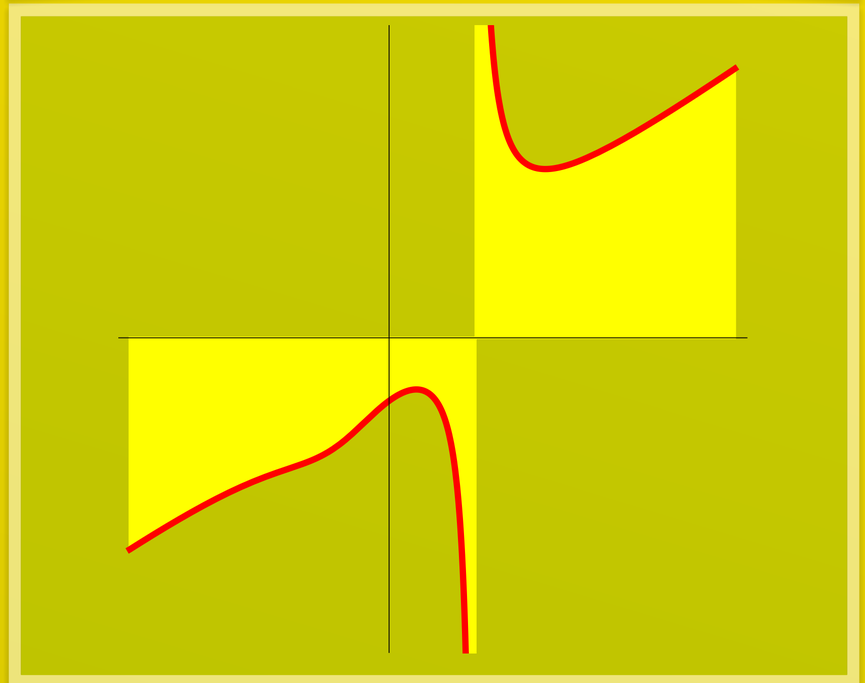
$$f(x) = \frac{x^2 + 1}{x^3 - 1}$$

Which is the correct graph?









JAM

Evaluate

	$\log_{10}(x)$	$\sqrt{x}$	$\frac{\log_{10}(x)}{\sqrt{x}}$
x= 1			
x= 10000			
x= 1'000'000			

THE END